A Static Enthalpy Equilibrium Approach to the Stability of Saturated Sands

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Abstract

The stability of saturated sands can be described as a continuous energetic spectrum ranging from dense, stable configurations to the onset of liquefaction in loose deposits. Classical stress-based approaches distinguish these regimes empirically, but do not explicitly account for the underlying energy balance that governs the transitions between them.
This study introduces a **static enthalpy equilibrium** as a general physical condition **for defining sand stability**.
The formulation expresses equilibrium as the equality between the structural enthalpy released by the granular framework and the hydraulic work capacity of the pore water:

$$H(n_0) - H(n_f) = \rho_w \cdot g \cdot h \qquad (1)$$

This condition applies to all saturated sands regardless of density or stress history. Loose sands, where the structural energy exceeds the hydraulic capacity, may collapse spontaneously; dense sands require external work to achieve the same balance. Spontaneous liquefaction therefore represents only a special case within a general energetic framework that unifies liquefaction, induced failure, and densification. The proposed formulation establishes a physical basis for understanding soil stability as an energy-equilibrium problem rather than a stress-path phenomenon, providing a consistent foundation for future theoretical and numerical developments.

Keywords: Saturated sands, enthalpy equilibrium, structural collapse, energetic stability, soil liquefaction, granular physics, thermodynamic soil mechanics

1 Introduction – Stability of saturated sands as an energetic problem

The mechanical behaviour of saturated sands - ranging from stable dense states to the onset of liquefaction in loose deposits - can be interpreted as points along a continuous energetic spectrum rather than as distinct mechanical regimes.

Classical soil mechanics has described these conditions primarily in terms of effective stress and strain, but not in terms of the underlying energy balance that governs transitions between them.
This paper introduces a static enthalpy equilibrium as a general physical condition defining the stability of saturated sands.
The formulation unifies spontaneous liquefaction, induced liquefaction, and densification within a single energetic framework by expressing stability as the balance between the structural energy stored in the granular skeleton and the hydraulic work capacity of the pore water.
The conceptual roots of this approach can be traced back to the foundations of classical soil mechanics.
In his Erdbaumechanik (1925), **Terzaghi** interpreted soil stability as a problem of hydraulic equilibrium, in which the balance between self-weight and pore-water pressure determines whether a structure remains stable or undergoes uplift. **Casagrande** (1936, 1975) extended this view to granular materials and proposed that the behaviour of sands is governed by a critical void ratio separating stable from unstable arrangements of particles.
While these formulations successfully described observed behaviour, they implicitly contained an energetic idea that was never made explicit: the notion that mechanical stability arises when the potential energy of the solid framework and the hydraulic energy of the pore water are in balance.

The conceptual foundation of this work follows the classical line of soil mechanics established by Terzaghi and Casagrande but extends it to an explicit energetic formulation. Terzaghi's interpretation of soil stability as a hydraulic equilibrium and Casagrande's empirical definition of a critical void ratio both implicitly relied on a balance between the potential energy of the solid framework and the hydraulic work of the pore water. The present study makes this relationship explicit by defining a static enthalpy equilibrium that expresses this balance in energetic terms. In doing so, the formulation preserves the spirit of the early concepts while providing a general physical condition applicable to all saturated sands.

2 Theoretical Framework – Definition of the static enthalpy equilibrium

The behaviour of saturated sands can be interpreted as an exchange of energy between two coupled subsystems: the granular framework and the pore-water phase. The first stores structural energy in the form of contact and gravitational potential, while the second performs hydraulic work in response to changes in pore volume and pressure.
A condition of static equilibrium is reached when the potential of one subsystem to perform work equals the capacity of the other to absorb it.

This balance can be expressed in terms of a specific enthalpy per unit volume $H(n)$, defined as

$$H(n) = (1-n)\cdot\rho_s\cdot g\cdot h + n\cdot K_{eff}\cdot ln\left(\frac{n_0}{n}\right) \qquad (2)$$

where n is the porosity, ρ_s the grain density, g the acceleration of gravity, h the representative vertical energy distance, and K_{eff} the effective bulk modulus of the saturated mixture (Houlsby and Puzrin, 2007; Collins, 2016).
All quantities have the dimension of energy per unit volume [J/m^3].

The first term represents the gravitational potential energy of the solid skeleton, and the second describes the hydraulic work associated with a change in pore volume from n_0 $to\ n$.
The second term therefore represents not a compression of the pore fluid itself, but the hydraulic work associated with the expulsion of water as the granular structure rearranges into a denser configuration.
The natural logarithm expresses the relative change of porosity during this process, which is continuous and quasi-static under the given boundary of the coupled fluid–solid system.
Equilibrium between the two subsystems is obtained when

$$H(n_0) - H(n_f) = \rho_w \cdot g \cdot h \quad (3)$$

a condition consistent with the thermodynamic interpretation of soil instability proposed by Buscarnera and Einav (2012), who described liquefaction as an energy conversion between mechanical and hydraulic work.
The effective modulus K_{eff} controls the rate at which hydraulic work is performed during a change in pore volume, but it does not appear explicitly in the equilibrium condition of Eq. (3) because, at equilibrium, the total structural enthalpy and hydraulic work are already equalised.
The modulus therefore governs the evolution toward equilibrium, not the final energetic state itself.
In this expression h denotes the representative vertical energy distance, defined as the characteristic depth over which the potential energy of the granular structure is balanced by the hydraulic work of the pore water.
It is an energetic rather than geometric measure and depends both on the grain size, which controls permeability and hydraulic response, and on the grain density ρ_s, which determines the gravitational potential energy stored per unit pore volume.
Consequently, materials with higher grain density possess a larger effective energy distance, reflecting a greater capacity for gravitational work within the same porosity range.
In analogy to a thermodynamic phase transition, the static enthalpy equilibrium can be regarded as a reversible energetic state in which mechanical stability corresponds to the equality of structural and hydraulic work potentials.
The condition is general and applies to all saturated sands.
Loose, over-energetic structures naturally evolve toward this equilibrium through collapse and densification, whereas dense sands can reach it only through the supply of external work.
The equilibrium therefore defines the universal energetic boundary between stability and instability in saturated granular materials.

3 Limiting States – Energetic spectrum of sand behaviour

The static enthalpy equilibrium defines a continuous energetic spectrum that characterises all possible mechanical states of saturated sands (Roscoe et al., 1958; Wan and Guo, 2018).

Each state corresponds to a specific relationship between the structural enthalpy of the granular framework and the hydraulic work capacity of the pore water. Depending on whether the total system energy is above, equal to, or below the equilibrium condition, the sand exhibits fundamentally different behaviours.

3.1 Over-energetic state – Loose sands

When the structural enthalpy exceeds the hydraulic work capacity,

$$H(n_0) - H(n_f) > \rho_w \cdot g \cdot h \quad (4)$$

the granular framework stores more potential energy than the pore-water system can immediately absorb. Such a configuration is metastable: it may persist temporarily but is energetically predisposed to collapse. The release of the excess structural enthalpy triggers a redistribution of pore pressure and volume until the enthalpy equilibrium is reached.
In this regime, spontaneous or easily triggered liquefaction may occur without significant external work input.

3.2 Equilibrium state – Critical porosity

At the condition

$$H(n_0) - H(n_f) = \rho_w \cdot g \cdot h \quad (5)$$

the total enthalpy of the soil–water system is minimised, and the structure is in energetic balance. This state represents the critical porosity, n_f , where gravitational potential, hydraulic work, and structural configuration are in harmony. Mechanical perturbations around this point do not lead to progressive instability but are dissipated within the system.

It is therefore the physical boundary between stability and instability.

3.3 Under-energetic state – Dense sands

If the structural enthalpy is less than the hydraulic work capacity,

$$H(n_0) - H(n_f) < \rho_w \cdot g \cdot h \quad (6)$$

the granular framework is energetically stable: no spontaneous collapse can occur because the system lacks sufficient stored energy to overcome the hydraulic resistance. A collapse or further densification can only be achieved if external work is supplied, for instance, through cyclic or seismic loading, vibration, or mechanical compaction. Once the external work raises the hydraulic energy to the equilibrium level, the ensuing collapse proceeds by the same physical mechanism as in loose sands.

Spontaneous and induced liquefaction therefore represent opposite directions of the same energetic process, distinguished only by the origin of the energy that restores equilibrium. The energetic states can, in principle, be represented schematically in the space of porosity and structural energy. However, because the relations are fully defined by the analytical expressions, no graphical illustration is required.

4 Physical Interpretation – Energy continuum and metastability

The transition between the energetic states defined in Section 3 is not instantaneous. When the equilibrium condition $H(n_0) - H(n_f) = \rho_w \cdot g \cdot h$ is violated, the granular-fluid system enters a metastable state in which the stored structural energy exceeds the hydraulic work that can be dissipated at a given time or permeability.

The framework remains apparently stable while the excess energy is temporarily balanced by contact friction and local pore-pressure fluctuations.

4.1 Metastable behaviour and delayed collapse

In this over-energetic regime, the soil structure behaves as a metastable energy reservoir. Local rearrangements of grains or transient hydraulic fluctuations may progressively reduce the enthalpy difference until a threshold is reached where the remaining structural energy is suddenly released. This results in a rapid but spatially limited collapse event, accompanied by a short-lived increase in pore pressure and a redistribution of energy within the surrounding material. Such micro-collapses can occur repeatedly, each reducing the total enthalpy and moving the system closer to equilibrium.

4.2 Energetic relaxation and local dissipation

The relaxation process is governed by the competition between energy release and local dissipation.
If the dissipation capacity - controlled by permeability, viscosity, and grain friction - is high, the released energy is consumed locally, and the collapse remains confined to a small region.
If dissipation is low and the material is homogeneous, the released energy may propagate and trigger collapse in adjacent zones, leading to a more extensive failure. Thus, the extent of liquefaction is determined not by a distinct mechanical threshold but by the spatial balance between energy transmission and dissipation within the granular medium.

4.3 Approach to equilibrium

With each collapse episode the system loses a portion of its excess structural enthalpy, and the energy deficit between the solid and fluid phases decreases. Ultimately, the metastable evolution terminates when

$$H(n_0) - H(n_f) = \rho_w \cdot g \cdot h \quad (7)$$

At this point the total enthalpy of the soil–water system has reached a stationary state; no further exchange of structural or hydraulic energy is possible under the given boundary conditions.
This state acts as an energetic attractor toward which all structural and hydraulic processes inevitably converge, regardless of the number or magnitude of intermediate collapse events. Once the enthalpy balance is fully satisfied, only local elastic or dissipative

responses remain possible; a global collapse can no longer occur without additional external energy input.

5 Discussion – Physical implications and generality

The static enthalpy equilibrium provides a unified physical interpretation of the stability of saturated sands.

It describes all observable mechanical regimes - spontaneous liquefaction, induced collapse, and densification - as different manifestations of the same energetic condition.

In this view, soil behaviour is governed not by discrete stress-path boundaries but by the continuous redistribution of energy between the granular and fluid phases.

5.1 Relation to classical soil mechanics

The equilibrium formulated here extends the classical concepts of Terzaghi and Casagrande by making their implicit energy balance explicit.
Terzaghi's hydraulic equilibrium and Casagrande's critical-void-ratio concept can both be interpreted as specific expressions of the same enthalpy condition.
At the critical porosity n_f, the gravitational potential of the solid framework and the hydraulic work of the pore water are in exact balance.
Thus, the present formulation does not replace the traditional stress framework but provides its physical foundation, clarifying why stability or liquefaction occurs under given conditions of density and permeability.

The ideas introduced by Terzaghi and Casagrande marked the beginning of a physical interpretation of soil stability, even though they were formulated in mechanical terms. Their notions of hydraulic equilibrium and critical void ratio anticipated the existence of an underlying energy balance between the solid framework and the pore water.

The static enthalpy equilibrium developed here formalises this balance and extends it to a general physical condition that applies to all saturated sands. In this sense, the present formulation represents a natural evolution of their concepts - from empirical observation and mechanical reasoning to an explicit energetic description of stability.

5.2 Consistency with granular physics

The proposed model is consistent with modern theories in the physics of granular matter, which describe structural transitions as energetic exchanges within dissipative particle assemblies.
In granular-physics terms, the structural enthalpy corresponds to the configurational energy stored in grain contacts, while the hydraulic work represents the dissipative energy flux through the interstitial fluid.
The static enthalpy equilibrium therefore links geotechnical mechanics with the thermodynamics of granular systems and provides a macroscopic expression of their shared energy-balance principle.

5.3 Physical parameters and measurability

All quantities in the formulation - porosity n, grain density ρ_s, water density ρ_w, effective modulus K_{eff}, and energy distance h - are measurable physical parameters. No empirical calibration or material-specific fitting constant is required.
This makes the equilibrium condition directly testable through controlled laboratory or field observation: if the measured energy change associated with structural rearrangement equals the hydraulic work computed from pore-pressure gradients, the material is at the enthalpy equilibrium.
Deviations from this balance identify either an over-energetic (potentially unstable) or under-energetic (stable) state.

5.4 Broader implications

Interpreting sand behaviour as an energy-equilibrium process has several broader implications.
It unifies spontaneous and externally induced liquefaction within one physical framework; it explains the metastable relaxation observed in cyclic or seismic loading as the progressive reduction of excess structural enthalpy; and it provides a foundation for energetic design concepts, in which densification and compaction can be quantified in terms of required external work rather than empirical energy input.
Ultimately, the static enthalpy equilibrium defines the universal energetic boundary between stability and instability in saturated granular soils, forming the physical basis on which existing constitutive or numerical models may be interpreted.

6 Conclusions

This study introduces a static enthalpy equilibrium as a general energetic condition governing the stability of saturated sands.

The formulation expresses equilibrium as the equality between the structural enthalpy released by the granular framework and the hydraulic work capacity of the pore water.

It provides a single physical criterion that describes all states of saturated sand - from loose, metastable configurations to dense, externally compacted ones - within one continuous energy framework.

Three characteristic energetic regimes are identified:

(1) an over-energetic state, typical of loose sands, where excess structural energy may lead to spontaneous or easily triggered collapse,

(2) the equilibrium state, corresponding to the critical porosity at which total enthalpy is minimised; and

(3) an under-energetic state, representative of dense sands, where collapse or further densification can occur only if external work is supplied.

Spontaneous and induced liquefaction thus emerge as opposite directions of the same energetic process, distinguished only by the source of the energy that restores equilibrium.

The static enthalpy equilibrium extends the classical concepts of Terzaghi's hydraulic equilibrium and Casagrande's critical-void-ratio condition by making their implicit energy balance explicit.

It links soil mechanics with the thermodynamics of granular matter and provides measurable physical parameters - porosity, grain and water density, effective modulus, and energy distance - by which the energetic state of a sand can be defined.

When the enthalpy balance is satisfied, the system is stable; when violated, it evolves toward equilibrium through local dissipation and metastable relaxation.

Ultimately, the static enthalpy equilibrium defines the universal energetic boundary between stability and instability in saturated granular soils.

It provides a physically consistent foundation on which existing constitutive or numerical models can be interpreted and from which new energetic design and densification concepts may evolve.

References

(a) Classical soil mechanics and foundation concepts

- Terzaghi, K. (1925). Erdbaumechanik auf bodenphysikalischer Grundlage. Franz Deuticke, Vienna.
- Casagrande, A. (1936). Characteristics of cohesionless soils affecting the stability of slopes and earth fills. Journal of the Boston Society of Civil Engineers, 23, 13–32.
- Casagrande, A. (1975). Liquefaction and cyclic deformation of sands - A critical review. In Proceedings of the 5th Panamerican Conference on Soil Mechanics and Foundation Engineering, 79–133. Buenos Aires.
- Roscoe, K. H.; Schofield, A. N.; Wroth, C. P. (1958). On the yielding of soils. Géotechnique, 8(1), 22–53.
- Seed, H. B.; Lee, K. L. (1966). Liquefaction of saturated sands during cyclic loading. Journal of the Soil Mechanics and Foundations Division (ASCE), 92(SM6), 105–134.

(b) Thermomechanical and energetic formulations in geomechanics

- Houlsby, G. T.; Puzrin, A. M. (2007). Principles of thermomechanical energy methods in geomechanics. Acta Geotechnica, 2, 143–160.
- Buscarnera, A.; Einav, I. (2012). The yielding and liquefaction of saturated granular soils as thermodynamic instabilities. Acta Geotechnica, 7(2), 83–106.
- Collins, I. F. (2016). An energy-based interpretation of rate-independent plasticity in geomaterials. Acta Geotechnica, 11, 255–271.
- Puzrin, A. M.; Germanovich, L. N. (2010). The growth of shear bands in catastrophic landslides. Acta Geotechnica, 5, 71–89.

(c) Granular physics and energetic analogues

- Nowak, E. R.; Knight, J. B.; Ben-Naim, E.; Jaeger, H. M.; Nagel, S. R. (1998). Density fluctuations in vibrated granular materials. Physical Review E, 57, 1971–1982.
- Richard, P.; Nicodemi, M.; Delannay, R.; Ribiere, P.; Bideau, D. (2005). Slow relaxation and compaction in granular systems. Nature Materials, 4, 121–128.
- Blumenfeld, R.; Edwards, S. F. (2009). Granular entropy: Explicit calculations for planar assemblies. Physical Review Letters, 90, 114303.

(d) Modern interpretations and applications

- Wan, R.; Guo, P. (2018). Instabilities and strain localization in granular materials. Acta Geotechnica, 13, 449–465.
- Li, X.; Zhang, L. (2021). Energy-based interpretation of liquefaction triggering in sands. Acta Geotechnica, 16, 1821–1838.