

Earthquakes within Earthquakes: Patterns in Rupture Complexity

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Earthquake source time functions carry information about the complexity of seismic rupture. We explore databases of source time functions of earthquakes and find that source time functions are composed of distinct peaks that we call subevents. We observe that earthquake complexity, as represented by the number of subevents, grows with earthquake magnitude. We find that subevent magnitudes are nearly proportional to their corresponding main event magnitude. We show that the main event magnitude can be estimated after observing only the first few subevents.

1 **1 line summary** Pattern in the complexity of earthquake source time function suggests a self organization in
2 ruptures.

3 Whether earthquake complexity contains underlying structure is fundamental to the issue of earthquake pre-
4 dictability. Recent work has suggested that such structure permits a sense of determinism whereby observations
5 made in a short period of time at the beginning of an earthquake may be used to infer the overall event size and
6 style (1–3). Previous studies have found that early P waves arrivals inform the overall earthquake size (1, 4), but
7 these correlations bear large uncertainties that necessitate additional information, such as attenuation, for meaning-
8 ful early magnitude estimate (5–7). The first P wave arrival, however, represents only a small fraction of potentially
9 deterministic information (8).

10 Earthquake seismic signals are the convolution of the source time function (STF), or spatially-averaged slip
11 rate history, and the Green’s function, which represents the effects of wave propagation from the earthquake fault
12 to the seismometer (9). The basic structure of the STF is an initial acceleration of seismic moment followed its
13 deceleration. When averaged over many events, STFs reach a maximum value around 30-50% of their duration
14 (2, 10, 11), which suggests a “weak determinism” whereby the moment of the earthquake is at least twice that
15 released when the moment-rate decreases. Although this trend describes large ensembles of events, STFs exhibit
16 significant variation between events as well as complexity within a single event (11). Here, we take advantage of
17 existing STF catalogues ((12, 13); see Supplementary Materials) to explore this complexity.

18 We describe STFs as a sum of “subevents,” or time-domain Gaussian pulses (14). We refer to our iterative
19 fitting procedure as a subevent decomposition (Supplementary Materials). Examples of such decomposition are
20 shown in Figure 1. The subevent decomposition tunes the duration and amplitude of each subevent to better
21 represent the catalogued STF. We consider only subevents that are greater than 10% of the STF maximum value
22 and longer than 1 s in duration. We refer to each subevent as having a duration and moment, keeping in mind the
23 inherent non-uniqueness of the STF due to its spatially averaged character.

24 Our main result is that subevent moment M_S scales with the total moment M_0 (Figure 2a) according to
25 $\log_{10} M_S = 0.8 \log_{10}(M_0) + 3.06$. This scaling implies that subevents do not have an intrinsic length scale,
26 but instead have uniform scaling with total event moment. We note several caveats of this analysis. First, a lower
27 bound on subevent moment is determined by our detection threshold. We find, however, that the scaling relation is
28 not sensitive to the choice of threshold (see Supplementary Materials). Second, the amplitude of the scaling could
29 be biased toward lower values because we ignore the smallest subevents that have a duration less than 1 second.
30 Thus, we believe that the scaling is between 0.8 and 1. The data quality being limited to signals of duration longer
31 than 1 s, we retain the threshold of 0.1 and the implication of an artificial lower bound in subevent size. Addition-
32 ally, the use of a symmetric Gaussian function for the pulse shape may not be not physical (15), but the reliable
33 frequency content of the database limits us to use smooth functions.

34 Using the number of subevents as a measure of earthquake complexity, we find our second main result: that

35 earthquake complexity scales with earthquake size (Figure 3a). In general, M5.5 earthquakes are constituted of
36 about 1-4 subevents, while the M8+ earthquakes are composed of 8-10 subevents. This trend is clearer than that
37 found by (16) who used the zero crossings in the time derivative of STFs in two databases (10, 16) to quantify
38 complexity. The growth in the number of observed subevents is more pronounced for the magnitudes greater than
39 M6.75, which source dimension of ~ 15 km corresponds to the seismogenic depth of crustal earthquakes. The
40 M9.0 2011 Tohoku Earthquake is the outlier of the SCARDEC dataset, with only few subevents given its size. The
41 Tohoku earthquake is well documented for being a single and large slip-patch (17, 18) hence bears a low complexity
42 index in our analysis. Strike slip earthquakes tend to have greater complexity than other modes (see Supplementary
43 Materials), perhaps due to their complex multi-fault geometry (Kokoxili 2001 (19, 20), Denali 2002 (21), Sumatra
44 2012 (22,23), Kaikoura 2016 (24)). Strike slip event complexity may be over estimated, however, by contamination
45 of the direct seismic phases with their reverberation in the water column (25).

46 These two main results are self-consistent. Note, for example, that ratio between seismic moments of a M8
47 and an M6 is about 1000, yet the M8 has about only 5 additional subevents. Thus, it is not possible for a M8
48 earthquake to be composed of subevents of the same size as the ones found in a M6 earthquake. Instead, the
49 observations imply that the subevent size grows with the main event size.

50 To better understand our observations, we turn to earthquake dynamic rupture simulations. Subevents occur
51 naturally in these simulations when incorporating a self-similar fault pre-stress distribution (power-law exponent
52 of 0.8, a random phase (26–28)). Friction is described using the linear slip weakening model on a two-dimensional
53 fault loaded out of the plane. The fault is 200 km long and the nucleation length is 1.6269 km (29). By randomly
54 selecting a nucleation location, we construct a suite of 600 events to reproduce a spread in event magnitude of three
55 orders of magnitudes. We reconstruct the STF by integrating the function that described the rate of moment per
56 unit of fault width over the fault dimension (see Figure 1). However, the simulated STF contain a high frequency
57 content (about 20 Hz) whereas the observed SCARDEC STFs are only reliable up to 1 Hz due to data limitation.
58 We thus smooth the simulated moment-rate with a Gaussian filter of width 2 s.

59 We perform the same subevent decomposition for both observed and modeled STFs. We find similar patterns

60 between our modeled and observed STF (Figure 2b and 3b). We find greater variability in the number of subevents
61 observed than modeled, which can be due in part to modeling strategies (such as a lower degree of structural and/or
62 fault property heterogeneity than reality) or to the over-simplified Green's function used in SCARDEC. Neverthe-
63 less, in both observed and simulated sets of STF, the number of subevents in general grows with earthquake size
64 (Fig. 2).

65 Earthquake Early Warning (EEW) is a natural application of the scaling between subevent and main event size.
66 Specifically, our results suggest that the final magnitude may plausibly be estimated by observing only a small
67 number of initial subevents. To test this idea, we extract an STF constructed from the September 28, 2018 M7.5
68 Palu Earthquake in Indonesia from the USGS database (30), accessed on 19/10/2018, Figure 4). This event was
69 not used in forming the main scaling relation. We begin scanning the STF at time $t = 0$ and we fit the first subevent
70 at $t = 1.8$ s. At this time we estimate the subevent moment and duration by fitting a Gaussian, and then apply the
71 above-described scaling relation to estimate the total event moment.

72 We find that our algorithm predicts a magnitude of M7.2 at 1.8 s of rupture time, while only M6.3 equivalent of
73 moment magnitude was released, and while the final size of the event is M7.5 (Fig. 4). Upon the second subevent
74 detection at 3.6 s, which we constrain as being no smaller than 15% of the amplitude of the first peak, we calculate
75 an updated moment magnitude estimate and take the mean of both estimates. We iterate as the rupture evolves
76 and refine the magnitude estimates based on the distributions of the previous individual estimates by taking the
77 mean value. This allows us to stabilize the magnitude estimates as the variability in subevent size will fluctuate
78 the estimates. An alternative approach to rapid moment estimation based on observing the maximum moment
79 rate (11) would require approximately an additional 10 s to our approach. This calculation ignores latency in wave
80 propagation and EEW systems (31).

81 We have performed a similar exercise as in Figure 4 for the entire SCARDEC database and find that we can
82 predict the final magnitude with an uncertainty of 0.27 in moment magnitude within the first 20% of rupture
83 duration (see Supplementary Materials). We find that the largest subevent is detected within 30-50% of the rupture
84 time, which is related to the slight asymmetrical shape found by (2, 10, 11). Nevertheless, these biases can be

85 accounted for in a future improvement and application of this scaling.

86 Our work is mainly limited by the globally averaged attenuation model and the simplicity in the Green's
87 function used to reconstruct the source time function from the teleseismic seismograms. Because we are limited to
88 seismic waves with frequencies lower than 1 Hz, improved considerations of 3D Green's functions and in particular
89 in attenuation models will enhance the data set at lower magnitudes. Improved methodology might also better
90 inform whether fault heterogeneity can be constrained by subevent distributions. But overall, our work suggests
91 that earthquake ruptures, seen through their complexity, are self organized and have a sense of determinism, and
92 that patterns in their complexity may be explained by physical models.

93 **References and Notes**

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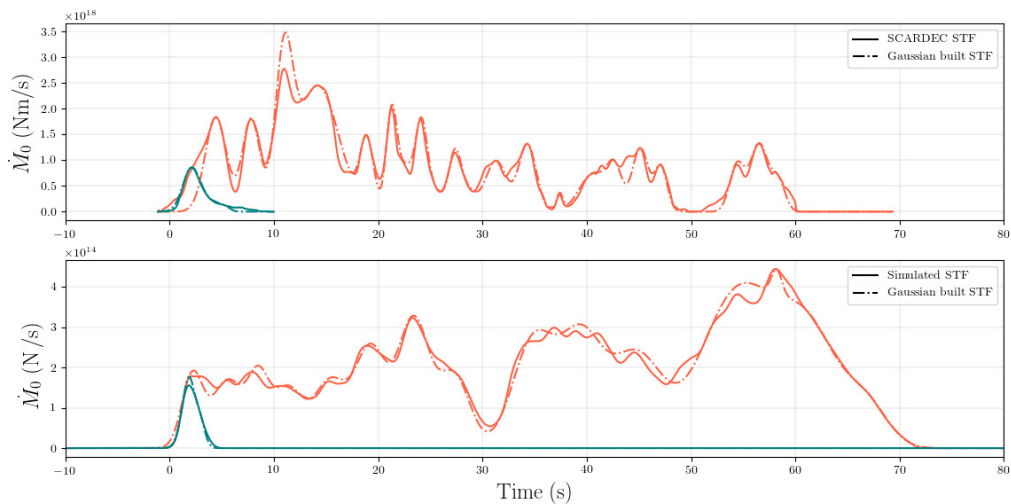


Figure 1: **Complexity in observed and modeled STF.** (Top) Two observed STF from the SCARDEC database from the 2015/07/03 M6.2 Leyte Earthquake in green and from the 1994/03/14 M7.1 Central Mid-Atlantic Ridge Earthquake in orange. (Bottom) Two modeled STF in moment-rate per unit of fault width for a large (orange) and small (green) events. In both figures, solid lines show the original SCARDEC STF and dashed lines show to the STF reconstructed with the Gaussian subevent functions. In both modeled and observed cases, the larger event exhibits more and bigger subevents than that of the small event.

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 132 draft. J.Y. performed the simulation analysis and helped editing the manuscript. B.P.L. advised the simulations part
 133 and helped with the manuscript. M.D designed and managed the project, served an advisory role to J.Y. and P.D,
 134 and edited manuscript and figures. None of the authors have any conflict or competing interest. The SCARDEC
 135 database is open to the public <http://scardec.projects.sismo.ipgp.fr/> and the SBIEMLAB codes
 136 are maintained by JP Ampuero at <https://github.com/jpampuero/semlab>. The python and matlab
 137 scripts are available in supplementary materials.

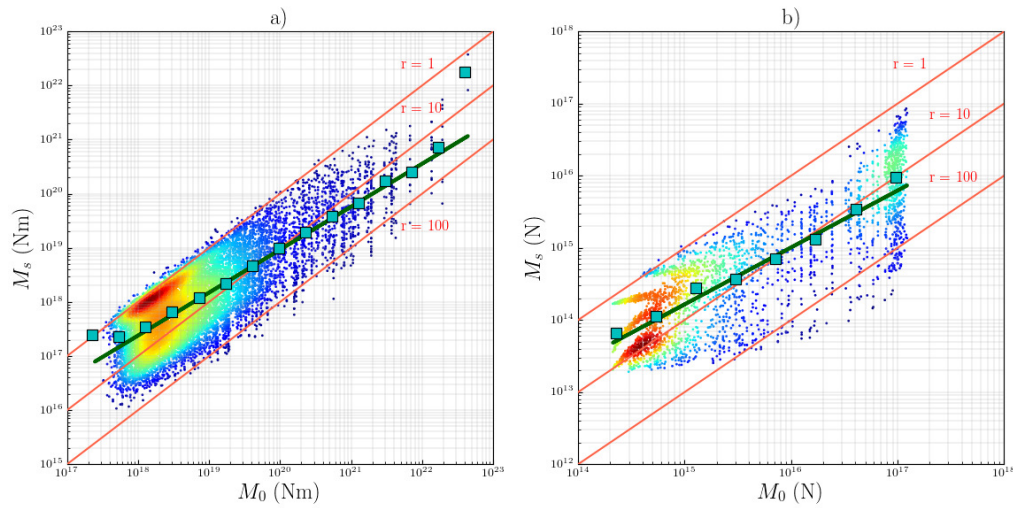


Figure 2: **Scaling of subevent moment with earthquake size**: individual subevent moments against main earthquake moment for observed (a) and modeled (b) STFs. In both figures, dots represent a single measurement, squares represent the medians over moment bins. Red lines indicate a ratio of seismic moment of subevent to earthquake r of 1, 10, and 100 respectively. The dots are colored according to their probability density. The green solid thick line is a linear regression performed over the the log-log distribution of individual measurements, $M_S \propto M_0^{0.8}$ for the observations and for the simulations, which we later use for rapid magnitude estimates (see Figure 4).

138 **Supplementary materials**

139 Materials and Methods

140 Supplementary Text

141 Figs. S1 to S7

142 Tables S1

143 Movie S1

144 References (32-41)

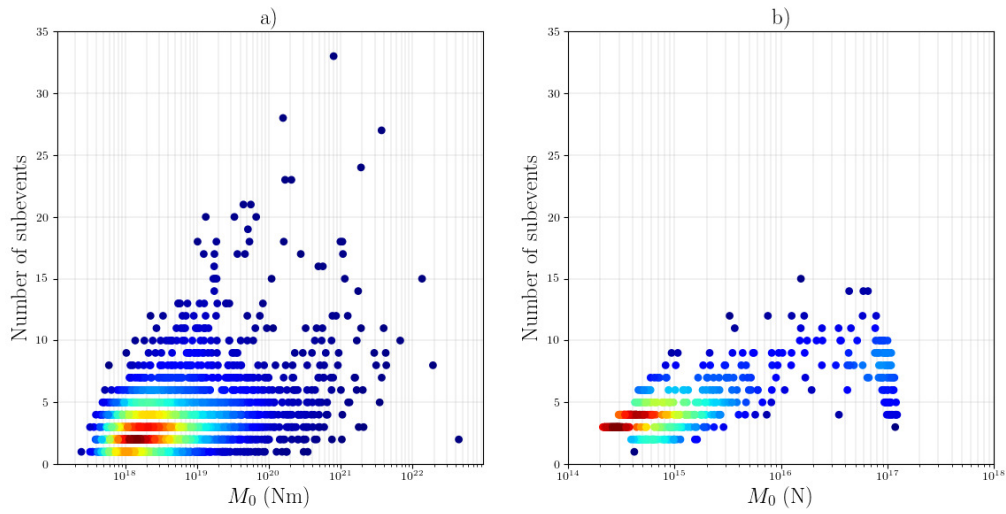


Figure 3: Earthquake complexity grows with earthquake size: Number of subevents as a function of seismic moment of the main event for (a) the observed STFs and (b) the modeled STFs. Dots are the individual earthquakes, orange squares are median in moment bins. Dots are colored according to the probability density function. The growth in complexity is monotonic with earthquake size for the observations, except for the M9.0 2011 Tohoku earthquake, and the growth is noticeable for the simulations.

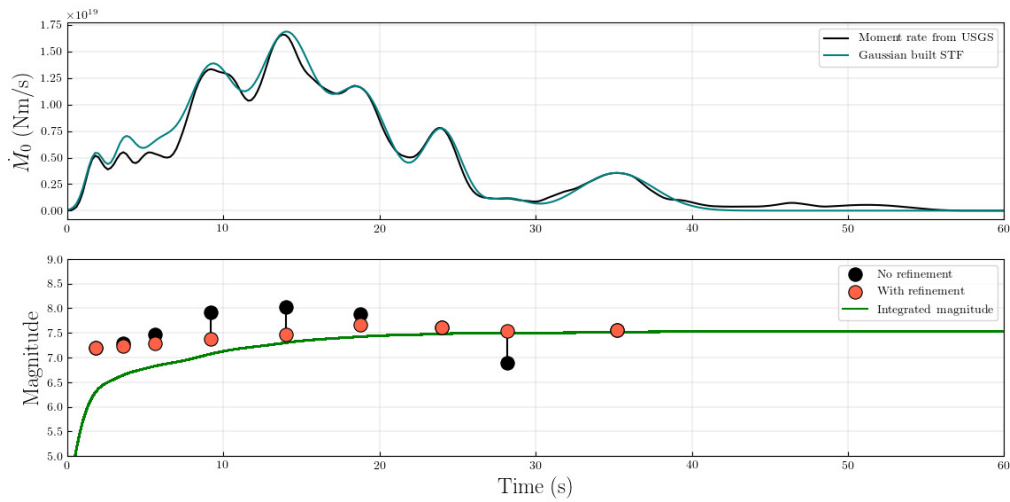


Figure 4: **Magnitude prediction in the recent 2018 M7.5 Palu Earthquake, Indonesia.** (Top) USGS Source time function estimate (https://earthquake.usgs.gov/archive/product/finite-fault/us1000h3p4/us/1539813424642/moment_rate.mr, last accessed on 10/19/2018) in black, and our Gaussian-built STF in green dash line. (Bottom) “Real time” magnitude estimate throughout the rupture: green curve is the magnitude difference from the cumulative seismic moment, black dots are the magnitude estimates from individual subevents fitting (using scaling statistics from Figure 3), red dots are the median of all previous magnitude estimates. See Supplementary Materials for the detailed algorithm.



Supplementary Materials for

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This PDF file includes:

Materials and Methods
Figs. S1 to S7
Tables S1
Movie S1: captions

Materials and Methods

1. Source time functions databases.

Our results use the SCARDEC database of source time functions <http://scardec.projects.sismo.ipgp.fr/>, last accessed on 11/11/2018. The database contains over 3,000 events of magnitudes 5.5 and greater from 1992 to 2017. The database is well established whereby earthquake source parameters have been compared with other published catalogs (12,32-36). The SCARDEC dataset of STF, as published on the website, are constructed from global P phases (P, pP, sP, PP, PcP) whereby wave propagation and attenuation are compensated with simple Green's function calculated in radially symmetric global Earth model. Because attenuation is better constrained for frequencies lower than 1Hz, we do not interpret signals that are shorter than 1s in the following. The sampling rate in the database is 0.0703s.

To validate our results, we use the United State Geological Survey (USGS) STF database that contains 180 events (13, to date of 12/4/2018). The STF from this dataset are constructed from kinematic inversion for slip based on the Ji et al. (2002) (37) algorithm that is a non-linear inversion that solves for rupture velocity, slip, and slip rate. The waveforms used in the fit are P, S, and surface waves (13). This database has also been validated for various earthquake-focus publications, is the official data product provided by the USGS, and commonly serves as a baseline for source time function and finite fault model comparison. The STFs in the USGS database have variable sampling rates. Figure S1 shows the same analysis as in the main manuscript and it serves as a validation with an independent database.

2. Gaussian decomposition algorithm

In the following, we elaborate on the Gaussian decomposition algorithm. It is similar in spirit as in Zhan et al., (2014) (14). We perform the decomposition forward in time. The algorithm starts from time zero:

- a. Go forward in time and detect an amplitude peak S_s (local maximum over 3 points) at time t_s that satisfies $S_s > 0.1 \max(\text{STF})$
- b. Fit a Gaussian function centered around t_s the STF by setting its amplitude to S_s and by varying its width with a grid search over the RMS width σ .
- c. Subtract to the STF the fitted Gaussian function (some residual peaks may be made detectable).
- d. Go forward and back to step a.

If the detection duration is less than 1s, we ignore the peak. Otherwise, the detection is considered a subevent and its moment M_s is the value of the Gaussian function integrated over time.

The choice of a threshold is necessary to ignore spurious residuals that are not resolvable by the data (high frequency or low amplitudes). We tried different thresholds (0.01, 0.05, 0.10, 0.15) as shown Figure S2 for the 0.01 threshold. The spread in subevent moment is large for this low threshold, whereby smaller subevents can be detected. However, if the spread of subevent moment changes, the scaling of subevent vs main event moment does not change. Due to the limitations

in the SCARDEC data set at high frequency, we choose a threshold of 0.1 to only retain the main subevents.

3. Application to Earthquake Early Warning (EEW).

In a real-time application of our findings, we do not have prior knowledge of the maximum value of the STF, which is known *a posteriori*. Thus, we modify the detection algorithm, and show one example in main manuscript Figure 4. In order to avoid fitting residuals, we proceed as follow:

- 1) Detect a first peak:
 - a) Compute its moment 1M_S and duration 1T_S .
 - b) Use the scaling relation to get 1M_0 from 1M_S and combine with magnitude-moment relation: ${}^1M_W = 0.8 \log_{10}({}^1M_S) - 9.3$
- 2) Detect the following peak “i” if it has at least 0.15 times the amplitude of the first peak (for the Palu Earthquake application to EEW) or with the 10% of the maximum STF criteria when applied to the whole SCARDEC database:
 - a) Compute its moment iM_S and duration iT_S .
 - b) Use the scaling relation to get iM_0 from iM_S and combine with magnitude-moment relation: ${}^iM_W = 0.8 \log_{10}({}^iM_S) - 9.3$
 - c) Take the median of the individual magnitude estimates ${}^iM_W = \text{median} (({}^{1 \rightarrow i})M_W)$.
- 3) Go back to 2.

This is the method implemented for Figure 4 of main manuscript. We apply this to all events in SCARDEC database. We show the difference between the magnitude ground truth and the predicted value against rupture time, which we normalize to the final source duration. We take the definition of duration similar to (33) whereby the duration is measured until the STF amplitude becomes lower than 10% the peak of the STF, to avoid long tail that cannot be resolved due to unmodeled near-source scattering.

In Figure S3, each dot is a magnitude estimate at the peak time of a subevent ${}^iM_W - M_{\text{true}}$. The color of the dot is chosen to illustrate the power density function of the dots (blue is few events, red is a large number of events). First estimates can come in as early as 5% of the source duration with an underprediction of about 0.5 magnitude increment. Between 30-50%, our method overpredicts the magnitude by an increment of 0.5. This is because the largest subevent tends to be in the central part of the STF (11). This is also visible in Figure 4. After 50% of the source duration, the uncertainty in magnitude estimates is 0.27 in magnitude estimate.

1. Properties of the subevents

To verify that the subevents have physically consistent values of duration T_S and moment magnitude M_S , we estimate their stress drop as if they were small independent quakes and compare them to the main event stress drop. Note that subevents are not independent, but rather dynamically connected through fracture mechanics. Following (34, 38, 39), the model-dependent stress drop is:

$$\Delta\sigma = \frac{7}{16} * (M/(k\beta)^3) \text{ where } fc = 0.6/T, \quad k = 0.32, \text{ and } \beta = 3900 \text{ m/s.}$$

The duration T is equal to the width where the Gaussian STF is at least 10% of the maximum value for the subevents, and as the time between zero and the last time where the STF reaches 10% of the maximum STF amplitude for the main event. We compute the stress drop for all subevents, their main event and take their ratio.

We show these estimates in Figure S4. We find a median value of 8 MPa for the subevents stress drop, whereas the median value of these calculated for the main events is 0.3 MPa, a low and model-dependent value similar to the findings of (34). The median ratio of subevent stress drop to main event stress drop is 19. One interpretation is that these peaks in moment-rate are events of larger stress drop than the rest of the event.

As mentioned in main text, complexity of earthquake as a count of subevents is stronger for strike slip than for dip slip earthquakes (Figure S5). The number of subevents for events with a dip higher than 70° (mostly strike slip) is higher than for those of lower dip (normal and thrust), especially at high magnitudes. This might argue for the need of statistics like in Figure 3 but made for subsets of main events, depending on their type, to add precision in the statistics performed.

2. Numerical simulations of dynamic ruptures.

Our simulations are based on 2D mode III dynamic crack model, efficiently solved by the spectral boundary integral methods (SBIEMLAB, code developed by Jean-Paul Ampuero, <http://web.gps.caltech.edu/~ampuero/software.html>). The ingredients for our simulations are listed below:

a) Basic parameters in the simulations

Model domain is 400 km long, with 200 km long fault domain and 200 km boundary domain, and is uniformly discretized by 4096 grids with grid size $\Delta x = 97.7$ m. Other basic parameters are listed in the Table S1.

b) Constitutive relation

We choose the basic linear slip weakening law to prescribe the friction on the fault:

$$\mu = \begin{cases} \frac{\mu_d - \mu_s}{D_C} s + \mu_s & s \leq D_C \\ \mu_d & s > D_C \end{cases}$$

where s is the slip, μ_d and μ_s are the dynamic and static friction coefficients, respectively. D_C is the critical slip distance, we choose $D_C = 0.8$ m in the main text but also test $D_C = 0.4$ m and 1.2 m. The frictional parameters μ_d and μ_s are uniform on the fault in our model.

c) Heterogeneous pre-stress distribution

We produce a heterogeneous pre-stress distribution using the power spectral density PSD obeys a self-similar power law:

$$\text{PSD}(k) = k^{-\gamma},$$

where k is the wavenumber, γ is the Hurst exponents and we choose $\gamma = 0.8$, which is constrained by the fault roughness observations (40). The phase of the pre-stress distribution

is randomly generated from a uniform distribution in $[0, 2\pi]$. Combining the PSD and random phases, we can produce a large population of statistically identical distributions.

Then we set the negative part of these distributions, normalize to the range between $[\mu_d\sigma_0, \mu_s\sigma_0]$. To avoid the artifacts in STF from abrupt stopping of rupture at the fault boundary in the spectral boundary integral solutions, we further apply a Tukey taper on the pre-stress distributions (Fig.S6 (a)). To make sure the post-processing on the pre-stress distribution does not distort the original PSD, we also compare the spectra of unprocessed and processed pre-stress distributions in the frequency domain (Fig.S6 (b)). Within the length scale of the entire active fault zone (200 km), the self-similarity, that is, the slope of the pre-stress with post-processing is mostly preserved with slight distortion.

d) 2D spontaneous dynamic rupture model (Mode III crack)

To nucleate the spontaneous dynamic rupture, we randomly and artificially set an over-stressed nucleation area. We choose the region whose pre-stress peak is within $x = [-30, 30]$ km on the fault and nucleate rupture at that location. This setting helps to avoid the boundary effects of pre-stress distribution and thus ensure that most of the simulated events evolve on a stress condition that is statistically similar during the event. The nucleation length is based on the relation from (41):

$$L_C \approx \frac{1.158\mu D_C}{(\mu_s - \mu_d)\sigma_0}.$$

For $D_C = 0.8$ m, the corresponding $L_C = 1626.9$ m $\approx 17\Delta x$, which also ensures the sufficient spatial resolution of our simulations. We set the size of nucleation zone to be $1.5L_C = 2440.4$ m and this is to guarantee the ending of quasi-static stage and beginning of dynamic unstable slip, after the nucleation (41).

For each D_C , we produce 600 pre-stress distributions that qualify the condition mentioned before, and finally get 600 simulated STFs with different event sizes.

S wave velocity V_s (km/s)	3.46
Density ρ (kg/m ³)	2,670
Shear modulus μ (GPa)	32
Normal stress σ_0 (MPa)	120
Dynamic friction μ_d	0.525
Static friction μ_s	0.677
D_C (m)	0.8

power law exponent γ	0.8
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Table 1: properties of the simulations

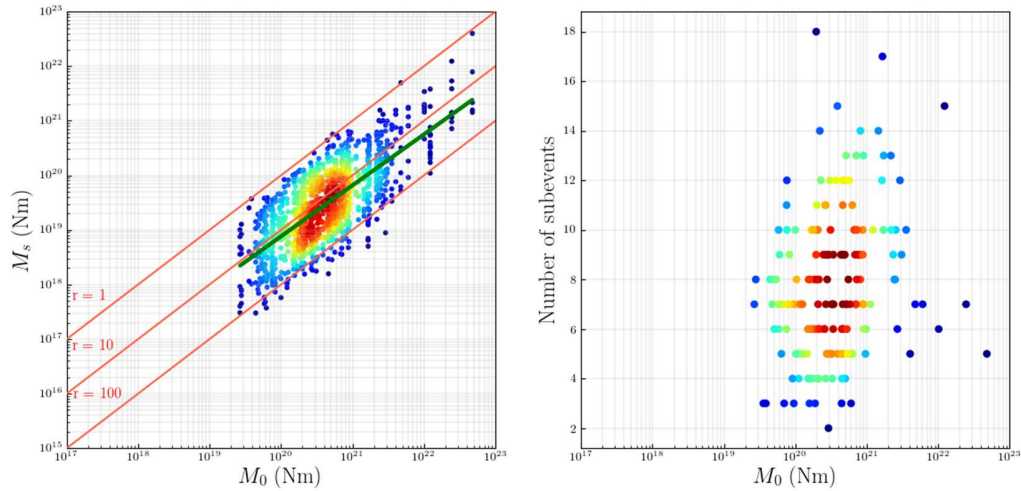


Figure S1. Results for the USGS STF (Hayes, 2017). (a) Subevent moment M_s as a function of main event moment M_0 for a peak detection with a 10% threshold. Green points are individual measurements of the moment, and squares are median results per 0.25 magnitude bins. The green line represents the result of the regression performed over the whole dataset. Finally, red lines represent the $r=1$, 10 and 100 ratios between the two moments. Despite a wider scatter, results still show a scaling similar to the SCARDEC results: $M_s \sim M_0^{0.9}$. (b) Number of subevents as a function of main event moment, dots representing individual measurements, colored by their density distribution. There is an increase of complexity with main moment, less pronounced than for SCARDEC, maybe because of the lack of data.

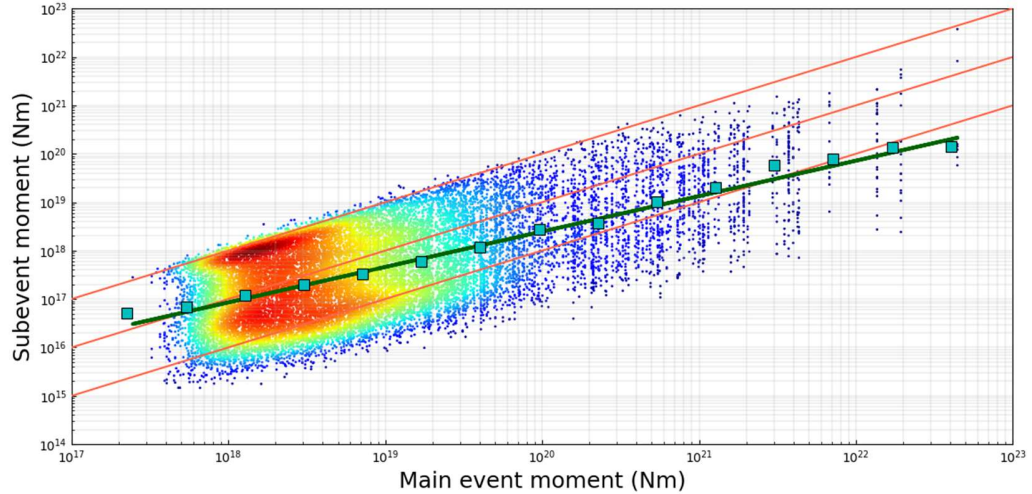


Figure S2. Subevent moment M_S against main event M_0 : Similar to Figure 2 of main manuscript but with a 0.01 threshold. Dots are individual subevent measurements of given their main even moment. Dots are colored to highlight the probability density function (blue few elements, red many elements). Blue squares are median results per 0.25 magnitude bin, the error bar in the median of their bootstrapped median are smaller than the marker size. The green line is the linear regression performed over the whole dataset in a log-log space. Red lines represent the $r=1$, 10, and 100 ratios moments. Despite a wider scatter, results still show a scaling similar to the 0.1 threshold results: $M_S=M_0^{0.8}$. It also shows that our detection becomes sensitive to the residuals from the Gaussian fitting, hence lower subevent moments.

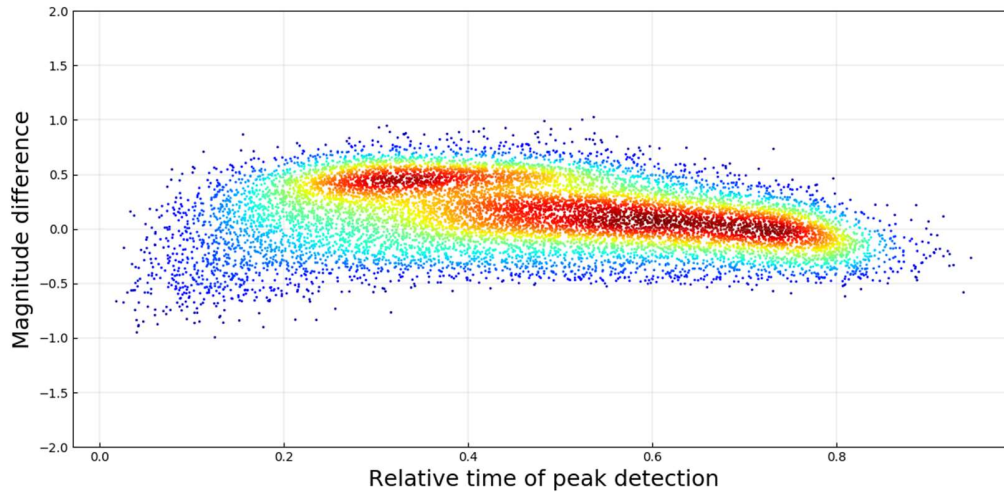


Figure S3. Magnitude estimates for all SCARDEC STF. Similar to Figure 4 of main manuscript. Each dot is the difference between the SCARDEC moment magnitude and the one estimated from the moment scaling (Figure 3a) for each subevent of every event. For each earthquake, we proceed with the algorithm described in section 3. Between 20% and 40 % of STF duration, our algorithm overpredicts the magnitude by 0.5 unit due to the fact that STFs tend to peak at that time, and thus experience the largest subevent. The estimates have an uncertainty of 0.27 magnitude before 20%, of 0.31 magnitude between 20 and 50%, and of 0.27 magnitude after 50%.

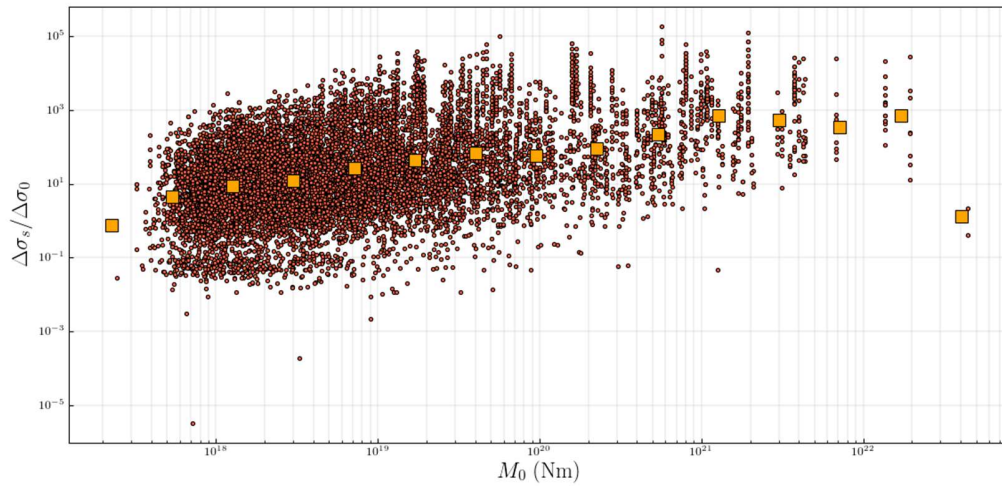


Figure S4. Ratio of subevent stress drop over main event stress drop. Each dot is an individual measurement of the ratio, squares represent the median ratio of 0.25 magnitude bins. The increase in the ratio is due to a growth of subevent stress drop instead of main event stress drop, as those exhibit a nearly constant stress drop around 0.3 MPa.

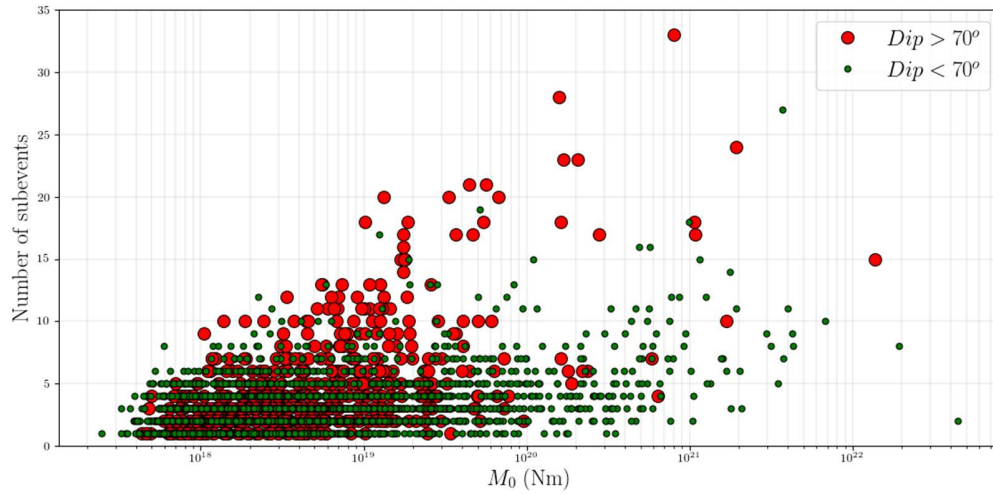


Figure S5. Number of subevents as a function of main event moment. We represent earthquakes with a dip higher than 70° in red dots, and in green dots the results for main events with a dip lower than 70° . We are therefore able to see the influence of dip over rupture complexity, and this highlights the higher complexity for strike slip events.

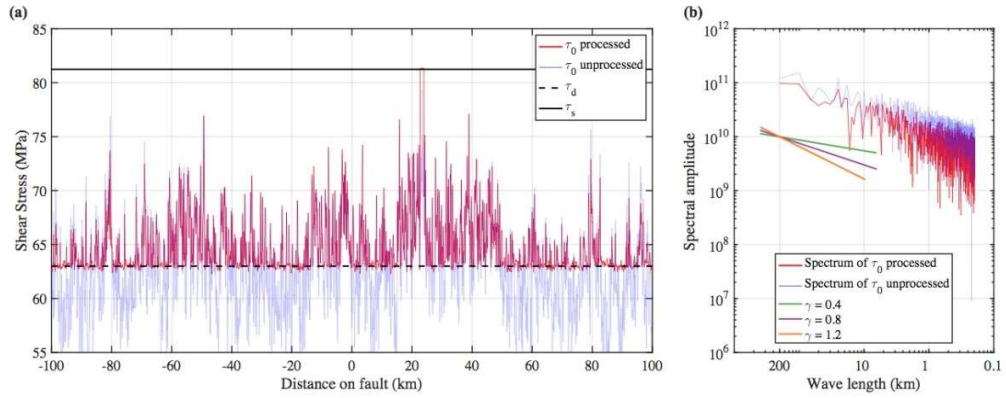


Figure S6: Pre-stress distribution and corresponding amplitude spectrum. (a) One example of the stochastically generated pre-stress distribution. Blue line shows the pre-stress distribution without any post-processing, while red line is the pre-stress after processing and used in the dynamic simulation. Black solid and dashed lines are the uniform fault strength and dynamic friction, respectively. (b) Comparison between the processed (red) and unprocessed (blue) pre-stress distributions in the frequency domain. Green, purple and orange lines show the references of different Hurst exponents of 0.4, 0.8 (used in this study) and 1.2, respectively.

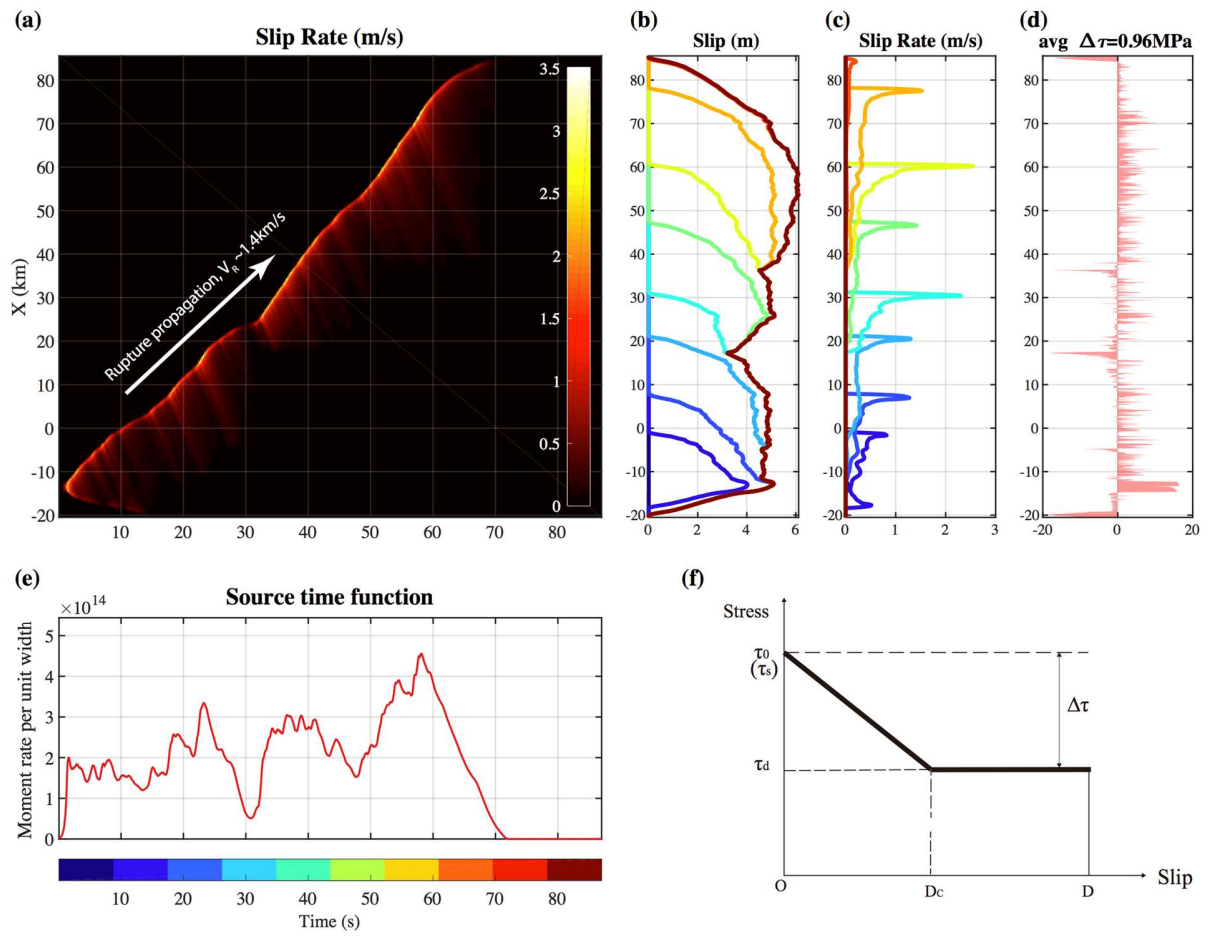


Figure S7: example of numerical simulation of the dynamic rupture. (a) shows the time-space distribution of slip rate, (b) the slip profile across the fault, (c) the slip rate as a function of time, (d) the stress change on the fault, (e) the final STF, (f) the linear slip weakening.

Movie 1: Application of our results to the 09/28/2018 Palu Earthquake, Indonesia. We apply the Gaussian decomposition as detailed above, the algorithm described in section 3 to the STF of the Palu Earthquake, the final result being shown in Figure 4.

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