Source effects in higher-order ambient seismic field correlations

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Key Points:

- Higher-order correlations facilitate the use of asynchronous station recordings and can enhance correlation functions for imaging.
- The re-correlation of direct waves increases the sensitivity to source effects which can lead to significant velocity estimation errors.
- The limited availability of "virtual sources" for re-correlation can introduce additional velocity bias.

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Abstract

Seismic interferometry of the ambient seismic field is widely used for surface wave imaging. It typically requires synchronous station recordings and assumes uniform noise source distributions. Higher-order correlations, such as the re-correlation of direct waves (C_2), have been suggested to facilitate imaging with asynchronous data and to improve an incomplete source distribution. Using field data and simulations, we show that C_2 surface wavefields are instead highly sensitive to the original source distribution and even amplify the effects associated with the directional incidence. This can lead to systematic errors in the obtained velocity estimates and the downstream subsurface images. Tested strategies for selecting auxiliary stations in the re-correlation process do not mitigate this bias but can introduce additional wavefield distortions. Local and far-field imaging approaches using higher-order C_2 correlation wavefields are affected by significant and systematic velocity estimation errors. Our results show that the re-correlation of direct waves is not an all-purpose correlation wavefield enhancement technique, and highlight the need for a careful consideration of source effects for improved imaging.

Plain Language Summary

Seismic interferometry is a technique that uses background vibrations, or "ambient seismic noise", to create images of the Earth's subsurface. Traditionally, this method works best when all sensors record ground vibrations at the same time and when the noise sources are spread out evenly. A derived technique referred to as higher-order correlation has been suggested to allow imaging with data from sensors that were not recording at the same time, and to better handle uneven noise source distributions. Using field data and wavefield simulations, we however find that these higher-order techniques do not reduce the problems caused by uneven or isolated noise sources. In fact, they can amplify these problems, leading to significant errors in estimates of structural properties. Hence if source effects are not carefully studied and accounted for, the application of higher-order correlations can result in systematically biased subsurface images.

Keywords: Surface waves and free oscillations, Time series analysis, Wave propagation, Theory, Tomography

1 Introduction

The ambient seismic field is routinely used to image and monitor Earth structure (Shapiro et al., 2005; Sens-Schönfelder & Wegler, 2006; Poli et al., 2012; Lu et al., 2018; Mao et al., 2025). Seismic interferometry of this complex wavefield yields a new wavefield—the C_1 correlation wavefield—that is related to the Green's function of the medium when sources are homogeneously distributed or, equivalently, when the illumination is isotropic (Wapenaar et al., 2005; Snieder et al., 2008). For commonly observed heterogeneous source distributions, the C_1 phase and associated travel time estimates are biased (Tsai, 2009; Froment et al., 2010; Wang et al., 2016; Schippkus et al., 2022). Strategies to mitigate this influence include temporal averaging (Sabra et al., 2005), correlation lag time averaging (Bensen et al., 2007), informed correlation lag time selection (Schippkus et al., 2022), filtering (Seydoux et al., 2017), jointly inverting for sources and structure (Fichtner et al., 2017; Sager et al., 2020), or excitation pattern based error modeling for potential consideration in the inversion (Giammarinaro et al., 2023).

Higher-order correlations are an alternative data-driven processing strategy for correlation wavefield enhancement (Stehly et al., 2008; Froment et al., 2011; Sheng et al., 2018). Higher-order correlation refers to the re-correlation of C_1 wavefields to obtain a new correlation wavefield with improved characteristics for imaging. For a regular C_1 correlation the recordings of two stations are required, which we refer to as the receiver station and the master station or "virtual source" (Figure 1a). For a higher-order correlation at least one additional auxiliary station is required (Figure 1b). This approach is therefore also referred to as three-station interferometry (S. Zhang et al., 2020). Higher-order correlations have been used successfully to yield more reliable velocity estimates (S. Zhang et al., 2020; Nouibat et al., 2022). In addition to the reported imaging improvements, this strategy facilitates the processing of asynchronously recorded data. This can significantly expand the applicability of seismic noise interferometry to extended configurations under the assumption of constant medium properties (Ma & Beroza, 2012; Chen & Saygin, 2020; S. Zhang et al., 2020).

Higher-order correlations too can yield robust Green's function estimates. The C_2 wavefield is obtained by re-correlating the direct waves in the C_1 wavefields reconstructed with persistent auxiliary stations ($\mathbf{r}_a$ in Figure 1b), whereas the C_3 wavefield results from the re-correlation of the multiple scattered C_1 coda wavefield. To obtain C_2 Green's func-

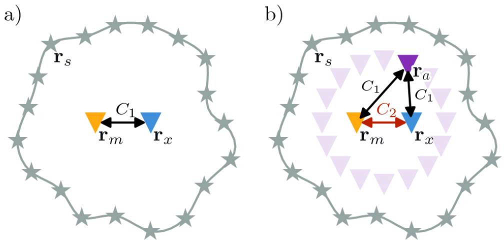

Figure 1. Cross-correlation concepts. (a) The C_1 cross-correlation of the receiver station (blue triangle) and master station (orange triangle) data is proportional to the Green's function $G(\mathbf{r}_x, \mathbf{r}_m)$ for homogeneously distributed noise sources along a boundary (dark gray stars). (b) The higher-order C_2 cross-correlation involves the re-correlation of the direct waves in the two C_1 wavefields between an auxiliary station (purple triangle) and the master and receiver station, respectively. Stacking over homogeneously distributed auxiliary stations is equivalent to a homogeneous virtual source distribution. In that case, C_2 also yields the Green's function $G(\mathbf{r}_x, \mathbf{r}_m)$, but the requirement of simultaneous data acquisition at $\mathbf{r}_x$ and $\mathbf{r}_m$ is relaxed.

tions the locations of auxiliary stations must meet certain requirements. They can either be homogeneously distributed, which reproduces the ideal situation of an isotropic illumination associated with a homogeneous distribution of original sources for the C_1 Green's function estimate (Figure 1b). Alternatively, auxiliary stations located in the endfire lobes of each master-receiver station pair can achieve the same quality estimates (S. Zhang et al., 2020), again in analogy to C_1 seismic interferometry (Roux et al., 2004; Wapenaar et al., 2010). In contrast, C_3 wavefields approximate the Green's function because the scattered C_1 coda waves approach isotropic illumination (Stehly et al., 2008). Robust C_3 results can hence be achieved with a significantly smaller number of auxiliary stations compared to C_2 processing. However, the C_2 approach provides better control on the illumination and the ballistic wave propagation solutions facilitate numerical analysis.

The $C_{1,2,3}$ terminology used here follows Stehly et al. (2008) and Sheng et al. (2018), but different naming conventions have been used (S. Zhang et al., 2020). The concepts and jargon used to exchange about higher-order correlations can vary and the obtained results can be counter-intuitive. This includes the view on wavefield directionality. It can be described from a local observational viewpoint based on beamforming or other directional measurements, or it is discussed from the perspective of the distant sources and their distribution. Here we adopt a source centered discussion to develop our arguments (Figures 1, 2, Sections 2, 3) but we frequently make the connection to the local surface wavefield incidence.

The most common subsurface imaging strategy using the ambient seismic field is surface-wave tomography (Shapiro et al., 2005; Lin et al., 2008; Lu et al., 2018; Schippkus et al., 2018; Nouibat et al., 2022), because surface waves tend to dominate the original and hence the C_1 wavefields (Toksöz & Lacoss, 1968; Harmon et al., 2008; Juretzek & Hadziioannou, 2016). For tomography, surface wave group or phase velocity dispersion is measured from the reconstructed Rayleigh and Love propagation in the C_1 wavefield. The dispersion data obtained between stations that are separated by two or more wavelengths (Bensen et al., 2007) is then inverted for lateral velocity distributions and 3D shear wave velocity models (Barmin et al., 2001; X. Zhang et al., 2018). Local phase velocity dispersion based on spatial auto-correlation fields or focal spots provide complementary observations that can extend the resolution of dense array surface wave imaging (Aki, 1957; Hillers et al., 2016; Tsarsitalidou et al., 2024).

Here, we investigate how the common observational situation of non-isotropic source distributions affect higher-order correlations. We focus on the C_2 wavefield properties obtained from the re-correlation of direct Rayleigh surface waves and demonstrate with observations and simulations that these higher-order correlations amplify the source signature in the C_1 wavefields. We analyze higher-order correlation focal spots and evaluate the impact of this source effect enhancement on velocity estimates. For this, we investigate C_1 wavefield focal spots obtained from field data and simulations (Section 2) and the relationship between the C_1 wavefield source signature and the C_2 wavefield properties (Section 3). We evaluate source effect mitigation strategies (Section 4) and the impact on imaging applications, including focal spot imaging and tomography (Section 5).

2 Cross-correlation of the ambient seismic field (C_1)

First, we study source effects in a C_1 correlation wavefield obtained from field data and then simulate these wavefields. We adopt a conceptual framework that discriminates between the original wavefield excitation from boundary sources and isolated sources (Schippkus et al., 2022). Consider the wavefield record u at location $\mathbf{r}_x$ that is induced by a source at $\mathbf{r}_s$. Neglecting an explicit frequency ω dependence, u is

$$u(\mathbf{r}_x) = N_s G(\mathbf{r}_x, \mathbf{r}_s), \tag{1}$$

where N_s is the source spectrum and G the Green's function. The C_1 cross-correlation of the records from the master station at $\mathbf{r}_m$ and a receiver station at $\mathbf{r}_x$ is

$$C_1(\mathbf{r}_x, \mathbf{r}_m) = \langle u(\mathbf{r}_x) u^*(\mathbf{r}_m) \rangle_s \,, \tag{2}$$

where $\langle \cdot \rangle_s$ indicates ensemble averaging over all sources (Wapenaar et al., 2010). If the wavefield is excited by sources that are homogeneously distributed along a boundary S and by one additional isolated noise source at $\mathbf{r}_I$ with source spectrum N_I, the C_1 correlation wavefield is given by (Schippkus et al., 2022, eq. 6)

$$C_1(\mathbf{r}_x, \mathbf{r}_m) = \underbrace{\frac{\rho c |N_s|^2}{2} \big(G(\mathbf{r}_x, \mathbf{r}_m) + G^*(\mathbf{r}_x, \mathbf{r}_m) \big)}_{\text{boundary source contribution}} + \underbrace{G(\mathbf{r}_x, \mathbf{r}_I) G^*(\mathbf{r}_x, \mathbf{r}_I) |N_I|^2}_{\text{isolated source contribution}} . \tag{3}$$

The boundary source contribution to the C_1 wavefield are wavefronts that converge towards the master station at negative lapse times, refocus, and then diverge at positive lapse times (Wapenaar et al., 2005). The contribution from the isolated source is a wavefront that is excited at negative lapse time $\tau = -|\mathbf{r}_m - \mathbf{r}_I|/c$ at $\mathbf{r}_I$ and that arrives at the master station at lapse time 0 s (Schippkus et al., 2022). In the following, we describe a C_1 wavefield computed from field data and demonstrate how the conceptual framework of boundary sources and isolated sources helps explain the observations. Note that the distinction between boundary and isolated sources is merely conceptual but facilitates the synthesis of directivity effects.

2.1 Field data

We use field data recorded by a large-N (N=4907) seismic deployment in the Vienna basin, Austria (Schippkus et al., 2022, Figure 2). Each station is equipped with 12 or 24 10-Hz vertical-component geophones. The data are stacked on the hardware level to increase the signal-to-noise ratio, particularly at low frequencies. We select a subset of 1990 receiver stations in a square configuration, 304 auxiliary stations around the receiver stations, and a central master station (Figure 2b). These stations were deployed in March 2019 and recorded seismic noise for four weeks. We cut the records into non-overlapping 1 hr windows, apply spectral whitening (Bensen et al., 2007), compute cross-correlations between all receiver stations and the master station for all time windows, and stack the correlations of all time windows linearly to obtain the C_1 correlation wavefield.

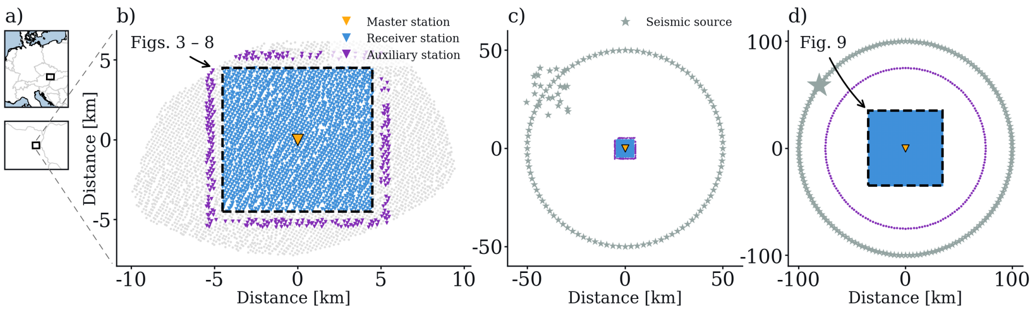

Figure 2. Station distribution and simulation configuration. (a) The overview map and the zoomed-in view show the Austrian-Slovakian border area of the data acquisition. (b) The map shows the spatial distribution of all stations (light gray dots) in the dataset, and the master station (orange triangle), receiver stations (blue triangles), and the auxiliary stations (purple triangles) used in this study are highlighted. (c) Configuration for the local wavefield simulations. The distribution of the synthetic sources (gray stars) are a combination of homogeneously distributed sources along a circle and additional isolated sources clustered in the Northwest. (d) The configuration of the far-field wavefield simulations involves a single, stronger isolated source in the Northwest.

Snapshots of the 0.3 Hz narrowband-filtered C_1 Rayleigh wavefield between the master station and the selected receiver stations at three lapse times $\tau = [-1, 0, 1]$ s show the effects of a non-isotropic energy flux that is associated with a non-homogeneous source distribution (Figure 3a–c). We choose 0.3 Hz for demonstration purposes because the coherent microseisms energy supports the reconstruction of clean C_1 wavefields. The corresponding wavelength is large compared to the array aperture, which results in our prioritization on local auto-correlation or focal spot patterns instead of propagating far-field signals in our analysis and illustrations, however, without loss of generality. For a homogeneous source distribution and an isotropic medium the narrowband focal spot at $\tau = 0$ s takes the shape of a rotationally symmetric Bessel function in space and the correlation wavefield is lapse-time symmetric (Aki, 1957). However, we observe a focal spot elongation in the Northeast-Southwest direction (Figure 3b) and an asymmetric correlation wavefield (Figure 3a, c), which are indicators of an inhomogeneous source distribution (Ermert et al., 2016; Giammarinaro et al., 2023).

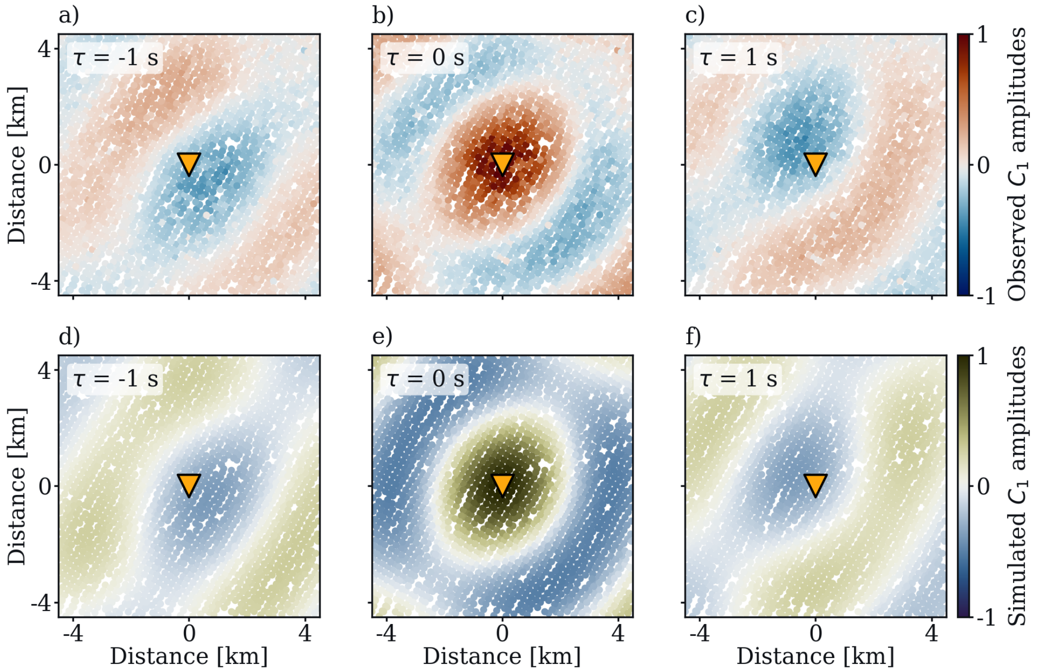

Figure 3. Snapshots of the 0.3 Hz narrowband-filtered C_1 correlation wavefield at three lapse times τ. (a–c) The C_1 correlation wavefield is estimated from field data. (d–f) The C_1 correlation wavefield is estimated from simulations. The observed lapse-time asymmetry is matched by the simulations that consider a directional illumination from the Northwest (Figure 2c).

In this area, wind turbines in a few hundred meters distance to the auxiliary stations in the Northeast of the deployment have been shown to act as isolated sources at frequencies above 0.5 Hz (Schippkus et al., 2022). At 0.3 Hz, however, we find no evidence of significant influence of these wind turbines. Instead, the patterning of the obtained C_1 surface wavefield suggests isolated sources to the Northwest (Figure 3a–c). The excitation of secondary ocean microseisms (Ardhuin et al., 2015) is compatible with an isolated noise source, which induces C_1 wavefield components that are referred to as spurious arrivals (Snieder et al., 2008; Zeng & Ni, 2010; Retailleau et al., 2017; Schippkus et al., 2023; Safarkhani et al., 2025). Near the master station, this implies wavefield interference between the two C_1 components of the boundary sources and the isolated sources for all station pairs. The observed C_1 wavefield pattern is thus governed by the ocean microseisms sources in the Northern Atlantic Ocean (Kedar et al., 2008; Hillers et al., 2012; Juretzek & Hadziioannou, 2016).

2.2 Synthetic wavefields

To confirm this hypothesis, we simulate C_1 surface wavefields by first computing wavefield synthetics u (Equation 1) and then their cross-correlations. We model synthetic Green's functions analytically for a homogeneous acoustic medium, $G(\mathbf{r}_x, \mathbf{r}_s) = \exp\big(-i\omega t(\mathbf{r}_x, \mathbf{r}_s)\big)$, where $t(\mathbf{r}_x, \mathbf{r}_s)$ is the travel time between source and receiver. We neglect amplitude terms such as geometrical spreading or intrinsic attenuation. We do not expect amplitude effects to play a governing role in our observed C_1 wavefields reconstructions, hence, in our analysis, we focus on the phase characteristics of the correlation wavefields. Travel times are computed for a medium velocity of 1.9 km/s, estimated from the observed focal spot at 0.3 Hz. The velocity is constrained by a linear regression of the spatial autocorrelation data using a Bessel function model (Hillers et al., 2016; Tsarsitalidou et al., 2024). Our analytical simulation approach neglects the response of heterogeneous structure, but it effectively targets first-order source- and illumination-related effects.

We configure synthetic sources around the stations that follow the above concept (Equation 3). Boundary sources are homogeneously distributed along a circle, and a cluster of isolated noise sources is located in the Northwest (Figure 2c) that models secondary ocean microseisms in the Northern Atlantic Ocean. For simplicity, we put these sources at ∼50 km distance compared to a more realistic ∼2000 km distance. This implies a small difference in the curvature of the wavefronts across the field and the synthetic array, however, we find no feature in our results that indicates a related effect. All sources are equivalent, i.e., $N_I = N_s$. The source term N_s is a Ricker wavelet with 0.3 Hz peak frequency.

The synthetic C_1 wavefield is the cross-correlation field of the simulated waveforms u. No additional pre-processing is applied. The C_1 wavefield is obtained by computing the correlation for each synthetic source individually. The correlations are stacked linearly to avoid correlation cross-terms between different sources (Wapenaar et al., 2010). The synthetic C_1 wavefield effectively reproduces the field data observations (Figure 3d–f). The remarkably high degree of similarity therefore suggests that the observed non-isotropic C_1 wavefield characteristic is a source-related phenomenon.

3 Higher-order correlation of direct waves (C_2)

We compute higher-order C_2 correlations of direct waves and investigate their sensitivity to source effects that are exposed in the C_1 wavefields. For homogeneously distributed sources, the direct waves in C_1 wavefields are propagating between the master and receiver stations (Equation 3, Figure 1a), i.e., the master station acts as a virtual source (Mehta et al., 2008). If several virtual sources are arranged homogeneously around the domain, the combination of these C_1 wavefields results in isotropic illumination (Figure 1b). Although the term virtual source can be helpful to communicate the physics, we advise caution about its application in the discussion of higher-order correlation phenomena. In the regular C_1 correlation procedure and context, the master station is interpreted to be the virtual source (Figure 1a). In the C_2 correlation procedure each auxiliary station is viewed as a virtual source for the C_1 computation (Figure 1b). In the final C_2 correlation wavefield stacked over all used auxiliary stations, however, the master station can again be addressed as the virtual source.

The higher-order C_2 correlation between a reference station at $\mathbf{r}_x$ and master station at $\mathbf{r}_m$, using auxiliary stations at $\mathbf{r}_a$ as virtual sources, is

$$C_2(\mathbf{r}_x, \mathbf{r}_m) = \langle C_1(\mathbf{r}_x, \mathbf{r}_a) C_1^*(\mathbf{r}_m, \mathbf{r}_a) \rangle_a \,. \tag{4}$$

Inserting Equation (2) yields

$$C_2(\mathbf{r}_x, \mathbf{r}_m) = \langle \langle u(\mathbf{r}_x) u^*(\mathbf{r}_a) \rangle_s \langle u^*(\mathbf{r}_m) u(\mathbf{r}_a) \rangle_s \rangle_a \,, \tag{5}$$

where $\langle \cdot \rangle_s$ indicates ensemble averaging over all original sources and $\langle \cdot \rangle_a$ indicates ensemble averaging over all auxiliary stations or virtual sources. This demonstrates that there are two mechanisms that introduce source effects and can affect the reconstruction of C_2 Green's functions: the distribution of original sources and the distribution of auxiliary stations.

We compute C_2 wavefields for the central master station and each receiver station from C_1 wavefields for all auxiliary stations and ensemble average over all obtained data (Figure 4). We treat the causal and anti-causal parts of the C_1 functions separately to avoid cross-terms, i.e., spurious arrivals that can emerge from the correlation between causal and anti-causal arrivals (Wapenaar et al., 2010). The C_2^+ wavefield from the causal part of the C_1 functions and the C_2^- wavefield from the anti-causal part are averaged for the C_2 wavefield (Sheng et al., 2018). We do not consider cross-component correlations—

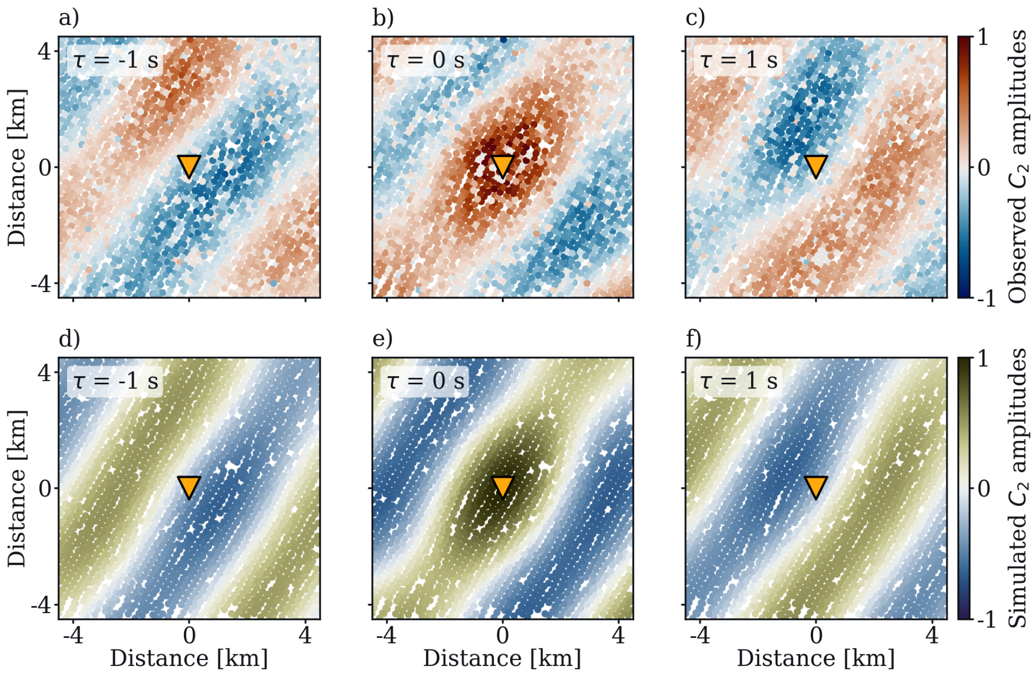

Figure 4. Snapshots of the 0.3 Hz narrowband-filtered higher-order C_2 correlation wavefield at three lapse times τ, averaged over all auxiliary stations. (a–c) The C_2 wavefield is estimated from field data. (d–f) The C_2 wavefield is estimated from simulations. In both cases, near-plane waves arriving from Northwest dominate the C_2 wavefield compared to the C_1 wavefield (Figure 3). The source effect of isolated ocean microseisms sources is amplified.

the correlation between causal and anti-causal parts of the C_1 wavefield—because one of the key characteristics of the effect of isolated sources on C_1 wavefields is the time-lapse asymmetry (Equation 3). Compared to the C_1 wavefield (Figure 3), the resulting C_2 wavefield shows an amplified source effect both for field data and simulations (Figure 4). The obtained stronger azimuthal asymmetry is associated with yet stronger isolated sources in the Northwest, which results in a strong plane wave component across the array. In other words, the amplitude of the isolated-source wavefront is amplified relative to the boundary-source wavefront in the C_2 wavefield compared to the C_1 wavefield. Note that this partitioning of isolated and boundary source contributions is insensitive to wave propagation effects, because we neglect amplitude terms in our simulations.

We use these wavefield simulation results to quantify the source effect induced by isolated sources (Figure 5). For this, we compute synthetic C_1 and C_2 wavefields for an ideal source scenario that excludes the isolated sources in the Northwest (Figure 2c). The

C_1^{ideal} focal spot is a rotationally symmetric Bessel function in space (Figure 5a), as expected for isotropic illumination (Aki, 1957). The difference to the simulated C_1 wavefield including isolated sources (Figures 3e, 5b) then illustrates the source effect (Figure 5c). Amplitudes in the Northeast and Southwest quadrants are moderately increased, whereas amplitudes in the Northwest and Southeast quadrants show the opposite behavior. The comparison of the C_2^{all} focal spot (Figures 4e, 5e) with the C_2^{ideal} wavefield (Figure 5d), both averaged over all auxiliary stations, confirms that the source effect in the C_1 wavefield (Figure 5c) is amplified in the C_2 wavefield (Figure 5f). Again, the observed effects result from the local anisotropic flux, but are ultimately controlled by the source distribution. The imperfect distribution of auxiliary stations in this data set has its own weak effect on the wavefield shown in Figure 5g, but it does not govern the here discussed features. Last, in Figure 5h we compare the C_2^{all} focal spot against the C_1 reference observation that is, however, also affected by the flux directivity. In the next three sections we study the C_2 wavefield properties for three scenarios to understand the amplification of the directivity effect. The three scenarios include one original isolated source, homogeneously distributed boundary sources, and both source types combined.

3.1 C_2 properties for an isolated source

In the case of a single original source and a single auxiliary station, Equation 5 simplifies to

$$C_2(\mathbf{r}_x, \mathbf{r}_m) = u(\mathbf{r}_x)u^*(\mathbf{r}_m)|u(\mathbf{r}_a)|^2. \tag{6}$$

Inserting Equation 1 yields

$$C_2(\mathbf{r}_x, \mathbf{r}_m) = \underbrace{|N_s|^2 G(\mathbf{r}_x, \mathbf{r}_s) G^*(\mathbf{r}_m, \mathbf{r}_s)}_{\text{isolated source in eq. 3}} \underbrace{|N_s|^2 |G(\mathbf{r}_a, \mathbf{r}_s)|^2}_{\text{influence of aux. station}}, \tag{7}$$

where the subscripts x, m, a, and s refer to the receiver station, master station, auxiliary station, and source, respectively. Only the first term carries phase information. This term is equivalent to the C_1 wavefield contribution of an isolated source for the same configuration (Equation 3), i.e., the phase is stationary with respect to the auxiliary station (Sheng et al., 2018). The C_2 wavefield exhibits the wavefront excited by the isolated source, independent of the auxiliary station. We compute C_2 spatial auto-correlation wavefields for four different auxiliary stations to confirm the phase is stationary (Figure 6a–d). The observed weak differences in the spatial variation of the amplitudes across the four panels are related to the influence of the auxiliary station. This shows that C_2 wave-

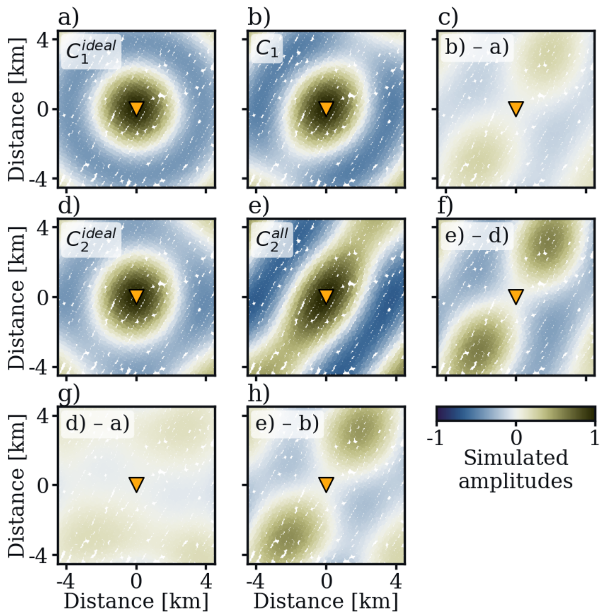

Figure 5. Synthetic focal spots illustrate source amplification effects. (a) The reference C_1^{ideal} focal spot for homogeneously distributed sources. (b) A C_1 focal spot in response to anisotropic illumination, as in Figure 3e. (c) The difference between the C_1 solution from panel (b) and the reference C_1^{ideal} from panel (a) illustrates the effect associated with the isolated sources added in the configuration in (b). (d) The C_2^{ideal} focal spot for the ideal source case averaged over all auxiliary stations. Weak azimuthal variations are controlled by the incomplete, square auxiliary station distribution (Figure 2b). (e) The C_2^{all} focal spot for anisotropic incidence averaged over all auxiliary stations, as in Figure 4e. (f) The source effect in the C_2 wavefield is amplified compared to the effect in the C_1 solutions in panel (c). (g) The amplitude effects that are controlled by the imperfect auxiliary station distribution are weak compared to the isolated source effect. (h) The difference between the C_2^{all} and the corresponding C_1 solutions indicates the amplification of the source effect in C_1 in panel (c).

fields associated with one isolated source but arbitrary auxiliary stations have the same phase characteristics. This further implies that averaging over different auxiliary stations, as we do for Figures 4 and 5, leads to constructive interference of the isolated source correlation wavefield contribution.

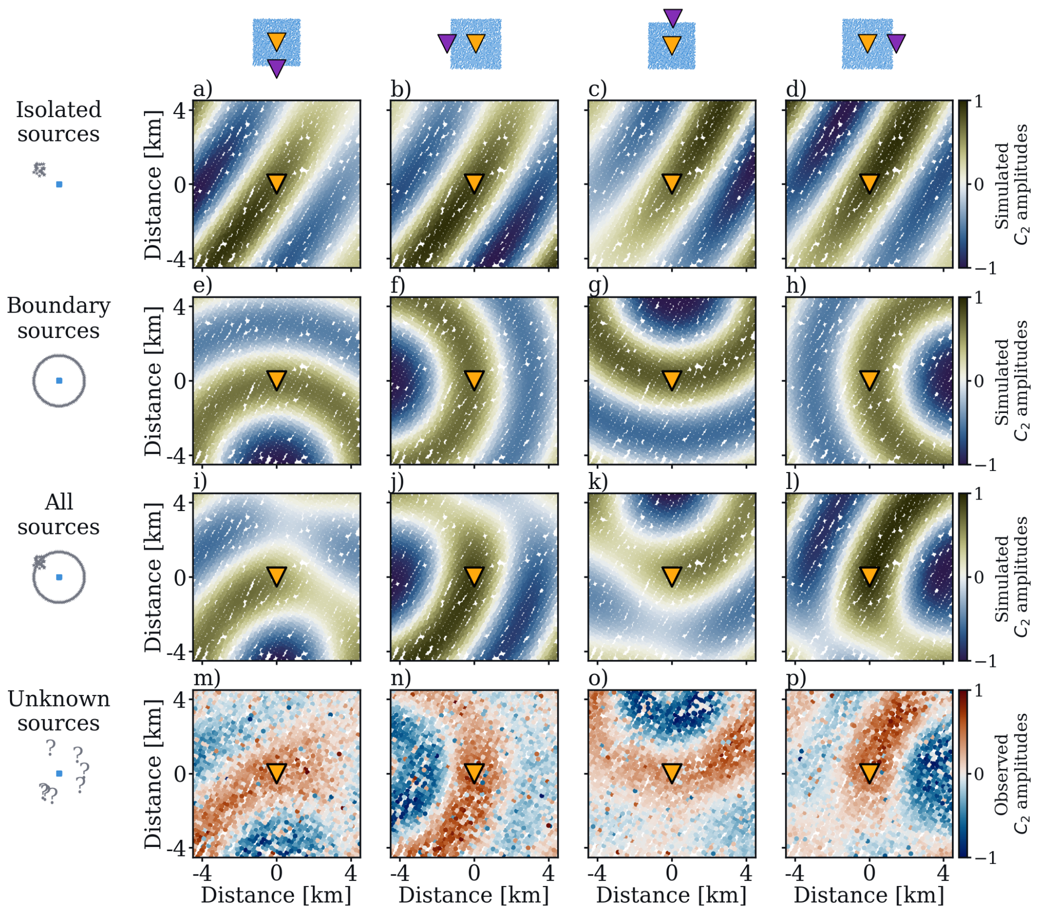

Figure 6. C_2 focal spots for (left to right) four single auxiliary stations and (top to bottom) four different excitation patterns. Cartoons at the top illustrate the purple indicated auxiliary station configurations. Cartoons on the left summarize the source configurations. (a–d) C_2 spatial auto-correlations in response to isolated sources or unidirectional incidence. The phase characteristics of the C_2 wavefield contributions controlled by the isolated sources are independent of the location of the auxiliary station. (e–h) For homogeneously distributed boundary sources the auxiliary station acts as virtual source. (i–l) The C_2 solutions for anisotropic incidence. The solution combines the patterns from the first and second row. (m–p) The C_2 focal spots obtained from field data exhibit a high degree of similarity to the synthetic results in row three, which implies a similar incidence pattern with a stronger excitation to the Northwest.

3.2 C_2 properties for boundary sources

When the original wavefield is excited by sources that are homogeneously distributed along a closed boundary S, the resulting C_1 and C_2 correlation wavefields behave differently compared to an isolated source. We refer to this as the "ideal" source case. The C_2 wavefield (Equation 4) for boundary sources is

$$C_2(\mathbf{r}_x, \mathbf{r}_m) = \oint_S C_1(\mathbf{r}_x, \mathbf{r}_a) C_1^*(\mathbf{r}_m, \mathbf{r}_a) d\mathbf{r}_s, \tag{8}$$

where integrating over the boundary sources accomplishes ensemble averaging. We apply the surface integral to both C_1 correlations separately and insert the boundary source contribution from Equation 2 to find

$$C_2(\mathbf{r}_x, \mathbf{r}_m) = \oint_S G(\mathbf{r}_x, \mathbf{r}_s) G^*(\mathbf{r}_a, \mathbf{r}_s) d\mathbf{r}_s |N_s|^2 \left(\oint_S G(\mathbf{r}_m, \mathbf{r}_s) G^*(\mathbf{r}_a, \mathbf{r}_s) d\mathbf{r}_s |N_s|^2 \right)^* . \tag{9}$$

Following Equation 11 of Wapenaar et al. (2005), the surface integrals resolve to

$$C_2(\mathbf{r}_x, \mathbf{r}_m) = \frac{\rho c}{2} |N_s|^2 [G(\mathbf{r}_x, \mathbf{r}_a) + G^*(\mathbf{r}_x, \mathbf{r}_a)] \left(\frac{\rho c}{2} |N_s|^2 [G(\mathbf{r}_m, \mathbf{r}_a) + G^*(\mathbf{r}_m, \mathbf{r}_a)] \right)^* . \tag{10}$$

We carry the complex conjugate $(\cdot)^*$ of the second term on the outside to avoid confusion about which Green's function terms relate to the causal and anti-causal parts of the C_1 wavefield. $G(\mathbf{r}_x, \mathbf{r}_a)$ and $G(\mathbf{r}_m, \mathbf{r}_a)$ correspond to the causal part of the C_1 wavefield, whereas $G^*(\mathbf{r}_x, \mathbf{r}_a)$ and $G^*(\mathbf{r}_m, \mathbf{r}_a)$ correspond to the anti-causal part. As said, we treat these parts separately to avoid correlation cross-terms. The C_2^+ wavefield from the causal part of the C_1 wavefield, and the C_2^- wavefield from the anti-causal part are

$$C_2^+(\mathbf{r}_x, \mathbf{r}_m) = \left(\frac{\rho c}{2} \right)^2 |N_s|^2 \underbrace{|N_s|^2 G(\mathbf{r}_x, \mathbf{r}_a) G^*(\mathbf{r}_m, \mathbf{r}_a)}_{C_1(\mathbf{r}_x, \mathbf{r}_m) \text{ for a source at } \mathbf{r}_\mathrm{a} \text{ (eq.3)}} \tag{11}$$

$$C_2^-(\mathbf{r}_x, \mathbf{r}_m) = \left(\frac{\rho c}{2} \right)^2 |N_s|^2 \underbrace{|N_s|^2 G^*(\mathbf{r}_x, \mathbf{r}_a) G(\mathbf{r}_m, \mathbf{r}_a)}_{C_1(\mathbf{r}_m, \mathbf{r}_x) \text{ for a source at } \mathbf{r}_\mathrm{a} \text{ (eq.3)}} . \tag{12}$$

The phase of the C_2^+ and C_2^- wavefields for homogeneously distributed original sources and a single auxiliary station is equivalent to the phase of the C_1 wavefield for a single isolated original source for the same probing configuration. In other words, the auxiliary station acts as a virtual source for the C_2 wavefield reconstruction, similar to the role of an isolated source for C_1 (Equation 3). This differs from a C_2 wavefield in which the original wavefield is excited by an isolated source (Figures 6a–d), where the auxiliary station has no impact on the phase of the C_2 wavefield. C_2^+ and C_2^- are furthermore symmetric in lapse time, i.e., $C_2^+ = (C_2^-)^*$, unlike the asymmetric contribution of an isolated source (Equation 7). The source power spectrum $|N_s|^2$ now appears twice, which is not important for our narrowband investigations, but it is relevant for our analysis of broadband far-field wavefields discussed below.

We simulate the C_2 wavefield for this ideal source scenario, averaged over C_2^+ and C_2^-, to confirm that the auxiliary station acts as a virtual source in C_2 wavefields when the original sources are homogeneously distributed. The spatial auto-correlations in Figures 6e–h clearly show wavefronts that appear to be excited by the auxiliary stations in the South, West, North, and East, respectively.

3.3 C_2 properties for combined boundary and isolated sources

We argue that the observed C_1 and C_2 wavefield properties can be explained by a combination of boundary and isolated source effects. This conceptual framework adopted in the previous sections predicts the C_1 and C_2 wavefield amplitude patterns observed in the field data (Figures 3, 4). The patterns in Figure 6i–l demonstrate that the C_2 wavefield can be understood as a linear combination of the contributions from isolated and boundary sources for each auxiliary station. When the phase of the C_2 wavefield depends on the auxiliary station (Figures 6e–h), averaging over an azimuthally homogeneous auxiliary station distribution yields a correlation that approaches a rotationally symmetric Bessel function in response to isotropic illumination (Figure 5d). However, this applies only to the situation where the original source distribution is also homogeneous and thus the ambient flux isotropic. When the phase of the C_2 wavefield is instead stationary with respect to the auxiliary station as for isolated sources (Equation 7, Figure 6a–d), averaging over auxiliary stations amplifies this wavefield contribution. This mechanism governs our original observation of the amplified source effect in C_2 (Figure 4) compared C_1 (Figure 3).

We construct C_2 wavefields from field data for the four individual auxiliary stations (Figure 6m–p). The good agreement between the field data results in Figure 6m–p and the patterns in the corresponding panels in the row above indicates the efficiency of our source distribution concept. More generally, this good agreement shows that our conceptual framework of boundary and isolated sources is helpful in understanding the direction-dependent energy distribution and the resulting properties of higher-order correlations. However, this also implies a trade-off in the computation of higher-order correlations. On one hand, multiple auxiliary stations are required to yield symmetric C_2 wavefields even for isotropic illumination. The benefit of stacking contributions from many auxiliary stations is also indicated by the fairly poor signal-to-noise ratio obtained for a single auxiliary station (Figure 6m–p). On the other hand, each auxiliary station amplifies the effect of isolated sources further.

The isolated source distribution for our simulations is chosen to maximize the qualitative similarity between the synthetic C_1 and C_2 and the field data (Figures 3–6). We do not estimate the direction to and the strength or number of the contributing sources from the field data systematically because we focus on the first-order mechanisms that govern source effect amplification in higher-order correlations. A more quantitative approach to generate matching C_1 and C_2 synthetics can be developed from our results here. In reverse, such an approach can lead to the utilization of higher-order correlations for improved isolated source detection and characterization.

4 Auxiliary station distributions to mitigate C_2 wavefield asymmetry

The application of higher-order correlations serves two goals. It allows the use of asynchronously deployed stations for imaging and it can be a correlation wavefield enhancement technique to achieve better signal-to-noise ratio for more reliable measurements. Figure 4 illustrates the significantly amplified source effect for homogeneously distributed auxiliary stations, and Figure 6 shows the source effect in C_2 wavefields obtained with single auxiliary stations. Hence, averaging over all auxiliary stations is a suitable strategy only if the original source distribution is also azimuthally homogeneous. In practice, however, the distribution of stations that can be used as auxiliary stations is unlikely homogeneous, which can limit the data driven mitigation approach.

Alternatively, auxiliary stations that are organized in line with each receiver-master station pair can be used to estimate C_2 Green's functions (S. Zhang et al., 2020; Nouibat et al., 2022). This strategy follows the same arguments that support the configuration of original sources located in line with station pairs for C_1 correlations, which is referred to as regions of stationary phase or endfire lobes (Snieder, 2004; Roux et al., 2004; Wapenaar et al., 2010). It has become clear, however, that the transfer of this argument from C_1 to C_2 wavefield reconstruction assumes a homogeneous original source distribution or isotropic illumination (Figure 6e–h).

Therefore, we investigate how the source effect associated with isolated sources affects C_2 solutions obtained with different selection strategies for station-pair-dependent auxiliary stations. For the selected stations to be in line with a receiver-master station

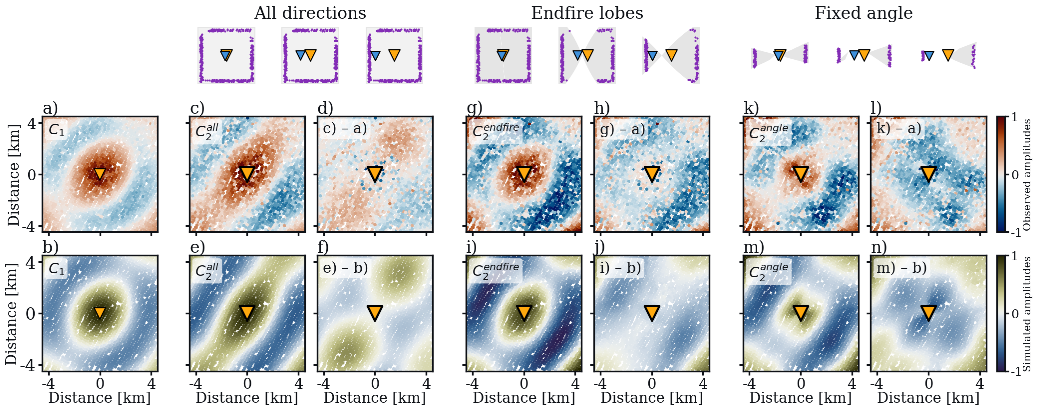

Figure 7. Influence of the auxiliary-station selection strategy on the source effect in C_2 wavefields. The cartoons sketch three receiver station examples (blue triangles) for the three auxiliary station (purple triangles) selection strategies. The gray area indicates the considered auxiliary station region. (a–b) The observed and simulated C_1 focal spots are reproduced from Figures 3b and 3d for reference. (c–f) C_2 focal spots for (c) field data and (e) simulations when auxiliary stations are located in all directions as in Figures 4, 5, as well as the (d, f) difference to the reference C_1 focal spots. (g–j) The effect of the endfire-lobe selection. The width of the selection area depends on inter-station distance. (k–n) The effect of the fixed-angle selection. We use $\pm 20^\circ$ relative to the inter-station azimuth. The variable observed focal spot characteristics controlled by the different selection strategies are well reproduced by the synthetic results for combined boundary and isolated sources.

pair, we can define endfire lobes (Roux et al., 2004) or a fixed-angle cone (S. Zhang et al., 2020; Nouibat et al., 2022). We compare synthetic and observed C_1 wavefields with C_2 wavefields that are reconstructed with three auxiliary-station configurations that include stations in all directions, in endfire lobes, and in fixed-angle regions (Figure 7).

First, we select stations from all directions (Figure 7c–f). For comparison, we plot again the biased C_2^{all} focal spot estimated from field data (Figures 7c, 4b) and simulations (Figures 7e, 4e). The source effect relative to the C_1 wavefield we predict from simulations (Figures 7f, 5h) agrees well with the features in the C_2 field data correlations (Figure 7d). This confirms again that the all-directions strategy amplifies the C_1 wavefield source effect governed by the isolated sources.

Second, we select auxiliary stations from endfire lobes. Endfire lobes are simplified descriptions of source sensitivity kernels. We use equation (3) of Roux et al. (2004) to define their area

$$B(\delta\theta) = 1 - \frac{\delta\theta^4}{8}\left(\frac{|\mathbf{r}_m - \mathbf{r}_x|}{c}\right)^2\left(\omega^2 + \frac{\Delta\omega^2}{12}\right), \tag{13}$$

where B is the directivity pattern, $\delta\theta$ is the angle difference to the axis of the receiver-master station pair, $|\mathbf{r}_m - \mathbf{r}_x|$ is the distance between receiver and master station, c denotes the medium velocity, ω the central angular frequency, and $\Delta\omega$ the frequency bandwidth. The selections for three example receiver stations in the central section of Figure 7 illustrate that endfire lobes narrow with increasing distance between the station pair. We further limit the selection of endfire-lobe auxiliary stations to ensure the same number of auxiliary stations on both sides of every station pair to avoid any additional effect related to virtual source asymmetry. This asymmetry is related to different numbers of stations in the selection regions on each side. We average C_2 wavefields from the selected auxiliary stations and find that the resulting $C_2^{endfire}$ focal spots from field data (Figure 7g) and from simulations (Figure 7i) have not improved. They exhibit yet a different amplitude pattern (Figure 7h, j) compared to the pattern resulting from the all-directions stacking (Figure 7d, f). This strategy is therefore not suitable to achieve isotropic illumination and mitigate the isolated source effect.

Third, a further simplification of the endfire-lobe selection strategy is to define a cone with a fixed-angle opening and select auxiliary stations within that cone area. In contrast to the endfire-lobe strategy, the width of the selection area is independent of the inter-station distance. At short inter-station distances, this results in a significantly narrower selection area compared to the selection based on endfire lobes (Figure 7). An auxiliary station is included if it satisfies equation (3) of S. Zhang et al. (2020)

$$||\mathbf{r}_a - \mathbf{r}_m| - |\mathbf{r}_a - \mathbf{r}_x|| \geq (1-\alpha)|\mathbf{r}_m - \mathbf{r}_x|, \tag{14}$$

where $\alpha = 1 - \cos(\theta)$ is the control on the opening of the cone and θ is half the opening angle. We choose $\theta = 20°$ (Nouibat et al., 2022). Similar to the endfire-lobe selection strategy, we also balance the number of auxiliary stations here. We note that for large inter-station distances, the endfire lobe areas and the fixed-angle cone areas can be similar. However, for the short inter-station distances compared to the wavelength here, the fixed-angle cones are much narrower (Figure 7). The resulting focal spots and amplitude patterns in Figure 7k–n show that at very short inter-station distances and in directions perpendicular to the Northwest-Southeast illumination additional amplitude variations are introduced compared to the endfire-lobe selection strategy.

None of the tested auxiliary station distributions mitigates the source effects observed in the C_1 wavefield. Selecting auxiliary stations from all directions amplifies the amplitude pattern in the C_2 functions, and the patterns obtained with the endfire lobe and fixed angle configurations do similarly not show an improvement towards an isotropic distribution (Figure 7). This suggests that anisotropic flux cannot be corrected for by re-correlating direct waves. Consequently, the application of higher-order correlations of direct waves depends on the careful consideration of the imaging situation.

5 Impact on imaging

We address the impact of source effect amplification in higher-order correlations on passive imaging results and quantify associated velocity measurement errors. We assess phase velocity estimates from correlation or focal spot data at local distances and group velocity measurements obtained from propagating waves that form the basis for surface-wave tomography. The comparison to a ground truth reference requires an analysis of these effects using simulations.

5.1 Phase velocity estimates from focal spot data for local imaging

The amplitude of the narrowband-filtered time domain focal spot or spatial autocorrelation field in a homogeneous medium for isotropic illumination is the classic SPAC result (Aki, 1957; Yokoi & Margaryan, 2008)

$$A(\mathbf{r}_x, \mathbf{r}_m) = J_0(k|\mathbf{r}_x - \mathbf{r}_m|) = J_0\left(\frac{\omega|\mathbf{r}_x - \mathbf{r}_m|}{c}\right), \tag{15}$$

where J_0 is the Bessel function of order zero, k is the wavenumber, c is the target phase velocity, and $|\mathbf{r}_x - \mathbf{r}_m|$ is the azimuthally independent distance between virtual source and receiver. We investigate the focal spots of the simulated ideal C_1^{ideal}, skewed C_1, all-direction-averaged C_2^{all}, endfire-lobe-averaged $C_2^{endfire}$, and fixed-angle-averaged C_2^{angle} solutions (Figure 8a–e).

We estimate the medium velocity c using a non-linear least squares regression between the time domain amplitude data and the isotropic J_0 parametrization (Figure 8f–j). For the regression, we use stations within 4.5 km distance to the master station, in-

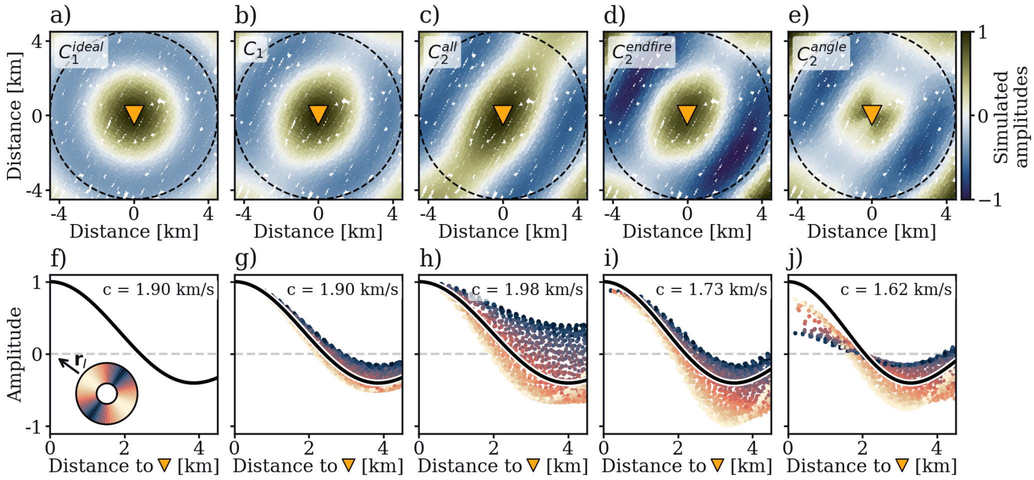

Figure 8. Impact of the source effect on focal spot regression results. (a–e) Simulated C_1^{ideal}, biased C_1, all-direction-averaged C_2^{all}, endfire-lobe-averaged $C_2^{endfire}$, and fixed-angle-averaged C_2^{angle} focal spots. We use stations within 4.5 km distance (dashed circles) for the regression to reduce potential azimuthal sampling artifacts. (f–j) Bessel function regression results assuming isotropic illumination and medium properties. The medium velocity c estimate is indicated. Samples are colored by the angle of the station pair axis relative to the direction towards the isolated sources at $\mathbf{r}_I$, indicated by the inset in panel (f). The reference velocity is 1.9 km/s.

dicated by the dashed circles, to establish homogeneous azimuthal sampling. We do not account for a distant dependent sample increase that can influence the sensitivity (Giammarinaro et al., 2023). The data range of $\sim 0.7\lambda$ emphasizes the local focal spot imaging approach (Giammarinaro et al., 2024; Tsarsitalidou et al., 2024).

We colorcode the simulated amplitudes in Figures 8f–j using the angle deviation of each station pair axis from the backazimuth to the isolated sources at $\mathbf{r}_I$ at about N300°E. The panels illustrate the systematic amplitude effects governed by isolated sources described above. The reference medium velocity of 1.9 km/s is well recovered in the ideal C_1^{ideal} case, i.e., for homogeneously distributed sources that result in isotropic illumination (Figure 8f). This implies that a C_2^{ideal} focal spot too facilitates a low-error velocity estimate under the condition that the original sources and the auxiliary stations are distributed isotropically (Figure 5d). The skewed C_1 focal spot resulting from a combination of circularly arranged sources and additional isolated sources (Figure 8g) facilitates the recovery of the reference speed. This is compatible with numerical sensitivity tests that synthesize a similar illumination situation (Giammarinaro et al., 2023). In both cases, the azimuthal average using the isotropic J_0 model (Eq. 15) results in an unbiased estimate (Aki, 1957). Spatially variable focal spot sampling at reference stations located away from the array center, or irregular array shapes, can yield variable estimates that depend on the imaging configuration (Tsarsitalidou et al., 2022).

The three different auxiliary-station selection approaches for computing the C_2 wavefield produce different focal spot patterns (Figure 8c–e). These result in large velocity measurement errors up to 15% compared to the C_1 focal spot regression (Figure 8h–j). The C_2^{all} solution overestimates the reference velocity but yields a better result compared to the two sectorial strategies $C_2^{endfire}$ and C_2^{angle}, which both underestimate the velocity. This further indicates that the distribution of available auxiliary stations has a significant impact on the accuracy of the velocity estimate.

Again, a complete azimuthal C_1 focal spot average for strongly anisotropic or even plane wave incidence results in a correct velocity estimate (Aki, 1957; Giammarinaro et al., 2023). SPAC models accounting for anisotropic incidence (Nakahara, 2006; Haney et al., 2012) can resolve the surface wave velocity from azimuthally incomplete focal spots. However, the usefulness of these models for C_2 data regression can be limited considering the C_2 sensitivity to auxiliary station patterns. To conclude, for common in-situ anisotropic illumination situations, the advantage of higher-order correlations to utilize asynchronously collected data is offset by yielding a biased estimate.

5.2 Group velocity estimates from Rayleigh wave propagation for tomography

Surface-wave tomography is the dominant imaging approach based on correlations of the ambient seismic field. Our discussion of the amplified source effects in C_2 wavefields is not limited to the spatial auto-correlation analysis at near field distances. However, our field data analysis is limited by the array aperture and the very short inter-station distances compared to the target microseisms wavelength (Figure 2). This wavelength is 6.3 km at the 0.3 Hz analysis frequency and all stations in the square array are thus located within approximately one wavelength. In ambient noise surface-wave tomography, station pairs with inter-station distances shorter than two or three wavelengths are commonly discarded to avoid the interference between the causal and anti-causal part of the C_1 wavefield (Bensen et al., 2007). The station configuration does therefore not support a data analysis of the enhanced illumination effect on propagation at the same frequency, hence we use simulations.

To avoid a separate discussion on isolated source characteristics at higher frequencies and shorter wavelengths, we adapt our simulation geometry and consider a larger domain with receiver stations in a square 70×70 km^2 grid with 500 m inter-station distance and place sources and auxiliary stations at larger distances. In Figure 2d, the 180 gray indicated boundary sources are organized on circle with 100 km radius. One isolated source that is six times stronger than a boundary source is located in the Northwest of this circle. The choice of the isolated source strength is arbitrary, as long as it leads to an intended weak C_1 wavefield contribution that then dominates the C_2 wavefield after averaging over many auxiliary stations. How dominant an isolated source contribution appears in C_2 wavefields ultimately depends on its relative source strength and the number of auxiliary stations. The 180 purple indicated auxiliary stations are distributed evenly along a smaller circle with 75 km radius.

We simulate C_1, C_2^{all}, $C_2^{endfire}$, and C_2^{angle} wavefields as described above for this adapted simulation geometry and filter the waveforms between 0.2 and 0.4 Hz (Figure 9). The results in the second to fourth row of Figure 9 clearly illustrate the variable partitioning of the isolated and boundary source correlation wavefields that depends on the auxiliary-station selection strategy. The C_2 wavefield snapshots also show that higher-order correlations are characterized by the squared source power spectrum $|N_s|^4$ (Equations 7, 11, 12) compared to the $|N_s|^2$ power spectrum in C_1 (Figure 9a–c). For each correlation wavefield, we estimate group arrival times at the maxima of the waveform envelopes for each receiver-master station pair and convert that to group velocity. The resulting patterns of group-velocity errors are shown in the right column panels in Figure 9. Positive values indicate overestimates that we explain with the interference of the two wavefronts generated by the boundary sources and the isolated source that are illustrated in the three left columns. Schippkus et al. (2022) explain the emergence of this C_1 velocity bias pattern from interference.

Compared to the C_1 wavefield in the top row in Figure 9, all higher-order correlation wavefields shown in the other rows exhibit darker colors, indicative of larger amplitudes, for the isolated source wavefront that propagates across the array. The strongest amplification of the isolated source contribution relative to the circular boundary source component is observed for the all-direction auxiliary station configuration (Figure 9e–g). This results from averaging over significantly more auxiliary stations compared to the other two selection strategies. The group arrival times are then always measured on the wavefront excited by the isolated source because it has the largest amplitudes. For most receiver stations, this results in spurious low travel time estimates and high errors (Figure 9h). At the extreme, the estimate at all stations that the isolated source wavefront reaches at $\tau = 0$ s (Figure 9f) approaches infinity. In contrast, for receiver stations that are inline with the master station and the isolated source we can make accurate estimates, because the correlation wavefield contributions of boundary and isolated sources are in phase along the white indicated region in Figure 9h.

The endfire lobes and the fixed-angle cones configurations also induce enhanced velocity bias (Figure 9l, p). Because endfire lobes for large inter-station distances are fairly narrow, the auxiliary stations selected with both strategies are similar in the far field, which results in the shown similar $C_2^{endfire}$ and C_2^{angle} wavefields in Figures 9i–k and 9m–o, respectively. This indicates that a fixed-angle configuration can be a valid strategy for propagation analysis. Note, though, that the two configurations yield different velocity errors, because the number of utilized auxiliary stations still varies. This results in systematically different relative weighting of the isolated source contribution.

Group velocity errors occur where the two wavefield contributions from isolated and boundary sources interfere. We show that this interference and these errors emerge already in the C_1 solutions (Figure 9a–d) and increase in the C_2 patterns (Figure 9e–p). Independent of the auxiliary station configuration, however, the phase of the boundary source contribution is accurately reconstructed in the C_2 wavefield. That is, C_2 contains the Green's function $G(\mathbf{r}_x, \mathbf{r}_m)$, here, the circular wavefront converging to and diverging away from the master station, in all cases. For strongly unidirectional illumination it is therefore possible to reduce the measurement bias by estimating velocities only from either the causal or anti-causal part of the C_2 wavefield. This utilizes the asymmetric lapse time contribution of isolated sources (Schippkus et al., 2022). At anti-causal lag times the stations in the Southeast are unaffected by interference (Figure 9a, e, i, m), whereas stations in the Northwest are unaffected at causal lag times (Figure 9c, g, k, o). Of course, this geometry depends on the relative position of the isolated sources and the direction of the energy flux.

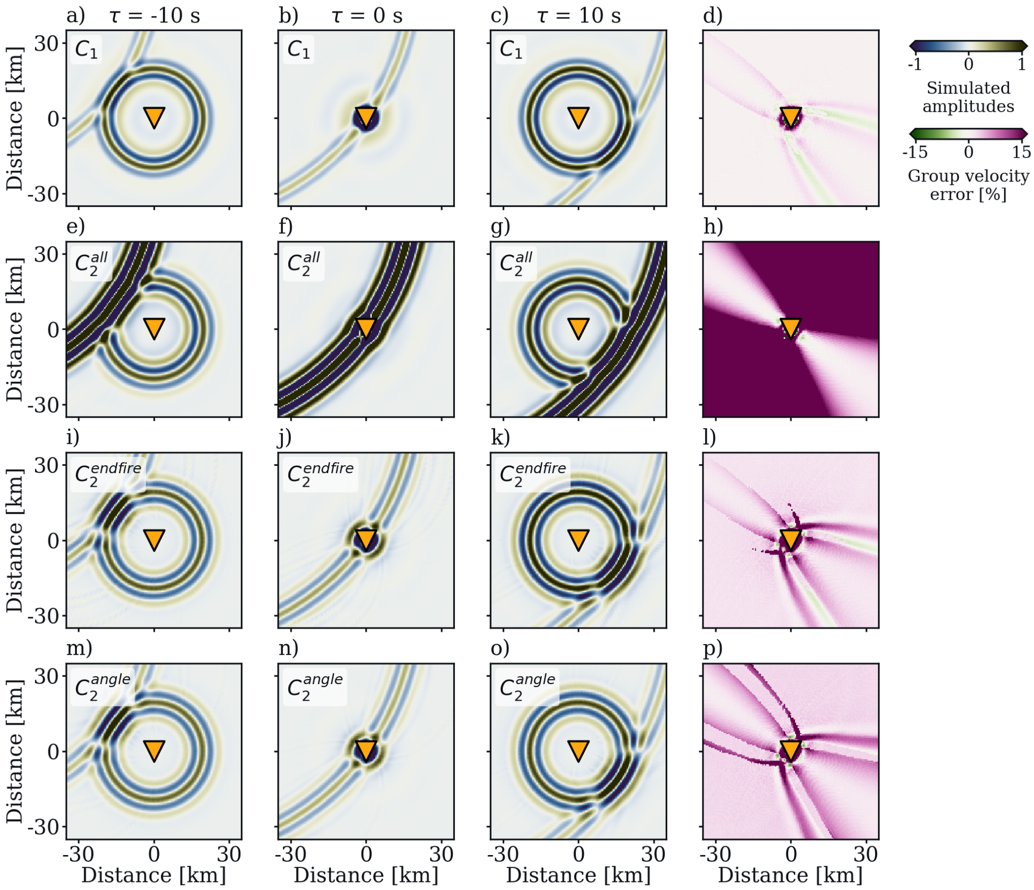

Figure 9. Impact of the isolated source or directional incidence effect on group velocity measurements using the configuration from Figure 2d. (a–c) The 0.2−0.4 Hz simulated C_1 wavefield. All wavefield snapshots are normalized to the peak amplitude of the boundary-source contribution at $\tau = 10$ s. (d) The corresponding error of the group velocity estimates. The interference between different correlation wavefield contributions controls the typical bias pattern (Schippkus et al., 2022). (e–h) Simulated C_2^{all} solutions for the all-direction selection of auxiliary stations and the corresponding group velocity error. (i–l) Results for the endfire-lobe selection. (m–p) Results for the fixed-angle selection. Panels (h), (l), and (p) show that all auxiliary-station selection strategies enhance the group velocity measurement bias associated with the source effects.

Reducing the number of auxiliary stations mitigates the impact of the isolated sources, which is suggested by the observed smaller bias resulting from the endfire-lobe and fixed-angle configurations compared to the all-direction-averaging (Figure 9). However, the signal-to-noise ratio is poorer when averaging over fewer auxiliary stations and even a single auxiliary station does not eliminate the isolated source contribution in C_2 wavefields (Figure 6). Because the group-velocity errors are governed by the reconstructed wavefield interference, estimates of other wave properties including phase velocity or amplitude are similarly affected.

6 Discussion and conclusions

Non-homogeneous noise source distributions and the resulting directional surface wave incidence lead to incomplete approximations of the inter-station Green's functions through cross-correlation. Various strategies (Sabra et al., 2005; Bensen et al., 2007; Seydoux et al., 2017; Fichtner et al., 2017; Sager et al., 2020; Schippkus et al., 2022; Giammarinaro et al., 2023) have been proposed to assess and mitigate the associated bias (Tsai, 2009; Froment et al., 2010; Wang et al., 2016; Schippkus et al., 2022) in the travel times estimates obtained from the C_1 noise correlation wavefield. In contrast to filtering and other invasive strategies, data driven approaches take advantage of the actual signal anatomy and statistics. Higher-order correlations or re-correlation is an alternative data-driven processing strategy for correlation wavefield enhancement (Stehly et al., 2008; Froment et al., 2011; Sheng et al., 2018) that offers the further advantage of integrating asynchronous records.

In this work we use data from a 2D seismic array that consists of nearly 5000 sensors and results from 2D acoustic numerical experiments to study properties of re-correlated wavefields. Our analysis is mostly based on time-domain spatial auto-correlations or focal spots, which differs from the conventional approach to study reconstructed propagating waves. This is motivated by the compact array size compared to the microseisms wavelength, but it demonstrates more generally the effectiveness of focal spot analysis for imaging and wavefield characterization. We find that the re-correlation of direct surface waves from C_1 solutions amplifies the effect of a non-homogeneous noise source distribution and hence directional incidence in the resulting higher-order C_2 correlations. The accuracy of medium velocity estimates from higher-order correlation wavefields can thus be significantly affected by the directional incidence (Figures 8, 9). We conclude that C_2 wavefields are a useful imaging resource, however, the re-correlation of direct waves is not an all-purpose correlation wavefield enhancement solution.

In addition to effects associated with the directional illumination, C_2 wavefields are sensitive to the configuration of auxiliary stations or virtual sources used for stacking

(Figures 2, 6, 7). We test the selection of auxiliary stations from all directions, from endfire lobes, and from fixed-angle cones, which yield the C_2^{all}, $C_2^{endfire}$, and C_2^{angle} wavefields, respectively. All strategies yield isotropic C_2 wavefields in the case of homogeneously distributed original sources. However, they can not mitigate directional incidence effects. The persistent incidence effect is governed by C_2 wavefield contributions that are stationary in phase with respect to the auxiliary stations (Sheng et al., 2018, Equation 7, Figure 6a–d). This means that auxiliary stations have no influence on the reconstructed propagation of wavefronts induced by isolated sources in C_2 wavefields. When the original sources are distributed homogeneously though, propagation does depend on the auxiliary stations and they can be considered as virtual sources (Figure 6e–h). Averaging over several auxiliary stations follows the necessity to improve the signal-to-noise ratio and achieve virtual source distributions that support seismic interferometry. This, however, amplifies the effects of isolated sources due to constructive interference.

In our synthetic propagation experiments the reference wave velocity is accurately estimated only at stations located in line with the master station in the center and the isolated source in the Northwest (Figure 9). Data at all other stations is affected negatively. The significant velocity errors, e.g., shown in Figure 9p, result from interference of the two C_2^{angle} wavefronts seen in Figures 9m–o that are excited by boundary sources and the isolated source, respectively. For the stations in the Southwest and Northeast these wavefronts do not interfere. There, setting constraints on acceptable recovered medium velocities can ensure that velocities are measured on the correct wavefront. For simple distributions of boundary sources and isolated source, these negative effects can be further avoided entirely by locating the isolated source and then estimating the speed either at the anti-causal or causal part of the C_2 wavefield depending on the station pair orientation (Figure 9), which is consistent with C_1 behavior (Schippkus et al., 2022). For our simulation configuration, stations in the Southeast are unaffected by interference at anti-causal lapse times (Figure 9e) and stations in the Northwest are unaffected at causal lapse times (Figure 9g). This also applies for the other C_1, C_2^{all}, and $C_2^{endfire}$ wavefields.

For spatial auto-correlation based velocity estimates using the shape of the focal spot we find that none of the tested auxiliary-station selection strategies allows an accurate recovery of the reference medium velocity (Figure 8). All tested C_2 strategies result in focal spots that yield inaccurate velocity estimates with errors up to 15%. This is because all wavefield components refocus on the master station at $\tau = 0$ s (Figure 9f), which leads to skewed estimates if the assumptions behind the J_0 parametrization are violated (Giammarinaro et al., 2023). In other words, all local stations are affected by wavefront interference. Developments to account for anisotropic illumination for focal spot data regression (Nakahara, 2006; Haney et al., 2012) can be adapted to the enhanced illumination effect in C_2 wavefields studied here by considering the role of auxiliary stations. This can facilitate the robust application of subsurface focal spot imaging for extensive station geometries by integrating data from asynchronously deployed stations.

Higher-order correlations were introduced in the framework of C_3 wavefields that are obtained by the re-correlation of C_1 function coda waves (Stehly et al., 2008). For C_3, the multiple scattered coda waves are targeted to reduce potential source effects associated with inhomogeneous source distributions. Recent results indicate, however, that isolated noise sources with self-correlated source terms can induce spurious correlation wavefield contributions at late C_1 coda lapse times that are erroneously considered as random coda arrivals of scattered waves (Schippkus et al., 2023; Safarkhani et al., 2025). Such spurious C_1 wavefield contributions are likely similarly amplified in the higher-order correlations and thus influence the reconstructed ballistic waves. Our results imply that higher-order correlation is not an all-purpose mitigation strategy but an effective processing tool to analyze directional flux properties and the associated isolated sources. Every iteration of higher-order correlation (Froment et al., 2011), e.g., adding another correlation step by re-correlating C_2 wavefields, further enhances the correlation wavefield contribution that is excited by isolated sources. This allows the targeted analysis of isolated sources, either to better characterize them, to utilize them, or for developing a strategy to mitigate their impact on imaging results.

Open Research

Data availability statement. To support reproducibility of our results, we provide all processing codes as Jupyter notebooks (Schippkus, 2025), as well as the linearly stacked C_1 cross-correlation functions computed from the field data (Schippkus et al., 2025). The raw field data are not public and the rights lie with OMV E&P GmbH. We provide a complete set of Jupyter notebooks that implement all higher-order correlation processing, cover all wavefield simulations, and generate the figures.

CRediT author contributions. Conceptualization: SS, GH; Data curation: SS; Formal analysis: SS; Funding acquisition: SS, GH, CH; Investigation: SS; Methodology: SS, GH, CH; Resources: SS; Software: SS; Validation: SS; Visualization: SS; Writing – original draft: SS, GH; Writing – review & editing: SS, GH, CH.

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