Infiltration drives a Heavy–Tailed Distribution of Combined Sewer Overflow Spill Durations

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Alex G. Lipp[1*] **and Barnaby Dobson**[2]

[1]Department of Earth Sciences, University College London, UK
[2]Department of Civil and Environmental Engineering, Imperial College London, London, UK

Abstract

Combined sewer overflows (CSOs) discharge untreated wastewater into natural water bodies during periods of excess sewer flow, posing environmental risks. Using ~ 4 million CSO spill events recorded by Event Duration Monitors across England (2020–2024), we characterize the statistical distribution of spill durations. We find that while spills exceeding four hours represent only 6.4 % of events, they account for 78.2 % of total spill time, suggesting a disproportionate environmental impact. Formal likelihood ratio tests indicate that this heavy-tailed distribution is well described by a stretched exponential model with a shape parameter of 0.18, a result that is consistent for data from different Water & Sewerage Companies. The fitted model shows periodic residuals correlated with diurnal fluctuations in domestic water use. Hydraulic modelling reproduces the observed heavy tail only when groundwater infiltration is included, suggesting that long-duration spills are primarily driven by infiltration rather than exceptional precipitation events. We further show that the observed scaling can be approximated by the first passage times of a sewer head time-series modelled as fractional Brownian motion. Given the outsized influence of long-duration spills, we recommend explicitly incorporating tail behavior into hydraulic model calibration. We propose the use of the parameters of the stretched exponential distribution as metrics for this calibration and for assessing CSO performance more widely.

Keywords Sewage Pollution · Heavy tailed distributions · Stretched Exponential · Event Duration Monitoring · Combined Sewer Overflows

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1 Introduction

Combined sewer systems transport wastewater from domestic, industrial & commercial sources alongside run-off from precipitation (Reichard 2024). This combination of wastewater and stormwater is transported to treatment plants before being returned to the environment or recycled. Under extreme conditions, the flow through the system can breach the network capacity. When this occurs, the excess flow is discharged into the environment at select locations called combined sewer overflows (CSOs). Whilst CSOs are necessary to prevent overflow upstream in the sewerage network, they are associated with negative environmental impacts. By discharging untreated, albeit often diluted, waste-water, CSOs are associated with, *inter alia*, oxygen depletion, sewage-fungus, contamination with pharmaceuticals, microplastics, and bacteria harmful to human health (Soller et al. 2010; Riechel et al. 2016; Munro et al. 2019; Woodward et al. 2021; Albini et al. 2023; Usher 2023; Zan et al. 2023; National Engineering Policy Centre 2024; Uhlhorn et al. 2025). Taken together, excessive CSO discharge results in ecological damage and reduced trust in the safety of water bodies by recreational users. Managing CSO operation is therefore a major goal of environmental policy globally, such as the 1991 European Urban Waste Water Treatment Directive 91/271/EEC and the UK's 2021 Environment Act.

A challenge to CSO management is that their operation is frequently not directly monitored resulting in limited data on spill volume, duration and frequency excepting localised studies of monitored CSOs (e.g., Bizer and Kirchhoff 2022; Jalbert et al. 2024). Instead, hydraulic modelling of sewer networks is typically calibrated to select monitoring points (Kleidorfer et al. 2009), and used to simulate spill frequency & volume under different scenarios (e.g., Abdellatif et al. 2015).

In England, The 2021 Environment Act mandated the deployment of Event Duration Monitors (EDM) on CSOs nationwide. EDMs record when the sewer head exceeds the known spill-level of a CSO. In this way, EDM records when, and for how long, a CSO spills into the environment. This monitoring has resulted in ~ 15,000 CSOs having continuous data on spill frequency and duration. Such dense monitoring of CSO operations is unusual, and beyond its regulatory purposes, has provided an unprecedented opportunity to understand spill drivers which are traditionally poorly constrained (e.g., Giakoumis and Voulvoulis 2023).

Since this new data-gathering, EDM data on spill durations by water & sewerage companies (WASCs) in the UK has featured prominently, and generally critically, in national media, and has been the topic of significant political discussion (e.g., Environmental Audit Committee 2022). A representative headline in *The Guardian* reports: '*Nearly 4 million hours of raw sewage dumped in England's waters last year*' (Laville and Goodier 2025). Whilst such eye-catching statistics have raised the salience of CSOs as an environmental stressor, it is less clear how to meaningfully interpret these values of spill duration, as there has been a lack of independent statistical analysis on EDM data.

In this study, we characterise the statistical distribution of CSO spill durations using England's very large catalogue of recorded spills. Our motivation is threefold. First, by defining the underlying population from which an individual spill is drawn, we can inform the interpretation of EDM data. i.e., is the 4

*correspondence: a.lipp@ucl.ac.uk

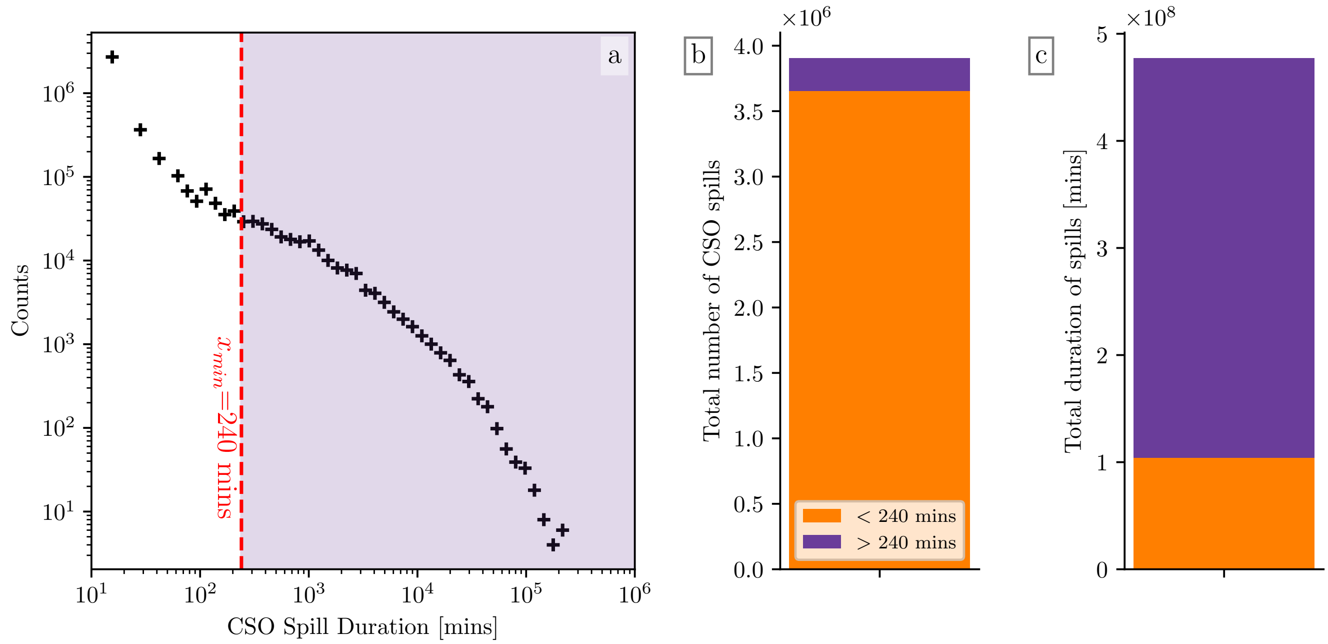

Figure 1: a) Empirical distribution of CSO spill durations from 3,905,013 events recorded by EDM between 2020 and 2024 in England. Durations are rounded up to nearest integer multiple of 15. A logarithmic spacing of bins is used. Note break in scaling behavior at a duration of 240 minutes. Purple area indicates the 'tail' of the distribution, relative to the 'body' in orange; b) Barchart of total number of spills present in the body (orange) and tail (purple); c) Barchart of total duration of spilling present in the body (orange) and tail (purple). Note greater importance of the tail in controlling total spill duration, which is likely linked to spill volume, and hence, environmental impact.

million hours of spilling made up of a handful of long-events, or very many short-events? These contrasting scenarios have very different implications for mitigation. Second, by exploring how the empirical distribution of spill durations can(not) be reproduced using process-based models of sewer hydraulics, we will improve our understanding of the drivers of CSO spills. Finally, our calibrated statistical model for spill durations can be used as part of surrogate generators of CSO forcing in catchment management models (e.g., Liu et al. 2024).

2 Data

EDM stop-start data was obtained via Environmental Information Regulation requests from eight of the nine WASCs operating in England. In general, data was provided covering between 2020 and 2024, but for some companies some complete years were not provided. For the remaining WASC, Thames Water, stop-start data was obtained from their application programming interface (`https://data.thameswater.co.uk/s/`) using the `POOPy` software (Lipp et al. 2025). EDM sensors often record and/or transmit data on discretised time intervals, most often to two or 15 minutes. As a result, spill durations that are integer multiples of these intervals are over-represented within these datasets, as depicted in Supplementary Figure S1 which shows the frequency distribution of raw spill durations. To avoid these artefacts influencing our results we round each spill's duration up to the nearest integer multiple of 15. This value was chosen as 15 minutes is the longest discretisations time interval observed in our dataset. As such, choosing this value results in data with no significant observable artefacts in the rounded data (For instance, comparing Figure 1a to Figure S1). Repeating our analysis without rounding has only a minor impact on fitted parameter values (Figure S2).

Anglian Water's 2024 data was excluded from study as it follows a different reporting convention, whereby spill durations exceeding 24 hours are discretised into repeated 24-hour events plus a final remainder (e.g., a 49 hour duration spill is recorded as three spills, two of which are 24 hours in duration and one 1 hour spill). In total, we have a compilation of 3,905,013 CSO spills, the frequency distribution of which is shown in Figure 1a. Consistent with Burnett and Bolton (2025), these data follow a significantly right skewed distribution.

For the remainder of this study we will focus on the 'tail' of this distribution defined here as any spill lasting longer than 240 minutes ($N = 249,493$). This threshold is based, first, on a qualitatively observed change in the scaling behaviour of spill durations which is observed roughly in the range 100 - 400 minutes (Figure 1a). Second, and to choose our threshold value more precisely and objectively, we employ the parameter stability approach described by Coles (2001b). This approach fits a given distribution to the tail of a distribution above a range of different threshold values. The chosen 'optimal' threshold value is the one above which the fitted parameters become stable. Applying this approach to our dataset (See Figure S3 and description in Supplementary Information for further details) suggests a value of 240 is appropriate, and that the precise value has only a minor impact on our results.

Table 1: Table of different heavy-tailed distributions considered in this study, modified from Table 1 of Clauset et al. (2009). The probability density function for each function, PDF(x), is given by $Cf(x)$, where C is a constant determined by the minimum x value (in this study, $x_0 = 240$), such that $\int_{x_0}^{\inf} Cf(x)\mathrm{d}x = 1$

Distribution	f(x)	C
Power Law	$x^{-\alpha}$	$(\alpha-1)x_0^{\alpha-1}$
Exponential	$e^{-\lambda x}$	$\lambda e^{-\lambda x_0}$
Log-Normal	$\frac{1}{x}\exp\left[-\frac{(\ln x-\mu)^2}{2\sigma^2}\right]$	$\sqrt{\frac{2}{\pi\sigma^2}}\left[\mathrm{erfc}(\frac{\ln x_0-\mu}{\sqrt{2}\sigma})\right]^{-1}$
Stretched Exponential	$(\lambda x)^{\beta-1}e^{(-\lambda x)^{\beta}}$	$\beta\lambda e^{(\lambda x_0)^{\beta}}$

Whilst only 6.4 % of spill *events* are longer than 240 minutes in duration, we find that they account for 78.2 % of total spilling *time*. Making the explicit assumption that spilling time is likely related to overall environmental impact, we argue that focussing on these low-frequency, high-magnitude events is justified.

3 Methods

3.1 Statistical Modelling

We now identify the probability distribution that best fits these data. We consider four common heavy-tailed distributions for this purpose: power law, exponential, log-normal and, stretched exponential distributions. The form of the probability density functions (PDFs) for these distributions are given in Table 1. These functions have been successfully used to describe diverse heavy-tailed probability distributions across multiple fields such as the populations of cities in the United States, the intensities of solar flares, earthquakes, and rainfall, temperature variations over 100,000+ year timescales and the energy emissions of galaxies, so present a reasonable set of distributions to consider initially (Laherrère and Sornette 1998; Wilson and Toumi 2005; Clauset et al. 2009).

Following Clauset et al. (2009) we opt for a maximum likelihood approach to fitting PDFs to our data. We use the `Python` implementation of these methods developed by Alstott et al. (2014). To compare which of these candidate distributions is optimal, we perform a series of likelihood ratio tests (Clauset et al. 2009). If we consider two candidate distributions which give the probability of a given observation x occurring as $p(x)$, the likelihood of that distribution $L = \prod_{i=1}^{n} p(x_i)$. Consequently, given two distributions with likelihood L_1 and L_2 we can calculate the log-likelihood ratio $R = \ln\left(\frac{L_1}{L_2}\right)$. If R is positive, the first distribution is more likely and *vice versa*.

We note that an assumption underlying our distribution fitting procedure is that individual CSO spills are statistically independent of each other. In reality, this is unlikely as CSO spills are highly clustered temporally during the winter months, or periods of high rain-fall (e.g., storms). This clustering violates the independence assumptions of these distributions, which could introduce biases into chosen parameter values. Fitting distributions to such dependent variables is possible, for instance by using 'declustering' approaches that take the maximum value within each cluster (e.g., Coles 2001a), although, we argue that more advanced methods, whilst important, are beyond the scope of this particular contribution.

3.2 Hydraulic Modelling

To recreate our empirical spill duration distributions from first principles we generate a suite of synthetic CSO spills with the widely used hydraulic modelling software, Storm Water Management Model (SWMM; Rossman 2010). We use three distinct models from three different locations. Kingston Bagpuize, UK, was selected because it is a wastewater catchment with 244 hours of spills in 2021 on average lasting 6 hours, representing a 'typical' Thames Water spill site. Additionally it is a small catchment conducive to running long hydraulic simulations. Chesham, UK, was selected because it is a wastewater catchment with 1814 hours of spills in 2021 on average lasting 15 hours, making it one of Thames Water's most severe spill sites. It is a large and complex catchment currently under investigation for infiltration into the sewer network. Bellinge, Denmark was selected because sewer network data is published in SWMM model format as part of an openly available dataset (Nedergaard Pedersen et al. 2021). The use of these three models examines how results may be location specific or attributable to unique model features. There are three important differences, besides location, between these SWMM models (Table 2).

The first was that groundwater infiltration processes were included in the Chesham model but not in the Bellinge or Kingston Bagpuize models. Infiltration to the Chalk aquifer below Chesham was modelled using a Modified Green-Ampt equation, with a power law stage-discharge relationship to describe lateral flow from the aquifer into pipes. The second difference was the method used to derive the model network and parameters. The Bellinge model was derived from a sewer network survey. Meanwhile, the Chesham and Kingston Bagpuize models were created using the SWMManywhere software, an-open source sewer network generator (Dobson et al. 2025a,b). The third difference was of the data types used by both models. The Bellinge model contained a wide variety of edge types (conduits, orifices, weirs, and pumps with rating curves) and cross-section types (circular, rectangular, closed, open) as surveyed, however, ¿99 % of nodes were treated as junctions, neglecting any storage they may provide. The Chesham and Kingston Bagpuize models contained only circular conduits and ideal pumps, owing to the synthetic nature of the model, however, Bellinge represented nodes as storages as befitting the storage provided by manholes.

We convert each simulated sewer head at the outfall into a series of CSO events by considering a spill level at the 90th percentile level for each modelled site. This threshold results in a spill time of 10 % which is slightly higher than the fractional spill time of an average CSO in England (2.9 %; Burnett and Bolton 2025). This higher value was chosen to allow the models to produce a statistically significant number of CSO spills to accurately fit distributions. Our EDM data from ~ 20,000 monitors over four years results in a total monitored timespan of nearly 100,000 years. Modelling this timespan is not feasible, and so, some trade-offs are required to produce a meaningful synthetic catalogue of spills. Nonetheless, we recognise that this parameter

Table 2: Comparing the model parameterisations between the Bellinge, Kingston Bagpuize & Chesham SWWM models

Feature	**Bellinge**	**Kingston Bagpuize**	**Chesham**
Routing	Dynamic Wave	Dynamic Wave	Dynamic Wave
Simulation Period	2010-01-06 – 2016-01-06	1994-02-28 – 2022-07-23	1998-01-01 – 2023-12-25
Area (km^2)	1.7	1.5	12
Node Type	Junction	Storage	Storage
Edge Type	Conduit, Weir, Orifice, Pump	Conduit, Pump	Conduit, Pump
Dry Weather Flow	Diurnal	Diurnal	Diurnal
Groundwater Infiltration	None	None	Modified Green-Ampt
Head Loss Model	Darcy-Weisbach	Hazen-Williams	Hazen-Williams
RDII	None	None	None
Mean annual precipitation (mm)	1400	860	1100
Sewer network creation	Survey	SWMManywhere	SWMManywhere

choice results in non-actualistic behaviour. A sensitivity test of this value is discussed below.

The Chesham model was run once, whilst the Kingston Bagpuize was run multiple times with distinct configurations. The Bellinge model was run once, but outfall data was extracted at points across the network representing the subsystems describe by Farina et al. (2023). We collate the 'recorded' spill durations from each model run into a single set, which simulates a suite of observed CSO spills from a small sewer network.

4 Results

4.1 A stretched exponential model for CSO spill durations

The empirical distribution is compared against the best-fitting form of the candidate distributions in Figure 2. The stretched exponential distribution provides the best visual fit to the data. The exponential distribution significantly under-predicts the likelihood of long-lasting spill events (i.e., > 1 week) whereas the log-normal and power law distributions over-predict these events. The likelihood ratio tests confirm these observations; when compared with each of the log-normal, power law and exponential distributions the stretched exponential is preferred with respective R values of 18.3, 49.2, & 88.5. In each case $p < 0.05$, with specific p-values given by $ERFC(x)$, for x = 12.9, 34.8, and 62.6 respectively, where $ERFC$ is the complementary error function. An alternative method for assessing the goodness-of-fit for a distribution to data is the Kolmogorov-Smirnov (KS) distance, which is the maximum distance between the empirical cumulative distribution function of the data, and that of the fitted model. The KS distance for the stretched exponential distribution is lowest at 0.0173 relative to 0.0186, 0.0509 and 0.2576 for the log-normal, power law and exponential distributions respectively. We therefore conclude that the stretched exponential is the best model for describing the probability of a CSO spill of a given duration.

The stretched exponential is described by a scale parameter λ and a shape parameter β (Table 1). Of these, β, which takes values between 0 and 1 is most important in controlling the

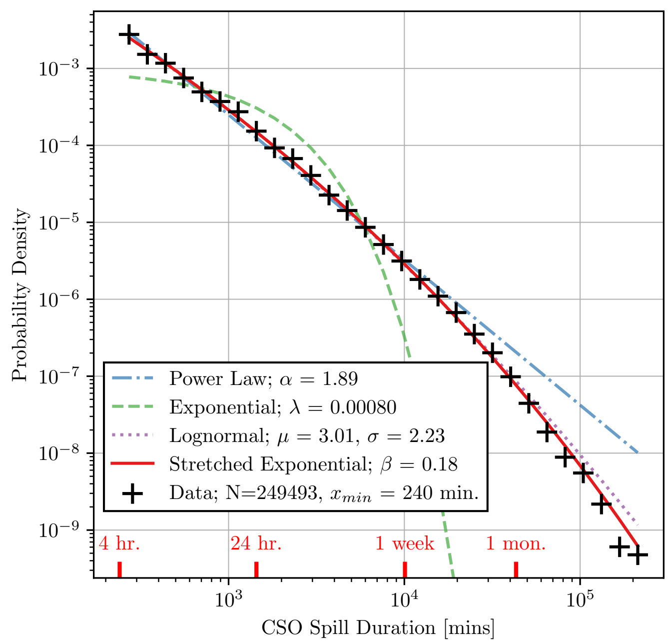

Figure 2: Comparing empirical probability distribution of CSO durations (black crosses) against four candidate models. Bin-widths for the empirical probability distribution are logarithmic-linearly spaced. Note logarithmic horizontal and vertical axes. See Table 1 for description of model parameters.

form of the distribution. When $\beta = 1$, the function reduces to the simple exponential distribution. However, when $\beta < 1$ the probability decays increasingly slowly which 'stretches' the distribution, indicating greater frequency of high-magnitude events. Our finding of a stretched exponential with $\beta \sim 0.18$ is robust to subsetting of our data-set, for instance breaking up our dataset into spills from different WASCs which operate in distinct hydrological catchments (Figures 3 & 4). All WASC datasets show a β in the range 0.1 – 0.3.

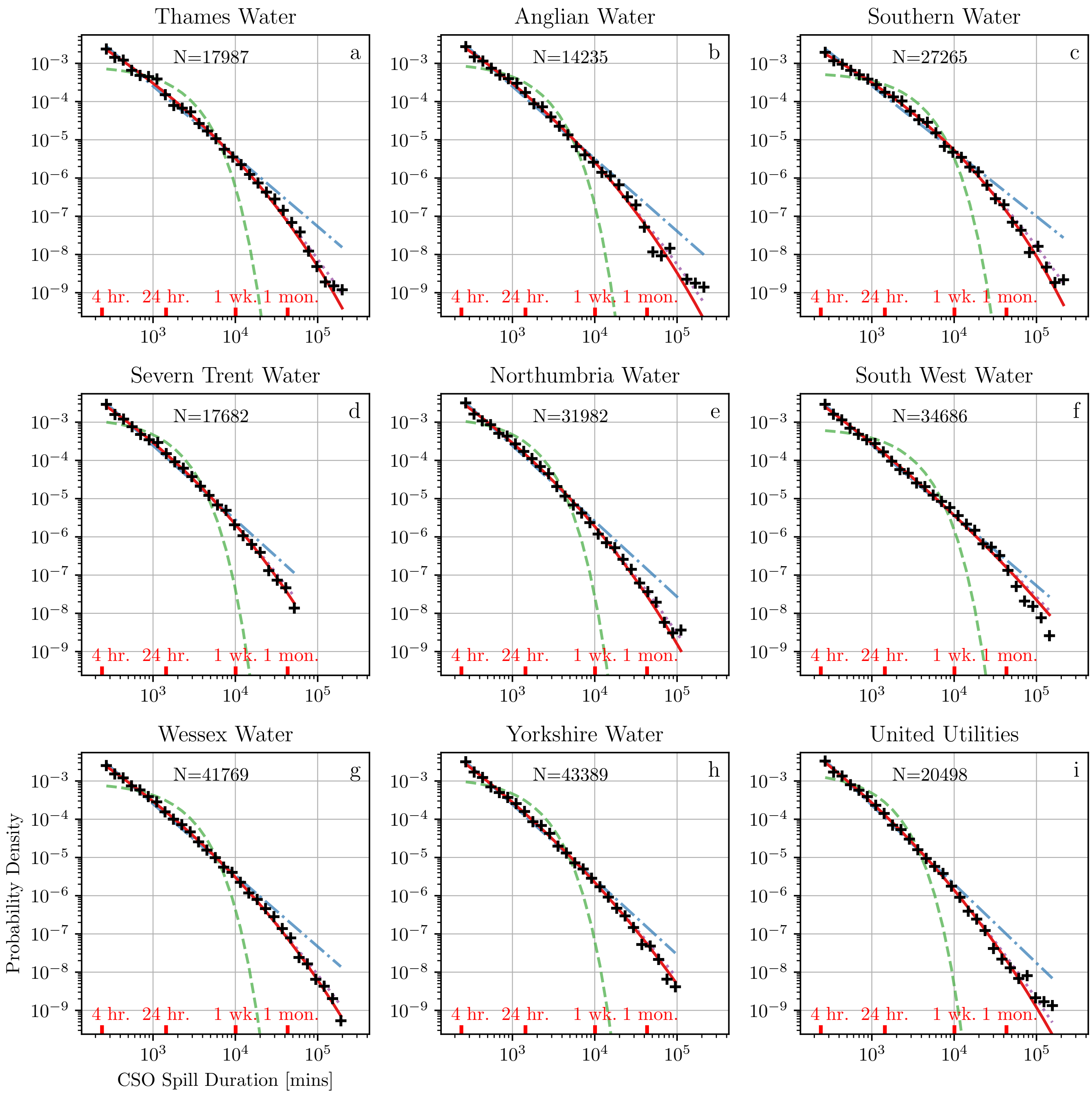

Figure 3: a – i) Empirical and fitted probability distributions for CSO durations for individual Water and Sewerage Companies (WASC) in England. See Figure 2 for legend of different statistical models. Shape parameter, β, values for each WASC are shown in Figure 4.

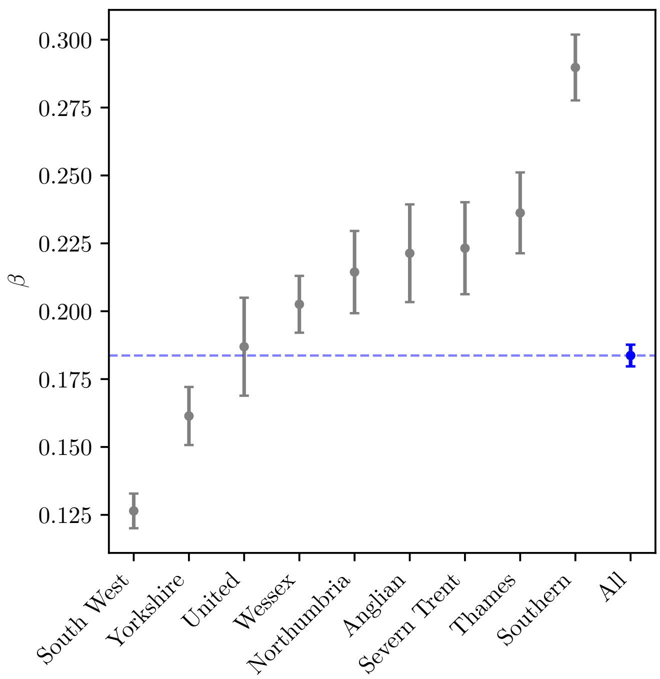

Figure 4: Range of observed β values for each WASC approximately all distributed at ~ 0.18. 95 % (2 σ) uncertainties on β generated via 200 bootstrap resamples of the underlying distributions. Horizontal dashed line shows β for overall compilation (Figure 2).

4.2 *Deviations from statistical model*

Whilst this distribution provides a very good fit to the data shown in Figure 2 there are notable deviations when viewed on a linear duration axis as shown in Figure 5. We observe periodic variability of observations above and below the fitted trend with approximate period of 24 hours. We interpret this variability as being driven by diurnal variations in sewer flow driven by domestic water usage (e.g., Figure 5c). Daily water usage fluctuations mean that, generally, in each 24 hour cycle there is always one nadir sewer level. Consequently, the probability of any given CSO spill being truncated 'early' at this daily nadir is elevated. In the example sewer head time-series in Figure 5c, this nadir occurs at $\sim$ 2 am every day. The consequence of this reliable daily minimum is that the probability of a CSO being of duration just less than integer multiples of 24 hours is, $\sim$ 20 % (according to our results) higher than the background (Figure 5b). Consequently, we tentatively propose that daily low-flows could be actively exploited to truncate long-lasting spills via operational adjustments as these periods are already passively performing this role.

5 Discussion

5.1 *Infiltration control on low-frequency high-magnitude spills*

We now seek to reproduce this distribution from hydraulic models of sewer flow. In Figure 6, the empirical distribution is compared to the probability distribution of spills returned by each of our SWMM simulations. The SWMM models from Bellinge and Kingston Bagpuize also follow a stretched exponential distribution but the β value is notably higher, at 0.61 (Bellinge) and 0.53 (Kingston Bapguize). The consequence is that the distribution of spills from these SWMM simulations have much greater curvature predicting fewer long-duration spills than observed in reality. By contrast, the Chesham SWMM model, whilst less well fitted due to a smaller catalogue of spills, produces a heavier tailed distribution approaching that of the observed data. To test the sensitivity of our results to our chosen spill threshold value, we repeat our analysis with thresholds of 80, 85, and 95 %. These results are shown in Figures S4–6 in the Supplementary Information. Using lower thresholds does not impact our results. However, choosing a value above our chosen thresold of 90 % does significantly reduce the number of long duration spills from the Chesham model. However, the much reduced number of total recorded spills (379 relative to 782; Table S1 in Supplementary Information) from this change in threshold significantly reduces statistical significance & stability of this outcome. Consequently, whilst this indicates some sensitivity to the chosen threshold value, we use our central value of 90 % as this is the highest, most actualistic, threshold value where fitted parameters are consistent.

Given the different parameterisations of these models (Table 2) we argue that these discrepancies are best explained by the presence of groundwater infiltration in the Chesham model but not in the Kingston Bagpuize and Bellinge models. Notably, the shape of the Chesham and Bellinge distributions are similar to the probability distribution of heavy rainfall. Wilson and Toumi (2005) show that the tail distribution of daily rainfall also follows a stretched exponential with $\beta \approx \frac{2}{3}$, a scaling that can also be predicted from the physics of rain formation. If CSOs are purely driven by heavy rainfall events, and assuming a linear relationship between rainfall intensity and spill duration, this would result in a distribution with β equal to ~ 0.67, as broadly observed in the Kingston Bagpuize and Bellinge models. We hypothesise then that these models are successful in modelling CSO spills in response to heavy rainfall, but are missing an underlying process that can produce spills days, or longer, in length. By contrast, the Chesham model includes groundwater infiltration which would allow for longer duration rises and falls in head, creating the conditions required for long-duration spills. To further investigate the potential role of groundwater we repeat the Chesham model run with all parameters indentical, with the exception that groundwater infiltration into the sewer network is turned off. The results from this model run are shown in Figure 6, and show a notably lighter tail than the identical run which does include infiltration. This controlled trial supports our hypothesis that infiltration drives the heavy-tail.

The discrepancy between the models that include groundwater (Chesham) and those that don't (Bagpuize, Bellinge) is striking. A spill lasting one day (24 hours) under the Kingston Bagpuize model is 100× less likely than suggested by empirical data, whereas is approximately well matched by the Chesham model. Whilst acknowledging the uncertainties in extrapolating beyond the range of the SWMM spill simulations, a CSO spill lasting a week is $\sim$ 1 billion times less likely in the Kingston Bagpuize distribution than in the empirical distribution. Our empirical dataset contains 1691 spills lasting 6 – 8 days in length, whereas the results from the Kingston Bagpuize & Bellinge SWMM

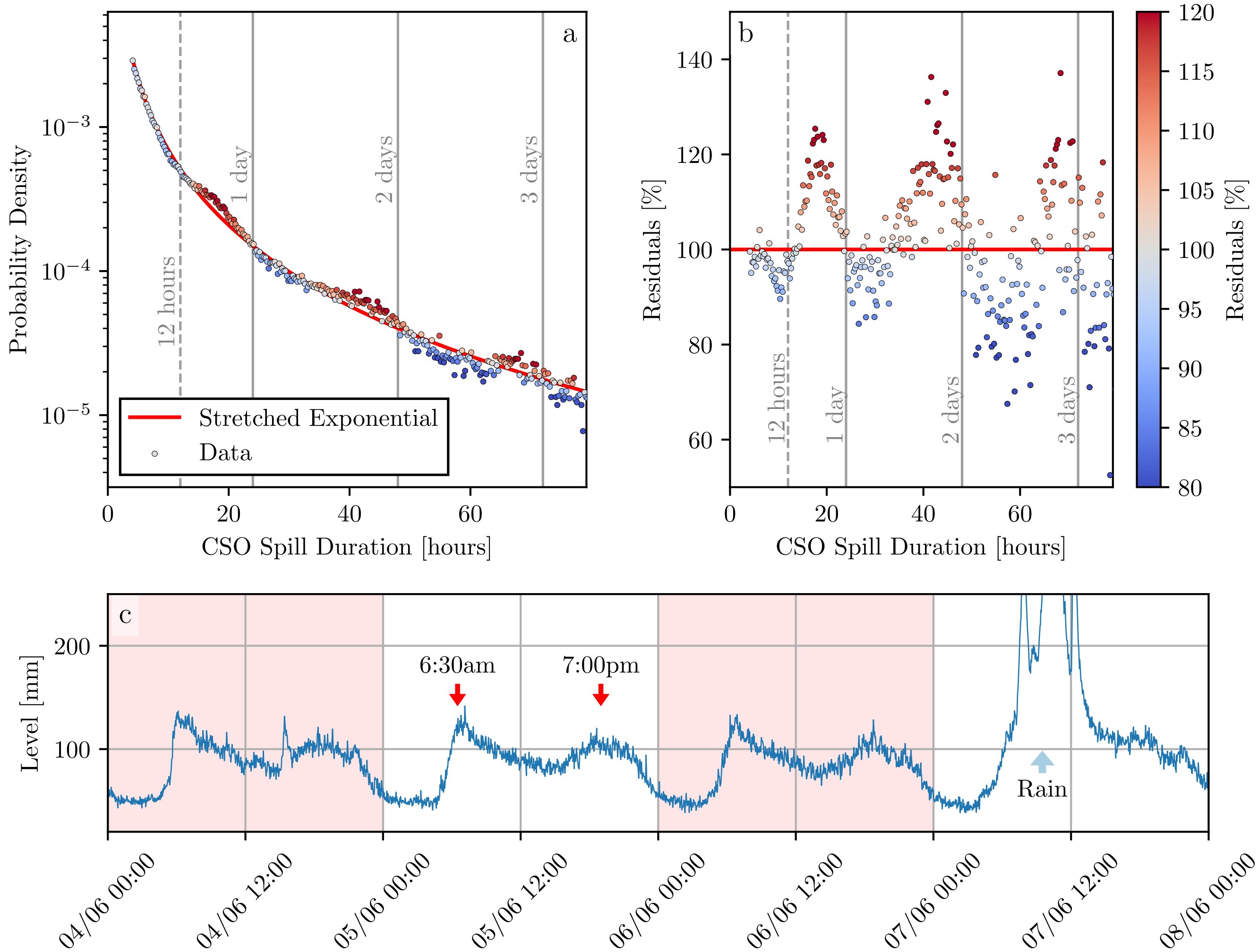

Figure 5: a) Comparing best fitting stretched exponential distribution (red line) against empirical data with linearly spaced bins 15 minutes. Note linear horizontal axis and logarithmic vertical access. Colour of points corresponds to difference between empirical probability and fitted probability at same duration (see colour bar in panel b). b) Plot of residuals between predicted and observed probability density for CSO spills of different durations. Red lines indicates best-fit stretched exponential model (panel a). Colour of points is relative residual. Note periodic variations around modelled line with period of 24 hours. c) An example sewer head time-series provided by Wessex Water from Monitor E5175 Bath, England (see `https://marketplace.wessexwater.co.uk/dataset/sewage-pumping-station-run-stop-example-data`). Pink shading indicates 24 hour periods. Red arrows indicate peak in flow associated with high domestic water consumption. Blue arrow indicates higher sewer head associated with rain.

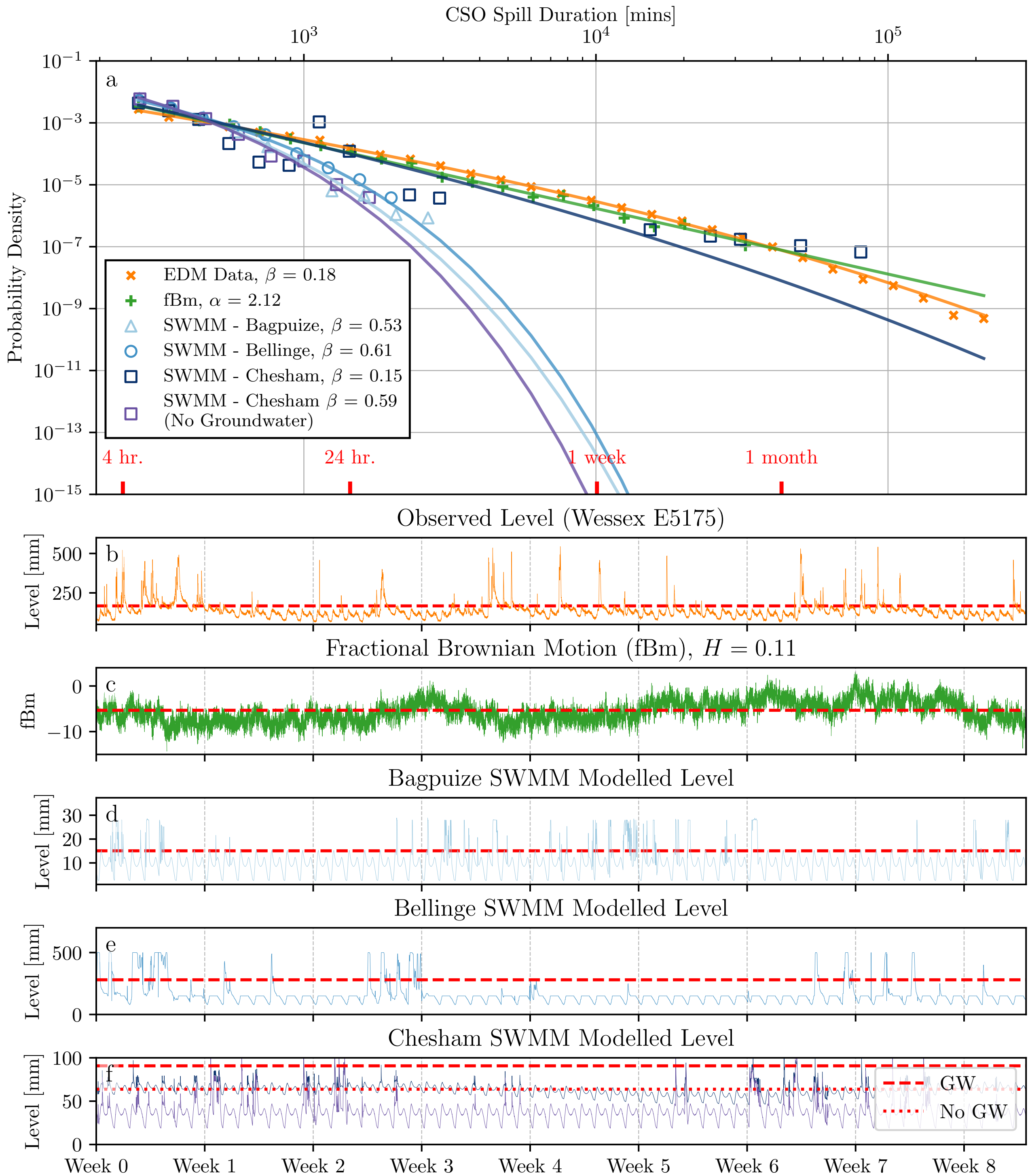

Figure 6: a) Comparing empirical probability distribution of CSO spill durations (orange crosses) against i) a fBm time-series crossing a threshold (green plus symbols) ii) simulated spill durations from three SWMM hydraulic models (blue open shapes). See body text for details of different SWMM model setups. Continuous lines indicate best fitted stretched exponential model to data b) An example time-series of real sewer head data from the E5175 level monitor operated by Wessex Water in Bath, England. See caption of Figure 5 for details. Red-dashed line indicates chosen spill level for CSOs. c) A time-series generated by a fBm process with $H = 0.19$. d) Example of simulated sewer head from the Bagpuize SWMM model for the same length of time as panel b. e) Same as panel d but from the Bellinge SWMM model. f) Same as panels d but for the Chesham SWMM model. GW = with groundwater; No GW = without groundwater. Note the week long fluctuations in sewer head in Chesham SWMM model when groundwater is present, not present in the model runs without groundwater.

models, when applied to a dataset of our size predicts 0 such spills would occur (specifically, 1.2×10^{-7}), i.e., practically speaking it should never occur. Given then importance of including infiltration processes in creating realistic spill duration distributions, we argue that accurately representing groundwater and other infiltration processes in sewer networks is essential for sensibly modelling the environmental impact of CSO spills.

5.2 *An approximate generative mechanism for heavy-tailed spill duration distributions*

Laherrère and Sornette (1998) demonstrate that stretched exponential distributions can be generated by a multiplicative chain of random variables. Given that our shape parameter appears consistent across diverse hydrological catchments (e.g., Figure 3), identifying whether such a fundamental generative mechanism exists may be fruitful, but none could be identified by the authors at the time of writing. However, we do propose a generative mechanism that *approximates* the observed heavy-tailed distribution.

For spills up to one month in duration, a power law distribution with $\alpha = 1.89$ presents a reasonable approximation of the data, albeit over-estimating longer-duration events (Figure 2). Let us hypothesise that sewer head is a time-series that can be described by fractional Brownian motion (fBm; Mandelbrot and Van Ness 1968). A fBm is described by one parameter, the Hurst exponent, H, which controls the long-range 'memory' of the variable. If fBm describes a time series $B(t)$, the covariance structure of B is given by:

$$E[B(t)B(s)] = \frac{1}{2}\left(|t|^{2H} + |s|^{2H} - |t-s|^{2H}\right).$$

H ranges from 0 to 1, and when $H = 0.5$, the motion is a random walk (i.e., increments are uncorrelated). When $H < 0.5$, the time-series has the property of mean reversion (i.e., subsequent increments are anti-correlated), whereas when $H > 0.5$, the time-series shows trending or persistent behaviour (i.e., subsequent increments are positively correlated). Whilst fBm has not, as far as we are aware, been widely used to describe sewer heads, we note that it has previously been used to describe the discharge of rivers, a superficially analogous system to sewer networks (e.g., Turcotte 1997; Koscielny-Bunde et al. 2006). Indeed, the name of the Hurst exponent is derived from the work of Hurst (1951) studying historical river flows in the Nile.

A CSO spill occurs when the level of the sewer, described by the fBm $B(t)$, exceeds some threshold. The duration of that spill corresponds to the time it takes for the sewer head to cross back below that same threshold. In other-words the distribution of CSO spill durations corresponds to the distribution of what are known as 'first passage times' for the underlying stochastic sewer head variable (Redner 2001). Ding and Yang (1995) demonstrate that for fBm, the distribution of first passage times is described by a power law with $\alpha = 2 - H$. Given our fitted α value this would suggest that level in a sewer can be simulated with a $H \sim 0.11$, i.e., a time-series with mean-reversion behaviour.

Figure 6 contains the results of a synthetic actualisation of this mechanism. Using the method of Davies and Harte (1987) implemented by Christopher Flynn (2019), we generate an instance of fBm with $H = 0.11$, lasting for the equivalent of 10 years at intervals of 1 minute. The generated fBm time-series is shown in comparison to actual sewer head data and the outputs of the hydraulic SWMM models in Figure 6b–f. As theoretically predicted (i.e., Ding and Yang 1995) the spill durations follow a power law distribution with slope ~ -1.9. This simple model provides a reasonably accurate approximation for durations up to two weeks in duration, beyond which it is an over-estimate. Whist this model is non-physical it is consistent with our findings from the hydraulic modeling that groundwater infiltration is important at controlling the heaviness of the spill duration distribution. Notably, the fBm time-series allows for fluctuations in sewer head that exceed one day in length (Figure 6c), which are not represented by the Bellinge & Kingston Bagpuize hydraulic models (Figure 6d–e), but can be recreated by groundwater infiltration as in the Chesham model (Figure 6f).

5.3 *Implications for hydraulic modelling and the interpretation of event duration monitoring*

The data presented in Figure 6a suggests that infiltration into sewers is a major driver of low-frequency, long-duration spills, which account for a significant proportion of total spilling time nationwide. In Chesham, it is likely that this infiltration originates in the phreatic zone due to its location atop a major chalk aquifer. Indeed, in Thames Water's 2021 EDM data, spills above aquifers were $\sim$ 30% longer than in areas with a rock character described as 'essentially no groundwater' (British Geological Survey 2020). However, over half of Thames Water's longest duration spills were still in areas with essentially no groundwater. In these cases we expect instead infiltration to be originating in the vadose zone, often referred to in urban drainage as rainfall driven inflow and infiltration (RDII) (Zeydalinejad et al. 2024).

In England, CSOs are only expected to operate under 'exceptional circumstances' generally interpreted as heavy rainfall or rapid snowmelt (Department for Environment Food and Rural Affairs 2025b). However, we find that long-duration CSO spills are likely instead driven by infiltration, not directly in response to an exceptional event. This finding aligns with those of Giakoumis and Voulvoulis (2023), who report that limited capacity of the sewer network, rather than exceptional circumstances, are the dominant driver of English CSO spills.

Our results highlight that CSO spill predictions from hydraulic models are highly sensitive to hydraulic processes beyond rainfall. This is concerning because the latest UK guidance on hydraulic modelling for drainage and wastewater management focuses almost exclusively on storm event return periods, with minimal consideration of infiltration (Department for Environment Food and Rural Affairs 2025a). The guidance states that 'the majority of [...] sewerage systems will require hydraulic models', that 'all overflows must be represented', and that suspected infiltration should trigger a survey (Titterington et al. 2017; Department for Environment Food and Rural Affairs 2025a). Taken together, these requirements imply that developing hydraulic models for all wastewater catchments will be extremely costly if surveys are needed to accurately constrain infiltration, yet without such data, the models will be insufficient at capturing long-duration spills.

Furthermore, we propose that poorly calibrated hydraulic models that fail to accurately simulate groundwater processes may significantly underestimate the likelihood of long-duration spills, with serious implications for catchment management. For example, a catchment source apportionment model of contaminants would substantially underestimate contributions from sewer overflows unless the tail of the spill duration distribution is properly characterised (Clist et al. 2015). Such mischaracterisation could have major consequences for the use of hydraulic models in managing the downstream impacts of CSOs (see, e.g., Environment Agency 2018).

Predicting the ecological impact of temporally varying stressors remains a major challenge (Jackson et al. 2021). Nonetheless, we propose that low-frequency long-duration CSOs may represent a qualitatively distinct ecological stressor than short-duration CSOs in response to high run-off. For instance, spills that exceed a day in length begin to exceed the generation times of the smallest organisms within a fluvial ecosystem (e.g., phytoplankton). In addition, there is less 'recovery time' for the ecosystem to recover (Albini et al. 2024). As a consequence, the CSO pollution stressor (e.g., elevated oxygen consumption, nutrient levels, and other contaminants) may transition from being 'discrete' to 'continuous' in nature, with impacts that could propagate, lagged, further up the ecosystem (Jackson et al. 2021). Moreover, such long-duration spills may no longer coincide with the period of elevated river flow associated with heavy-rainfall, resulting in a more limited effect of dilution increasing the ecological impacts of wastewater pollution (Rice and Westerhoff 2017). Given these potentially elevated ecological impacts of long-duration spills, ensuring that they are adequately represented in hydraulic models is essential.

Notwithstanding the above, we emphasise that the data provided by EDM is highly limited with respect to constraining environmental impact. Ultimately, the best predictor of environmental impact from any point source pollutant is volumetric discharge, coupled with observations of contaminant concentration or intensity in the effluent. EDM, however, provides no direct information on either of this parameters, only duration. As a consequence, any inferences on environmental impact from EDM data relies on the assumption that duration is correlated with environmental impact. This assumption is an explicit limitation of the results presented here, and an implicit one in the use of EDM more widely. Existing studies have identified a possible non-linear relationship between spill-frequency and water quality impact (Lau et al. 2002). We argue that further research relating CSO spill duration to volume and effluent quality would be highly valuable.

Moving forwards, we argue that hydraulic model validation must explicitly interrogate the tail-distribution of predicted CSO spills. A 2024 report by UK Water Industry Research (UKWIR; Alasdair Fraser et al. 2023) performed such a validation exercise, however, only the total *number* of CSO spills predicted by models was evaluated against EDM data. Such a comparison is dominated by the body of the distribution and as such is largely insensitive to the tail. Consequently, any model validated in this way could significantly under (or, over-) estimate the likelihood of high-magnitude events which are of potentially greater environmental harm.

Finally, given the heavy-tailed nature of CSO durations, we advise caution when using summary statistics which may implicitly assume, or be interpreted in the context of, a gaussian distribution. For instance, the mean can be highly misleading when applied to heavy-tailed distributions. In the case of our best fitted power-law distribution, as $\alpha < 2$, the mean is in fact not even defined, although it is calculable for the stretched exponential. In addition, we caution against using simple spill frequency as a metric of CSO performance as this masks the very wide range of observed spill durations and assumed environmental impact (Sriwastava 2018). Given the results of this study, we propose that the scale, λ, and shape, β, parameters of a stretched exponential distribution are more informative statistics for characterising the distribution of CSO spill durations. An example `Python` script to calculate λ and β values from EDM data is provided at https://github.com/AlexLipp/cso_scaling. For instance, we propose that the shape parameter of the stretched exponential appears to be diagnostic of the mechanisms driving CSO spilling. By calculating the β parameter for data from all EDMs managed by a WASC, it may be possible to objectively identify which parts of the network require interventions focussed on minimising infiltration (i.e., those with low β values) and those which focus on mitigating storm runoff (i.e., those with high β values). By contrast, the scale parameter λ, which controls the total duration of spilling for a given shape parameter, could be used to objectively prioritise finite resources to mitigating the CSOs with distributions with the largest λ values. Finally, by assessing how λ and β change following interventions (e.g., replacing fractured pipes; Staufer et al. 2012) it is possible to observe and quantify the impact of the intervention. A challenge for all of these proposed usages is that many CSOs record relatively small numbers of spills, making it difficult to accurately fit distributions to EDM data for an individual monitor. If this is the case, using bootstrapping, or other approaches, to produce confidence intervals on parameter values would be of great importance.

6 Conclusions

We analysed the durations of ~ 4 million combined sewer overflow spill events from England to examine the frequency distribution of spill durations. Long-duration spills (> 4 hours) represent less than 10 % of events but account for ~ 80 % of total spill time. Formal likelihood tests show that this heavy tail is best described by a stretched exponential distribution with shape parameter $\beta \approx 0.18$, a result largely consistent across individual water companies in distinct hydrological domains. Daily fluctuations in domestic water use introduce periodic deviations above and below this model for spills of duration near integer multiples of 24 hours. Hydraulic modelling reproduces this heavy tail only when groundwater infiltration is included, indicating that contrary to conditions of their operation, the highest duration CSO events are not driven in response to 'exceptional' weather events. The heavy-tailed scaling behaviour of CSO distributions can be approximately reproduced by simulating sewer head as a fractional Brownian motion crossing a threshold level. Given a potentially elevated environmental impact of long-duration spills, we recommend explicitly incorporating tail behaviour into hydraulic model calibration. We propose the shape and scale parameters of the stretched exponential distribution as useful statistics for this purpose.

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Data Access

Stop/start times obtained by Environmental Information Regulation requests cannot be shared by the authors but can be obtained by contacting the relevant WASCs. An example Python script for fitting a stretched exponential distribution to an example Event Duration Monitoring dataset is available at: https://github.com/AlexLipp/cso_scaling and archived at the point of submission at: https://doi.org/10.5281/zenodo.17832639.

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