Liquefaction as an energetic instability of saturated granular systems

Density control and static enthalpy equilibrium

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This manuscript is a preprint and has not undergone peer review.

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Date: December 2025 – Version 1.0

Preprint notice: This manuscript has been made publicity available as a preprint on EartArXiv to document the conceptual development of the work. The final version of the pater will be submitted to Acta Geotechnica (Springer Nature) for peer review and publication.

Abstract

Liquefaction of saturated granular materials is commonly interpreted within stress-based frameworks that rely on the existence of an intact grain skeleton. At the onset of liquefaction, however, the contact network collapses and effective stress ceases to be a meaningful state variable. This work reformulates liquefaction as an enthalpy-driven instability of the coupled grain–water system and introduces a stability concept based on **Static Enthalpy Equilibrium (SEE)**.

Within this framework, a saturated granular assembly occupies a local minimum of total enthalpy under the constraints of gravity, buoyancy and volume constancy. Liquefaction may be triggered by a single strong excitation **and/or** by multiple smaller excitation events occurring in rapid succession, provided that their **cumulative energy input** exceeds the porosity-dependent enthalpy barrier associated with SEE. Once this threshold is exceeded, the subsequent collapse of the grain skeleton proceeds spontaneously in a **post-trigger sense**, driven by internally released gravitational–buoyancy enthalpy rather than by continued external forcing.

Two complementary stability controls emerge naturally from the enthalpy formulation. First, a **density-controlled energetic limit** defines a unique stabilised porosity that depends solely on grain density and follows directly from buoyancy constraints. Second, an **energetic liquefaction** curve specifies the minimum external energy required to destabilise a configuration at a given porosity. The intersection of these two conditions determines both the accessibility and the termination of liquefaction.

The resulting framework provides a **physically grounded optimization of admissible system states**, derived from the first principles and valid across the entire transition from intact skeleton to fully liquefied suspension. It offers a consistent alternative to stress-based liquefaction criteria without invoking effective stress in regimes where it is not physically defined.

Keywords: liquefaction, saturated granular materials, static enthalpy equilibrium, enthalpy-driven instability, pore-water pressure build-up, density-controlled stability limit, state space, pore-water pressure build up

1. Introduction

Liquefaction of saturated granular materials is one of the most striking instability phenomena in geosystems. Classical interpretations describe liquefaction as a stress-path problem governed by effective stress, cyclic shear loading and pore-water pressure accumulation (Seed & Idriss, 1971); (Ishihara, 1993). These concepts provide a useful macroscopic description as long as an intact load-bearing grain skeleton exists.

At the instant of liquefaction, however, the granular contact network collapses and effective stress lose its physical meaning. This conceptual limitation has long been recognised (Gudehus, 1996), (Gudehus, 2011), (Iverson, 2005), (Borja, 2006) yet most existing liquefaction criteria remain tied to stress variables that are undefined once the skeleton disintegrates.

The present work focusses on the **first principles** and therefore cites prior studies selectively, emphasizing conceptual relevance rather than completeness, as it adopts a different perspective on liquefaction.

Liquefaction is treated not as a stress failure, but as an **energetic instability** of the coupled grain–water system. The governing quantity is the **total enthalpy of the saturated granular assembly**, and stability is defined by the existence of a local enthalpy minimum (Wittig, 2025). This concept, referred to as **Static Enthalpy Equilibrium (SEE)**, remains meaningful irrespective of whether an intact skeleton exists.

2. Static enthalpy equilibrium of saturated granular systems

A saturated granular medium is considered as a coupled two-phase system consisting of a solid grain phase and pore water. The total enthalpy comprises gravitational–buoyancy contributions of the grains, hydraulic contributions of the pore water and a structural contribution associated with the internal organisation of the contact network.

A configuration is in Static Enthalpy Equilibrium if the total enthalpy is at a **local minimum** with respect to all admissible rearrangements of grains and pore water under the constraints of gravity, buoyancy and volume constancy. In this state, the system is mechanically and hydraulically quiescent.

SEE generalises the classical notion of equilibrium. In the pre-collapse regime, it is compatible with effective stress descriptions. In the liquefied regime, where no grain skeleton exists, SEE remains well defined, whereas stress-based variables do not.

Throughout this work, **enthalpy is used as a purely mechanical system quantity; thermal aspects are not considered**.

3. Energetic triggering and post-trigger collapse

3.1 Cumulative triggering

Liquefaction does not require a single, large excitation event. Within the Static Enthalpy Equilibrium (SEE) framework, triggering is controlled by the **cumulative external energy input**. A structural collapse is initiated whenever a single strong excitation and/or multiple smaller excitation events occurring within a time interval shorter than the characteristic hydraulic dissipation time supply sufficient energy to overcome the SEE barrier, i.e.

$$\sum_i E_{\mathrm{input},i} > E_{\mathrm{crit}}(n_0). \tag{1}$$

Equation (1) defines a purely energetic triggering condition that is independent of stress amplitude, stress path or cycle count.

3.2 Post-trigger spontaneous collapse

Once this condition is satisfied, the subsequent evolution of the system no longer requires continued external forcing. The collapse of the grain skeleton and the associated rearrangement of the granular phase proceed spontaneously in a **post-trigger sense**, driven by internally released gravitational–buoyancy enthalpy and governed by hydraulic dissipation.

In this framework, the term spontaneous collapse therefore denotes a **self-driven process after triggering**, not an instability occurring without an external cause. The driving force of the process is the tendency of the system to reduce its total enthalpy.

4. Collapse-generated enthalpy and post-collapse evolution

Once SEE is violated, the granular contact network destabilises and collapses. Collapse is not driven by pore-water pressure; rather, excess pore pressure arises as a **hydraulic consequence** of collapse-generated enthalpy.

The collapse of the grain phase corresponds to a downward shift of its centre of mass by Δz_s. The gravitational–buoyancy enthalpy released per unit area is

$$E_{\mathrm{grain}} = (1-n)\cdot(\rho_s-\rho_w)\cdot g\cdot\Delta z_s, \tag{2}$$

where ρ_s and ρ_w are the grain and water densities, respectively. This internally generated energy drives pore-water displacement and governs the subsequent enthalpy dissipation of the system.

Within the SEE framework liquefaction is described entirely in terms of the **evolution of total system enthalpy**. Observable mechanical responses, including stress redistribution or pore-pressure changes, arise as **secondary manifestations** of enthalpy-driven structural reorganisation and are not treated as governing state variables.

5. Density control and energetic liquefaction curve

5.1 Density-controlled stabilised porosity

The saturated density of a grain–water mixture with porosity n is

$$\rho_{\mathrm{sat}} = (1 - n) \cdot \rho_s + n \cdot \rho_w. \tag{3}$$

Equation (3) characterises the gravitational-buoyancy state of the coupled grain-water system and therefore its available source of potential enthalpy. Changes in this quantity directly reflect changes in the amount of gravitational-buoyancy enthalpy that can be released by internal structural reorganization of the granular assembly.

Within the SEE framework, collapse-generated enthalpy vanishes once the saturated density falls below twice the water density. The stabilised porosity therefore follows from $\rho_{\mathrm{sat}}(n^*) = 2 \cdot \rho_w$, which yields

$$n^*(\rho_s) = \frac{\rho_s - 2 \cdot \rho_w}{\rho_s - \rho_w}. \tag{4}$$

Equations (3) and (4) thus define the energetic source term for pore-water pressure build-up. In the liquefied regime, pore-water pressure can increase only insofar as gravitational–buoyancy enthalpy is released by internal collapse and reorganisation of the granular structure. Because this enthalpy source is finite, the mobilizable pore-water pressure is inherently bounded.

The porosity n^* therefore marks the point at which no further pore-water pressure can be generated by spontaneous collapse, not because a geometric packing limit is reached, but because the enthalpy available for hydraulic work is exhausted. Beyond this limit, additional changes in porosity may still occur through external mechanical compaction, but they are no longer driven by spontaneous enthalpy release.

This density-controlled limit defines the admissible domain of enthalpy-driven liquefaction and forms the energetic foundation for the liquefaction curve introduced in Section 5.2.

The enthalpy-driven accessibility of collapse, the density-controlled SEE limit, and the exhaustion of the enthalpy source are summarized schematically in figure 1.

5.2 Energetic liquefaction curve

The energetic liquefaction curve introduced below is defined only within the density-admissible domain, as illustrated in figure 1.

Within the density-admissible range defined by n^*, the energetic liquefaction curve determines whether collapse is accessible under a given external excitation. The density criterion specifies whether collapse-generated enthalpy is possible, but it does not determine how easily collapse can be triggered.

The energetic liquefaction curve $E_{\text{crit}}(n)$ represents the SEE barrier and scales with the geometric displacement of the solid-phase centre of mass required to destabilise the grain skeleton and follows the relation

$$E_{\text{crit}}(n) \propto \frac{n_L - n}{1 - n}, \tag{5}$$

where n_L is a loose reference porosity.

Calibration at the stabilised porosity yields

$$E_{\text{crit}}(n^*) = \sum E_{\text{input}}, \tag{6}$$

so that the fully calibrated energetic liquefaction curve becomes

$$E_{\text{crit}}(n) = \left(\sum E_{\text{input}}\right) \cdot \frac{(n_L - n)/(1 - n)}{(n_L - n^*)/(1 - n^*)}. \tag{7}$$

Equations (5)-(7) provide a unified energetic triggering criterion applicable to equally to single-strong-pulse and cumulative excitation scenarios. The distinction between isolated impulses and cyclic loading is irrelevant in the SEE framework; only the cumulative energy input relative to the enthalpy barrier governs the onset of collapse.

6. Relation to established experimental observations

Numerous well-documented laboratory and model-scale experiments provide observational evidence that is fully consistent with an energetic interpretation of liquefaction, even though these studies were originally analysed using stress-based descriptors. Resonant column tests, cyclic torsion experiments and shaking table studies repeatedly report a gradual build-up of pore-water pressure under small-amplitude excitation, followed by an accelerated transition into collapse without a sharply defined stress threshold, e.g. (Ishihara, 1993); (Towhata, 2008).

Limitations of stress-based state descriptions for granular materials undergoing structural reorganisation have been discussed previously, in particular with respect to the non-uniqueness of stress as a state variable once the contact network degrades or collapses (Gudehus, 1996); (Gudehus, 2011). Within the Static Enthalpy Equilibrium framework, the observed experimental behaviour arises naturally from the cumulative erosion of an enthalpy barrier separating metastable configurations from collapse-accessible states. External excitation progressively increases the energetic accessibility of collapse, while the subsequent rise of pore-water pressure reflects a secondary hydraulic response to collapse-generated gravitational–buoyancy enthalpy rather than a triggering cause.

The same body of experimental work further documents a systematic attenuation of liquefaction response under repeated excitation, often described as cyclic hardening. In SEE, this observation follows directly from the progressive exhaustion of internally available collapse-generated enthalpy as the system approaches the density-controlled stabilised porosity. In this sense, the SEE framework represents a continuation and unifying reinterpretation of established experimental findings, elevating their analysis from stress-based descriptors to an energetically defined state perspective.

7. Discussion

The enthalpy-based formulation resolves several long-standing ambiguities in liquefaction theory by redefining stability and instability at the level of total system enthalpy rather than stress-based state variables. It reverses the classical causal interpretation by identifying collapse as the origin of pore-pressure response, not its consequence. It provides a physically meaningful stability criterion in regimes where effective stress is undefined and explains the attenuation of liquefaction response under repeated excitation without invoking empirical hardening rules.

The SEE-framework deliberately avoids stress-based descriptions, including both effective and total stresses, as primary explanatory variables. In systems involving structural collapse and reformation, stresses do not uniquely characterize the state of the system. The governing quantity is instead the total enthalpy of the coupled grain-water system, from which observable mechanical responses emerge as secondary effects.

The SEE framework further explains why spontaneous collapse terminates at a porosity significantly higher than the densest achievable packing. Termination is controlled by a **density-dependent energetic limit**, not by geometric packing constraints. In this sense, liquefaction is understood as a **state optimisation process**, not as a failure mechanism.

An immediate implication of the enthalpy-based formulation is the dependence of the density-controlled termination of liquefaction on grain density. According to the Static Enthalpy Equilibrium framework, the porosity n^* at which enthalpy-driven collapse terminates increases with increasing grain density. For otherwise comparable saturated granular materials, higher grain densities therefore expand the admissible porosity range over which enthalpy-driven liquefaction can occur. This dependence follows directly from first principles and does not require empirical calibration or material-specific fitting. The mechanical consequences of this density dependence are addressed in subsequent work.

8. Conclusions

Liquefaction of saturated granular systems is fundamentally an **enthalpy-driven instability**. Static Enthalpy Equilibrium (SEE) defines stability as a local minimum of total system enthalpy, while instability corresponds to energetic accessibility of lower-enthalpy configurations.

Density control determines the termination of spontaneous collapse, and the energetic liquefaction curve governs triggering through single and/or cumulative excitation. Mechanical quantities such as stresses or pore-water pressure arise as secondary manifestations of underlying enthalpy evolution rather than as governing variables.

The SEE framework may therefore be understood as a **physically grounded optimization of admissible system states**, providing a consistent alternative to stress-based liquefaction criteria across intact and liquefied regimes.

Figures

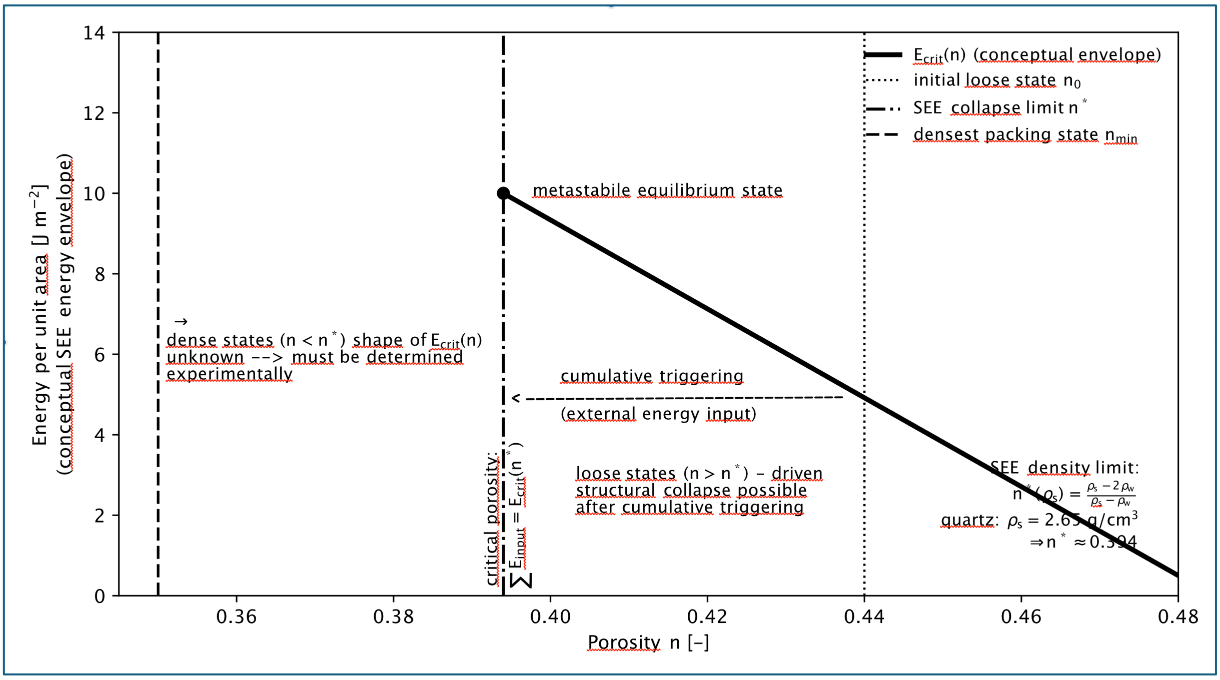

Figure 1.
Conceptual representation of liquefaction as an enthalpy-driven instability within the Static Enthalpy Equilibrium (SEE) framework.
The porosity axis defines the admissible state space of the saturated granular system, while grain density determines a density-controlled energetic limit $n^*(\rho_s)$, representing the *maximum* extend of enthalpy-driven densification.
Within this domain, the energetic liquefaction curve $E_{\text{crit}}(n)$ indicates the external energy required to *initiate* structural collapse under **single and/or cumulative excitation**.
Such excitation may trigger liquefaction and partial densification; however, neither the onset of liquefaction nor the occurrence of spontaneous collapse implies the attainment of the SEE limit.
The SEE boundary therefor represents an energetic upper bound of admissible states, rather than a necessary or typical outcome, corresponding to complete exhaustion of the enthalpy source for further pore-water pressure build-up

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Appendix A — Illustrative example: loose saturated granular system and cumulative triggering

This appendix provides a concise illustrative example demonstrating the application of the Static Enthalpy Equilibrium (SEE) framework to a loose, fully saturated granular fill. The example is intended to clarify the logical sequence of density control, energetic triggering and post-trigger collapse described in Sections 3 and 5 and illustrated in Figure 2.

A.1 Initial state

Consider a loose, fully saturated granular system with an initial porosity

$n_0 = 0.43,$

and a grain density representative to quartz,

$$\rho_s = 2.65\,\mathrm{g\,cm^{-3}}, \qquad \rho_w = 1.0\,\mathrm{g\,cm^{-3}}.$$

In the absence of external excitation, the system occupies a Static Enthalpy Equilibrium and remains mechanically stable.

A.2 Density-controlled collapse limit

Within the SEE framework, the density-controlled limit for spontaneous structural collapse is given by

$$n^*(\rho_s) = \frac{\rho_s - 2\rho_w}{\rho_s - \rho_w}. \tag{A1}$$

For quartz grains $\rho_s = 2.65 \cdot g/cm^3$ these yields $n^* = 0.394$.

This value represents an energetic upper bound for spontaneous collapse within the SEE framework. Below this porosity, collapse-generated gravitational–buoyancy enthalpy vanishes and spontaneous structural collapse is energetically impossible. Importantly, this limit does not correspond to the densest achievable packing of the grains.

A.2.1 Energy normalisation

For consistency across laboratory, column and field-scale configurations, energetic quantities are expressed per unit horizontal area. The gravitational–buoyancy enthalpy released during structural collapse can be written as

$$E_{\text{grain}} = (1 - n) \cdot (\rho_s - \rho_w) \cdot g \cdot \Delta z_s,$$

where Δz_s denotes the downward displacement of the solid-phase centre of mass. This normalisation yields energy densities in units of $J \cdot m^{-2}$, which provide a natural measure for comparing enthalpy release and external excitation across different geometries and system sizes.

A.3 Energetic barrier at the initial state

Because $n_0 > n^*$, the initial configuration lies within the density-admissible domain for energetic liquefaction. An energetic barrier $E_{\text{crit}}(n_0)$ therefore exists, as defined by the energetic liquefaction curve introduced in Section 5.2.

The exact numerical value of $E_{\text{crit}}(n_0)$ depends on calibration; its physical meaning, however, is unambiguous: it represents the minimum cumulative external energy per unit area required to lift the system out of its local enthalpy minimum and destabilise the granular contact network.

A.4 Single and/or cumulative triggering

Liquefaction may be initiated by a single sufficiently strong excitation or by a limited number of cumulative excitation pulses,

$$\sum_i E_{\text{input},i} > E_{\text{crit}}(n_0), \qquad (A2)$$

provided that the individual enthalpy barrier $E_{\text{crit}}(n_0)$ is exceeded.

Accordingly, a single sufficiently strong excitation and/or multiple smaller excitation events may trigger collapse, while weaker or widely separated events do not.

A.5 Post-trigger collapse and termination

Once the energetic barrier is exceeded, the granular contact network collapses and the subsequent evolution proceed spontaneously in a post-trigger sense, driven by internally released gravitational–buoyancy enthalpy rather than by continued external forcing.

Collapse and associated rearrangement continue until the porosity reaches the density controlled SEE limit n^*. At this point, collapse-generated enthalpy vanishes and further spontaneous structural collapse becomes energetically impossible. Any additional reduction in porosity beyond this limit would require external mechanical compaction rather than self-driven collapse.

A.6 Interpretation

This illustrative example highlights three central features of the SEE framework:

1. Liquefaction is governed by **energy**, not by stress variables.
2. Triggering may occur through a single strong excitation **and/or** through cumulative smaller excitations.
3. The termination of spontaneous collapse is controlled by a density-dependent energetic limit rather than by the densest achievable packing state.