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TITLE

Introducing Difference: from Euclidean Space to Geological Limits

AUTHORS

Kris Piessens, Geological Survey of Belgium, Royal Belgian Institute of Natural Sciences. E-mail: Kris.Piessens@naturalsciences.be

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Kris Piessens

Geological Survey of Belgium, Royal Belgian Institute of Natural Sciences. E-mail: Kris.Piessens@naturalsciences.be, tel. +32 2 788 76 34

The Lithotectonic Framework (LTF) provides a systematic approach to describing regional geology in terms of geological history. While LTF has been applied to vocabulary development and regional geological description, its theoretical foundations have remained undocumented. This paper introduces the Spatio-Temporal Framework (STF), which extends Euclidean geometry by adding 'difference' as a primitive notion alongside space and time. This enables defining 'event' without circularity, from which lithotectonic limits and units can be formally derived. The five LTF axioms then establish operational relationships between these concepts. STF thus provides LTF with a formal foundation analogous to how Euclidean definitions underpin geometric reasoning.

Introduction

Mathematics has long provided the conceptual framework for physics, since before they were seen as strictly separate disciplines. In geology similar conceptualization is a much more recent evolution that started in the field of structural geology (e.g. seminal work of Ramsay, 1967). This major step forward relied on the physics-based approach: describing reality geometrically to transition to mathematical description that in turn allows, for example, strain analysis.

When describing regional geology, geometrical description is helpful, and provides 2D or 3D shapes that can be visualized as a map or model. But it does not allow conceptualizing geological history. In recent decades semantic modelling became increasingly important, and several attempts were made to apply this to regional geology (e.g. Jähne, 2014; Németh, 2021; Le Bayon et al., 2022; Piessens et al., 2024). In Europe this evolution resulted in the theory of Lithotectonic Framework (LTF) .

LTF uses two closely related concepts: planar limits and volumetric units. Limits are the core concepts and their definition is intimately linked to structural, sedimentological or other events that are preserved in the geological record. Limits characterize units, that then become blocks with a history distinct from that of adjacent geology. Secondary limits may further alter or offset a unit, adding to its history, although its primary identity remains.

LTF is being used for improving geological vocabularies and to describe actual regional geology through initiatives such as the former European project GeoERA-GeoConnect^{3}d and the ongoing Geological Service for Europe (GSEU). In Belgium, this momentum led to the establishment of the Lithotectonic Working Group in 2023 within the National Commission on Stratigraphy. LTF is based on theory, mostly translated into practical rules of thumb and examples that serve as templates. This paper documents the most fundamental roots of LTF theory which so far remained undocumented.

Current Frameworks

Euclidean geometry seems to offer all that is needed to conceptualize the physical world. It describes space in as many dimensions as needed, and can be easily extended to include time as a temporal dimension. This provides us with the where and when reference frames. It usefully outlines spatial configurations as well as processes. Groundwater is a geological domain that explicitly uses all of this, and so do many more like geochemical modelling, physics of plate-tectonics, geophysical research, etc.

But it doesn't lend itself very well to what is possibly the most basic geological discipline: describing the geological history embedded in a rock sequence. History clearly happens at a place and moment in time, but is more. What lies preserved in the geological record are events, separating different geology on either side.

Several frameworks exist to describe geology. Stratigraphy, in whatever form or flavor, successfully orders and correlates geology and is very useful where mapping is concerned. However, it captures only very partially the geological history.

Structural frameworks are much more targeted and can document the geometry and relations of a system of structural elements. However, their scope is narrow. They are mostly used for analytical rather than descriptive work. [refs are needed here]

None of these approaches seem able to describe geology framed in regional geological history. Assigning a place (where) and time (when) to a geological element is possible, but not the difference, the 'what' that happened. LTF is set up to specifically allow for all three, adding the missing ingredient for documenting geological history.

Primitive Notions

The Lithotectonic Framework can be grounded in a more general Spatio-Temporal Framework (STF) that extends Euclidean geometry with time and difference as primitives. This provides LTF with a formal foundation analogous to how Euclidean definitions underpin geometric reasoning.

Primitives are pre-theoretical concepts that are recognized directly, and can not be otherwise derived. As such they are central concepts that allow axioms to be formulated, but themselves remain undefined. Euclidean geometry describes space and has the concepts point, line, plane and volume. Depending on which theoretical approach is taken (e.g. Hilbert, 1902; Birkhoff, 1932), at least **point** is a primitive notion, alongside relational primitives such as 'contains'. Euclidean geometry describes three-dimensional space, which for many applications requires a temporal extension. For non-relativistic purposes, this can be done by introducing a fourth dimension for time. Time then becomes an additional primitive.

The still missing primitive notion is the one introduced here as **difference**. It can simply be understood as a recognizable contrast. This is the primitive that allows to construct the spatio-temporal framework from which the lithotectonic framework is derived.

Where the spatial concepts are quantitative or metric primitives (continuous, orderable), difference is a qualitative or categorical primitive (like incidence in geometry).

Table 1. Primitive notions as needed in this paper. Difference is newly introduced.

Primitive	Description
Space	Euclidean geometry (point, line, plane, volume) - inherited, in general use
Time	Temporal extension of the Euclidean framework - existing, in general use
Difference	Recognizable contrast; no characterization of what differs required - new

Spatio-Temporal Framework

For our purposes, the spatio-temporal framework (STF) requires defining only a set of derived concepts. The STF is not domain specific. It is a generic framework that is here used to define concepts that are not geology specific.

The central concept is **event** that is defined as *that which creates difference at a location-moment*. With that, event combines the three primitives we established: space, time and difference. STF mixes quantitative and qualitative primitives already in its starting concept.

From here we can continue to define a **spatio-temporal point** as *a point in space marked by* *an event*. Point in essence means a Euclidean point, but one that is associated with an event, rather than an arbitrary point. This context is important and allows us to build up further.

The **spatio-temporal plane** is a *collection of spatio-temporal points sharing the same event*. Since a plane is a localized approximation of a surface, this also defines **spatio-temporal surface**. The purpose of this construct is facilitating the definition of the next and much more meaningful concept that will have a meaningful equivalent in the lithotectonic framework.

A **spatio-temporal limit** is a *spatio-temporal surface where the event is exclusive to that surface*. Or else, the event does not reach outside the surface.

The STF can accommodate other geometric elements (lines, volumes) following the same logic if required for other applications. For the Lithotectonic Framework, the planar element (limit) is the relevant construct, and also event is given an equivalent.

Lithotectonic Framework

The Lithotectonic Framework is the application of the spatio-temporal Framework to geology. At its core LTF theory consists of two definitions and five axioms.

Lithotectonic Framework Definitions

The LTF relies on events that are recognizable in the geological record as planar features, such as faults, unconformities, etc. These are referred to as the lithotectonic limits (LTLs) and are a conceptual representation of the actual geological features. The definition in the vocabulary for LTF users reads *A lithotectonic limit is a geological boundary representing testimony of a geological event*. This approach works, but makes an intuitive leap.

The now established STF offers the theoretical basis for defining this core concept directly: *A lithotectonic limit is a spatio-temporal limit applied to the geological domain*. Event does not need to be mentioned, since that reference is inherited. The core concept on which LTF is built, can be traced back to its primitive constructs.

Where LTL is an adaptation of the spatio-temporal limit, the definition of lithotectonic unit (LTU) is based on event, or more precisely the sum of events that give it a unique identity: *A lithotectonic unit is a volume in the geological record whose identity is marked by geological events*. The number of events can be one, but not zero.

Here also, the foundational definition of LTU is not the one used by practitioners. Instead, axiom 1 is cited that defines the relationship between limits and units (see below). It is this relation that matters most when applying the LTF.

It would have been possible, as was necessary for the LTL, to already define a unit concept as part of the STF. That would make sense if elaboration of the STF becomes a goal, here we use it merely as a useful theoretical step and settle for the above shortcut.

Lithotectonic Framework Axioms

The axioms establish operational relationships between LTF entities.

Axiom 1: identity through limits

A lithotectonic unit is defined by its primary lithotectonic limits.

Axiom 1 establishes the most fundamental relation between limits and units, and is also known as the 'limits come first' principle as practical application rule. The word 'defined' does not indicate that the axiom is a definition. It is here used to emphasize that the identity of an LTU is obtained through its primary limits and the events with which they are associated. This aspect is further deepened in axiom 2.

The concepts *primary* and *secondary* were not separately defined because these are generally understood in geology as pertaining to the origin (primary) or to later changes (secondary).

Axiom 2: events as the basis of limits

A primary lithotectonic limit represents an event that differentiates a lithotectonic unit from *other lithotectonic units*.

This is a claim of uniqueness. Even if two faults form at the same time, during one faulting episode, then the unique event of each of these faults is the formation of that fault. The definition of the fault block in between those faults thus indirectly references two events.

Moreover, two LTUs with identical geometrical shape, and therefore coinciding limits, are still different concepts. Consider an active sedimentary basin and cover unit, both LTUs defined by their initial (lower) unconformity. Typically, sedimentation of the cover would extend beyond the basin limits (which are defined by the spill lines of the basin). But in the situation where sedimentation is limited to the basin and does not reach spill lines, both are identical in 3D-space and time. Still the events, as defined for sedimentary basin and cover, are slightly different: the cover requires sedimentation on top of an unconformity, a basin requires start of preferential sedimentation but does not require unconformity. This is enough to define, where or when needed, both the sedimentary basin and cover separately as non-identical LTUs.

Axiom 3: identity persistence

A lithotectonic unit retains its earliest identity until a destructive event eliminates its defining limits.

Axiom 3 provides clarity on the specific situation that is crucial when describing geological history. Once a unit is formed, how does later deformation, metamorphism, erosion... change the identity of that unit? As long as it is recognizable, even when it only partly survives, the initial definition or identity still holds. When the primary limits are erased from the geological record, then also the definition of the unit is lost.

What destruction can entail is not specified to keep the statement fundamental, but erosion, melting or intense metamorphism are valid processes, whereas faulting only results in displacement, not destruction.

Axiom 4: secondary limits as overprinting

A secondary lithotectonic limit is a geological testimony of an overprinting event.

Axiom 4 can be read as a principle of structural geology, and should come intuitive to geologists. Faulting mentioned earlier is an overprinting or modifying process. It doesn't change the primary definition of the LTU, but does form part of the geological history of that unit after it was formed.

Axiom 5: recursive nature of limits

A secondary lithotectonic limit is a primary lithotectonic limit to a more recent lithotectonic unit.

It seems straightforward that any secondary limit also indicates the presence of a lithotectonic unit to which it is a primary limit, and in geological practice this is always the case. However, it is not a corollary from axiom 1, or any axiom. Nevertheless, it is possible that Axiom 5 could be derived from the primitives and other axioms in a non-straightforward way. This would make it a theorem and is therefore ranked last.

From Foundational to Applied Lithotectonic Framework

The rest of LTF theory is derivable from the above theory, where needed combined with other mathematical theory. The rules for defining different more practical concepts (generic and real world concepts), the strict rules for building hierarchical relations between concepts, in which vocabularies poly-hierarchy is allowed, how type definitions of real-world LTUs and LTLs can or can not be assigned..., all can be brought back to the core lithotectonic framework as above introduced.

A particular proposition that follows from core theory, is that LTUs can overlap (proof involves absence of violation to overlap). This is noteworthy because it is different from how stratigraphic units are defined. By definition each boundary is shared by two adjacent stratigraphic units, preventing overlap of units of equal rank. The necessary feature that LTUs should be allowed to overlap (and often do) poses interesting visualization challenges.

In applied form LTF ontology exists of two main levels. The first are generic lithotectonic limits and units. Both are type-of hierarchies that define under limits faults and other tectonic contacts, unconformities, intrusive contacts, or plate boundaries, and under units common terms such as sedimentary basins, deformation belts, igneous units, or plate tectonic units. Compared to traditional definitions, LTF definitions are more accurate and stable (Piessens et al., 2024).

Based on these, mappable real-world lithotectonic elements are defined at conceptual level, with the goal to create a lithotectonic map of Europe by 2028 [GSEU project]. This is extensive work with the first published results being the regional definition of basements and orogens in North-West Europe (Piessens et al., Submitted).

Discussion

LTF is designed as a logical classification system, as opposed to pragmatic systems that often have a more organic origin. It is the hidden layer of STF that provides LTF with a formal foundation analogous to how Euclidean definitions underpin geometric reasoning. Treating 'difference' as primitive enables defining 'event' without circularity, which in turn allows geological limits to be defined descriptively while axioms specify their operational roles. Together this creates a comprehensive deductive and logical system as a stable basis for lithotectonic classification, even if this is implicit for most of its users.

Using limits as the central defining elements is an explicit design choice in LTF. This excludes certain types of geological units, most notably metamorphic units. These are not lithotectonic because the event does not create limits, but affects a rock volume. Metamorphic contours are just gradients in that volume, not event limits. While putting limits central is a useful choice for LTF, STF does provide a solid basis for setting up parallel frameworks with different or more relaxed constraints, for example by foreseeing a volumetric extension in STF.

LTF is heavily influenced by structural geology, where description comes before interpretation. Structural nomenclature was purged from interpretative connotation decades ago, and now relies strongly on geometrical description. Exceptions are interpretations that are foregone conclusions. An example is a normal fault, where fault movement is arguably observed rather than interpreted. LTF adheres to this philosophy as strictly as possible. An event is a change, like a fault movement, not the geological episode that provides the possible explanation for the faulting event. Any absolute timing or broader context is interpretation and not part of a lithotectonic definition. As such concept and definition are given stability, providing inertness against ever evolving or radically changing interpretations.

Conclusion

The Spatio-Temporal Framework provides the Lithotectonic Framework with formal foundations by introducing 'difference' as a primitive notion. This enables event-based definitions that are descriptive rather than interpretative, while axioms establish operational relationships. With its theoretical basis now documented, LTF stands as a comprehensive deductive system for geological classification, currently being applied to create a lithotectonic map of Europe.

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