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4 TITLE

5 **Introducing Difference: from Euclidean Space to Geological Limits**

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11 **Introducing Difference: from Euclidean Space to Geological Limits**

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18 *The Lithotectonic Framework (LTF) provides a systematic approach to describing*
19 *regional geology in terms of geological history. While LTF has been applied to vocabulary*
20 *development and regional geological description, its theoretical foundations have remained*
21 *undocumented. This paper introduces the Spatio-Temporal Framework (STF), which extends*
22 *Euclidean geometry by adding 'difference' as a primitive notion alongside space and time. This*
23 *enables defining 'event' without circularity, from which lithotectonic limits and units can be*
24 *formally derived. The five LTF axioms then establish operational relationships between these*
25 *concepts. STF thus provides LTF with a formal foundation analogous to how Euclidean*
26 *definitions underpin geometric reasoning.*

27 **Introduction**

28 Mathematics has long provided the conceptual framework for physics, since before they were
29 seen as strictly separate disciplines. In geology similar conceptualization is a much more recent
30 evolution that started in the field of structural geology (e.g. seminal work of Ramsay, 1967).
31 This major step forward relied on the physics-based approach: describing reality geometrically
32 to transition to mathematical description that in turn allows, for example, strain analysis.

33 When describing regional geology, geometrical description is helpful, and provides 2D or 3D
34 shapes that can be visualized as a map or model. But it does not allow conceptualizing

35 geological history. In recent decades semantic modelling became increasingly important, and
36 several attempts were made to apply this to regional geology (e.g. Jähne, 2014; Németh, 2021;
37 Le Bayon et al., 2022; Piessens et al., 2024). In Europe this evolution resulted in the theory of
38 Lithotectonic Framework (LTF) .

39 LTF uses two closely related concepts: planar limits and volumetric units. Limits are the core
40 concepts and their definition is intimately linked to structural, sedimentological or other events
41 that are preserved in the geological record. Limits characterize units, that then become blocks
42 with a history distinct from that of adjacent geology. Secondary limits may further alter or
43 offset a unit, adding to its history, although its primary identity remains.

44 LTF is being used for improving geological vocabularies and to describe actual regional
45 geology through initiatives such as the former European project GeoERA-GeoConnect^{3d} and
46 the ongoing Geological Service for Europe (GSEU). In Belgium, this momentum led to the
47 establishment of the Lithotectonic Working Group in 2023 within the National Commission on
48 Stratigraphy. LTF is based on theory, mostly translated into practical rules of thumb and
49 examples that serve as templates. This paper documents the most fundamental roots of LTF
50 theory which so far remained undocumented.

51 **Current Frameworks**

52 Euclidean geometry seems to offer all that is needed to conceptualize the physical world. It
53 describes space in as many dimensions as needed, and can be easily extended to include time
54 as a temporal dimension. This provides us with the where and when reference frames. It
55 usefully outlines spatial configurations as well as processes. Groundwater is a geological
56 domain that explicitly uses all of this, and so do many more like geochemical modelling,
57 physics of plate-tectonics, geophysical research, etc.

58 But it doesn't lend itself very well to what is possibly the most basic geological discipline:
59 describing the geological history embedded in a rock sequence. History clearly happens at a
60 place and moment in time, but is more. What lies preserved in the geological record are events,
61 separating different geology on either side.

62 Several frameworks exist to describe geology. Stratigraphy, in whatever form or flavor,
63 successfully orders and correlates geology and is very useful where mapping is concerned.
64 However, it captures only very partially the geological history.

65 Structural frameworks are much more targeted and can document the geometry and relations
66 of a system of structural elements. However, their scope is narrow. They are mostly used for
67 analytical rather than descriptive work. [refs are needed here]

68 None of these approaches seem able to describe geology framed in regional geological history.
69 Assigning a place (where) and time (when) to a geological element is possible, but not the
70 difference, the 'what' that happened. LTF is set up to specifically allow for all three, adding the
71 missing ingredient for documenting geological history.

72 **Primitive Notions**

73 The Lithotectonic Framework can be grounded in a more general Spatio-Temporal Framework
74 (STF) that extends Euclidean geometry with time and difference as primitives. This provides
75 LTF with a formal foundation analogous to how Euclidean definitions underpin geometric
76 reasoning.

77 Primitives are pre-theoretical concepts that are recognized directly, and can not be otherwise
78 derived. As such they are central concepts that allow axioms to be formulated, but themselves
79 remain undefined. Euclidean geometry describes space and has the concepts point, line, plane
80 and volume. Depending on which theoretical approach is taken (e.g. Hilbert, 1902; Birkhoff,

81 1932), at least **point** is a primitive notion, alongside relational primitives such as 'contains'.
 82 Euclidean geometry describes three-dimensional space, which for many applications requires
 83 a temporal extension. For non-relativistic purposes, this can be done by introducing a fourth
 84 dimension for time. Time then becomes an additional primitive.

85 The still missing primitive notion is the one introduced here as **difference**. It can simply be
 86 understood as a recognizable contrast. This is the primitive that allows to construct the spatio-
 87 temporal framework from which the lithotectonic framework is derived.

88 Where the spatial concepts are quantitative or metric primitives (continuous, orderable),
 89 difference is a qualitative or categorical primitive (like incidence in geometry).

90

91 Table 1. Primitive notions as needed in this paper. Difference is newly introduced.

Primitive	Description
Space	Euclidean geometry (point, line, plane, volume) - inherited, in general use
Time	Temporal extension of the Euclidean framework - existing, in general use
Difference	Recognizable contrast; no characterization of what differs required - new

92

93 **Spatio-Temporal Framework**

94 For our purposes, the spatio-temporal framework (STF) requires defining only a set of derived
 95 concepts. The STF is not domain specific. It is a generic framework that is here used to define
 96 concepts that are not geology specific.

97 The central concept is **event** that is defined as *that which creates difference at a location-*
 98 *moment*. With that, event combines the three primitives we established: space, time and
 99 difference. STF mixes quantitative and qualitative primitives already in its starting concept.

100 From here we can continue to define a **spatio-temporal point** as *a point in space marked by*

101 *an event*. Point in essence means a Euclidean point, but one that is associated with an event,
102 rather than an arbitrary point. This context is important and allows us to build up further.

103 The **spatio-temporal plane** is a *collection of spatio-temporal points sharing the same event*.

104 Since a plane is a localized approximation of a surface, this also defines **spatio-temporal**
105 **surface**. The purpose of this construct is facilitating the definition of the next and much more
106 meaningful concept that will have a meaningful equivalent in the lithotectonic framework.

107 A **spatio-temporal limit** is a *spatio-temporal surface where the event is exclusive to that*
108 *surface*. Or else, the event does not reach outside the surface.

109 The STF can accommodate other geometric elements (lines, volumes) following the same logic
110 if required for other applications. For the Lithotectonic Framework, the planar element (limit)
111 is the relevant construct, and also event is given an equivalent.

112 **Lithotectonic Framework**

113 The Lithotectonic Framework is the application of the spatio-temporal Framework to geology.
114 At its core LTF theory consists of two definitions and five axioms.

115 ***Lithotectonic Framework Definitions***

116 The LTF relies on events that are recognizable in the geological record as planar features, such
117 as faults, unconformities, etc. These are referred to as the lithotectonic limits (LTLs) and are a
118 conceptual representation of the actual geological features. The definition in the vocabulary for
119 LTF users reads *A lithotectonic limit is a geological boundary representing testimony of a*
120 *geological event*. This approach works, but makes an intuitive leap.

121 The now established STF offers the theoretical basis for defining this core concept directly: *A*
122 *lithotectonic limit is a spatio-temporal limit applied to the geological domain*. Event does not
123 need to be mentioned, since that reference is inherited. The core concept on which LTF is built,

124 can be traced back to its primitive constructs.

125 Where LTL is an adaptation of the spatio-temporal limit, the definition of lithotectonic unit
126 (LTU) is based on event, or more precisely the sum of events that give it a unique identity: *A*
127 *lithotectonic unit is a volume in the geological record whose identity is marked by geological*
128 *events*. The number of events can be one, but not zero.

129 Here also, the foundational definition of LTU is not the one used by practitioners. Instead,
130 axiom 1 is cited that defines the relationship between limits and units (see below). It is this
131 relation that matters most when applying the LTF.

132 It would have been possible, as was necessary for the LTL, to already define a unit concept as
133 part of the STF. That would make sense if elaboration of the STF becomes a goal, here we use
134 it merely as a useful theoretical step and settle for the above shortcut.

135 ***Lithotectonic Framework Axioms***

136 The axioms establish operational relationships between LTF entities.

137 ***Axiom 1: identity through limits***

138 *A lithotectonic unit is defined by its primary lithotectonic limits.*

139 Axiom 1 establishes the most fundamental relation between limits and units, and is also known
140 as the 'limits come first' principle as practical application rule. The word 'defined' does not
141 indicate that the axiom is a definition. It is here used to emphasize that the identity of an LTU
142 is obtained through its primary limits and the events with which they are associated. This aspect
143 is further deepened in axiom 2.

144 The concepts *primary* and *secondary* were not separately defined because these are generally
145 understood in geology as pertaining to the origin (primary) or to later changes (secondary).

146 ***Axiom 2: events as the basis of limits***

147 *A primary lithotectonic limit represents an event that differentiates a lithotectonic unit from*

148 *other lithotectonic units.*

149 This is a claim of uniqueness. Even if two faults form at the same time, during one faulting
150 episode, then the unique event of each of these faults is the formation of that fault. The
151 definition of the fault block in between those faults thus indirectly references two events.
152 Moreover, two LTUs with identical geometrical shape, and therefore coinciding limits, are still
153 different concepts. Consider an active sedimentary basin and cover unit, both LTUs defined by
154 their initial (lower) unconformity. Typically, sedimentation of the cover would extend beyond
155 the basin limits (which are defined by the spill lines of the basin). But in the situation where
156 sedimentation is limited to the basin and does not reach spill lines, both are identical in 3D-
157 space and time. Still the events, as defined for sedimentary basin and cover, are slightly
158 different: the cover requires sedimentation on top of an unconformity, a basin requires start of
159 preferential sedimentation but does not require unconformity. This is enough to define, where
160 or when needed, both the sedimentary basin and cover separately as non-identical LTUs.

161 ***Axiom 3: identity persistence***

162 *A lithotectonic unit retains its earliest identity until a destructive event eliminates its defining*
163 *limits.*

164 Axiom 3 provides clarity on the specific situation that is crucial when describing geological
165 history. Once a unit is formed, how does later deformation, metamorphism, erosion... change
166 the identity of that unit? As long as it is recognizable, even when it only partly survives, the
167 initial definition or identity still holds. When the primary limits are erased from the geological
168 record, then also the definition of the unit is lost.

169 What destruction can entail is not specified to keep the statement fundamental, but erosion,
170 melting or intense metamorphism are valid processes, whereas faulting only results in
171 displacement, not destruction.

172 ***Axiom 4: secondary limits as overprinting***

173 *A secondary lithotectonic limit is a geological testimony of an overprinting event.*

174 Axiom 4 can be read as a principle of structural geology, and should come intuitive to
175 geologists. Faulting mentioned earlier is an overprinting or modifying process. It doesn't
176 change the primary definition of the LTU, but does form part of the geological history of that
177 unit after it was formed.

178 ***Axiom 5: recursive nature of limits***

179 *A secondary lithotectonic limit is a primary lithotectonic limit to a more recent lithotectonic*
180 *unit.*

181 It seems straightforward that any secondary limit also indicates the presence of a lithotectonic
182 unit to which it is a primary limit, and in geological practice this is always the case. However,
183 it is not a corollary from axiom 1, or any axiom. Nevertheless, it is possible that Axiom 5 could
184 be derived from the primitives and other axioms in a non-straightforward way. This would
185 make it a theorem and is therefore ranked last.

186 **From Foundational to Applied Lithotectonic Framework**

187 The rest of LTF theory is derivable from the above theory, where needed combined with other
188 mathematical theory. The rules for defining different more practical concepts (generic and real
189 world concepts), the strict rules for building hierarchical relations between concepts, in which
190 vocabularies poly-hierarchy is allowed, how type definitions of real-world LTUs and LTLs can
191 or can not be assigned..., all can be brought back to the core lithotectonic framework as above
192 introduced.

193 A particular proposition that follows from core theory, is that LTUs can overlap (proof involves
194 absence of violation to overlap). This is noteworthy because it is different from how

195 stratigraphic units are defined. By definition each boundary is shared by two adjacent
196 stratigraphic units, preventing overlap of units of equal rank. The necessary feature that LTUs
197 should be allowed to overlap (and often do) poses interesting visualization challenges.

198 In applied form LTF ontology exists of two main levels. The first are generic lithotectonic
199 limits and units. Both are type-of hierarchies that define under limits faults and other tectonic
200 contacts, unconformities, intrusive contacts, or plate boundaries, and under units common
201 terms such as sedimentary basins, deformation belts, igneous units, or plate tectonic units.
202 Compared to traditional definitions, LTF definitions are more accurate and stable (Piessens et
203 al., 2024).

204 Based on these, mappable real-world lithotectonic elements are defined at conceptual level,
205 with the goal to create a lithotectonic map of Europe by 2028 [GSEU project]. This is extensive
206 work with the first published results being the regional definition of basements and orogens in
207 North-West Europe (Piessens et al., Submitted).

208 **Discussion**

209 LTF is designed as a logical classification system, as opposed to pragmatic systems that often
210 have a more organic origin. It is the hidden layer of STF that provides LTF with a formal
211 foundation analogous to how Euclidean definitions underpin geometric reasoning. Treating
212 'difference' as primitive enables defining 'event' without circularity, which in turn allows
213 geological limits to be defined descriptively while axioms specify their operational roles.
214 Together this creates a comprehensive deductive and logical system as a stable basis for
215 lithotectonic classification, even if this is implicit for most of its users.

216 Using limits as the central defining elements is an explicit design choice in LTF. This excludes
217 certain types of geological units, most notably metamorphic units. These are not lithotectonic

218 because the event does not create limits, but affects a rock volume. Metamorphic contours are
219 just gradients in that volume, not event limits. While putting limits central is a useful choice
220 for LTF, STF does provide a solid basis for setting up parallel frameworks with different or
221 more relaxed constraints, for example by foreseeing a volumetric extension in STF.
222 LTF is heavily influenced by structural geology, where description comes before interpretation.
223 Structural nomenclature was purged from interpretative connotation decades ago, and now
224 relies strongly on geometrical description. Exceptions are interpretations that are foregone
225 conclusions. An example is a normal fault, where fault movement is arguably observed rather
226 than interpreted. LTF adheres to this philosophy as strictly as possible. An event is a change,
227 like a fault movement, not the geological episode that provides the possible explanation for the
228 faulting event. Any absolute timing or broader context is interpretation and not part of a
229 lithotectonic definition. As such concept and definition are given stability, providing inertness
230 against ever evolving or radically changing interpretations.

231 **Conclusion**

232 The Spatio-Temporal Framework provides the Lithotectonic Framework with formal
233 foundations by introducing 'difference' as a primitive notion. This enables event-based
234 definitions that are descriptive rather than interpretative, while axioms establish operational
235 relationships. With its theoretical basis now documented, LTF stands as a comprehensive
236 deductive system for geological classification, currently being applied to create a lithotectonic
237 map of Europe.

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