Spontaneous liquefaction in saturated granular deposits:

State controlled boundary and surface reconfiguration

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The preprint addresses post-liquefaction ground reconfiguration in saturated granular deposits. It focusses on the post-liquefaction state and spatial boundary determination, not on triggering or cyclic loading.

It is intended for scientific discussion and does not represent an opinion or commentary paper.

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Abstract

In the case of water-saturated, granular deposits that are at risk of liquefaction, engineers need reliable information about the spatial extent of soil deformation in the event of liquefaction. It is not so important for them to know the exact location of the first failure. However, existing analyses primarily deal with the triggering of liquefaction and offer only limited information on how large the areas affected by liquefaction can become.

This study presents a **state-based interpretation** of soil deformation after liquefaction, in which the boundary of terrain deformation is determined by the energy distribution at the system level, with the starting point of liquefaction playing a subordinate role. The analysis focuses explicitly only on the state after liquefaction and does not address the triggering conditions that ultimately led to the onset.

The concept is supported by **Event 42**, a large-scale field event in Lusatia that can be interpreted as a natural experiment. The ground surface of an inner dump of a former lignite mine was deliberately designed with very shallow initial slopes (approx. 3°) to avoid local triggering due to slope instability. Despite these conditions, liquefaction occurred, spreading over a large part of the site until a clearly defined and reproducible boundary, which is obviously characteristic of the prevailing material condition, was reached. High-resolution surface surveys before and after the event made it possible to quantitatively determine the geometry, while independently available historical aerial photographs confirm the large-scale extent of the newly configured area in terms of size.

The observed invariance of the boundary and heights with respect to the orientation of working profiles shows that the final shape is independent of the location where liquefaction began. This suggests that the geometric changes to the terrain after liquefaction are a state-controlled

process. Based on this interpretation, the spatial extent of the change in slope geometry can be determined on the basis of the initial geometry and soil mechanical parameters without knowing where liquefaction began. A proposal for the practical calculation of the boundaries of an area affected by liquefaction is included in the appendices.

Keywords: Spontaneous liquefaction, post-liquefaction ground reconfiguration, state-controlled deformation, energy landscape, saturated granular systems, surface adjustment, metastable stability, hydraulic coupling, liquefaction boundary, natural experiment

List of symbols and variables

The following symbols are used to describe geometric, hydraulic, and material state properties. No explicit numerical evaluation is performed in the present study.

Geometric quantities

- z Vertical coordinate or surface elevation
- Δz Vertical surface change (uplift or subsidence)
- A Area of the liquefaction-affected domain
- Ω Liquefaction-prone domain under consideration
- z_b Geometrical lower constraint limiting vertical displacement

Hydraulic quantities

- z_w Groundwater level
- γ_w Unit weight of water
- u Porewater pressure

Material state variables

- n Porosity
- n_c Critical porosity separating liquefiable and stable states
- γ' Effective (buoyancy-reduced) unit weight of the granular material

Energetic quantities

- E State-dependent static energy of the system
- ∇E Gradient of the energy landscape

Conceptual terms

- **Energy landscape**: Spatial distribution of static energy governing admissible post-liquefaction states
- **State-controlled boundary**: Boundary separating energetically admissible and inadmissible surface configurations
- **Geometrical lower constraint**: Given geometric limitation of vertical displacement imposed by surrounding terrain and non-liquefied zones.

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1. Introduction

Liquefaction of saturated granular deposits is commonly assessed in engineering practice with respect to its initiation, i.e. whether liquefaction is triggered under given loading conditions. In contrast, the **spatial extent of ground reconfiguration after liquefaction**—the question of where deformation ultimately ends—is often treated implicitly or left unresolved. For practical decision-making, however, the latter question is frequently the more relevant one: infrastructure planning, hazard zoning, and remediation require knowledge of the **affected area**, not merely the location of first failure.

Most established approaches address liquefaction through **triggering-based concepts**, relying on local stress conditions, cyclic loading parameters, or pore-pressure generation criteria. These methods are well suited to determine whether liquefaction may occur at a given point, but they provide little guidance on how deformation propagates once liquefaction has taken place, or how the spatial limits of post-liquefaction ground reconfiguration are defined. In particular, they do not constrain the deformation boundary in cases where the initiation location is unknown, variable, or deliberately suppressed (Terzaghi, 1925), (Casagrande, 1976).

Field observations indicate that post-liquefaction behaviour often exhibits characteristics that are difficult to reconcile with purely local or path-dependent interpretations. Ground reconfiguration may extend over large areas, develop under very small surface gradients, and terminate at clearly defined boundaries that are reproducible across different cross sections. These features suggest that post-liquefaction evolution is governed not only by local failure mechanisms, but by **system-level constraints** acting on the liquefied domain as a whole.

In this study, a **state-based interpretation** of post-liquefaction ground reconfiguration is adopted (Wittig, 2024). Once liquefaction has occurred and a hydraulically connected granular domain exists, redistribution is assumed to be controlled by a system-level **energy landscape** rather than by the specific location of initiation. Within this framework, the deformation boundary corresponds to an energetically neutral limit for the prevailing material state, beyond which further redistribution is no longer admissible. Post-liquefaction stabilization is therefore understood as **conditional and state-dependent**, rather than necessarily permanent. The focus of this work is explicitly on the **post-liquefaction state**; conditions for liquefaction triggering and initiation are not addressed (Ishihara, 1993).

The central evidence is provided by **Event 42**, a large-scale field event that can be interpreted as a natural experiment. The site geometry had been deliberately designed with very shallow initial slopes (approximately 3°) in order to avoid localized initiation. Despite these conditions, liquefaction occurred and expanded across the site until a distinct and reproducible boundary of ground reconfiguration was reached. High-resolution surface surveys before and after the event constrain the geometry quantitatively, while independent historical aerial imagery confirms the large-scale spatial extent of the affected area.

The objective of this paper is to demonstrate that, in a liquefaction-prone domain, **the spatial extent of post-liquefaction ground reconfiguration can be determined without knowledge of where initial failure occurs**. By combining field evidence with a state-based interpretation, the study shows that the deformation boundary is a property of the system state rather than of the triggering path.

2. Conceptual framework: post-liquefaction ground reconfiguration

Once liquefaction has occurred, a saturated granular deposit undergoes a fundamental change in mechanical behaviour. Load-bearing grain contacts are largely lost, effective stresses are strongly reduced, and the material transitions from a friction-dominated skeleton to a **hydraulically coupled granular–fluid system**. In this state, deformation is no longer governed by classical shear failure along discrete planes, but by redistribution of mass within a connected domain under system-level constraints (Gudehus, 2011).

A key consequence of this transition is that **post-liquefaction deformation becomes a state problem rather than a path problem**. The evolution of the system is no longer controlled by the location or orientation of the initial failure, but by the admissible configurations of the liquefied domain under given boundary conditions. These boundary conditions include geometry, overburden load, confinement, hydraulic connectivity, and the current material state.

2.1 System-wide hydraulic coupling

After liquefaction, pore water and grains form a coupled system with very low bulk compressibility. Pressure changes induced locally are therefore transmitted rapidly throughout the liquefied domain. As a result, the system responds as a whole, even though surface deformation may develop progressively over time.

This hydraulic coupling implies that local initiation does not remain a local phenomenon. Instead, it modifies the effective boundary conditions of the entire liquefied region. Redistribution is therefore not driven by directional failure propagation, but by the global compatibility of the system under the imposed constraints.

2.2 Energy landscape as governing state variable

To describe this behaviour, the concept of an **energy landscape** is introduced. In the present context, the energy landscape represents the spatial distribution of static enthalpy that governs the energetic admissibility of post-liquefaction redistribution (Andreotti, Forterre, & Pouliquen, 2013), (Duran, 2000). It is a state-dependent field determined by elevation relative to the underlying base, effective unit weight, overburden load, geometric confinement, and the evolving saturation and porosity state.

The energy landscape is **not** a geometric surface and does not represent a flow path. Instead, it defines which redistributions are energetically favourable, neutral, or unfavourable once liquefaction has removed local shear resistance. Redistribution proceeds as long as it reduces the system's energy for the current state and ceases where further redistribution would no longer be energetically admissible.

2.3 Definition of the deformation boundary

Within this framework, the spatial boundary of post-liquefaction ground reconfiguration is defined as the **energetically neutral limit** of redistribution for the prevailing material state. At this boundary, additional mass transfer would not reduce the system's energy and is therefore not realized.

Importantly, this boundary is a **state property**. It does not depend on the location of the initial failure, the sequence of deformation, or the direction of analysis. Once the liquefied domain, boundary conditions, and material state are specified, the deformation boundary is uniquely determined for that state.

2.4 Two-dimensional representation of a three-dimensional state

Although post-liquefaction deformation is inherently three-dimensional, the governing state is system-level and direction-independent. Consequently, two-dimensional cross sections provide valid representations of the prevailing configuration. If the deformation boundary represents a true state property, independent cross sections taken in different orientations must converge to the same boundary.

This property explains why robust agreement between two-dimensional sections and measured three-dimensional geometry can be obtained, even though the underlying process is three-dimensional. Such invariance is characteristic of state-controlled systems and would not be expected for path-dependent failure mechanisms.

3. Event 42 as field evidence of state-controlled boundary formation

Event 42 (former open-pit Schlabendorf, Germany) provides a rare opportunity to examine post-liquefaction ground reconfiguration under **exceptionally constrained boundary conditions**. The site can be interpreted as a natural experiment in which geometric and hydraulic conditions were deliberately configured to suppress localized initiation, thereby isolating system-level behaviour.

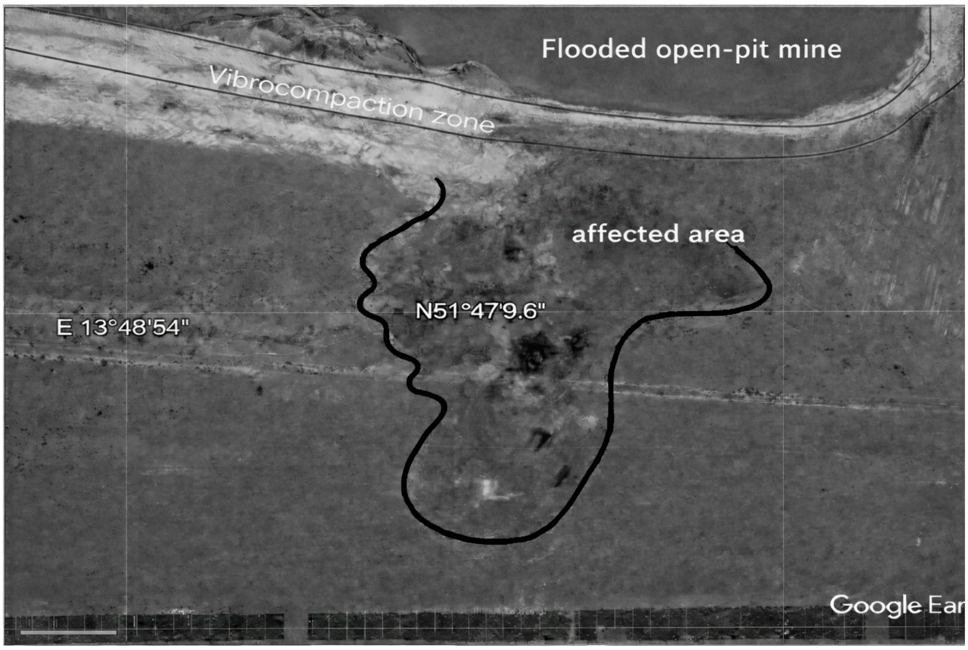

Figure 1. Site context and affected area of Event 42

Base imagery: © Google Earth; used for contextual reference only.

Satellite view of the Event 42 site showing the spatial extent of post-liquefaction ground re-configuration. The outline marks the affected area as mapped from surface observations and surveys. The vibrocompacted zone is indicated along the crest, and the flooded open pit mine is shown at the northern boundary. Material loss occurred across the vibrocompaction zone into the flooded open-pit mine at this northern boundary. The figure provides spatial context only; no quantitative analysis or numerical reconstruction is implied; quantitative evaluation is based on high-resolution surface survey data.

3.1 Boundary conditions of the natural experiment

During the rehabilitation of the opencast mine, the ground surface was deliberately designed to be **very slightly sloped** because of the awareness of the risk of liquefaction. The embankment was constructed with an initial slope of approximately **3°**. This geometry was chosen to avoid a preferred mode of failure and to minimise classic deformation mechanisms caused by slopes. From a conventional mechanical point of view, such a slight incline would not be expected to control large-scale ground movements.

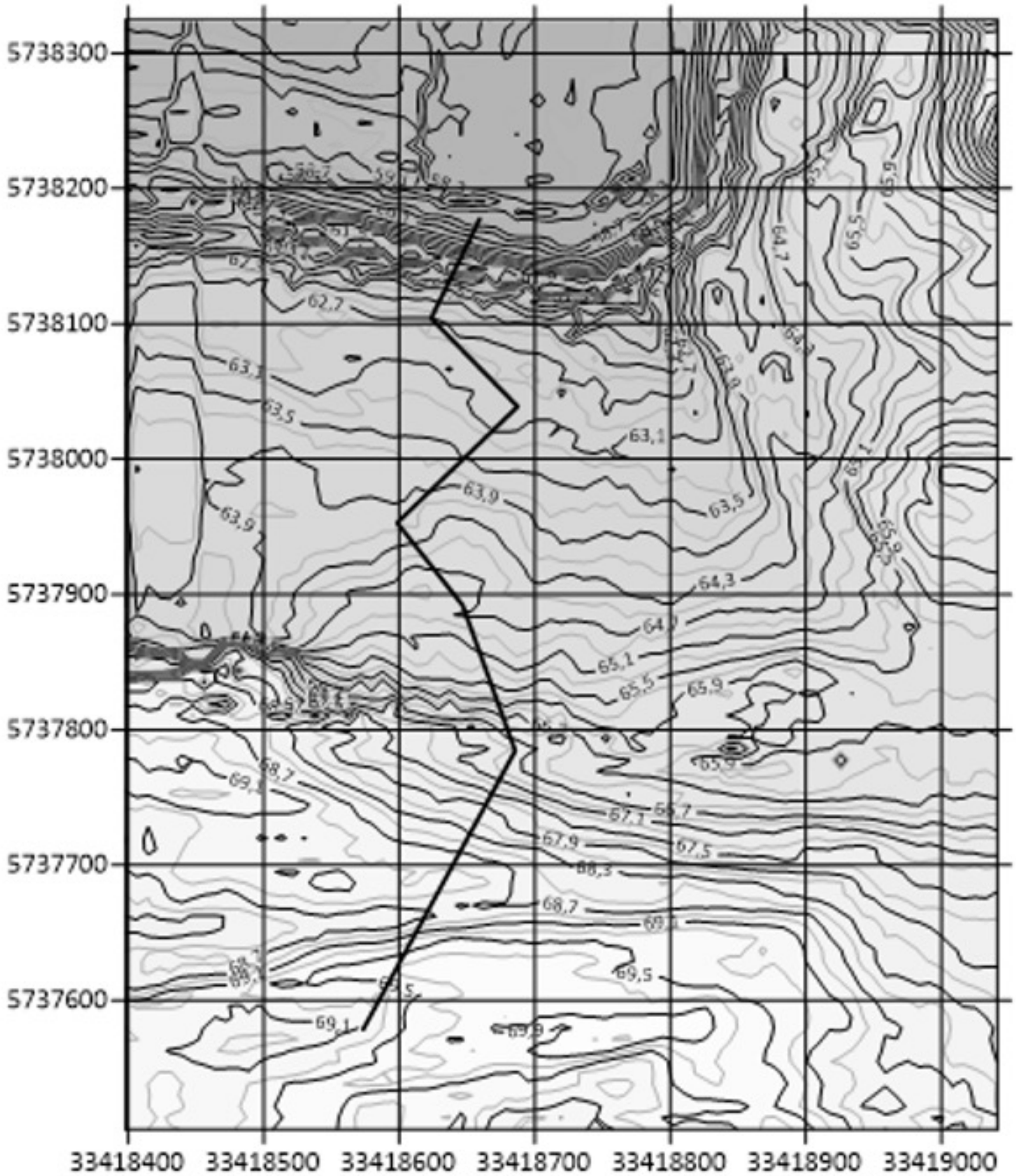

Figure 2. Contour map of the measured ground surface prior to liquefaction at Event 42.

The irregular profile line is intended solely to illustrate the presumed absence of a dominant geometric axis or preferred direction of deformation. The illustration merely documents the original surface geometry and serves as a reference state for subsequent observations; no interpretation or reconstruction is implied.

Despite these favourable conditions, liquefaction occurred, spreading across a large, contiguous area. The driving energy associated with the low slope of the surface was therefore extremely low, but sufficient to cause extensive terrain to change after liquefaction. This combination – minimal slope, no external trigger and large spatial response – makes Event 42 particularly suitable for testing state-based interpretations.

Under these conditions, liquefaction occurred without any discernible static, dynamic or cyclic influence and is therefore classified here as **spontaneous liquefaction**, caused by internal state instability rather than external influences.

3.2 Observed configuration and boundary

High-resolution terrain surveys conducted before and after the event show a clearly defined and coherent configuration of terrain deformation for the prevailing material state after liquefaction. The affected area has a distinct spatial boundary that is sharp at the observation level and stable over the observation period.

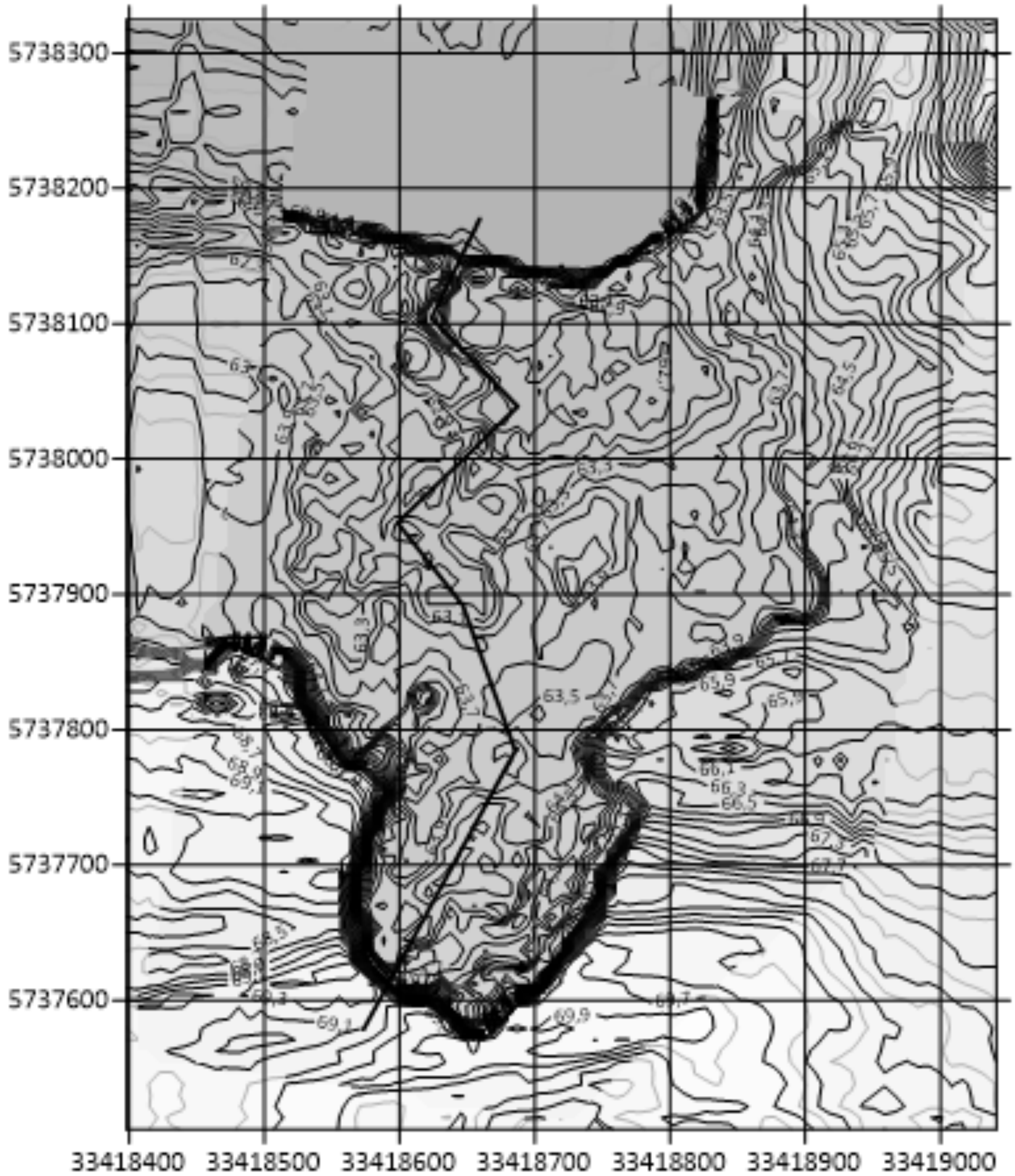

Figure 3. Contour map of the measured ground surface after Event 42, documenting the spatial extent of surface reconfiguration.

The boundary of the affected area is clearly defined and reproducible. The shown geometry reflects the surface configuration after liquefaction and subsequent material loss across the vibrocompaction zone into the flooded open-pit mine; it does not represent the immediate post-liquefaction configuration. The figure documents the observed surface state only.

Independent two-dimensional cross sections extracted from the three-dimensional survey data show consistent agreement with the measured geometry, regardless of section orientation or lateral position. This invariance indicates that the observed configuration does not reflect a direction-dependent deformation path, but rather a **system-level state**.

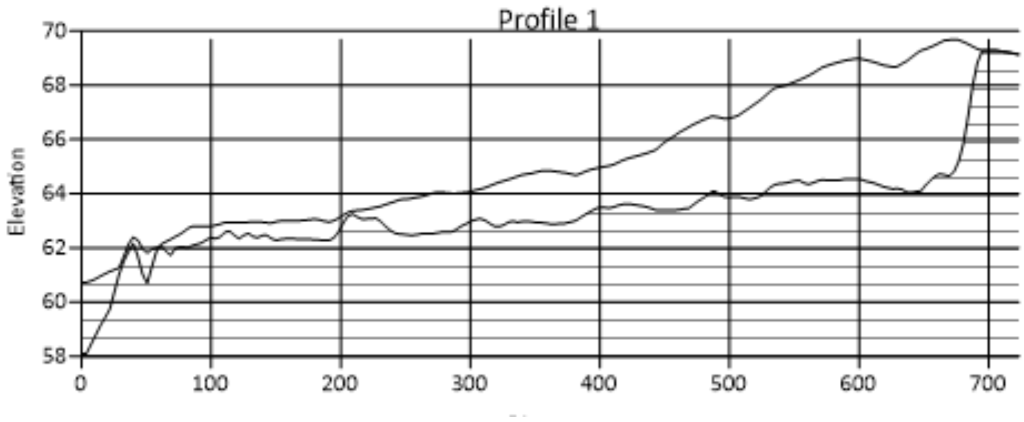

Figure 4. Ground-surface profiles extracted along a representative oblique cross-section before and after Event 42.

Despite the non-orthogonal orientation of the section, the profiles exhibit a form-preserving vertical adjustment within the affected area and a consistent boundary location. The agreement across the affected domain illustrates the directional invariance of the observed state-controlled boundary. The profiles represent measured surface configurations and do not imply any numerical reconstruction.

In addition to the quantitative survey data, the spatial extent of the reconfigured area is independently visible in publicly available historical aerial imagery. While such imagery is not used for quantitative analysis, it confirms the large-scale nature of the deformation and provides external verification of the mapped boundary.

3.3 Implications of the field evidence

Two observations are central. First, the **location of initial failure is not known** and is not required to interpret the observed configuration. Second, the deformation boundary is **robust and reproducible**, despite minimal geometric gradients and the absence of a dominant initiation point.

Together, these observations exclude explanations based on localized triggering, progressive failure paths, or slope-controlled mechanisms. Instead, Event 42 demonstrates that, once liquefaction has occurred, the spatial extent of ground reconfiguration is governed by **state-dependent, system-level constraints**.

4. Engineering implication: determination of the post-liquefaction boundary

From an engineering perspective, the principal challenge in liquefaction-prone areas is not the identification of the initiation point, but the determination of the **spatial extent of ground reconfiguration** relevant for design and hazard assessment. Planning decisions depend on knowing **where deformation is energetically admissible** under the prevailing state, even when the triggering mechanism or initiation location is uncertain or intentionally suppressed.

Within the energy-landscape framework, the deformation boundary corresponds to the locus at which further redistribution no longer reduces the system's energy for the current material state. This boundary is therefore independent of the initiation location, independent of the direction of analysis, and determined by geometry, overburden conditions, confinement, and material state variables.

Because the governing state is system-level and direction-invariant, two-dimensional analyses provide reliable representations of the three-dimensional boundary for a given state. A practical engineering procedure for determining this boundary is provided in the appendices.

5. Discussion

5.1 Event 42 as a natural experiment

Event 42 represents an unusually clean field configuration for examining post-liquefaction ground reconfiguration. The site geometry was deliberately designed with very shallow initial slopes (approximately 3°) in order to suppress localized initiation and classical slope-driven mechanisms. The occurrence of liquefaction and subsequent ground response under these conditions therefore isolates the post-liquefaction state as the governing regime.

Because neither dynamic excitation nor a dominant geometric driving direction is present, Event 42 can be interpreted as a natural experiment in which system-level behavior becomes observable without being masked by stronger external or geometric asymmetries.

5.2 Role of small gradients in the energy landscape

Event 42 demonstrates that very small surface inclinations are sufficient to structure post-liquefaction behaviour. Although the geometric surface was nearly horizontal, the governing energy landscape was not flat. Even minor elevation differences generated gradients that controlled **internal redistribution within the liquefied domain**, once local shear resistance had been lost.

This internal redistribution is not **directly observable** at the surface. Instead, it manifests as a **state-controlled vertical surface response**, expressed through patterns of subsidence and uplift. In the closed-system interpretation adopted here, the dominant observable effect is therefore vertical surface adjustment, while the underlying redistribution occurs within the liquefied material and remains hidden.

The spatial extent of this vertical surface response is controlled not by the absolute energy level, but by the structure of the energy landscape and the point at which further internal redistribution becomes energetically neutral for the prevailing material state.

5.3 Independence from initiation and path

The observed deformation boundary in Event 42 is invariant with respect to the unknown location of initial failure and with respect to section orientation. Multiple independent two-dimensional cross sections extracted from the three-dimensional survey data converge on the same boundary location.

This invariance is incompatible with path-dependent or initiation-controlled explanations. If post-liquefaction behaviour were governed by progressive failure paths or directional mechanisms, significant variability between sections would be expected. Instead, the observed consistency indicates that the configuration represents a **state-controlled condition**.

5.4 Local material overtopping and interpretation of the boundary

Limited local material transfer across the confining embankment was observed in Event 42. Such overtopping does not contradict the state-controlled boundary identified here. The boundary defines the extent of the liquefied, hydraulically connected domain for the prevailing state, not an impermeable geometric barrier for individual particles.

Overtopping occurs only if the **hydraulic head (pressure height) within the liquefied material exceeds the elevation of the confining structure**. Under this condition, local material discharge across the boundary becomes mechanically possible. This process is therefore **pressure-controlled**, not energetically driven, and does not imply an expansion of the liquefied domain beyond the state-controlled boundary.

From a system perspective, the liquefied domain remains **energetically closed** under normal conditions. It becomes **temporarily open** only when excess hydraulic head is present, allowing limited material release until pressure levels equilibrate. Once hydraulic heads have adjusted, the system effectively closes again, and the state-controlled boundary of the liquefied domain remains unchanged.

This behaviour reflects a clear separation between **system-level state constraints governing the extent of liquefaction-induced reconfiguration and secondary, transient mechanical processes associated with pressure equilibration across fixed geometric boundaries**.

5.5 Temporary stability and repeated post-liquefaction reconfiguration

Field evidence from Event 42 indicates that liquefaction-induced stabilization may be **temporary** rather than final. Following liquefaction, surface deformation may cease and the system may appear stable under the prevailing conditions. However, this apparent stability represents a **metastable state**, not a terminal configuration.

The energy landscape that existed prior to liquefaction does not disappear during liquefaction. Instead, liquefaction leads to a local **reduction of energy contrasts**: surface elevations are lowered and depressions are partially filled, while the overall ground-surface contour is preserved. The resulting configuration therefore represents a **smoothed, but not fully levelled, energy landscape**.

Because vertical displacement is constrained by a geometrical lower constraint imposed by the surrounding terrain and adjacent non-liquefied zone, the configuration attained immediately after liquefaction can be understood as a form-preserving downward adjustment of the pre-existing surface geometry. Relative elevations are maintained, while absolute levels are reduced within the limits imposed by the fixed base. As a result, liquefaction smooths energy contrasts but does not erase the geometric imprint of the original surface.

This partial equalization has important consequences for subsequent state evolution. Material that has subsided below the groundwater level becomes progressively saturated, whereas material that has been lifted above the groundwater level loses buoyant support. These post-liquefaction changes modify effective unit weight and local energetic conditions **without requiring further geometric change**.

If the porosity of the saturated granular material remains **above the critical threshold** for the given grain system, the system may again become energetically admissible for internal redistribution. In this case, post-liquefaction reconfiguration can **repeat in a staged manner**, driven by renewed state instability rather than by external forcing.

Importantly, the observed surface reconfiguration precedes the subsequent densification and re-stabilization of the liquefied material. The geometrically visible adjustment therefore represents a **pre-stabilization response** of the system. In Event 42, this temporal sequence cannot be demonstrated quantitatively because part of the material was discharged into the residual pit, preventing strict volume conservation. However, other liquefaction events with effectively constant volume clearly exhibit the same ordering, with surface adjustment occurring prior to measurable densification of the liquefied layer.

Repeated reconfiguration may continue until one of two terminal conditions is reached: either the energy landscape becomes effectively flat due to progressive geometric smoothing, or densification reduces porosity below the critical threshold, restoring true structural stability. Only under these conditions does **permanent stabilization** occur.

5.6 Regional constraint and scope of applicability

The Lausitz region in Germany is not subject to significant seismic activity. Dynamic excitation due to earthquakes can therefore be excluded as a contributing factor. Liquefaction and subsequent ground reconfiguration in Event 42 must be interpreted in the absence of cyclic or sustained dynamic loading.

The framework presented here applies to post-liquefaction behaviour under conditions of saturation, hydraulic connectivity, and absence of sustained external excitation. It does not address liquefaction triggering or redistribution under continued dynamic loading.

6. Conclusions

This study addresses the question relevant to engineering of how large the area affected by liquefaction will be once it has occurred, rather than where the liquefaction began. With a focus on **spontaneous liquefaction** due to internal instability, the following conclusions are drawn:

1. The change in the terrain surface after liquefaction is state-controlled.

Once liquefaction has occurred, the spatial extent of the surface adjustment is determined by energetic constraints at the system level and not by the location of the onset, the deformation path or external influences.

2. For a given material state, there is a clear boundary of the affected area.

Field observations from Event 42 show a clearly defined and reproducible boundary that is independent of the point of onset and invariant with respect to the orientation of the section.

3. Small geometric gradients are sufficient to structure the response.

Even slightly sloping terrain can develop a distinct boundary of remodelling once shear strength is lost.

4. The change in the surface is completely finished before its consolidation begins.

The observable adjustment of the soil surface occurs before the subsequent compaction and re-stabilisation of the liquefied material, suggesting that the surface change is a reaction prior to stabilisation rather than a consequence of material consolidation.

5. The stabilisation that occurs through terrain levelling may be temporary.

Liquefaction smooths out energy contrasts but does not erase the previously existing energy landscape. Continued changes in the condition of the terrain, including saturation changes and porosity reduction, can nevertheless restore the energetic permissibility of liquefaction and lead to repeated liquefaction associated with terrain adjustments.

6. Permanent stabilisation is only achieved under physically given end conditions.

True stability occurs when the energy landscape effectively flattens out or when consolidation reduces porosity below the critical threshold.

In summary, spontaneous liquefaction in saturated granular deposits leads to a **state-dependent, geometrically limited change in the surface of the affected terrain**, characterised by a clear boundary and potential metastability. Event 42 provides clear field evidence that this behaviour occurs without recourse to initiation scenarios, external dynamic excitation or explicit numerical description.

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Appendix A: Conceptual background – state-based interpretation of post-liquefaction behaviour

A.1 Purpose of this appendix

This appendix provides the conceptual background underlying the state-based interpretation adopted in the main text. Its purpose is to clarify the physical meaning of state, boundary, and stability in the context of post-liquefaction ground reconfiguration, without introducing triggering concepts or time-dependent flow models.

The appendix supports the interpretation presented in the main text and establishes the conceptual basis for the formal SEE framework (Appendix C) and the engineering procedure (Appendix D).

A.2 State versus process

In classical geotechnical analysis, instability is commonly treated as a process problem, in which deformation evolves along specific paths governed by local failure criteria, stress redistribution, or progressive weakening. In contrast, the interpretation adopted here distinguishes clearly between:

- the process of transition (how the system moves), and
- the state of the system (which configurations are admissible).

Post-liquefaction ground reconfiguration is treated primarily as a state problem. Once liquefaction has occurred and the granular skeleton has lost load-bearing capacity, the system is no longer constrained by local shear resistance. Redistribution then proceeds until an admissible configuration for the prevailing state is reached.

This distinction is essential: while transition paths may vary, the admissible configurations of the system are governed by global constraints and are therefore path-independent.

A.3 Meaning of stability in a state-based framework

Within a state-based framework, stability does not imply immobility or finality. Instead, stability refers to the existence of a configuration that is energetically admissible for the current state variables.

A configuration that appears stable at a given time may therefore be only conditionally stable. If state variables such as saturation, porosity, or effective unit weight evolve after deformation has ceased, the energetic admissibility of the configuration may change.

Consequently, post-liquefaction stabilization should be understood as state-dependent and potentially metastable, rather than as a permanent restoration of stability.

A.4 Energy landscape as a state descriptor

To capture this behaviour, the concept of an energy landscape is used. In this context, the energy landscape represents the spatial distribution of static enthalpy governing post-liquefaction redistribution under given boundary conditions.

The energy landscape is:

- a state-dependent field,
- determined by geometry, overburden, confinement, and material properties,
- independent of deformation paths or initiation history.

It is not a geometric surface and does not describe a flow trajectory. Instead, it defines which redistributions are energetically favourable, neutral, or unfavourable for the prevailing state.

A.5 Definition of the boundary in a state-based sense

Within the energy-landscape framework, the boundary of post-liquefaction ground reconfiguration corresponds to the energetically neutral limit for the prevailing state. At this boundary, further redistribution does not reduce the system's energy and is therefore not realized.

This boundary:

- is a state property,
- is independent of the location of initial failure,
- is independent of the direction of analysis,
- may evolve if the system state evolves.

It should be distinguished from geometric or structural boundaries, which may control local mechanical behaviour but do not define the extent of the liquefied state itself.

A.6 Closed and temporarily open systems

From a state perspective, the liquefied domain is energetically closed under normal conditions. Redistribution occurs internally until the energetically admissible configuration for the prevailing state is reached.

Temporary opening of the system may occur if local hydraulic conditions permit material discharge across a geometric boundary (e.g. overtopping). Such opening is mechanically and hydraulically controlled, not energetically driven, and persists only until pressure levels equilibrate.

This distinction allows separation between:

- system-level state constraints, which define the liquefied domain, and
- secondary mechanical processes, which may occur locally without altering the system state.

A.7 State evolution and repeated reconfiguration

Because the energy landscape depends on state variables, it may evolve after apparent stabilization. Progressive saturation, porosity change, or densification can modify the energy landscape and reintroduce energetic imbalance.

As long as the material remains above the critical porosity for the given grain system, renewed redistribution remains energetically admissible. Post-liquefaction ground reconfiguration may therefore occur in stages, separated by metastable pauses, until either:

- geometric smoothing renders the energy landscape effectively flat, or
- densification reduces porosity below the critical threshold.

Only then does true terminal stability exist.

A.8 Relation to the main text

The main text applies this state-based interpretation to field evidence from Event 42. Appendix A provides the conceptual foundation for that interpretation, while Appendix C formalizes the framework and Appendix D translates it into an engineering procedure.

Together, these components establish a consistent view of post-liquefaction ground reconfiguration as a state-evolution problem, rather than as a sequence of local failure events.

Appendix B: Pressure coupling and incompressibility in post-liquefaction systems

B.1 Purpose of this appendix

This appendix clarifies the role of hydraulic coupling and low compressibility in saturated granular systems after liquefaction. Its purpose is to explain why post-liquefaction redistribution acts at the system level, why local initiation does not remain local, and why pressure-controlled effects (including temporary opening) can occur without violating the state-controlled boundary.

The discussion complements the conceptual background in Appendix A and supports the interpretation of Event 42 in the main text.

B.2 Saturated granular media as weakly compressible systems

After liquefaction, a saturated granular deposit behaves as a granular–fluid mixture with very low bulk compressibility. While neither the pore fluid nor the grains are strictly incompressible, their combined compressibility is sufficiently small that pressure changes are transmitted rapidly across the hydraulically connected domain.

As a consequence, local deformation or mass redistribution cannot be accommodated by local volume change alone. Instead, pressure adjusts at the domain scale, enforcing mechanical compatibility across the liquefied region.

This property underlies the transition from localized behaviour prior to liquefaction to system-wide response after liquefaction.

B.3 Pressure as a coupling variable, not a driving criterion

Within the SEE framework, pressure does not serve as an independent triggering or failure criterion. Instead, it functions as a coupling variable that enforces equilibrium within the liquefied domain.

Pressure responds to:

- mass redistribution,
- geometric constraints,

- overburden load,
- and boundary conditions.

It does not determine the admissibility of redistribution by itself. Energetic admissibility is governed by the state-dependent energy landscape; pressure adjusts to satisfy that state.

This distinction is essential to avoid conflating pressure-driven mechanics with state-controlled stability.

B.4 System-wide pressure adjustment after liquefaction

Once liquefaction has occurred, pressure changes induced at any location propagate rapidly throughout the hydraulically connected domain. As a result:

- redistribution at one location modifies pressure conditions elsewhere,
- the system responds coherently rather than through directional propagation,
- the final configuration reflects global compatibility, not local paths.

This behaviour explains why post-liquefaction deformation boundaries are invariant with respect to section orientation and independent of initiation location, as observed in Event 42.

B.5 Pressure limits imposed by overburden and confinement

Pressure within the liquefied domain is constrained by overburden load and confinement. These constraints define the maximum admissible pressure level and prevent unbounded pressure growth.

As redistribution proceeds and the system approaches an energetically admissible configuration for the prevailing state, pressure gradients diminish. Apparent stabilization occurs when pressure redistribution no longer drives further mass transfer under the current state variables.

This pressure-limited behaviour is consistent with the observation that post-liquefaction redistribution can cease temporarily even though the material remains liquefaction-prone.

B.6 Pressure-controlled temporary opening of the system

Although the liquefied domain is energetically closed under normal conditions, it may become temporarily open when hydraulic head within the domain exceeds the elevation of a geometric boundary (e.g. an embankment crest).

Under this condition:

- local material discharge becomes mechanically possible,
- pressure is reduced through limited release,
- and the system transitions back to a closed state once hydraulic heads equilibrate.

Such temporary opening is pressure-controlled, not energetically driven, and does not imply an extension of the liquefied domain beyond the state-controlled boundary.

B.7 Relation to metastable post-liquefaction behaviour

Because pressure adjusts rapidly while material state variables (such as saturation and porosity) evolve more slowly, the system may enter metastable configurations. Pressure equilibration can temporarily suppress redistribution, even though the energy landscape may later change as saturation increases or porosity evolves.

This separation of time scales explains why:

- apparent stabilization can occur,
- redistribution may resume without renewed external forcing,
- and post-liquefaction reconfiguration can proceed in stages.

B.8 Relation to the main text and other appendices

The main text uses Event 42 to demonstrate system-level behaviour that follows naturally from hydraulic coupling and low compressibility. Appendix B provides the physical explanation for this coupling, while Appendix A establishes the state-based interpretation and Appendix C formalizes the SEE framework. Appendix D translates these concepts into an engineering workflow.

Appendix C: Formal SEE framework for post-liquefaction state determination

C.1 Purpose and role of the SEE formulation

This appendix provides the formal representation of the state-based interpretation used throughout the main text. It introduces the Static Energy Equilibrium (SEE) framework as a means of defining admissible configurations of a liquefied granular system for a given material state.

The SEE framework is not intended to describe transition dynamics, triggering, or time-dependent flow. Its sole purpose is to determine which configurations are energetically admissible for a given state and to identify the state-controlled boundary of post-liquefaction ground reconfiguration.

C.2 System definition

We consider a saturated granular domain Ω that has undergone liquefaction and is hydraulically connected. The system is characterized by:

- grain density and size distribution,
- porosity n,
- degree of saturation,
- effective unit weight (buoyancy-reduced),
- geometric constraints and confinement,
- overburden load.

After liquefaction, grain contacts no longer provide shear resistance, and redistribution is governed by system-level constraints rather than local strength criteria.

C.3 Static energy as state variable

Within SEE, the relevant state variable is the static energy (static enthalpy) of the system, representing the potential for internal redistribution under the prevailing state.

This static energy incorporates:

- gravitational potential of the granular mass,

- effects of buoyancy through effective unit weight,
- geometric constraints imposed by the base and confinement,
- material state variables such as porosity and saturation.

Importantly, static energy is evaluated for a given state, not along a deformation path.

C.4 Energy landscape and admissible configurations

The energy landscape is defined as the spatial distribution of static energy across the liquefied domain. For any admissible redistribution of mass, the system evolves toward configurations that reduce static energy for the prevailing state.

An admissible configuration satisfies:

- redistribution leads to non-increasing static energy,
- mechanical compatibility is maintained across the domain,
- geometric and confinement constraints are respected.

Redistribution ceases when no further energy reduction is possible under the given state variables.

C.5 Definition of the SEE boundary

The SEE boundary is defined as the locus within the domain where redistribution becomes energetically neutral for the prevailing state.

At this boundary:

- further redistribution does not reduce static energy,
- redistribution is therefore not realized,
- the configuration is admissible but marginal

This boundary defines the spatial extent of post-liquefaction ground reconfiguration for the given state.

C.6 Independence from initiation and path

Because the SEE boundary is defined entirely by state variables and boundary conditions, it is:

- independent of the location of initial liquefaction,
- independent of deformation sequence,
- independent of analysis direction.

Different transition paths may lead to the same admissible configuration, but the boundary itself is unique for a given state.

This property explains the observed invariance of the deformation boundary with respect to cross-section orientation in Event 42.

C.7 Relation between SEE and two-dimensional representations

Although redistribution occurs in three dimensions, the SEE boundary is a system-level state property. Any two-dimensional section through the domain represents a projection of the same state.

Consequently:

- consistent boundary locations across independent sections indicate a valid SEE state,
- discrepancies would indicate that the system has not reached an admissible configuration for the prevailing state.

This provides a practical validation criterion for SEE using field data.

C.8 State evolution and metastability

SEE describes admissible configurations for a given state but does not assume that the state itself is static.

State variables such as saturation and porosity may evolve after apparent stabilization. If such evolution modifies the energy landscape, previously admissible configurations may become energetically unfavourable.

In this case:

- a new SEE boundary emerges,
- redistribution may resume without renewed external forcing,
- stabilization is revealed as metastable rather than terminal.

True terminal stability is reached only when further state evolution does not reintroduce energetic imbalance.

C.9 Relation to other appendices

- Appendix A provides the conceptual interpretation of state, stability, and boundary.
- Appendix B explains pressure coupling and incompressibility, which enforce system-wide response.
- Appendix C (this appendix) defines the formal SEE framework for admissible configurations.
- Appendix D translates SEE into a practical engineering procedure.

Together, these appendices establish a consistent framework for interpreting post-liquefaction ground reconfiguration as a state-evolution problem.

Appendix D: Engineering procedure for determining the spatial extent of post-liquefaction ground reconfiguration

D.1 Purpose of this appendix

This appendix provides a practical engineering procedure for determining the spatial extent of ground reconfiguration following liquefaction. The procedure is designed for liquefaction-prone, saturated granular systems and explicitly does not require knowledge of the initiation location.

The procedure applies to the post-liquefaction state. It does not address triggering, cyclic loading, or time-dependent flow dynamics. Its objective is to identify the state-controlled boundary of ground reconfiguration relevant for hazard assessment, zoning, and design.

D.2 Preconditions for application

The procedure is applicable when the following conditions are satisfied:

1. The granular material is saturated or forms a hydraulically connected saturated domain.
2. The material is liquefaction-prone (i.e. structural stability can be lost).
3. No sustained external dynamic excitation dominates the post-liquefaction evolution.
4. Geometric boundaries, confinement, and overburden conditions can be reasonably defined.

If these conditions are met, post-liquefaction ground reconfiguration can be treated as a state-controlled problem.

D.3 Definition of the liquefaction-prone domain

The first step is the identification of the liquefaction-prone domain Ω:

- include saturated granular deposits,
- exclude rigid or non-liquefiable zones (e.g. bedrock, fixed structures),
- include all areas where post-liquefaction redistribution is physically possible.

This step relies on standard geotechnical information and does not require identification of the initiation point.

D.4 Construction of the energy landscape

For the defined domain Ω, an energy landscape is constructed.

This is a scalar state field representing the static energy governing post-liquefaction redistribution.

At each location, the energy landscape is determined by:

- elevation relative to the underlying base,
- effective (buoyancy-reduced) unit weight,
- overburden load,
- geometric confinement and fixed boundaries,
- prevailing material state (saturation, porosity).

The energy landscape is not a flow surface and does not depend on direction. It represents energetic admissibility for the current state.

D.5 Criterion of energetic admissibility

Redistribution of material is energetically admissible if it leads to a reduction of static energy for the prevailing state.

For engineering purposes:

- locations where redistribution reduces energy belong to the reconfiguration domain,
- locations where redistribution is energetically neutral define the boundary,
- locations where redistribution would increase energy lie outside the affected domain.

The boundary corresponds to an iso-energy condition for the given state.

D.6 Determination of the state-controlled boundary

The boundary of post-liquefaction ground reconfiguration is obtained by:

1. Evaluating the energy landscape across \Omega.
2. Identifying locations were redistribution transitions from energetically favourable to neutral.
3. Connecting these locations to form a closed boundary.

This boundary is:

- independent of the initiation location,
- independent of analysis direction,
- robust with respect to local irregularities.

D.7 Use of two-dimensional sections

Although redistribution is three-dimensional, the governing state is system-level. Consequently:

- two-dimensional cross sections provide valid representations of the boundary,
- multiple independent sections should converge on the same boundary,
- agreement across sections confirms that the boundary represents a state property, not a path-dependent mechanism.

This provides a practical means of validation using field data.

D.8 Treatment of local overtopping

Local material overtopping across geometric boundaries may occur only if the hydraulic head within the liquefied domain exceeds the elevation of the confining structure.

Such overtopping:

- is pressure-controlled, not energetically driven,
- represents a temporary opening of an otherwise closed state system,
- does not imply expansion of the liquefied domain beyond the state-controlled boundary.

Once hydraulic heads equilibrate, the system effectively closes again, and the state-controlled boundary remains unchanged.

D.9 Interpretation of results

The outcome of the procedure is a state-dependent spatial boundary of post-liquefaction ground reconfiguration. This boundary represents:

- the maximum extent of redistribution admissible for the prevailing material state,
- a conservative and robust basis for engineering decisions,
- a boundary that may evolve if the material state evolves (e.g. due to saturation or densification).

Apparent stabilization should therefore be interpreted as conditional unless terminal state conditions are reached.

D.10 Relation to the main text and appendices

- Appendix A establishes the conceptual meaning of state and stability.
- Appendix B explains pressure coupling and incompressibility.
- Appendix C provides the formal SEE framework.
- Appendix D (this appendix) translates SEE into a practical engineering workflow.

Together, these elements provide a complete framework for analysing post-liquefaction ground reconfiguration as a state-evolution problem.