

Sampling variability under extreme skewness: sample size guidance for future methane measurement campaigns

William S. Daniels^{1,2}, Dorit M. Hammerling^{1,3}

¹Colorado School of Mines, Golden CO, USA

²Current Affiliation: Johns Hopkins University, Baltimore MD, USA

³Energy Emissions Modeling and Data Lab, The University of Texas at Austin,
Austin TX, USA

Corresponding author: William Daniels
Email: wdanie16@jh.edu

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Sampling variability under extreme skewness: sample size guidance for future methane measurement campaigns

William S. Daniels*,†,‡ and Dorit M. Hammerling†,¶

*†Department of Applied Mathematics and Statistics, Colorado School of Mines,
Golden, Colorado 80401, United States*

†Current Affiliation: Department of Environmental Health and Engineering, Johns Hopkins University, Baltimore, Maryland 21218, United States

*¶Energy Emissions Modeling and Data Lab, The University of Texas at Austin,
Austin, Texas 78712, United States*

* E-mail: wdanie16@jh.edu

Abstract

Methane emissions from the oil and gas sector follow highly right-skewed distributions, making it hard to accurately quantify average emissions with a limited number of measurements. In this study, we probe the statistical implications of sampling (i.e., measuring) from these highly right-skewed distributions, using six US oil and gas basins as an example. For each basin, we provide a minimum sample size that bounds error in the average emission rate estimate introduced by sampling variability. We find that the largest emissions drive sample behavior, and by extension, sample size requirements; samples will underestimate (overestimate) average emissions if super-emitters are observed below (above) their true frequency. Importantly, we show that very large sample sizes can be necessary to mitigate this sampling effect. Furthermore, we find

12 that a one-size-fits-all sampling strategy across basins is suboptimal; differences in
13 super-emitter characteristics between basins necessitate a more tailored sampling ap-
14 proach. To increase the practical applicability of this study, we provide a web tool that
15 both reproduces our findings and performs the same analysis on any user-uploaded dis-
16 tribution of emission rates; the flexibility of this tool enables highly targeted sample
17 size guidance. This work has broad utility across fields where samples are taken from
18 right-skewed distributions.

19 Introduction

20 Methane is a potent greenhouse gas with a relatively short lifetime in the atmosphere;¹
21 this makes reducing methane emissions a key component of short-term climate action.^{2,3}
22 The oil and gas sector accounts for 21% of global anthropogenic methane emissions⁴ and
23 is viewed as a tractable opportunity for emission reduction.⁵ Methane emissions represent
24 a loss of product for many oil and gas operators, providing an economic motivation for
25 emission reduction. Furthermore, certain emission sources within the oil and gas supply
26 chain have clear mitigation pathways: emissions from malfunctioning equipment can often be
27 eliminated once the leaks are identified and engineering improvements can reduce emissions
28 from normally operating equipment like pneumatic controllers.⁵

29 Traditional activity-based, bottom-up estimates of methane emissions from the oil and
30 gas sector are known to underestimate total emissions.⁶⁻¹⁰ As such, efforts to reduce oil and
31 gas methane emissions increasingly rely on direct measurements from ground-based, aerial,
32 or satellite platforms.¹¹ One such effort is the creation of measurement-based emissions in-
33 ventories, in which direct measurements are used to estimate, e.g., annual total emissions
34 from a given oil and gas basin.¹² However, due to the distributed nature of oil and gas in-
35 frastructure, it is extremely challenging to measure methane at all possible source locations
36 within a basin continuously throughout the year; in practice, any measurement-based inven-
37 tory of methane emissions is created using a sample of measurements that lacks complete

38 coverage in time or across sites (or both). As such, total emissions are often estimated by
39 extrapolating an average emission rate from a limited sample of measurements (the “sample
40 mean”) to the temporal and spatial scale of the measurement-based inventory.¹³ This extrap-
41 olation introduces uncertainty in the inventory, raising a key question for future measurement
42 campaigns: how large of a sample is necessary to keep error in the sample mean introduced
43 by sampling variability below an acceptable threshold? In other words, how many sites in a
44 basin must be measured to obtain an accurate measurement-based inventory for that basin?

45 Determining an appropriate sample size is complicated by the fact that methane emissions
46 from the oil and gas sector follow highly right-skewed distributions. In particular, we know
47 from previous campaigns that methane emissions at a single point in time (e.g., from an aerial
48 or satellite technology) across many sites follow a right-skewed distribution.^{10,14,18} This is
49 likely because methane emissions over time on individual sites also follow a right-skewed
50 distribution.^{19,22} The difference between measurements over time on a single site and at a
51 single point in time over many sites is irrelevant for sample statistics (e.g., the sample mean)
52 if methane emissions follow an ergodic process.¹² However, no study (to our knowledge) has
53 tested this assumption, as doing so requires dense coverage both in time and across sites. We
54 do not test the ergodic assumption in this paper and instead illustrate findings that apply
55 to both samples over time on a single site (e.g., from continuous monitoring systems) and
56 samples at a single point in time over many sites (e.g., from an aerial or satellite technology).

57 Importantly, samples from right-skewed distributions do not behave in the same way as
58 samples from distributions with minimal skew. Principle among these differences is the rate
59 at which the sample mean converges to a normal distribution, a property guaranteed by the
60 central limit theorem. In the absence of skew, the distribution of sample means (i.e., the
61 distribution obtained by taking the mean of many repeated samples) is approximately normal
62 even if the sample size is small.²³ If the underlying distribution is skewed, however, then a
63 much larger sample size is necessary for the sample mean to approximately follow a normal
64 distribution.²⁴ In fact, the Berry–Esseen theorem states that the rate of this convergence is

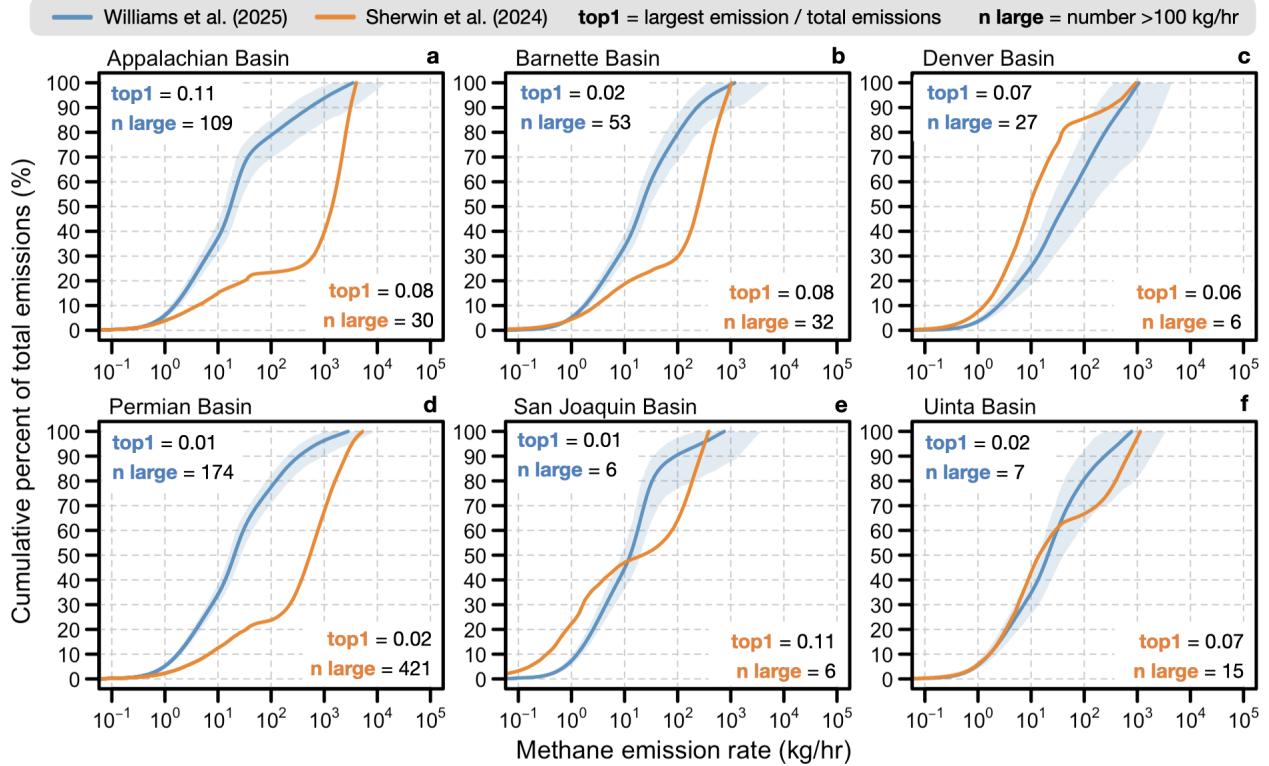


Figure 1: (a) thru (f) The basin-level emission rate distributions used in this study. Williams et al. ²⁶ provide 500 realizations of each distribution; the inner 95% of these realizations is plotted as a shaded region and the average as a solid line. Sherwin et al. ¹⁰ provide multiple distributions for each basin that each correspond to a separate measurement campaign; the campaign that most closely aligns in time with the Williams et al. ²⁶ distribution is selected for use in this study. See the Supporting Information file for details. Two metrics related to the largest emissions are shown for each reference distribution; “top1”: the magnitude of the largest emission relative to the sum of all emissions (“total emissions”), and “n large”: the number of super-emitters (emissions >100 kg/hr).

65 a direct function of skewness.²⁵ As a result, many common practices that assume normality
 66 of the sample mean cannot be used when analyzing highly right-skewed methane emissions;
 67 doing so may misrepresent average emissions or underestimate uncertainty.¹⁷

68 In this article, we quantify some of the statistical implications of sampling from highly
 69 right-skewed methane emission rate distributions. We then provide basin-specific sample
 70 size guidance for future methane measurement campaigns that bounds error in the sample
 71 mean caused by sampling variability. We conduct this analysis for six United States oil
 72 and gas basins by taking many repeated samples from two state-of-the-art emission rate

73 distributions: Williams et al.²⁶ and Sherwin et al.¹⁰. These distributions are shown in
74 Figure 1. The basin-level data provided by these studies allow for basin-specific sample
75 size guidance, expanding previous analysis in Brandt et al.¹⁷. Additionally, using both sets
76 of reference distributions provides a novel comparison of these contemporary estimates of
77 methane emissions from the US oil and gas sector.^{10,26} Finally, we provide a user-friendly
78 web tool that performs the resampling analysis from this paper. This tool can be used to
79 both reproduce our findings using the Williams et al.²⁶ and Sherwin et al.¹⁰ distributions
80 and to conduct the same analysis on any user-uploaded distribution of emission rates or on a
81 selection of parametric distributions. Using this tool, our sample size guidance can be made
82 more specific to a subregion or collection of sites where users have information a priori about
83 the distribution of emission rates.

84 Results

85 Behavior of the sample mean under extreme skewness

86 Figure 2a illustrates a defining feature of highly right-skewed emission rate distributions:
87 small samples that include rare, extremely large emissions will substantially overestimate
88 the true population mean, while the majority of samples that do not contain these large
89 emissions will slightly underestimate. Figure 2a shows the emission rate distribution from
90 Sherwin et al.¹⁰ for the Denver-Julesburg basin ($n = 7,000$) and three samples of size $n =$
91 200 taken without replacement. This distribution is used for illustrative purposes in Figure
92 2; basin-specific sample size guidance is provided later in this section. In Figure 2a, Sample
93 2 includes the largest emission in the population distribution (942 kg/hr) and its mean
94 consequently overestimates the true mean by an order of magnitude. This is because Sample
95 2 is not large enough to contain a sufficient number of small emissions to average out the
96 influence of the largest emission. Samples 1 and 3 do not include the largest emissions and,
97 as a result, underestimate the true average emission rate.

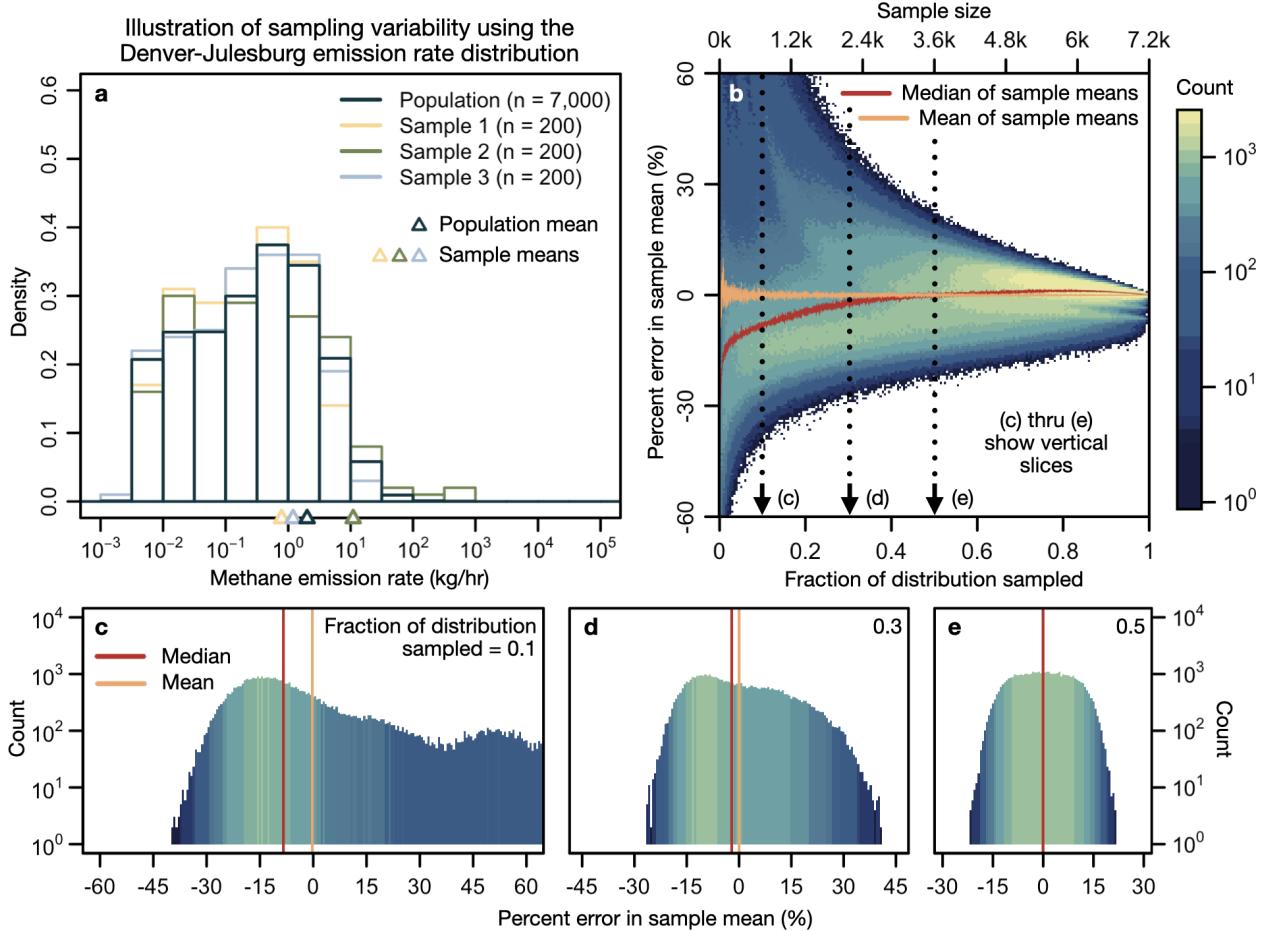


Figure 2: Illustration of sampling variability using the Denver-Julesburg emission rate distribution from Sherwin et al.²⁷. (a) Population distribution and three samples of size $n = 200$ plotted on a log-scale. Average values are shown as triangles along the horizontal axis. (b) Distribution of 1,000 repeated sample means (shown as percent errors) at sample sizes ranging from one to the length of the population distribution. The bottom axis shows sample size as a fraction of the length of the distribution, and the top axis shows sample size as a number of measurements. The color scale shows the number of samples within each grid cell. (c) thru (e) Three vertical “slices” from (b) at different sample sizes (equivalently, different sampled fractions). Both vertical axis and color scale show the number of samples within each percent error bin. Note that the mean and median are visually the same in (e).

98 Figure 2b shows the behavior of 1,000 samples taken at sample sizes ranging from a
 99 single measurement to the size of the full distribution. These samples were again taken
 100 without replacement; see the Methods section for a discussion of this choice. Figures 2c
 101 thru e show three vertical “slices” from Figure 2b as histograms. For the Denver-Julesburg
 102 basin, a surprisingly large sample is required to mitigate the influence of the rare super-

103 emitters. Sampling 50% of the distribution (the threshold for comprehensive spatial coverage
104 in Sherwin et al.^[10]) results in a 95% confidence interval for the sample mean between -15.0%
105 and 15.3% of the true mean and a maximum error of 23.0% in the sample mean. Note
106 that the median becomes slightly positive when sampling over 50% of the distribution; this
107 behavior is explained in the following section.

108 Smaller samples amplify the effects of sampling variability: with a sample of size $n = 200$
109 (as used in Figure 2a), the 95% confidence interval for the sample mean is between -40.5%
110 and 209.6% and the maximum error is 378.5%. Furthermore, at this sample size, there
111 is a 72% chance that the sample mean will underestimate by failing to capture the rare,
112 large emissions in the population distribution. In summary, for the Denver-Julesburg basin
113 (according to the Sherwin et al.^[10] distribution), measuring 200 sites results in substantial
114 variability in the sample mean, and even a relatively large sample covering half of the basin
115 is not enough to fully mitigate these sampling effects.

116 We pause here to highlight two important points. First, errors of the magnitude seen
117 in Figure 2 are not uncommon in individual emission rate estimates, regardless of the mea-
118 surement technology (e.g., ground-based, aerial, satellite).^[27,31] However, the percent errors
119 in Figure 2 are in the *mean* of many emission rate estimates, with sample sizes ranging from
120 one to the length of the full distribution. Second, the behavior of the sample mean shown in
121 Figure 2 is not a result of *bias* in the sample mean; the sample mean is an unbiased estimator
122 of the population mean by definition.^[23] As such, the mean of many repeated sample means
123 will equal the true population mean (seen in Figure 2). However, in practice it is often
124 infeasible to take many repeated samples, making it important to understand the behavior
125 of an individual sample under different sample sizes and super-emitter regimes.

126 Impact of super-emitters on sampling variability

127 Figure 3 highlights the outsized influence of super-emitters on sampling variability. Again
128 using the Denver-Julesburg distribution from Sherwin et al.^[10], Figure 3 shows three error

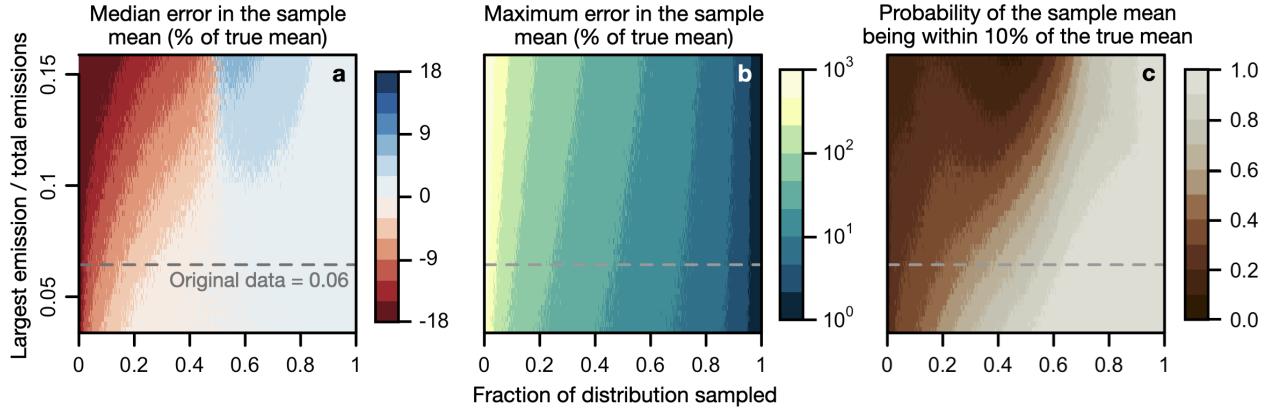


Figure 3: Impact of the largest emission in the population distribution on sampling variability. The subplots show three error metrics for the sample mean using the Denver-Julesburg emission rate distribution from Sherwin et al.^[10] as an example. The horizontal axis shows sample size as a fraction of the population. The vertical dimension shows how the error metrics change as the magnitude of the largest emission in the population distribution is adjusted. The vertical axis gives the magnitude of the largest emission in the population distribution relative to total emissions. The true ratio of largest emission to total emissions for the Sherwin et al.^[10] Denver-Julesburg basin is shown as a horizontal dashed line. All other rows are generated by artificially replacing the largest emission with a different value and taking new samples.

129 metrics for the sample mean across a range of sample sizes and as the magnitude of the
 130 largest emission in the population distribution changes. We manually adjust the largest
 131 emission in the Sherwin et al.^[10] distribution to illustrate this behavior.

132 At sample sizes below 50% of the distribution, the sample mean becomes more likely to
 133 underestimate as the largest emission increases (Figure 3a). This is because average (or,
 134 equivalently, cumulative) emissions are sensitive to outliers, making it increasingly necessary
 135 to observe the largest emission as its magnitude, and therefore influence on the mean, in-
 136 creases. Interestingly, sample sizes above 50% have a slight tendency to overestimate. This
 137 is because these samples are all more likely to observe the largest emission than to miss it,
 138 and once the largest emission is observed, a large number of “normal” emissions closer to the
 139 bulk of the distribution must be observed to counteract its influence.

140 A single sample becomes less likely to be within 10% of the true mean as the magnitude
 141 of the largest emission in the underlying distribution increases (Figure 3c). This metric is
 142 particularly important for determining the sample size of future measurement campaigns,

143 as a campaign provides just one “sample” from the true emission rate distribution. As seen
144 in Figure 3c, it becomes increasingly necessary to measure almost all sites within a basin as
145 the magnitude of the largest emission increases relative to total emissions.

146 **Basin-specific sample size guidance**

147 Figure 4 provides sample size guidance for future methane measurement campaigns in six
148 US oil and gas basins based on three error metrics related to the sample mean. For each
149 metric, we show the sample size required to keep the error introduced by sampling variability
150 below a given threshold. The specific thresholds shown in Figure 4 are not meant to be an
151 exhaustive list, rather they are meant to demonstrate how sample size requirements vary by
152 basin, reference distribution, and error tolerance.

153 We highlight three findings from Figure 4. First, the behavior of the largest emissions
154 drives sampling variability and, by extension, sample size requirements across the three
155 error metrics studied here. In particular, the magnitude of the largest emission in the true
156 population distribution relative to total emissions is a significant predictor ($p < 0.01$) of
157 required sample size based on a simple linear regression for all three error metrics. This is a
158 direct result of the behavior shown in Figure 3: the presence of super-emitters that are both
159 large and rare make it hard to accurately characterize average emissions. Interestingly, basins
160 with more super-emitters (>100 kg/hr) require smaller samples to accurately characterize
161 average emissions. This is because a measurement campaign is more likely to observe super-
162 emitters in basins where they are more common, like the Permian, than in basins where they
163 are relatively rare, like the Denver-Julesburg. As a result, it easier to characterize both the
164 bulk and the tail of the emission rate distribution in the Permian, leading to a more accurate
165 estimate of average emissions at smaller sample sizes. This fact has important implications
166 for measurement campaign design; if the objective is to detect as many super-emitters as
167 possible (e.g., for emission mitigation), then it is desirable to allocate more measurements
168 to basins with more super-emitters. Conversely, if the objective is to characterize average

169 emissions as accurately as possible (e.g., for inventory development), then it is desirable to
 170 allocate more measurements to basins with fewer super-emitters where they are harder to
 171 find.

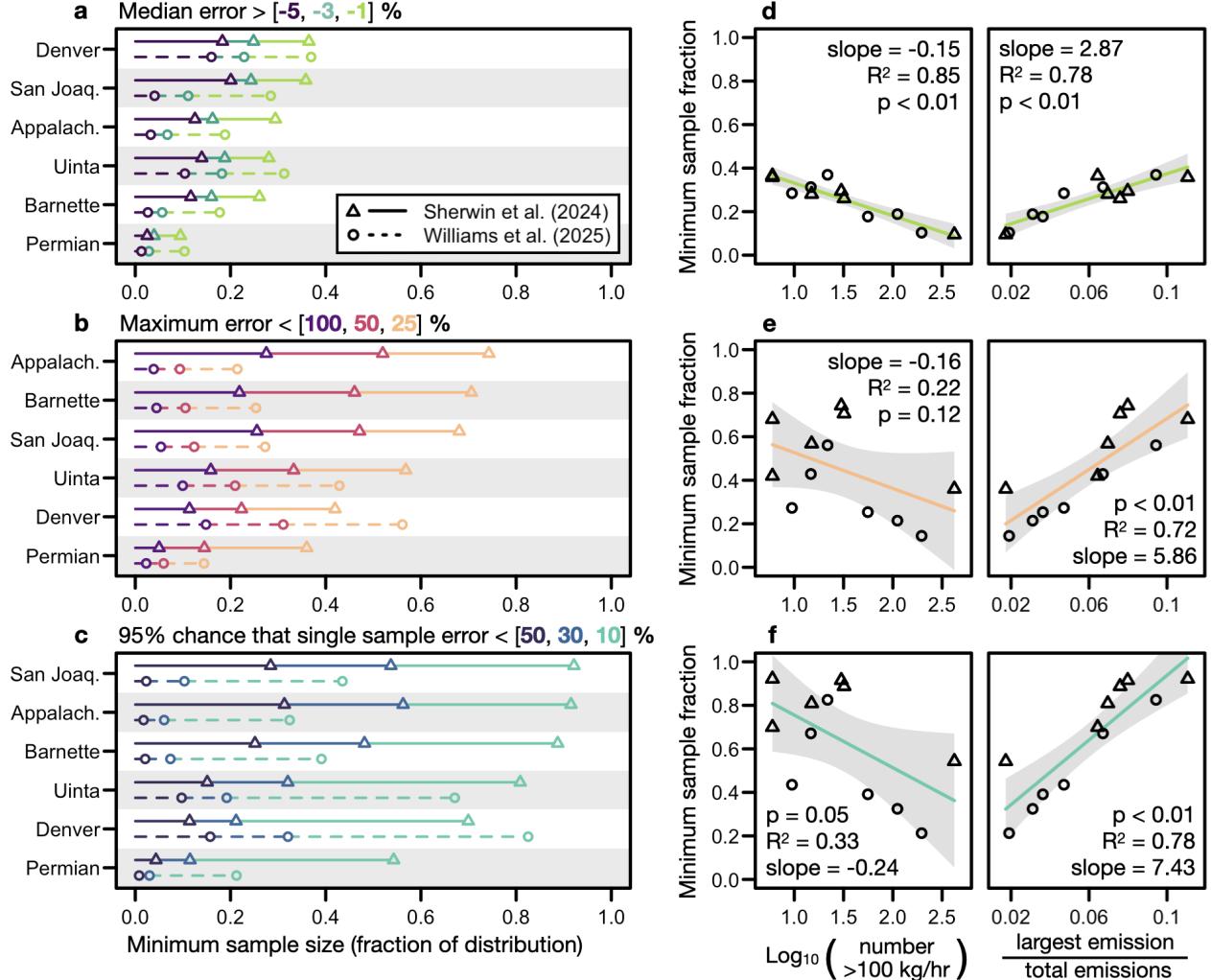


Figure 4: Basin-specific sample size guidance according to three different error metrics for the sample mean: (a) median error, (b) maximum error, and (c) single sample error. Each point shows the minimum sample size fraction required to satisfy the corresponding error threshold listed in the plot title. Correspondence between points and error thresholds is shown with color. Point and line type denote the reference distribution. Basins on the vertical axis are sorted according to the Sherwin et al.^[10] values within each plot. (d) thru (f) Linear relationship between two features of the emission rate distributions and the sample size required to meet the strictest error thresholds from (a) thru (c). Each point corresponds to a basin-level distribution from either Williams et al.^[26] or Sherwin et al.^[10]. Vertical axis shows the sample size required to satisfy the strictest error threshold from (a) thru (c), and the horizontal axes shows the feature values: the base-10 logarithm of the number of super emitters (>100 kg/hr) and the magnitude of the largest emission relative to total emissions.

172 Second, despite variability between basins, very large samples are often required to keep
173 the error in a single sample below 10% (at the 95% confidence level). As stated above,
174 this is an important metric for measurement campaign design, as each campaign provides
175 one “sample” of the emission rate distribution at a snapshot in time. For example, when
176 using the Sherwin et al.^[10] distributions, campaigns must measure over 80% of sites in the
177 San Joaquin, Appalachian, Barnette, and Uinta basins to keep the error in a single sample
178 below 10% (at the 95% confidence level). When using the Williams et al.^[26] distributions,
179 campaigns must measure over 80% of sites in the Denver-Julesburg basin to meet the same
180 error threshold.

181 Third, as illustrated by the previous example, there are substantial differences between
182 the Williams et al.^[26] and Sherwin et al.^[10] emission rate distributions. In particular, the
183 Williams et al.^[26] distributions have a larger contribution from emissions <100 kg/hr than
184 the Sherwin et al.^[10] distributions in all but the Denver-Julesburg basin. Similarly, the
185 Sherwin et al.^[10] distributions have a larger contribution to total emissions from the single
186 largest emission than the Williams et al.^[26] distributions in four of the six basins. Because
187 the largest emissions have an outsized influence on the behavior of the sample mean, the
188 Sherwin et al.^[10] distributions often have more variability in the sample mean than the
189 Williams et al.^[26] distributions, which ultimately results in larger sample size requirements.
190 Given the large difference in sample size requirements between these two sets of distributions,
191 it is clear that more work is needed to definitively characterize the distribution of methane
192 emission rates in US oil and gas basins.

193 Discussion

194 Despite recent federal deregulation in the United States, ongoing efforts to measure methane
195 from the US oil and gas sector will likely continue. From an economic perspective, methane
196 emissions often represent a loss of product to oil and gas operators, incentivizing measurement-

197 based surveys to quickly identify leaks. From a reporting perspective, voluntary global pro-
198 grams, such as OGMP 2.0,³² and recent import requirements set by the European Union³³
199 both require measurement-based emissions accounting. The latter has demonstrated that
200 participation in parts of the global natural gas market may depend on the ability to measure
201 and verify emission intensities. In addition, measurement-based reporting requirements may
202 resume in the US, such as the methane fee for large release events previously introduced by
203 the US Environmental Protection Agency.³⁴

204 Whether or not they are intended to satisfy the reporting programs discussed above, all
205 future methane measurement campaigns will need to select a sample size. In this study,
206 we have provided specific sample size requirements to bound errors introduced by sampling
207 variability in six US oil and gas basins. We find that sample size requirements vary by
208 basin, largely being driven by differences in super-emitter characteristics between basins.
209 Furthermore, we find notably different sample size requirements when using different ref-
210 erence distributions.^{10,26} Improved sample size guidance will require reconciliation of these
211 state-of-the-art estimates of methane emissions from the US oil and gas sector.

212 To broaden the applicability of this work, we have created a web tool to both reproduce
213 our analysis using the Williams et al.²⁶ and Sherwin et al.¹⁰ distributions and to perform
214 the same analysis on any user-uploaded distribution or a selection of parametric distribu-
215 tions. Using this tool, sample size requirements can be made more specific to a subregion or
216 collection of sites (e.g., all sites owned by a specific operator) where users have information
217 a priori about the distribution of emission rates. Additionally, this tool can be used to cal-
218 culate sample size requirements at any user-specified error thresholds, not just the selection
219 of thresholds shown in Figure 4.

220 Our method for estimating sampling variability, made publicly available in the web tool
221 discussed above, also has broad applicability beyond methane emissions from the oil and
222 gas sector. Any measurement campaign focused on atmospheric pollutants or trace gasses
223 that is unable to visit all possible source locations will need to make a sample size deter-

224 mination. Ignoring the impact of sampling variability for any such campaign may lead to
225 misinterpretation of uncertainties in the downstream analysis.

226 While the focus of this article was on the distribution of emissions over sites at a single
227 point in time, the methodology (and web tool) are equally applicable for distributions of
228 emissions over time on a single site. In this context, a similar question often emerges:
229 how often do I need to measure a given site to accurately estimate the long-term average
230 emission rate? The answer to this question has implications for, e.g., the number of fixed
231 point sensors that must be installed around the fenceline of a given facility; more sensors
232 mean more coverage but come at a higher cost. As estimates for emission rate distributions
233 over time evolve (e.g., from continuous point sensors), they can be uploaded to the web tool
234 directly for temporal sample size guidance.

235 Finally, it is important to reiterate that the sample size guidance presented in this study
236 only considers sampling variability. There may be other considerations that contribute to
237 measurement campaign design, such as a desired stratification across facility types.^{12,35} Fur-
238 thermore, this study (intentionally) does not consider variability introduced by temporal
239 intermittency, or the fact that methane emissions vary over time. Other studies have made
240 advancements in this direction by conducting repeat measurement campaigns of the same
241 region, finding that aggregated methane emissions are different (to varying degrees) be-
242 tween repeat campaigns.^{10,35} These differences are a result of both temporal variability and
243 sampling variability (because only a subset of sites were measured). By isolating errors in
244 the sample mean caused solely by sampling variability, we provide a way to estimate the
245 contribution of temporal variability alone when conducting repeat measurement campaigns.

246 Methods

247 Reference emission rate distributions

248 Williams et al.²⁶ used data from 16 studies to create basin-level methane emission rate

249 distributions. Approximately 85% of these data are from site-level measurements using
250 technologies with low detection thresholds around 0.1 kg/hr, such as tracer-based releases,
251 EPA Other Test Method (OTM 33), or Gaussian dispersion modeling. The other 15% of
252 the data are from aggregated component-level measurements, such as Hi-Flow samplers or
253 flux chambers. These measurement data are scaled to the basin-level within a probabilistic
254 framework that leverages activity data and flaring detections.

255 Sherwin et al.^[10] used aerial data from Kairos Aerospace (now Insight M) and Carbon
256 Mapper (CM) together with the bottom-up simulation tool from Rutherford et al.^[8] to create
257 methane emission rate distributions at the basin-level. Both Kairos and CM have higher de-
258 tection thresholds (~ 10 kg/hr) than the measurement technologies used in Williams et al.^[26].
259 Sherwin et al.^[10] provide multiple distributions per basin, each corresponding to a different
260 measurement campaign. For the analysis in this paper, we select the Sherwin et al.^[10] distri-
261 butions that most closely align in time with the Williams et al.^[26] basin-level distributions.
262 See the Supporting Information file for details.

263 Resampling methodology

264 For both the Williams et al.^[26] and Sherwin et al.^[10] emission rate distributions, we resample
265 the data to quantify sampling variability under different sample sizes, roughly following the
266 procedure in Chen et al.^[36]. That is, for each of the distributions shown in Figure 1, we
267 sample 1,000 times without replacement at sample sizes ranging from a single measurement
268 to the size of the entire distribution. We then compare the 1,000 sample means to the true
269 distribution mean using three metrics: the median percent error (Q_{50}), the maximum percent
270 error (ϵ_{\max}), and the probability of a single sample being within 10% of the truth (\mathbb{P}_{10}). For
271 a given sample size and distribution, if we let $\bar{\mathbf{x}}^* = \{\bar{x}_1^*, \dots, \bar{x}_{1000}^*\}$ represent the 1,000 sample
272 means and μ the true distribution mean, then we compute the percent error in each sample
273 mean as

$$\epsilon = 100 \times (\bar{\mathbf{x}}^* - \mu) / \mu$$

274 and the three metrics as

$$Q_{50} = \text{median}(\epsilon), \quad \epsilon_{\max} = \max(\epsilon), \quad \mathbb{P}_{10} = \frac{\text{number of } |\epsilon| < 10}{\text{length of } \epsilon}.$$

275 We sample without replacement to isolate sampling variability from other causes of variabil-
276 ity, such as temporal intermittency. The physical interpretation of this approach depends
277 on whether the true population distribution represents emissions over time on a single site
278 or emissions over sites at a single point in time. For emissions over time, sampling without
279 replacement ensures that the estimate of long-term average emissions approaches the true
280 value as measurements are taken at a higher frequency (and, in the limit, approach a truly
281 continuous measurement system). In this context, sampling 50% of the distribution can
282 be interpreted as measuring emissions half of the time. For emissions over sites, sampling
283 without replacement ensures that measuring all sites results in a correct estimate of average
284 emissions at a snapshot in time, which is the desired behavior if we ignore temporal variabil-
285 ity. In this context, sampling 50% of the distribution can be interpreted as measuring half
286 of the sites within a given domain. This paper focuses on the latter situation.

287 This resampling procedure assumes that the distributions from Williams et al.^[26] and
288 Sherwin et al.^[10] are the true underlying emission distributions in their respective regions,
289 despite these distributions being (very large) samples themselves. The resampling procedure
290 also assumes that emissions of all sizes can be detected, while in reality all measurement tech-
291 nologies have a probability of detection that decreases with emission magnitude. However,
292 emissions below the detection threshold of the measurement technology can be estimated
293 (usually via a bottom-up inventory) and added to the estimate of total emissions.

294 Data Availability

295 Methane emission rate distributions from Sherwin et al.^[10] can be obtained from their “Source
296 data” section. Methane emission rate distributions from Williams et al.^[26] can be obtained

297 from Williams³⁷.

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301 Author Contributions

302 W.S.D.: Conceptualization, Analysis, Visualization, Writing - Original Draft, Writing -
303 Editing and Review. D.M.H.: Conceptualization, Writing - Editing and Review, Funding
304 Acquisition, Project Supervision.

305 Competing Interests

306 The authors have no competing interests to declare.

307 Supporting Information

308

- Supporting information file.

309

- Web tool: <https://mbasanese-sampling.share.connect.posit.cloud/>

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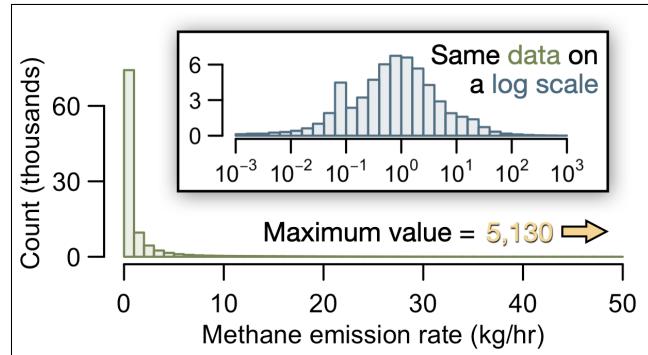
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455 TOC Graphic

456



Supporting Information for:
**Sampling variability under extreme skewness:
sample size guidance for future methane measurement campaigns**

William S. Daniels^{1,2} and Dorit M. Hammerling^{1,3}

¹Department of Applied Mathematics and Statistics, Colorado School of Mines,
Golden, Colorado 80401, United States

²Current Affiliation: Department of Environmental Health and Engineering, Johns Hopkins University,
Baltimore, Maryland 21218, United States

³Energy Emissions Modeling and Data Lab, The University of Texas at Austin,
Austin, Texas 78712, United States

Email: wdanie16@jhu.edu

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S1 Details on selected reference distributions

Sherwin et al. (2024) [1] provide one to five emission rate distributions per basin, each corresponding to a different measurement campaign. Williams et al. (2025) [2] provide 500 realizations of the methane emission rate distribution for each basin. These realizations are meant to represent uncertainty from their modeling framework for the emission rate distribution for 2021. Because the Sherwin distributions span multiple years, we select one Sherwin distribution per basin that most closely aligns in time with the Williams distributions. This was done instead of averaging the Sherwin distributions to mitigate the effects of temporal variability when comparing between the two studies. Table [1] lists the Sherwin distributions that were selected for use in this study.

Justification for our selection is as follows. Sherwin et al. (2024) provide only one distribution for the Appalachian, Barnette, and Uinta basins. For the Denver-Julesburg, we select the latest of the two distributions (Fall 2021). For the Permian, we select the campaign with the best coverage (Carbon Mapper 2019) over the more recent campaigns (Carbon Mapper 2020-2021). This is because we treat the Sherwin and Williams distributions as the true population distributions for their respective basins in this study, so having as large a sample helps mitigate sampling variability in our choice of population distribution. For the San Joaquin, we select the latest distribution (Fall 2021).

For all of our analysis using the Williams et al. (2025) distributions, we use the average of the 500 realizations produced by their simulation framework. To perform this average, we first sort each realization from the smallest to the largest emission. We then average the smallest emission from all 500 realizations, the second smallest emission from all 500 realizations, et cetera. This procedure works because all 500 realizations have the same length.

Table 1: Exact reference distributions used in this study.

Basin name used in this study	Distribution from Sherwin et al. (2024)	Distribution from Williams et al. (2025)
Appalachian	Carbon Mapper Pennsylvania 2021	Appalachian
Barnette	Kairos Fort Worth 2021	Barnette
Denver-Julesburg	Carbon Mapper Denver-Julesburg Fall 2021	Denver-Julesburg
Permian	Carbon Mapper Permian 2019	Permian
San Joaquin	Carbon Mapper San Joaquin Fall 2021	San Joaquin
Uinta	Carbon Mapper Uinta 2021	Uinta

S2 Distribution of sample means for each basin

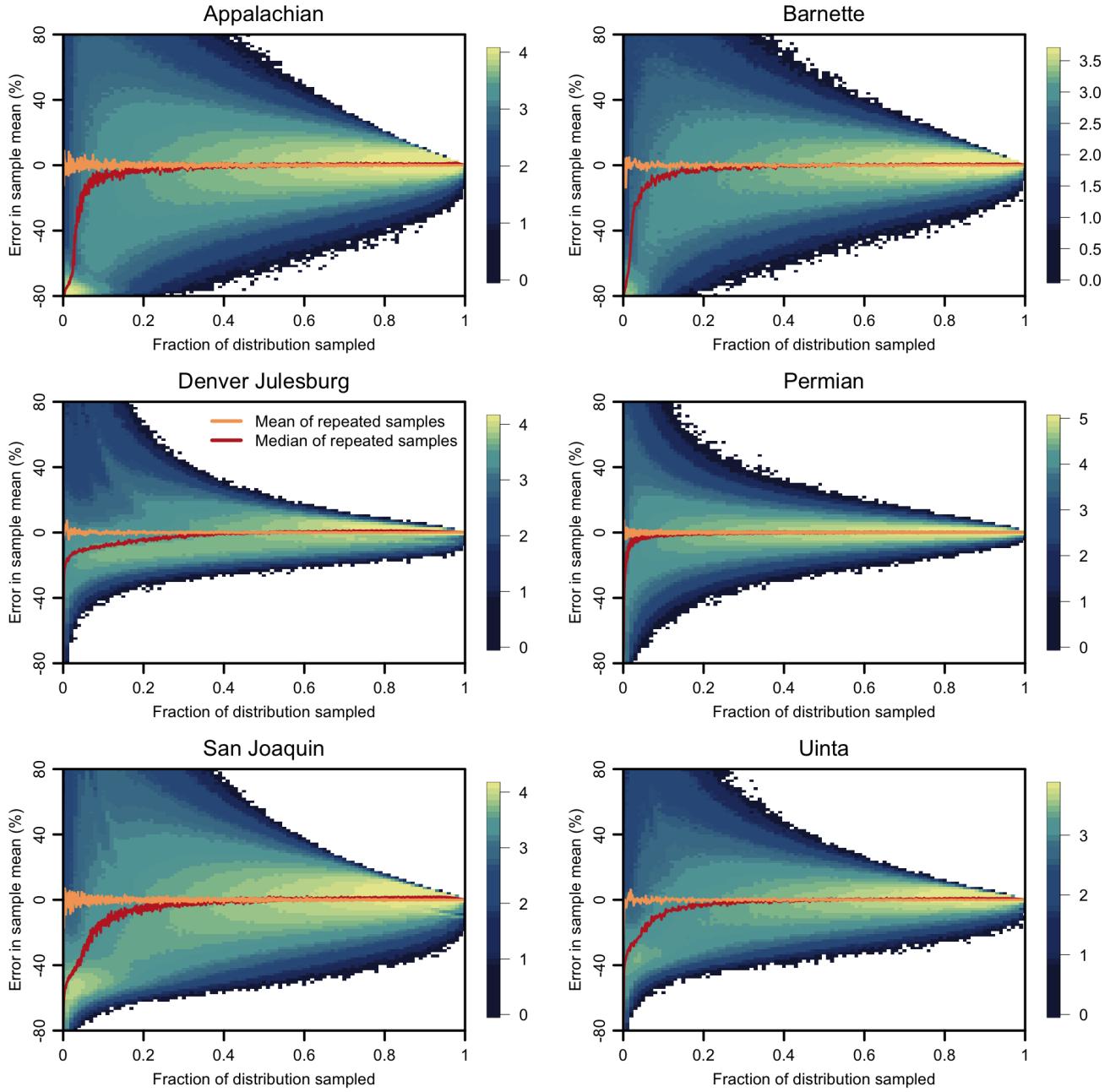


Figure S1: Distribution of sample means at different sample sizes using the Sherwin et al. (2024) II_{I} distributions. Vertical axis shows percent error between sample mean and true population mean. Horizontal axis shows the sample size as a percent of the length of the population distribution. Color scale shows the number of samples within each grid cell.

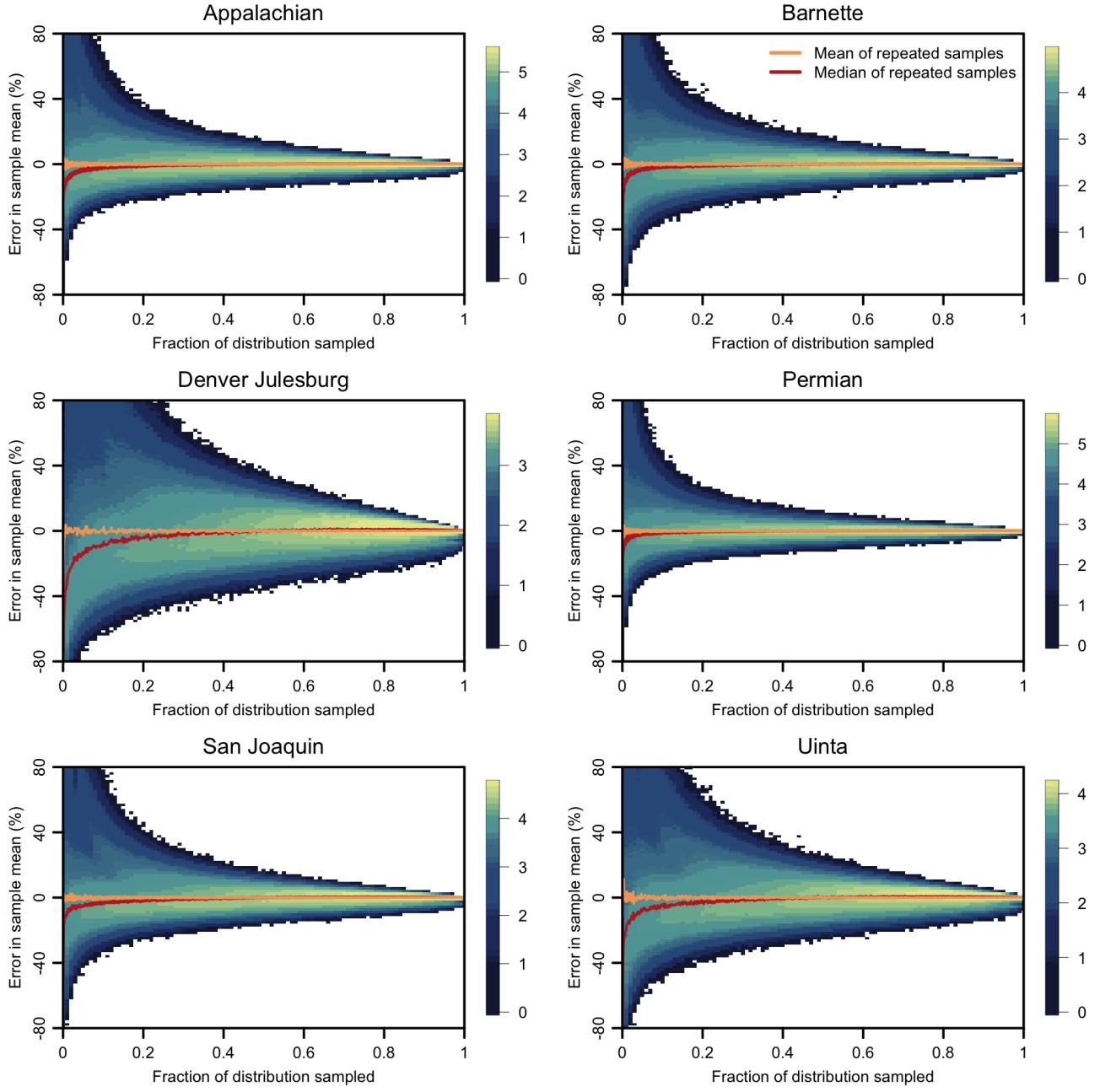


Figure S2: Distribution of sample means at different sample sizes using the Williams et al. (2025) [2] distributions. Vertical axis shows percent error between sample mean and true population mean. Horizontal axis shows the sample size as a percent of the length of the population distribution. Color scale shows the number of samples within each grid cell.

S3 Sample mean metrics at all sample sizes

Figures S3 through S14 show the sample mean error metrics used in Figure 4 in the main text evaluated at all sample sizes. See the main text for a definition of each metric. Note that sample size is expressed as a fraction of the full distribution. In all of the following plots, the light shaded lines show the metric value at every sample size, and the darker lines show the centered moving average with window size equal to 0.1% of the length of the distribution.

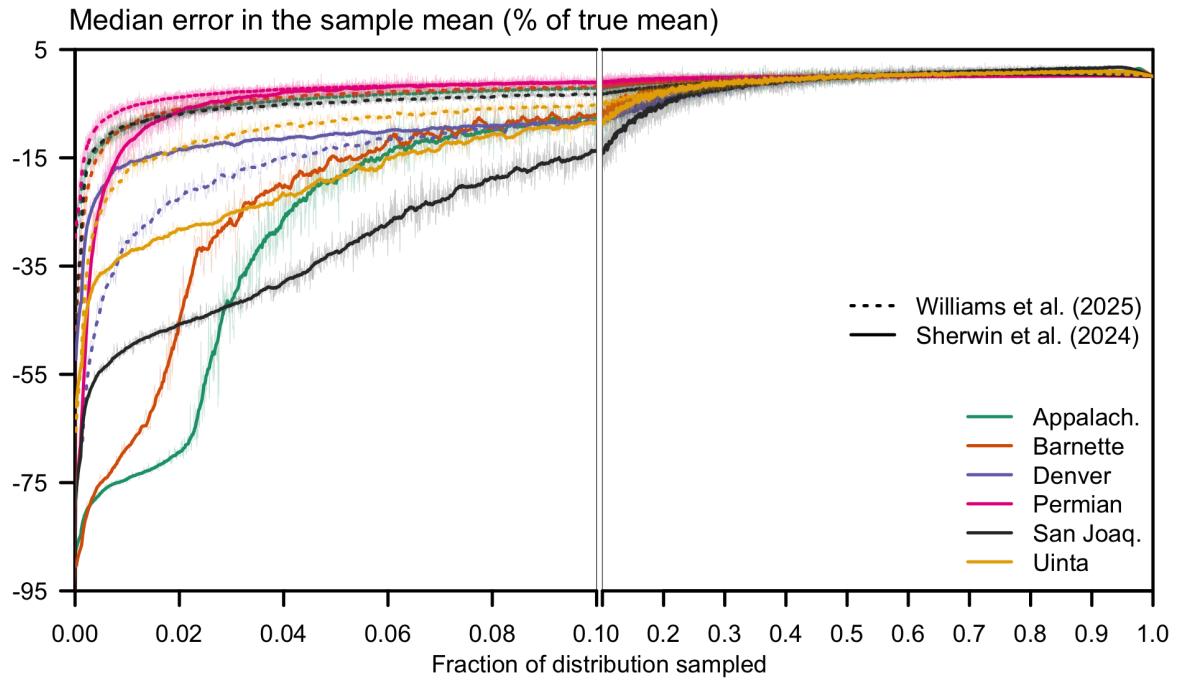


Figure S3: Median error in the sample mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

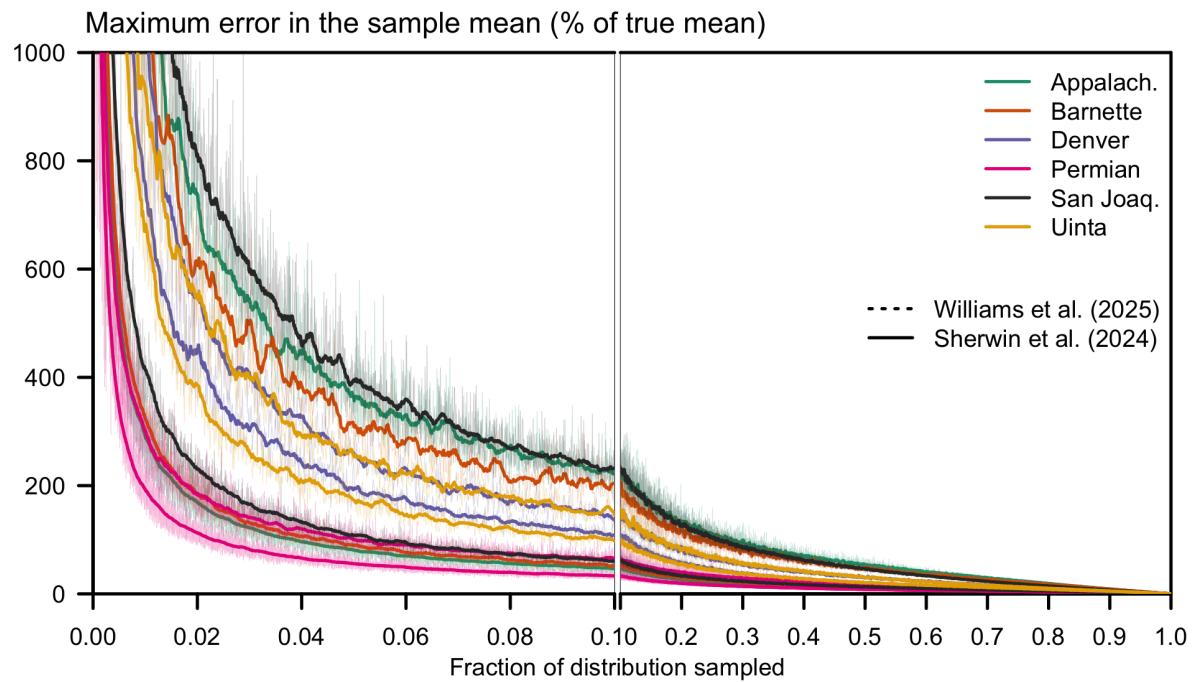


Figure S4: Maximum error in the sample mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

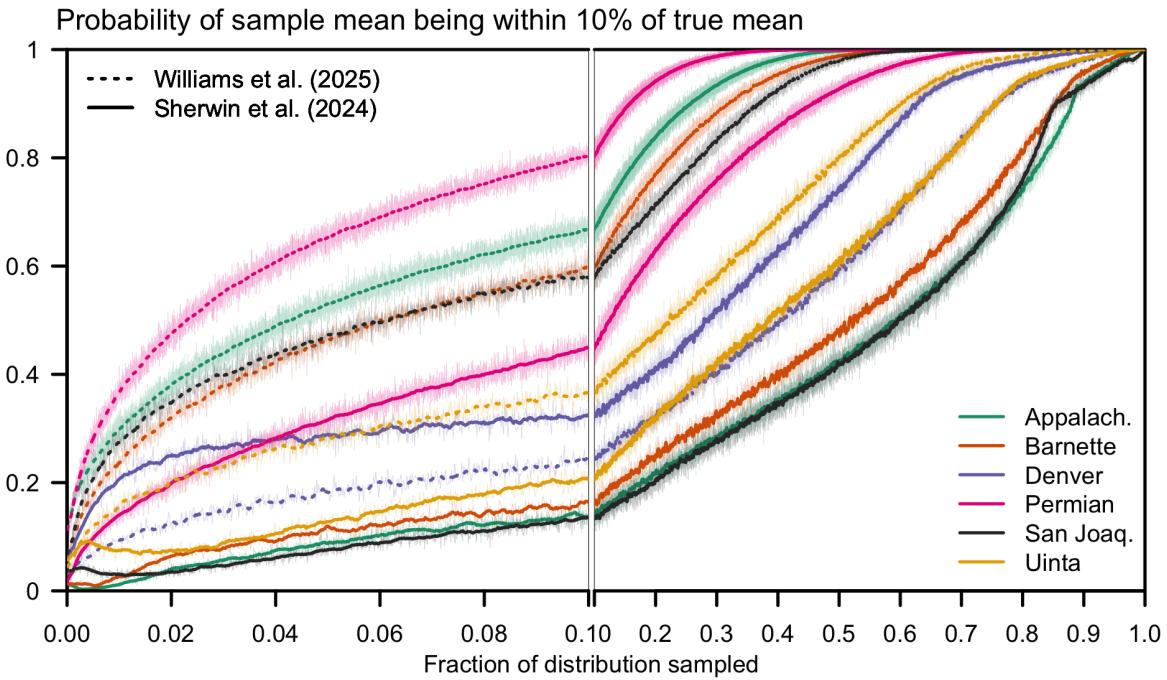


Figure S5: Probability of the sample mean having error within 10% of the true mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

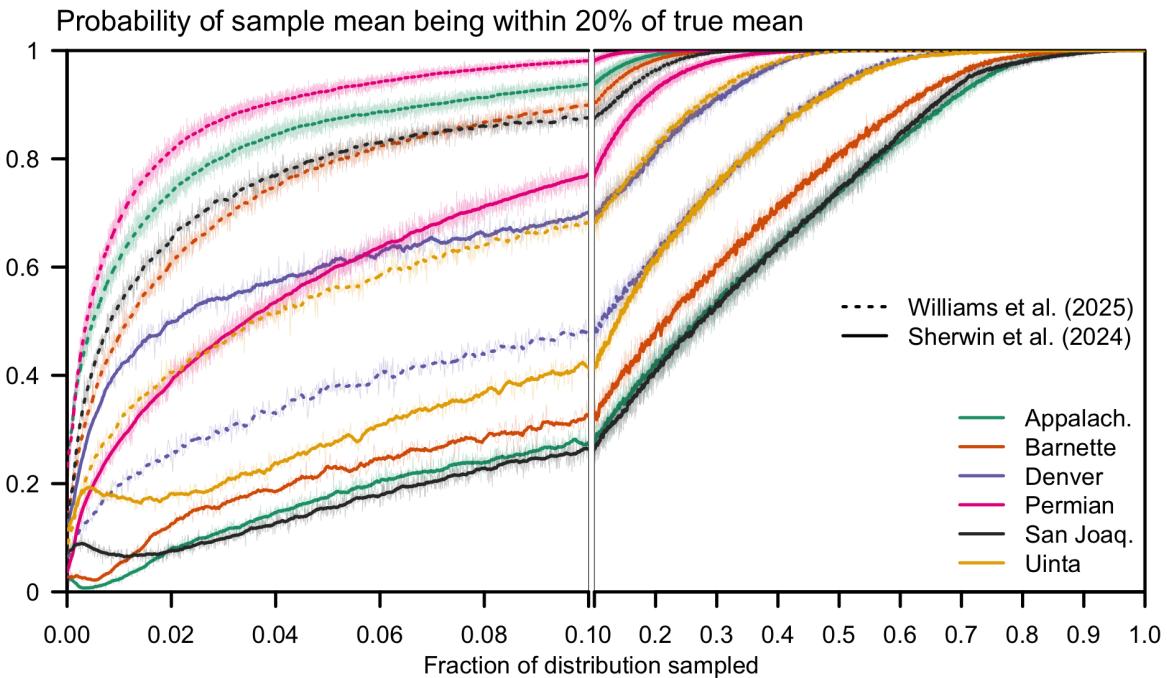


Figure S6: Probability of the sample mean having error within 20% of the true mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

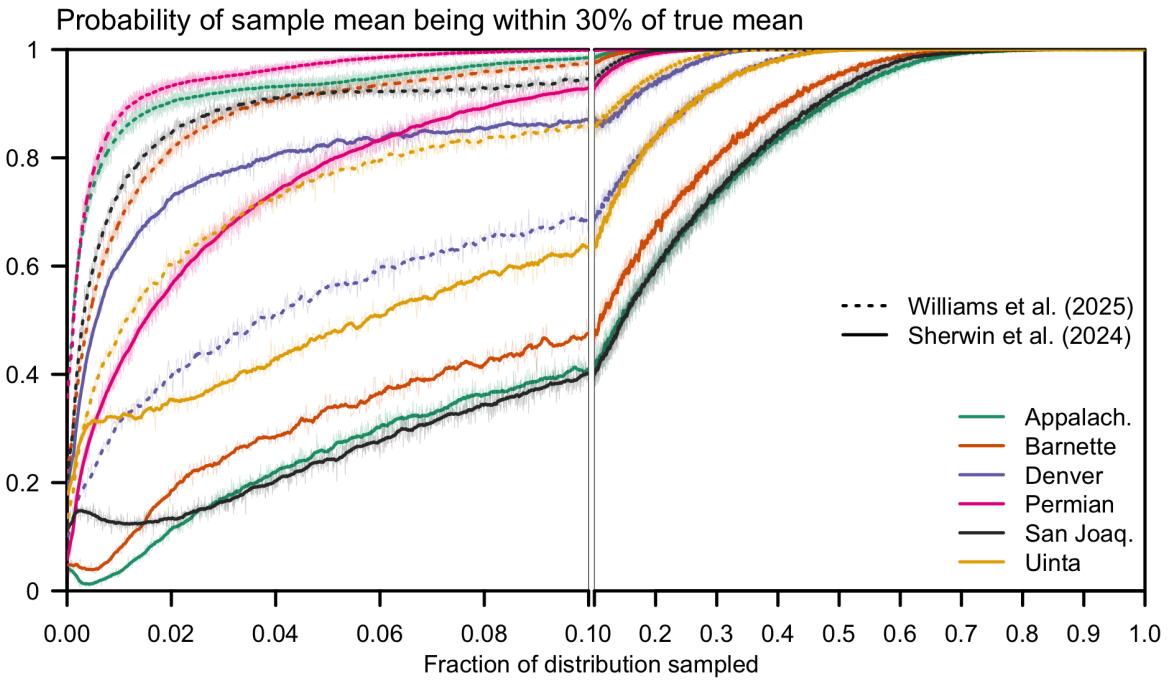


Figure S7: Probability of the sample mean having error within 30% of the true mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

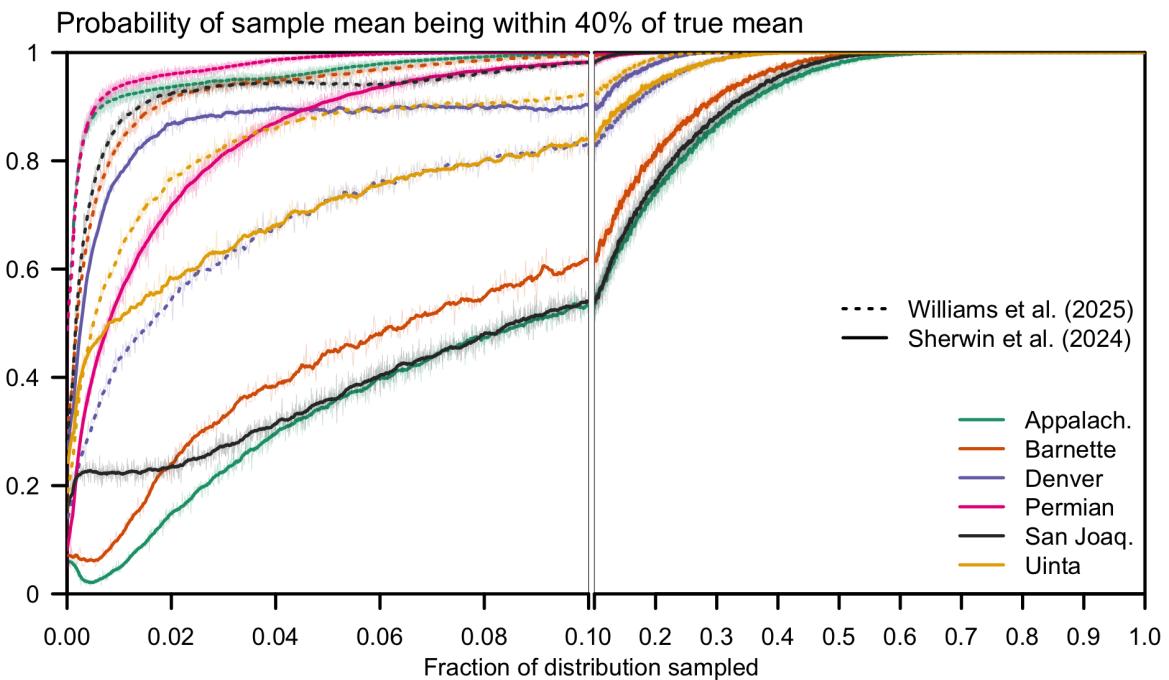


Figure S8: Probability of the sample mean having error within 40% of the true mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

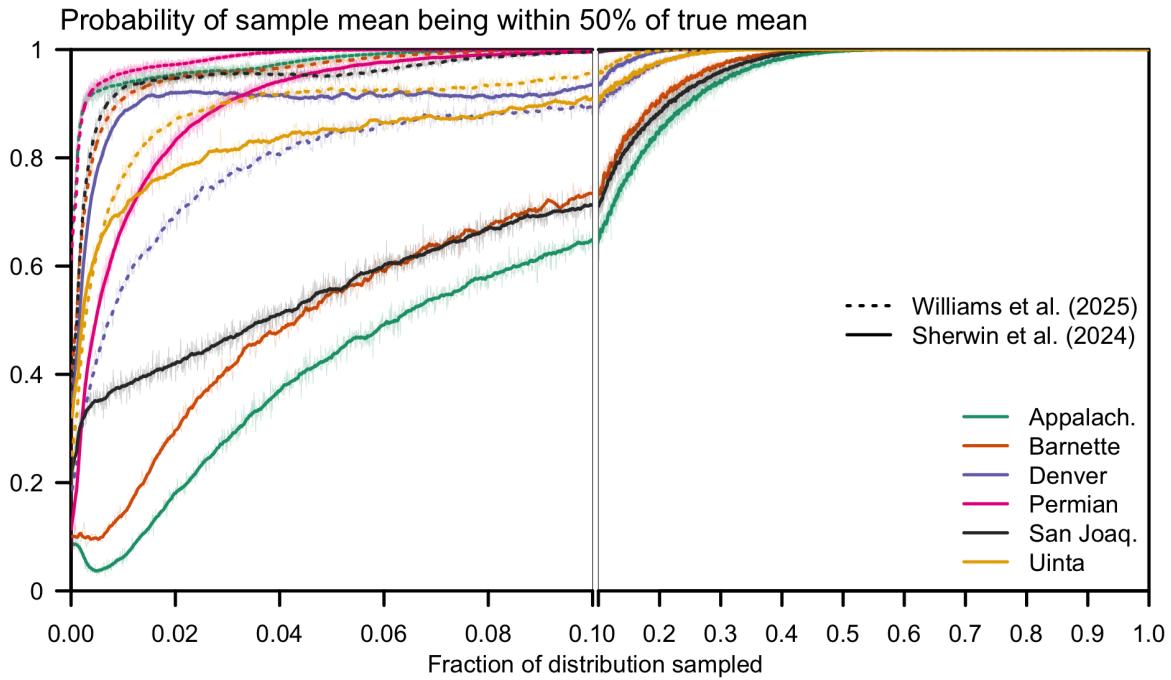


Figure S9: Probability of the sample mean having error within 50% of the true mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

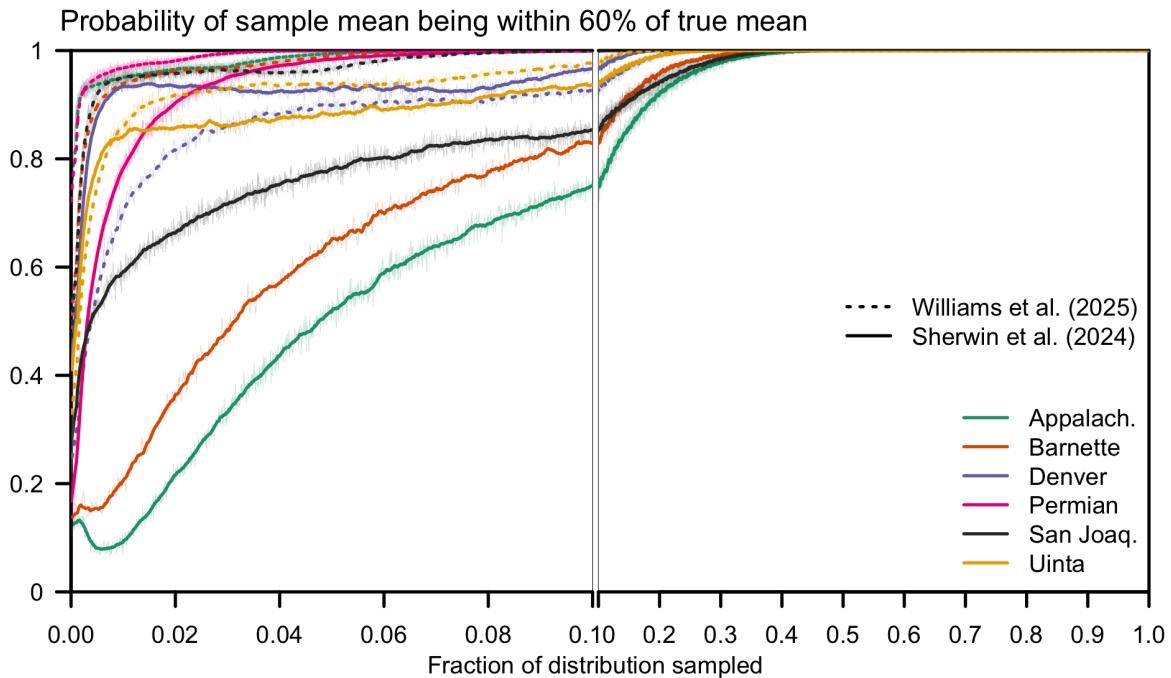


Figure S10: Probability of the sample mean having error within 60% of the true mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

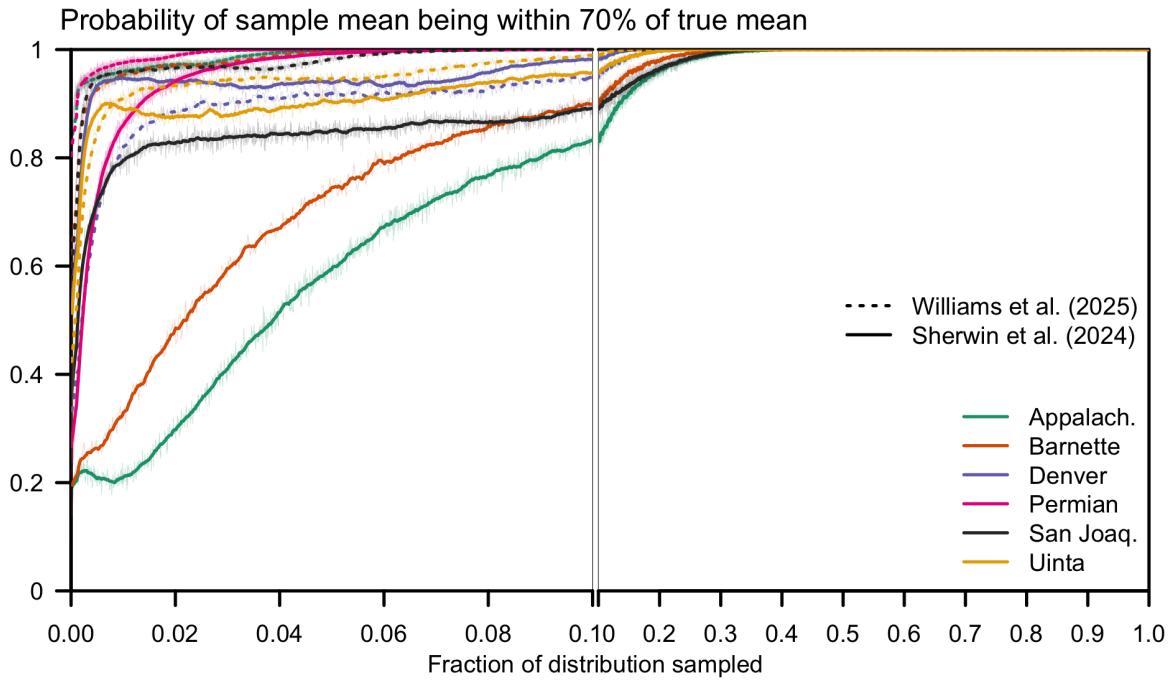


Figure S11: Probability of the sample mean having error within 70% of the true mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

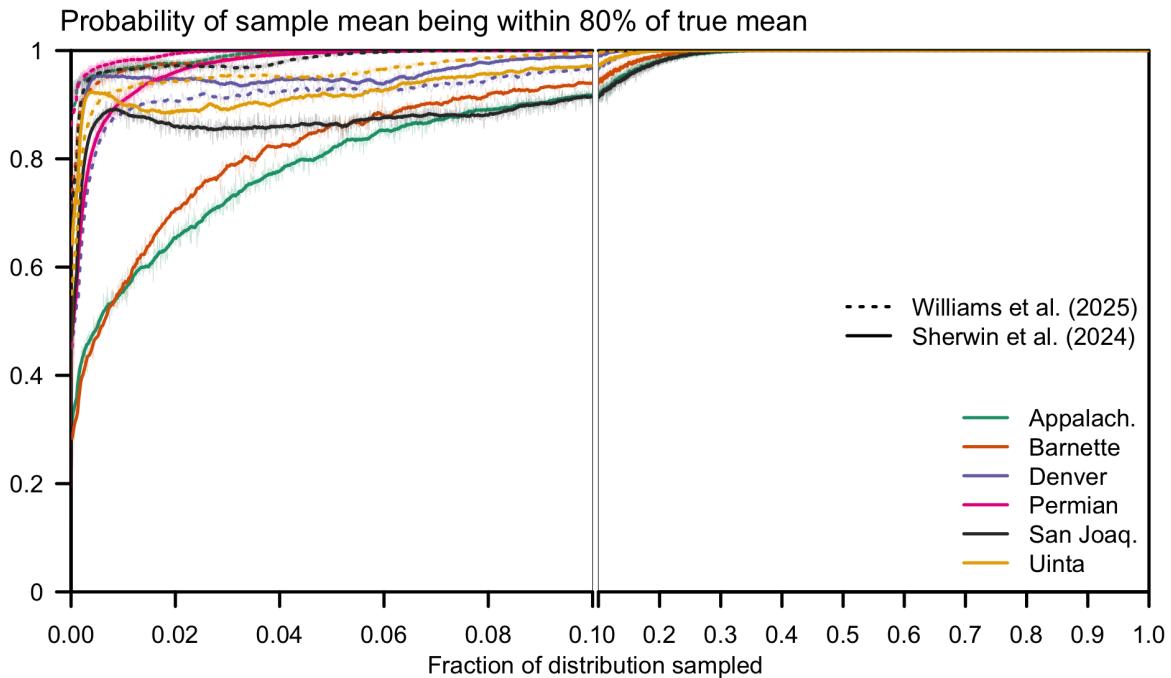


Figure S12: Probability of the sample mean having error within 80% of the true mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

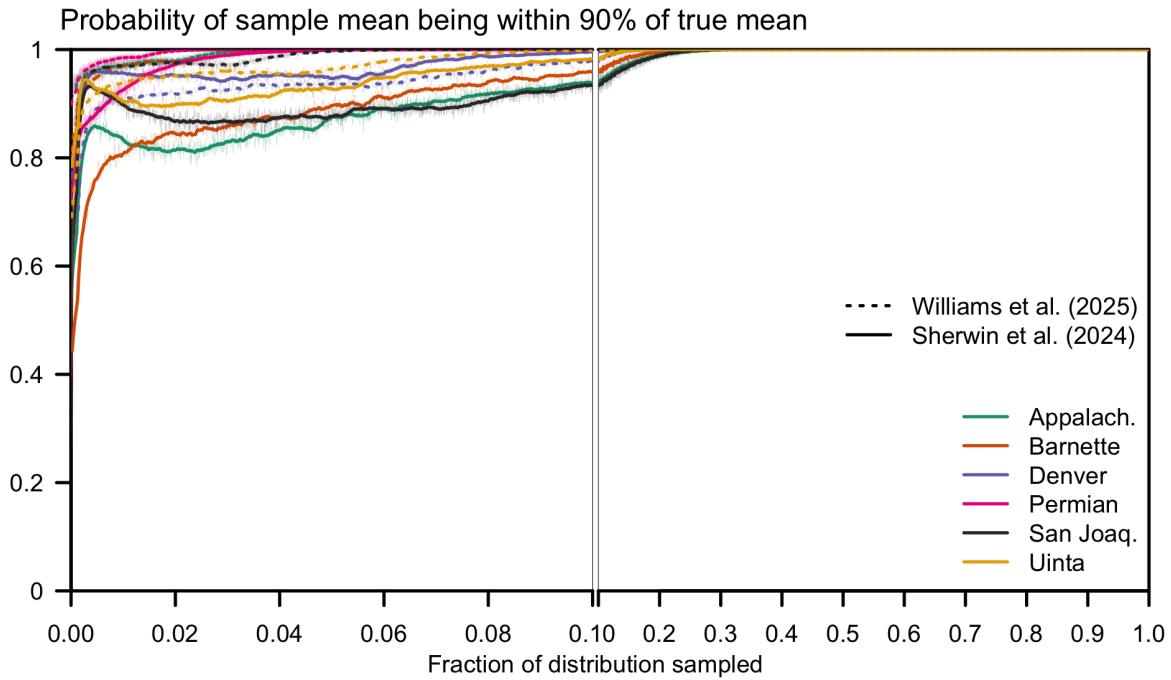


Figure S13: Probability of the sample mean having error within 90% of the true mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

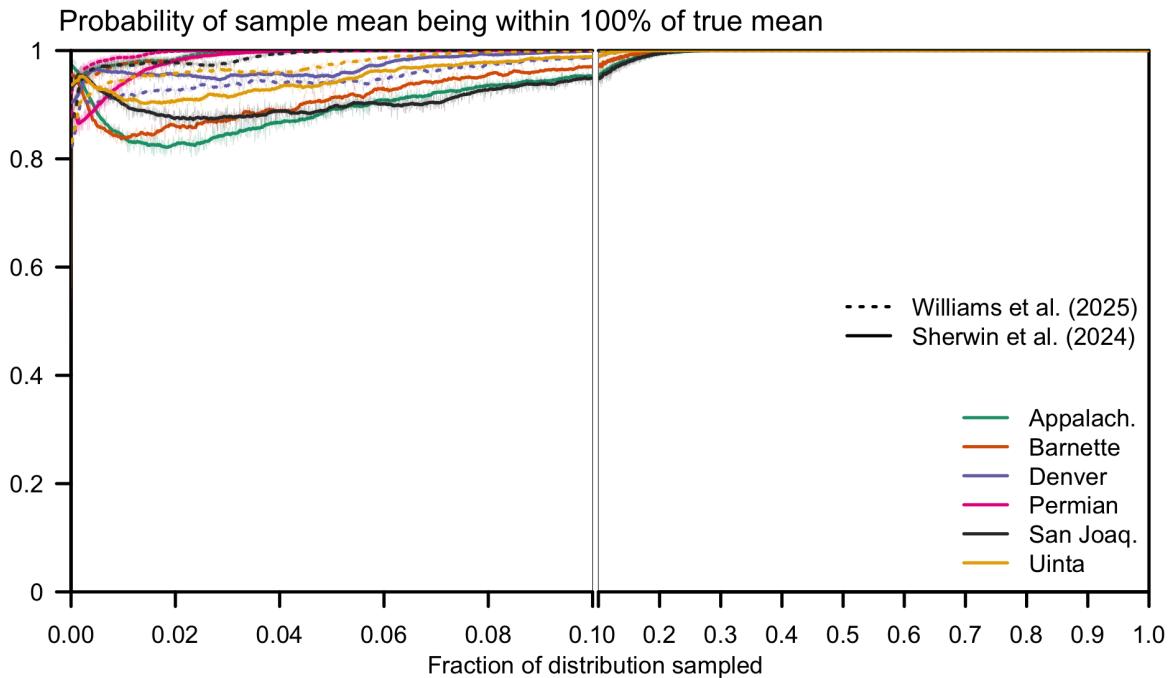


Figure S14: Probability of the sample mean having error within 100% of the true mean as a function of sample size. Note that the horizontal axis zooms in on the [0.0, 0.1] region to show detail.

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