

An Anisotropy-Aware Framework for Directional Response Spectra and Rotation-Invariant Ground-Motion Measures

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Abstract

Horizontal earthquake ground motion is inherently two-dimensional, yet most engineering applications rely on scalar intensity measures. Rotation-invariant response spectra such as RotD50, MaxRotD50, and RotD100 are widely used to remove dependence on sensor orientation, and are often treated as direction-free scalars, which they are not. In this study, directional pseudo-spectral acceleration (PSA) is treated instead as a stochastic field on the circle, and rotation-invariant measures are interpreted as directional selection operators acting on that field. Building on empirical evidence that the squared directional response is strongly dominated by its first admissible angular harmonic, we derive closed-form approximations for RotD50, MaxRotD50, and RotD100. This removes the need for explicit directional sampling while making clear what directional information each operator retains. Validation against a large ground-motion dataset shows that the closed-form predictions are essentially unbiased across periods and preserve record-to-record variability. A key finding is that max-type measures depend on an effective anisotropy that includes a stochastic contribution from directional peak variability, even when geometric anisotropy is weak. As a result, ratios such as RotD100/RotD50 can be elevated in motions that are nearly isotropic in the root-mean-square sense. The results identify anisotropy of PSA, evaluated along the principal directions of root-mean-square response, as a latent directional state variable that controls the behaviour of rotation-invariant measures and their ratios across records and periods. Rather than removing directionality, rotation-invariant operators transform it. Recognising this distinction provides a more transparent and physically interpretable basis for using rotation-invariant intensity measures in ground-motion modelling and seismic design.

Keywords: rotation-invariant response spectra, RotD50, RotD100, ground motion directionality

1. Introduction

Horizontal earthquake ground motion is inherently two-dimensional. At a site, the two horizontal components define a vector process whose intensity depends on the orientation along which the motion is projected. Most engineering workflows nevertheless rely on scalar representations of horizontal ground motion. Ground-motion models (GMMs) commonly predict a single horizontal intensity measure (IM); Probabilistic Seismic Hazard Assessment (PSHA) aggregates hazard in terms of scalar spectral ordinates; record selection and scaling use scalar response spectra as targets; and design codes prescribe scalar spectra applied along structural axes and combined using algebraic rules, such as those in ASCE/SEI 7-22 (ASCE, 2022) and Eurocode 8 (CEN, 2004). This scalarisation is practical and deeply embedded in current practice.

A scalar horizontal IM, however, is not the ground motion itself. It is a reduction of an underlying directional response field. For a fixed oscillator period T , let $Y(T, \theta)$ denote the pseudo-spectral acceleration, PSA, obtained by projecting the two horizontal components onto an axis oriented at angle θ . The function $Y(T, \theta)$ is smooth and π -periodic in orientation. Rotation-invariant measures such as RotD50, MaxRotD50, and RotD100 are therefore not independent scalar quantities. They are operators acting on this directional field. RotD50 selects a central directional level, RotD100 selects the maximum over orientation, and MaxRotD50 selects the median over orientation of the larger response in each orthogonal pair.

The standard computational procedure for evaluating these measures is well established. The two recorded horizontal components are rotated through a dense set of non-redundant orientations, the response spectrum is computed at each orientation, and the desired statistic is extracted. This procedure underlies orientation-

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independent measures introduced by Boore et al. (2006), the RotDnn family introduced by Boore (2010), and later measures such as MaxRotD50 proposed by Poulos and Miranda (2022). It is numerically straightforward and, with current computing resources, dense angular sampling is not a serious computational burden. The limitation is instead conceptual. Dense angular sampling can make the directional response appear to be a collection of angular observations, whereas for a given record and period, $Y(T, \theta)$ is a deterministic field generated by the same two horizontal response histories. Neighbouring values are not independent samples. They are constrained by smoothness, periodicity, and the geometry of the two-component response field.

This distinction between angular samples and a directional field has practical consequences. If the angular ordinates are treated only as sampled values, their dispersion may be interpreted as component-to-component variability, scalar ratios such as RotD100/RotD50 may be modelled as empirical conversion factors, and maximum-direction measures may be deployed as if they were ordinary component spectra. In a field view, variation with orientation is a geometric property of a single two-component response, and stochasticity enters through record-to-record variation of field parameters such as scale, anisotropy, and orientation. This perspective clarifies why orthogonal directional responses are coupled rather than independent, why scalar ratios between rotation-invariant measures can combine persistent directional structure with peak-selection effects, and why the meaning of a scalar measure depends on how it is translated into a design action. These implications motivate the present formulation, although the focus of this paper is narrower: to determine whether the directional PSA field is sufficiently low-dimensional that the main rotation-invariant measures can be derived as operators acting on a reduced field representation.

The first step toward such a representation is to move beyond scalar IMs and ask what the simplest useful directional object is. A scalar spectrum corresponds geometrically to a circle: the same amplitude is assigned to every horizontal orientation. The simplest anisotropic extension is an ellipse, described by an overall scale, a shape or anisotropy, and an orientation. This is not merely a visual analogy. The root-mean-square (RMS) response of a linear oscillator is exactly elliptical because it is determined by the quadratic form of the two-component response covariance matrix. Thus, the RMS response field has an exact principal-axis representation. PSA is not identical to RMS response because it is a peak quantity. It contains peak-selection effects in addition to second-order response geometry. Nevertheless, PSA inherits the π -periodicity of the directional projection and therefore admits a Fourier representation in orientation. This leads to the central question of the paper: is the squared directional PSA field a high-dimensional angular object requiring dense sampling, or can it be represented with sufficient accuracy by the simplest non-scalar, first-harmonic field?

This paper tests whether this first-harmonic representation is sufficient for the directional information used by current rotation-invariant measures. The first admissible harmonic retains a modulation amplitude and orientation while preserving the required periodicity. Across the records and periods considered, squared directional PSA is strongly dominated by the first admissible harmonic, and the residual higher-harmonic content produces only bounded errors in RotD50, MaxRotD50, and RotD100.

This result indicates that the directional PSA field is not effectively a set of many unrelated angular ordinates. To leading order, the information relevant to these scalar measures is contained in a reduced representation defined by scale and anisotropy. We express the first-harmonic structure through the RMS principal axes, obtained directly from the covariance matrix of the two response histories, which provide a physically meaningful coordinate system in which PSA evaluated along the principal directions yields compact scale and anisotropy parameters. The closed-form expressions for RotD50, MaxRotD50, and RotD100 follow from this representation and are evaluated against dense directional sampling; their agreement shows both that numerical rotation can be avoided and that the reduced representation captures the directional information these measures retain.

The objectives of the paper are therefore threefold. First, we formulate directional PSA as a structured periodic field rather than as a collection of independent angular samples. Second, we test whether the first admissible harmonic in squared-PSA space is sufficient for the directional information selected by RotD50, MaxRotD50, and RotD100. Third, we express this reduced harmonic structure in terms of geometrically interpretable scale and anisotropy parameters based on the RMS principal axes and derive closed-form expressions for the three rotation-invariant operators. The results clarify what directional information is retained by each scalar measure and provide a foundation for future directionality-aware ground-motion modelling, record selection, and bidirectional demand assessment.

The remainder of the paper is organised as follows. Section 2 formulates directional PSA as a periodic field and derives the first-harmonic representation. Section 3 evaluates the empirical adequacy of the first harmonic using the directional PSA fields of recorded ground motions. Section 4 introduces the RMS principal axes, develops the geometric scale and anisotropy parameters, and derives closed-form expressions for RotD50, MaxRotD50, and RotD100 from the reduced field representation. Section 5 validates the closed-form construction against dense

directional sampling. Section 6 discusses implications for ground-motion modelling, record selection, and interpretation of directionality in scalar IMs.

2. Directional PSA as a periodic field

Engineering applications have long relied on scalar horizontal IMs to represent two-component ground motion. Common definitions include a single arbitrary horizontal component, the larger of two recorded components, the geometric mean of two orthogonal components, and measures derived by rotating the horizontal pair through all non-redundant orientations. These definitions can yield similar median amplitudes in many cases, but they differ in dispersion, interpretation, and relationship to structural demand. Beyer and Bommer (2006) systematically quantified relationships among median values and aleatory variabilities for different horizontal-component definitions and emphasised the need for consistency between the IM used in a GMM and the IM required in engineering applications.

A major motivation for orientation-independent measures is the dependence of traditional component definitions, especially the as-recorded geometric mean, on the azimuth of the recording sensors. Boore et al. (2006) introduced orientation-independent geometric-mean measures by rotating the two recorded components and identifying a rotation-independent median measure. Boore (2010) subsequently introduced the RotDnn percentile family of non-geometric-mean orientation-independent response spectra. In this framework, RotD50 is the median of the spectral ordinates over all non-redundant orientations, while RotD100 is the maximum over orientation. These definitions are now widely used because they remove arbitrary dependence on instrument orientation.

MaxRotD50 was proposed by Poulos and Miranda (2022) to reflect the probability that at least one of two orthogonal structural axes experiences an unfavourable orientation. Defined as the median over all orientations of the larger of two orthogonal spectral ordinates, it lies between RotD50 and RotD100 and retains orthogonal-pair demand information suppressed by RotD50 without selecting the absolute maximum direction as RotD100 does. Following the NGA-West2 project, RotD50 became the predominant horizontal-component definition in many GMMs (Ancheta et al., 2014; Bozorgnia et al., 2014), while design provisions in the United States have emphasised maximum-direction spectra (ASCE, 2022; Stewart et al., 2011). The difference between median-type and maximum-type measures has motivated empirical conversion relationships, including models for RotD100/RotD50 (Shahi and Baker, 2014) and broader relationships among horizontal-component IMs (Boore and Kishida, 2017).

In the present work, these IMs are interpreted as reductions of a directional field rather than only as alternative component definitions. RotD50, MaxRotD50, and RotD100 are not interchangeable summaries: RotD50 retains the central level of the field, RotD100 the extreme directional lobe, and MaxRotD50 a feature of the upper envelope of orthogonal pairs. All three are invariant to coordinate rotation, but this invariance removes dependence on arbitrary sensor orientation; it does not mean the horizontal ground-motion field lacks directional structure.

The directional variation of response spectra reflects physical mechanisms such as rupture directivity, radiation pattern, path effects, basin effects, topographic effects, and directional site response. Directional site resonances have been documented by Vidale et al. (1991), directional topographic response by Spudich et al. (1996), and rupture-directivity effects by Somerville et al. (1997). Near-fault studies have also shown that maximum-direction response can be particularly important in the presence of strong directivity effects (Huang et al., 2009). These observations support the view that directionality is not merely numerical scatter over rotation angle, but a structured property of horizontal ground motion.

Rotation-invariant formulations based on resultant oscillator response provide a complementary perspective. Rupakhety and Sigbjörnsson (2013, 2014a, 2014b) developed rotation-invariant measures of response spectra and strong-motion duration by exploiting quantities that are invariant under rotation of the horizontal coordinate system. In particular, the RMS response is governed by the covariance matrix of the two horizontal response components and therefore has an exact elliptical representation. This observation suggests that the ellipse is the minimal anisotropic extension of the scalar representation: a scalar IM corresponds to a circle, while a two-parameter anisotropic representation corresponds to an ellipse with scale and shape.

Despite these developments, most practical treatments of rotation-invariant spectra remain scalar: a GMM predicts one measure, such as RotD50, and others are obtained through empirical conversion factors. This does not explicitly represent the directional field from which the scalar measures derive, and it makes different rotation-invariant quantities appear as separate empirical objects rather than as operators applied to the same field.

We now define the directional field explicitly. Let $x_1(t; T, \xi)$ and $x_2(t; T, \xi)$ denote the two horizontal pseudo-acceleration response histories of a linear oscillator at period T and damping ratio ξ . In this study, ξ is fixed at 5%

of critical damping and is omitted from the notation hereafter. These histories are obtained by solving the linear single-degree-of-freedom response to the two horizontal ground-motion components and multiplying the relative displacement response by $\omega^2 = (2\pi/T)^2$. The response projected along orientation θ is

$$x_\theta(t; T) = x_1(t; T)\cos\theta + x_2(t; T)\sin\theta \quad (1)$$

The directional PSA field is

$$Y(T, \theta) = \max_t |x_\theta(t; T)| \quad (2)$$

Because reversing the projection axis changes only the sign of $x_\theta(t; T)$, the absolute peak response is unchanged. Therefore,

$$Y(T, \theta + \pi) = Y(T, \theta) \quad (3)$$

and the directional PSA field is π -periodic. The non-redundant angular domain is therefore $[0, \pi)$. For notational convenience, we work with the squared PSA field,

$$Q(T, \theta) = Y^2(T, \theta) \quad (4)$$

Since $Y(T, \theta)$ is π -periodic, $Q(T, \theta)$ is also π -periodic. Let us define the median of $Q(T, \theta)$ as

$$Q_{50}(T) = \text{median}_\theta Q(T, \theta) \quad (5)$$

Because $Q(T, \theta) = Y^2(T, \theta)$ and the square is monotonic for positive values,

$$Q_{50}(T) = \text{RotD50}^2(T) \quad (6)$$

We define the normalised median-centred squared field

$$g(T, \theta) = \frac{Q(T, \theta)}{Q_{50}(T)} - 1. \quad (7)$$

By construction, the median of $g(T, \theta)$ over orientation is zero. The scalar part of the field has therefore been removed, and $g(T, \theta)$ contains only directional modulation relative to RotD50. Because $g(T, \theta)$ is π -periodic, it admits a Fourier representation with even angular harmonics,

$$g(T, \theta) = a_0(T) + \sum_{n=1}^{\infty} [a_n(T)\cos 2n\theta + b_n(T)\sin 2n\theta] \quad (8)$$

The constant coefficient a_0 is small when the angular mean and angular median of Q are close. It is retained in the exact representation but is not part of the directional modulation. The first admissible harmonic is

$$g_1(T, \theta) = a_1(T)\cos 2\theta + b_1(T)\sin 2\theta \quad (9)$$

For clarity, we write

$$a(T) = a_1(T), \quad b(T) = b_1(T) \quad (10)$$

The corresponding amplitude and phase are

$$\rho(T) = \sqrt{a^2(T) + b^2(T)} \quad \text{and} \quad \theta_Y(T) = \frac{1}{2}\tan^{-1}\left(\frac{b(T)}{a(T)}\right) \quad (11)$$

with the usual quadrant convention. The first-harmonic approximation is therefore

$$Q(T, \theta) \approx Q_{50}(T)[1 + \rho(T)\cos 2(\theta - \theta_Y(T))] \quad (12)$$

This expression has a simple interpretation. $Q_{50}(T)$ is the squared central level of the field. $\rho(T)$ is the leading directional anisotropy. $\theta_Y(T)$ is the orientation of the dominant lobe. A scalar representation keeps only Q_{50} . The first harmonic is the minimal extension that retains directional structure.

The rotation-invariant measures follow immediately under this first-harmonic model. The median of $\cos 2\theta$ over a non-redundant angular range is zero, so

$$\text{RotD50}(T) = \sqrt{Q_{50}(T)} \quad (13)$$

The maximum of $Q(T, \theta)$ occurs when the cosine term is one, giving

$$\text{RotD100}(T) \approx \sqrt{Q_{50}(T)[1 + \rho(T)]} \quad (14)$$

182 For MaxRotD50, we consider an orthogonal pair. Since

$$Q(T, \theta + \pi/2) \approx Q_{50}(T)[1 - \rho(T)\cos 2(\theta - \theta_Y(T))] \quad (15)$$

183 the larger squared response in the orthogonal pair is

$$\max\{Q(T, \theta), Q(T, \theta + \pi/2)\} \approx Q_{50}(T)[1 + \rho(T) |\cos 2(\theta - \theta_Y(T))|] \quad (16)$$

184 The median of $|\cos 2\theta|$ over $[0, \pi)$ is $1/\sqrt{2}$. Therefore,

$$\text{MaxRotD50}(T) \approx \sqrt{Q_{50}(T) \left[1 + \frac{\rho(T)}{\sqrt{2}}\right]} \quad (17)$$

185 These expressions are exact for a purely first-harmonic squared field. They show that the three common rotation-
 186 invariant measures are determined by two rotation-invariant quantities: the scale Q_{50} and the leading anisotropy
 187 ρ . The phase θ_Y is needed to reconstruct the field in physical orientation, but it drops out of RotD50, MaxRotD50,
 188 and RotD100 because these operators are invariant under angular shift. A useful property of the first-harmonic
 189 field is the orthogonal-pair invariant. For

$$Q(T, \theta) = Q_{50}(T)[1 + a(T)\cos 2\theta + b(T)\sin 2\theta] \quad (18)$$

190 the average squared response in any two orthogonal directions is independent of θ :

$$\frac{Q(T, \theta) + Q(T, \theta + \pi/2)}{2} = Q_{50}(T) \quad (19)$$

191 3. Empirical adequacy of the first-harmonic field

192 The preceding section introduced the first-harmonic representation as the minimal directional extension of a scalar
 193 PSA field. This section tests whether that representation is adequate for recorded ground motions. The purpose is
 194 not to assume that directional PSA is exactly elliptical or exactly first harmonic. It is to determine whether the
 195 leading harmonic captures the part of the directional field to which the common rotation-invariant operators are
 196 primarily sensitive.

197 3.1 Data and response calculations

198 The empirical analysis uses horizontal strong-motion records from the Engineering Strong-Motion database, ESM
 199 (Luzi et al., 2016; Lanzano et al., 2021). ESM provides uniformly processed accelerometric records from
 200 European and surrounding-region earthquakes and has been widely used for GMM development. The dataset used
 201 here consists of 2182 pairs of horizontal records from 229 earthquakes. Records were selected using moment
 202 magnitude $M_w > 5.2$ and Joyner-Boore distance $R_{JB} < 110\text{km}$. These criteria retain a broad range of moderate-
 203 to-large events and source-to-site distances while avoiding very weak or very distant records for which long-
 204 period response may be poorly constrained.

205 For each record, 5% damped linear elastic single-degree-of-freedom response was computed over 100 oscillator
 206 periods logarithmically spaced between 0.01 s and 10 s. At each oscillator period T , the two horizontal pseudo-
 207 acceleration response histories were rotated through the non-redundant half-circle using 1° increments. This gives
 208 180 directional ordinates for each record and period. The dense angular grid is used here only as a reference
 209 calculation for testing the field representation. The objective is not to replace one numerical grid by another, but
 210 to determine how much of the directional information in that grid is carried by the leading harmonic. For each
 211 record $j = 1, \dots, N$, with $N = 2182$, the squared directional field $Q_j(T, \theta)$ was evaluated using Eq. (4). The
 212 squared RotD50 level $Q_{50,j}(T)$, and the normalised median-centred field $g_j(T, \theta)$, were then obtained from Eqs.
 213 (5) and (7), respectively.

214 3.2 First-harmonic projection

215 For each record and period, the normalised squared field $g_j(T, \theta)$ is projected onto the first non-constant
 216 admissible harmonic, $g_{1,j}(T, \theta)$, using Eq. (9). The coefficients $a_j(T)$ and $b_j(T)$ are obtained by least-squares
 217 projection over the angular grid. Equivalently, for a uniform angular grid over $[0, \pi)$, they are the discrete Fourier
 218 coefficients associated with $\cos 2\theta$ and $\sin 2\theta$. In continuous notation,

$$a_j(T) = \frac{2}{\pi} \int_0^\pi g_j(T, \theta) \cos 2\theta d\theta \quad \text{and} \quad b_j(T) = \frac{2}{\pi} \int_0^\pi g_j(T, \theta) \sin 2\theta d\theta \quad (20)$$

For the discrete 1° grid used here, the corresponding sums are used. The first-harmonic amplitude $\rho_j(T)$, and the phase of the dominant lobe $\theta_{v,j}(T)$, are computed using Eqs. (11) and (12), respectively. The resulting median-anchored first-harmonic approximation to the squared directional field is

$$\hat{Q}_{1,j}(T, \theta) = Q_{50,j}(T) [1 + \rho_j(T) \cos 2(\theta - \theta_{v,j}(T))] \quad (21)$$

This form uses $Q_{50,j}(T)$, the actual angular median of $Q_j(T, \theta)$, as the reference level. The approximation therefore describes the leading directional modulation around RotD50. It should not be interpreted as a full Fourier reconstruction of $g_j(T, \theta)$, because no separate constant term is fitted. This choice is deliberate: the objective is to isolate the dominant directional mode relative to the RotD50 level, which is the scalar reference used by the rotation-invariant operators considered below. All remaining content is collected in the residual field $r_j(T, \theta) = g_j(T, \theta) - \hat{Q}_{1,j}(T, \theta)$. This residual includes higher harmonics and any constant offset arising from the difference between the angular mean and angular median of $Q_j(T, \theta)$. Thus, the first-harmonic approximation is exact only in the ideal case where the median-anchored squared field is fully described by the first admissible harmonic.

3.3 Record-level and ensemble visual diagnostics

The first diagnostic is visual and is intended to show what the first-harmonic projection represents at the level of individual records. Figure 1 shows examples of directional PSA fields at three oscillator periods and three levels of first-harmonic modulation. It illustrates two features that are central to the remainder of the paper. First, directional PSA is not generally isotropic about the RotD50 level. Second, the dominant departure from the RotD50 circle is well captured by a single two-lobed angular mode over a wide range of modulation amplitudes. The approximation is not exact, and local departures remain, but the leading directional structure is already evident at the record level.

A stronger ensemble diagnostic is obtained by aligning all records according to the phase of their fitted first harmonic. For each record and period, the angular coordinate is shifted so that the maximum of the fitted first harmonic occurs at zero aligned angle. Figure 2 shows the aligned ensemble diagnostic. The close agreement between the median actual field and the median first-harmonic model shows that the leading angular structure is systematic across records. The percentile band narrows near aligned angles of $\pm 45^\circ$, where the leading harmonic crosses the RotD50 level for every record independently of the modulation amplitude; this nodal structure is therefore present in the actual directional fields, not only in the model. The remaining spread reflects higher harmonics and residual record-specific structure. Together, these results indicate that the dominant variation of directional PSA is a low-dimensional angular structure: a scalar level, a first-harmonic amplitude, and a phase.

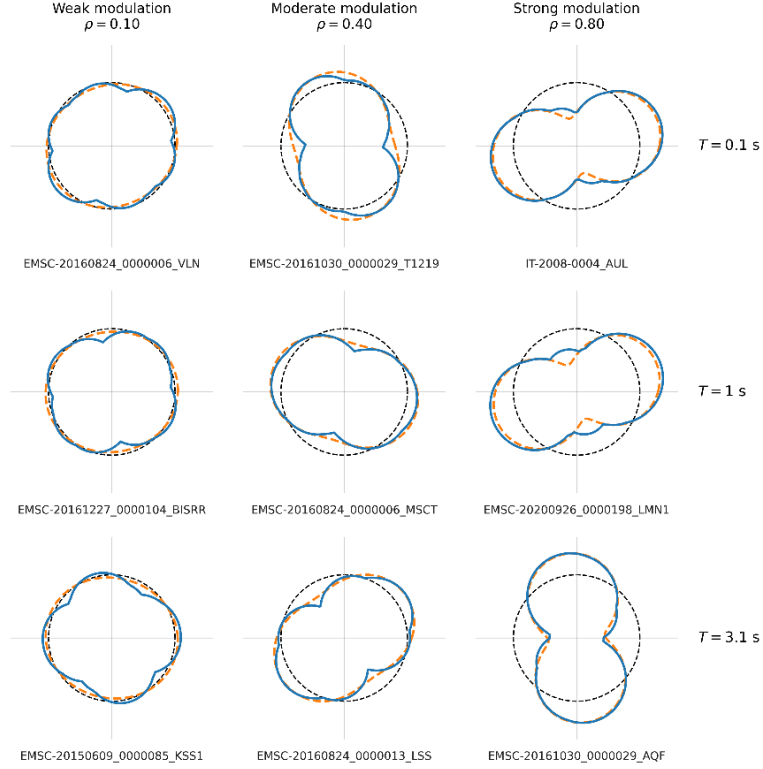


Figure 1. Record-level examples of directional PSA fields in polar form. The radial coordinate is $Y(\theta)/\text{RotD50}$. The blue solid curves show the actual directional PSA field, the orange dashed curves show the first-harmonic model fitted in squared-PSA space and plotted in PSA-amplitude form, and the black dashed circle marks the RotD50 reference level. Columns show examples with weak, moderate, and strong first-harmonic modulation; rows correspond to the oscillator periods indicated on the right.

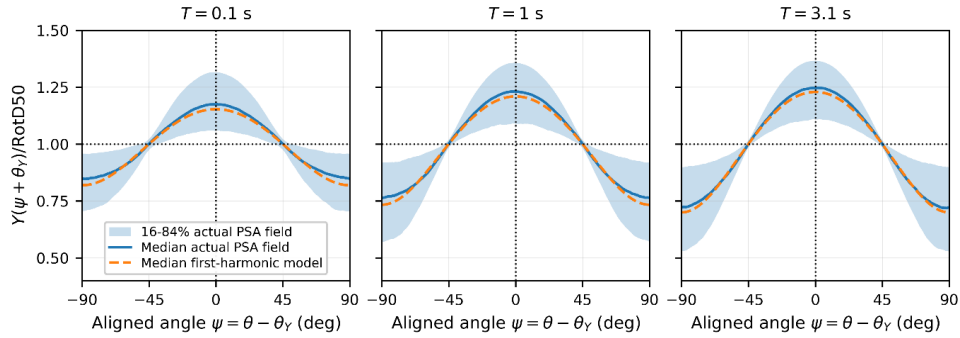


Figure 2. Ensemble diagnostic after first-harmonic phase alignment. Directional PSA fields are normalised by RotD50 and shifted so that the fitted first-harmonic maximum occurs at zero aligned angle. The blue solid curves show the median actual PSA field, the shaded blue bands show the 16th to 84th percentile range, the orange dashed curves show the median first-harmonic model.

3.4 Quantitative diagnostics for first-harmonic adequacy

The adequacy of the first harmonic representation is now quantified in two complementary ways. The first set of diagnostics evaluates the field approximation itself, by measuring the fraction of angular variation explained by the first harmonic and the amplitude of the residual field. The second evaluates the rotation-invariant operators that depend on the directional field. The mean-square energy of the first-harmonic component is defined as

$$E_{1,j}(T) = \langle g_{1,j}^2(T, \theta) \rangle_\theta = \frac{a_j^2(T) + b_j^2(T)}{2} \quad (22)$$

where $\langle \cdot \rangle_\theta$ denotes angular averaging. The mean-square residual energy is defined as

$$E_{\text{res},j}(T) = \langle r_j^2(T, \theta) \rangle_\theta \quad (23)$$

264 A convenient first-harmonic fraction is then

$$F_{1,j}(T) = \frac{\langle g_{1,j}^2(T, \theta) \rangle_\theta}{\langle g_j^2(T, \theta) \rangle_\theta} \quad (24)$$

265 For a uniform angular grid, the numerator is $(a_j^2 + b_j^2)/2$. Values of $F_{1,j}$ close to one indicate that the angular
 266 modulation is dominated by the first admissible harmonic. The residual field is also summarised by its maximum
 267 absolute amplitude, $\Delta_j(T) = \max_\theta |r_j(T, \theta)|$. This diagnostic is intentionally conservative. It measures the
 268 largest normalised departure of the squared directional field from the first-harmonic approximation. Since
 269 rotation-invariant measures involve angular medians, maxima, and orthogonal-pair selections, the maximum
 270 residual amplitude provides a direct indication of the possible influence of higher harmonics.

271 The second set of diagnostics evaluates the rotation-invariant operators themselves. In the present median-
 272 anchored representation, $Q_{50,j}(T)$ is defined as the angular median of $Q_j(T, \theta)$. The RotD50 level is therefore
 273 preserved by construction and is not used as an error metric. The nontrivial operator predictions $\widehat{\text{RotD100}}_j(T)$
 274 and $\widehat{\text{MaxRotD50}}_j(T)$, are computed from Eqs. (14) and (17), respectively. These predictions are compared with
 275 the corresponding values obtained directly from the 1° angular sampling. For an operator M , the percent residual
 276 is defined as $\delta_{M,j}(T) = 100[M_{j,\text{actual}}(T)/\widehat{M}_{j,1}(T) - 1]$, where M denotes either MaxRotD50 or RotD100. A value
 277 of zero indicates exact agreement between the directly sampled rotation-invariant value and the first-harmonic
 278 prediction. Positive values indicate that the directly sampled value is larger than the first-harmonic prediction.

279 Figure 3 summarises the field-level diagnostics over the full period range. The median first-harmonic fraction is
 280 already high at short periods, with $F_1 \approx 0.81$ at $T = 0.1\text{s}$, and increases to about 0.90 at $T = 1\text{s}$ and 0.93 at $T =$
 281 3.1s . Thus, the first admissible harmonic captures most of the angular mean-square variation of the normalised
 282 squared field across the period range considered. This first-harmonic dominance of the squared PSA field is
 283 consistent with, and complementary to, the finding of Rupakhety and Hernández-Aguirre (2026), who decompose
 284 PSA into an RMS response and a peak factor and show, on the same strong-motion database, that the directional
 285 peak-factor fluctuation field is itself dominated by its first admissible harmonic. The present result is obtained by
 286 working directly on the squared PSA field, without separating the RMS and peak-factor components, and therefore
 287 does not depend on that decomposition.

288 The maximum residual amplitude in Figure 3 is intentionally conservative, because it records the largest pointwise
 289 departure over orientation for each record and period. Its median remains close to 0.2 over most of the period
 290 range. Since the residual is defined in the normalised squared field $Q/Q_{50} - 1$, this corresponds to a worst-case
 291 squared-field deviation of about 20% of Q_{50} . In PSA-amplitude terms, this is roughly a 10% local deviation near
 292 the RotD50 level. The corresponding RMS residuals at representative periods are smaller, about 0.09 to 0.10 in
 293 the normalised squared field, indicating that the typical residual amplitude is substantially below the maximum
 294 residual.

295 Figure 4 evaluates whether the residual field materially affects the rotation-invariant operators. The first-harmonic
 296 prediction reproduces MaxRotD50 with very small bias. The median percent residual is approximately +0.6% at
 297 $T = 0.1\text{s}$ and $T = 1\text{s}$, and approximately +0.5% at $T = 3.1\text{s}$. RotD100 shows somewhat larger residuals, as
 298 expected. Unlike MaxRotD50, RotD100 depends on the single largest orientation and is therefore more sensitive
 299 to local angular peaks and higher-harmonic content. The median residual decreases from about +3.5% at $T =$
 300 0.1s to about +1.8% at $T = 3.1\text{s}$. The positive residual indicates that the directly sampled operator is, on average,
 301 slightly larger than the first-harmonic prediction, consistent with higher harmonics and local peaks raising the
 302 maximum directional ordinate beyond the smooth first-harmonic envelope. Overall, the operator residuals are
 303 small compared with the directional modulation itself, indicating that the median-anchored first harmonic captures
 304 the components of the directional field that most strongly control MaxRotD50 and RotD100.

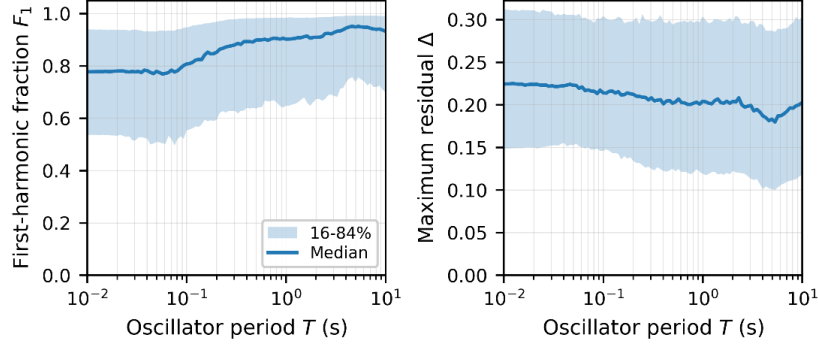


Figure 3. Quantitative diagnostics of the median-anchored first-harmonic field approximation.

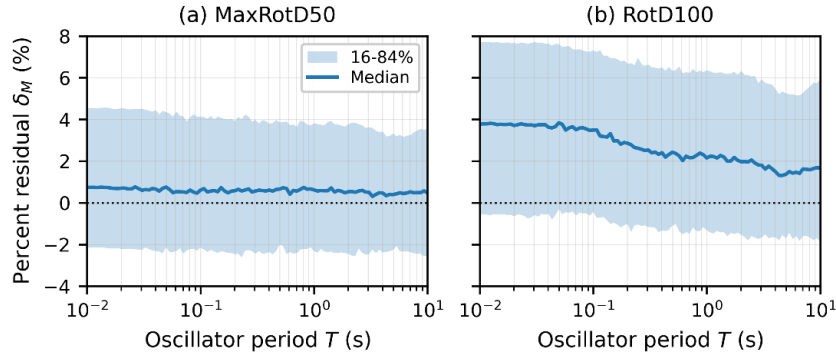


Figure 4. Percent residuals of MaxRotD50 and RotD100 predicted from the median-anchored first-harmonic approximation.

These results suggest a practical reinterpretation of rotation-invariant spectra. The different RotD measures need not be regarded as unrelated scalar ordinates requiring separate empirical models. Instead, they can be viewed as different functionals of a common directional field. In the median-anchored first-harmonic representation, the field is controlled, for rotation-invariant purposes, by a scalar level Q_{50} , or equivalently RotD50, and a modulation amplitude ρ . MaxRotD50, RotD100, random component, or any other percentile measure then follow from the same field approximation. At this stage, however, ρ has been estimated from the densely sampled directional PSA field itself. The remaining question is how this directional modulation can be interpreted and parametrised in terms of quantities that are available without treating the angular response as an arbitrary collection of sampled ordinates. The next section addresses this question by introducing the RMS response covariance frame as a geometric coordinate system for the directional field.

4. Geometric parametrization through RMS principal axes

The median-anchored first-harmonic representation of Section 3 demonstrates the low-dimensional structure of the directional field, but it is not the most convenient form for engineering interpretation: the harmonic coefficients depend on the coordinate system of the two horizontal components, and the phase describes orientation rather than directional strength. A practical parametrization should separate scale, anisotropy, and orientation in a coordinate system defined intrinsically by the two response histories. The RMS principal axes provide such a coordinate system. They follow from the covariance matrix of the two horizontal oscillator response histories and therefore require no angular search, unlike axes defined by the maxima or minima of PSA, which would reintroduce directional sampling. They are the exact principal axes of the RMS response ellipse and provide a natural frame in which to decompose the directional PSA field.

4.1 Coordinate dependence of the harmonic coefficients

Let the leading harmonic in a fixed horizontal coordinate system $g_1(T, \theta)$, be written as in Eq. (9). The pair (a, b) is coordinate-dependent. If the horizontal coordinate system is rotated by an angle φ , the coefficient vector transforms as

$$\begin{pmatrix} a \\ b \end{pmatrix} \rightarrow \begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ -\sin 2\varphi & \cos 2\varphi \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \quad (25)$$

Thus, the individual coefficients do not have intrinsic physical meaning. Their magnitude, $\rho(T)$ (Eq. 11), is invariant under rotation of the recording axes, while the phase changes with the coordinate system. This motivates expressing the field in a coordinate system that is itself defined by the response. The RMS principal frame provides this coordinate system.

4.2 The RMS response ellipse

Let the two horizontal pseudo-acceleration response histories at oscillator period T be assembled into the vector

$$\mathbf{x}(t; T) = \begin{bmatrix} x_1(t; T) \\ x_2(t; T) \end{bmatrix} \quad (26)$$

The response covariance matrix is

$$\mathbf{C}(T) = \langle \mathbf{x}(t; T) \mathbf{x}^\top(t; T) \rangle \quad (27)$$

where $\langle \cdot \rangle$ denotes averaging over time. For a unit vector $\mathbf{e}_\theta = [\cos \theta; \sin \theta]$, the squared RMS response in orientation θ is $\sigma^2(T, \theta) = \mathbf{e}_\theta^\top \mathbf{C}(T) \mathbf{e}_\theta$. This relation is exact. Since $\mathbf{C}(T)$ is symmetric and positive semidefinite, it has real non-negative eigenvalues $\lambda_1(T) \geq \lambda_2(T)$ and orthogonal eigenvectors. Let $\theta_0(T)$ denote the direction of the eigenvector associated with $\lambda_1(T)$. Then the RMS field can be written as

$$\sigma^2(T, \theta) = \sigma_0^2(T) [1 + \kappa_{\text{rms}}(T) \cos 2\{\theta - \theta_0(T)\}] \quad (28)$$

where

$$\sigma_0^2(T) = \frac{\lambda_1(T) + \lambda_2(T)}{2} \quad \text{and} \quad \kappa_{\text{rms}}(T) = \frac{\lambda_1(T) - \lambda_2(T)}{\lambda_1(T) + \lambda_2(T)} \quad (29)$$

The RMS response field is therefore exactly first harmonic in squared response. Its scale and anisotropy are determined by the eigenvalues, and its principal directions are determined by the eigenvectors. No directional PSA sampling is involved in defining this geometry.

4.3 PSA invariants in the RMS principal frame

The RMS axes define the coordinate system, but the engineering quantity of interest is PSA. We therefore evaluate the squared PSA field on the RMS principal axes. Let $Q_{\parallel}(T) = Q(T, \theta_0)$ and $Q_{\perp}(T) = Q(T, \theta_0 + \pi/2)$, where $Q(T, \theta) = Y^2(T, \theta)$. The subscripts $\parallel$ and $\perp$ refer to the RMS major and minor axes, respectively. These two values define a squared PSA scale and a signed anisotropy parameter:

$$Q_0(T) = \frac{Q_{\parallel}(T) + Q_{\perp}(T)}{2}, \quad \kappa_s(T) = \frac{Q_{\parallel}(T) - Q_{\perp}(T)}{Q_{\parallel}(T) + Q_{\perp}(T)} \quad (30)$$

The sign of κ_s indicates whether the PSA value is larger on the RMS major axis or on the RMS minor axis. For scalar rotation-invariant measures, only the magnitude is relevant, and we define $\kappa(T) = |\kappa_s(T)|$. Equivalently, if $S_1(T) \geq S_2(T)$ denote the two PSA amplitudes on the RMS principal axes ordered by magnitude, then

$$Q_0(T) = \frac{S_1^2(T) + S_2^2(T)}{2} \quad \text{and} \quad \kappa(T) = \frac{S_1^2(T) - S_2^2(T)}{S_1^2(T) + S_2^2(T)} \quad (31)$$

The pair (Q_0, κ) is a pair of PSA-based invariants in the RMS frame. They do not depend on the arbitrary orientation of the recorded horizontal components, because the RMS axes rotate with the response. The quantity Q_0 gives the scale of the squared PSA field on the RMS principal pair, while κ gives the anisotropy of peak response in that frame. It is important to distinguish κ from κ_{rms} . The latter is an RMS-response anisotropy defined from covariance eigenvalues. The former is a PSA anisotropy defined from peak responses evaluated on the RMS principal axes.

4.4 The unresolved quadrature component

The two PSA invariants Q_0 and κ do not, in general, determine the full first-harmonic directional field. To see this, we express the leading harmonic in the RMS principal basis. Let $\phi = \theta - \theta_0$ be the orientation measured from the RMS major axis. Define the Q_0 -normalised squared PSA field as

$$g_0(T, \phi) = \frac{Q(T, \theta_0 + \phi)}{Q_0(T)} - 1 \quad (32)$$

The leading harmonic in this frame can be written as

$$g_0(T, \phi) \approx \kappa_s(T) \cos 2\phi + \eta(T) \sin 2\phi \quad (33)$$

The coefficient κ_s is the resolved component of the first harmonic. It is determined entirely by the two PSA values on the RMS axes. The coefficient η is the quadrature component. It is the part of the leading harmonic that is rotated by 45° in harmonic phase relative to the RMS-axis contrast. The total first-harmonic amplitude in the Q_0 -normalised RMS frame is $\rho_0(T) = \sqrt{\kappa_s^2(T) + \eta^2(T)}$. Thus, $\kappa = |\kappa_s|$ is only the resolved part of the first-harmonic amplitude. The full amplitude satisfies $\rho_0(T) \geq \kappa(T)$, with equality only when $\eta(T) = 0$.

The reason η cannot be recovered from the two RMS-axis PSA values is structural. At the two RMS principal axes, $\phi = 0$ and $\phi = \pi/2$, and therefore $\sin 2\phi = 0$. Hence the quadrature term vanishes at both axes. The two-axis PSA pair measures only the cosine component. It contains no information about $\eta(T)$. In other words, the quadrature component lies in the null space of the two-axis measurement. Recovering it would require evaluating the PSA field away from the RMS axes, for example near $\phi = \pi/4$, where the sine component is maximal. That is precisely the type of directional sampling that the geometric parametrization seeks to avoid.

This also explains why the RMS-axis invariants work best when κ is large. If $|\kappa_s|$ is large compared with $|\eta|$, then $\rho_0 = \sqrt{\kappa_s^2 + \eta^2} \approx \kappa$ and the unresolved quadrature has little influence. If κ is small, however, even a modest value of $|\eta|$ can control the total harmonic amplitude. In the limiting case $\kappa_s = 0$ and $\eta \neq 0$, the two RMS-axis PSA values are equal, but the directional field is still modulated. This is the regime in which max-type rotation-invariant measures cannot be predicted accurately from Q_0 and κ alone.

4.5 Empirical characterisation of the quadrature component

The quadrature component is unobservable from the two RMS-axis PSA values alone, but it can be measured from the directional fields used for validation in this study. For each record and period, the squared PSA field is expressed in the RMS principal frame and normalised by $Q_0(T)$. The leading harmonic is then projected onto $\cos 2\phi$ and $\sin 2\phi$, yielding the resolved coefficient $\kappa_s(T)$ and the quadrature coefficient $\eta(T)$. This empirical extraction is used only to characterise the statistical magnitude of the unresolved component. It is not required in an application where a closed-form approximation is used. In such an application, Q_0 and κ are obtained from the two PSA values on the RMS axes, while the effect of the unobserved η is represented statistically. Figure 5 shows the empirical quadrature component obtained by projecting the Q_0 -normalised squared PSA field in the RMS principal frame. The signed coefficient η is centred close to zero at all representative periods, indicating that there is no preferred sense of rotation of the PSA harmonic relative to the RMS major axis. The relevant quantity for scalar rotation-invariant operators is therefore its magnitude. The median value of $|\eta|$ is remarkably stable with period, remaining close to 0.12 from short to long periods. At $T = 0.1, 1.0$, and 3.1 s, the corresponding median values are approximately 0.114, 0.121, and 0.117, respectively. This supports the use of a simple representative quadrature scale $q_0 \approx 0.12$.

The scale q_0 should be interpreted as the typical magnitude of the first-harmonic component that is structurally invisible to the RMS-axis PSA pair. It is not an additional directional-response model. Rather, it is an empirical summary of the component omitted when the full directional field is reduced to the two invariants Q_0 and κ . This distinction is important: the resolved anisotropy κ is measured directly from the RMS-axis pair for each record and period, whereas q_0 represents only the typical unresolved quadrature amplitude needed when no off-axis PSA values are evaluated.

4.6 Effective anisotropy and closed-form operators

A closed-form approximation based only on the RMS-axis pair requires a statistical representation of the unresolved quadrature contribution. We represent this missing contribution by an equivalent quadrature amplitude $q_i(T)$, where the subscript i denotes the rotation-invariant operator. This gives the effective anisotropy

$$\kappa_{\text{eff},i}(T) = \sqrt{\kappa^2(T) + q_i^2(T)} \quad (34)$$

Here $i = 50$ denotes MaxRotD50 and $i = 100$ denotes RotD100. The operator dependence of q_i does not imply that separate directional fields are fitted for different RotD measures. It reflects the fact that the same unresolved quadrature component affects different functionals of the field differently. MaxRotD50 is a median over orthogonal-pair maxima and is therefore relatively insensitive to isolated local peaks. RotD100 is the maximum

over orientation and is more sensitive to local angular irregularity, higher-harmonic content, and the tail of the unresolved quadrature distribution. Thus, the same empirical quadrature population can correspond to different equivalent amplitudes for different operators. We therefore write $q_i(T) = c_i q_0$, where $q_0 \simeq 0.12$ is the representative quadrature scale and c_i is a dimensionless operator-sensitivity factor. This notation separates the measured scale of the unresolved quadrature from the way in which each operator responds to it. With this closure, the unresolved total anisotropy is replaced by $\kappa_{\text{eff},i}$ when evaluating max-type operators.

The construction has a direct geometric interpretation. The resolved anisotropy κ is the component of the leading harmonic measured by the RMS-axis PSA pair. The effective quadrature q_i represents the component structurally omitted by that pair, expressed as an operator-equivalent amplitude. The effective anisotropy is the length of the resulting two-component harmonic vector. Thus, the closure supplies only the missing directional component needed to evaluate max-type operators from the two PSA invariants Q_0 and κ , rather than fitting unrelated empirical relations for different RotD measures.

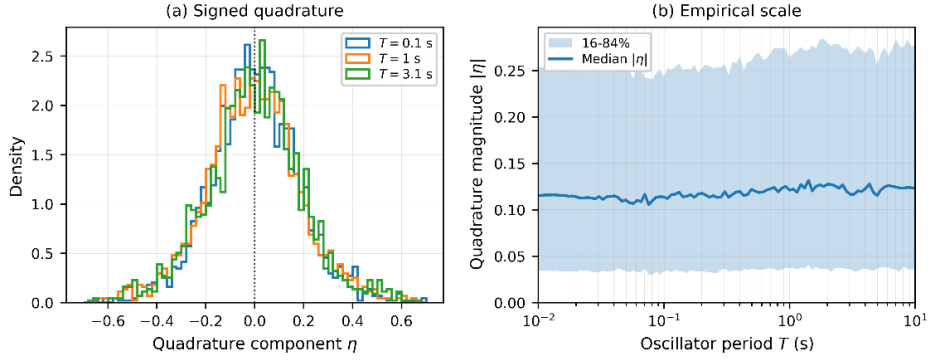


Figure 5. Empirical quadrature component in the RMS principal frame. Panel (a) shows the distribution of the signed quadrature coefficient η at representative periods. Panel (b) shows the period dependence of $|\eta|$.

With the RMS-frame quantities defined above, the rotation-invariant operators can be written directly in closed form. The RotD50 approximation follows from the RMS-axis scale,

$$\widehat{\text{RotD50}}(T) = \sqrt{Q_0(T)} \quad (35)$$

For the max-type operators, the resolved anisotropy $\kappa(T)$ is replaced by the operator-specific effective anisotropy $\kappa_{\text{eff},i}(T)$, which accounts for the unresolved quadrature component. This gives

$$\widehat{\text{MaxRotD50}}(T) = \sqrt{Q_0(T) \left[1 + \frac{\kappa_{\text{eff},50}(T)}{\sqrt{2}} \right]} \quad \text{and} \quad \widehat{\text{RotD100}}(T) = \sqrt{Q_0(T) [1 + \kappa_{\text{eff},100}(T)]} \quad (36)$$

These equations are the practical closed-form framework proposed here. They require only the two PSA amplitudes on the RMS principal axes, through Q_0 and κ , together with the compact quadrature closure introduced in Section 4. They do not assume that PSA extrema occur on the RMS principal axes. The RMS axes define the reference frame; the PSA values on those axes define the resolved scale and anisotropy; and the effective anisotropy accounts for the unresolved quadrature component of the leading harmonic. The validation below therefore focuses on whether these closed-form expressions recover the directly sampled rotation-invariant operators.

5. Validation of the closed-form formulation

This section evaluates the closed-form formulation against the rotation-invariant measures obtained from dense directional sampling. The validation has three aims. First, we calibrate and test the operator-equivalent quadrature amplitudes entering the effective anisotropy. Second, we examine whether the resulting closed-form expressions remain unbiased across oscillator period and across the range of resolved anisotropy. Third, we assess whether the approximation preserves record-to-record spectral amplitudes.

5.1 Calibration and validation of the effective anisotropy closure

The closed-form expressions in Section 4 require the operator-equivalent quadrature amplitudes q_i . The empirical quadrature scale $q_0 \simeq 0.12$ was obtained in Section 4 from the distribution of $|\eta|$. Here we determine how this unresolved component enters MaxRotD50 and RotD100. Calibration is performed after normalising each max-

type operator by the RMS-frame scale $\sqrt{Q_0}$. The parameter q_i is chosen so that, over all record-period samples satisfying $\kappa \leq 0.10$, the median closed-form value of $\ln A_i$ equals the median empirical value of $\ln A_i$, where A_i denotes the corresponding normalised operator. This gives $q_{50} = 0.1278$, $q_{100} = 0.2133$. Expressed relative to the empirical quadrature scale $q_0 = 0.12$, these correspond to $c_{50} = 1.065$, $c_{100} = 1.778$. Thus, MaxRotD50 requires an operator-equivalent quadrature amplitude very close to the typical observed value of $|\eta|$, whereas RotD100 requires a larger effective amplitude. This difference is expected because RotD100 is a maximum over orientation and is therefore more sensitive to local angular peaks and unresolved off-axis structure.

Figure 6 shows the empirical amplification factors as functions of the resolved anisotropy κ . The empirical medians vary smoothly with κ . The resolved-only model underpredicts the amplification at low κ , especially for RotD100. Introducing the nominal quadrature amplitude $q_i = q_0$ removes most of this discrepancy for MaxRotD50, but not for RotD100. The operator-equivalent closure $q_i = c_i q_0$ reproduces the low- κ amplification while preserving the high- κ behaviour. The convergence of all curves at large κ shows that the quadrature correction does not alter the dominant deterministic anisotropy regime. Its role is confined mainly to weakly anisotropic records, where the resolved RMS-axis pair contains little directional contrast and the omitted quadrature component has a proportionally larger effect.

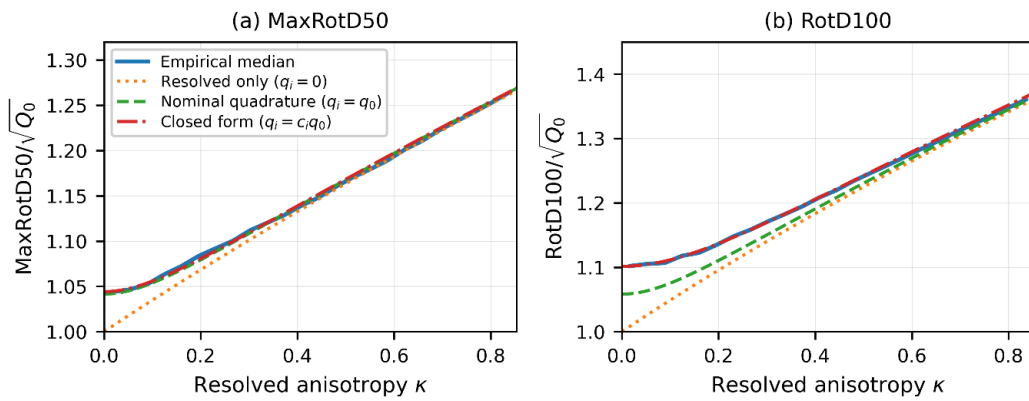


Figure 6. Empirical normalised MaxRotD50 and RotD100 as a function of the resolved PSA anisotropy κ . Blue curves show binned empirical medians. The resolved-only model neglects the unresolved quadrature component, the nominal-quadrature model uses $q_i = q_0$, and the closed-form model uses the operator-equivalent amplitudes $q_i = c_i q_0$.

5.2 Bias with oscillator period

The calibrated closed-form formulation is next evaluated as a function of oscillator period. For each rotation-invariant measure, the percentage bias is defined as $\delta_M(T) = 100[M_{ds}(T)/M_{cf}(T) - 1]$, where M_{ds} denotes the value obtained from dense directional sampling and M_{cf} denotes the corresponding closed-form prediction. Positive values therefore indicate that the dense-sampled value is larger than the closed-form estimate. Figure 7 shows that the median bias remains close to zero over the full period range for all three operators. RotD50 exhibits a small negative median bias, of order 1%, reflecting the slight difference between the RMS-frame scale $\sqrt{Q_0}$ and the angular median RotD50. At representative periods, the median RotD50 bias is -1.22% at $T = 0.1s$, -1.05% at $T = 1.0s$, and -0.70% at $T = 5.0s$. The two max-type operators show even smaller median bias after the effective anisotropy closure is applied. For MaxRotD50, the median bias is -0.51% , -0.16% , and -0.17% at $T = 0.1$, 1.0 , and $5.0s$, respectively. For RotD100, the corresponding values are -0.19% , -0.24% , and -0.38% . No systematic drift with period is observed. The 16th–84th percentile bands quantify record-to-record residual scatter. Their widths remain modest and broadly stable across the period range. This indicates that the closed-form expressions preserve the main period dependence of the dense-sampled rotation-invariant spectra, while leaving only small residual deviations at the record level.

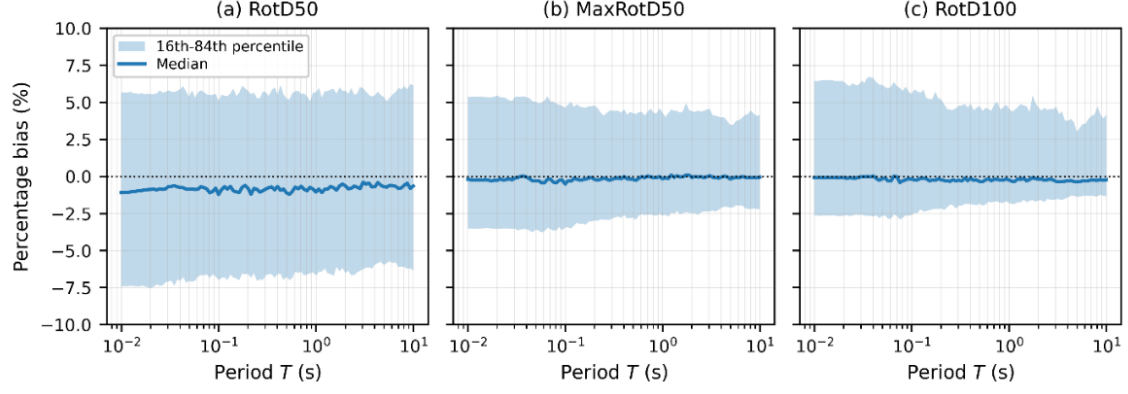


Figure 7. Percentage bias of the closed-form rotation-invariant operators relative to dense directional sampling as a function of oscillator period. Bias is defined as $100(M_{ds}/M_{cf} - 1)$, where M_{ds} is obtained from dense directional sampling and M_{cf} is the closed-form prediction.

5.3 Bias conditioned on resolved anisotropy

We next condition the same bias shown in Figure 7 on the resolved anisotropy κ . This provides a more direct test of the effective-anisotropy closure, because the quadrature correction is expected to be most influential when the resolved anisotropy is small. Figure 8 shows the percentage bias for MaxRotD50 and RotD100 after pooling all record-period samples and binning them by κ . The median residual remains close to zero over the full anisotropy range for both operators. In the low-anisotropy range $\kappa < 0.05$, the median bias is -0.20% for MaxRotD50 and -0.05% for RotD100. For $0.05 \leq \kappa < 0.10$, the corresponding values are -0.12% and -0.11% . At larger anisotropy, the median residuals remain similarly small; even for $0.60 \leq \kappa < 0.85$, the median biases are only -0.20% and -0.33% , respectively.

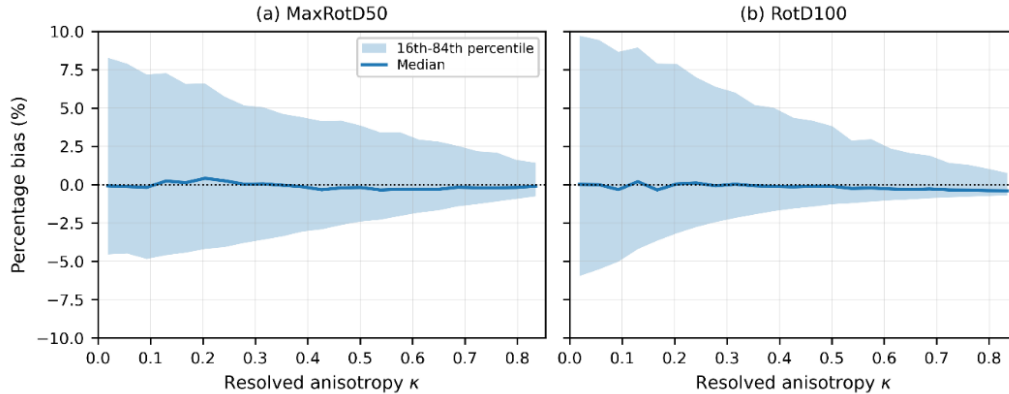


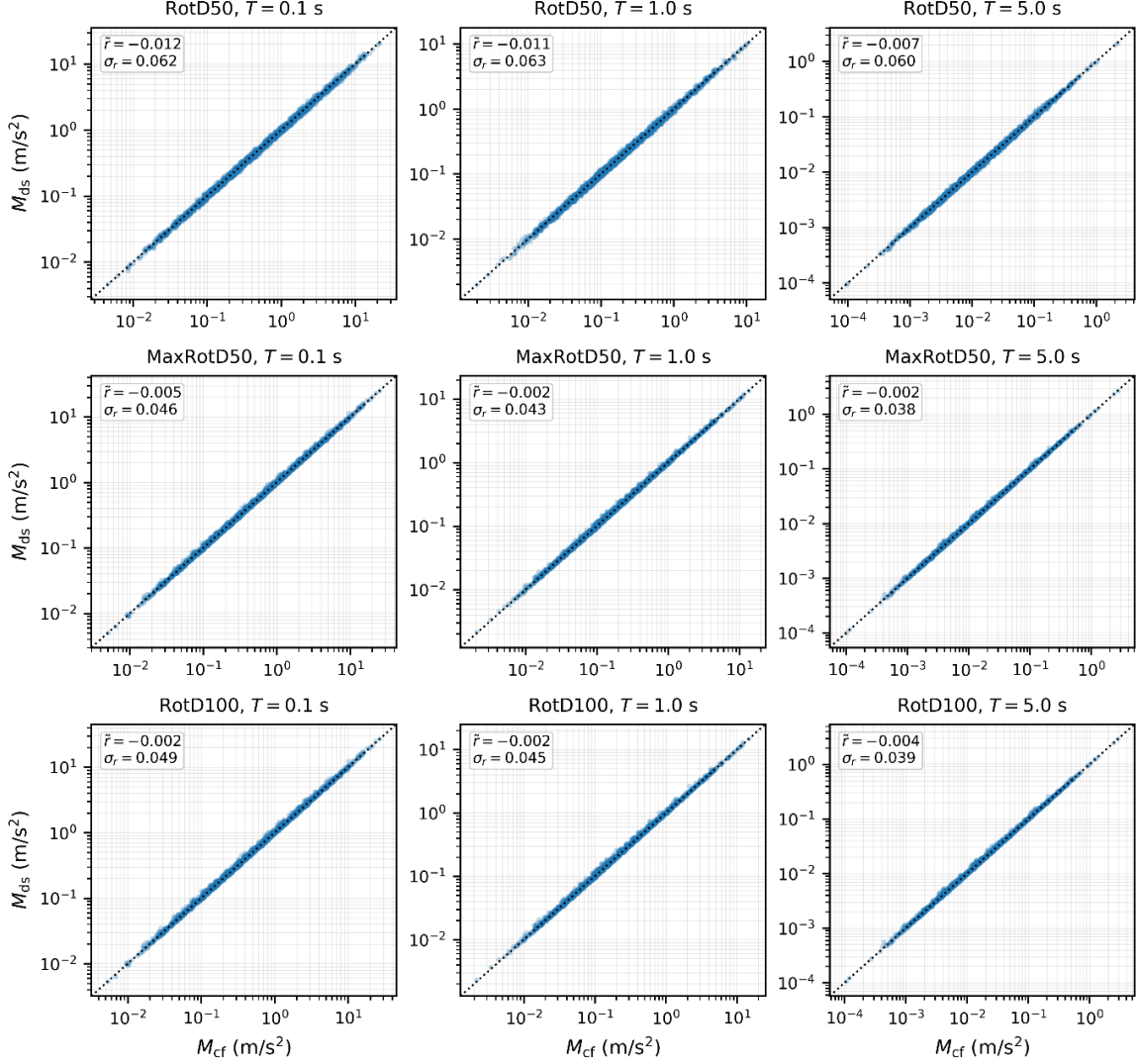
Figure 8. Percentage bias of the closed-form predictions for MaxRotD50 and RotD100 as a function of resolved anisotropy κ . The bias definition is the same as in Fig. 7, $100(M_{ds}/M_{cf} - 1)$. Samples are pooled over all records and periods and binned by κ . Solid curves show binned medians, and shaded bands show the 16th–84th percentile range.

The residual spread is largest at small κ , where the resolved RMS-axis pair contains little directional contrast and the unresolved quadrature component represents a larger fraction of the total directional modulation. As κ increases, the 16th–84th percentile band narrows, consistent with the resolved anisotropy becoming the dominant contribution. The near-zero median trend across the full range of κ confirms that the effective-anisotropy closure removes the low- κ bias without introducing systematic bias at moderate or large anisotropy.

5.4 Record-level performance

The preceding diagnostics show that the closed-form formulation is unbiased in aggregate, both as a function of oscillator period and as a function of resolved anisotropy. We next examine whether the same formulation preserves record-to-record amplitudes. This is important because rotation-invariant measures are commonly used in ground-motion modelling and hazard applications, where preserving the relative spectral levels of individual records is as important as reproducing the median trend. For each operator and representative period, Figure 9 compares the dense-sampled value M_{ds} with the corresponding closed-form prediction M_{cf} . The comparison is made on logarithmic axes in physical units, and the residual is defined as $r = \ln(M_{ds}/M_{cf})$. The scatter clouds

512 closely follow the one-to-one line for all three operators and all representative periods. This agreement extends
 513 over several orders of magnitude in spectral amplitude, indicating that the closed-form formulation preserves the
 514 record-to-record scaling of the dense-sampled measures. The median residuals are consistent with the small biases
 515 reported in Figures. 7 and 8. The residual scatter is also small. The standard deviation of r is approximately 0.06
 516 for RotD50 and about 0.04 to 0.05 for MaxRotD50 and RotD100. Thus, the closed-form expressions introduce
 517 only modest record-level deviations while preserving the dominant spectral amplitude and ranking of individual
 518 records. This confirms that the proposed reduction to Q_0 , κ , and the effective quadrature closure does not merely
 519 reproduce ensemble averages, but also retains the record-specific information needed for practical applications.



520
 521 *Figure 9. Record-level comparison between dense directional sampling and closed-form predictions for RotD50, MaxRotD50,*
 522 *and RotD100 at representative periods $T = 0.1, 1.0$, and 5.0 s. Each panel shows M_{ds} versus M_{cf} on logarithmic axes, with*
 523 *the dotted line indicating equality. Annotations report the median and standard deviation of the log residual $r = \ln(M_{ds}/M_{cf})$.*

524 5.5 Example spectra for best- and worst-matching records

525 As a final record-level diagnostic, Figure 10 compares full spectra for the records with the smallest and largest
 526 typical discrepancies between dense directional sampling and the closed-form predictions. For each record, the
 527 mismatch score is defined as

$$S_j = \text{median}_{T,i} \left| \ln \left(\frac{M_{ds,j,i}(T)}{M_{cf,j,i}(T)} \right) \right| \quad (37)$$

528 where $i \in \{\text{RotD50, MaxRotD50, RotD100}\}$. The three records with the smallest S_j are shown in the top row, and
 529 the three records with the largest S_j are shown in the bottom row. The best-matching records show almost complete
 530 overlap between the dense-sampled and closed-form spectra. For the worst-matching records, the differences are

still modest and mainly appear as small offsets over limited period ranges. The overall spectral shapes and relative amplitudes are preserved for all three rotation-invariant measures. This confirms, at the level of complete spectra, that the closed-form formulation retains the record-specific information contained in the dense directional sampling.

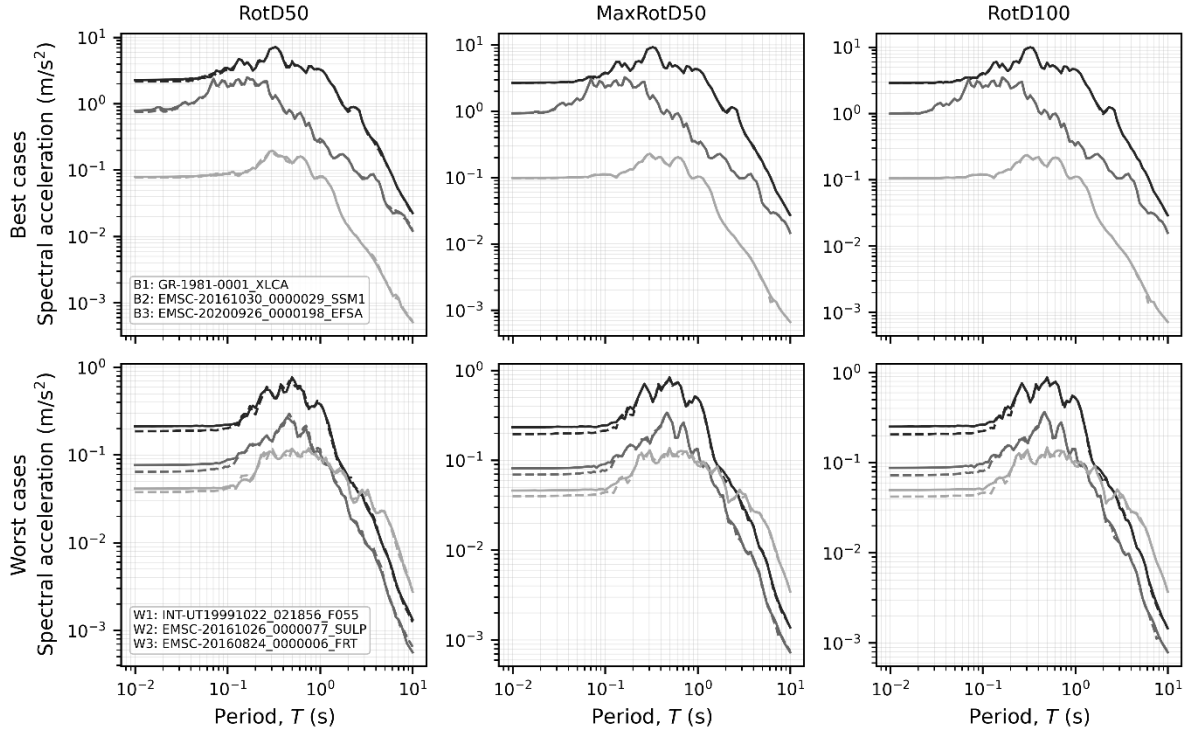


Figure 10. Best- and worst-matching spectral comparisons. Records are selected using the median absolute log residual S_j over all periods and the three rotation-invariant operators. Solid curves show dense-sampled spectra and dashed curves show closed-form predictions. The columns show RotD50, MaxRotD50, and RotD100.

6. Discussion

The preceding sections established that the directional PSA field is low dimensional. After normalisation by a scalar level, most of the angular variation of the squared field is carried by the first admissible harmonic, so that what appears under dense angular sampling to be a large set of orientation-dependent ordinates is, to leading order, described by a scale, an anisotropy, and a phase. This is the central contribution of the paper. The value of the formulation is not that it avoids dense directional sampling, which is not prohibitive with modern computing power. Its value is conceptual: it identifies the minimum directional information required to treat horizontal ground motion as a two-dimensional object. The closed-form expressions for RotD50, MaxRotD50, and RotD100 then follow as consequences of the reduced field representation. They are not the primary result, but operators acting on the field scale and anisotropy.

This change in representation clarifies a set of difficulties that are often treated separately: how to account for component-to-component variability in seismic hazard analysis, how to interpret maximum-direction spectra as design actions, and how to convert between rotation-invariant measures across period. In a scalar formulation, these appear as separate empirical corrections: an additional variance term, a scalar envelope, or a conversion ratio. In the field formulation, they are different reductions of the same object: a coupled, low-dimensional, per-record deterministic directional field. We discuss these implications in turn, beginning with scalar variance inflation and its loss of per-record geometric bounds.

6.1 Directional variability as field geometry

When a scalar IM such as RotD50 is used in a GMM, the predicted logarithmic intensity is represented as a median plus residual variability, usually decomposed into between-event and within-event components, $\sigma_{\text{GMM}}^2 = \tau^2 + \phi^2$. If the target is instead the arbitrary, randomly oriented horizontal component, an additional component-to-component term, σ_{c2c} , is often introduced to represent the dispersion of directional ordinates about the scalar reference, and the total standard deviation is then written as $\sigma_{\text{arb}}^2 = \tau^2 + \phi^2 + \sigma_{c2c}^2$. This treatment is motivated by a correct observation: structures respond to orientation-dependent intensities rather than to RotD50 itself

(Girmay and Miranda, 2026). However, the scalar variance-inflation approach changes the object being propagated. The arbitrary component is a marginal sample from a directional field, not an independent scalar residual. A record carries a complete directional field; once one orientation is sampled, the orthogonal orientation is not an independent second draw but the companion value on the same field. The directional variation is conditional geometry of a record; not independent scatter attached to a scalar model.

This distinction has a consequence that can be stated without any reference to the first-harmonic approximation. Let $Y(\theta)$ be the directional pseudo-spectral acceleration at a fixed period and let RotD100 and RotD50 be its maximum and median over orientation. For any two-component ground motion, $\text{RotD100} \leq \sqrt{2} \text{RotD50}$. Equality occurs only in the limit case of perfect polarisation. In that case, the maximum directional response equals the peak resultant (R), while the median over orientation falls to $R/\sqrt{2}$. Any departure from perfect polarisation fills the orthogonal directions, raises the median, and lowers the ratio.

This ceiling exposes the difficulty with scalar variance inflation. The arbitrary component sampled from a record is bounded by that record's directional envelope: it cannot exceed RotD100, and RotD100 itself is bounded relative to RotD50. A lognormal model centred on RotD50 with an inflated standard deviation has no such bound. It assigns positive probability to arbitrary-component intensities larger than $\sqrt{2} \text{RotD50}$, a state that no two-component ground motion can produce from the same field. This is not a minor technical detail. The upper tail of the intensity distribution is the part that governs the low mean annual exceedance rates used for design and maximum-considered-event checks, so scalar inflation does not merely widen the distribution slightly beyond the physical range: it places probability on physically impossible intensities at the return periods that control design.

The defect cannot be repaired within the scalar standard deviation, because the missing information is not another scalar; it is the shape of the field. In a first-harmonic squared field $Q(\theta) = Q_0[1 + \rho_0 \cos 2(\theta - \theta_0)]$ the admissible maximum is $\text{RotD100} = \sqrt{Q_0(1 + \rho_0)}$, so the envelope depends on the anisotropy of the individual record. Once the field has been replaced by a scalar RotD50 and an inflated standard deviation, the record-specific anisotropy is lost, and a truncation imposed after the fact has no way to know where the true envelope lies.

The same point appears if the component-to-component variability is examined through the field model. For a RotD50-centred definition,

$$\sigma_{c2c}^2 = \langle [\ln Y(\theta) - \ln \text{RotD50}]^2 \rangle_\theta,$$

With $Q(\theta) = Q_0[1 + \rho_0 \cos 2\theta]$ and $\text{RotD50} = \sqrt{Q_0}$,

$$\ln Y(\theta) - \ln \text{RotD50} = \frac{1}{2} \ln(1 + \rho_0 \cos 2\theta)$$

so that to leading order in the anisotropy $\sigma_{c2c} \approx \frac{\rho_0}{2\sqrt{2}}$. Thus, to leading order, component-to-component variability is the field anisotropy expressed as a standard deviation. It is not independent of directional shape; it is a fixed multiple of it. The empirical observation that σ_{c2c} increases at periods where response is more polarised is therefore the same observation, in different terms, as the growth of directional anisotropy with period. The quantity treated as additional aleatory variability is, in the field view, a scalar projection of structured directional geometry.

The operation underlying σ_{c2c} can also be interpreted as a two-stage average. For each record j and period T one first computes the angular spread of the directional ordinates about the scalar reference,

$$s_j^2(T) = \langle [\ln Y_j(T, \theta) - \ln \text{RotD50}_j(T)]^2 \rangle_\theta$$

and then averages this within-record spread over the record population, $\sigma_{c2c}^2(T) = \langle s_j^2(T) \rangle_j$. The first average is over orientation within a fixed record, where $Y_j(T, \theta)$ is deterministic once the two histories are known. The second average is over records. Therefore, it produces a population-average angular variability but collapses the record-to-record variation of field shape into a single number. However, the variable needed to retain that shape, the anisotropy, is averaged away. Within-record directional variation is geometry; between-record variation of the field parameters is stochastic; the two-stage average fuses them, and after the fusion the bound, which is a per-record geometric fact, is no longer recoverable.

Combining σ_{c2c} with the ground-motion dispersion in quadrature does not alter this conclusion. The physical ceiling constrains the arbitrary component relative to the RotD50 of the same record, whereas the GMM dispersion describes the scatter of that record's RotD50 about the predicted median. Writing the realised arbitrary component as

$$\ln Y_{\text{arb}} = \ln \text{RotD50}_{\text{med}} + \varepsilon_{\text{GMM}} + \varepsilon_{c2c}, \text{RotD50}_{\text{samp}} = \text{RotD50}_{\text{med}} e^{\varepsilon_{\text{GMM}}}$$

the geometric ceiling is the statement that $Y_{\text{arb}}/\text{RotD50}_{\text{samp}} \leq \sqrt{2}$. The GMM residual ε_{GMM} scales the record's RotD50, and the ceiling moves up with it; it is common to the arbitrary component and to the bound and therefore cancels from their ratio. Only ε_{c2c} moves the arbitrary component relative to its own RotD50, and it is this term that is geometrically constrained. The probability assigned by the scalar inflation model to inadmissible values is therefore

$$P\left(\frac{Y_{\text{arb}}}{\text{RotD50}} > \sqrt{2}\right) = P(\varepsilon_{c2c} > \ln \sqrt{2}) = 1 - \Phi\left(\frac{\ln \sqrt{2}}{\sigma_{c2c}}\right),$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. This consequence can be quantified from the published component-to-component variabilities. The component-to-component standard deviation grows with period as the field polarises, reaching values approximately 0.30 to 0.40 in natural-log units for RotD50 at long period (Girmay and Miranda, 2026). For $\sigma_{c2c} \approx 0.13$, typical of short periods, the inadmissible fraction is below 1%, and the inflation is essentially admissible. By $T \approx 5\text{s}$, where $\sigma_{c2c} \approx 0.30$, it is about 12%, and by $T = 10\text{s}$ about 19%. Thus, at long periods, a substantial fraction of the modelled directional variability can be placed on arbitrary-component intensities that no two-component ground motion can produce from the same field.

The field representation avoids this inconsistency by keeping stochastic variability and directional geometry separate. The stochastic quantities are the record-to-record distributions of the scale Q_0 , the anisotropy κ , and the field orientation. Directional uncertainty is propagated by sampling these field parameters and rotating the field, not by replacing the field with an unbounded scalar residual. Every realisation carries its own scale, anisotropy, orientation, and envelope, so the arbitrary component satisfies $Y_{\text{arb}} \leq \sqrt{2} \text{RotD50}$ for each realisation, and the exceedance rate of the arbitrary component at level y cannot exceed the exceedance rate of RotD50 at level $y/\sqrt{2}$. The bound holds automatically because the coupling between directions has not been discarded. This is why the anisotropy is not an optional refinement but the minimum additional information a directional model requires: the scale describes how large the field is, the anisotropy describes how concentrated it is, and without both a model cannot represent the envelope, the orthogonal coupling, or the boundedness of the directional ordinates.

6.2 Scalar measures and bidirectional design actions

The same reduction underlies the interpretation of maximum-direction spectra in design. RotD100 is the largest value attained by the directional response spectrum over orientation. It is an envelope property of the field not a pair of compatible component spectra. If RotD100 is applied simultaneously along two orthogonal structural axes, the implied demand state places both axes at the maximum-direction intensity. For an anisotropic field this state generally cannot occur, because the maximum direction and its orthogonal direction are coupled by the same field geometry that produces the $\sqrt{2}$ bound: when the response is large along one axis, the orthogonal response is constrained, not free to equal it. Applying the scalar envelope to both axes replaces the directional field with a circle of radius RotD100, and that circle contains demand states outside the original field. The difficulty is not that RotD100 is incorrect but that a scalar envelope is being used as if it carried the bidirectional structure of the motion. Practical difficulties in design applications and record selection arising because of deploying an envelope spectrum as a component action is discussed in more detail in Rupakhety (2026).

The same holds for any scalar spectrum deployed as a bidirectional action. RotD50, MaxRotD50, and RotD100 are all circles when applied as isotropic design spectra; changing the scalar changes the radius but does not recover the anisotropy or orientation of the field. This is why the effective conservatism of a scalar target depends on the analysis procedure and combination rule: a scalar contains no information about the orientation of the field relative to the structure, so any procedure requiring that information must impose it indirectly or ignore it. The field representation locates the missing information explicitly. A design action specified by a scale and an anisotropy carries the two principal levels of the field; the demand along any structural orientation follows by rotation, and the orthogonal companion demand is determined by the same field rather than assigned the same scalar envelope.

6.3 Anisotropy and rotation-invariant ratios

The third difficulty concerns the empirical ratios used to convert between rotation-invariant measures, of which RotD100/RotD50 is the most consequential because it links hazard products in terms of RotD50 to design spectra in terms of RotD100. These ratios are commonly modelled as functions of period (Poulos and Miranda, 2022; Boore and Kishida, 2017). The field representation shows that period is not the primary variable that controls them: the ratios increase because the field becomes more anisotropic, and period enters only because the

distribution of anisotropy changes with period. Under the first-harmonic model the ratio is governed by the anisotropy alone $\frac{\text{RotD100}}{\text{RotD50}} \approx \sqrt{1 + \rho_0}$ reaching the bound $\sqrt{2}$ as $\rho_0 \rightarrow 1$.

Figure 11 shows this directly. The pooled ratios over all records reproduce the published population-average conversion models, but once the data are stratified by the resolved anisotropy κ the curves separate systematically and the period dependence within each stratum is largely removed. Records with small κ remain near the isotropic limit across all periods, while records with large κ have substantially larger ratios that are themselves nearly flat in period. The pooled period-dependent curve is a mixture over records of differing anisotropy, and its rise with period reflects the shift of that mixture toward higher anisotropy at long periods rather than any period dependence of the operator relationship itself.

The effect is large enough to matter in practice. At $T = 1\text{ s}$ the median RotD100/RotD50 ratio is about 1.10 for records with $\kappa < 0.05$ and about 1.29 for records with $\kappa \geq 0.30$, an 18% difference in maximum-direction demand at the same RotD50 level; for MaxRotD50/RotD50 the corresponding medians are about 1.04 and 1.21. Two records matched to an identical RotD50 spectrum therefore impose meaningfully different maximum-direction demands if their anisotropies differ, and a conversion applied through a single period-dependent ratio cannot distinguish them.

Figure 12 establishes a further point that bears on how directionality should be measured. Although the ratios are organised by κ in the median, the ratio RotD100/RotD50 is not itself a clean directionality metric for an individual record, because RotD100 is an extremal operator sensitive to local angular peaks rather than to the persistent geometry of the field. Of the 30,631 record-period samples with $\kappa < 0.1$, some 3,539 (about 12%) nonetheless have RotD100/RotD50 exceeding 1.2, so an elevated maximum-direction ratio does not identify a strongly anisotropic field. The distribution shifts upward with κ , from a median of about 1.10 for $\kappa < 0.1$ to about 1.24 for $\kappa \geq 0.1$, but the overlap is substantial.

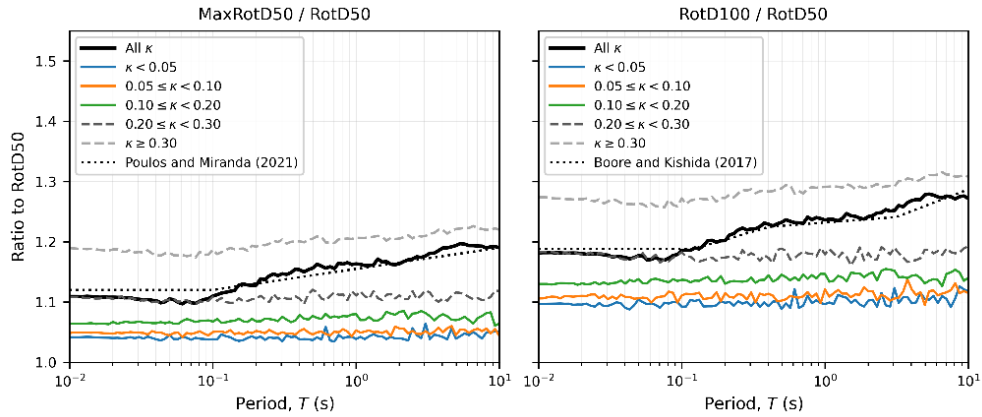


Figure 11. Rotation-invariant ratios stratified by resolved PSA anisotropy κ . Curves show dense-sampled median ratios as functions of period for all records and for selected κ bins. Published population-average models are shown as dotted reference curves: Poulos and Miranda (2022) for MaxRotD50/RotD50 and Boore and Kishida (2017) for RotD100/RotD50. The pooled ratios follow the expected period-dependent trends, but conditioning on κ reveals systematic separation by directional anisotropy.

The application to record selection follows directly. Selecting records to match a target RotD50 spectrum controls the scalar level but not the directional shape, and two records with the same RotD50 spectrum may impose very different bidirectional demands. A field-aware procedure matches both the scalar spectrum and the distribution of κ appropriate to the hazard scenario, so that the directional structure of the selected motions is correct for the scenario; and conversion from RotD50-based hazard to maximum-direction design spectra can be conditioned on the anisotropy expected for the controlling motions rather than applied through a period-averaged ratio that mixes scenarios of differing directional character.

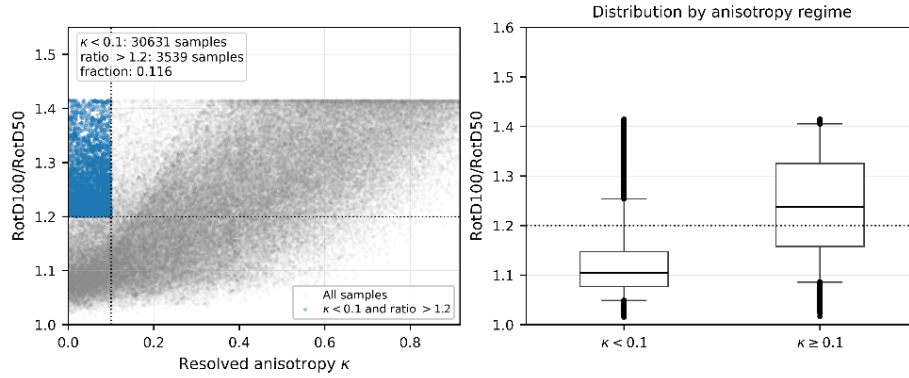


Figure 12. Elevated RotD100/RotD50 ratios at low resolved anisotropy. Left panel shows RotD100/RotD50 versus κ for all record-period samples; highlighted points satisfy $\kappa < 0.1$ and RotD100/RotD50 > 1.2 . Right panel compares the ratio distributions for low- and higher- κ regimes. Although the distribution shifts upward with κ , 11.6% of the low- κ samples exceed a ratio of 1.2, showing that large maximum-direction ratios do not uniquely identify strong persistent directional anisotropy.

6.4 Toward scale-and-shape aware ground-motion models

The field representation suggests a direct path to modelling directional structure: retain the directional information and represent the field by the minimum variables it requires, a scale and an anisotropy. These constitute a minimal vector-valued IM that contains the information from which RotD50, MaxRotD50, and RotD100 can be recovered, while preserving the directional coupling that scalar measures discard.

This changes how directional information can be organised probabilistically. Existing characterisations often describe directionality through distributions of directional ratios at each of the nonredundant orientations, a high-dimensional object, one distribution per angle. The result here is that this characterisation is over-parametrized: the field has far fewer degrees of freedom, so the directional information is carried not by a family of angular distributions but by the distribution of a single anisotropy parameter together with the scale. The contribution is the dimensional reduction of the directional characterisation, justified empirically by first-harmonic dominance rather than assumed.

This reduction makes the representation practical. A model for the scale, or for a closely related scalar such as RotD50, describes the overall response level; a companion model for κ describes the directional shape; the common rotation-invariant measures become derived operators rather than separate empirical targets. The two components may depend on physical predictors differently. Directivity, rupture radiation pattern, basin geometry, and site-specific directional response may influence the anisotropy more strongly than the scale. A scale-and-shape model can ask which predictors control amplitude and which control directional concentration. The same representation provides a diagnostic for physics-based simulation: a simulation may reproduce the observed RotD50 spectra while producing systematically too little or too much anisotropy, a discrepancy invisible to a scalar comparison but immediate in the distribution of κ . The anisotropy is therefore not merely a parameter for closed-form approximation; it is the simplest observable descriptor of directional shape and a natural target for modelling, selection, and validation.

6.5 Scope and limitations

The results are established for 5% damped linear elastic PSA using the ESM dataset and the period range considered in this study. The first-harmonic representation captures the dominant angular structure of the squared directional field but is not exact; higher harmonics, local angular notches, and peak-selection effects remain, and their influence is most visible for the maximum-type operators, particularly RotD100. The quadrature closure should accordingly be understood as an engineering reduction of the unresolved directional component rather than a claim that the full field is exactly harmonic. The empirical quadrature scale and the operator-equivalent amplitudes are estimated from the present dataset, and their stability should be checked for other datasets, damping levels, and IMs.

Several natural directions for further work follow from the representation. The record-to-record distribution of the anisotropy should be modelled and related to magnitude, distance, site condition, faulting mechanism, directivity, and regional effects, supplying the directional component of a vector-valued ground-motion representation. The consequences for hazard analysis, record selection, spectral matching, and bidirectional structural demand should be evaluated explicitly, including the construction of hazard estimates for the arbitrary component that respect the physical directional bound.

7. Conclusions

Horizontal ground motion is two-dimensional, but its directional response spectrum is not the high-dimensional angular object that dense sampling appears to produce. This paper treated directional pseudo-spectral acceleration as a field on orientation and examined the rotation-invariant measures RotD50, MaxRotD50, and RotD100 as operators acting on that field. The main findings are as follows.

First, the squared directional PSA field is strongly dominated by its first admissible angular harmonic. After normalisation by a scalar level, most of the directional variation is described by a scale, an anisotropy, and a phase. This low-dimensional structure explains why rotation-invariant measures that are normally computed by dense directional sampling can be recovered from a compact field representation.

Second, the scalar RotD measures are not independent IMs. They are different operators applied to the same directional field. RotD50 represents the central level of the field, whereas MaxRotD50 and RotD100 depend on directional modulation and therefore on anisotropy. The closed-form expressions derived in this paper make this dependence explicit.

Third, two rotation-invariant parameters are sufficient for practical reconstruction of the common RotD measures: a scale Q_0 and an anisotropy κ , both evaluated from the RMS principal-axis pair. The unresolved quadrature component can be represented by a compact empirical closure. With this closure, the closed-form predictions of RotD50, MaxRotD50, and RotD100 are essentially unbiased across oscillator period and preserve record-to-record spectral amplitudes.

Fourth, anisotropy is the organising variable behind rotation-invariant ratios. Ratios such as MaxRotD50/RotD50 and RotD100/RotD50 are often represented as functions of period, but the present results show that their systematic variation is primarily controlled by the directional shape of the field. Conditioning on κ separates records with weak and strong directional modulation and explains why records matched to the same RotD50 spectrum can impose different maximum-direction demands.

The broader implication is that directional variability should not be treated as information external to scalar IMs. It is part of the ground-motion field itself. A directional model therefore requires, at minimum, both scale and anisotropy. This is a modest extension of current scalar practice, but it preserves the orthogonal coupling and boundedness that scalarisation discards. The directional response spectrum is, to leading order, a scale-and-shape field, and Recognising this structure provides a basis for directionality-aware ground-motion modelling, record selection, hazard analysis, and bidirectional design.

Data availability:

All ground motion data used in this study are available at the Engineering Strong Motion Database (<https://esm-db.eu/#/home>)

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