Estimating flood quantiles at ungauged sites using nonparametric regression methods with spatial components

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Abstract

Prediction of flood quantiles at ungauged sites has been investigated using several nonparametric regression methods including: local regression based on regions of influence, neural networks and generalized additive models (GAM). These methods were used to describe the relationship between run-off variables and catchment descriptors to predict flood quantiles. Previous work reported the presence of spatial correlation in the residuals for these models. To this end, this study proposes and investigates ways of incorporating spatial components. An L-moments regression technique (LRT) is developed to predict L-moments of target sites and flood quantiles are derived by aggregating quantiles from multiple candidate distributions. The predictive power of the proposed methods is evaluated on a large database of Canadian rivers using cross-validation. The results are examined inside different provinces and hydrological regions to assess the behaviour of the methods. The results show that GAM and local regression using respectively thin plate spline and kriging provide the best predictive powers among the considered methods. Additionally,

the LRT method is found to improve prediction power over the well-known index-flood model and has similar results to quantile regression techniques (QRT) when using the same nonparametric regression approaches.

Keywords: Kriging, Flood, Frequency analysis, Generalized additive models, Region of influence, Ungauged site, Canada.

1. Introduction

Estimating the statistical distribution of extreme events is a crucial task in designing safe and cost-efficient infrastructure. For a given catchment, the amount of streamflow data may not be sufficient to carry out a proper at-site frequency analysis with the desired level of accuracy. In such a situation, regional frequency analysis is commonly used to improve the quality of the estimates by transferring additional information from nearby catchments with similar characteristics. In particular, when a target site is ungauged, flood quantiles associated with a given return period must be deduced from the relationship with existing catchment descriptors. Quantile regression technique (QRT) (Thomas and Benson, 1975) is an approach where the model is directly linking the at-site flood quantiles to the catchment descriptors. This method avoids the subjective task of selecting a best distribution at the site of interest, which impacts the extrapolation for longer return periods. However, this approach may lead to incoherent estimations among different return periods as separate models are necessary for every desired return period. Alternatively, parameter regression techniques (PRT) can be used to link the parameters of a target distribution to its catchment descriptors. In terms of performance for estimating flood quantiles at ungauged sites, recent comparison studies concluded that both techniques led to similar results and that the choice between the two methods depends mostly on practical considerations (Ahn and Palmer, 2016; Haddad and Rahman, 2012).

Among existing parameter regression techniques, the index-flood method (Hosking and Wallis, 1997) is commonly used to estimate flood quantiles based on the assumption that inside a homogenous region all sites follow the same distribution up to a scale factor, called the index-flood. In particular, a neighborhood defines a homogenous region that is unique to a target site and where the pooled catchments are the nearest catchments according to a given metric (Acreman and Sinclair, 1986). Several comparative studies showed that this type of region, commonly called Region Of Influence (ROI), outperforms the use of fixed regions derived from common clustering techniques (Burn, 1990; GREHYS, 1996; Tasker et al., 1996). For gauged-site analysis, the notion of homogeneity of a region is crucial to the validity of the model. For that purpose, the degree of homogeneity is generally evaluated according to statistics comparing the properties of the flood responses (Burn, 1997; Castellarin et al., 2008; Requena et al., 2017; Viglione et al., 2007). When there are no streamflow data, the degree of homogeneity of a region in regards of the target site cannot be directly assessed. However, useful regions can still be delineated using geographical distance or similarity between catchment descriptors (Eng et al., 2007; Zrinji and Burn, 1994), even though the resulting regions may lack hydrological similarities (Burn et al., 1997; Oudin et al., 2010). Nonetheless, the delicate task of delineating and validating homogenous regions can be avoided if one assumes instead that the catchment attributes evolve smoothly inside the descriptor space (Chokmani and Ouarda, 2004; Laio et al., 2011).

Linear regression equations are frequently employed to characterize the relationship between run-off variables and catchment descriptors (Pandey and Nguyen, 1999). When the number of gauged sites is limited, the lack of information may prevent the utilization of more complex models, but in general, a nonlinear relationship should be expected (Wittenberg, 1999). Nonparametric regression is a general term associated with various methods for which the relationship between a response variable and some explanatory variables is sufficiently flexible to accommodate for arbitrary patterns, if sufficient information is available. Therefore, the use of nonparametric regression methods for the prediction of a flood quantile at ungauged sites can avoid the formation and validation of homogenous regions by providing a single model valid for all considered catchments.

Local regression is one example of nonparametric regression methods where a low-degree polynomial is fitted inside small neighborhoods to locally approximate the global pattern (Altman, 1992; Cleveland and Devlin, 1988). This strategy is well known in regional flood frequency analysis as it includes as a special case the utilization of multiple regression inside a region of influence. Another example of a nonparametric regression method is a Neural Network (NN) that is inspired by the way neurons are mapped inside the human brain. For prediction at ungauged sites, neural networks were found to outperform the use of linear models inside homogenous regions in different contexts (Aziz et al., 2014; Dawson et al., 2006; Shu and Burn, 2004; Shu and Ouarda, 2007). Apart from these two better known methods, other nonparametric regression methods investigated in regional frequency analysis include regression trees (Laaha and Blöschl, 2006; Schnier and Cai, 2014), support-vector regression (Gizaw and Gan, 2016) and Generalized Additive Models (GAM) (Latraverse et al., 2002; Rahman et al., 2018).

Catchment descriptors used as explanatory variables in a regression model can be classified in two groups. One is composed of catchment attributes that are specific to a given location on a river. This group includes, among others, drainage area, channel length and mean slope. The second group represents physio-climatic attributes that are continuous in space, such as rainfall, temperature, forest canopy and soil characteristics. In the United Kingdom, the study of Kjeldsen and Jones (2009) showed that even when this second group of descriptors is used in a regression model, spatial correlation may still be found in the residuals, which indicates that the information passed to the models does not fully characterize the flood-generating process. In particular, gathering information on the underground characteristics can be a major challenge (Oudin et al., 2008). In such case, spatial proximity can be used as a surrogate and several strategies have been considered to efficiently combine information derived from catchment descriptors and spatial proximity. Oudin et al. (2008) in France and Steinschneider et al. (2015) in the United States integrated spatially autocorrelated errors in their regression models. Similarly, Archfield et al. (2013) compared kriging techniques (Castiglioni et al., 2009; Skøien and Blöschl, 2007) in the United States and found that a two-step procedure that performs kriging in the descriptor space followed by kriging of the residuals in geographical space works better than both techniques taken separately.

The main objective of the present study is to investigate strategies that can improve the treating of geographical information inside nonparametric regression models for the objective of predicting flood quantiles at ungauged sites. A special interest is given to local regression, which is largely applied in practice, and GAM modelling. The latter possesses a practical mathematical representation and provides straightforward interpretation of the catchment descriptors (Chebana et al., 2014). Due to its practical form, GAM can be directly combined with kriging techniques without requiring a two-step procedure as in the suggestion of Archfield et al. (2013). Alternatively, thin plate splines (TPS) (Green and Silverman, 1993) are a nonparametric regression technique that is specifically designed to model spatial data and Wood (2003) introduced an efficient way of incorporating TPS in GAM modelling. These two approaches extended the class of GAM models by adding a spatial component in a manner that has not been investigated in the context of flood frequency analysis. Similar extensions are also developed for local regression. In this study, these two regression approaches are compared together, and with neural networks, to identify which combinations of nonparametric regression methods and spatial components should be recommended.

This study also pursues secondary objectives. In Canada, water policies fall under provincial jurisdiction and for this reason Canadian studies have traditionally been performed at the provincial level (El-Jabi et al., 2016; GREHYS, 1996; Sandrock et al., 1992), even though administrative boundaries are derived for political reasons and do not often have hydrological relevance. Recent studies have illustrated the interest of performing regional frequency analysis at national and even global scale to take advantage of the increasing amount of information and to assess the model performance in different conditions (Durocher et al., 2018; Salinas et al., 2014; Smith et al., 2015). In particular, Canada has diverse climatic and hydrological environments that result in different types of flood regimes (Burn et al., 2016; Buttle et al., 2016). Therefore, the present study considers a large dataset of 770 hydrometric stations across Canada, to investigate how the considered methods perform in different hydrological environments.

The present methodology will be denoted L-moment regression technique (LRT) as the L-moments of ungauged sites are predicted on the basis of catchment descriptors (Laio et al., 2011). This strategy can be seen as a special case of parameter regression technique, but where catchments with different distributions can be mixed together and aggregated to compute flood quantiles from multiple candidate distributions. In this framework, the application of GAM is similar to that of GAM location-scale-shape (GAMLSS) (Rigby and Stasinopoulos, 2005), that was used in flood frequency analysis to model nonstationarity in time series (López and Francés, 2013; Villarini et al., 2009). However, the present study focuses on prediction at ungauged sites and develops particular strategies for extending this basic framework for incorporating spatial components and mixing at-site distributions. In particular, a new weighting procedure is investigated for evaluating the relative contribution of each candidate distribution. The proposed method will be compared to the index-flood model and quantile regression techniques to determine which of these three approaches is best.

The paper is organized as follows. In section 2, the proposed methodology, including kriging and nonparametric regression, is presented. In section 3, the investigated methods are applied and compared on a large dataset of Canadian rivers. Finally, discussion and general conclusions are provided in Section 4.

2. Methodology

The general methodology consists in fitting separate nonparametric regression models to the first three L-moments of the annual maxima of a river discharge. More precisely, the sample mean (L1), the L-coefficient of variation (LCV), and the L-coefficient of skewness (LSK) are selected.

A logarithm transformation is applied to the first two L-moments to obtain more normal-shape distributions and to enforce positive values. The L-moments are then used to estimate the flood quantiles of ungauged sites for multiple candidate distributions and a final prediction is obtained by averaging the output of each candidate. This section starts by presenting the nonparametric regression methods and continues by describing the procedure to estimate the flood quantiles from the L-moments predicted at target sites. At the end of the section, criteria are proposed for assessing the quality of estimated flood quantiles.

2.1 Universal kriging estimator

Kriging is a powerful geostatistical technique for predicting a variable at unknown location s_0 when data are generated by a Gaussian random field. In the present context, kriging can be used to obtain L-moments at ungauged sites. Let's consider a set of gauged sites $\mathbf{s} = (s_1, \ldots, s_n)$ with a matrix of descriptors $\mathbf{X}(\mathbf{s})$ and a vector of descriptors $\mathbf{x}(s_0)$ at the ungauged site. Similarly, note $\mathbf{z}(\mathbf{s})$ the vector of the sample L-moment of a specific order. According to this notation, the kriging model is written

(1)
$$\begin{aligned} \mathbf{z}(\mathbf{s}) &= \mathbf{X}(\mathbf{s})\beta + \varepsilon(\mathbf{s}), \\ z(s_0) &= \mathbf{x}(s_0)'\beta + \varepsilon(s_0) \end{aligned}$$

where $\varepsilon \sim N(0, \Sigma)$ is an error term and β is a set of parameters. For this kriging model, the impact of the catchment descriptors is described by the deterministic trend $\mathbf{x}(s_0)'\beta$ and the spatial component is characterized by the covariance matrix Σ of the error terms. This separation provides a coherent and distinct decomposition of the model influences.

The objective of kriging is to predict $\hat{z}(s_0) = \mathbf{a}'\mathbf{z}(\mathbf{s})$ as a linear combination of observed values. The kriging estimator is obtained by finding the kriging weights $\mathbf{a}$ that minimize the predicting variance. If the error term follows a known covariance model, then it is possible to compute the covariance $\sigma = \mathrm{Cov}\{z(s_0), \mathbf{z}(\mathbf{s})\}$ between the gauged and target sites and the variance $\sigma_0 = \mathrm{Var}\{z(s_0)\}$ at the target site. Under these conditions, the universal kriging predictor has the form

(2) $$\mathbf{a}'\mathbf{z}(\mathbf{s}) = x(s_0)'\hat{\beta} + \sigma'\Sigma^{-1}\left\{\mathbf{z}(\mathbf{s}) - \mathbf{X}(\mathbf{s})\hat{\beta}\right\},$$

where $\hat{\beta} = \left\{\mathbf{X}(\mathbf{s})'\Sigma^{-1}\mathbf{X}(\mathbf{s})\right\}^{-1}\mathbf{X}(\mathbf{s})'\mathbf{z}(\mathbf{s})$ is the generalized least square estimator (Schabenberger and Gotway, 2004). In simple words, the universal kriging estimator is the sum of a trend and corrected residuals derived from the spatial covariance structure. In practice, the covariance matrix Σ is not known and must be estimated. To this end, a semivariogram model representing the strength of the covariance as a function of distance can be fitted. See for instance Schabenberger and Gotway (2004).

2.2 Generalized additive models

Nonparametric regression models assume that the relationship between the response variable and explanatory variables is characterized by arbitrary, but continuous functions. For a regression model with a single explanatory variable the estimated function is chosen from flexible classes, such as kernel smoothers, spline polynomials or radial basis function (Hastie et al., 2009). One way to extend nonparametric regression to include several explanatory variables is GAM modelling. Following the established notation, a GAM trend describing a response variable is

$$z(s) = \mu + \sum_{k=1}^{p} f_k \left\{ x_k(s) \right\} \qquad (3)$$

where μ is the global mean, $\mathbf{x}(s) = \left\{ x_1(s), \ldots, x_p(s) \right\}$ are explanatory variables and f_i are smooth univariate functions that respect basic conditions that ensure their uniqueness relative to the global mean (Hastie and Tibshirani, 1986).

Spline polynomials are formed as a series of low-order polynomials joined together (under some regularity conditions) at control points. For well-chosen control points, spline polynomials represent an attractive modelling strategy that can be assimilated to a linear model. Therefore, a GAM trend using spline polynomials can be incorporated directly in the kriging model (1). However, the choice of the control points plays an important role in the quality of the fitting and using a large number of control points provides more flexibility, but is likely to overfit the data. In that case, regulation can be used to control the smoothness of the estimated function f_i by adding further restrictions, or penalties. A common choice is $\int f_k''(t)^2 dt < \lambda_k$ that restrict unnecessary fluctuations of the second derivative (Green and Silverman, 1993). In practice, different values of the calibration parameter λ_k are tried and a final model is chosen according to an objective criterion such as the Generalized Cross-Validation criteria (Craven and Wahba, 1978). However, adding such regularization falls outside the framework of the universal kriging estimator in equation (2).

The motivation for using the sum of univariate smooth functions in GAM modelling is that the effect on the response variable with respect to each explanatory variable is straightforward and it was shown that this strategy adapts reasonably well to various nonlinear forms encountered in flood frequency analysis (Chebana et al., 2014). However, that restriction creates limitations. For instance, it means that a GAM model cannot correctly characterize geographical coordinates, because they are treated separately. Consequently, for two or more dimensions, an alternative approach is needed. In this line, for a set of r bivariate control points $c_1, \ldots, c_r$, thin plate spline (TPS) is better suited for modelling spatial data and is defined as a radial basis function

$$f(s) = \sum_{j=1}^{r} w_j \phi\left(\left\| s - c_j \right\|\right) \tag{4}$$

where $\phi(t) = t^2 \log(t)$. Wood (2003) showed that thin plate splines can be used to expand the capability of GAM models by incorporating a bivariate function that characterizes the interaction between two explanatory variables, while keeping the general interpretability of the GAM model. For predicting L-moments at ungauged sites, a GAM model can be considered where the spatial component is described by thin plate splines. In this case, the deterministic trend includes both the spatial component and the relationship with the catchment descriptors and consequently the error term is assumed to be spatially independent.

2.3 Other nonparametric regression models

The GAM models described above contain useful strategies for the spatial component in the prediction of L-moments at ungauged sites. Other regression procedures may not share the same mathematical simplicity, but can also lead to accurate predictions. Local regression where a multiple regression model is performed inside a region of influence is one example. In the present study, weights proportional to the distance of a gauged site from the target site are used to give more importance to the closer sites. Formally, if $s_{(i)}$ are the gauged sites of a neighborhood of site $i = 1, \ldots, n$, ordered by distance $h_{(i)}$ with the target then the weights are

(5) $$w_{(i)} = 1 - \left(h_{(i)} / h_{(n)} \right)^2 .$$

Note that only the number of sites in the neighborhood must be calibrated to use the model. Durocher et al. (2018) indicated that using constant neighborhood sizes by hydrological regions can improve the overall quality of the method in comparison to individual calibrations. This recommendation is followed in the present study. Also, as the addition of one site is not likely to bring substantial changes in the fitting, the best calibration is searched among neighborhoods of sizes from 25 to 200 by steps of 5.

Although locally linear, the local regression as a whole cannot be treated as a linear model, and therefore the universal kriging estimator (2) with a ROI trend cannot be evaluated. However, a ROI/KRG procedure can be considered by extracting the trend in a first step and then using kriging (or TPS) in a second step to further predict the residuals based on spatial proximity. Here, the mean of the residuals is known to be zero and that assumption is incorporated as a constraint in the simple kriging estimator (Schabenberger and Gotway, 2004). This gives rise to a model having a similar form as the universal kriging estimator, but with a different trend and spatial component.

Another alternative to GAM is neural networks (NN). In general, neural networks take different forms, but in the present study, feedforward neural networks with a single hidden layer are considered as they represent a flexible class of models that can approximate any continuous function with the desired precision (Bishop, 1995). The only parameter required for calibration is the number of units or neurons. One challenge with neural networks is that the fitting algorithm often converges to locally optimal solutions. To avoid this behaviour and improve the predictive capability of the model, the neural networks are incorporated in a resampling strategy known as bagging (Hastie et al., 2009; Shu and Burn, 2004). Accordingly, the neural networks are fitted on 20 bootstrap samples and the final predictions are taken as the average of the individual outputs. Moreover, Shu and Ouarda (2007) successfully used canonical correlation analysis as a pre-treatment of the explanatory variables to improve the learning rate of a neural network. A similar strategy is adopted here, but with principal component analysis to create standardized and uncorrelated input variables from the catchment descriptors. Finally, the present study investigates the relevance of combining neural network with the kriging (or TPS) of the residuals in a two-step procedure as with local regression.

2.4 Distributions at target sites

After the prediction of the first three L-moments at a target site, the present methodology follows by estimating the parameters of four candidate distributions. They are the Generalized Extreme Value (GEV), Pearson type III (PE3), Generalized Normal (GNO) and Generalized Logistic (GLO) distributions (Hosking and Wallis, 1997). Similarly to Laio et al. (2011), instead of selecting a single best distribution among the candidates, the final flood quantiles are computed as the average flood quantile of the candidate distributions. However, the present methodology differs as weights are used to represent the relevance of each candidate distribution. These weights are evaluated according to the frequency of each candidate distribution being selected for at-site frequency analysis in the proximity of the target site. The rationale is that if the same distribution is selected for all nearby sites when at-site analyses are performed, it is reasonable to choose the same distribution for the target site. The final weights used for aggregating the flood quantiles are proportional to the sums of the weights w_i in equation (5) for each candidate distribution among the sites in a neighborhood.

In context of at-site flood frequency analysis, Zhang et al. (2018) performed a comparison study of several statistical distributions in Canada using several criteria and came to the conclusion that the GEV is generally the best choice for Canadian rivers. Therefore, based on these practical considerations, this methodology intends to select alternative distributions only when this choice is supported by the data. In a first step, the adopted procedure selects the best at-site distribution as the one having the lowest Akaike Information Criterion (AIC) (Di Baldassarre et al., 2009). Afterward, the AIC of the best distribution is compared to the one of the GEV. If they are judged similar, then the GEV is used; otherwise, the best distribution is kept. The present study accepts that two distributions are similar when the difference between their AIC is lower than two. Note that such threshold is not largely biased towards the GEV. In particular, that difference corresponds to the impact of removing one parameter from a nested model; Burnham and Anderson (2002) also suggest that differences greater than 4 should be considered meaningful.

An alternative to the presented methodology for evaluating flood quantiles at a target site using L-moments is the index-flood model (IF) using the L-Moment algorithm (Hosking and Wallis, 1997). The index-flood method computes the flood quantile of a site s as $q(s) = z(s) \times q$, where $z(s)$ is the index-flood factor taken here as the mean of the site annual maxima s_i (L1) and q represents the flood quantile of a dimensionless distribution, or regional growth curve. With the L-Moment algorithm, the regional growth curves are estimated using neighborhoods of size 20 and by averaging the LCV and LSK using weights proportional to the record lengths. Hosking and Wallis (1997) reported that neighborhoods of size larger than 20-25 sites do not usually provide more information. Traditionally, the L-Moment algorithm selects a single regional distribution based on a Z-statistic that identifies the distribution having theoretically the most coherent L-kurtosis with the sample. Note that in the present situation, the homogeneity of the regions used for computing the regional growth curve is not verified as it is assumed instead that the L-moments are smoothly evolving in descriptor space.

2.5 Model evaluation and uncertainty

The assessment of a method for flood frequency analysis is mostly focused on evaluating the capacity of such method in estimating flood quantiles of specific return periods. Leave-one-out cross-validation is often used in that context to evaluate in turn the prediction error made when one site is assumed ungauged. For local regression, such strategy can be efficiently implemented by excluding the target site in each neighborhood. However, for other nonparametric regression methods such as GAM and neural network, refitting a model many times is inefficient. Consequently, k-fold cross-validation is preferred here to limit the computational burden (Hastie et al., 2009). This cross-validation procedure does essentially the same as a leave-one-out cross-validation scheme. In turns, one of k group of sites with similar sizes is considered as ungauged and these sites are predicted by the remaining sites. For the present study $k = 10$ and n=770 sites. Therefore, this strategy reduces by 77 times the number models to be fitted.

Based on the residuals of the cross-validation scheme, two criteria are used to evaluate the magnitude of the prediction error. Let $\tilde{z}(s_i)$ be a response variable predicted at the i-th of n sites when considered as ungauged and $z(s_i)$ the observed value. Two well-known criteria are the Nash-Sutcliffe

(6) $$NSH = 100 \times \left[1 - \frac{\sum_{i=1}^{n} \left\{ \tilde{z}(s_i) - z(s_i) \right\}^2}{\sum_{i=1}^{n} \left\{ \tilde{z}(s_i) - n^{-1} \sum_{j=1}^{n} z(s_j) \right\}^2} \right]$$

and the mean absolute deviation

(7) $$MAD = n^{-1} \sum_{i=1}^{n} \left| \tilde{z}(s_i) - z(s_i) \right| .$$

The NSH criterion provides a measure of the quadratic error relatively to the performance of the model represented by a global mean. Values close to 100 % indicate good predictive performance. Alternatively, the MAD criterion is based on absolute errors, which leads to a criterion that is less affected by large discrepancies and so more robust to potential outliers. Note that the MAD criterion provides a measurement of the prediction error in the same units as the predicted variable, while NSH is a dimensionless measure.

In general, for two competing methods it is difficult to interpret the meaning of small differences in performance measures as they can represent either a real gain of prediction skill or be simply the consequence of random sampling. In the context of hydrological model forecasting, it was shown that the assessment of significantly better forecasting skills can be determined by hypothesis testing (DelSole and Tippett, 2014; Hamill, 1999). The same strategy is adopted here to evaluate prediction skills using the Wilcox signed-rank (WSR) test. For the first two predicted L-moments

(L1 and LCV), the loss differential of two competing methods with predictions $\tilde{z}(s_i)$ and $\hat{z}(s_i)$ is defined as

$$(8) \qquad d_i = \log\left\{\tilde{z}(s_i)/z(s_i)\right\}^2 - \log\left\{\hat{z}(s_i)/z(s_i)\right\}^2$$

and for the L-coefficient of skewness

$$(9) \qquad d_i = \left\{\tilde{z}(s_i) - z(s_i)\right\}^2 - \left\{\hat{z}(s_i) - z(s_i)\right\}^2 \ .$$

The Wilcox signed-rank test verifies the hypothesis of a zero-mean rank of the loss differential against a significant (bilateral) difference. Therefore, a small p-value of this test provides evidence that one competing method has superior predictive skills. In particular, as a nonparametric test, the outcome is invariant to the distribution of the loss differential.

In the proposed methodology, regression models are fitted directly on the sample L-moments of each site. Consequently, the resulting criteria NSH and MAD do not account for a sampling source of error that could lead to underestimate the true uncertainty of the model (Tasker and Stedinger, 1989). Therefore, the fitting of each model is performed on 1000 bootstrap samples to achieve better evaluation of the model uncertainty. In particular, the cross-validation is performed on each bootstrap iteration to obtain reliable measures of the predictive uncertainty. In Canada, to create spatially dependent bootstrap samples, a multivariate Normal copula was found to well represent the strength of the intersite correlation among streamflow data (Durocher et al., 2018). To simulate from the Normal copula, the correlation coefficients of the matrix are deduced from a known power exponential model. The process is repeated to obtain spatially dependent samples inside the uniform interval [0, 1] that are rescaled using the inverse cumulative distribution function estimated from at-site frequency analysis of each site. To compare the level of uncertainty across sites, the coefficient of variation of the estimated flood quantiles is computed and an average coefficient of variation (ACV) is used to describe the overall uncertainty for a specific group of sites.

3. Results

3.1 Data and hydrological environments

The investigated nonparametric regression methods are applied to a case study of 770 gauged sites in Canada. The hydrometric data were extracted from the Water Survey of Canada (WSC, 2018) and the catchment descriptors were provided by Environment and Climate Change Canada. Every site is verified to have at least 20 years of record with no significant trend according to the Mann-Kendall test. The descriptors considered for the prediction of the L-moments include: drainage area, mean annual precipitation, percentage of watershed covered by waterbodies, stream density, catchments mean slope, site elevation, longitude and latitude. A summary of these descriptors is presented in Table 1. These descriptors were the explanatory variables used by Durocher et al. (2018) for the same database and were selected by cross-validation using local regression.

Canada is a vast country with many distinct hydrological environments. To improve the interpretability of the results, the sites are further divided into hydrological regions based on the drainage area and mean annual precipitation. These two catchment attributes were shown to be related to the sample moments of flood distributions (Basu and Srinivas, 2015; Blöschl and

Sivapalan, 1997; Meigh et al., 1997). The hydrological regions were formed by hierarchal classification using Ward's method (Ward, 1963) and their total number is determined based on their overall parsimony and interpretability. Figure 1 illustrates the geographical locations of the 770 gauged sites regrouped in 7 hydrological regions. Region 1 represents catchments located along the Pacific coast, which receive the largest amount of precipitation in Canada. Regions 2 to 4 are wetter catchments mostly found in eastern Canada and in British Columbia, while regions 5 to 7 are drier catchments mainly located in the Prairies and northern Canada. One can see that British Columbia is hydrologically diverse as at least one member of every region is found in its territory. Notice also that large watersheds are generally found at the more northerly locations.

3.2 Regionalization of the L-moments

The quality of the models is assessed by the cross-validation criteria and is reported in Table 2. When no spatial component is included, the GAM method is found to have the least predictive power for the mean (L1) with low NSH and high MAD. This suggests that the assumption of independence between the smooth functions limit the model flexibility. In that regard, ROI offers the best predicting power, with results slightly outperforming NN. Similar observations are made for LCV and LSK. By default, ROI uses the Mahalanobis distance between catchment descriptors to delineate the neighborhood of a target site. Similarly, ROI/GEO is the same method, but using instead the geographical distance. One can see that the change of metric improves the predictive capability of the local regression approach, although based on a WSR test the difference of predictive skills is not significant for L1 and LCV at a significance level of 0.05.

Table 2 reveals that the addition of a spatial component (KRG or TPS) increases the accuracy of all nonparametric regression methods. In particular, examining all pairwise comparison with the WSR test, at a significance level of 0.05, leads to the conclusion of superior predictive skill. Based on the MAD criterion, the GAM/TPS method is best for predicting the LCV and GAM/KRG is found to better describe LSK. For L1, the choice of a best approach depends on which cross-validation criteria is examined. This may point toward either GAM/TPS or ROI/KRG. Overall, the difference between methods having both a trend and a spatial component appears relatively small, except for GAM/KRG that does not perform as well as the others for L1. For ROI and NN, kriging appears to do slightly better than TPS. The NSH criterion indicates that L1 and LCV are globally well predicted, with values roughly around 95 and 81 respectively for the best methods. In particular, notice that the NSH value of the GAM method for LCV passes from 70.3 to 81.8 when adding TPS. Similarly, from GAM to GAM/KRG, the NSH criterion for LCV improves by 11.7. These important gains show the importance of geographical proximity in the determination of the scale and shape of the target distribution.

3.3 Evaluation of the aggregated weights

Section 2.4 described a strategy to use weights for estimating flood quantiles from multiple candidate distributions. For that purpose, the Mahalanobis distance between descriptors was proposed to delineate neighborhoods of size 20. This size of neighborhood was found optimal in terms of cross-validation criteria. Table 3 presents the contingency table of the distributions selected by at-site frequency analysis. As expected, the GEV distribution is preferred in a large proportion (78%). However, Region 5 (smaller and drier) shows a substantial percentage of sites (47%) where the GNO and PE3 distributions are selected. Due to the prevalence of GEV, weights greater than 0.5 are found for that distribution in 84% of the neighborhoods. At the same time, in Region 5 the cumulative weight of the GNO and PE3 is greater than 0.5 in 42% of the cases. This shows that the proposed methodology preserves the specificity of the distribution selected inside hydrological regions. Consequently, in a majority of the sites outside of Region 5, the estimated flood quantiles are essentially those of a GEV distribution, while inside Region 5 a compromise is made between the relevant distributions.

3.4 Estimation of the flood quantiles

In the following, the focus is set on the GAM/TPS and ROI/KRG methods that were found to be the best methods overall for predicting the L-moments at target sites. The cross-validation criteria of the estimated flood quantiles of return period of 10 and 100 years are presented in Table 4. Overall, it is seen that neither of the two methods is systematically superior to the other. Indeed, according to the WSR test, only hydrological region 7 is significantly better modelled by ROI/KRG for a significance level of 0.05. Notice that the NSH of the hydrological regions is always lower than the NSH of all sites, because the predicting errors are compared to the regional means and not the global mean. The hydrological region with the lowest NSH is Region 7 with 70.3 for Q100. However, this region has a MAD of 0.293 that is comparable with Region 2 (0.310), even though the latter has a NSH of 84.0. According to MAD, Region 5 (smaller and drier) is the hydrological region estimated with the lowest accuracy.

For evaluating the quality of the prediction considering the sampling error, the coefficient of variation of the estimated flood quantiles for each site is evaluated and averaged by hydrological regions. For Q100, apart from Regions 5-6, the average coefficient of variation (ACV) of

GAM/TPS is relatively constant between 10% and 12% . The ACV criterion also shows that the flood quantiles in Region 5 are the less accurately predicted with an ACV of almost double that of the others (19.4). The ACV of Q100 for the ROI/KRG model is less constant across hydrological regions (between 8.6 and 13.7), but they are overall slightly lower than those of GAMP/TPS for all sites. For Q10, according to ACV the ROI/KRG method led to more accurate estimates than GAM/TPS, except for Region 2.

3.5 Comparison of different approaches for computing flood quantiles

Table 5 reports the comparison of different approaches for computing the flood quantiles. The results indicate that the index-flood (IF) method is underperforming when estimating L1 by both GAM/TPS and ROI/KRG. According to the WSR test, the GAM/TPS method using the LRT approach has significantly better prediction skill than the index-flood method to predict Q100 with a p-value less than 0.001. Similar results are observed for ROI/KRG. One reason for this discrepancy is that the index-flood method does not include a spatial component for LCV and LSK, which was shown in Table 2 to provide important information. A second factor could be the choice of the distribution using the Z-statistic. In particular, the adopted procedure for selecting the at-site distribution has chosen the GEV distribution in 78% of the cases, while the GEV distribution is selected in 49% of the neighborhoods using the Z-statistic. To measure the impact of this selection procedure, a variant of the index-flood model (IFW) is considered where the estimated flood quantiles are taken as a weighted average of candidate distributions, like LRT. These results, presented in Table 5, reveal that for Q10 the two variants of the index-flood method are essentially the same in terms of cross-validation criteria, but a slight discrepancy is observed for Q100. This is coherent with the fact that the selection of the regional growth curve has more impact in the extrapolation to the longer return periods and that the averaging approach is more coherent with the at-site selection procedure. However, LRT has also significantly better prediction skill than IFW, according to the WSR test. This shows that the choice of the regional growth curve is not the most important factor to explain the gap in prediction skill between IF and LRT.

Previous studies suggested that parameter regression techniques (similar to LRT) have prediction power comparable to quantile regression techniques, QRT (Ahn and Palmer, 2016; Haddad and Rahman, 2012). Table 5 revisits these conclusions and arrives at similar results. In particular, notice that the highest difference in NSH between LRT and QRT is only 0.03, which shows a similar predictive power of these two methods. These findings are also corroborated by the WSR tests with p-values greater than 0.14.

The comparison between the various methods have shown overall very similar results in terms of cross-validation when examined at the national level. One reason is that this case study includes a large number of sites and so nonparametric regression methods having comparable features converge to similar solutions. As mentioned, flood frequency analyses in Canada are often performed at the provincial level for political reasons. Therefore, it is useful to investigate the impact of imposing administrative boundaries and limiting the number of sites. The ROI/KRG and the GAM/TPS methods are thus carried out individually for each province. To ensure a minimum of sites by province, the Atlantic provinces (ATL), the three North territories (NT) and Manitoba-Saskatchewan (MS) are pooled together. Also, even though Labrador is part of the Atlantic province of Newfoundland and Labrador, these sites are regrouped with the adjacent province of

Quebec (QC). Note that for ROI/KRG, the hydrological regions are not used in this context and a constant neighborhood size is selected for each province. Table 6 shows the cross-validation criteria by “province”, using national and provincial strategies. Overall, the national analysis appears to outperform the provincial analysis when looking at the cross-validation criteria, but for the vast majority of the provinces and for the whole of Canada, the WSR test does not identify a significant difference in prediction skill at a significance level of 0.05. For GAM/TPS, the North territories (NT) and Quebec (QC), having respectively 51 and 58 sites, the cross-validation criteria are clearly greater using the national analysis. At the opposite side, the predictive powers in British Columbia (BC) and Ontario (ON), having respectively 192 and 128 sites, are slightly better when using the provincial analysis. These results suggest that the number of sites in a province may influence the quality of the GAM/TPS outputs. For ROI/KRG, the national analysis also provides slightly more accurate flood quantiles according to NSH and MAD, but the discrepancy between the national and provincial analysis for the northern territories and Quebec are less important. Moreover, the three provinces with most sites (AB, BC and ON) are slightly better predicted by the national analysis. This suggests that ROI/KRG is more stable than GAM/TPS when restricted by administrative boundaries.

4. Conclusions

In this study, nonparametric regression models with spatial components were investigated for predicting flood quantiles at ungauged sites. The proposed methodology used three separate regression models to predict the first three L-moments of a target distribution. The flood quantiles were then computed as a weighted average of the flood quantile derived by multiple candidate distributions and a weighting scheme was developed to represent the relative importance of each distribution in the proximity of the target site. The overall performance of the nonparametric regression methods was evaluated using cross-validation on a case study of 770 sites in Canada.

The results were summarized at the national level, but also inside 7 relevant hydrological and 7 administrative regions. This allowed verification of how the considered methods behave in different situations. It was shown that all nonparametric regression methods worked best when a spatial component was added. In particular, a GAM model combined with thin plate spline (GAM/TPS) and a local regression model using kriging (ROI/KRG) were shown to provide globally the most accurate flood quantile estimates. Overall, the cross-validation criteria used to compare these two methods did not identify a better method and the WSR tests showed no significant difference in prediction skills among them. Each method was found to perform slightly better than the other depending on the context and criteria examined. This resemblance in terms of predictive power could be attributed to the large number of sites present in this case study, which allowed them to converge toward similar solutions. Dividing the analysis by administrative regions revealed a small reduction in the predictive power, but this reduction was not found significant according to the WSR test. Nonetheless, these restrictions brought about by use of administrative regions appear to have affected the performance of GAM/TPS more than ROI/KRG.

Further comparisons showed that the proposed L-moment regression techniques outperform the index-flood method by incorporating spatial information in the L-moments of second and third order. On the other hand, the L-moment regression techniques showed predictive power similar to that of quantile regression techniques. Therefore, without loss of efficiency, the L-moment regression techniques have an interesting capacity of mixing sites with different candidate distributions. Like the quantile regression techniques and parametric regression techniques, it provides coherent estimates of flood quantiles when evaluated for multiples return periods.

Globally, GAM/TPS and ROI/KRG could be recommended for performing flood frequency analysis at ungauged sites. The GAM method has the advantage of having more straightforward descriptions of the relationship between the L-moments and the catchment descriptors. On the other hand, because frequency analysis using local regression can be limited to close neighborhoods and administrative boundaries do not greatly affect its predictive power, ROI/KRG may be attractive in practice when only one or few sites are of interest. In particular, this would greatly facilitate the resampling strategy necessary for evaluating model uncertainty affected by sampling error.

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References

Acreman, M., Sinclair, C., 1986. Classification of drainage basins according to their physical characteristics; an application for flood frequency analysis in Scotland. Journal of Hydrology 84, 365–380.

Ahn, K.-H., Palmer, R., 2016. Regional flood frequency analysis using spatial proximity and basin characteristics: Quantile regression vs. parameter regression technique. Journal of Hydrology 540, 515–526. https://doi.org/10.1016/j.jhydrol.2016.06.047

Altman, N.S., 1992. An Introduction to Kernel and Nearest-Neighbor Nonparametric Regression. The American Statistician 46, 175–185. https://doi.org/10.1080/00031305.1992.10475879

Archfield, S.A., Pugliese, A., Castellarin, A., Skøien, J.O., Kiang, J.E., 2013. Topological and canonical kriging for design flood prediction in ungauged catchments: an improvement over a traditional regional regression approach? Hydrol. Earth Syst. Sci. 17, 1575–1588. https://doi.org/10.5194/hess-17-1575-2013

Aziz, K., Rahman, A., Fang, G., Shrestha, S., 2014. Application of artificial neural networks in regional flood frequency analysis: A case study for Australia. Stochastic Environmental Research and Risk Assessment 28, 541–554.

Basu, B., Srinivas, V.V., 2015. A recursive multi-scaling approach to regional flood frequency analysis. Journal of Hydrology 529, 373–383. https://doi.org/10.1016/j.jhydrol.2015.07.037

Bishop, C.M., 1995. Neural networks for pattern recognition. Oxford university press.

Blöschl, G., Sivapalan, M., 1997. Process controls on regional flood frequency: Coefficient of variation and basin scale. Water Resour. Res. 33, 2967–2980. https://doi.org/10.1029/97WR00568

Burn, D.H., 1997. Catchment similarity for regional flood frequency analysis using seasonality measures. Journal of Hydrology 202, 212–230. https://doi.org/10.1016/S0022-1694(97)00068-1

Burn, D.H., 1990. An appraisal of the “region of influence” approach to flood frequency analysis. Hydrological Sciences Journal 35, 149–165. https://doi.org/10.1080/02626669009492415

Burn, D.H., Whitfield, P.H., Sharif, M., 2016. Identification of changes in floods and flood regimes in Canada using a peaks over threshold approach. Hydrol. Process. 30, 3303–3314. https://doi.org/10.1002/hyp.10861

Burn, D.H., Zrinji, Z., Kowalchuk, M., 1997. Regionalization of Catchments for Regional Flood Frequency Analysis. Journal of Hydrologic Engineering 2, 76–82. https://doi.org/10.1061/(ASCE)1084-0699(1997)2:2(76)

Burnham, K.P., Anderson, D.R., 2002. Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach, 2nd ed. Springer-Verlag, New York.

Buttle, J.M., Allen, D.M., Caissie, D., Davison, B., Hayashi, M., Peters, D.L., Pomeroy, J.W., Simonovic, S., St-Hilaire, A., Whitfield, P.H., 2016. Flood processes in Canada: Regional and special aspects. Canadian Water Resources Journal / Revue canadienne des ressources hydriques 41, 7–30. https://doi.org/10.1080/07011784.2015.1131629

Castellarin, A., Burn, D.H., Brath, A., 2008. Homogeneity testing: How homogeneous do heterogeneous cross-correlated regions seem? Journal of Hydrology 360, 67–76. https://doi.org/10.1016/j.jhydrol.2008.07.014

Castiglioni, S., Castellarin, A., Montanari, A., 2009. Prediction of low-flow indices in ungauged basins through physiographical space-based interpolation. Journal of Hydrology 378, 272–280. https://doi.org/10.1016/j.jhydrol.2009.09.032

Chebana, F., Charron, C., Ouarda, T.B.M.J., Martel, B., 2014. Regional frequency analysis at ungauged sites with the generalized additive model. Journal of hydrometeorology 15, 2418–2428. https://doi.org/10.1175/JHM-D-14-0060.1

Chokmani, K., Ouarda, T.B.M.J., 2004. Physiographical space-based kriging for regional flood frequency estimation at ungauged sites. Water Resources Research 40. https://doi.org/10.1029/2003WR002983

Cleveland, W.S., Devlin, S.J., 1988. Locally Weighted Regression: An Approach to Regression Analysis by Local Fitting. Journal of the American Statistical Association 83, 596–610. https://doi.org/10.2307/2289282

Craven, P., Wahba, G., 1978. Smoothing noisy data with spline functions. Numerische Mathematik 31, 377–403.

Dawson, C.W., Abrahart, R.J., Shamseldin, A.Y., Wilby, R.L., 2006. Flood estimation at ungauged sites using artificial neural networks. Journal of Hydrology 319, 391–409. https://doi.org/10.1016/j.jhydrol.2005.07.032

DelSole, T., Tippett, M.K., 2014. Comparing Forecast Skill. Mon. Wea. Rev. 142, 4658–4678. https://doi.org/10.1175/MWR-D-14-00045.1

Di Baldassarre, G., Laio, F., Montanari, A., 2009. Design flood estimation using model selection criteria. Physics and Chemistry of the Earth, Parts A/B/C, Recent developments of statistical tools for hydrological application 34, 606–611. https://doi.org/10.1016/j.pce.2008.10.066

Durocher, M., Burn, D.H., Mostofi Zadeh, S., 2018. A nationwide regional flood frequency analysis at ungauged sites using ROI/GLS with copulas and super regions. EarthArXiv. https://doi.org/10.31223/osf.io/qrnfe

El-Jabi, N., Caissie, D., Turkkan, N., 2016. Flood analysis and flood projections under climate change in New Brunswick. Canadian Water Resources Journal 41, 319–330. https://doi.org/10.1080/07011784.2015.1071205

Eng, P. C. Milly, Gary D. Tasker, 2007. Flood Regionalization: A Hybrid Geographic and Predictor-Variable Region-of-Influence Regression Method. Journal of Hydrologic Engineering 12, 585–591. https://doi.org/10.1061/(ASCE)1084-0699(2007)12:6(585)

Gizaw, M.S., Gan, T.Y., 2016. Regional Flood Frequency Analysis using Support Vector Regression under historical and future climate. Journal of Hydrology 538, 387–398. https://doi.org/10.1016/j.jhydrol.2016.04.041

Green, P.J., Silverman, B.W., 1993. Nonparametric regression and generalized linear models: a roughness penalty approach. Chapman & Hall/CRC.

GREHYS, 1996. Presentation and review of some methods for regional flood frequency analysis. Journal of Hydrology 186, 63–84. https://doi.org/10.1016/S0022-1694(96)03042-9

Haddad, K., Rahman, A., 2012. Regional flood frequency analysis in eastern Australia: Bayesian GLS regression-based methods within fixed region and ROI framework – Quantile Regression vs. Parameter Regression Technique. Journal of Hydrology 430–431, 142–161. https://doi.org/10.1016/j.jhydrol.2012.02.012

Hamill, T.M., 1999. Hypothesis Tests for Evaluating Numerical Precipitation Forecasts. Wea. Forecasting 14, 155–167. https://doi.org/10.1175/1520-0434(1999)014<0155:HTFENP>2.0.CO;2

Hastie, T., Tibshirani, R., 1986. Generalized additive models. Statistical science 297–310.

Hastie, T., Tibshirani, R., Friedman, J.H., 2009. The elements of statistical learning: data mining, inference, and prediction, Springer series in statistics. Springer.

Hosking, J.R.M., Wallis, J.R., 1997. Regional frequency analysis: an approach based on L-moments. Cambridge Univ Pr.

Kjeldsen, T.R., Jones, D.A., 2009. An exploratory analysis of error components in hydrological regression modeling. Water Resour. Res. 45, W02407. https://doi.org/10.1029/2007WR006283

Laaha, G., Blöschl, G., 2006. A comparison of low flow regionalisation methods—catchment grouping. Journal of Hydrology 323, 193–214. https://doi.org/10.1016/j.jhydrol.2005.09.001

Laio, F., Ganora, D., Claps, P., Galeati, G., 2011. Spatially smooth regional estimation of the flood frequency curve (with uncertainty). Journal of Hydrology 408, 67–77. http://dx.doi.org/10.1016/j.jhydrol.2011.07.022

Latraverse, M., Rasmussen, P.F., Bernard, B., 2002. Regional estimation of flood quantiles: Parametric versus nonparametric regression models. Water Resources Research 38, 1–1. https://doi.org/10.1029/2001WR000677

López, J., Francés, F., 2013. Non-stationary flood frequency analysis in continental Spanish rivers, using climate and reservoir indices as external covariates. Hydrol. Earth Syst. Sci. 17, 3189–3203. https://doi.org/10.5194/hess-17-3189-2013

Meigh, J.R., Farquharson, F.A.K., Sutcliffe, J.V., 1997. A worldwide comparison of regional flood estimation methods and climate. Hydrological Sciences Journal 42, 225–244. https://doi.org/10.1080/02626669709492022

Oudin, L., Andréassian, V., Perrin, C., Michel, C., Le Moine, N., 2008. Spatial proximity, physical similarity, regression and ungaged catchments: A comparison of regionalization approaches based on 913 French catchments. Water Resources Research 44. https://doi.org/10.1029/2007WR006240

Oudin, L., Kay, A., Andréassian, V., Perrin, C., 2010. Are seemingly physically similar catchments truly hydrologically similar? Water Resources Research 46, n/a–n/a. https://doi.org/10.1029/2009WR008887

Pandey, G.R., Nguyen, V.-T.-V., 1999. A comparative study of regression based methods in regional flood frequency analysis. Journal of Hydrology 225, 92–101. https://doi.org/10.1016/S0022-1694(99)00135-3

Rahman, A., Charron, C., Ouarda, T.B.M.J., Chebana, F., 2018. Development of regional flood frequency analysis techniques using generalized additive models for Australia. Stoch Environ Res Risk Assess 32, 123–139. https://doi.org/10.1007/s00477-017-1384-1

Requena, A.I., Chebana, F., Ouarda, T.B.M.J., 2017. Heterogeneity measures in hydrological frequency analysis: review and new developments. Hydrol. Earth Syst. Sci. 21, 1651–1668. https://doi.org/10.5194/hess-21-1651-2017

Rigby, R.A., Stasinopoulos, D.M., 2005. Generalized additive models for location, scale and shape. Journal of the Royal Statistical Society: Series C (Applied Statistics) 54, 507–554. https://doi.org/10.1111/j.1467-9876.2005.00510.x

Salinas, J.L., Castellarin, A., Viglione, A., Kohnová, S., Kjeldsen, T.R., 2014. Regional parent flood frequency distributions in Europe – Part 1: Is the GEV model suitable as a pan-European parent? Hydrol. Earth Syst. Sci. 18, 4381–4389. https://doi.org/10.5194/hess-18-4381-2014

Sandrock, G., Viraraghavan, T., Fuller, G.A., 1992. Estimation of Peak Flows for Natural Ungauged Watersheds in Southern Saskatchewan. Canadian Water Resources Journal / Revue canadienne des ressources hydriques 17, 21–31. https://doi.org/10.4296/cwrj1701021

Schabenberger, O., Gotway, C.A., 2004. Statistical methods for spatial data analysis. CRC Press.

Schnier, S., Cai, X., 2014. Prediction of regional streamflow frequency using model tree ensembles. Journal of Hydrology 517, 298–309. https://doi.org/10.1016/j.jhydrol.2014.05.029

Shu, C., Burn, D.H., 2004. Artificial neural network ensembles and their application in pooled flood frequency analysis. Water Resour. Res. 40, W09301. https://doi.org/10.1029/2003WR002816

Shu, C., Ouarda, T.B.M.J., 2007. Flood frequency analysis at ungauged sites using artificial neural networks in canonical correlation analysis physiographic space. Water Resources Research 43. https://doi.org/10.1029/2006WR005142

Skøien, J.O., Blöschl, G., 2007. Spatiotemporal topological kriging of runoff time series. Water Resources Research 43, n/a–n/a. https://doi.org/10.1029/2006WR005760

Smith, A., Sampson, C., Bates, P., 2015. Regional flood frequency analysis at the global scale. Water Resources Research 51, 539–553. https://doi.org/10.1002/2014WR015814

Steinschneider, S., Yang, Y.-C.E., Brown, C., 2015. Combining regression and spatial proximity for catchment model regionalization: a comparative study. Hydrological Sciences Journal 60, 1026–1043. https://doi.org/10.1080/02626667.2014.899701

Tasker, G., Stedinger, J., 1989. An operational GLS model for hydrologic regression. Journal of Hydrology 111, 361–375. https://doi.org/10.1016/0022-1694(89)90268-0

Tasker, G.D., Hodge, S.A., Barks, C.S., 1996. Region of Influence Regression for Estimating the 50-Year Flood at Ungaged Sites1. JAWRA Journal of the American Water Resources Association 32, 163–170. https://doi.org/10.1111/j.1752-1688.1996.tb03444.x

Thomas, D., Benson, M., 1975. Generalization of streamflow characteristics from drainage-basin characteristics. US Geological Survey Water-Supply Paper.

Viglione, A., Laio, F., Claps, P., 2007. A comparison of homogeneity tests for regional frequency analysis. Water Resour. Res. 43, W03428. https://doi.org/10.1029/2006WR005095

Villarini, G., Serinaldi, F., Smith, J.A., Krajewski, W.F., 2009. On the stationarity of annual flood peaks in the continental United States during the 20th century. Water Resour. Res. 45, W08417. https://doi.org/10.1029/2008WR007645

Ward, J.H., 1963. Hierarchical Grouping to Optimize an Objective Function. Journal of the American Statistical Association 58, 236–244. https://doi.org/10.1080/01621459.1963.10500845

Wittenberg, H., 1999. Baseflow recession and recharge as nonlinear storage processes. Hydrological Processes 13, 715–726. https://doi.org/10.1002/(SICI)1099-1085(19990415)13:5<715::AID-HYP775>3.0.CO;2-N

Wood, S.N., 2003. Thin plate regression splines. Journal of the Royal Statistical Society: Series B (Statistical Methodology) 65, 95–114. https://doi.org/10.1111/1467-9868.00374

WSC, 2018. Water Survey of Canada [WWW Document]. URL http://www.wsc.ec.gc.ca/applications/H2O/index-eng.cfm

Zhang, Z., 2018. Identification of a broadly accepted statistical distribution for at-site flood frequency analysis in Canada.

Zrinji, Z., Burn, D.H., 1994. Flood frequency analysis for ungauged sites using a region of influence approach. Journal of Hydrology 153, 1–21. https://doi.org/10.1016/0022-1694(94)90184-8

Zhang, Z., Stadnyk, T.A., Burn, D.H., 2018. Identification of a broadly accepted statistical distribution for at-site flood frequency analysis in Canada. Presented at the 71rst CWRA National Conference, May 28 – June 1, Victoria, BC, Canada.

Tables

Table 1: Summary statistics of streamflow data and catchment descriptors.

Variables	Abrv.	Min	Q1	Med	Avg	Q3	Max
Record length (yr)		20	25	36	39	48	111
mean annual maximum discharge (m^3/s)		0.2	13.4	45.6	206.9	174.1	5068.3
Drainage area (km^2)	area	1	146	460	2829	1992	48867
Basin mean slope (%)	slope	<0.1	1.2	3.6	10.5	17.1	59.0
Waterbody area (%)	wb	<0.1	0.4	1.3	3.7	4.5	38.3
Stream density (km^{-1})	dens	<0.1	0.6	1.0	1.2	1.6	3.4
Elevation at site (m)	elev	1	181	382	474	731	1699
Mean annual precipitation (mm)	map	213	498	761	836	1052	3216

Table 2: Cross-validation criteria for predicted L-moments.

		NSH			MAD		
Trend	**Spatial**	**L1**	**LCV**	**LSK**	**L1**	**LCV**	**LSK**
GAM	(*)	92.5	70.3	28.4	0.370	0.203	0.090
	KRG	93.5	81.0	**40.0**	0.340	0.158	**0.081**
	TPS	95.1	**81.8**	39.3	**0.296**	**0.154**	0.082
	KRG	93.5	81.0	**40.0**	0.340	0.158	**0.081**
ROI	(*)	94.5	78.1	30.6	0.327	0.172	0.090
	GEO	94.6	79.5	33.6	0.317	0.163	0.086
	KRG	**95.2**	81.6	36.8	0.300	0.157	0.085
	TPS	95.0	81.2	34.5	0.305	0.158	0.087
NN	(*)	93.9	75.2	28.0	0.337	0.183	0.091
	KRG	95.1	81.6	36.3	0.298	0.156	0.085
	TPS	94.9	81.3	34.2	0.305	0.158	0.087

(*) indicates no spatial component is included…
Bold indicates the best criteria in columns

Table 3: Contingency table of the selected at-site distributions for geographical regions indicated in Figure 1.

Region	GEV	GLO	GNO	PE3	Total
1	21	0	0	1	22
2	34	0	0	3	37
3	165	5	1	7	178
4	81	5	1	1	88
5	114	1	27	73	215
6	104	3	5	20	132
7	85	8	1	4	98
Total	604	22	35	109	770

Table 4: Cross-validation criteria for estimated flood quantiles by hydrological regions.

Model	Criteria		1	2	3	4	5	6	7	All
GAM/TPS	**NSH**	**Q10**	**84.8**	**86.4**	**92.8**	89.6	81.1	**90.8**	74.7	**94.7**
		Q100	**77.9**	**84.0**	89.2	84.1	**77.0**	**88.3**	69.4	**92.7**
	MAD	**Q10**	0.236	**0.267**	0.237	0.344	**0.375**	**0.325**	0.252	**0.298**
		Q100	0.286	**0.297**	0.276	0.393	0.442	**0.344**	0.293	0.343
	ACV	**Q10**	0.063	**0.069**	0.061	0.057	0.141	0.099	0.067	0.090
		Q100	**0.107**	**0.121**	**0.101**	0.097	**0.197**	0.148	0.106	0.137
ROI/KRG	**NSH**	**Q10**	83.5	85.0	92.2	**92.4**	**81.2**	89.6	**79.0**	**94.7**
		Q100	77.2	80.3	**89.6**	**87.9**	76.2	88.0	**73.2**	**92.7**
	MAD	**Q10**	**0.232**	0.270	**0.235**	**0.262**	0.386	0.358	**0.236**	0.300
		Q100	**0.279**	0.334	**0.250**	**0.334**	**0.439**	0.368	**0.281**	**0.340**
	ACV	**Q10**	**0.062**	0.084	**0.060**	**0.049**	**0.138**	**0.087**	**0.060**	**0.086**
		Q100	**0.107**	0.137	0.102	**0.086**	0.202	**0.123**	**0.096**	**0.133**

*Bold indicates best of each criteria in column between models.

Table 5: Cross-validation criteria for estimated flood quantiles using different methods for evaluating the flood quantiles.

		NSH		MAD	
Model	**Method**	**Q10**	**Q100**	**Q10**	**Q100**
GAM/TPS	**LRT**	**94.68**	**92.72**	**0.298**	**0.343**
	IF	94.44	91.57	0.304	0.368
	IFW	94.43	91.73	0.304	0.365
	QRT	94.66	92.70	0.299	0.345
ROI/KRG	**LRT**	**94.67**	92.73	**0.300**	**0.340**
	IF	94.56	91.92	0.304	0.367
	IFW	94.53	92.03	0.305	0.363
	QRT	94.64	**92.74**	0.301	0.341

*Bold indicates best method in columns for each model.

Table 6: Cross-validation criteria for estimated flood quantiles by provinces with (Provincial) and without (National) the use of administrative boundaries.

Model	Analysis			AB	ATL	BC	MS	NT	ON	QC	All
			nb. sites	**177**	**83**	**192**	**81**	**51**	**128**	**58**	**770**
GAM/TPS	**National**	**NSH**	**Q10**	**90.2**	**96.0**	95.5	86.0	**94.2**	93.3	**97.5**	**94.7**
			Q100	**86.9**	**94.3**	94.4	**80.1**	**91.6**	92.0	**95.8**	**92.7**
		MAD	**Q10**	**0.353**	0.227	0.296	**0.360**	**0.338**	0.251	**0.228**	**0.298**
			Q100	**0.421**	**0.261**	0.327	**0.399**	**0.410**	0.278	**0.286**	**0.343**
	Provincial	**NSH**	**Q10**	89.3	94.7	**96.4**	**88.4**	89.0	**94.6**	91.2	94.1
			Q100	85.6	93.6	**95.2**	79.8	86.3	**93.6**	86.0	91.6
		MAD	**Q10**	0.371	**0.220**	**0.253**	0.346	0.454	**0.235**	0.315	0.301
			Q100	0.424	0.269	**0.291**	0.418	0.505	**0.264**	0.393	0.350
ROI/KRG	**National**	**NSH**	**Q10**	**91.2**	**95.9**	**95.6**	86.3	**90.4**	**94.5**	96.3	**94.7**
			Q100	**88.2**	**94.4**	93.8	**82.2**	**88.1**	**93.2**	94.8	**92.7**
		MAD	**Q10**	**0.35**	**0.209**	**0.277**	0.382	**0.431**	**0.237**	0.267	**0.300**
			Q100	**0.403**	**0.255**	**0.318**	0.407	**0.455**	**0.260**	0.322	**0.340**
	Provincial	**NSH**	**Q10**	90.4	95.6	95.5	**87.2**	89.6	93.6	**96.7**	94.4
			Q100	87.4	93.5	**93.9**	81.6	86.0	92.7	**95.2**	92.3
		MAD	**Q10**	0.361	0.229	0.287	**0.363**	0.447	0.254	**0.258**	0.309
			Q100	0.406	0.268	0.332	**0.406**	0.509	0.275	**0.305**	0.350

* Bold indicates best criteria between national and provincial analysis for the same model.

Figures

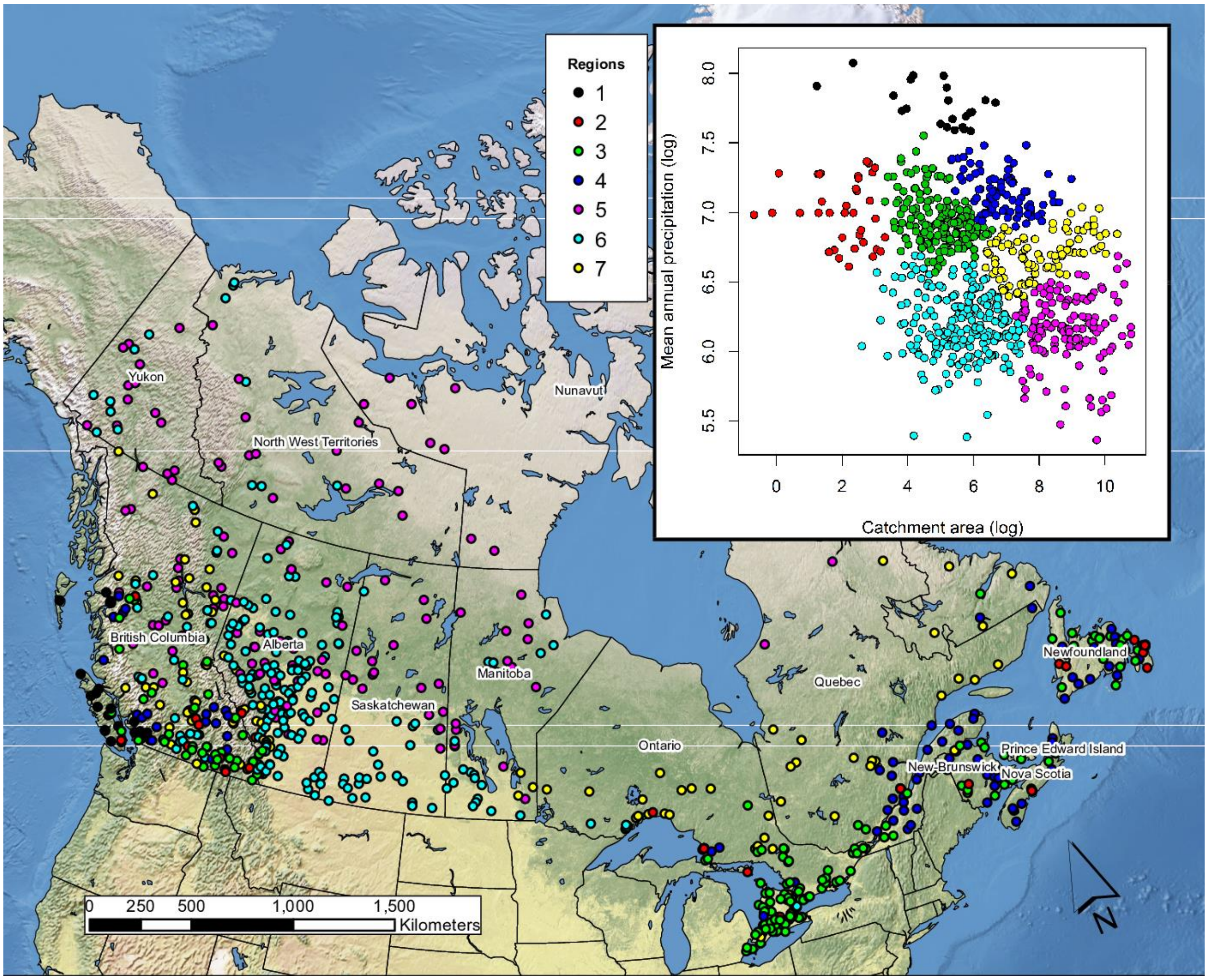

Figure 1: Locations of 770 hydrometric stations in Canada and descriptors space.