

A mathematical morphology approach to the identification of drought events in space and time

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Abstract

Drought events occur worldwide and possibly incur severe consequences. Trying to understand and characterizing drought events is of primordial importance in order to improve the preparedness for coping with future events. In this paper, drought events are characterized by exploiting their spatio-temporal nature. Operators borrowed from mathematical morphology are applied to represent drought events as connected components in space and time. Characteristics reflecting the affected area, duration, and intensity are extracted from the proposed representation of a drought event. As an illustration, drought events are identified on the basis of a 35-year data set of daily soil moisture values covering Australia.

Keywords: mathematical morphology, drought identification, space and time, connected component

1. Introduction

1 Drought events are caused by a lack of precipitation over a large area
2 and a long period of time. On-site, it is relatively easy to tell whether or not
3 one is experiencing a drought event. Yet, properly defining a drought event
4 is not an easy task. A drought event can be regarded as a creeping hazard
5 with no clear start and ending, it furthermore moves around and changes in
6 space and time. Drought events occur worldwide and sometimes have severe
7

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8 socio-economic consequences. At the worst, they cause catastrophes such as
9 famines and may be the immediate reason of severe conflicts or even wars.
10 In order to improve the preparedness for future drought event occurrences
11 by e.g. adjusting or drawing up water management plans, it is essential to
12 first try to understand and characterize these phenomena.

13 In this paper, an attempt is undertaken to characterize drought events
14 by exploiting their spatio-temporal nature. Operators borrowed from math-
15 ematical morphology (Serra, 1986) are first applied such that drought events
16 can be represented as connected components in space and time. Next, of the
17 resulting events, characteristics describing the affected area, duration and
18 intensity are extracted. This drought event representation and characteriza-
19 tion can serve as a support for water managers and users in the field. Future
20 research will then focus on the comparison of an ongoing drought event with
21 historical drought events on the basis of the delineated characteristics.

22 In scientific literature, much effort has already been made to try to quan-
23 titatively characterize drought events. However, capturing all aspects of a
24 drought event is still a difficult task, as it concerns a moving and chang-
25 ing phenomenon. Tallaksen and Van Lanen (2004) pointed out that a
26 drought event is a spatio-temporal phenomenon of which the space- and
27 time-components are to be considered. Yet, only recently an increasing
28 number of studies have reported on the characterization of drought events
29 as a spatio-temporal connected component. Andreadis et al. (2005) were
30 among the first to introduce a spatial identification procedure and took
31 into account the fact that multiple drought events at one time step can
32 merge into a larger drought event at a subsequent time step. Similarly, they
33 consider that one single drought event can break up into multiple smaller
34 drought events. Sheffield et al. (2009) also employed this spatial identifica-
35 tion procedure for drought area identification, whereas Lloyd-Hughes (2011)
36 further elaborated on this procedure and extended it from the spatial do-
37 main to the space-time domain to extract coherent space-time structures.
38 Herrera-Estrada et al. (2017) also employed the method of Andreadis et al.
39 (2005) to try to track the movement of drought events across regions. In a
40 parallel path of trying to understand the impact of climate extremes on ter-
41 restrial ecosystems and corresponding land-atmosphere fluxes, Zscheischler
42 et al. (2013) identified large spatio-temporal contiguous extreme events by
43 identifying connected components through the use of a flood-fill algorithm.

44 Generally, whenever drought events are characterized, one relies upon
45 drought indices such as the popular standardised precipitation index (McKee
46 et al., 1993) or the Palmer drought severity index (Palmer, 1965) in order
47 to be able to evaluate drought characteristics of interest (Mishra and Singh,

48 2010). Next, relationships between drought characteristics such as severity-
49 intensity-duration or severity-area-duration are generally established (see,
50 among others, the work of Sridhar et al. (2008); Mishra et al. (2009)), some
51 by means of copulas (e.g. Wong et al. (2010)), such that the dependence
52 between characteristics can be summarized, thus facilitating a frequency
53 analysis of the events by means of return periods (see e.g. Halwatura et al.
54 (2015); Reddy and Ganguli (2011); Serinaldi et al. (2009)). Nevertheless, one
55 has to bear in mind that all drought indices have their own advantages and
56 shortcomings (Sheffield et al., 2004; Sheffield and Wood, 2011). Applications
57 of these indices may hence also suffer from these inadequacies (Sheffield
58 et al., 2004). Furthermore, Sheffield and Wood (2007, 2011) suggest to use
59 percentile values of a drought variable in order to overcome inconsistencies
60 between severity classes assigned by different drought indices and to provide
61 a sound probabilistic grounding for a drought index. As percentile values
62 represent probabilities of occurrence, using percentile values also allows for a
63 straightforward comparison between values at different locations (Sheffield
64 and Wood, 2007, 2011).

65 Sheffield et al. (2004) and Sheffield and Wood (2007) state that soil
66 moisture is a useful drought index. It is a key variable in the hydrological
67 cycle as it controls the majority of processes in the hydrological cycle such as
68 evaporation, runoff, infiltration and drainage. It also reflects the impact of
69 meteorological variables such as temperature and radiation. Furthermore,
70 soil moisture values in the top layer of the soil are related to short-term
71 precipitation, whereas soil moisture values in the root zone indicate the
72 amount of water available for plant growth, and soil moisture values in the
73 deeper soil layers represent how much water is available for recharge to
74 aquifers and rivers (Sheffield et al., 2004).

75 Hence, in the present study, drought characteristics are determined on
76 the basis of percentile values of soil moisture. Furthermore, a long time
77 series of spatial soil moisture data is required, such that the spatial and
78 temporal components of a drought event can be captured. A time series
79 of ca. 35 years of daily values of GLEAM-estimated (Miralles et al., 2011;
80 Martens et al., 2017) soil moisture data at a resolution of 0.25° meets these
81 requirements and is employed in this study. As Australia is regularly af-
82 fected by drought events, data covering Australia were selected from this
83 global data set. It was decided to focus on daily values in contrast to the
84 generally-used monthly values, as in this way drought events that last less
85 than one month and are terminated by a single storm (Byun and Wilhite,
86 1999; Sheffield et al., 2004) can still be detected. Furthermore, drought
87 events are identified in space and time using operators from mathematical

88 morphology (Serra, 1986). Further, characteristics reflecting the spatial and
89 temporal components and the severity level of the drought events are de-
90 termined. In order to make a small comparison to the general approach
91 of using monthly data, the method employing operators from mathematical
92 morphology to identify drought events in space and time will also be applied
93 to weekly data, such that drought events with a minimal duration of *ca.* one
94 month will be retained.

95 Section 2 first elaborates on the data and the study region chosen for
96 this research. Section 3 then explains the data pre-processing, the selection
97 of the threshold used as a basis for identifying drought events and illustrates
98 the basics of mathematical morphology and its application in the identifi-
99 cation of drought events. Section 4 further elaborates on the determined
100 characteristics of the resulting drought events. Section 5 then formulates
101 the conclusions that are drawn from the results.

102 **2. Data and study region**

103 In order to be able to characterize an ongoing drought event, one has
104 to be able to compare it to historical drought events. To that end, a long
105 time series of historical data is required. Ideally, such time series should
106 be available at large scales in order to also capture the spatial character-
107 istics of the events. With the emergence of satellite remote sensing data
108 in the late seventies, obtaining information at a high temporal and spatial
109 resolution has become easier. The Global Land Evaporation Amsterdam
110 Model (GLEAM) (Miralles et al., 2011) maximally benefits from the use
111 of satellite-derived observations to estimate terrestrial evaporation and soil
112 moisture. Its resulting data sets of evaporation and soil moisture have al-
113 ready been used in modelling studies or in evaluations w.r.t. other data
114 sets (see e.g. Tobin and Bennett (2017); Roy et al. (2017); Majozi et al.
115 (2017); Lopez et al. (2017); McCabe et al. (2016); Liu et al. (2016); Lorenz
116 et al. (2014); Trambauer et al. (2014)). The data set employed in this paper
117 (the GLEAM v3.0a data set (Martens et al., 2017)) spans a period of 35
118 years (from 1/1/1980 till 31/12/2014) of global daily root-zone soil mois-
119 ture values at a 0.25° resolution. The data has been estimated on the basis
120 of satellite-observed soil moisture, vegetation optical depth and snow water
121 equivalents, reanalysis air temperature and radiation and a multi-source pre-
122 cipitation product (Martens et al., 2017) and shows a slightly higher quality
123 compared to other GLEAM data sets when evaluated against in situ mea-
124 sured soil moisture data. The depth of the root zone employed in GLEAM
125 is a function of the land-cover type and can consist of up to three layers

126 (0–10 cm, 10–100 cm and 100–250 cm). Three (resp. two) layers are taken
127 into account for the fraction of tall (resp. small) vegetation, whereas only
128 the first layer is taken into account for the bare soil fraction.

129 As Australia is vulnerable to the effects of climate change, in particular to
130 the expected drying trend for the next 50-100 years (McCarthy et al., 2001),
131 daily data covering Australia were selected from this GLEAM data set. At
132 present, substantial agricultural areas are affected by periodic drought events
133 and Australia contains large areas of arid and semi-arid land. Unlike in
134 many other drought characterization studies, it was decided not to convert
135 the daily values to monthly values as these daily values will allow for the
136 detection of drought periods lasting less than one month. As pointed out
137 by Byun and Wilhite (1999), an affected drought region can return to normal
138 conditions with only one day’s rainfall.

139 **3. Data pre-processing and drought identification**

140 As one of the drought characteristics that will be determined throughout
141 this paper is linked to the spatial component of the event, the soil moisture
142 values were first reprojected to the Lambert Azimuthal Equal Area coordi-
143 nate system such that areas can be accurately calculated. These reprojected
144 data, with a resolution of $27.442 \text{ km} \times 29.079 \text{ km}$, are then further used as
145 the basis for the drought characterization in the remainder of the paper.

146 *3.1. Selection of the drought threshold*

147 As pointed out by Sheffield and Wood (2011), characterizing a drought
148 event is a challenging task because it varies in many dimensions: its spatial
149 components describe the area it covers, its temporal component reflects the
150 time it persists and its intensity changes both in space and time. Ideally, the
151 characterization of a drought event should reflect all of these components
152 and should furthermore be transferable across regions and through time.
153 Therefore, the suggestion of Sheffield and Wood (2007) and Sheffield and
154 Wood (2011) to use a percentile level as threshold for defining a drought
155 event is followed in this paper. A value below the threshold then indicates
156 that drought conditions are met. A value of 10% was chosen, which then
157 reflects that drought conditions are observed 10% of the time. This value
158 can also be regarded as the value that separates moderate from severe and
159 more extreme drought events (Andreadis et al., 2005). Furthermore, to take
160 into account that a drought event is not restricted to a single location and
161 to allow that the soil moisture values of the neighbouring locations also take

162 part in determining the soil moisture value of the threshold, a neighbour-
163 hood was identified around the location under consideration. In this research
164 study, three different sizes of neighbourhoods N , *i.e.* a 3×3 -, 5×5 -, and
165 7×7 -neighbourhood, were used. In this way, a smoother transition between
166 the soil moisture values of the thresholds for neighbouring pixels is estab-
167 lished. In order to set the soil moisture value corresponding to the percentile
168 threshold of 10% for the pixel at hand, the empirical cumulative distribution
169 function of all soil moisture values observed within this neighbourhood was
170 established. In this way, determining a threshold corresponding to the 10th
171 percentile indicates that the soil moisture value at the location should drop
172 below the soil moisture value corresponding to the 10th percentile of the
173 entire neighbourhood before it is regarded as dry. This idea can easily be
174 extended when one aims at taking into account larger neighbourhoods such
175 as regions with the same land cover.

176 3.2. Applicaton of mathematical morphology

177 After selecting only those pixels with a value below the 10th percentile
178 value a time series of binary maps that indicate which locations possibly
179 belong to a drought event are obtained (see Figure 1 for an example of such
180 a map for a 3×3 -neighbourhood). It is clear that applying the threshold
181 results in a scattered pattern. Single pixels are denoted as dry, while larger
182 dry regions contain pixels that are not denoted to be dry. This also occurs in
183 the time-dimension. Pixels can be denoted as dry for one time step, whereas
184 for some following time steps, they are denoted as not dry, followed by being
185 denoted as dry in the time steps thereafter. Hence, a processing procedure is
186 required in order to smooth away these irregularities. A method that is well
187 suited for this purpose is mathematical morphology (Serra, 1986), whose
188 operations aim at simplifying images by retaining the essential shape char-
189 acteristics and removing irrelevancies (Haralick et al., 1987). Applications
190 and extensions of mathematical morphology w.r.t. image filtering, image
191 segmentation, *etc.* have already been reported in the processing of remote
192 sensing data (Soille and Pesaresi, 2002) and medical image analysis (Dufour
193 et al., 2013).

194 By using the basic operators from mathematical morphology, *i.e.* erosion
195 and dilation, the salt-and-pepper noise, *i.e.* the holes within the larger dry-
196 denoted regions and the smaller dry-denoted regions can be filled or removed.
197 To apply these operators, a structuring element should first be determined,
198 the size of which influences the size of the dry-denoted regions that will be
199 removed and the holes that will be filled. As a drought event has a spatio-
200 temporal character, it is chosen to employ a structuring element that has

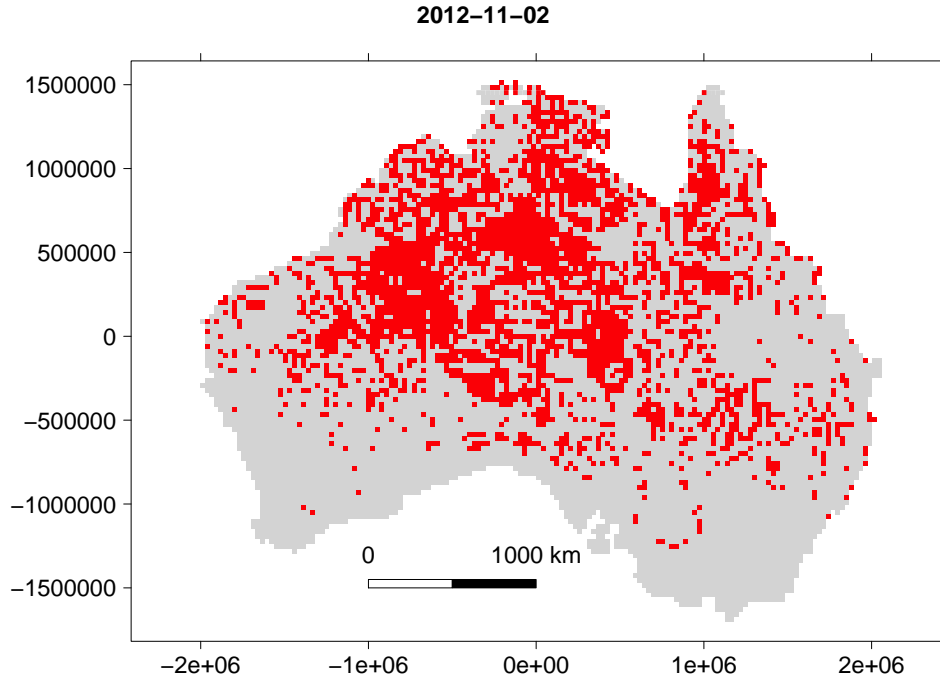


Figure 1: Possible drought locations (red) after thresholding a percentile map

201 space-time dimensions. To this end, the thresholded maps of the time series
 202 are placed one after the other, and a three-dimensional structuring element
 203 can hence be applied to this series. Different sizes of structuring elements
 204 were used of which the smallest a $3 \times 3 \times 3$ - and the largest a $7 \times 7 \times 7$ -box.
 205 The first two dimensions indicate the spatial size of the structuring element,
 206 the last one indicates the number of time steps that is taken into account.

207

208 For reasons of simplicity, the erosion and dilation operators are first ex-
 209 plained in two dimensions. Consider the space E of all pixels, of which the
 210 set A of dry-denoted pixels at a particular time step is a subset. With each
 211 point x of the space E , a two-dimensional structuring element $B(x)$, e.g.
 212 a square, is associated. Figure 2(a) shows the space E , the set A of dry-
 213 denoted pixels indicated in grey and a 3×3 structuring element (red). The

214 erosion ϵ of A and dilation δ are then:

$$\epsilon(A) = \{x \in E \mid B(x) \subseteq A\}, \quad (1)$$

$$\delta(A) = \{x \in E \mid B(x) \cap A \neq \emptyset\}. \quad (2)$$

215 The application of the above operators on A is performed as follows. The
 216 structuring element is positioned with its center at each pixel x of E , *i.e.*
 217 $B(x)$. Regarding the erosion operator, a grey pixel x will remain grey if
 218 the pixels covered by the structuring element, positioned at x , are a subset
 219 of A . Regarding the dilation operator, a white pixel x becomes grey if the
 220 intersection of the pixels covered by the structuring element positioned at
 221 x with A is not empty. Figures 2 and 3 illustrate these two morphological
 222 operators. An original map, for which the dry-denoted pixels (the set A) are
 223 indicated in grey, is given in Figure 2(a) together with the 3×3 structuring
 224 element (red). In Figure 2(b), the structuring element is positioned at the
 225 pixel corresponding to its central pixel, indicated in light grey. As by this
 226 positioning, the pixels covered by the structuring element do not make up
 227 a subset of dry-denoted pixels of the original map, this pixel remains white,
 228 *i.e.* not dry, after the erosion process. Similarly, the pixel corresponding to
 229 the structuring element's central pixel in Figure 2(c) will remain grey, *i.e.*
 230 dry, after the erosion process, as for this pixel, the set of pixels covered by
 231 the structuring element is a subset of the original set. Figure 2(d) shows
 232 the final result after positioning the structuring element at all pixels of the
 233 space E . Analogously, Figure 3 illustrates the application of the dilation
 234 operator.

235 The compositions $\gamma = \delta \circ \epsilon$ and $\phi = \epsilon \circ \delta$ are called the morphological
 236 opening and closing, respectively. By first applying a morphological opening
 237 followed by a morphological closing, an open-close filter is obtained and the
 238 salt-and-pepper noise can be removed in the following sequential way:

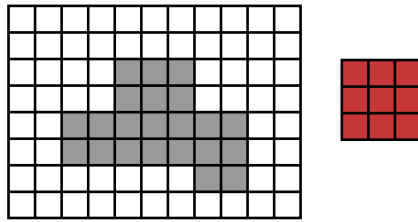
$$\epsilon(A) = \{x \in E \mid B(x) \subseteq A\}, \quad (3)$$

$$\gamma(A) = (\delta \circ \epsilon)(A) = \{x \in E \mid B(x) \cap \epsilon(A) \neq \emptyset\}, \quad (4)$$

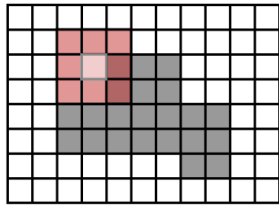
$$(\delta \circ \gamma)(A) = \{x \in E \mid B(x) \cap \gamma(A) \neq \emptyset\}, \quad (5)$$

$$(\phi \circ \gamma)(A) = (\epsilon \circ \delta \circ \gamma)(A) = \{x \in E \mid B(x) \subseteq (\delta \circ \gamma)(A)\}. \quad (6)$$

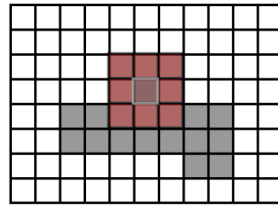
239 For applying the above-described operators in the three dimensions of the
 240 time series of thresholded images, the space E now consists of all pixels at
 241 all time steps in this time series, and A is the time series of dry-denoted
 242 pixels. The three-dimensional structuring element, e.g. a $3 \times 3 \times 3$ -box, is
 243 then positioned at each pixel x in E , and the operators can be applied as



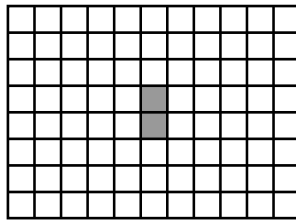
(a)



(b)

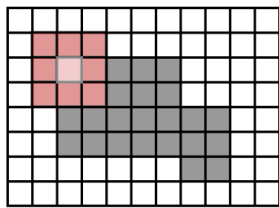


(c)

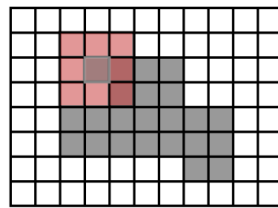


(d)

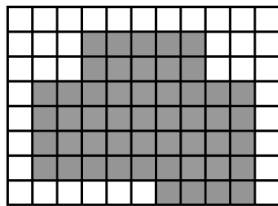
Figure 2: Map and structuring element (a), two steps (b) and (c) picked out of the erosion process and result (d) of the erosion process.



(a)



(b)



(c)

Figure 3: Two steps (a) and (b) picked out of the dilation process on the image in Fig 2(a) and result (c) of the dilation process.

244 described before. By first applying the erosion operator (see Eq. (3)), dry-
245 denoted regions that are strictly smaller than the structuring element are
246 removed. Larger dry-denoted regions are diminished and existing holes will
247 be, initially, enlarged. By applying the dilation operator (see Eq. (4)) in the
248 second step, the diminished regions are enlarged and the enlarged holes are
249 diminished. The application of the dilation and erosion operator in the third
250 and fourth step, respectively (see Eq. (5) and Eq. (6)), aims at removing
251 holes that are strictly smaller than the structuring element.

252 The resulting dry-denoted regions that are connected in space and time
253 are then regarded as separate drought events. An operational definition of
254 a drought event is thus obtained:

255 *A single drought event is defined as a connected component in space and*
256 *time after application of the morphological operators.*

257 Figure 4 illustrates the resulting drought events obtained at November
258 the 2nd, 2012, after applying an open-close filter to the thresholded time
259 series with a $3 \times 3 \times 3$ -box. Different colors are used to illustrate the different
260 drought events. This figure clearly shows that at the given time step, the
261 green-coloured drought event is not spatially contiguous. However, as in
262 former or later time steps, the currently isolated green parts merge, these
263 parts belong to the same drought event. For the GLEAM data set spanning
264 35 years, 1859 drought events were identified in this way. The smallest and
265 shortest drought event of these identified events corresponds to the size of the
266 chosen structuring element. It should be noted however, that by enlarging
267 the size of the structuring element to e.g. a $5 \times 5 \times 5$ -box, fewer drought events
268 will be identified of which the smallest drought event will hence correspond
269 to the size of this applied structuring element. Furthermore, as the dilation
270 operator is part of the procedure to identify drought events, it is inevitable
271 that pixels that are originally denoted as not dry, *i.e.* their value is higher
272 than the threshold, will become part of the identified drought event.

273

274 It is furthermore to be noted that by applying a larger structuring el-
275 ement, e.g. a $7 \times 7 \times 7$ -box, larger parts of a possible drought event will
276 be eliminated when the structuring element does not fit between the holes,
277 *i.e.* the salt noise, in the event. This means that in the application of the
278 erosion operator (Eq. (3)), the structuring element does not entirely overlap
279 the dry-denoted pixels resulting in the removal of these pixels. For instance,
280 after application of an open-close filter with a $7 \times 7 \times 7$ -box on the thresh-
281 olded time series for the date corresponding to Figure 1, no drought event

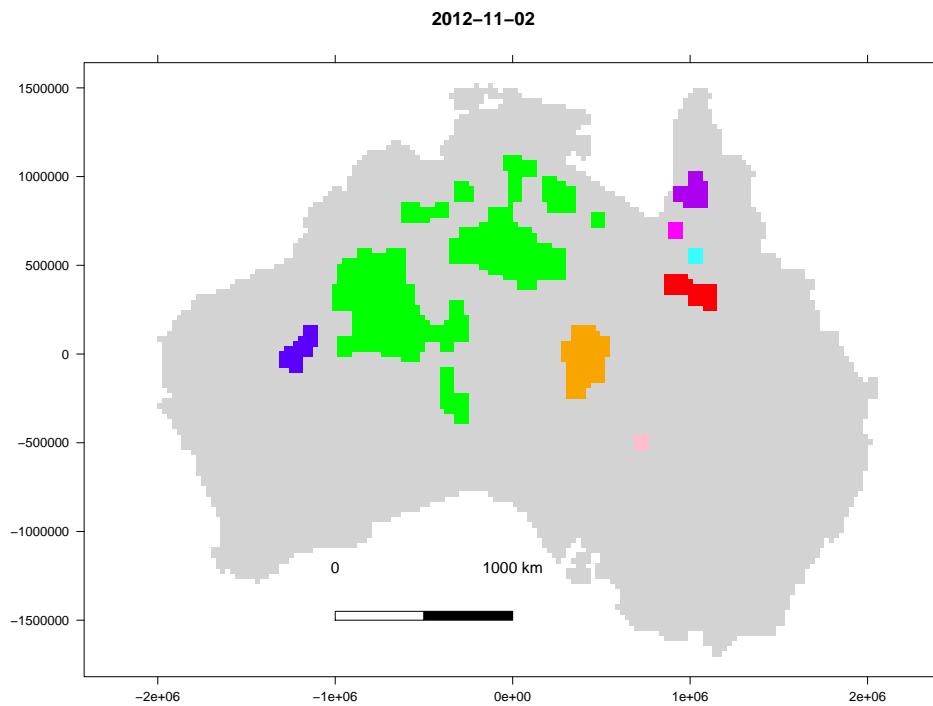


Figure 4: Drought events identified after application of an open-close filter to the thresholded time series. Different colours indicate different drought events. From the drought events identified, only one day is shown as illustration.

282 was identified. Such a result is not desired as Figure 1 clearly shows that
 283 a quite large contiguous area, apart from some isolated pixels, is dry. This
 284 side-effect can be alleviated by gradually eliminating the noise components
 285 by using an open-close filter sequentially, starting from a small structuring
 286 element in the first iteration and by enlarging the structuring element for
 287 subsequent iterations. Such filter is called an alternating sequential filter
 288 (ASF) (Serra and Vincent, 1992):

$$\text{ASF}_i = (\phi_i \circ \gamma_i) \circ (\phi_{i-1} \circ \gamma_{i-1}) \circ \dots \circ (\phi_1 \circ \gamma_1), \quad (7)$$

289 in which i represents the i -th iteration, γ_1 and ϕ_1 represent the opening,
 290 respectively closing operator with the smallest structuring element. ASF_1
 291 hence corresponds to the open-close filter using a $3 \times 3 \times 3$ -box. ASF-filters
 292 were applied in order to compare the results of directly applying a larger
 293 structuring element to the results of gradually enlarging the structuring
 294 element, *i.e.* by applying ASF_2 and ASF_3 respectively with a $3 \times 3 \times 3$ -
 295 and a $5 \times 5 \times 5$ - and a $3 \times 3 \times 3$ -, a $5 \times 5 \times 5$ - and a $7 \times 7 \times 7$ -box.
 296 Figure 5 illustrates the obtained result after applying ASF_2 and ASF_3 on
 297 the time series resulting from a 3×3 -neighbourhood. It can be seen that
 298 the larger drought event (green-coloured in Figure 4) also appears in the
 299 result after applying ASF_3 . This favours the application of an ASF-filter
 300 instead of directly applying an open-close filter with a larger structuring
 301 element. The splitted drought event, purple-coloured in the top panel of
 302 Figure 5, is identified by ASF_2 , however, its upper part is too small to be
 303 identified by ASF_3 . Furthermore, it is noted that the larger drought event
 304 (green-coloured in Figure 4) has been split in smaller events (green- and
 305 purple-coloured in Figure 5) after application of an ASF. The next sections
 306 will further elaborate on the influence of the size of the structuring elements
 307 or the filter used (open-close vs. ASF).

308

309 4. Drought characteristics

310 For each drought event identified by the above-described method, char-
 311 acteristics reflecting its spatial and temporal components and its intensity
 312 level are determined. With respect to the spatial component, one needs a
 313 characteristic that summarizes the extent reached by the event. The maxi-
 314 mal area covered by the drought event could characterize the affected area.
 315 However, it might be more informative to aggregate the τ largest daily areal
 316 extents. In order to be in line with the size of the structuring elements

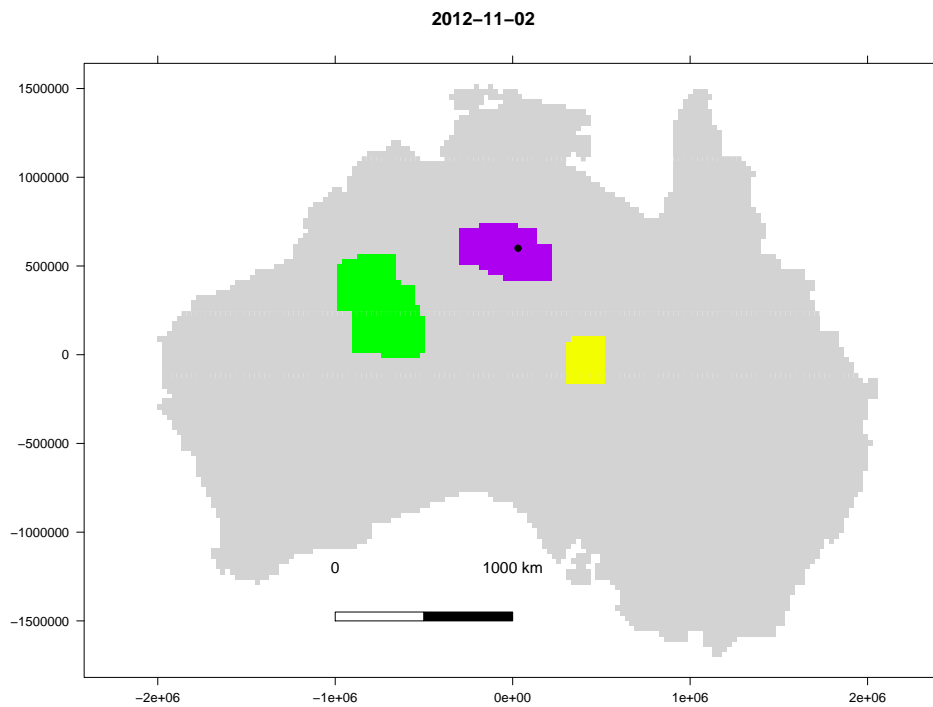
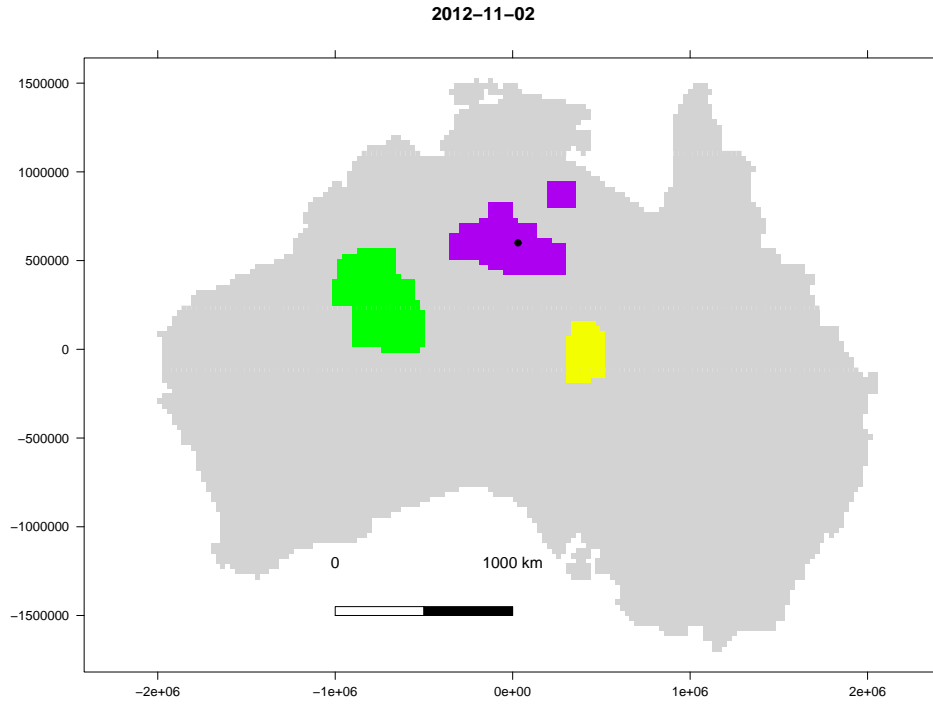


Figure 5: Drought events identified after application of an alternating sequential filter with a $3 \times 3 \times 3$ - and $5 \times 5 \times 5$ -box (top panel) and a $3 \times 3 \times 3$ -, a $5 \times 5 \times 5$ - and a $7 \times 7 \times 7$ -box (bottom panel). From the identified drought events, only one day is shown as illustration.

317 that were employed in the previous section, it was chosen to retain as many
 318 largest daily areal extents as the size τ of the structuring element in the
 319 time dimension. With respect to the characteristic reflecting the temporal
 320 component, as is already performed in many studies and is quite straightfor-
 321 ward, the drought duration is chosen. In order to summarize the intensity
 322 reached by the drought event, the daily percentile values q were first con-
 323 verted to survival percentiles, *i.e.* $1-q$, to express the intensities. In this way,
 324 higher values correspond to a higher intensity. Similarly as for the affected
 325 area, the τ largest daily intensity values were aggregated. The aggregation
 326 is performed using an ordered-weighted-averaging (OWA) operator (Yager,
 327 1988).

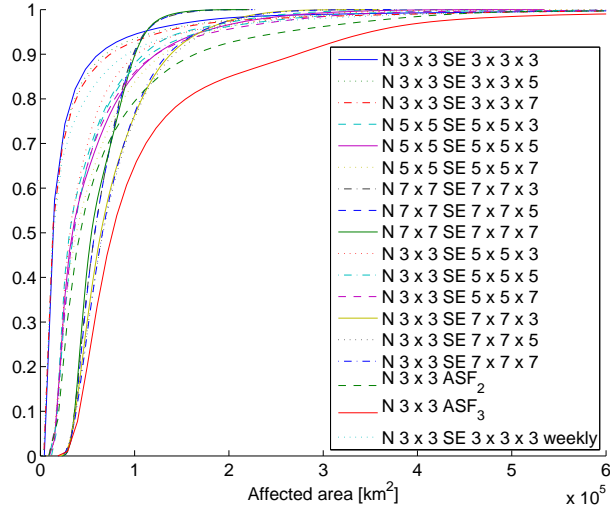
328 The OWA operator shows a great flexibility to model a wide variety of
 329 aggregators (Ahn, 2006) as by using a different weighting vector, a differ-
 330 ent type of aggregation is performed. The OWA operator has already been
 331 used in data mining applications (Torra, 2004), decision making (Vigier
 332 et al., 2017), regression problems (Yager and Beliakov, 2010), classifica-
 333 tion (Mohammed et al., 2016) and outlier reduction (Beliakov et al., 2016).
 334 An OWA operator $F : \mathbb{R}^n \rightarrow \mathbb{R}$ of arity n has a weighting vector $\mathbf{W} =$
 335 $(w_1, w_2, \dots, w_n)^T \in [0, 1]^n$ associated with it such that $\sum_{i=1}^n w_i = 1$, and
 336 takes the following form:

$$F(a_1, a_2, \dots, a_n) = \sum_{i=1}^n w_i b_i, \quad (8)$$

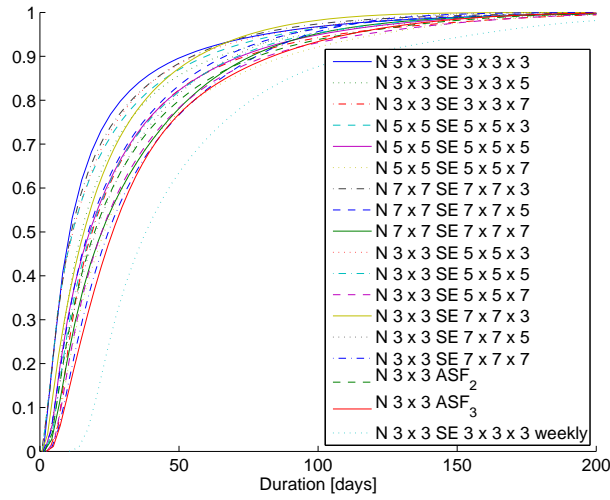
337 with b_j the j -th largest element of $\{a_1, a_2, \dots, a_n\}$.

338 If one would only be interested in the maximum (resp. the minimum)
 339 of the values, the vector of associated weights would be $\mathbf{W} = (1, 0, \dots, 0)$
 340 (resp. $\mathbf{W} = (0, 0, \dots, 1)$). When all weights are equal, the OWA operator
 341 corresponds to the arithmetic mean. Corresponding to the weighting vector,
 342 a measure of orness of the aggregation is associated, which reflects the degree
 343 to which the aggregation behaves like an or-operator. An orness of 1, (resp.
 344 0), corresponds to taking the maximum, (resp. the minimum) of the values.
 345 An orness of 0.5 corresponds to the arithmetic mean. In the current study,
 346 it was chosen to give more weight to the larger daily areal extents, therefore,
 347 an orness of 0.75 was used. The method of Fullér and Majlender (2001) was
 348 then used to obtain weights corresponding to these orness-values assuring
 349 that the dispersion of the weights is maximal, *i.e.* the degree to which all
 350 information in the aggregation is taken into account is maximal. Table 1
 351 lists these weights for OWA operators of arity 3, 5 and 7.

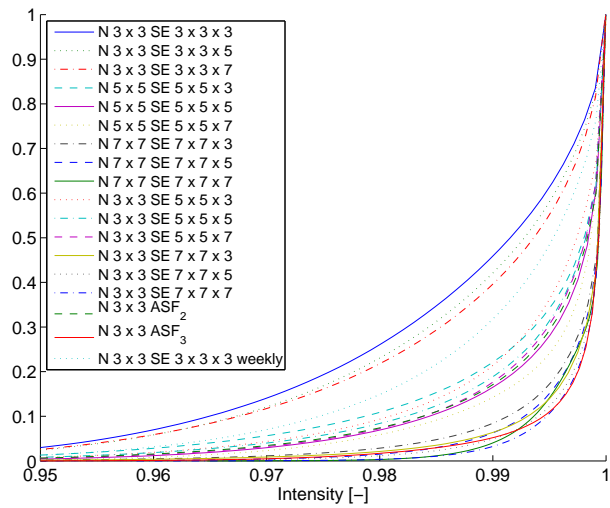
352 Table 2 lists the number of completed drought events obtained after



(a)



(b)



(c)

Figure 6: Cumulative distribution functions of the obtained drought characteristics for different sizes of neighbourhoods (N), structuring elements (SE) and morphological operators. (a) Affected area, (b) Drought duration and (c) Drought intensity.

Table 1: Weights used by the OWA operators of different arities corresponding to the length τ of the structuring element B in the time dimension.

orness=0.75			
τ	3	5	7
w_1	0.6162	0.4594	0.3637
w_2	0.2676	0.2608	0.2390
w_3	0.1162	0.1480	0.1556
w_4	-	0.0840	0.1012
w_5	-	0.0477	0.0659
w_6	-	-	0.0429
w_7	-	-	0.0279

353 application of the morphological operators using different sizes of the struc-
354 turing elements on data stemming from different neighbourhood sizes as de-
355 scribed in the previous section. Also, the corresponding values for the first
356 quartile, the median, the third quartile and the maximum for each of the
357 characteristics determined on these events are listed. Figure 6 furthermore
358 shows the cumulative distribution functions of these obtained drought char-
359 acteristics. From Table 2, it can be seen that the size of the neighbourhood
360 does not majorly influence the number of identified drought events, however,
361 as to be expected, the larger the size of the structuring element, the lower
362 the number of events that are identified. Furthermore, the largest drought
363 events obtained with the smaller structuring element have a larger affected
364 area than the largest drought events obtained with the larger structuring el-
365 ements (see Table 2). Similarly, the longest drought becomes shorter when
366 larger structuring elements are used. This is probably due to the fact that
367 the structuring element has to fit entirely, in space and time, within the time
368 series of thresholded maps, before a pixel being dry is retained. This also
369 holds for the tail ends of the drought events, in which smaller structuring
370 elements more easily fit, resulting in the fact that the largest and longest
371 drought event is identified when a smaller structuring element is used.

372 Furthermore, as the use of smaller structuring elements also results in
373 the identification of more, smaller and shorter drought events, the majority
374 of the drought events has a smaller affected area when they are identified by
375 smaller structuring elements. A similar observation can be made w.r.t. the
376 duration of the events. Figure 6 shows that the drought events identified

Table 2: Number of completed drought events for different sizes of neighbourhoods (N) and structuring elements (SE) and values corresponding to the first quartile (Q_1), the median, the third quartile (Q_3) and the maximum for the affected area, duration and intensity of the events. Results obtained on the basis of daily data are given in the top panel, results on the basis of weekly data are given in the bottom panel.

Daily data														
N	SE	# events	affected area (km ²)				duration (days)				intensity (-)			
			Q_1	median	Q_3	max	Q_1	median	Q_3	max	Q_1	median	Q_3	max
3 × 3	3 × 3 × 3	1859	7.18 10 ³	1.20 10 ⁴	2.71 10 ⁴	1.07 10 ⁶	5	11	23	333	0.9798	0.9914	0.9979	1.0000
	3 × 3 × 5	1391	7.18 10 ³	1.28 10 ⁴	2.87 10 ⁴	0.90 10 ⁶	8	15	31	333	0.9815	0.9927	0.9981	0.9999
	3 × 3 × 7	1157	7.18 10 ³	1.28 10 ⁴	2.95 10 ⁴	0.90 10 ⁶	10	19	38	333	0.9826	0.9935	0.9983	0.9998
	5 × 5 × 3	470	2.00 10 ⁴	2.9 10 ⁴	5.59 10 ⁴	0.57 10 ⁶	5	12	26	191	0.9918	0.9972	0.9996	1.0000
	5 × 5 × 5	328	2.39 10 ⁴	3.4 10 ⁴	6.06 10 ⁴	0.57 10 ⁶	9	18	39	191	0.9930	0.9981	0.9996	0.9999
	5 × 5 × 7	270	2.39 10 ⁴	3.35 10 ⁴	6.62 10 ⁴	0.57 10 ⁶	11	21.5	42	191	0.9932	0.9981	0.9996	0.9998
	7 × 7 × 3	97	4.47 10 ⁴	6.24 10 ⁴	9.58 10 ⁴	0.22 10 ⁶	8	16	31	88	0.9978	0.9994	0.9999	1.0000
	7 × 7 × 5	76	4.47 10 ⁴	6.33 10 ⁴	9.54 10 ⁴	0.22 10 ⁶	9.5	20.5	39.5	96	0.9980	0.9995	0.9998	0.9999
	7 × 7 × 7	63	5.03 10 ⁴	6.30 10 ⁴	9.57 10 ⁴	0.22 10 ⁶	13.25	25	45	110	0.9983	0.9994	0.9997	0.9998
		ASF ₂	348	2.75 10 ⁴	4.07 10 ⁴	8.08 10 ⁴	0.86 10 ⁶	10	19.5	41	191	0.9939	0.9983	0.9997
	ASF ₃	118	5.58 10 ⁴	7.62 10 ⁴	1.20 10 ⁵	1.0 10 ⁶	15	26.5	45	155	0.9983	0.9994	0.9997	0.9998
5 × 5	5 × 5 × 3	389	2.00 10 ⁴	2.87 10 ⁴	6.16 10 ⁴	0.34 10 ⁶	5	12	29	194	0.9909	0.9982	0.9996	1.0000
	5 × 5 × 5	285	1.99 10 ⁴	3.19 10 ⁴	6.99 10 ⁵	0.34 10 ⁶	9	18	32.25	194	0.9945	0.9984	0.9996	0.9999
	5 × 5 × 7	222	1.99 10 ⁴	3.63 10 ⁴	7.50 10 ⁴	0.34 10 ⁶	12	23	41	194	0.9956	0.9988	0.9996	0.9998
7 × 7	7 × 7 × 3	1034.47 10 ⁴	5.98 10 ⁴	7.82 10 ⁴	0.14 10 ⁶	5.25	11	25.75	156	0.9967	0.9994	0.9999	1.0000	
	7 × 7 × 5	74	4.47 10 ⁴	5.59 10 ⁴	7.82 10 ⁴	0.14 10 ⁶	9	17	34	156	0.9970	0.9995	0.9998	0.9999
	7 × 7 × 7	60	4.47 10 ⁴	5.03 10 ⁴	7.90 10 ⁴	0.14 10 ⁶	11	20.5	45	156	0.9969	0.9995	0.9997	0.9998
Weekly data														
N	SE	# events	affected area (km ²)				duration (weeks)				intensity (-)			
			Q_1	median	Q_3	max	Q_1	median	Q_3	max	Q_1	median	Q_3	max
3 × 3	3 × 3 × 3	660	7.18 10 ³	1.31 10 ⁴	3.57 10 ⁴	8.80 10 ⁵	3	5	9	48	0.9871	0.9951	0.9990	0.9999

377 with ASF_3 generally have a larger affected area. Regarding the identified
378 drought intensities, it can be seen that the intensity values generally be-
379 come larger, *i.e.* only the more severe drought events are retained, for larger
380 structuring elements. It should furthermore be mentioned that the values
381 of 1.000 in Table 2 are in fact smaller than 1, but appear as such because
382 of the rounding. Table 2 furthermore shows that more drought events are
383 identified after applying an ASF than after applying an open-close filter with
384 a comparable structuring element (*i.e.* ASF_2 vs. a $5 \times 5 \times 5$ -box and ASF_3
385 vs. a $7 \times 7 \times 7$ -box). Furthermore, as expected, the affected areas become
386 larger after application of an ASF. These observations might indicate that
387 by using smaller structuring elements, drought events might be too easily
388 withheld, whereas the use of larger structuring elements might be too strict.
389 The identification of drought events by means of ASF_2 or ASF_3 hence might
390 serve as a golden mean. Zhao and Lu (2017) also reported in their medical
391 image analysis study that using ASF_2 obtains good results for most images.
392 Yet, these drought events are still the result of an operational procedure
393 such that identified droughts may not entirely correspond to the opinion of
394 water managers or on-site experience.

395 Generally, characterization of drought events is performed on the basis
396 of monthly data (Byun and Wilhite, 1999). In order to compare the above-
397 described results to what could be obtained for monthly data, for each pixel
398 its weekly average of soil moisture data was calculated. To these weekly
399 data, a threshold of 10% was chosen for a 3×3 -neighbourhood, and an
400 open-close filter with a $3 \times 3 \times 3$ -box was applied. By doing so, dry-denoted
401 areas need a minimum duration of three weeks (more or less comparable to
402 one month) before they will be identified as a drought event. Table 2 shows
403 that only 660 events were identified compared to 1859 on the basis of daily
404 data. Naturally, the majority of the events has a longer duration compared
405 to the events identified on a daily basis. The longest drought event has a
406 comparable duration. With respect to the affected areas and the intensities,
407 comparable values as for the smaller structuring elements are obtained.

408 5. Conclusions

409 It has been shown in this paper that mathematical morphology can be
410 used to operationally identify drought events taking into account their spa-
411 tial and temporal nature. This has been illustrated using the GLEAM data
412 set, spanning a period of 35 years of daily root-zone soil moisture values at
413 a 25° resolution. Daily data covering Australia have been selected on the
414 basis of which soil moisture values corresponding to the 10th percentile of a

415 neighbourhood have been used as threshold to determine whether drought
416 conditions are possibly met. Operators from mathematical morphology em-
417 ploying a structuring element in space and time dimensions have then been
418 applied to this spatio-temporal data series to operationally identify drought
419 events. Different sizes of neighbourhoods and structuring elements have
420 been used to identify drought events of which characteristics reflecting their
421 spatio-temporal nature have been determined. Drought affected area, dura-
422 tion and intensity have been calculated. In order to summarize the affected
423 area covered by and the intensity level of the drought event, an ordered-
424 weighted-averaging operator was used taking into account as many values
425 as the time dimension of the structuring element used.

426 Results show that the largest and longest drought event was obtained
427 by using the smallest structuring element. However, larger structuring ele-
428 ments generally identify larger, longer, and more severe drought events. Yet,
429 less events are obtained with the larger structuring elements. This might
430 indicate that smaller structuring elements identify drought events too eas-
431 ily whereas larger structuring elements are too strict in the identification
432 of drought events. Results showed that a golden mean might be offered by
433 applying an alternating sequential filter. However, one has to be aware that
434 these identified drought events are the result of an operational identifica-
435 tion procedure, from which the resulting drought events may not entirely
436 correspond to the on-site experience or to the opinion of water managers.

437 In future work, it will be illustrated how the spatio-temporal charac-
438 teristics of drought events as identified on the basis of mathematical mor-
439 phology can be used to relate an ongoing event to historical drought events
440 and estimate its severity by taking into account the dependence between its
441 characteristics.

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