Analysis of Orbit Determination Error Impact on Clutter Doppler Frequency for Space-Based Early Warning Radar

Xiaobin Huang [1], Yan Zhang [1] and Shupeng Jin

Abstract*—The accuracy of clutter Doppler frequency in space-based early warning radar (SBR) directly determines moving target detection performance, and satellite orbit determi-nation error is a important cause of clutter Doppler deviation. Taking satellite position and velocity in the Geocentric Celestial Reference System (GCRS) as core variables, this paper derives the analytical expression of clutter Doppler frequency through co-ordinate transformations from GCRS to the International Terrestrial Reference System (ITRS) and then to the North-East-Zenith (NEZ) local coordinate system. The multi-variate function gradient analysis method is used to quantitatively reveal the influ-ence laws of GCRS position and velocity errors on Doppler frequency. Results indicate that satellite velocity error is the linear dominant term of Doppler variation with a constant gradient magnitude of 2/λ, and only the velocity component along the satel-lite–ground line-of-sight contributes to frequency shift. Satellite position error acts as a nonlinear modulation term, whose gradient magnitude is inversely proportional to the satellite–ground distance and coupled with satellite velocity. Under typical SBR oper-ating parameters and BeiDou receiver orbit determination errors, a velocity error of 0.1 m/s causes a Doppler deviation below 1 Hz, and a 10 m position error results in a de-viation less than 1.6 Hz, both negligible in engineering. This research clarifies orbit de-termination error propagation characteristics and provides a theoretical foundation for clutter Doppler accuracy control and orbit precision design of SBR.

***Index Terms*—space-based early warning radar; clutter Doppler frequency; orbit determination er-ror; gradient analysis; coordin -ate transformation**

I. INTRODUCTION

SPACE-BASED early warning radar (SBR) is severely affected by ground clutter during moving target detection [1,2]. Accurate estimation of clutter Doppler frequency is the core prerequi-site for frequency-domain moving target detection and clutter suppression algorithm design, and this estimation accuracy strongly depends on the precision of the position and velocity parameters of the radar satellite platform itself [3,4]. In engineering ap-plications, real-time satellite position and velocity information are usually provided by satellite-borne GNSS navigation receivers [5]. However, due to combined effects of navigation system measurement errors, ephemeris errors, clock biases, and complex space environments, the satellite state parameters output by navigation receivers in-evitably contain orbit determination errors [6]. Such errors are directly propagated in-to the clutter Doppler frequency calculation process, causing deviations between pre-dicted and actual frequencies, which further degrade clutter suppression performance and reduce radar's moving target detection capability. Therefore, quantitatively ana-lyzing the influence of orbit determination error on clutter Doppler frequency of SBR and revealing the mechanism between satellite state parameter errors and Doppler frequency deviation are of great theoretical significance and engineering value for im-proving the overall performance of SBR.

Current research on clutter Doppler characteristics of space-based radar mostly focuses on clutter modeling and suppression algorithms [7], lacking quantitative anal-ysis of orbit determination error propagation to Doppler frequency in the GCRS iner-tial frame, and cannot clarify the influence weight and mechanism of position and ve-locity errors. To solve this problem, this paper takes satellite GCRS position and veloc-ity as core variables, derives the analytical expression of clutter Doppler frequency with respect to GCRS state parameters via GCRS→ITRS→NEZ coordinate transfor-mations, and then uses multivariate function gradient analysis to quantitatively reveal the influence characteristics of satellite orbit determination errors (i.e., GCRS position and velocity errors) on clutter Doppler frequency. The results show that satellite ve-locity error is the linear dominant term of Doppler frequency variation, with a con-stant gradient magnitude of 2/λ, and only the velocity component along the satellite–ground line of sight contributes to frequency shift; satellite position error is a nonlinear modulation term, whose gradient magnitude is inversely proportional to satellite–ground distance, and its modulation effect is coupled with satellite velocity. For typical SBR parameters and orbit determination errors from satellite-borne Beidou receivers, theoretical calculations and simulations show that a velocity error of 0.1 m/s leads to Doppler deviation less than 1 Hz, and a position error of 10 m results in deviation less than 1.6 Hz, so the influence of orbit determination error on clutter Doppler frequency can be ignored in engineering practice. This research clarifies the propagation charac-teristics of orbit

This research was funded by the Fundamental Theory Project, grant number KJ2024C009. Conceptualization, Xiaobin Huang; Writing—Original Draft Preparation, Xiaobin Huang; Vali-dation and Writing—Review, Yan Zhang; Editing, Jin Shupeng. All authors have read and agreed to the published version of the manuscript.

Xiaobin Huang is Air Force Early Warning Academy(e-mail: 532454132@qq.com).
Yan Zhang is Air Force Early Warning Academy(e-mail: 549712493@qq.com).
Shupeng Jin is Air Force Early Warning Academy(e-mail: 445723121@qq.com).

determination error and provides a theoretical basis for clutter Dop-pler frequency accuracy control and orbit determination precision design of SBR.

II. Symbol Conventions

To standardize symbol usage in formula derivation and analysis, unify definitions of vectors, matrices, coordinate systems, and derivative operations, and clarify sym-bols, physical meanings, and value properties of physical quantities, this section sum-marizes main symbols and operation rules used in the paper.

A. Basic Rules for Vectors and Matrices

1)Vectors are 3×1 column vectors; matrices and vectors are written in bold italic, and scalars in regular italic.

2)Transpose of matrix/vector is denoted as $\boldsymbol{A}^{\mathrm{T}}$, $\boldsymbol{a}^{\mathrm{T}}$; vector magnitude as $|\boldsymbol{a}|$; vector dot product as $\boldsymbol{a}\cdot\boldsymbol{b}$ (output scalar); vector cross product as $\boldsymbol{a}\times\boldsymbol{b}$ (output 3×1 column vector).

3)Orthogonal matrices satisfy that the inverse equals the transpose, i.e., $\boldsymbol{A}^{-1}=\boldsymbol{A}^{\mathrm{T}}$, and the product of orthogonal matrices remains orthogonal.

4)Skew-symmetric cross-product matrix: for any 3D vector $\boldsymbol{a}=[a_1,a_2,a_3]^{\mathrm{T}}$, its skew-symmetric cross-product matrix is defined as

$$\boldsymbol{a}^{\times}=\begin{bmatrix}0 & -a_3 & a_2\\ a_3 & 0 & -a_1\\ -a_2 & a_1 & 0\end{bmatrix}$$

which satisfies

$$\boldsymbol{a}\times\boldsymbol{x}=\boldsymbol{a}^{\times}\boldsymbol{x},\quad (\boldsymbol{a}^{\times})^{\mathrm{T}}=-\boldsymbol{a}^{\times}$$

For Earth rotation rate vector $\boldsymbol{\omega}_{\oplus}=[0,0,\omega_{\oplus}]^{\mathrm{T}}$, its skew-symmetric matrix is

$$\boldsymbol{\omega}_{\oplus}^{\times}=\begin{bmatrix}0 & -\omega_{\oplus} & 0\\ \omega_{\oplus} & 0 & 0\\ 0 & 0 & 0\end{bmatrix},\quad \omega_{\oplus}=7.292115\times10^{-5}\ \mathrm{rad/s}$$

B. Definition of Derivative Operations

Vector derivatives in this paper follow the denominator-layout rule: let $\boldsymbol{y}\in\mathbb{R}^{m}$ be an m-dimensional column vector and $\boldsymbol{x}\in\mathbb{R}^{n}$ an n-dimensional column vector, then the derivative $\partial\boldsymbol{y}/\partial\boldsymbol{x}$ is an $n\times m$ matrix with elements

$$\left(\frac{\partial\boldsymbol{y}}{\partial\boldsymbol{x}}\right)_{ij}=\frac{\partial y_j}{\partial x_i}$$

Main derivative rules used in gradient analysis of this paper are as follows.

1)Linear transformation derivative: for constant matrix $\boldsymbol{A}$ and variable vector $\boldsymbol{x}$,

$$\frac{\partial(\boldsymbol{A}\boldsymbol{x})}{\partial\boldsymbol{x}}=\boldsymbol{A}^{\mathrm{T}}$$

2)Vector dot-product derivative: for constant vector $\boldsymbol{a}$ and variable vector $\boldsymbol{x}$,

$$\frac{\partial(\boldsymbol{a}\cdot\boldsymbol{x})}{\partial\boldsymbol{x}}=\boldsymbol{a}$$

3)Composite function dot-product derivative: for vector-valued functions $\boldsymbol{p}(\boldsymbol{x})$, $\boldsymbol{q}(\boldsymbol{x})$,

$$\frac{\partial(\boldsymbol{p}\cdot\boldsymbol{q})}{\partial\boldsymbol{x}}=\left(\frac{\partial\boldsymbol{p}}{\partial\boldsymbol{x}}\right)\boldsymbol{q}+\left(\frac{\partial\boldsymbol{q}}{\partial\boldsymbol{x}}\right)\boldsymbol{p}$$

4)Vector cross-product derivative: for constant vector $\boldsymbol{a}$, constant matrix $\boldsymbol{A}$ and variable vector $\boldsymbol{x}$,

$$\frac{\partial(\boldsymbol{a}\times\boldsymbol{A}\boldsymbol{x})}{\partial\boldsymbol{x}}=-\boldsymbol{A}^{\mathrm{T}}\boldsymbol{a}^{\times}$$

where $\boldsymbol{a}^{\times}$ is the skew-symmetric cross-product matrix of vector $\boldsymbol{a}$.

5)Vector magnitude derivative: for orthogonal matrix $\boldsymbol{T}$, constant vector $\boldsymbol{c}$ and variable vector $\boldsymbol{x}$,

$$\frac{\partial|\boldsymbol{T}\boldsymbol{x}-\boldsymbol{c}|}{\partial\boldsymbol{x}}=\frac{\boldsymbol{T}^{\mathrm{T}}(\boldsymbol{T}\boldsymbol{x}-\boldsymbol{c})}{|\boldsymbol{T}\boldsymbol{x}-\boldsymbol{c}|}$$

6)Quotient rule: for variable vector $\boldsymbol{x}$,

$$\frac{\partial}{\partial\boldsymbol{x}}\left(\frac{N}{L}\right)=\frac{1}{L^2}\left(L\frac{\partial N}{\partial\boldsymbol{x}}-N\frac{\partial L}{\partial\boldsymbol{x}}\right)$$

C. Definition of Rotation Operators

Coordinate transformations are described by rotation operator $\boldsymbol{R}_n(\theta)$, representing an orthogonal rotation matrix rotating by angle θ around axis n (n=1,2,3 for x, y, z axes respectively), expressed as

$$\boldsymbol{R}_1(\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{bmatrix},\quad \boldsymbol{R}_2(\theta)=\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix},\quad \boldsymbol{R}_3(\theta)=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

Note: positive for clockwise rotation viewed along the positive direction of the rotation axis.

D. Symbols and Physical Meanings

Satellite-related physical quantities use subscript s, ground-point-related quantities use subscript D; superscripts G (GCRS), I (ITRS), N (NEZ) denote the coordinate system. Table 1 lists main symbols for reader reference.

TABLE I

List of Physical Symbols

Symbol	Physical Meaning	Property/Value
GCRS	Geocentric Celestial Reference System	Inertial coordinate system
ITRS	International Terrestrial Reference System	Earth-fixed coordinate system
NEZ	North-East-Zenith Coordinate System	Topocentric coordinate system
$\boldsymbol{r}_{\mathrm{s}}^{\mathrm{G}}, \boldsymbol{v}_{\mathrm{s}}^{\mathrm{G}}$	Satellite position and velocity vectors in GCRS	3×1 column vectors
$\boldsymbol{r}_{\mathrm{s}}^{\mathrm{I}}, \boldsymbol{v}_{\mathrm{s}}^{\mathrm{I}}$	Satellite position and velocity vectors in ITRS	3×1 column vectors
$\boldsymbol{r}_{\mathrm{s}}^{\mathrm{N}}, \boldsymbol{v}_{\mathrm{s}}^{\mathrm{N}}$	Satellite position and velocity vectors in NEZ	3×1 column vectors
$\boldsymbol{r}_{\mathrm{D}}^{\mathrm{G}}, \boldsymbol{r}_{\mathrm{D}}^{\mathrm{I}}$	Position vectors of ground point D in GCRS and ITRS	Constant, 3×1 column vectors
v_r	Radial velocity of	Scalar, m/s

Symbol	Physical Meaning	Property/Value
	the satellite relative to the ground point	
f_d	Clutter Doppler frequency of the ground point relative to the satellite	Scalar, Hz
λ	Wavelength of radar transmitted signal	System constant, scalar, m
$\boldsymbol{W},\boldsymbol{R},\boldsymbol{N},\boldsymbol{P},\boldsymbol{B}$	Polar motion, Greenwich rotation, nutation, precession, frame bias matrices	Orthogonal matrices, 3×3
$\boldsymbol{P}_1$	NEZ coordinate correction matrix	$\boldsymbol{P}_1$=diag[−1,1,1], 3×3 diagonal matrix
$\boldsymbol{R}_n(\theta)$	Rotation operator rotating by angle θ around axis n	Orthogonal rotation matrix, 3×3
λ_D,ϕ_D	Astronomical longitude and latitude of ground point D	Constant, scalar, rad
L,B,H	Geodetic longitude, latitude, height of ground point D	Constant, scalar, rad (m)
$\boldsymbol{\omega}_\oplus$	Earth rotation rate vector in ITRS	$\boldsymbol{\omega}_\oplus=[0,0,\omega_\oplus]^T$; $\omega_\oplus\approx7.292115\times10^{-5}$ rad/s
$R_\oplus$	Earth equatorial radius	System constant, $R_\oplus$=6378.137 km
$e_\oplus$	Eccentricity of Earth ellipsoid	System constant, scalar
N	Prime vertical radius of curvature of Earth ellipsoid	Scalar, $N=R_\oplus/\sqrt{1-e_\oplus^2\sin^2 B}$, m
μ	Gravitational constant of Earth	Celestial mechanical constant, μ=3.986004418×10^{14} m^3/s^2
$\boldsymbol{T}_{G\to I}$	Coordinate transformation matrix from GCRS to ITRS	Orthogonal matrix, $\boldsymbol{T}_{G\to I}=\boldsymbol{W}^T\boldsymbol{R}^T\boldsymbol{N}^T\boldsymbol{P}^T\boldsymbol{B}^T$
$\boldsymbol{T}_{I\to N}$	Coordinate transformation matrix from ITRS to NEZ	Orthogonal matrix, $\boldsymbol{T}_{I\to N}=\boldsymbol{P}_1\boldsymbol{R}_2(90^\circ-\varphi_D)\boldsymbol{R}_3(\lambda_D)$
$\boldsymbol{T}$	Total transformation matrix from GCRS to NEZ	Orthogonal matrix, $\boldsymbol{T}=\boldsymbol{T}_{I\to N}\boldsymbol{T}_{G\to I}$
$\boldsymbol{c}$	Constant vector of ground point	Constant, $\boldsymbol{c}=\boldsymbol{T}_{I\to N}\boldsymbol{r}_D^I$, 3×1 column vector
$\boldsymbol{\Omega}(\boldsymbol{r}_s^G)$	Earth rotation coupling term	3×1 column vector related to $\boldsymbol{r}_s^G$
(a,e,i,Ω,ω,f)	Classical elements of the satellite (semi-major axis, eccentricity, inclination, right ascension of ascending node, argument of perigee, true anomaly)	Six parameters describing satellite motion

III. Coordinate Transformation and Doppler Frequency Calculation

Clutter Doppler frequency calculation for SBR centers on satellite–ground relative radial velocity, requiring GCRS→ITRS→NEZ coordinate transformations to map satellite GCRS state parameters to the NEZ topocentric frame [8], so as to solve radial velocity and compute clutter Doppler frequency. This section sequentially gives definitions of relevant coordinate systems, coordinate transformation relations, and Doppler frequency formulas, laying a foundation for deriving analytical expressions of GCRS parameters.

A. Definition of Relevant Coordinate Systems

Clutter Doppler frequency calculation for SBR involves three core coordinate systems: GCRS, ITRS, and NEZ. The transformation from satellite orbital elements to GCRS state parameters requires an orbital coordinate system. The origin, reference plane, and coordinate axis directions of each system are defined in Table 2 [9], with physical meanings and application scenarios as follows.

1)GCRS: Geocentric Celestial Reference System, an inertial frame for accurately describing satellite position and motion, a fundamental reference in astronomy and aerospace orbital dynamics.

2)ITRS: International Terrestrial Reference System, an Earth-fixed frame rotating with the Earth, a basic system for describing positions on and near Earth, widely used in geodesy and satellite navigation.

3)NEZ: North-East-Zenith topocentric coordinate system (left-handed), centered at a ground observation point, consistent with ground-measured spatial orientation conventions, the target frame for satellite–ground relative radial velocity and clutter Doppler frequency calculation.

4)Orbital coordinate system: Geocentric orbit-fixed frame coinciding with the satellite orbital plane, an intermediate frame for converting satellite orbital elements to GCRS state parameters.

TABLE Ⅱ

Definition of Coordinate Systems

Coordinate System	Origin	Reference Plane	Cartesian Axis Direction
GCRS	Geocenter	Mean equator at	*X*-axis: Mean vernal equinox at J2000.0 *Z*-axis: Mean pole at J2000.0 *Y*-axis: In J2000.0 mean

			equator, right-handed with X, Z
ITRS	Earth mass center	Conventional equator (orthogonal to geocenter–CIO line)	X-axis: Intersection of reference plane and mean Greenwich meridian Z-axis: Points to CIO Y-axis: Right-handed with X, Z
NEZ	Station center	Geoid	x-axis: North z-axis: Zenith y-axis: Left-handed with x, z
Orbital Coordinate System	Geocenter	Satellite orbital plane	x-axis: Perigee (eccentricity vector direction) z-axis: Aligns with orbital normal $\boldsymbol{h}$ y-axis: In orbital plane, right-handed with x, z

B. Orbital Elements to GCRS State Parameters

Satellite orbital elements are basic parameters describing orbital motion and must be converted to GCRS position and velocity vectors $(\boldsymbol{r}_s^G, \boldsymbol{v}_s^G)$ for subsequent coordinate transformation and Doppler frequency calculation. Figure 1 shows the reference diagram for orbital elements to GCRS state parameters, where *O-XYZ* is GCRS and *O-xyz* is the orbital coordinate system.

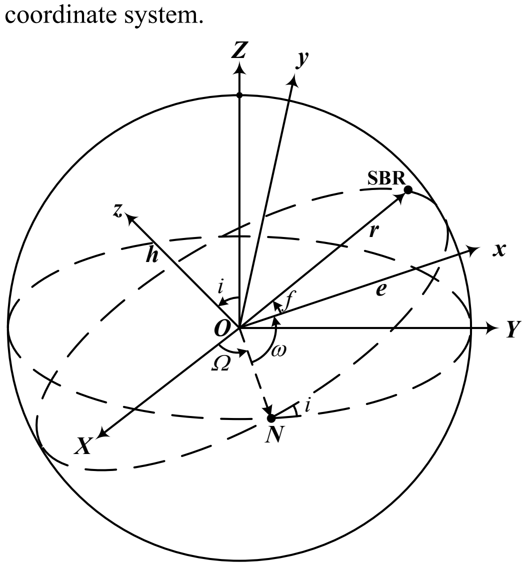

Fig 1. Reference diagram for orbital elements to GCRS state parameters transformation.

Combined with rotation operator definitions in Section 2.3, three rotations map the orbital coordinate system to GCRS, and the expressions of GCRS state parameters in terms of satellite orbital elements are derived as [10]

$$\begin{cases} \boldsymbol{r}_s^G = \boldsymbol{R}_3(-\Omega)\boldsymbol{R}_1(-i)\boldsymbol{R}_3(-\omega)\begin{bmatrix} r\cos f \\ r\sin f \\ 0 \end{bmatrix} = r\cos f\cdot\boldsymbol{P} + r\sin f\cdot\boldsymbol{Q} \\ \boldsymbol{v}_s^G = \boldsymbol{R}_3(-\Omega)\boldsymbol{R}_1(-i)\boldsymbol{R}_3(-\omega)\begin{bmatrix} -\frac{h}{p}\sin f \\ \frac{h}{p}(e+\cos f) \\ 0 \end{bmatrix} = -\frac{h}{p}\sin f\cdot\boldsymbol{P} + \frac{h}{p}(e+\cos f)\cdot\boldsymbol{Q} \end{cases} \quad (1)$$

where $r=a(1-e^2)/(1+e\cos f)$ is satellite geocentric distance, $p=a(1-e^2)$ is semi-latus rectum of elliptical orbit, $h=(p\mu)^{1/2}$ is specific orbital angular momentum, μ is gravitational constant of Earth, $\boldsymbol{P}$ and $\boldsymbol{Q}$ are unit vectors of orbital coordinate system x and y axes in GCRS, expressed as

$$\boldsymbol{P} = \begin{bmatrix} \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i \\ \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i \\ \sin\omega\sin i \end{bmatrix}, \quad \boldsymbol{Q} = \begin{bmatrix} -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i \\ -\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i \\ \cos\omega\sin i \end{bmatrix} \quad (2)$$

C. Satellite State Parameter Transformation from GCRS to ITRS

GCRS is an inertial frame and ITRS is an Earth-fixed frame rotating with the Earth. Their transformation considers combined corrections of polar motion, Greenwich rotation, nutation, precession, and frame bias. Position vectors undergo pure rotation, and velocity vectors require additional Earth rotation correction. Transformation formulas are [11]

$$\begin{cases} \boldsymbol{r}_s^I = \boldsymbol{W}^T\boldsymbol{R}^T\boldsymbol{N}^T\boldsymbol{P}^T\boldsymbol{B}^T\boldsymbol{r}_s^G \\ \boldsymbol{v}_s^I = \boldsymbol{W}^T\left\{\boldsymbol{R}^T\boldsymbol{N}^T\boldsymbol{P}^T\boldsymbol{B}^T\boldsymbol{v}_s^G - \boldsymbol{\omega}_\oplus \times \boldsymbol{W}\boldsymbol{r}_s^I\right\} \end{cases} \quad (3)$$

where $\boldsymbol{r}_s^I, \boldsymbol{v}_s^I$ are satellite state parameters in ITRS; $\boldsymbol{W}$, $\boldsymbol{R}$, $\boldsymbol{N}$, $\boldsymbol{P}$, $\boldsymbol{B}$ are polar motion, Greenwich rotation, nutation, precession, and frame bias matrices in order; $\boldsymbol{\omega}_\oplus$ is Earth rotation rate vector, and the cross-product term is the Earth-fixed correction of satellite velocity due to Earth rotation.

D. ITRS Position Solution of Ground Point

Position of ground point D is given by geodetic coordinates (longitude L, latitude B, height H) and must be converted to ITRS position vector $\boldsymbol{r}_D^I$ as a reference constant for satellite ITRS-to-NEZ transformation. Conversion formula is [12]

$$\boldsymbol{r}_D^I = \begin{bmatrix} (N+H)\cos B\cos L \\ (N+H)\cos B\sin L \\ \left[N(1-e^2)+H\right]\sin B \end{bmatrix} \quad (4)$$

where Earth ellipsoid prime vertical radius $N = R_\oplus / \sqrt{1-e_\oplus^2\sin^2 B}$, $R_\oplus$ is Earth equatorial radius, $e_\oplus$ is Earth ellipsoid eccentricity, both being Earth geometric constants.

E. Satellite State Parameter Transformation from ITRS to NEZ

Figure 2 shows the reference diagram for ITRS-to-NEZ coordinate transformation, where *O-XYZ* is ITRS and *O-xyz* is NEZ. Transformation of satellite ITRS state parameters to

NEZ first translates position based on ITRS position $\boldsymbol{r}_{\mathrm{D}}^{\mathrm{I}}$ of ground point D, then aligns coordinate systems via longitude–latitude rotations, and introduces NEZ left-handed correction matrix $\boldsymbol{P}_1$ for calibration. Transformation formulas are

$$\begin{cases} \boldsymbol{r}_{\mathrm{s}}^{\mathrm{N}} = \boldsymbol{P}_1\boldsymbol{R}_2(90^\circ - \varphi_{\mathrm{D}})\boldsymbol{R}_3(\lambda_D)\left(\boldsymbol{r}_{\mathrm{s}}^{\mathrm{I}} - \boldsymbol{r}_{\mathrm{D}}^{\mathrm{I}}\right) \\ \boldsymbol{v}_{\mathrm{s}}^{\mathrm{N}} = \boldsymbol{P}_1\boldsymbol{R}_2(90^\circ - \varphi_{\mathrm{D}})\boldsymbol{R}_3(\lambda_D)\boldsymbol{v}_{\mathrm{s}}^{\mathrm{I}} \end{cases} \tag{5}$$

where $\boldsymbol{r}_{\mathrm{s}}^{\mathrm{N}}, \boldsymbol{v}_{\mathrm{s}}^{\mathrm{N}}$ are satellite state parameters in NEZ; λ_{D}, φ_{D} are astronomical longitude and latitude of ground point D, serving as angular parameters for rotation; $\boldsymbol{R}_2$, $\boldsymbol{R}_3$ are rotation matrices around corresponding axes; $\boldsymbol{P}_1$=diag[−1,1,1] is NEZ left-handed correction matrix.

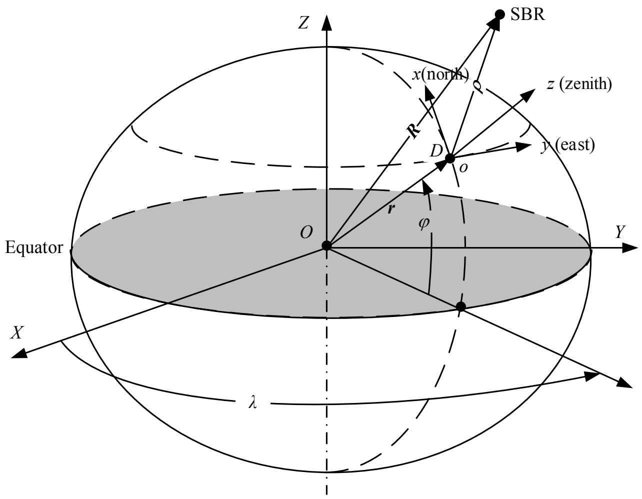

Fig. 2. Reference diagram for ITRS to NEZ coordinate transformation.

F. Clutter Doppler Frequency Calculation

Clutter Doppler frequency of SBR is calculated from relative radial velocity v_r between satellite and ground point in NEZ coordinates. Radial velocity is the projection of satellite NEZ velocity onto the satellite–ground line of sight, reflecting motion speed along the line of sight. Calculation formula is

$$v_r = \frac{\boldsymbol{r}_{\mathrm{s}}^{\mathrm{N}} \cdot \boldsymbol{v}_{\mathrm{s}}^{\mathrm{N}}}{\left|\boldsymbol{r}_{\mathrm{s}}^{\mathrm{N}}\right|} \tag{6}$$

(5)

Clutter Doppler frequency of ground point relative to satellite is determined by radial velocity and radar transmitted wavelength. Based on radar Doppler effect principle, formula is

$$f_d = -\frac{2v_r}{\lambda} \tag{7}$$

When the satellite approaches ground point D, $\boldsymbol{r}_{\mathrm{s}}^{\mathrm{N}} \cdot \boldsymbol{v}_{\mathrm{s}}^{\mathrm{N}} < 0$, radial velocity v_r<0, clutter Doppler frequency f_d>0, and the echo has positive frequency shift; when the satellite moves away, the opposite occurs with negative frequency shift.

Ⅳ. NEZ State Parameters Expressed by GCRS State Parameters

To reveal the inherent influence of satellite GCRS state parameters on clutter Doppler frequency, the intermediate ITRS variables are eliminated, and satellite position and velocity vectors in NEZ are directly expressed as functions of GCRS position and velocity vectors. This section defines coordinate transformation matrices and simplified ground-point constant vectors to clarify the functional relationship.

A. Definitions Related to Coordinate Transformation

To simplify GCRS-to-NEZ transformation expressions, three orthogonal transformation matrices and one ground-point constant vector are defined, with definitions and properties as follows.

1)GCRS-to-ITRS transformation matrix

$$\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}} \equiv \boldsymbol{W}^{\mathrm{T}}\boldsymbol{R}^{\mathrm{T}}\boldsymbol{N}^{\mathrm{T}}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}} \tag{8}$$

Polar motion, Greenwich rotation, nutation, precession, and frame bias matrices are all orthogonal, so $\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}$ remains orthogonal by orthogonality of matrix products. Substitute Eq. (8) into Eq. (3), GCRS-to-ITRS position transformation simplifies to

$$\boldsymbol{r}_{\mathrm{s}}^{\mathrm{I}} = \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_{\mathrm{s}}^{\mathrm{G}} \tag{9}$$

2)ITRS-to-NEZ transformation matrix

$$\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}} \equiv \boldsymbol{P}_1\boldsymbol{R}_2(90^\circ - \varphi_{\mathrm{D}})\boldsymbol{R}_3(\lambda_{\mathrm{D}}) \tag{10}$$

(6)

Obviously $\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}$ is also orthogonal. Substitute Eq. (10) into Eq. (5), ITRS-to-NEZ transformation simplifies to

$$\begin{cases} \boldsymbol{r}_{\mathrm{s}}^{\mathrm{N}} = \boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\left(\boldsymbol{r}_{\mathrm{s}}^{\mathrm{I}} - \boldsymbol{r}_{\mathrm{D}}^{\mathrm{I}}\right) \\ \boldsymbol{v}_{\mathrm{s}}^{\mathrm{N}} = \boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{v}_{\mathrm{s}}^{\mathrm{I}} \end{cases} \tag{11}$$

3)GCRS-to-NEZ total transformation matrix

$$\boldsymbol{T} \equiv \boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}} \quad (12$$

Product of orthogonal matrices remains orthogonal, and $\boldsymbol{T}$ satisfies dot-product invariance of orthogonal matrices, i.e., $(\boldsymbol{Ta})\cdot(\boldsymbol{Tb})=\boldsymbol{a}\cdot\boldsymbol{b}$ for any 3D vectors $\boldsymbol{a}$,$\boldsymbol{b}$.

4)Ground-point constant vector

$$\boldsymbol{c} \equiv \boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{r}_{\mathrm{D}}^{\mathrm{I}} \tag{13}$$

Since ITRS position $\boldsymbol{r}_{\mathrm{D}}^{\mathrm{I}}$ of ground point D is constant and $\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}$ is fixed, $\boldsymbol{c}$ is a 3D constant vector independent of satellite state parameters.

B. NEZ Position Vector Expression Based on GCRS Position

Substitute Eq. (9) into the position vector of Eq. (5), and simplify using definitions of matrix $\boldsymbol{T}$ and ground-point constant vector $\boldsymbol{c}$:

$$\boldsymbol{r}_{\mathrm{s}}^{\mathrm{N}} = \boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\left(\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_{\mathrm{s}}^{\mathrm{G}} - \boldsymbol{r}_{\mathrm{D}}^{\mathrm{I}}\right) = \boldsymbol{T}\boldsymbol{r}_{\mathrm{s}}^{\mathrm{G}} - \boldsymbol{c} \tag{14}$$

Eq. (14) shows that satellite NEZ position vector depends only on GCRS position vector, as a superposition of linear orthogonal rotation and fixed translation of GCRS position, independent of GCRS velocity.

C. NEZ Velocity Vector Expression Based on GCRS State Parameters

Substitute velocity vector of Eq. (3) into velocity vector of Eq. (11), replace ITRS position vector with Eq. (9), expand and simplify using definition of $\boldsymbol{T}$:

$$\begin{aligned}\boldsymbol{v}_\mathrm{s}^\mathrm{N} &= \boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{W}^\mathrm{T}\left\{\boldsymbol{R}^\mathrm{T}\boldsymbol{N}^\mathrm{T}\boldsymbol{P}^\mathrm{T}\boldsymbol{B}^\mathrm{T}\boldsymbol{v}_\mathrm{s}^\mathrm{G}-\boldsymbol{\omega}_\oplus\times\boldsymbol{W}\boldsymbol{r}_\mathrm{s}^\mathrm{I}\right\}\\ &= \boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{W}^\mathrm{T}\boldsymbol{R}^\mathrm{T}\boldsymbol{N}^\mathrm{T}\boldsymbol{P}^\mathrm{T}\boldsymbol{B}^\mathrm{T}\boldsymbol{v}_\mathrm{s}^\mathrm{G}-\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{W}^\mathrm{T}\left(\boldsymbol{\omega}_\oplus\times\boldsymbol{W}\boldsymbol{r}_\mathrm{s}^\mathrm{I}\right)\\ &= \boldsymbol{T}\boldsymbol{v}_\mathrm{s}^\mathrm{G}-\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{W}^\mathrm{T}\left(\boldsymbol{\omega}_\oplus\times\boldsymbol{W}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_\mathrm{s}^\mathrm{G}\right)\end{aligned} \tag{15}$$

Define Earth rotation coupling term

$$\boldsymbol{\Omega}(\boldsymbol{r}_\mathrm{s}^\mathrm{G}) \equiv \boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{W}^\mathrm{T}\left(\boldsymbol{\omega}_\oplus\times\boldsymbol{W}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_\mathrm{s}^\mathrm{G}\right) \tag{16}$$

This coupling term depends only on satellite GCRS position vector, with no velocity variables, integrating additional corrections of celestial parameters such as Earth rotation, polar motion, and nutation, belonging to a position-related nonlinear modulation term.
Substitute Earth rotation coupling term into Eq. (15), NEZ velocity vector becomes

$$\boldsymbol{v}_\mathrm{s}^\mathrm{N} = \boldsymbol{T}\boldsymbol{v}_\mathrm{s}^\mathrm{G}-\boldsymbol{\Omega}(\boldsymbol{r}_\mathrm{s}^\mathrm{G}) \tag{17}$$

Eq. (17) shows that satellite NEZ velocity vector consists of two decoupled terms: the linear term is orthogonal rotation of GCRS velocity, the core part of NEZ velocity; the nonlinear modulation term is determined by GCRS position vector, an additional correction of celestial parameters to NEZ velocity, with amplitude much smaller than the linear term.

V. Clutter Doppler Frequency Expressed by GCRS State Parameters

Substitute Eqs. (14) and (17) into Eq. (6), simplify using dot-product invariance of orthogonal matrices:

$$\begin{aligned}v_r &= \frac{\left(\boldsymbol{T}\boldsymbol{r}_\mathrm{s}^\mathrm{G}-\boldsymbol{c}\right)\cdot\left(\boldsymbol{T}\boldsymbol{v}_\mathrm{s}^\mathrm{G}-\boldsymbol{\Omega}(\boldsymbol{r}_\mathrm{s}^\mathrm{G})\right)}{\left|\boldsymbol{T}\boldsymbol{r}_\mathrm{s}^\mathrm{G}-\boldsymbol{c}\right|} = \frac{\boldsymbol{T}\boldsymbol{r}_\mathrm{s}^\mathrm{G}\cdot\boldsymbol{T}\boldsymbol{v}_\mathrm{s}^\mathrm{G}-\boldsymbol{T}\boldsymbol{r}_\mathrm{s}^\mathrm{G}\cdot\boldsymbol{\Omega}(\boldsymbol{r}_\mathrm{s}^\mathrm{G})-\boldsymbol{c}\cdot\boldsymbol{T}\boldsymbol{v}_\mathrm{s}^\mathrm{G}+\boldsymbol{c}\cdot\boldsymbol{\Omega}(\boldsymbol{r}_\mathrm{s}^\mathrm{G})}{\left|\boldsymbol{T}\boldsymbol{r}_\mathrm{s}^\mathrm{G}-\boldsymbol{c}\right|}\\ &= \frac{\boldsymbol{r}_\mathrm{s}^\mathrm{G}\cdot\boldsymbol{v}_\mathrm{s}^\mathrm{G}-\left(\boldsymbol{T}\boldsymbol{r}_\mathrm{s}^\mathrm{G}-\boldsymbol{c}\right)\cdot\boldsymbol{\Omega}(\boldsymbol{r}_\mathrm{s}^\mathrm{G})-\boldsymbol{c}\cdot\boldsymbol{T}\boldsymbol{v}_\mathrm{s}^\mathrm{G}}{\left|\boldsymbol{T}\boldsymbol{r}_\mathrm{s}^\mathrm{G}-\boldsymbol{c}\right|}\end{aligned} \tag{18}$$

Define four scalar auxiliary terms

$$\begin{cases}K_1(\boldsymbol{r}_\mathrm{s}^\mathrm{G},\boldsymbol{v}_\mathrm{s}^\mathrm{G}) = \boldsymbol{r}_\mathrm{s}^\mathrm{G}\cdot\boldsymbol{v}_\mathrm{s}^\mathrm{G}\\ K_2(\boldsymbol{r}_\mathrm{s}^\mathrm{G}) = \left(\boldsymbol{T}\boldsymbol{r}_\mathrm{s}^\mathrm{G}-\boldsymbol{c}\right)\cdot\boldsymbol{\Omega}(\boldsymbol{r}_\mathrm{s}^\mathrm{G}) = \boldsymbol{r}_\mathrm{s}^\mathrm{N}\cdot\boldsymbol{\Omega}(\boldsymbol{r}_\mathrm{s}^\mathrm{G})\\ K_3(\boldsymbol{v}_\mathrm{s}^\mathrm{G}) = \boldsymbol{c}\cdot\boldsymbol{T}\boldsymbol{v}_\mathrm{s}^\mathrm{G}\\ L(\boldsymbol{r}_\mathrm{s}^\mathrm{G}) = \left|\boldsymbol{T}\boldsymbol{r}_\mathrm{s}^\mathrm{G}-\boldsymbol{c}\right| = \left|\boldsymbol{r}_\mathrm{s}^\mathrm{N}\right|\end{cases} \tag{19}$$

where K_1 is the dot product of satellite GCRS position and velocity, the core variable term in GCRS; K_2 is GCRS position coupling term, related only to satellite GCRS position vector; K_3 is GCRS velocity linear term, related only to satellite GCRS velocity vector; L is satellite–ground distance, related only to satellite GCRS position vector.

Eq. (19) shows that the satellite GCRS velocity $\boldsymbol{v}_\mathrm{s}^\mathrm{G}$ appears only in the linear terms K_1 and K_3, with no nonlinear coupling to the GCRS position $\boldsymbol{r}_\mathrm{s}^\mathrm{G}$; the GCRS position appears in both the linear term K_1 and the nonlinear terms K_2 and L, which constitutes the source of the nonlinear variation in the Doppler frequency.

Substitute four scalar auxiliary terms into Eq. (18)

$$v_r = \frac{K_1-K_2-K_3}{L} \tag{20}$$

Substitute Eq. (20) into Eq. (7), define $N(\boldsymbol{r}_\mathrm{s}^\mathrm{G},\boldsymbol{v}_\mathrm{s}^\mathrm{G}) = K_1-K_2-K_3$, and finally obtain the analytical expression of clutter Doppler frequency in terms of GCRS state parameters

$$f_d = -\frac{2}{\lambda}\frac{N}{L} \tag{21}$$

VI. Quantitative Influence of GCRS State Parameters on Clutter Doppler Frequency

The clutter Doppler frequency is treated as a vector function of satellite GCRS position and velocity. Using total differentiation of multivariate functions and gradient analysis, we quantitatively characterize how variations in GCRS position and velocity affect the clutter Doppler frequency.

A. Total Differential Expression of Clutter Doppler Frequency

Small change Δf_d of Doppler frequency is contributed by small changes in satellite GCRS position and velocity, with first-order approximation

$$\Delta f_d \approx \left(\nabla_{\boldsymbol{r}_\mathrm{s}^\mathrm{G}} f_d\right)^\mathrm{T}\Delta\boldsymbol{r}_\mathrm{s}^\mathrm{G}+\left(\nabla_{\boldsymbol{v}_\mathrm{s}^\mathrm{G}} f_d\right)^\mathrm{T}\Delta\boldsymbol{v}_\mathrm{s}^\mathrm{G} \tag{22}$$

(7)

where $\nabla_{\boldsymbol{r}_\mathrm{s}^\mathrm{G}} f_d = \partial f_d/\partial\boldsymbol{r}_\mathrm{s}^\mathrm{G}$ is the gradient of f_d with respect to satellite GCRS position, representing Doppler frequency change per unit position change; $\nabla_{\boldsymbol{v}_\mathrm{s}^\mathrm{G}} f_d = \partial f_d/\partial\boldsymbol{v}_\mathrm{s}^\mathrm{G}$ is the gradient of f_d with respect to satellite GCRS velocity, representing Doppler frequency change per unit velocity change.

B. Influence Analysis of Velocity Gradient

Satellite GCRS velocity $\boldsymbol{v}_\mathrm{s}^\mathrm{G}$ appears only in the numerator N of the clutter Doppler frequency analytical expression, and the denominator satellite–ground distance L is independent of velocity, so $\partial L/\partial\boldsymbol{v}_\mathrm{s}^\mathrm{G} = \boldsymbol{0}$. Combined with derivative rules, take partial derivative of f_d with respect to GCRS velocity to get velocity gradient expression

$$\nabla_{\boldsymbol{v}_\mathrm{s}^\mathrm{G}} f_d = -\frac{2}{\lambda}\frac{\boldsymbol{r}_\mathrm{s}^\mathrm{G}-\boldsymbol{T}^\mathrm{T}\boldsymbol{c}}{\left|\boldsymbol{T}\boldsymbol{r}_\mathrm{s}^\mathrm{G}-\boldsymbol{c}\right|} \tag{23}$$

Since $\boldsymbol{T}$ is orthogonal, vector magnitude is invariant under orthogonal transformation, so

$$\left|\boldsymbol{r}_\mathrm{s}^\mathrm{G}-\boldsymbol{T}^\mathrm{T}\boldsymbol{c}\right| = \left|\boldsymbol{T}\boldsymbol{r}_\mathrm{s}^\mathrm{G}-\boldsymbol{c}\right| = L \tag{24}$$

Thus velocity gradient becomes

$$\nabla_{\boldsymbol{v}_\mathrm{s}^\mathrm{G}} f_d = \frac{2}{\lambda}\boldsymbol{e}_r^\mathrm{G} \tag{25}$$

where $\boldsymbol{e}_r^{\mathrm{G}}$ is the unit vector along the satellite–ground line of sight in GCRS (from satellite to ground point).

Eq. (25) shows that the magnitude of velocity gradient is constant, and the influence of unit velocity change on Doppler frequency is consistent across the entire observation domain. Take a typical SBR operating at 1.5 GHz with wavelength 0.2 m as an example; satellite-borne Beidou receiver velocity error is less than 0.1 m/s. Calculated by Eq. (26), Doppler frequency change caused by velocity error is no more than 1 Hz.

$$\Delta f_d^v \le \left|\nabla_{\boldsymbol{r}_s^{\mathrm{G}}} f_d\right| \cdot \left|\Delta \boldsymbol{v}_s^{\mathrm{G}}\right| = 1\mathrm{Hz} \tag{26}$$

C. Influence Analysis of Position Gradient

Let $\boldsymbol{k}_2 = \partial K_2(\boldsymbol{r}_s^{\mathrm{G}}) / \partial \boldsymbol{r}_s^{\mathrm{G}}$, using properties in Sections Ⅱ.1 and Ⅱ.2, compute

$$\begin{aligned}
\boldsymbol{k}_2 &= \frac{\partial K_2(\boldsymbol{r}_s^{\mathrm{G}})}{\partial \boldsymbol{r}_s^{\mathrm{G}}} = \frac{\partial\left[\left(\boldsymbol{T}\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{c}\right)\cdot \boldsymbol{\Omega}(\boldsymbol{r}_s^{\mathrm{G}})\right]}{\partial \boldsymbol{r}_s^{\mathrm{G}}} = \frac{\partial\left(\boldsymbol{T}\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{c}\right)}{\partial \boldsymbol{r}_s^{\mathrm{G}}}\boldsymbol{\Omega}(\boldsymbol{r}_s^{\mathrm{G}}) + \frac{\partial \boldsymbol{\Omega}(\boldsymbol{r}_s^{\mathrm{G}})}{\partial \boldsymbol{r}_s^{\mathrm{G}}}\left(\boldsymbol{T}\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{c}\right) \\
&= \boldsymbol{T}^{\mathrm{T}}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{W}^{\mathrm{T}}\left(\boldsymbol{\omega}_\oplus \times \boldsymbol{W}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_s^{\mathrm{G}}\right) + \frac{\partial\left[\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{W}^{\mathrm{T}}\left(\boldsymbol{\omega}_\oplus \times \boldsymbol{W}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_s^{\mathrm{G}}\right)\right]}{\partial \boldsymbol{r}_s^{\mathrm{G}}}\left(\boldsymbol{T}\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{c}\right) \\
&= \boldsymbol{T}^{\mathrm{T}}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_s^{\mathrm{G}} + \frac{\partial\left[\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_s^{\mathrm{G}}\right]}{\partial \boldsymbol{r}_s^{\mathrm{G}}}\left(\boldsymbol{T}\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{c}\right) \\
&= \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}^{\mathrm{T}}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}^{\mathrm{T}}\left(\boldsymbol{T}\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{c}\right) \\
&= \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}^{\mathrm{T}}\boldsymbol{T}\boldsymbol{r}_s^{\mathrm{G}} + \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}^{\mathrm{T}}\boldsymbol{c} \\
&= \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}^{\mathrm{T}}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_s^{\mathrm{G}} + \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}^{\mathrm{T}}\boldsymbol{c} \\
&= \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}\boldsymbol{r}_s^{\mathrm{G}} + \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}^{\mathrm{T}}\boldsymbol{c} \\
&= \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}^{\mathrm{T}}\boldsymbol{T}_{\mathrm{I}\to\mathrm{N}}\boldsymbol{r}_{\mathrm{D}}^{\mathrm{I}} \\
&= \boldsymbol{T}_{\mathrm{G}\to\mathrm{I}}^{\mathrm{T}}\boldsymbol{W}^{\mathrm{T}}\boldsymbol{\omega}_\oplus^\times \boldsymbol{W}\boldsymbol{r}_{\mathrm{D}}^{\mathrm{I}}
\end{aligned} \tag{27}$$

Eq. (27) shows that $\boldsymbol{k}_2$ is a vector independent of satellite GCRS state parameters, determined only by Earth rotation rate vector, ITRS position vector of ground point, and celestial parameters. Physically, it is the projection of the ground-point convective velocity due to Earth rotation onto GCRS. At engineering meter-level precision, celestial parameter corrections to $\boldsymbol{k}_2$ direction can be neglected, and $\boldsymbol{k}_2$ direction is taken as the eastward rotational tangential direction of the ground point in GCRS, with magnitude $\left|\boldsymbol{\omega}_\oplus \times \boldsymbol{r}_{\mathrm{D}}^{\mathrm{I}}\right| \approx \omega_\oplus R_{\mathrm{e}} \cos B = 465 \cos B (\mathrm{m/s})$.

Using Eqs. (19), (21), and (27), derive position gradient expression

$$\begin{aligned}
\nabla_{\boldsymbol{r}_s^{\mathrm{G}}} f_d &= -\frac{2}{\lambda L^3}\left[L^2\left(\boldsymbol{v}_s^{\mathrm{G}} - \boldsymbol{k}_2\right) - N\boldsymbol{T}^{\mathrm{T}}\boldsymbol{r}_s^{\mathrm{N}}\right] \\
&= -\frac{2}{\lambda L}\left(\boldsymbol{v}_s^{\mathrm{G}} - \boldsymbol{k}_2\right) + \frac{2}{\lambda L^3} N\boldsymbol{T}^{\mathrm{T}}\left(\boldsymbol{T}\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{c}\right) \\
&= -\frac{2}{\lambda L}\left(\boldsymbol{v}_s^{\mathrm{G}} - \boldsymbol{k}_2\right) + \frac{2v_r}{\lambda L^2}\left(\boldsymbol{r}_s^{\mathrm{G}} - \boldsymbol{r}_{\mathrm{D}}^{\mathrm{G}}\right) \\
&= -\frac{2}{\lambda L}\left(\boldsymbol{v}_s^{\mathrm{G}} - \boldsymbol{k}_2\right) + \frac{2v_r}{\lambda L^2} L\boldsymbol{e}_r^{\mathrm{G}} \\
&= -\frac{2}{\lambda L}\left(\boldsymbol{v}_s^{\mathrm{G}} - \boldsymbol{k}_2 - v_r\boldsymbol{e}_r^{\mathrm{G}}\right) \\
&= \frac{2}{\lambda L}\left(\boldsymbol{k}_2 - v_\perp \boldsymbol{e}_\perp^{\mathrm{G}}\right)
\end{aligned} \tag{28}$$

where $\boldsymbol{e}_\perp^{\mathrm{G}}$ is the unit vector perpendicular to the satellite–ground line of sight in GCRS, $v_\perp$ is the component of satellite GCRS velocity perpendicular to the satellite–ground line of sight.

Eq. (28) shows that position gradient magnitude can be written as

$$\left|\nabla_{r_s^G} f_d\right| = \frac{2}{\lambda L}\left|\boldsymbol{k}_2 - v_\perp \boldsymbol{e}_\perp^{\mathrm{G}}\right| \tag{29}$$

According to the triangle inequality $\left|\boldsymbol{k}_2 - v_\perp \boldsymbol{e}_\perp^{\mathrm{G}}\right| \le \left|\boldsymbol{k}_2\right| + \left|v_\perp\right|$, substituting it into Eq. (29) yields the upper bound of the gradient magnitude

$$\left|\nabla_{r_s^G} f_d\right| \le \frac{2}{\lambda L}\left(\left|\boldsymbol{k}_2\right| + \left|v_\perp\right|\right) \tag{30}$$

For circular-orbit satellites, maximum perpendicular velocity approximates satellite velocity $v_s^{\mathrm{G}} = \sqrt{\mu / (H_s + R_\oplus)}$, and convective velocity amplitude is less than 465.1 m/s. Considering satellite–ground distance $L > H_s$ (satellite altitude), further obtain

$$\left|\nabla_{r_s^G} f_d\right| \le \frac{2}{\lambda H_s}\left(465.1 + \sqrt{\frac{\mu}{H_s + R_e}}\right) \tag{31}$$

From Eqs. (26) and (32), in practical engineering, orbit determination errors from satellite-borne Beidou receivers have negligible influence on clutter Doppler frequency.

Ⅶ. SIMULATION EXPERIMENTS

To verify the correctness of the derived influence law of orbit determination error on clutter Doppler frequency for SBR, the following verification strategy is adopted: first calculate the reference clutter Doppler frequency using unbiased GCRS position and velocity parameters by Eq. (7); after introducing orbit determination errors, compute Doppler frequency deviation by two methods and compare them — Method 1 recalculates Doppler frequency by Eq. (7) and takes the difference from the reference frequency; Method 2 directly calculates frequency deviation by Eq. (22). Consistency between the two methods validates the correctness of the theoretical analysis in this paper.

The simulation uses the WGS84 Earth ellipsoid model, radar operates in L-band at 1.5 GHz (wavelength λ=0.2 m). Satellite orbital elements are set in Table 3; ground clutter point is at 120°E longitude, 30°N latitude, 0 m altitude; orbit determination errors take typical engineering values: GCRS position error 10 m, velocity error 0.1 m/s, random error directions. Simulation start time is 2026-02-28 07:59:00 (UTC), time step 30 s, 27 sequential sampling points covering the full pass arc of the satellite over the target ground point.

TABLE Ⅲ

Satellite orbit elements.

Parameter Name	**Value**
Epoch time	2026-02-28 07:59:00 (UTC)
Semi-major axis	*a*=6378.137km (orbit height 500km)

Eccentricity	e=0
Orbit inclination	i=0°
Right ascension of ascending node	Ω=0°
Argument of perigee	ω=0°
True anomaly	f=0°

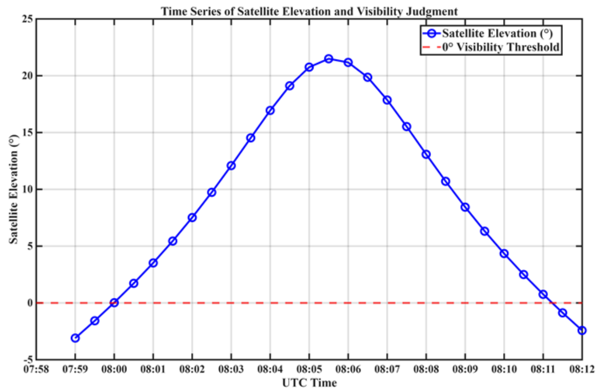

Fig. 3. Elevation angle of ground clutter point relative to the satellite (visible when elevation > 0°).

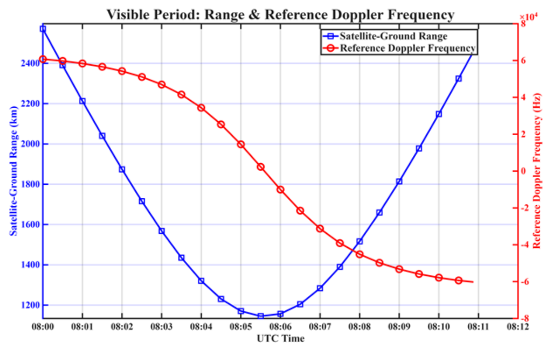

Fig.4. Satellite–ground distance and clutter Doppler frequency during visibility period.

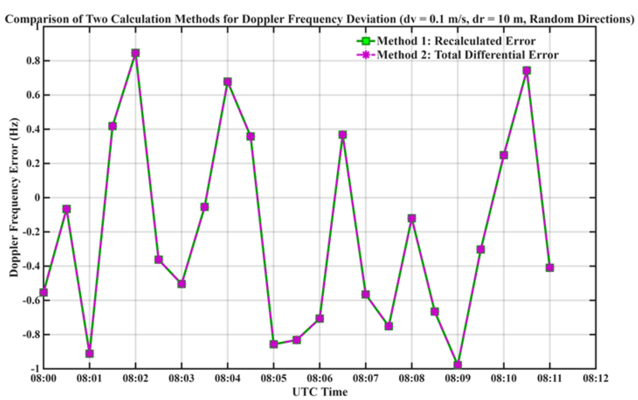

Fig 5. Comparison of two calculation methods for Doppler frequency deviation (dv=0.1m/s, dr=10m, random directions).

Fig. 3 shows the elevation angle curve of the ground clutter point relative to the satellite (visible when elevation >0°). Among 27 sampling points, 23 points (3rd to 25th) are in the visible phase between satellite and ground point. Figure 4 shows satellite–ground distance and reference clutter Doppler frequency during visibility: satellite–ground distance ranges 1145.2–2571.2 km, clutter Doppler frequency ranges −6051.2–6077.2 Hz. The clutter Doppler center frequency varies widely, posing challenges to moving target detection, but its variation pattern is clear and can support clutter suppression. Figure 5 compares Doppler frequency deviations calculated by the two methods; results are fully consistent. Under combined position and velocity errors, Doppler frequency deviation is less than 1 Hz, verifying the correctness of theoretical derivation.

Ⅷ. CONCLUSION

This paper derives the analytical expression of clutter Doppler frequency for SBR in terms of satellite GCRS state parameters through multi-stage GCRS→ITRS→NEZ coordinate transformations, and quantitatively reveals the influence law of orbit determination error on clutter Doppler frequency using multivariate function gradient analysis. The research clarifies the propagation characteristics of orbit determination error and provides theoretical support for Doppler frequency accuracy control and satellite-borne orbit determination system design of SBR. Future research can further explore the influence mechanism of orbital perturbations on clutter Doppler frequency in case of satellite-borne Beidou receiver failure.

REFERENCES

[1] DUAN, K.Q.; LI, X.; XING, K.; et al. Clutter mitigation in space-based early warning radar using a convolutional neural network. Journal of Radars, 2022, 11(3), 386–398. https://doi.org/10.12000/JR21161

[2] SHEN, W.; DUAN, K.Q.; XIE, H.; et al. Research on Grating Lobe Clutter Suppression Method for Distributed Space-Based Early Warning Radar. Journal of Signal Processing, 2025, 41, 1735–1749.

[3] QIU, Z.; DUAN, K.; YANG, X.; et al. Range-Ambiguous Clutter Suppression for Space-Based Early Warning Radar via EPC–FDA–MIMO With Nonorthogonal Waveforms. IEEE Transactions on Aerospace and Electronic Systems, 2024, 60(5), 7106-7124.

[4] Yan, X.; Chen, J.; Nies, H.; Loffeld, O. A Bistatic Analytical Approximation Model for Doppler Rate Estimation Error from Real-Time Spaceborne SAR Onboard Orbit Determination Data. Remote Sens. 2020, 12, 3156. https://doi.org/10.3390/rs12193156

[5] XU, K.X.; ZHOU, X.H.; PENG, H.L.; et al. HY2D Satellite Onboard BDS-3 Data Quality Evaluation and Post-precise Orbit Determination Accuracy Analysis. Acta Astronomica Sinica, 2024, 65(6), 100-109. https://doi.org/10.15940/j.cnki.0001-5245.2024.06.009

[6] KAN, Q.Y.; XU, J.W.; LIAO, G.S.; et al. Clutter modeling and analysis for air-space bistatic radar. Systems Engineering and Electronics, 2025, 47(6), 1806-1815.

[7] YAN, X.; CHEN, J.; NIES, H.; et al. A bistatic analytical approximation model for Doppler rate estimation error from real-time spaceborne SAR onboard orbit determination data. Remote Sensing, 2020, 12(19): 3156. https://doi.org/10.3390/rs12193156

[8] Huang, X.; Zhang, Y. Coordinate transformation method for beam grazing angle calculation of space-based early warning radar. Sci Rep (2026). https://doi.org/10.1038/s41598-026-42233-4

[9] Huang, X.B.; Xiao, R.; Zhang, Y. Fundamental Theory and Simulation Application of Orbital Space Objects; Huazhong University of Science and Technology Press: Wuhan, China, 2024.

[10]Huang, X.B.; Liu J.K.; Xiao, R. Principles and Applications of Space Object Surveillance; Huazhong University of Science and Technology Press: Wuhan, China, 2025.

[11]Vallado, D. A. Fundamentals of Astrodynamics and Applications, 5th ed.; McGraw-Hill Press: New York, USA, 2023.

[12]Huang, X.B.; Xiao, R.; Meng C.Z. Principles, Modeling and Application Practice of Space Target Orbit Simulation; Publishing House of Electronics Industry Press: Beijing, China, 2025.