

1 **A Vertical Equilibrium Model for CO₂ Migration in Depleted**
2 **Gas Fields**

3 Saeid Telvari^{1,*}, Hariharan Ramachandran¹, Gang Wang¹, Florian Doster¹

4 ¹Institute of GeoEnergy Engineering (IGE), School of Energy, Geoscience, Infrastructure
and Society, Heriot-Watt University, Edinburgh, EH14 4AS, UK

*Corresponding author: Saeid Telvari (st4014@hw.ac.uk)

5 **Preprint statement.**

6 This manuscript is a non-peer-reviewed preprint submitted to **EarthArXiv**. The
manuscript has also been submitted to *Water Resources Research* for peer review.
Subsequent versions of this preprint may differ in content following peer review; if
accepted, the final version will be available via a *DOI* link on this preprint page.
Please feel free to contact any of the authors with feedback.

7 **ORCID:** <https://orcid.org/0000-0002-4896-295X>

8 **Keywords:** Vertical equilibrium modeling; CO₂ storage; Depleted gas reservoirs; Reser-
voir simulation.

A Vertical Equilibrium Model for CO₂ Migration in Depleted Gas Fields

Saeid Telvari^{1,*}, Hariharan Ramachandran¹, Gang Wang¹, and Florian Doster¹

¹Institute of GeoEnergy Engineering (IGE), School of Energy, Geoscience, Infrastructure and Society, Heriot-Watt University, Edinburgh, EH14 4AS, UK

*Corresponding author: st4014@hw.ac.uk

Abstract

This study presents an extension of the vertical equilibrium (VE) modeling framework to simulate multi-phase flow involving CO₂, methane, and brine in depleted gas reservoirs. The primary motivation is to enable efficient and accurate simulation of CO₂ injection and migration in situations where methane is present, which introduces additional complexity not captured in traditional VE models. The proposed model integrates a black-oil approximation with VE assumptions, reducing the dimensionality of the problem and allowing for rapid simulation of large-scale CO₂ injection in depleted gas reservoirs. By representing CO₂ and methane as separate compressible phases, and treating brine as an incompressible phase, the model achieves a balance between computational speed and physical realism. Pressure-dependent density and viscosity relationships for CO₂ and methane enable accurate thermophysical property estimation without computationally intensive flash calculations. Three case studies validate the model: (1) a flat, layered reservoir benchmark demonstrating accurate CO₂ plume behavior under buoyancy control; (2) a dual-anticline scenario showing that the VE model reproduces the large-scale plume geometry and inter-anticline mass redistribution observed in compositional simulations; and (3) a field-scale application to the Hugin Formation East, where the VE model replicates key features of CO₂ and methane migration observed in compositional models. In all cases, the VE model runs orders of magnitude faster than full 3D simulations while capturing large-scale displacement dynamics effectively. Although the present formulation neglects some physics such as explicit mixing, dissolution, and capillary fringe effects, it still captures the first order displacement behaviour while offering up to 100× speedups. This makes the model a practical tool for rapid screening, uncertainty quantification, and optimisation workflows in CCS projects targeting depleted gas reservoirs.

This study extends the vertical equilibrium (VE) modeling framework to simulate multi-phase flow involving CO₂, methane, and brine in depleted gas reservoirs. Methane presence introduces complexity not captured in traditional VE models. The proposed model integrates a black-oil approximation with VE assumptions, reducing dimensionality and enabling rapid simulation of large-scale CO₂ injection. By representing CO₂ and methane as separate compressible phases, and treating brine as incompressible, the model balances computational speed and physical realism. Pressure-dependent density and viscosity relationships for CO₂ and methane enable accurate property estimation without flash calculations. Three case studies validate the model: (1) a flat, layered reservoir benchmark demonstrating accurate CO₂ plume behavior under buoyancy control; (2) a dual-anticline scenario demonstrating that the VE model reproduces the large-scale plume geometry and inter-anticline mass redistribution observed in compositional simulations; and (3) a field-scale application to the Hugin Formation East, where the VE model replicates key features of CO₂ and methane migration observed in compositional models. The VE model runs orders of magnitude faster than full 3D simulations while capturing large-scale displacement dynamics. Although the formulation neglects explicit mixing, dissolution, and capillary fringe effects, it captures first-order displacement behaviour with speedups up to 100×. This makes it a practical

56 tool for screening, uncertainty quantification, and optimisation in CCS projects targeting
57 depleted gas reservoirs.

58 Plain Language Summary

59 Storing carbon dioxide (CO₂) underground in depleted gas reservoirs is a promising way to
60 reduce greenhouse gas emissions. However, predicting how injected CO₂ moves in these reservoirs
61 is difficult because the pore space may still contain natural gas and water, and detailed computer
62 models can be very slow. In this study, we developed a faster modeling approach to simulate
63 how CO₂, methane, and brine move together after CO₂ injection. The method is based on
64 the idea that fluids can quickly arrange themselves vertically under gravity, which allows the
65 model to simplify the problem while still capturing the main large-scale behavior. We tested the
66 method in three cases, from simple idealized settings to a field-scale reservoir model. The results
67 show that the new model reproduces the main migration and trapping patterns seen in much
68 more detailed three-dimensional simulations, while running up to 100 times faster. Although it
69 does not yet include some finer-scale processes such as mixing and dissolution, it captures the
70 dominant flow behavior well. This makes it useful for rapid screening, uncertainty analysis, and
71 planning of carbon storage projects in depleted gas reservoirs.

72 1 Introduction

73 Carbon capture, utilization, and storage (CCUS) is a key mitigation strategy for reducing green-
74 house gas emissions and supporting global carbon neutrality efforts, particularly for industries
75 that are challenging to decarbonize, including steel, cement, and chemical manufacturing, as
76 well as potential large-scale production of bio-hydrogen [Méndez et al., 2023, Chen et al., 2022,
77 Rosa and Mazzotti, 2022, Wei et al., 2023]. CCUS entails capturing carbon dioxide (CO₂)
78 from industrial emissions or directly from the atmosphere, followed by its transportation for
79 either reuse or permanent underground storage [Krevor et al., 2023, Wareing et al., 2014, Chen
80 et al., 2022, Lau et al., 2021]. One of the most promising methods for permanent CO₂ storage
81 involves injecting it into geological formations such as deep saline aquifers or depleted oil and
82 gas reservoirs [Hepple and Benson, 2004, Kumar et al., 2005, Ringrose et al., 2021, Hamza
83 et al., 2021]. Utilizing depleted oil and gas fields generally offers secure long-term storage, as
84 these formations have retained hydrocarbons beneath impermeable caprocks for millions of
85 years. In some cases, it may also facilitate enhanced hydrocarbon recovery, which can help
86 offset some of the costs related to CO₂ sequestration. Compared to saline aquifers, which
87 can experience pressure build-up and integrity challenges during CO₂ injection, hydrocarbon
88 reservoirs provide a well-documented storage environment with lower pressures and extensive
89 geological data availability. Another significant advantage of depleted reservoirs is the presence
90 of existing infrastructure for CO₂ transport and injection [Hamza et al., 2021, Jenkins et al.,
91 2012, Muhammed et al., 2023, Patel et al., 2016].

92 A key difference between depleted reservoirs and saline aquifers is the presence of natural
93 gas in depleted gas reservoirs. This adds complexity to the process as contact between CO₂
94 and the gas causes fluid mixing, leading to the creation of a mixing zone that alters key fluid
95 properties, including density and viscosity, which in turn modifies displacement mechanisms
96 and can induce convective currents that further influence the migration and redistribution of
97 CO₂ and reservoir gas. Therefore, an accurate representation and study of their behavior and
98 interactions are required [Ma et al., 2019, Ghanbari et al., 2020, Liu et al., 2022]. Reservoir
99 simulation serves as a crucial tool for analyzing and studying CO₂ geological storage. Both
100 commercial and open-source simulators are available to represent various scenarios [Class et al.,
101 2009]. To be able to properly define and differentiate CO₂ and natural gas and capture the
102 mixing phenomena, we need to define a multi-component gas phase. Compositional simulations

103 use Equations of State (EoS) to model multi-component behavior in one or more phases. In
104 this context, CO₂ can be accurately differentiated from hydrocarbons, allowing for accurate
105 modeling of their interactions in the reservoir [Oldenburg and Org, 2003, Al-Hashami et al., 2005,
106 Ghanbari et al., 2020, Liu et al., 2022]. Compositional simulations are computationally intensive
107 because they must perform flash calculations in every gridblock, which typically require more
108 iterations for convergence than models that rely on simplified phase behavior representations.
109 Previous research has explored the application of black oil models to simulate thermodynamic
110 phase behavior in compositional simulations in order to speed up the simulations and maintain
111 accuracy as much as possible. This approach sacrifices some accuracy in favor of cost efficiency,
112 making it particularly useful for uncertainty analysis [Hassanzadeh et al., 2008, Amooie and
113 Moortgat, 2018]. Nonetheless, CO₂ storage simulations still remain computationally intensive
114 due to the large spatial scales and extended timeframes spanning several centuries.

115 In many cases, the horizontal extent of the reservoir is much larger than the vertical. When
116 there is a significant density difference between fluids and sufficient vertical permeability, gravity
117 causes them to segregate, with heavier fluids settling at the bottom and lighter ones rising to the
118 top. This leads to a state of vertical equilibrium (VE), where fluid saturations vary predictably
119 with depth. The vertical equilibrium assumption allows us to integrate the governing equations
120 over the vertical axis and express them in terms of vertically averaged parameters. As a result,
121 we can simplify a 3D model into a 2D one, which greatly reduces computational cost. This
122 approach, referred to as VE modeling, is increasingly used for simulating large-scale CO₂ plume
123 migration in the subsurface [Nordbotten and Celia, 2011, Bandilla et al., 2019]. Traditionally,
124 VE models applied to CO₂ storage problems have focused on two-phase systems, where the
125 reservoir is initially saturated with brine and CO₂ is injected as a single immiscible gas phase.
126 The most common simplification in early VE models involved complete segregation of fluids with
127 a distinctive interface [Gasda et al., 2009, Nilsen et al., 2016b, Court et al., 2012]. These models
128 have since been extended to include effects such as hysteresis [Doster et al., 2012, 2013, Gasda
129 et al., 2013b], heterogeneity [Guo et al., 2016, Moyner and Nilsen, 2019], dissolution [Gasda et al.,
130 2011, Mykkeltvedt and Nordbotten, 2012], capillary fringe [Nilsen et al., 2016a, Nordbotten and
131 Dahle, 2011], sub-scale topographic variations (rugosity) [Gasda et al., 2012, 2013a, Ahmadiania
132 et al., 2019], fault leakage [Ramachandran et al., 2024], geomechanics [Bjørnarå et al., 2016,
133 Andersen et al., 2017], compressibility [Andersen et al., 2015], and fractured (dual-continuum)
134 reservoirs [March Castaneda Neto, 2018, Guo et al., 2017, Tao et al., 2019]. Additionally,
135 several studies have aimed to relax the assumption of vertical equilibrium for wider applications,
136 including near-wellbore flow, well leakage, and large-scale plume migration in heterogeneous
137 formations [Guo et al., 2014, 2016, Becker et al., 2017, Zheng et al., 2021]. To address the
138 limitations of VE models, particularly near injection wells and heterogeneous reservoir regions,
139 other researchers have explored hybrid approaches that couple between VE and 3D models
140 [Moyner and Nilsen, 2019, Becker et al., 2022, Buntic et al., 2025].

141 The computational efficiency of VE models makes them particularly valuable for screening
142 studies and statistical analysis workflows that require the simulation of large ensembles. In carbon
143 storage assessment, uncertainty quantification demands evaluation across numerous geological
144 realizations, injection scenarios, and operational parameters—tasks that become computationally
145 impractical with full-physics compositional models. de Jonge Anderson et al. [2024] used
146 reduced-complexity VE models to perform well location and storage capacity assessments in
147 heterogeneous saline aquifers, running thousands of simulations within practical timeframes.
148 Similarly, VE frameworks facilitate the rapid screening of multiple storage sites, robust sensitivity
149 and uncertainty analysis of key parameters affecting plume migration, and probabilistic risk
150 assessment of leakage scenarios [Allen et al., 2018, Tadjer et al., 2021, Pettersson et al., 2022,
151 Anyosa et al., 2024]. For depleted gas reservoirs, where additional uncertainties arise from gas
152 saturation distributions, pressure depletion levels, and CO₂-methane displacement dynamics,
153 the ability to perform comprehensive statistical analysis becomes even more critical.

154 Building upon the foundational VE modeling work for two-phase systems, two important
155 early contributions extended VE concepts to three-phase flow. The first, by Virnovsky et al.
156 [1996], developed a vertically integrated model for gas-oil-water displacement fronts during water-
157 alternating-gas (WAG) operations. Assuming vertical equilibrium and incompressible, immiscible
158 fluids, the authors derived pseudo relative permeability and capillary pressure functions to reduce
159 2D gravity-segregated flow into a 1D formulation. This allowed for analytical exploration of
160 front stability through traveling-wave solutions and was among the earliest works to address
161 three-phase VE dynamics and their implications for vertical sweep efficiency. The second, by
162 Guerrero et al. [2013] and Donat et al. [2014], proposed a one-dimensional three-phase VE model
163 incorporating both buoyancy-driven hyperbolic terms and capillarity-driven parabolic terms.
164 They solved the governing equations using high-resolution Weighted Essentially Non-Oscillatory
165 (WENO) schemes coupled with Implicit–Explicit (IMEX) time-stepping strategies. Their work
166 highlighted the numerical challenges of simulating saturation fronts under VE and provided a
167 robust computational approach. Both studies assumed incompressible fluids and made significant
168 strides in capturing multi-phase dynamics within VE frameworks.

169 In this study, we build upon existing two-phase vertically integrated models by extending
170 them to a three-phase framework. In this approach, CO₂ and methane are treated as two
171 distinct compressible phases, while water is represented as the third, incompressible phase. Our
172 formulation simplifies the problem in two stages: first by reducing a compositional model to a
173 black-oil approximation, and then by collapsing the full 3D representation into a 2D vertically
174 integrated model. This dual simplification enables simulations that are orders of magnitude
175 faster while maintaining reasonable accuracy.

176 2 Three Phase Vertical Equilibrium Modeling

177 2.1 Overview and Assumptions

178 This study examines CO₂ injection into a depleted gas reservoir system that is vertically confined
179 at both upper and lower boundaries. The reservoir structure consists of one or more gas caps with
180 the remaining volume occupied by brine. Due to depletion, the reservoir pressure is anticipated
181 to be below its initial state. The density of CO₂ is substantially lower than brine but higher
182 than methane across all relevant pressure and temperature conditions. CO₂ exhibits non-ideal
183 behavior, with density and viscosity demonstrating significant variations with pressure and
184 temperature changes. The higher viscosity of CO₂ compared to methane effectively suppresses
185 viscous fingering phenomena and promotes the formation of a stable displacement front. The
186 interaction between CO₂ and methane creates a mixing zone at the front whose characteristics
187 are determined by the competing effects of diffusion and dispersion. At elevated pressures, the
188 density contrast between CO₂ and methane increases (see Figure 8), leading to a narrower
189 mixing zone, particularly under stable stratification conditions. Stable stratification develops
190 when the denser CO₂ layer is positioned beneath the lighter methane layer, preventing convective
191 mixing processes. In contrast, when the denser CO₂ layer overlies lighter methane, gravitational
192 instability develops, substantially accelerating mixing dynamics through convective mixing. In
193 configurations where methane is initially positioned above CO₂, mixing is governed by diffusion.
194 The higher viscosity of CO₂ relative to methane creates a mobility ratio that prevents the
195 development of viscous fingering instabilities [Oldenburg et al., 2001]. Numerical simulations
196 indicate that after CO₂ injection stops, the stored CO₂ continues to sink beneath the overlying
197 methane under density and pressure gradients, forming stratified layers while the mixing zone
198 gradually expands through molecular diffusion [Oldenburg et al., 2001]. Methane is far less dense
199 and viscous than water, so it separates quickly. By contrast, O₂ is less viscous than water but,
200 at higher reservoir pressures, can have a density approaching that of water. It is still typically
201 slightly buoyant and will disengage from the aqueous phase, though with a smaller density

202 contrast than methane [Ghanbari et al., 2020].

203 The conventional black oil modeling framework lacks the capability to define multi-component
204 gas phases. Compositional simulation approaches incorporate equations of state to predict
205 fluid properties and phase interactions under varying pressure and temperature conditions
206 by accounting for the characteristics of individual components. For computational simplicity,
207 this study assumes that the gas plume within the reservoir consists exclusively of methane.
208 The initial compositional model to simulate CO₂ injection into the depleted gas reservoir is
209 developed using the MATLAB Reservoir Simulation Toolbox (MRST) compositional module and
210 subsequently transferred to Jutul Darcy, a high-performance simulator that offers significantly
211 faster computational performance [Lie, 2019, Moyner, 2024]. The model initialization incorporates
212 two-phase saturations comprising an aqueous phase (water) and a gaseous phase (methane),
213 with methane occupying the upper reservoir zones and hydrostatic pressure distribution specified
214 throughout the domain. The system is characterized as a three-phase model encompassing
215 water, oil, and gas phases, where the oil and gas phases represent liquid and vapor phases
216 calculated through equation of state modeling. However, given that both methane and CO₂
217 possess extremely low critical temperatures of -82.59°C and 31.8°C respectively, both components
218 are represented as gas phase constituents in typical reservoir conditions. Consequently, no oil
219 (liquid) phase exists during the simulations, effectively reducing the system to a two-phase
220 water-gas system. To isolate the fundamental CO₂-methane-water displacement mechanisms,
221 water is treated as incompressible with no dissolution effects, and capillary pressure is neglected.
222 These simplifications enable focus on capturing the essential reservoir processes while serving as
223 an initial framework for subsequent detailed modeling efforts. Molecular diffusion and dispersion
224 processes are excluded from this analysis. Although these physical mixing mechanisms are
225 not explicitly incorporated in the model, a mixing zone between fluids can still develop during
226 simulation due to numerical dispersion, which is inherent to the discretization methods employed
227 for solving the governing flow equations. The magnitude of this numerical dispersion depends on
228 factors such as grid resolution, meaning that the observed mixing in simulation results reflects
229 the chosen numerical scheme and may not directly correspond to actual physical mixing processes
230 [Haajizadeh et al., 1999, Shrivastava, 2003, Ghanbari et al., 2018, 2020].

231 The VE model aims to simulate three-dimensional aquifer systems with substantial horizontal
232 extent using two-dimensional representations through the solution of integrated 2D governing
233 equations. This upscaled model formulation relies on the fundamental assumption of VE,
234 requiring the absence of flow perpendicular to the aquifer plane. This assumption must remain
235 valid across the relevant spatial and temporal scales of the system. While VE models can
236 accommodate various physical effects including hysteresis, solubility, and capillary phenomena,
237 the primary objective of this study is to develop a formulation that incorporates CO₂ and
238 methane in a manner consistent with 3D compositional simulations and to evaluate the resulting
239 model performance. Several simplifications are therefore adopted: rock and brine phases are
240 treated as incompressible, no flow occurs through top and bottom boundaries, dissolution
241 and capillary fringe effects are neglected, residual saturations are assumed to be zero, and
242 isothermal conditions are maintained throughout the reservoir. Additionally, mixing phenomena
243 are excluded from the vertical equilibrium model. While this approach ignores the miscible
244 behavior of CO₂ and methane, it preserves the computational benefits of VE modeling while
245 extending its capability to capture key multiphase flow phenomena. We are hence representing
246 the water-gas phase system with water, CO₂ and methane components as a three phase system
247 where each phase represents a component. We show with the help of representative scenarios
248 that due to strong gravity segregation of CO₂ and methane this mixing is a second order effect
249 while first order displacement dynamics, such as the CO₂ plume extent and gas–water contact
250 shape, are captured well with our approach.

251 The ad-core module within MRST provides an object-oriented framework for rapid prototyp-
252 ing of reservoir simulators based on automatic differentiation. This framework facilitates the

253 modification and extension of existing simulation models in MRST or the implementation of new
 254 ones. This framework is used to apply the three-phase VE formulation discussed in the following
 255 section. The development focuses on an h-formulation system of equations where the primary
 256 variables are plume heights rather than saturations, offering a more intuitive representation for
 257 VE models.

258 2.2 Governing Equations

259 This section presents the governing equations for a three-phase vertical equilibrium (VE) model
 260 designed for multiphase flow in porous media, with particular application to CO₂ storage in
 261 depleted gas reservoirs. The model considers a vertically stratified system containing methane,
 262 CO₂, and water, arranged according to their densities under the assumption of rapid vertical
 263 pressure equilibration. The emphasis is placed on the fundamental assumptions and simpli-
 264 fications inherent in the VE framework, rather than addressing the complexity of reservoir
 265 heterogeneity or boundary conditions. Throughout this work, lower-case symbols denote fine-
 266 scale, three-dimensional quantities, while the corresponding upper-case symbols (introduced
 267 later) represent vertically integrated, VE-scale quantities.

268 The system under consideration involves three immiscible phases flowing through a rigid
 269 porous medium under isothermal conditions. The fluid phases are denoted as brine (b), carbon
 270 dioxide (c), and methane (m), ordered by density such that $\rho_b > \rho_c > \rho_m$. Each phase occupies
 271 a portion of the pore space, and their respective saturations satisfy the constraint.

$$s_b + s_c + s_m = 1 \quad (1)$$

272 The mass conservation principle for each fluid phase $\alpha \in b, c, m$, representing brine, CO₂,
 273 and methane respectively, is expressed through the three-dimensional mass balance equation:

$$\frac{\partial}{\partial t} (\rho_\alpha \phi s_\alpha) + \nabla \cdot \rho_\alpha \mathbf{u}_\alpha = \psi_\alpha \quad (2)$$

274 In this formulation, ϕ represents the porosity, s_α denotes the saturation of phase α , \mathbf{u}_α is the
 275 corresponding Darcy velocity, and ψ_α accounts for sources or sinks. The velocity field \mathbf{u}_α follows
 276 Darcy's law:

$$\mathbf{u}_\alpha = -\frac{k k_{r\alpha}}{\mu_\alpha} (\nabla p_\alpha - \rho_\alpha \mathbf{g}) \quad (3)$$

277 Here, k denotes the absolute permeability, $k_{r\alpha}$ is the relative permeability to phase α , μ_α is
 278 the viscosity of phase α , p_α is the pressure of phase α , and \mathbf{g} is the gravitational acceleration
 279 vector. In order to derive a vertically integrated flow model, we integrate the mass balance
 280 equation (2) over the vertical coordinate, z , from the bottom boundary ζ_B to the top boundary
 281 ζ_T :

$$\int_{\zeta_B}^{\zeta_T} \left[\frac{\partial}{\partial t} (\rho_\alpha \phi s_\alpha) + \nabla \cdot (\rho_\alpha \mathbf{u}_\alpha) \right] dz = \int_{\zeta_B}^{\zeta_T} \psi_\alpha dz \quad (4)$$

282 The objective is to express the resulting upscaled equation based on variables that depend
 283 solely on the horizontal spatial coordinates (x, y) and time t . Assuming there is no vertical flow
 284 through the top and bottom boundaries (i.e., $\mathbf{u}_{\alpha,z} = 0$ at $z = \zeta_B$ and $z = \zeta_T$), and that the
 285 boundaries are stationary, the Leibniz rule for differentiation under the integral sign can be
 286 applied. This procedure yields the vertically integrated equation:

$$\frac{\partial}{\partial t} \left(\int_{\zeta_B}^{\zeta_T} \rho_\alpha \phi s_\alpha dz \right) + \nabla_{\parallel} \cdot \left(\int_{\zeta_B}^{\zeta_T} \rho_\alpha \mathbf{u}_{\alpha,\parallel} dz \right) = \int_{\zeta_B}^{\zeta_T} \psi_\alpha dz \quad (5)$$

287 where ∇_{\parallel} and $\mathbf{u}_{\alpha,\parallel}$ refer to the horizontal components of the gradient and velocity, respectively.
 288 To proceed with the formulation, it is necessary to introduce several quantities that have been

289 integrated over the vertical coordinate. These vertically integrated variables are defined as
 290 follows:

$$\Phi := \int_{\zeta_B}^{\zeta_T} \phi dz \quad (6)$$

$$S_\alpha := \frac{1}{\Phi} \int_{\zeta_B}^{\zeta_T} \phi s_\alpha dz \quad (7)$$

$$R_\alpha := \frac{1}{\Phi S_\alpha} \int_{\zeta_B}^{\zeta_T} \rho_\alpha \phi s_\alpha dz \quad (8)$$

$$\Psi_\alpha := \int_{\zeta_B}^{\zeta_T} \psi_\alpha dz \quad (9)$$

$$\mathbf{U}_\alpha := \int_{\zeta_B}^{\zeta_T} \mathbf{u}_{\alpha,\parallel} dz \quad (10)$$

$$\mathbf{F}_\alpha := \int_{\zeta_B}^{\zeta_T} \rho_\alpha \mathbf{u}_{\alpha,\parallel} dz \quad (11)$$

291 Here, Φ denotes the vertically integrated porosity, S_α is the upscaled saturation, and R_α
 292 refers to the upscaled density of phase α . The term \mathbf{U}_α corresponds to the total horizontal
 293 volumetric flux, while \mathbf{F}_α indicates the total horizontal mass flux of phase α across the vertical
 294 column. The term Ψ_α represents the integrated source or sink for phase α over the vertical
 295 domain. Substituting these expressions into equation (5) yields the vertically integrated mass
 296 balance equation:

$$\frac{\partial}{\partial t} (R_\alpha \Phi S_\alpha) + \nabla_{\parallel} \cdot \mathbf{F}_\alpha = \Psi_\alpha \quad (12)$$

297 This vertically integrated formulation accounts for variable density effects, enabling accurate
 298 modeling of density variations throughout the system [Andersen et al., 2015]. To evaluate the
 299 integral of the phase's lateral mass flux, we incorporate equation (3) into equation (11), yielding:

$$\mathbf{F}_\alpha = - \int_{\zeta_B}^{\zeta_T} \rho_\alpha \lambda_\alpha k_{\parallel} \nabla_{\parallel} p_\alpha dz \quad (13)$$

300 The evaluation of this integral requires the reconstruction of fine-scale quantities based on
 301 coarse scale (upscaled) variables, which can vary depending on the specific modeling approach
 302 and physical context. Certain variables like porosity ϕ and permeability k are predetermined
 303 inputs while others are reconstructed from upscaled quantities under specific assumptions. For
 304 the pressure gradient $\nabla_{\parallel} p_\alpha$ and the density ρ_α , these quantities are expressed relative to reference
 305 values, with the fundamental reference quantity being the reference pressure, termed upscaled
 306 pressure P_α in this formulation. This upscaled pressure represents a two-dimensional field
 307 that varies spatially across the horizontal domain (x_1, x_2) and serves as the primary pressure
 308 variable in the VE formulation. Under the VE assumption, the pressure of each phase p_α varies
 309 hydrostatically in the vertical direction according to:

$$\frac{\partial p_\alpha}{\partial z} = -\rho_\alpha g \quad (14)$$

310 The total hydrostatic pressure change between two vertical depths a and b is determined by
 311 integrating this expression:

$$\Delta p_\alpha = - \int_a^b \rho_\alpha g dz \quad (15)$$

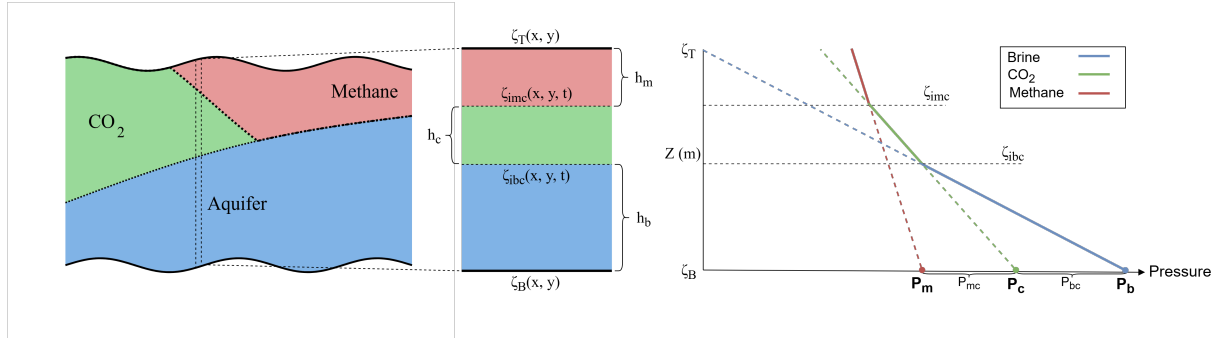


Figure 1: A schematic of phase interfaces and plume heights

312 This relationship allows the vertical pressure profile to be reconstructed from an upscaled
 313 phase pressure, typically taken at either the bottom or top of the column. Denoting the phase-
 314 specific pressure at the bottom as $P_\alpha(x_1, x_2, t)$, the vertical pressure distribution for each phase
 315 under equilibrium conditions is written as:

$$p_\alpha(x_1, x_2, z, t) = P_\alpha(x_1, x_2, t) - g \int_{\zeta_B}^z \rho_\alpha dz' \quad (16)$$

316 To proceed with the formulation, we must define the phase densities. In our case, brine is
 317 assumed to be incompressible, while the densities of CO₂ and methane vary with pressure under
 318 the assumption of constant temperature. We capture this relationship by fitting an equation
 319 to equation-of-state (EoS) data, as described in A. A key challenge arises because we are only
 320 solving for the VE reference (bottom) pressure rather than for the entire column, and we need
 321 to provide the density values across the interval. Consequently, we must incorporate a density
 322 profile that depends on the hydrostatic pressure distribution—which itself depends on the density
 323 at other points. This interdependence introduces a nonlocal effect. The most direct approach
 324 would involve numerically reconstructing the full vertical density profile at every timestep and
 325 integrating across the column, but this method is computationally expensive.

To avoid this computational cost, a Taylor series expansion is employed that allows representation of a function locally in terms of its value and derivatives at a reference point. In this context, the expansion is applied to the density functions of the compressible phases to approximate their density profile over their respective phase fluid columns. By integrating this approximated density function over each fluid column, an “upscaled” density equation is obtained that corresponds to the total mass of that phase in the column. We limit the Taylor series to second-order terms and expand around a reference pressure (i.e., the upscaled pressure), which leads to the following density equation:

$$\rho_\alpha(p) \approx \rho_\alpha(p_{ref}) - \frac{\rho'_\alpha(p_{ref})}{1!} \rho_\alpha(p_{ref}) g [z - \zeta_{ref}] + \frac{\rho''_\alpha(p_{ref})}{2!} (\rho_\alpha(p_{ref}) g [z - \zeta_{ref}])^2 \quad (17)$$

326 This expression is valid for any reference point situated within the reservoir’s height. In this
 327 study, we chose the bottom of the reservoir as the reference point, meaning the phase pressures
 328 at this depth are taken as the reference pressure. A similar Taylor series approach is adopted for
 329 representing viscosity. For flux calculations, the formulation is simplified by retaining only the
 330 zeroth-order terms of both density and viscosity, ignoring higher-order terms and evaluating
 331 them at a single reference pressure:

$$\rho_\alpha(p) \approx \rho_\alpha(p_{ref}), \quad \mu_\alpha(p) \approx \mu_\alpha(p_{ref}) \quad (18)$$

332 This approximation implies that density and viscosity variations along the vertical coordinate
 333 are negligible, enabling these terms to be factored out of vertical integrals.

334 To efficiently address the system of equations, standard practice involves selecting and solving
 335 explicitly for a single-phase pressure, with the pressures of other phases expressed relative to
 336 this reference through pressure differences. The vertical domain is divided into distinct regions
 337 by phase interfaces located at $\zeta_{i\alpha\beta}$, where adjacent phases α and β meet. At these interfaces,
 338 the pressure difference between two adjacent phases is defined by subtracting the hydrostatic
 339 contribution from each other:

$$p_{\alpha\beta} = P_{\alpha\beta} - g \int_{\zeta_B}^{\zeta_{i\alpha\beta}} (\rho_\alpha - \rho_\beta) dz \quad (19)$$

340 When fine-scale effects like capillary pressure are neglected, the pressure difference between
 341 each pair of phases at the interfaces becomes zero. This simplification reduces the equations to:

$$P_{\alpha\beta} = g \int_{\zeta_B}^{\zeta_{i\alpha\beta}} (\rho_\alpha - \rho_\beta) dz \quad (20)$$

342 Incorporating the density approximation eq (18) into the upscaled pressure difference equa-
 343 tions yields:

$$P_{\alpha\beta} = (\rho_\alpha(P_\alpha) - \rho_\beta(P_\beta)) g (\zeta_{i\alpha\beta} - \zeta_B) \quad (21)$$

344 Here, P_β refers to the phase β pressure at the bottom, and the subscripts α and β represent
 345 any two adjacent phases in the multi-phase system. However, this choice introduces a circular
 346 dependency: the upscaled pressure difference needed to calculate the phase β pressure from
 347 the phase α pressure depends on the phase β density, while the phase β density itself requires
 348 knowledge of the phase β pressure. Solving this implicit coupling directly would demand iterative
 349 reconstruction of the phase β pressure at every timestep, which is computationally expensive.
 350 To avoid this computational complexity, we expand $\rho_\beta(P_\beta)$:

$$\begin{aligned} \rho_\beta(P_\beta) &= \rho_\beta(P_\alpha + P_{\alpha\beta}) \\ &\approx \rho_\beta(P_\alpha) + \left. \frac{\partial \rho_\beta}{\partial P} \right|_{P_\alpha} \cdot P_{\alpha\beta} \end{aligned} \quad (22)$$

351 In our formulation, CO₂ pressure is the primary unknown variable we solve for (phase α),
 352 with water and methane as phase β . Since water is incompressible and methane is weakly
 353 compressible, the density derivative $\left. \frac{\partial \rho_\beta}{\partial P} \right|_{P_\alpha}$ becomes zero or negligible. Additionally, pressure
 354 differences are small relative to absolute pressures in the system. These conditions allow us to
 355 evaluate phase β density at the phase α pressure (P_α) rather than the phase β pressure (P_β),
 356 which eliminates the circular dependency and simplifies the pressure difference equations to
 357 depend only on phase α pressure. Additionally, the interface depths are subsequently expressed
 358 in terms of phase thicknesses as follows:

$$\begin{aligned} \text{brine} : \quad h_b &\equiv \zeta_{ibc} - \zeta_B \\ \text{carbon dioxide} : \quad h_c &\equiv \zeta_{imc} - \zeta_{ibc} \equiv H - h_m - h_b \\ \text{methane} : \quad h_m &\equiv \zeta_T - \zeta_{imc} \end{aligned} \quad (23)$$

359 Where $H = \zeta_T - \zeta_B$. Here, the CO₂ phase height is expressed as a function of the brine and
 360 methane heights, thereby reducing the number of independent unknowns, in a manner similar to
 361 three-phase black-oil models where saturations are the primary variables. These changes yield
 362 phase-specific equations for CO₂-brine and CO₂-methane as follows:

$$P_{bc} = (\rho_b - \rho_c(P_c)) g h_b \quad (24)$$

$$P_{mc} = (\rho_c(P_c) - \rho_m(P_c)) g (H - h_g) \quad (25)$$

363 Based on the new density and phase height expressions, we reformulate the pressure equations
 364 and compute the lateral pressure gradients for each phase. Assuming an incompressible brine
 365 and CO₂ and methane densities independent of the vertical coordinate, along with negligible
 366 lateral variations in density and gravitational effects, we obtain the following phase-specific
 367 expressions for $\nabla_{\parallel} p_{\alpha}$:
 368

$$\nabla_{\parallel} p_{\alpha} = \nabla P_{\alpha} + \rho_{\alpha}(P_{\alpha}) g \nabla \zeta_B \quad (26)$$

369 Note that the final pressure gradients are not a function of z and are taken out of the integral
 370 in equation (13). We now define other vertically integrated variables as follows:

$$K := \int_{\zeta_B}^{\zeta_T} k_{\parallel} dz \quad (27)$$

$$\Lambda_{\alpha} := \frac{1}{K} \int_{\zeta_B}^{\zeta_T} \lambda_{\alpha} k_{\parallel} dz \quad (28)$$

$$\tilde{R}_{\alpha} := \frac{1}{K \Lambda_{\alpha}} \int_{\zeta_B}^{\zeta_T} \rho_{\alpha} \lambda_{\alpha} k_{\parallel} dz \quad (29)$$

371 Where K and Λ_{α} are vertically integrated permeability and mobility terms, respectively. The
 372 variable \tilde{R}_{α} is introduced to account for density effects in the integrated flux term. Specifically,
 373 \tilde{R}_{α} is defined as the upscaled phase density weighted by mobility and permeability. When
 374 assuming either incompressible fluids or compressible fluids with no dependence on the vertical
 375 coordinate (i.e., equation (18)), the density is taken outside the integral, resulting in:

$$\tilde{R}_{\alpha} = \begin{cases} \rho_{\alpha}, & \alpha = b, \\ \rho_{\alpha}(P_{ref}), & \alpha \in \{c, m\}. \end{cases} \quad (30)$$

376 Having established the vertically integrated density \tilde{R}_{α} , we now turn to the pressure gradient
 377 term in Darcy's equation (3). In the special case of a fully homogeneous reservoir ($k_{\parallel} = k = \text{const}$),
 378 a sharp-interface model with no residual saturations (λ_{α} takes only saturation values of 0 or 1),
 379 the constant permeability moves out of the integrals. This permits analytical integration and
 380 allows Λ_{α} to be explicitly defined for each phase:

$$\Lambda_{\alpha} = \frac{k}{kH} \int_{\zeta_B}^{\zeta_T} \lambda_{\alpha} dz = \lambda_{\alpha}(1) \frac{h_{\alpha}}{H}. \quad (31)$$

381 The flow interaction between CO₂ and methane is modeled using a shared relative permeability
 382 curve for both components, consistent with the conventional compositional approach in which
 383 they are treated as a single phase. Consequently, the endpoint mobilities for the two components
 384 share the same relative permeability value at full saturation, meaning $k_{rm}(1) = k_{rc}(1)$. As given
 385 in equation (18), the viscosity terms in λ_{α} do not vary with the vertical coordinate and can
 386 therefore be pulled out of the integral.

387 The final upscaled mass flux terms for each phase are written using the coarse-scale perme-
 388 ability and mobilities as:

$$\mathbf{F}_{\alpha} = -\tilde{R}_{\alpha} \Lambda_{\alpha} K (\nabla P_{\alpha} + \rho_{\alpha}(P_{\alpha}) g \nabla \zeta_B). \quad (32)$$

389 Returning to the upscaled mass accumulation term in equation (8), for a homogeneous
 390 reservoir and given that saturation is either zero or one, we can restrict the integration domain
 391 for each phase to the vertical interval where its saturation is one. This simplifies equation (8) to:

$$R_\alpha \Phi S_\alpha = \phi \int_{\zeta_a}^{\zeta_b} \rho_\alpha dz, \quad \{a, b\} \in \{B, ibc, imc, T\} \quad (33)$$

Where ζ_a to ζ_b denotes the vertical region occupied entirely by the phase α . For brine, which is incompressible, density can be taken outside the integral, yielding a simplified expression. For CO₂ and methane density functions, however, rather than neglecting higher-order terms, we retain them and evaluate the vertical integral of the density function from equation (17), resulting in:

$$R_\alpha = \rho_\alpha(p_a)[\zeta_b - \zeta_a] - \frac{\rho'_\alpha(p_a)}{1!}(\rho_\alpha(p_a)g)\frac{[\zeta_b - \zeta_a]^2}{2} + \frac{\rho''_\alpha(p_a)}{2!}(\rho_\alpha(p_a)g)^2\frac{[\zeta_b - \zeta_a]^3}{3}, \quad \{\alpha\} \in \{c, m\} \quad (34)$$

This expression is then used in the mass accumulation term for CO₂ and methane, where the reference pressure matches that used in the upscaled phase density of the flux term. The final vertically integrated flow equations become the following:

$$-\Phi \frac{\partial(R_\alpha S_\alpha)}{\partial t} + \nabla \cdot \mathbf{F}_\alpha = \Psi_\alpha \quad (35)$$

The vertically integrated equations derived above were implemented in the MATLAB Reservoir Simulation Toolbox (MRST) using its Automatic Differentiation framework [Lie and Møyner, 2021]. The approach uses plume heights as the primary variables and calculates phase fluxes based on vertically averaged mobilities and hydrostatic pressure gradients, consistent with VE assumptions. This setup retains the analytical structure of the VE formulation while utilizing MRST's ADI capabilities for residual construction, Jacobian computation, and fully implicit time integration, ensuring both robustness and scalability.

3 Results and Discussion

This section shows the results from three case studies using the three-phase VE model developed in this work, and compares them to compositional simulations. The first case uses a flat, layered synthetic reservoir to benchmark the VE model against compositional simulations in an idealised geometry. The second looks at a dual-anticline structure to assess grid sensitivity and structural trapping behavior. The third applies the model to the real-world Hugin Formation East to evaluate its performance in a geologically realistic field setting. As described earlier, the initial compositional models were built with the MRST compositional module, then transferred and executed in Jutul Darcy. Simulations were carried out either on a local workstation with an Intel Core i9-14900K processor or on the DMOG HPC cluster. Some simulations also used MPI-based parallel computing to speed up the runtimes. Brine properties are treated as incompressible in both the compositional and VE models, using constant values of 1000 kg m⁻³ for density and 1 cP for viscosity. In the VE model, methane and CO₂ are treated as compressible fluids, with their density and viscosity defined by pressure-dependent correlations, as outlined in Section 2. The compositional model uses an equation of state (EoS) to compute the thermophysical properties of methane and CO₂.

3.1 Case 1 — Flat-Layered Gas–Brine Benchmark

The first case study features a simplified flat-layered synthetic reservoir model, designed as a benchmark scenario to study basic flow behavior in compositional simulations. The model has a uniform structure with no topographic variation and spans 10 kilometers in the horizontal direction with a constant vertical thickness of 50 meters. The domain is initialized with the upper half of the reservoir saturated with methane, while the lower half contains brine, representing a

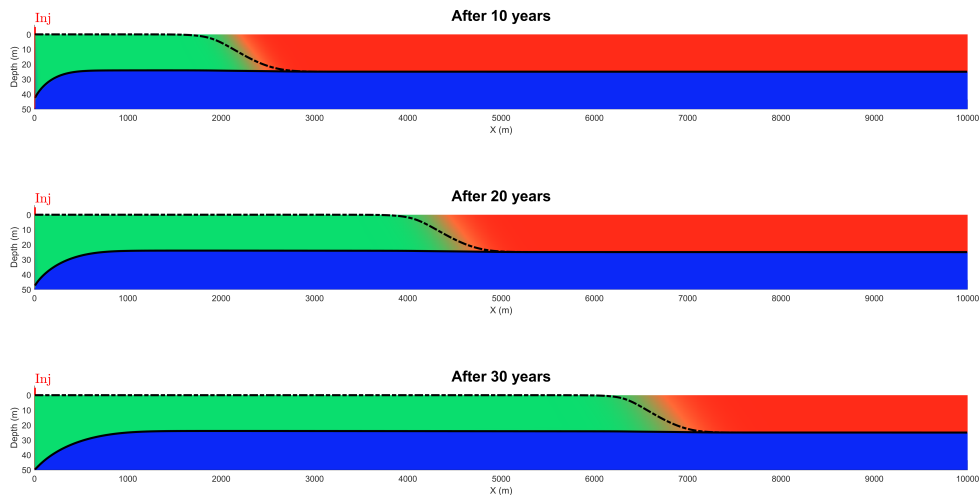


Figure 2: Compositional simulation (colored fields) versus VE model predictions (black lines) for Case 1. The VE model approximates the CO_2 (green) plume’s migration into methane (red) and along the brine (blue) interface at 10, 20, and 30 years.

429 stratified two-phase system. The reservoir pressure is set to 150 bar at the gas–water interface,
 430 with a hydrostatic pressure distribution applied throughout the domain. The porosity is set
 431 uniformly at 20%, and the permeability is isotropic, with values of 100 mD. The reservoir is
 432 discretized into a structured grid with 1000 horizontal cells ($dx = 10$ m) and 100 vertical layers
 433 ($dz = 0.5$ m), yielding a total of 100,000 grid cells. Injection is performed from a well at the
 434 left side at a constant rate of 2000 kg/day for a period of 30 years, while the right boundary
 435 maintains constant hydrostatic pressure.

436 Figure 2 compares the fine-grid compositional simulation to the three-phase VE model for
 437 the benchmark scenario. The compositional model shows a more diffuse and vertically mixed
 438 transition zone between CO_2 and methane, particularly farther from the injection well, whereas
 439 the VE model effectively approximates the leading front of the CO_2 plume as it displaces the
 440 methane gas at the top of the reservoir. Agreement is especially strong at the brine– CO_2
 441 interface, where the VE model closely follows the lower boundary of the migrating plume. These
 442 results confirm that, despite its simplified assumptions, the VE model captures the key large-scale
 443 displacement and plume geometry. Its sharp interface approximation provides a robust match
 444 to the dominant flow behavior observed in the compositional simulation.

445 A key observation in this scenario is that, despite CO_2 being co-injected into both brine- and
 446 methane-saturated zones, the majority of the CO_2 plume preferentially migrates upward into
 447 the methane-rich region, while its advancement within the brine zone is comparatively slower.
 448 This response is governed primarily by fluid mobility contrasts in a pressure-driven system, with
 449 gravity exerting an influence through gravitational segregation. In particular, CO_2 displacing
 450 methane is more favorable than CO_2 displacing water because the CO_2 –methane mobility ratio
 451 is closer to unity than the CO_2 –brine mobility ratio. This setup serves as a reference to validate
 452 fluid behavior and mobility under simplified conditions before moving on to more geologically
 453 complex cases.

454 3.2 Case 2 — Dual-Anticline Grid-Sensitivity Benchmark

455 In the second scenario, a two-dimensional reservoir with a sinuous geometry and two anticlines
 456 was used to examine CO_2 plume migration under geologically realistic conditions. The initial

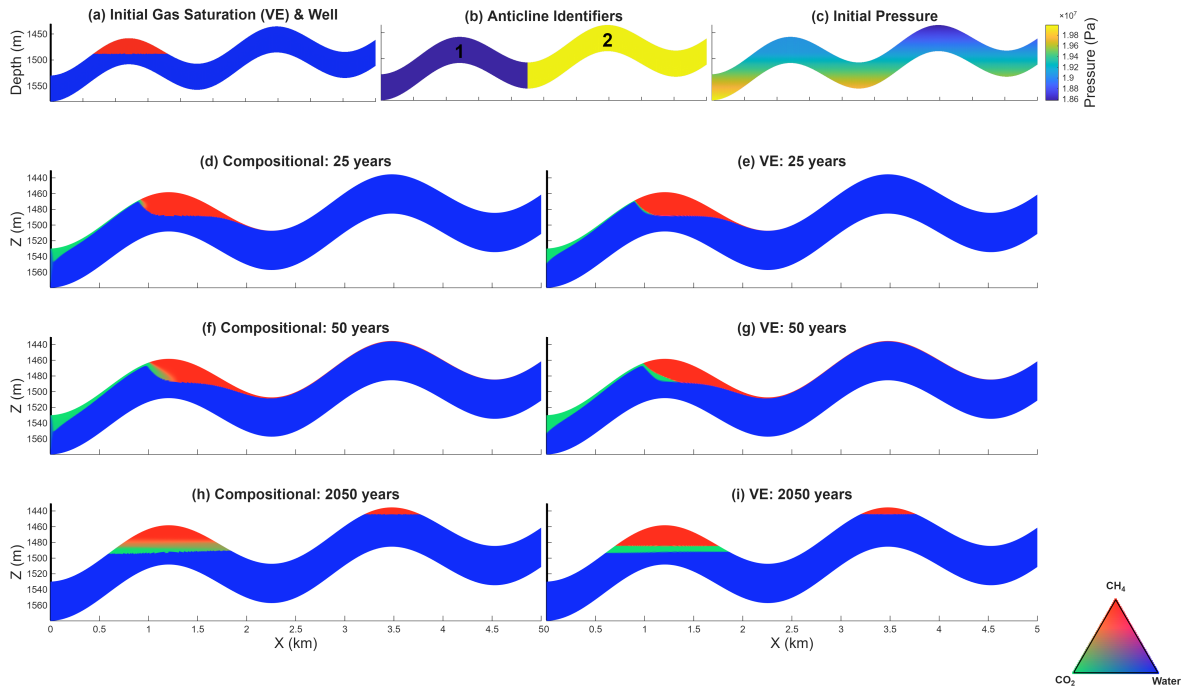


Figure 3: Structural reservoir model and temporal evolution of gas migration. Top row: (a) Initial gas saturation (methane shown in red) and injection well location; (b) Domain segmentation highlighting individual anticline structures used for mass balance tracking; (c) Initial hydrostatic pressure distribution (Pa) across the 5 km domain. Rows 2–4: Comparison of gas plume migration between the Compositional model (left column: d, f, h) and the VE model (right column: e, g, i) at 25, 50, and 2050 years. The snapshots illustrate the displacement of the initial methane (red) by the injected CO₂ (green) and the subsequent structural trapping within the anticline formations.

457 gas-brine configuration, with a 30-meter-thick gas cap in the left anticline and the remainder of
 458 the reservoir saturated with water, is shown in Figure 3a, while the subdivision of the domain
 459 into two indexed anticline compartments used for subsequent analysis is shown in Figure 3b.
 460 CO₂ is injected at a constant rate of 25 t/year for 50 years through a well located at the left
 461 boundary. A constant hydrostatic pressure was maintained at the right boundary, and all other
 462 boundaries were treated as no-flow. The simulation continued for 2000 years after injection
 463 ceased. The initial pressure at the deepest point in the reservoir was set to 200 bar, and the
 464 resulting hydrostatic pressure field is illustrated in Figure 3c, with pressures across the domain
 465 calculated based on local fluid saturations. The model domain extends 5 km laterally, 50 m
 466 vertically, and 1 m in width, with uniform rock properties defined by a porosity of 20% and a
 467 permeability of 10 mD.

468 Compositional simulations were performed on a refined grid to provide a converged reference
 469 solution for comparison with the VE model. The comparison focuses on the predicted redistri-
 470 bution of CO₂ and methane between the two anticlines, as illustrated by the time evolution
 471 of phase distributions in the bottom panels of Figure 3, where the compositional results are
 472 shown alongside the VE model. To further examine the behaviour in each structure, two indexed
 473 regions in the anticlines were tracked over time (see Figure 3b), and the corresponding evolution
 474 of CO₂ and methane mass in each compartment is summarized in Figure 4.

475 The mass balance evolution in each anticline, shown in Figure 3 and Figure 4, is used to
 476 compare the VE model against the compositional reference solution. The comparison focuses on

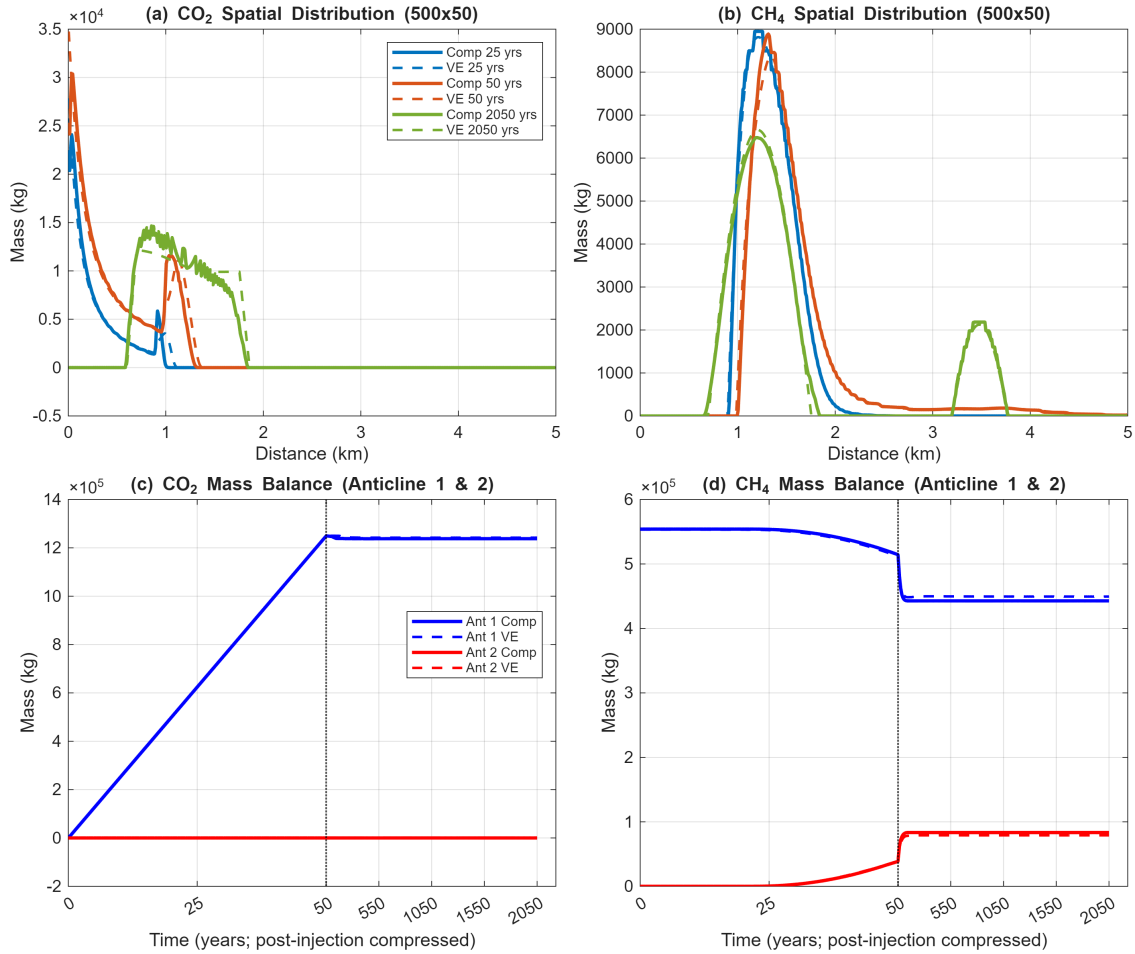


Figure 4: Spatial distribution and mass balance analysis for CO₂ and methane. (a) CO₂ spatial distribution along the domain at 25, 50, and 2050 years. (b) Methane spatial distribution showing the migration and attenuation of the gas plume over time. (c) Cumulative CO₂ mass balance within Anticline 1 and Anticline 2, showing primary containment in Anticline 1. (d) Methane mass balance illustrating the secondary migration from Anticline 1 to Anticline 2. Solid lines indicate the full compositional model (Comp), while dashed lines represent the Vertical Equilibrium (VE) approximation.

477 the redistribution of CO₂ and methane between the two structural compartments and on the
 478 corresponding component inventories over time. Overall, the VE results reproduce the main
 479 trends of the compositional model, including the partitioning of mass between the anticlines
 480 and its temporal evolution. Since the compositional model is taken here as the reference solution,
 481 the purpose of this comparison is to assess how well the VE formulation captures the large scale
 482 migration behaviour relevant to this case. It should also be noted that the compositional model
 483 does not include molecular diffusion or hydrodynamic dispersion.

484 3.3 Case 3 — Field-scale demonstration: Hugin Formation East

485 The third study uses the publicly available *Hugin Formation East* dataset from the CO₂ Storage
 486 Atlas of the Norwegian Continental Shelf [Bjørnstad et al., 2017]. The Hugin Formation, located
 487 in the South Viking Graben of the North Sea, comprises Middle Jurassic shallow-marine and
 488 marginal-marine sandstones deposited during a regional transgression. Here, we use the averaged
 489 formation values across the whole formation ($\phi = 12.45\%$, $k = 500$ mD). The simulation grid is
 490 a corner-point mesh that honours the complex geometry: 2264×20 cells (horizontal \times vertical)

491 for a total of 45 280 active cells, covering $\sim 2.26 \times 10^3 \text{ km}^2$ with layer thicknesses ranging from
 492 5.8 to 106.4 m.

493 The model was initialized with 15 % methane gas and 85 % brine in every cell. A preliminary
 494 run is performed so that pressure and saturations relax to hydrostatic equilibrium; this state
 495 becomes the initial condition for the main study. A single injector, placed near the structural
 496 center, injects 2.5 Mt yr^{-1} of CO_2 for 50 years. After shut-in, the simulation continues for 1 000
 497 years to capture the post-injection redistribution and trapping processes. Figure 6 shows a top
 498 view of the compositional simulations compared to the VE model after 10, 50 and 1050 years.

499 *10 years.* Both models predict nearly identical plume footprints. The VE solution appears
 500 smoother because it integrates over the vertical direction and is therefore insensitive to vertical
 501 resolution or the precise shape of the relative-permeability curves.

502 *50 years (end of injection).* The plume has migrated downdip beneath the pre-existing methane
 503 cap. Crucially, both models place the leading edge at virtually the same x - y location, confirming
 504 that the VE model captures the dominant buoyancy- and mobility-controlled dynamics.

505 *1050 years (end of migration).* CO_2 primarily occupies the smaller anticlines close to the injection
 506 site, displacing the methane that originally resided there into larger and shallower structural
 507 traps. The VE model captures the overall redistribution pattern well but underpredicts methane
 508 displacement in smaller anticlines, compared to the compositional model. This discrepancy
 509 arises in part from numerical dispersion in the compositional model, where coarse grid resolution
 510 leads to exaggerated mixing and overestimation of methane movement. However, the VE model
 511 also contributes to this difference by excluding mixing processes altogether, which limits its
 512 ability to fully represent gas displacement dynamics. The transition zone between CO_2 and
 513 methane within the larger gas caps is similarly influenced by both numerical dispersion in the
 514 compositional model and the absence of mixing in the VE formulation.

515 Another key difference lies in the amount of trapped gas predicted by the two models. The
 516 VE model uses an analytical approach to estimate vertical saturation, making it independent of
 517 vertical grid resolution. In contrast, the 3D compositional model relies on explicit gridding, which
 518 can lead to the uppermost cells in smaller anticlines being fully saturated with gas. As a result,
 519 the VE model tends to predict a thinner gas column in these regions. This discrepancy, therefore,
 520 is due to differences in vertical resolution and the level of detail in the grid discretization.

521 Overall, the VE model reproduces the dominant features of plume evolution, structural
 522 trapping, and long-term redistribution while running orders of magnitude faster than the full
 523 compositional calculation, as quantified by the comparison of simulation times in Table 1. This
 524 makes VE modelling a powerful tool for rapid field-scale screening and optimisation of prospective
 525 CO_2 storage sites, particularly where millennial-scale performance must be evaluated across
 526 many geological realizations.

Table 1: Comparison of Simulation Times for Compositional and VE simulations

Scenario	Compositional (s)	VE (s)
Case 1	15228	36
Case 2	65952	42
Case 3	72360	330

527 3.4 Discussion

528 The present study extends the well-established VE modeling framework to account for flow
 529 involving CO_2 , methane, and brine in depleted gas reservoirs. While previous applications of
 530 VE models have demonstrated their utility in simulating the large-scale migration of CO_2 in
 531 two-phase (i.e., CO_2 -brine) systems with significant computational savings (Gasda et al., 2009;

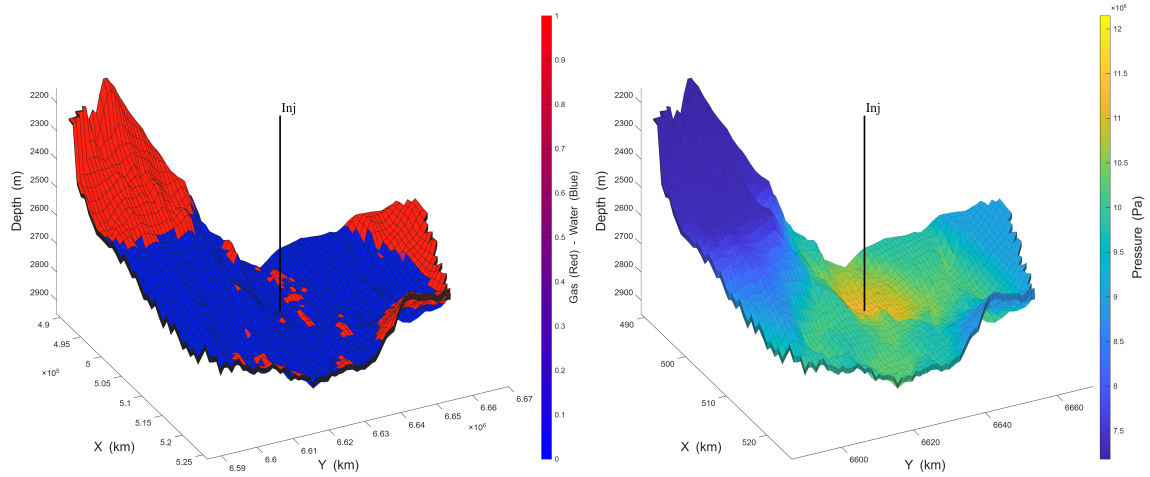


Figure 5: Case 3, Initial saturation and pressure distribution shown from two viewing angles.

532 Nordbotten and Celia, 2011; Court et al., 2012), their application to depleted gas fields has not
 533 previously been explored.

534 Across all three case studies, the three-phase VE formulation reproduced the large-scale
 535 behavior seen in compositional simulations. (i) *Flat, layered benchmark*: the VE model accurately
 536 captured the migration of CO_2 into the methane layer and the shape of the CO_2 -brine interface.
 537 (ii) *Dual anticlines*: the VE model compared well with the compositional simulations, reproducing
 538 the large-scale plume geometry and inter-anticline mass redistribution; a supplementary outcome
 539 of this test was that the comparison also revealed a strong sensitivity of compositional results
 540 to vertical grid resolution. (iii) *Field scale (Hugin Formation)*: the VE model successfully
 541 replicated the plume migration, structural trapping, and long-term redistribution predicted by
 542 the compositional model, confirming its applicability to realistic geological settings.

543 Across all three cases, certain model limitations became apparent, primarily stemming from
 544 the fundamental assumptions of the VE framework. The most significant limitation is the model's
 545 current inability to simulate mixing between CO_2 and methane. The current formulation treats
 546 CO_2 and methane as immiscible phases with a sharp interface. Consequently, the model cannot
 547 replicate phenomena driven by convective mixing, which may be important in scenarios involving
 548 direct injection into methane-saturated zones or in highly heterogeneous formations. These
 549 limitations are largely due to the core assumptions of the VE method itself, rather than just the
 550 fact that it doesn't model mixing. This simplification, while enabling computational speedup,
 551 resulted in some discrepancies in predicted volumes of displaced CO_2 and methane within
 552 structural traps, particularly when compared to compositional simulations. Additionally, the
 553 current formulation excludes other physical mechanisms known to enhance long-term trapping,
 554 namely the dissolution of CO_2 into brine and capillary fringe effects, though these mechanisms
 555 could be incorporated using established methods from two-phase VE systems [Andersen, 2017,
 556 Nilsen et al., 2016b].

557 4 Conclusion

558 This study introduces and evaluates a three-phase VE modeling framework for simulation of CO_2
 559 migration in depleted gas reservoirs, explicitly accounting for the presence of methane. The VE
 560 formulation simplifies the compositional model into a black-oil approximation and subsequently
 561 collapses the spatial domain into two dimensions with significant computational speedup.

562 Benchmarking across idealized and field-scale configurations shows that the model captures

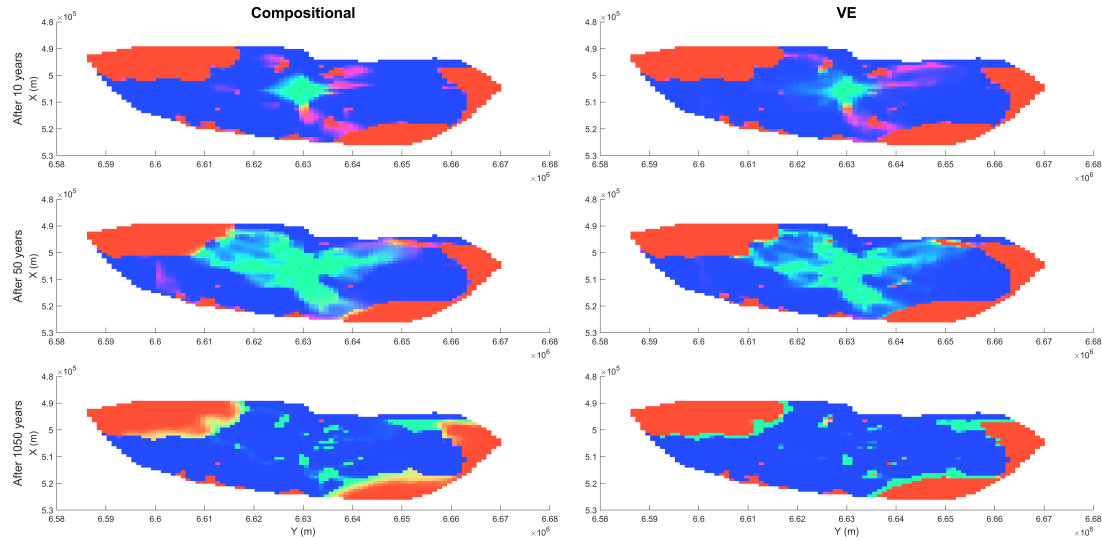


Figure 6: Case 3, Top-view comparison of CO₂ and methane plume migration after 10, 50, and 1050 years using compositional (left column) and VE (right column) models. The spatial extent of the plume is shown in the XY plane.

563 the key gravity-driven plume shape, lateral migration pathways, and locations of structural
 564 trapping that are most relevant for reservoir- to basin-scale assessment. Because vertical
 565 profiles are handled analytically, the formulation is largely insensitive to vertical grid refinement
 566 while coarse-grid compositional simulations are susceptible to numerical dispersion and can
 567 misrepresent the amount of fluid spilling across anticlines. Quantitatively, differences are most
 568 apparent near sharp contacts and in zones where small-scale mixing controls phase volumes, but
 569 the regional patterns and time scales of migration remain well reproduced.

570 Despite known limitations, the VE model delivers substantial computational gains, typically
 571 on the order of 10^2 – 10^3 relative to compositional models, cutting runtimes from hours or days
 572 to minutes or seconds and enabling simulations over long timeframes and large domains. This
 573 dramatic speed-up enables ensemble-centric workflows that would otherwise be impractical:
 574 rapid site screening over large geographic extents (especially valuable in early CCS project
 575 stages), systematic sensitivity studies and assessment of injection strategies, optimization under
 576 operational constraints, and workflows requiring several runs such as uncertainty quantification
 577 and history matching. For the early stages of CCS project development, where rapid screening of
 578 potential sites and assessment of various injection strategies are critical, such a tool is invaluable.
 579 While this study focused on CO₂ injection into depleted gas reservoirs, the framework is adaptable
 580 to other subsurface applications characterized by gravity segregation and extensive lateral scales,
 581 including energy storage, CO₂-enhanced gas recovery, and water-alternating-gas processes.

582 Open Research Section

583 The data associated with this manuscript will be made publicly available upon publication of
 584 the journal article.

585 Conflict of Interest disclosure

586 To the best of the authors' knowledge, there are no conflicts of interest associated with this
 587 work.

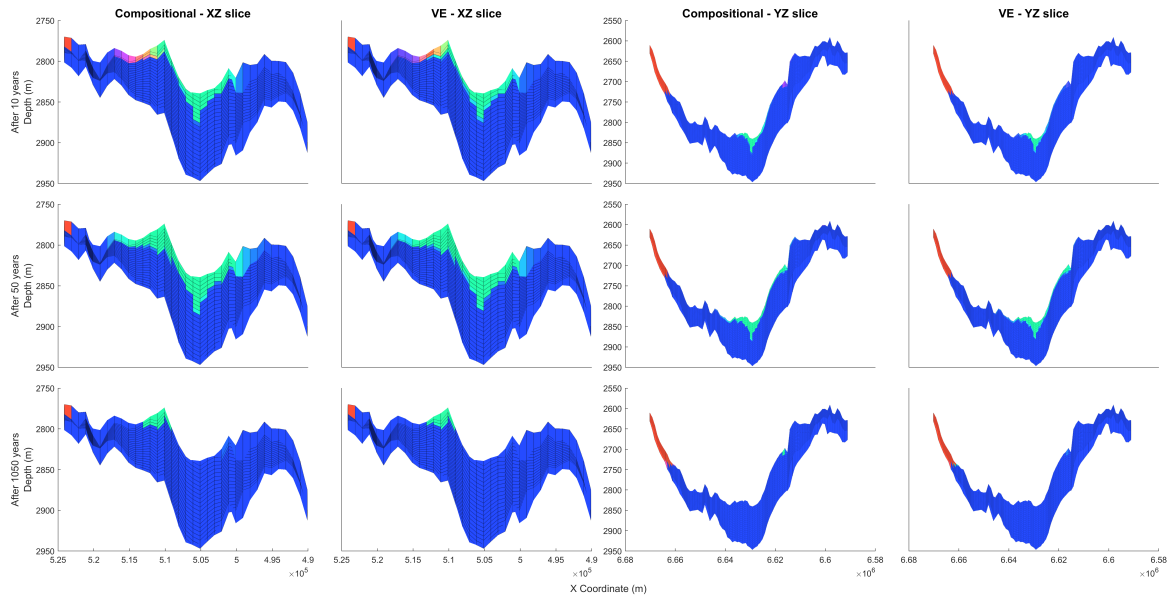


Figure 7: Case 3, Comparison of CO₂ and methane plume evolution at three time steps (10, 50, and 1050 years) for compositional and vertical equilibrium (VE) models. XZ and YZ 2D cross-sections are shown for each case.

Acknowledgments

Saeid Telvari acknowledges the support of a James Watt Scholarship from Heriot-Watt University. This work is part of the MuPSI project (Multiscale Pressure-Stress Impacts on fault integrity for multi-site regional CO₂ storage) which operates under the Clean Energy Transition Partnership (CETP).

Declaration of Generative AI Use

AI language models were used to assist with aspects of manuscript preparation. All content was reviewed, verified, and approved by the authors, who take full responsibility for the manuscript.

A Thermophysical Property Modeling of CO₂ and Methane

The density and viscosity properties of CO₂ and methane are determined through flash calculations using the equation of state across a range of pressures at 60°C. These computational results are subsequently used to develop fitted equations describing the pressure-dependent variations in density and viscosity for each component. The fitting procedure employs Fourier-type series equations with 3 to 8 terms, depending on the specific dataset characteristics. The accuracy of the fitted equations is quantified using root mean squared error analysis, as shown in Table 2. Notably, the errors associated with CO₂ are larger due to its more non-ideal behaviour compared to methane. The fitting equation takes the form:

$$f(p) = a_0 + \sum_{n=1}^8 [a_n \cos(n\omega p) + b_n \sin(n\omega p)] \quad (36)$$

Here, $f(p)$ represents the fitted property (density or viscosity) as a function of pressure p , a_0 , a_n , and b_n are fitting coefficients, ω is a frequency parameter, and n is the number of harmonic terms used, determined for each specific fluid and property being modeled.

Table 2: RMSE Errors between Fitted Equations and EoS Data

Property	CO ₂	CH ₄
Density	0.2137	4.1728e-05
Viscosity	2.23358e-08	2.4823e-10

Table 3: Fitted Fourier coefficients and frequency parameter (ω) for pressure-dependent property correlations

Coeff	ρ_c	ρ_m	μ_c	μ_m
a_0	4.647e+02	6.811e+01	4.717e-05	2.058e-05
a_1	-3.783e+02	-6.191e+01	-1.989e-05	-9.762e-06
b_1	-1.631e+02	1.372e+02	-2.211e-05	1.016e-06
a_2	-1.142e+02	-7.962e+00	-8.102e-06	1.302e-06
b_2	1.674e+01	-2.807e+01	-2.916e-06	7.019e-07
a_3	2.506e+00	-1.246e+00	-3.038e-06	6.118e-08
b_3	5.676e+01	2.240e+01	3.585e-06	-4.160e-07
a_4	2.021e+01	4.839e+00	-5.284e-07	–
b_4	2.251e+01	-7.902e+00	2.918e-06	–
a_5	6.303e+00	-1.827e+00	5.821e-07	–
b_5	-3.976e+00	2.764e-01	8.952e-07	–
a_6	-9.955e-01	–	5.955e-07	–
b_6	-4.553e+00	–	9.503e-09	–
a_7	-2.635e-01	–	2.104e-07	–
b_7	7.068e-01	–	-4.759e-08	–
a_8	4.358e-01	–	–	–
b_8	1.176e+00	–	–	–
ω	1.377e-07	4.909e-08	1.398e-07	6.545e-08

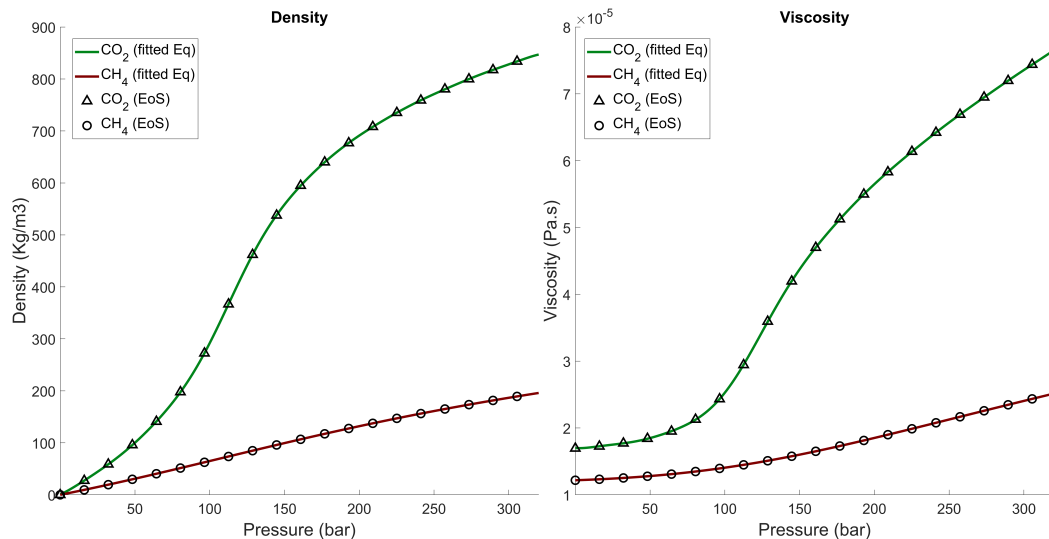


Figure 8: Density and viscosity of CO₂ and methane at T = 60°C, EoS data vs. fitted equation

References

- 608
- 609 M Ahmadinia, S M Shariatipour, O Andersen, and M Sadri. Benchmarking of vertically
 610 integrated models for the study of the impact of caprock morphology on CO₂
 611 migration. *International Journal of Greenhouse Gas Control*, 90, 2019. ISSN 1750-5836. doi:
 612 10.1016/j.ijggc.2019.102802. URL [GotoISI://WOS:000495026000009](https://www.wos.org/wos/doi/10.1016/j.ijggc.2019.102802).
- 613 A. Al-Hashami, S. R. Ren, and B. Tohidi. CO₂ Injection for Enhanced Gas Recovery and
 614 Geo-Storage: Reservoir Simulation and Economics. 6 2005. doi: 10.2118/94129-MS. URL
 615 <https://dx.doi.org/10.2118/94129-MS>.
- 616 Rebecca Allen, Halvor M. Nilsen, Knut Andreas Lie, Olav Møyner, and Odd Andersen. Using
 617 simplified methods to explore the impact of parameter uncertainty on CO₂ storage estimates
 618 with application to the Norwegian Continental Shelf. *International Journal of Greenhouse
 619 Gas Control*, 75:198–213, 8 2018. ISSN 17505836. doi: 10.1016/J.IJGGC.2018.05.017.
- 620 Mohammad Amin Amooie and Joachim Moortgat. Higher-order black-oil and compositional
 621 modeling of multiphase compressible flow in porous media. *International Journal of Multiphase
 622 Flow*, 105:45–59, 8 2018. ISSN 03019322. doi: 10.1016/J.IJMULTIPHASEFLOW.2018.03.016.
- 623 O. Andersen, S. E. Gasda, and H. M. Nilsen. Vertically Averaged Equations with Variable
 624 Density for CO₂ Flow in Porous Media. *Transport in Porous Media*, 107:95–127, 3 2015. ISSN
 625 15731634. doi: 10.1007/s11242-014-0427-z.
- 626 O A Andersen, H M Nilsen, and S E Gasda. Vertical Equilibrium Flow Models with Fully
 627 Coupled Geomechanics for CO₂ Storage Modeling, Using Precomputed Mechanical Response
 628 Functions. *Energy Procedia*, 114:3113–3131, 2017. ISSN 1876-6102. doi: [https://doi.org/
 629 10.1016/j.egypro.2017.03.1440](https://doi.org/10.1016/j.egypro.2017.03.1440). URL [https://www.sciencedirect.com/science/article/
 630 pii/S1876610217316259](https://www.sciencedirect.com/science/article/pii/S1876610217316259).
- 631 Odd Andersen. *Simplified models for numerical simulation of geological CO₂ storage*. PhD thesis,
 632 2 2017.
- 633 Susan Anyosa, Jo Eidsvik, and Dario Grana. Evaluating geophysical monitoring strategies for a
 634 CO₂ storage project. *Computers & Geosciences*, 185:105561, 3 2024. ISSN 0098-3004. doi:

- 635 10.1016/J.CAGEO.2024.105561. URL [https://www.sciencedirect.com/science/article/
636 pii/S009830042400044X](https://www.sciencedirect.com/science/article/pii/S009830042400044X).
- 637 Karl W Bandilla, Bo Guo, and Michael A Celia. A guideline for appropriate application of
638 vertically-integrated modeling approaches for geologic carbon storage modeling. *International
639 Journal of Greenhouse Gas Control*, 91:102808, 2019. ISSN 1750-5836. doi: [https://doi.org/10.
640 1016/j.ijggc.2019.102808](https://doi.org/10.1016/j.ijggc.2019.102808). URL [https://www.sciencedirect.com/science/article/pii/
641 S1750583619300684](https://www.sciencedirect.com/science/article/pii/S1750583619300684).
- 642 Beatrix Becker, Bo Guo, Karl Bandilla, Michael A. Celia, Bernd Flemisch, and Rainer Helmig.
643 A Pseudo-Vertical Equilibrium Model for Slow Gravity Drainage Dynamics. *Water Resources
644 Research*, 53:10491–10507, 12 2017. ISSN 19447973. doi: 10.1002/2017WR021644.
- 645 Beatrix Becker, Bo Guo, Ivan Buntic, Bernd Flemisch, and Rainer Helmig. An Adaptive Hybrid
646 Vertical Equilibrium/Full-Dimensional Model for Compositional Multiphase Flow. *Water
647 Resources Research*, 58(1):e2021WR030990, 2022. ISSN 0043-1397. doi: [https://doi.org/
648 10.1029/2021WR030990](https://doi.org/10.1029/2021WR030990). URL [https://agupubs.onlinelibrary.wiley.com/doi/abs/10.
649 1029/2021WR030990](https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2021WR030990).
- 650 Tore I. Bjørnarå, Jan M. Nordbotten, and Joonsang Park. Vertically integrated models for
651 coupled two-phase flow and geomechanics in porous media. *Water Resources Research*, 52:
652 1398–1417, 2 2016. ISSN 19447973. doi: 10.1002/2015WR017290.
- 653 Andreas Bjørnstad, Eva Halland, and Fridtjof Riis. Evaluation of the CO₂ Storage Potential of
654 the Norwegian Continental Shelf: Online Data Repository. In *Energy Procedia*, volume 114,
655 pages 4394–4399. Elsevier Ltd, 2017. doi: 10.1016/j.egypro.2017.03.1593.
- 656 Ivan Buntic, Martin Schneider, Bernd Flemisch, and Rainer Helmig. A fully-implicit solving
657 approach to an adaptive multi-scale model - coupling a vertical-equilibrium and full-dimensional
658 model for compressible, multi-phase flow in porous media. *Computational Geosciences*, 29:11,
659 4 2025. ISSN 1420-0597. doi: 10.1007/s10596-025-10351-z.
- 660 S Y Chen, J F Liu, Q Zhang, F Teng, and B C McLellan. A critical review on deployment
661 planning and risk analysis of carbon capture, utilization, and storage (CCUS) toward carbon
662 neutrality. *Renewable & Sustainable Energy Reviews*, 167, 2022. ISSN 1364-0321. doi:
663 10.1016/j.rser.2022.112537. URL [GotoISI://WOS:000830429400001](https://www.wos.org/doi/abs/10.1016/j.rser.2022.112537).
- 664 Holger Class, Anozie Ebigbo, Rainer Helmig, Helge K Dahle, Jan M Nordbotten, Michael A
665 Celia, Pascal Audigane, Melanie Darcis, Jonathan Ennis-King, Yaqing Fan, Bernd Flemisch,
666 Sarah E Gasda, Min Jin, Stefanie Krug, Diane Labregere, Ali Naderi Beni, Rajesh J Pawar,
667 Adil Sbai, Sunil G Thomas, Laurent Trenty, and Lingli Wei. A benchmark study on problems
668 related to CO₂ storage in geologic formations. *Computational Geosciences*, 13(4):409–434,
669 2009. ISSN 1573-1499. doi: 10.1007/s10596-009-9146-x. URL [https://doi.org/10.1007/
670 s10596-009-9146-x](https://doi.org/10.1007/s10596-009-9146-x).
- 671 B Court, K W Bandilla, M A Celia, A Janzen, M Dobossy, and J M Nordbotten. Applicability
672 of vertical-equilibrium and sharp-interface assumptions in CO₂ sequestration
673 modeling. *International Journal of Greenhouse Gas Control*, 10:134–147, 2012. ISSN 1750-5836.
674 doi: 10.1016/j.ijggc.2012.04.015. URL [GotoISI://WOS:000309790500013](https://www.wos.org/doi/abs/10.1016/j.ijggc.2012.04.015).
- 675 I de Jonge Anderson, H Ramachandran, U Nicholson, S Geiger, A Widyanita, and F Doster.
676 Determining CO₂ storage efficiency within a saline aquifer using reduced complexity mod-
677 els. *Advances in Geo-Energy Research*, 13(1):22–31, 2024. doi: 10.46690/ager.2024.07.
678 04. URL [https://www.scopus.com/inward/record.uri?eid=2-s2.0-85197686714&doi=
679 10.46690%2fager.2024.07.04&partnerID=40&md5=b1a6413ffb543b3b9e168b67dabbacf7](https://www.scopus.com/inward/record.uri?eid=2-s2.0-85197686714&doi=10.46690%2fager.2024.07.04&partnerID=40&md5=b1a6413ffb543b3b9e168b67dabbacf7).

- 680 R Donat, F Guerrero, and P Mulet. Implicit-explicit methods for models for vertical equilibrium
681 multiphase flow. *Computers & Mathematics with Applications*, 68(3):363–383, 2014. ISSN
682 0898-1221. doi: 10.1016/j.camwa.2014.06.006. URL [<GotoISI>://WOS:000340316100020](#).
- 683 F. Doster, J. M. Nordbotten, and M. A. Celia. Impact of capillary hysteresis and trapping on
684 vertically integrated models for CO₂ storage. *Advances in Water Resources*, 62:465–474, 12
685 2013. ISSN 03091708. doi: 10.1016/j.advwatres.2013.09.005.
- 686 Florian Doster, Jan M. Nordbotten, and Michael A. Celia. Hysteretic upscaled constitutive
687 relationships for vertically integrated porous media flow. *Computing and Visualization in
688 Science*, 15:147–161, 8 2012. ISSN 14329360. doi: 10.1007/s00791-013-0206-3.
- 689 S E Gasda, J M Nordbotten, and M A Celia. Vertical equilibrium with sub-scale analytical
690 methods for geological CO₂ sequestration. *Computational Geosciences*, 13(4):
691 469–481, 2009. ISSN 1420-0597. doi: 10.1007/s10596-009-9138-x. URL [<GotoISI>://WOS:
692 000272180000005](#).
- 693 S. E. Gasda, J. M. Nordbotten, and M. A. Celia. Vertically averaged approaches for CO₂
694 migration with solubility trapping, 2011. ISSN 00431397.
- 695 Sarah E. Gasda, Halvor M. Nilsen, Helge K. Dahle, and William G. Gray. Effective models for
696 CO₂ migration in geological systems with varying topography. *Water Resources Research*, 48,
697 2012. ISSN 00431397. doi: 10.1029/2012WR012264.
- 698 Sarah E. Gasda, Halvor M. Nilsen, and Helge K. Dahle. Impact of structural heterogeneity on
699 upscaled models for large-scale CO₂ migration and trapping in saline aquifers. *Advances in
700 Water Resources*, 62:520–532, 12 2013a. ISSN 03091708. doi: 10.1016/j.advwatres.2013.05.003.
- 701 Sarah E. Gasda, Elsa du Plessis, and Helge K. Dahle. Upscaled models for CO₂ injection and
702 migration in geological systems. In *Simulation of Flow in Porous Media*, pages 1–38. DE
703 GRUYTER, 10 2013b. doi: 10.1515/9783110282245.1.
- 704 S. Ghanbari, E. J. Mackay, and G. E. Pickup. Measurement of physical dispersion in random
705 correlated permeability fields and its application for upscaling. In *16th European Conference
706 on the Mathematics of Oil Recovery, ECMOR 2018*. European Association of Geoscientists
707 and Engineers, EAGE, 2018. doi: 10.3997/2214-4609.201802200.
- 708 Saeed Ghanbari, Eric J. Mackay, Niklas Heinemann, Juan Alcalde, Alan James, and Michael J.
709 Allen. Impact of CO₂ mixing with trapped hydrocarbons on CO₂ storage capacity and security:
710 A case study from the Captain aquifer (North Sea). *Applied Energy*, 278, 11 2020. ISSN
711 03062619. doi: 10.1016/j.apenergy.2020.115634.
- 712 F. Guerrero, R. Donat, and P. Mulet. Solving a model for 1-D, three-phase flow vertical
713 equilibrium processes in a homogeneous porous medium by means of a Weighted Essentially
714 Non Oscillatory numerical scheme. *Computers and Mathematics with Applications*, 66:1284–
715 1298, 10 2013. ISSN 08981221. doi: 10.1016/j.camwa.2013.07.027.
- 716 B Guo, K W Bandilla, J M Nordbotten, M A Celia, E Keilegavlen, and F Doster. A multiscale
717 multilayer vertically integrated model with vertical dynamics for CO₂ seques-
718 tration in layered geological formations. *Water Resources Research*, 52(8):6490–6505, 2016.
719 ISSN 0043-1397. doi: 10.1002/2016wr018714. URL [<GotoISI>://WOS:000383684400044](#).
- 720 Bo Guo, Karl W. Bandilla, Florian Doster, Eirik Keilegavlen, and Michael A. Celia. A vertically
721 integrated model with vertical dynamics for CO₂ storage. *Water Resources Research*, 50:
722 6269–6284, 8 2014. ISSN 19447973. doi: 10.1002/2013WR015215.

- 723 Bo Guo, Yiheng Tao, Karl Bandilla, and Michael Celia. Vertically Integrated Dual-porosity and
724 Dual-permeability Models for CO₂ Sequestration in Fractured Geological Formation. In *Energy*
725 *Procedia*, volume 114, pages 3343–3352. Elsevier Ltd, 2017. doi: 10.1016/j.egypro.2017.03.1466.
- 726 M. Haajizadeh, F. J. Fayers, A. P. Cockin, M. Roffey, and D. J. Bond. On the Importance
727 of Dispersion and Heterogeneity in the Compositional Simulation of Miscible Gas Processes.
728 Society of Petroleum Engineers (SPE), 10 1999. doi: 10.2118/57264-ms.
- 729 A Hamza, I A Hussein, M J Al-Marri, M Mahmoud, R Shawabkeh, and S Aparicio.
730 CO₂ enhanced gas recovery and sequestration in depleted gas reservoirs:
731 A review. *Journal of Petroleum Science and Engineering*, 196, 2021. ISSN 0920-4105. doi:
732 10.1016/j.petrol.2020.107685. URL [GotoISI://WOS:000600808100054](https://www.isi.net/WOS/000600808100054).
- 733 Hassan Hassanzadeh, Mehran Pooladi-Darvish, Adel M. Elsharkawy, David W. Keith, and Yuri
734 Leonenko. Predicting PVT data for CO₂-brine mixtures for black-oil simulation of CO₂
735 geological storage. *International Journal of Greenhouse Gas Control*, 2(1):65–77, 2008. ISSN
736 17505836. doi: 10.1016/S1750-5836(07)00010-2.
- 737 R. P. Hepple and S. M. Benson. Geologic storage of carbon dioxide as a climate change mitigation
738 strategy: performance requirements and the implications of surface seepage. *Environmental*
739 *Geology* 2004 47:4, 47(4):576–585, 11 2004. ISSN 1432-0495. doi: 10.1007/S00254-004-1181-2.
740 URL <https://link.springer.com/article/10.1007/s00254-004-1181-2>.
- 741 Charles R Jenkins, Peter J Cook, Jonathan Ennis-King, James Undershultz, Chris Boreham,
742 Tess Dance, Patrice de Caritat, David M Etheridge, Barry M Freifeld, Allison Hortle, Dirk
743 Kirste, Lincoln Paterson, Roman Pevzner, Ulrike Schacht, Sandeep Sharma, Linda Stalker,
744 and Milovan Urosevic. Safe storage and effective monitoring of CO₂ in depleted
745 gas fields. *Proceedings of the National Academy of Sciences*, 109(2):E35–E41, 2012. doi: 10.
746 1073/pnas.1107255108. URL <https://www.pnas.org/doi/abs/10.1073/pnas.1107255108>.
- 747 Samuel Krevor, Heleen de Coninck, Sarah E Gasda, Navraj Singh Ghaleigh, Vincent de Gooyert,
748 Hadi Hajibeygi, Ruben Juanes, Jerome Neufeld, Jennifer J Roberts, and Floris Swennenhuis.
749 Subsurface carbon dioxide and hydrogen storage for a sustainable energy future. *Nature Reviews*
750 *Earth & Environment*, 4(2):102–118, 2023. ISSN 2662-138X. doi: 10.1038/s43017-022-00376-8.
751 URL <https://doi.org/10.1038/s43017-022-00376-8>.
- 752 Ajitabh Kumar, Robin Ozah, Myeong Noh, Gary A. Pope, Steven Bryant, Kamy Sepehrmoori,
753 and Larry W. Lake. Reservoir Simulation of CO₂ Storage in Deep Saline Aquifers. *SPE*
754 *Journal*, 10(03):336–348, 9 2005. ISSN 1086-055X. doi: 10.2118/89343-PA. URL <https://dx.doi.org/10.2118/89343-PA>.
- 756 H C Lau, S Ramakrishna, K Zhang, and A V Radhamani. The Role of Carbon Capture and
757 Storage in the Energy Transition. *Energy & Fuels*, 35(9):7364–7386, 2021. ISSN 0887-0624.
758 doi: 10.1021/acs.energyfuels.1c00032. URL [GotoISI://WOS:000648878900028](https://www.isi.net/WOS/000648878900028).
- 759 Knut-Andreas Lie. *An Introduction to Reservoir Simulation Using MATLAB/GNU Octave:
760 User Guide for the MATLAB Reservoir Simulation Toolbox (MRST)*. Cambridge University
761 Press, Cambridge, 2019. ISBN 9781108492430. doi: DOI:10.1017/9781108591416. URL
762 <https://www.cambridge.org/core/product/F48C3D8C88A3F67E4D97D4E16970F894>.
- 763 Knut-Andreas Lie and Olav Møyner. *Advanced Modeling with the MATLAB Reservoir Sim-
764 ulation Toolbox*. Cambridge University Press, Cambridge, 2021. ISBN 9781316519967.
765 doi: DOI:10.1017/9781009019781. URL [https://www.cambridge.org/core/product/
766 7AC2425C73F6F729DB88DB1A504FA1E7](https://www.cambridge.org/core/product/7AC2425C73F6F729DB88DB1A504FA1E7).

- 767 Shezhan Liu, Lei Yuan, Changzhong Zhao, Yi Zhang, and Yongchen Song. A review of research
768 on the dispersion process and CO₂ enhanced natural gas recovery in depleted gas reservoir, 1
769 2022. ISSN 09204105.
- 770 Jianli Ma, Qi Li, Thomas Kempka, and Michael Kühn. Hydromechanical Response and Impact
771 of Gas Mixing Behavior in Subsurface CH₄ Storage with CO₂-Based Cushion Gas. *Energy &
772 Fuels*, 33(7):6527–6541, 7 2019. ISSN 0887-0624. doi: 10.1021/acs.energyfuels.9b00518. URL
773 <https://doi.org/10.1021/acs.energyfuels.9b00518>.
- 774 Rafael March Castaneda Neto. Modelling of CO₂ storage in naturally fractured reservoirs, 2018.
775 URL <http://hdl.handle.net/10399/3516>.
- 776 Carlos Méndez, Nicholas Simpson, Francis Johnson, and Arlene Birt. *Climate Change 2023:
777 Synthesis Report (Full Volume) Contribution of Working Groups I, II and III to the Sixth
778 Assessment Report of the Intergovernmental Panel on Climate Change*. 7 2023. doi: 10.59327/
779 IPCC/AR6-9789291691647.
- 780 O. Moyner. JutulDarcy.jl - a Fully Differentiable High-Performance Reservoir Simulator based on
781 Automatic Differentiation. In *ECMOR 2024*, volume 2024, pages 1–37. European Association
782 of Geoscientists & Engineers, 9 2024. doi: 10.3997/2214-4609.202437111. URL [https:
783 //www.earthdoc.org/content/papers/10.3997/2214-4609.202437111](https://www.earthdoc.org/content/papers/10.3997/2214-4609.202437111).
- 784 O Moyner and H M Nilsen. Multiresolution coupled vertical equilibrium model for fast flexible
785 simulation of CO₂ storage. *Computational Geosciences*, 23(1):1–20, 2019. ISSN
786 1420-0597. doi: 10.1007/s10596-018-9775-z. URL [GotoISI://WOS:000459423400001](https://www ISI.com/WOS/000459423400001).
- 787 N S Muhammed, M B Haq, D A Al Shehri, A Al-Ahmed, M M Rahman, E Zaman, and S Iglauer.
788 Hydrogen storage in depleted gas reservoirs: A comprehensive review. *FUEL*, 337, 2023. ISSN
789 0016-2361. doi: 10.1016/j.fuel.2022.127032.
- 790 Trine S. Mykkeltvedt and Jan M. Nordbotten. Estimating effective rates of convective mixing
791 from commercial-scale injection. *Environmental Earth Sciences*, 67:527–535, 9 2012. ISSN
792 18666280. doi: 10.1007/s12665-012-1674-3.
- 793 H M Nilsen, K A Lie, and O Andersen. Fully-implicit simulation of vertical-equilibrium models
794 with hysteresis and capillary fringe. *Computational Geosciences*, 20(1):49–67, 2016a. ISSN
795 1420-0597. doi: 10.1007/s10596-015-9547-y. URL [GotoISI://WOS:000373347600004](https://www ISI.com/WOS/000373347600004).
- 796 H M Nilsen, K A Lie, and O Andersen. Robust simulation of sharp-interface models for fast
797 estimation of CO₂ trapping capacity in large-scale aquifer systems. *Computa-
798 tional Geosciences*, 20(1):93–113, 2016b. ISSN 1420-0597. doi: 10.1007/s10596-015-9549-9.
799 URL [GotoISI://WOS:000373347600006](https://www ISI.com/WOS/000373347600006).
- 800 J. M. Nordbotten and H. K. Dahle. Impact of the capillary fringe in vertically integrated
801 models for CO₂ storage. *Water Resources Research*, 47, 2011. ISSN 00431397. doi: 10.1029/
802 2009WR008958.
- 803 Jan Nordbotten and Michael Celia. Geological Storage of CO₂: Modeling Approaches for Large-
804 Scale Simulation. *Geological Storage of CO₂: Modeling Approaches for Large-Scale Simulation*,
805 pages i–ix, 11 2011. ISSN 9780470889466. doi: 10.1002/9781118137086.fmatter.
- 806 C M Oldenburg, K Pruess, and S M Benson. Process Modeling of CO₂ Injection into Natural
807 Gas Reservoirs for Carbon Sequestration and Enhanced Gas Recovery. *Energy & Fuels*, 15(2):
808 293–298, 3 2001. ISSN 0887-0624. doi: 10.1021/ef000247h. URL [https://doi.org/10.1021/
809 ef000247h](https://doi.org/10.1021/ef000247h).

- 810 Curtis M Oldenburg and Escholarship Org. Carbon sequestration in natural gas reservoirs:
811 enhanced gas recovery and natural gas storage. 2003. URL [https://escholarship.org/
812 content/qt61b1p0gk/qt61b1p0gk.pdf](https://escholarship.org/content/qt61b1p0gk/qt61b1p0gk.pdf).
- 813 Milan J Patel, Eric F May, and Michael L Johns. High-fidelity reservoir simulations of enhanced
814 gas recovery with supercritical CO₂. *Energy*, 111:548–559, 2016. ISSN 0360-5442. doi: <https://doi.org/10.1016/j.energy.2016.04.120>. URL [https://www.sciencedirect.com/science/
815 //doi.org/10.1016/j.energy.2016.04.120](https://www.sciencedirect.com/science/article/pii/S0360544216305400). URL [https://www.sciencedirect.com/science/
816 article/pii/S0360544216305400](https://www.sciencedirect.com/science/article/pii/S0360544216305400).
- 817 Per Pettersson, Sverre Tveit, and Sarah E. Gasda. Dynamic estimates of extreme-case CO₂
818 storage capacity for basin-scale heterogeneous systems under geological uncertainty. *Inter-
819 national Journal of Greenhouse Gas Control*, 116:103613, 5 2022. ISSN 1750-5836. doi:
820 10.1016/J.IJGGC.2022.103613. URL [https://www.sciencedirect.com/science/article/
821 pii/S1750583622000329](https://www.sciencedirect.com/science/article/pii/S1750583622000329).
- 822 Hariharan Ramachandran, Iain de Jonge-Anderson, Ikhwanul Hafizi Musa, Uisdean Nicholson,
823 Chee Phuat Tan, Sebastian Geiger, and Florian Doster. Rapid Fault Leakage Modeling for
824 CO₂ Storage in Saline Aquifers. 8 2024. doi: 10.31223/X5S12N. URL [https://eartharxiv.
825 org/repository/view/7535/](https://eartharxiv.org/repository/view/7535/).
- 826 Philip S. Ringrose, Anne Kari Furre, Stuart M.V. Gilfillan, Samuel Krevor, Martin Landroslash,
827 Rory Leslie, Tip Meckel, Bamshad Nazarian, and Adeel Zahid. Storage of Carbon Dioxide in
828 Saline Aquifers: Physicochemical Processes, Key Constraints, and Scale-Up Potential. *Annual
829 Review of Chemical and Biomolecular Engineering*, 12:471–494, 6 2021. ISSN 19475446. doi:
830 10.1146/ANNUREV-CHEMBIOENG-093020-091447,. URL [https://pubmed.ncbi.nlm.nih.
831 gov/33872518/](https://pubmed.ncbi.nlm.nih.gov/33872518/).
- 832 Lorenzo Rosa and Marco Mazzotti. Potential for hydrogen production from sustainable biomass
833 with carbon capture and storage. *Renewable and Sustainable Energy Reviews*, 157:112123,
834 2022. ISSN 1364-0321. doi: <https://doi.org/10.1016/j.rser.2022.112123>. URL [https://www.
835 sciencedirect.com/science/article/pii/S136403212200051X](https://www.sciencedirect.com/science/article/pii/S136403212200051X).
- 836 Vijay Kumar Shrivastava. Physical Dispersion in Compositional Reservoir Simulation. Technical
837 report, 2003. URL <http://hdl.handle.net/1880/39841>.
- 838 Amine Tadjer, Aojie Hong, and Reidar B. Bratvold. A sequential decision and data ana-
839 lytics framework for maximizing value and reliability of CO₂ storage monitoring. *Jour-
840 nal of Natural Gas Science and Engineering*, 96:104298, 12 2021. ISSN 1875-5100. doi:
841 10.1016/J.JNGSE.2021.104298. URL [https://www.sciencedirect.com/science/article/
842 pii/S1875510021004959](https://www.sciencedirect.com/science/article/pii/S1875510021004959).
- 843 Yiheng Tao, Bo Guo, Karl W. Bandilla, and Michael A. Celia. Vertically integrated dual-
844 continuum models for CO₂ injection in fractured geological formations. *Computational
845 Geosciences*, 23:273–284, 4 2019. ISSN 15731499. doi: 10.1007/s10596-018-9805-x.
- 846 George A. Virnovsky, Hans Martin Helset, and Svein M. Skjæveland. Stability of displacement
847 fronts in WAG operations. *SPE Journal*, 1:383–393, 1996. ISSN 1086055X. doi: 10.2118/
848 28622-pa.
- 849 Christopher J. Wareing, Michael Fairweather, Samuel A.E.G. Falle, and Robert M. Woolley.
850 Validation of a model of gas and dense phase CO₂ jet releases for carbon capture and storage
851 application. *International Journal of Greenhouse Gas Control*, 20:254–271, 2014. ISSN
852 17505836. doi: 10.1016/j.ijggc.2013.11.012.

- 853 B Wei, B W Wang, X Li, M Aishan, and Y Ju. CO₂ storage in depleted oil and
854 gas reservoirs: A review. *Advances in Geo-Energy Research*, 9(2):76–93, 2023. ISSN 2207-9963.
855 doi: 10.46690/ager.2023.08.02. URL [GotoISI://WOS:001054316800002](https://doi.org/10.46690/ager.2023.08.02).
- 856 Tianyuan Zheng, Bo Guo, and Haibing Shao. A hybrid multiscale framework coupling multilayer
857 dynamic reconstruction and full-dimensional models for CO₂ storage in deep saline aquifers.
858 *Journal of Hydrology*, 600, 9 2021. ISSN 00221694. doi: 10.1016/j.jhydrol.2021.126649.