

1 **Steady state analysis of vegetation growth models with correlated white noises**

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8 **Key Points:**

- 9 • Comparison between logistic and linear vegetation growth model coupled with a
10 surface erosion model.
- 11 • Steady state equilibrium vegetation profile along slope suggests logistic growth model
12 is suitable.
- 13 • Stationary probability distribution shows the effect of Gaussian noises in vegetation
14 growth.
15

16 **Abstract**

17 Vegetation community plays a pivotal role in geomorphic processes. However, the growth of
18 vegetation intrinsically depends on the effective shear stresses exerted by the flow of material
19 (e.g. water or soil) along the slope. We comparatively assess the growth and decay of
20 vegetation using linear and logistic growth model coupled with a runoff erosion model. The
21 model parameters are calibrated with normalized vegetation cover along a slope from
22 Western Ghat escarpment. The deterministic model suggests that the logistic growth model is
23 better predictor of vegetation profiles along a slope transect. Additionally, we propose a
24 stochastic model to capture the role of internal or external factors in the dynamics of
25 vegetation growth using two Gaussian noises. The steady probability distribution functions
26 from the stochastic model provide insight about the role of different noises on the reaction of
27 the system and suggest that bio-environmental factors are difficult to separate out.

28 **Plain Language Summary**

29 Earth surface is shaped by different surface processes which are controlled by the plant
30 community. They restrict the erosion process by binding the soil. However, the vegetation
31 community is also removed by the same processes that shape the earth's surface while the
32 remaining vegetation tends to grow naturally. We are trying to model the balance between the
33 growth and decay which will eventually provide us the amount of vegetation on a slope.
34 While this is one part of the complex interrelated processes, the other aspect deals with the
35 randomness in growth and decay of vegetation. This randomness is primarily driven by either
36 the environmental factors (e.g. rainfall, solar radiation or diseases leading to destruction of
37 vegetation) or inherent to the vegetation species (sudden growth or mortality). Due to these
38 external or internal factors the aforementioned model of vegetation growth and decay falls
39 short. Our aim is to check, how the external or internal factors attribute to the change in
40 vegetation growth.

41 **1. Introduction**

42 Vegetation community is efficient to enrich its condition through growth, decay and
43 sustenance by virtue of inherent physico-chemical processes (Wilson & Agnew, 1992). The
44 spatio-temporal modulation in vegetation mass is greatly influenced by the coupled
45 amalgamation of fluvial hydrodynamic regime, hillslope configuration, climatic factors and
46 soil cover which acts as feedback mechanism to modify the geomorphic features of earth's
47 surface (Tucker & Bras, 1999). In addition to this, the response of vegetation to the
48 environmental elements affecting the geomorphic variabilities is rather complex with inherent
49 nonlinearities and stochasticity rooted within the system.

50 The earliest vegetation growth model of forest cover system was elaborated by Botkin et al.,
51 (1972) where the environment was considered as carrying capacity limited. Subsequently,
52 over the past few decades, there has been significant contribution of exploring the vegetation
53 growth utilizing linear (Collins et al., 2004), exponential or logarithmic relationship between
54 plant cover and biomass (Flanagan et al., 2007; Martinez et al., 2008) and predator-prey
55 models (Kallay & Cohen, 2008; Tanner, 1975; Yoshida et al., 2003). Thornes (1985)
56 initiated the pioneering step and introduced the concept of a coupled system for vegetation
57 growth with a logistic growth of vegetation and slope dependent erosion model. The intricate
58 details of the evolution of vegetation has been further explored using the CHILD numerical
59 tool (Tucker et al., 2001) in various hydro-climatic conditions which capture certain
60 complexities of the physical processes involved.

61 Deterministic models of many real-world phenomena are a difficult task owing to the fact
62 that the various variables and parameters of the system can behave randomly within a similar
63 environment. Therefore, in several instances, it fails to incorporate this stochasticity of
64 coupled biophysical systems. Noise induced phenomena for vegetation growth and resilience
65 have been widely examined by various scholars in differing hydro-climatic conditions. These
66 studies include the feedback mechanism between soil moisture (Borgogno et al., 2007;
67 D'Odorico et al., 2005), water table (Ridolfi et al., 2006), stream flows (Camporeale &
68 Ridolfi, 2007) or geomorphology (Muneepeerakul et al., 2007; Vesipa et al., 2015) with
69 riparian vegetation. Although, a significant amount of study has been undertaken, very
70 limited understanding has been provided with calibration of model parameters using actual
71 vegetation cover data set.

72 In this work, an attempt has been made to couple a logistic vegetation growth model with a
73 wash profile model (Tucker & Bras, 1999) to evaluate the model predictions with previously
74 available analytical solutions of linear growth model. The novelty of the present study lies in
75 the fact of calibration methodology of the coupled model of vegetation growth using actual
76 vegetation cover dataset. Furthermore, we have implemented a steady state stochastic model
77 to analyse the bioenvironmental stochasticity and their effect on the steady state distribution
78 of vegetation cover. The modelling approach and its results makes an effort to address two
79 major issues: (1) which is a better growth model (linear or logistic) in case of a coupled
80 system? (2) How does the noise-induced phenomena affect the steady state probability
81 distribution of the vegetation?

82 **2. Methods and Solution Scheme**

83 2.1. Deterministic vegetation growth model

84 We follow a modelling scheme on similar lines of Tucker & Bras (1999). However, the
85 present formulation takes into account the vegetation proliferation as a logistic growth model
86 that considers the growth of a particular vegetation species is dependent on the existing
87 fractional cover of vegetation (Collins & Bras, 2010).

88 *2.1.1. Logistic growth model*

89 Our modelling scheme utilizes the logistic vegetation growth (Collins et al., 2004) with a
 90 model of wash profile (Tucker & Bras, 1999). Unlike the linear growth models, logistic
 91 model captures the reproduction limited and resource limited condition (Thornes, 1990). This
 92 yields to the following mathematical relation

$$93 \quad \frac{dV_g}{dt} = K_{vg}V(1 - V) \quad (1)$$

94 V_g is vegetation growth, K_{vg} is rate of growth of vegetation on the unvegetated surface.
 95 Reciprocal of the vegetation regrowth rate implies the time taken by a plant community for
 96 regrowth.

97 In natural system, plant community are removed from the soil by various means. However,
 98 we consider that the loss of vegetation is primarily by virtue of the channel and riparian
 99 processes. The simplest physical process for removal of the vegetation cover will depend on
 100 the excess shear stress.

$$101 \quad \frac{dV_e}{dt} = -K_{vd}V(R_f\tau - \tau_c)^\eta \quad (2)$$

102 V_e denotes the vegetation erosion, K_{vd} is the species-dependent erosion parameter, R_f is the
 103 factor of friction, τ and τ_c are the shear stress and effective critical shear stress.

104 The effective critical shear stress is posed as a sum of critical shear stress for pure
 105 unvegetated surface (τ_{cs}) and critical shear stress under 100% vegetation cover (τ_{cv}).

$$106 \quad \tau_c(V) = \tau_{cs} + V\tau_{cv} \quad (3)$$

107 Combining the erosion and growth terms (Eq. (1) and Eq. (2)) the governing equation yields
 108 the following form:

$$109 \quad \frac{d(V_g - V_e)}{dt} = \frac{dV}{dt} = K_{vg}V(1 - V) - K_{vd}V(R_f\tau - \tau_c)^\eta \quad (4)$$

110 *2.1.2. Steady state solutions*

111 For simplicity purposes we assume $\eta = 1$ i.e., the erosion law follows linear function. τ_{cs} is
 112 considered as zero as we have idealized that bare soil surface does not introduce resistive
 113 shear stress. All the physical quantities, which have been taken into account to model the
 114 vegetation growth, have been converted to non-dimensional quantities for ease of
 115 computation.

116 The final form of the non-dimensional steady state equation for fractional vegetation cover
 117 (VCF) is

$$118 \quad V - V^2\{1 + N_v K_{rv}(N_e^q x'^q + V)\} + N_v V^2 = 0 \quad (5)$$

119 N_v is the vegetation number which describes the growth relative to destruction. K_{rv} signifies
 120 the friction coefficient, N_e is the erosion number that relates the shear stress with distance
 121 and q is the non-dimensional exponent that explains the non-linearity in the process involved.

122 Solution of Eq. (5) yields $V = 0$ and the other two roots are

$$123 \quad V = \frac{(N_v - N_v K_{rv} N_e^q x'^q) \pm \sqrt{(N_v - N_v K_{rv} N_e^q x'^q)^2 + 4N_v K_{rv}}}{2N_v K_{rv}} \quad (6)$$

124 The first solution ($V = 0$) corresponds to the specific condition where there is no vegetation
 125 along the slope. The positive root among the other two roots has been considered for
 126 evaluation of the VCF for steady state logistic growth model since there is no physical
 127 significance of negative vegetation cover.

128 The steady state solution of linear vegetation growth model has also been evaluated for the
 129 calibration procedure. The solution is

$$130 \quad V = \frac{1}{1 + N_v N_e^q x'^q} \quad (7) \text{ (Tucker \& Bras, 1999)}$$

131 2.2. Stochastic vegetation growth model

132 We consider two prominent sources of stochasticity in the evolution of vegetation. The
 133 inherent characteristics of the vegetation community has been coined as ‘*intrinsic*’ noise. On
 134 the other hand, the external factors, viz. inhomogeneity in precipitation amount, spatial
 135 variation of temperature, soil moisture retention capacity, ground-water table variability,
 136 aspect of slope etc. apparently serve as ‘*extrinsic*’ noise. In subsequent sections, we describe
 137 that the separation of the intrinsic and extrinsic noise is difficult owing to the fact of complex
 138 interrelationship between the external and internal factors with the system.

139 2.2.1. Formulation of stochastic model

140 The stochastic vegetation model with logistic growth is driven by two white Gaussian noises
 141 $\epsilon(t)$ and $\Gamma(t)$ termed as (negative) additive and multiplicative noise respectively. One-
 142 dimensional Langevin equation with two correlated Gaussian white noises $\epsilon(t)$ and $\Gamma(t)$ with
 143 a non-zero correlation between the multiplicative and negative additive noises leads to

$$144 \quad \frac{dV}{dt} = V + C_1 V^2 + C_2 V^3 + V\epsilon(t) - \Gamma(t) \quad (8)$$

145 where

$$146 \quad C_1 = N_v - N_v K_{rv} N_e^q x'^q - 1 \quad (9)$$

147 and

$$148 \quad C_2 = -N_v K_{rv} \quad (10)$$

149 The Gaussian noises have zero mean and are defined as,

$$150 \quad \langle \epsilon(t)\epsilon(t') \rangle = 2D\delta(t - t') \quad (11)$$

$$151 \quad \langle \Gamma(t)\Gamma(t') \rangle = 2\alpha\delta(t - t') \quad (12)$$

$$152 \quad \langle \epsilon(t)\Gamma(t') \rangle = \langle \epsilon(t')\Gamma(t) \rangle = 2\lambda\sqrt{D\alpha}\delta(t - t') \quad (13)$$

153 λ denotes the degree of correlation between the noises $\epsilon(t)$ and $\Gamma(t)$. D and α are the
 154 strength of the noises $\epsilon(t)$ and $\Gamma(t)$ respectively.

155 2.2.2. Steady state analysis

156 We derive the Fokker-Planck equation (FPE) (Ai et al., 2003; Da-Jin et al., 1994; Li et al.,
 157 2015) for estimation of steady state of probability density function corresponding to Eq. (8)
 158 which is of the following form,

$$159 \quad \frac{\partial P(V,t)}{\partial t} = \frac{\partial A(V)P(V,t)}{\partial V} + \frac{\partial^2 B(V)P(V,t)}{\partial V^2} \quad (14)$$

160 where $P(x, t)$ is the probability density and

161
$$A(V) = V + C_1V^2 + C_2V^3 + DV + \lambda\sqrt{D\alpha} \quad (15)$$

162
$$B(V) = DV^2 + 2\lambda\sqrt{D\alpha}V + \alpha \quad (16)$$

163 The stationary probability distribution of FPE is given by

164
$$P_{st}(V) = \frac{N}{B(V)} \exp \int_0^V \frac{A(V')}{B(V')} dV' \quad (17)$$

165 where N is a normalization constant. In addition, the extrema of $P_{st}(V)$ obeys a general
 166 equation $A(V) - \frac{dB(V)}{dV} = 0$. It leads to

167
$$C_2V^3 + C_1V^2 + (1 - D)V - \lambda\sqrt{D\alpha} = 0 \quad (18)$$

168 If $\lambda = 0$ then there exists no correlation between the two types of noises. This shows that
 169 there is no such dependency on (negative) additive noise at the extrema position $V = 0$ and

170 $V = \frac{-C_1 \pm \sqrt{C_1^2 - 4C_1(1-D)}}{2C_2}$ of the Stationary Probability Distribution (SPD) of FPE for zero
 171 correlation.

172 3. Data, Calibration and Parameterization

173 We use MODIS Vegetation Continuous Field (VCF) product (MOD44B) for the years 2000-
 174 2005 for calibration of the parameters for linear and logistic growth model. A small transect
 175 of Western Ghat escarpment is chosen for the current study. A swath average vegetation
 176 profile of 15 km wide and 80 km long stretch (as shown in Figure 1) has been accounted for
 177 in the present context. Observed vegetation data has been transformed into non-dimensional
 178 vegetation cover with respect to the maximum VCF value within the particular transect.
 179 Distance has been non-dimensionalized with respect to the total length of the transect. We
 180 have idealized that the linear shear stress model for vegetation erosion holds true for overland
 181 flow (Dietrich et al., 2003) as the swath average profile covers the channel as well as the
 182 valley region.

183 Our calibration scheme provides a simplified approach to validate the existing model
 184 outcomes with an available dataset. We have optimized the N_v value by a brute force for each
 185 model run so that the Root Mean Square Error (RMSE) between the modelled and the
 186 observed VCF is minimum. Once the optimal value of N_v is obtained, we reiterate the same
 187 scheme with variable K_{rv} . The erosion number N_e is primarily a function of uniform rate of
 188 erosion (E) and coefficient of erosion (K) (Tucker & Bras, 1999). Considering homogeneity
 189 and constant critical shear stress along the slope as well as uniform and constant erosion rate
 190 (E), we have relaxed the effectiveness of erosion number, N_e and assumed that the value of
 191 N_e is 10.

192 The integral in Eq. (17) has been estimated numerically, with the logistic growth model and
 193 varied the noise parameters and N_v . We have plotted the curves of the SPD after varying the
 194 value of one particular stochastic noise parameter among λ, D, α and fixing the value of other
 195 two parameters. Since, in the deterministic model, the vegetation cover V is a function of the
 196 normalized position x , therefore in stochastic model, SPD has been considered as an implicit
 197 function of V and x . The optimal value of N_v from the calibration of the logistic model
 198 provide the stable solution in terms of the probability distribution of the vegetation cover. We
 199 have considered those numeric values of N_v which optimize the minimum error obtained
 200 from the deterministic model as discussed in earlier section of the article. Also, the range of V
 201 has been taken based on the actual vegetation cover data to plot the SPD.

202 4. Results and Discussion

203 4.1. Steady state vegetation profiles and sensitivity of deterministic model parameters

204 Non-dimensional actual fractional vegetation profile reveals that for most of the years, ~ 60-
205 80% reduction of vegetation cover occurs within ~ 20-40% of the initial length of the total
206 transect (Figure 2). This suggests a steady decrease of actual vegetation cover in upstream
207 zone of the transect, although fluctuation of the VCF is easily observed along the entire
208 transect. The prominent observable fact is moderate increase of the vegetation cover after
209 ~60% of the total transect. It is worth to note that this moderate increase of vegetation cover
210 is more than the existing vegetation cover between ~20-40% of the total transect. Therefore, a
211 steady decrease of vegetation away from the divide does not always hold true.

212 Solutions for the best fit linear model exhibit that the equilibrium vegetation profile declines
213 ~50% within less than initial ~10% of the total transect i.e. adjacent to the hilltop region
214 (Figure 2). After ~20% of the transect length, the VCF for linear model shows a very low rate
215 of decrease in the downstream. The logistic model describes the steady decrease of vegetation
216 cover. We observed ~40-50% reduction of the non-dimensional vegetation cover takes within
217 ~30-40% of the total distance of the transect (Figure 2). The prime important fact to note is
218 that the vegetation cover decreases steadily for logistic growth model and tends to match
219 visually more similar than the vegetation profile of linear growth model.

220 In order to identify the commonalities and discrepancies between model and actual vegetation
221 cover data, we have assessed Root Means Square Error (RMSE) as a metric for error. RMSE
222 is lower in all the years for logistic growth model when compared with the linear one. Unlike
223 the linear model, the logistic model portrays a steady decrease of vegetation cover which can
224 be supported by the observed dataset. The best fit N_v values for the logistic growth model is
225 always higher ($N_v = 1 - 10$) than the linear model ($N_v = 0.4 - 2$). The difference of N_v
226 values can be considered as the inherent characteristics of the model formulation and
227 attributing the conceptualization of the modelling and solution scheme. Error is consistently
228 minimum for $K_{rv} = 0.7$. This indicates a high coefficient of resistance offered by vegetation
229 possibly due to higher vegetation cover in the Western Ghat.

230 The most interesting outcome of the present work is calibration of N_v and K_{rv} with the help
231 of the real vegetation cover dataset. The main driving force of the growth of the vegetation is
232 assessed as the availability of moisture content, slope aspect (Stephenson, 1998) or land
233 surface temperature (Weng et al., 2004). N_v is the critical parameter which controls the
234 growth and decay of the vegetation simultaneously and therefore it includes all of the
235 aforesaid effects ($N_v = \frac{K_{vd} \times \tau_{cv}}{K_{vg}}$). Inclusion of all the effects reduced the complicated problem
236 into a single vegetation number. In our results N_v reflects a very low vegetation number in
237 comparison to most of the model parameter values adopted in the other study (e.g., Collins et
238 al., 2004).

239 4.2. Role of noise induced phenomena in vegetation distribution

240 We show the effect of the Gaussian noises, degree of correlation between these two noises
241 and the vegetation number N_v parameter in Figure 3. In all three cases of the noise induced
242 system, the peak of SDP shifts left as the N_v increases. This feature is universal and common,
243 because vegetation number actually defines the ratio between decay and growth parameters.
244 Therefore, as N_v increases, the vegetation cover decreases and value of $P_{st}(V)$ peaks for
245 small vegetation cover. In other words, the overall vegetation cover disappears for high value
246 of N_v . However, the change in the strength of the noises with low N_v values does not affect
247 the position of the maxima of the SPD.

248 Figure 3, panel *a* represents of the effect of multiplicative noise (D) that acts as a
249 constructive force by increasing the vegetation cover. We find $P_{st}(V)$ is weakly affected by
250 the strength of D when the degree of λ and the strength of α is fixed corresponding to any
251 value of N_v . The prominent cause of the similarity of different SPD is primarily due to the
252 normalization factor that stretches the vegetation cover between 0 to 1. SPD can be
253 distinguished for small value of vegetation number ($N_v = 4$) while, when it is increased to
254 the tune of 80 the SPD are barely separable. At low vegetation cover ($< \sim 0.35$) $P_{st}(V)$
255 decreases; on the contrary at high vegetation cover ($\sim 0.4 - 0.8$) it increases as the strength
256 of D is increased. As value of N_v increases, the difference in $P_{st}(V)$ is indistinguishable as
257 destruction of vegetation is enhanced with higher decay coefficient. This reflects the fact that,
258 the vegetation cover along the transect is not significantly influenced by the multiplicative
259 noise when N_v value is quite high. This high value of N_v sets the stage for a certain extinction
260 of vegetation. One can appreciate another fact that with increasing N_v , $P_{st}(V)$ for higher
261 vegetation cover is always high.

262 The role of the (negative) additive noise strength (α) on the SPD with the fixed λ and D is
263 described in Figure 3, panel *b*. With increasing the strength of α , we observed that the peak of
264 the $P_{st}(V)$ reduces for any value of N_v . Although the magnitude of $P_{st}(V)$ decreases for
265 lower vegetation cover, it is actually higher for higher vegetation cover (Figure 3, panel *b*).
266 Therefore, as the strength of α is increased, the magnitude of $P_{st}(V)$ for small vegetation
267 cover decreased while for high vegetation cover increased. This is indicative to the fact that
268 the (negative) additive noise is actually equalizing the vegetation distribution along the
269 profile by reducing the $P_{st}(V)$ at small vegetation cover. Figure 3, panel *c* offers the effect of
270 correlation between the two Gaussian noises on the SPD. It is evident that as the correlation
271 strength (λ) increases, the probability for the smaller vegetation cover values increases, then
272 drops sharply around 30% of vegetation coverage. For smaller values of N_v , $P_{st}(V)$ increases
273 for lower VCF ($\sim < 0.35$) and decreases when the vegetation cover is higher ($> \sim 0.4 - 0.5$).
274 This implies that higher values of λ promotes the destruction of the overall vegetation pattern.
275 We observed that on increasing the strength of λ at low N_v value, position of peak of $P_{st}(V)$
276 remains stationary.

277 4.3. Implication of the proposed model

278 For the first time, the current study attempts to present a direct calibration of the coupled bio-
279 physical model by extracting the model parameters with the actual vegetation cover dataset.
280 Most of the previous studies (Collins & Bras, 2010; Collins et al., 2004; Istanbuluoglu &
281 Bras, 2005) fall short in calibrating the model parameters for the actual vegetation, as the
282 prime interest was to explore the effect of vegetation on the relief and drainage development.
283 In addition to this, we asserted that the logistic growth model dictates the actual vegetation
284 cover better than the linear growth model. Best fit models for the linear growth
285 underestimates vegetation cover in the upslope region, however, it overestimates the
286 vegetation cover in the downslope (Figure 2). Logistic growth model depicts the nature of
287 VCF distribution more accurately because of its inherent property of growth in resource
288 limited condition. Low value of the N_v suggests that integrated coefficient of vegetation
289 mortality and shear stress is not more than 10 times of the coefficient of vegetation growth
290 for this vegetation type.

291 All characteristic curves of Figure 3 indicate that the multiplicative noise does not act as a
292 drift term unlike discussed in Ai et al., (2003), and vegetation community remains stationary
293 with a fixed peak of $P_{st}(V)$. However, one can consider the (negative) additive noise as a
294 diffusive term which results in reduction of vegetation growth and flattens the peak of $P_{st}(V)$.
295 We also observed that the position of the maxima of $P_{st}(V)$ is not at all affected by the

296 strength of noises. Therefore, we suggest that the intensity of different Gaussian noises does
297 not effectively drive the system to an effective growth or destruction. However, these noises
298 effectively reshape the SPD by decreasing or increasing the magnitude of $P_{st}(V)$ at a certain
299 extremum of VCF. Segregation of the intrinsic and extrinsic factors in the evolution of the
300 vegetation is difficult due to their complicated behaviour. When the internal factors
301 predominantly influence the system, it results in an increase in vegetation growth and VCF.
302 The external factors on the contrary, delay the spread of the vegetation cover.

303 Additionally, the value of $P_{st}(V)$ at higher vegetation cover is also higher when we increased
304 the strength of α . Our results are on similar lines as that of the observations reported by Ai et
305 al. (2003). This could be attributed to the erosion model and steady distribution of the
306 vegetation profile. In the erosion model, erosion rate increases from upstream to downstream.
307 Therefore, the rate of vegetation destruction is lower in the upstream region. The amount of
308 vegetation cover is also higher in the upstream region. We suspect that the combined effect of
309 higher vegetation cover and the lower erosion rate in the upstream region results in lower
310 sensitivity of the vegetation destruction. This particular phenomena should be further
311 investigated.

312 4.4. Revisiting modelling assumptions

313 The spatial resolution of our vegetation dataset is 250 m and therefore it does not distinguish
314 between the vegetation within the channel and in the floodplains. In general, channels are
315 devoid of vegetation owing to the fact that the fluid motion does not promote vegetation
316 growth. The excess shear stress model, used in numerous other studies, has been previously
317 implemented as idealized cases of transport and incision process within the channel (Baldwin
318 et al., 2003; Whipple, 2004). However, numerical models take into consideration a single
319 transport law for both channel as well as surface wash processes (Dietrich et al., 2003). This
320 justifies our data preparation process elaborated in Section 3 and adaptation of the model.
321 Idealized value of N_e is another simplification of the erosion model as we do not consider
322 substantial change in erodibility downslope. Erodibility at a regional scale varies significantly
323 if the landscape encounters a set of different lithology or climatic condition. Similarly, we do
324 not consider that the friction factor k_{rv} changes substantially in order to retain simplification
325 of the model. We have also kept it constant keeping in view that the scale of the transect of
326 vegetation profile is small enough to idealize it as a constant friction condition.

327 The major issue with the logistic model is that the model does not implicate $V = 1$ at $x = 0$.
328 It is acknowledged as a small limitation of our model formulation with the logistic growth. In
329 spite of this, our model solution present better estimates than the existing linear model.
330 Additionally, we do not intend to present sensitivity of the growth, decay, friction, lithologic
331 and noise parameters and this is beyond the objective of the present work. However, the
332 sensitivity analysis can shed light on the role of noise levels on the steady distribution.
333 Western Ghat is considered as a biodiversity hotspot (Cincotta et al., 2000) dominated by
334 various species of flora. Therefore, the most important simplification of the present model is
335 the constant N_v that incorporates growth, decay and shear stress. This implies that the slope is
336 dominated only by a single particular type of vegetation and there is no inter species
337 interaction. We have lumped the factors of multiple species into one vegetation number and
338 did not consider model for intra or inter-species competition.

339 We have characterized the multiplicative noise as a positive role playing agent while the
340 additive noise plays a negative role. One can argue about the character of these noises and
341 may idealize them differently. Additionally, it is nearly impossible to segregate the internal
342 and external factors that lead to environmental stochasticity. Factors such as, solar radiation,
343 precipitation or nutrients generally augment the growth of the vegetation. However,

344 anomalous amount and intensity of these factors can lead to a probable destruction of
345 vegetation cover as well. For example, increased rainfall can lead to higher runoff which can
346 eventually result in vegetation destruction. Similarly, intrinsic character of the vegetation
347 species can simultaneously increase or decrease the vegetation cover along a slope.

348 **5. Conclusions**

349 Here, we have proposed and presented the solutions for a logistic growth model of vegetation
350 and a novel stochastic model with two Gaussian noises. We affirm that the logistic growth
351 model predicts a better estimate of the VCF along a slope. A low vegetation number
352 calibrated from the model needs further investigation to interpret the interaction between the
353 growth and decay of vegetation community. Biological evolution is always regarded as a
354 stochastic system and this gave the motivation to explore the effect of random noises in the
355 vegetation growth along a slope profile. The Gaussian noises and their correlation parameter
356 implicate a stable change in the SPD. Additionally, in the context of the noise level we have
357 chosen, the vegetation growth system does not shift towards immediate sporadic growth or
358 extinction. We observed anomalous effect of the (negative) additive noise which needs
359 further elaboration. We conclude that the effects of different intrinsic and extrinsic noises are
360 difficult to separate out due to complex interrelationship between the environment and
361 biological community.

362

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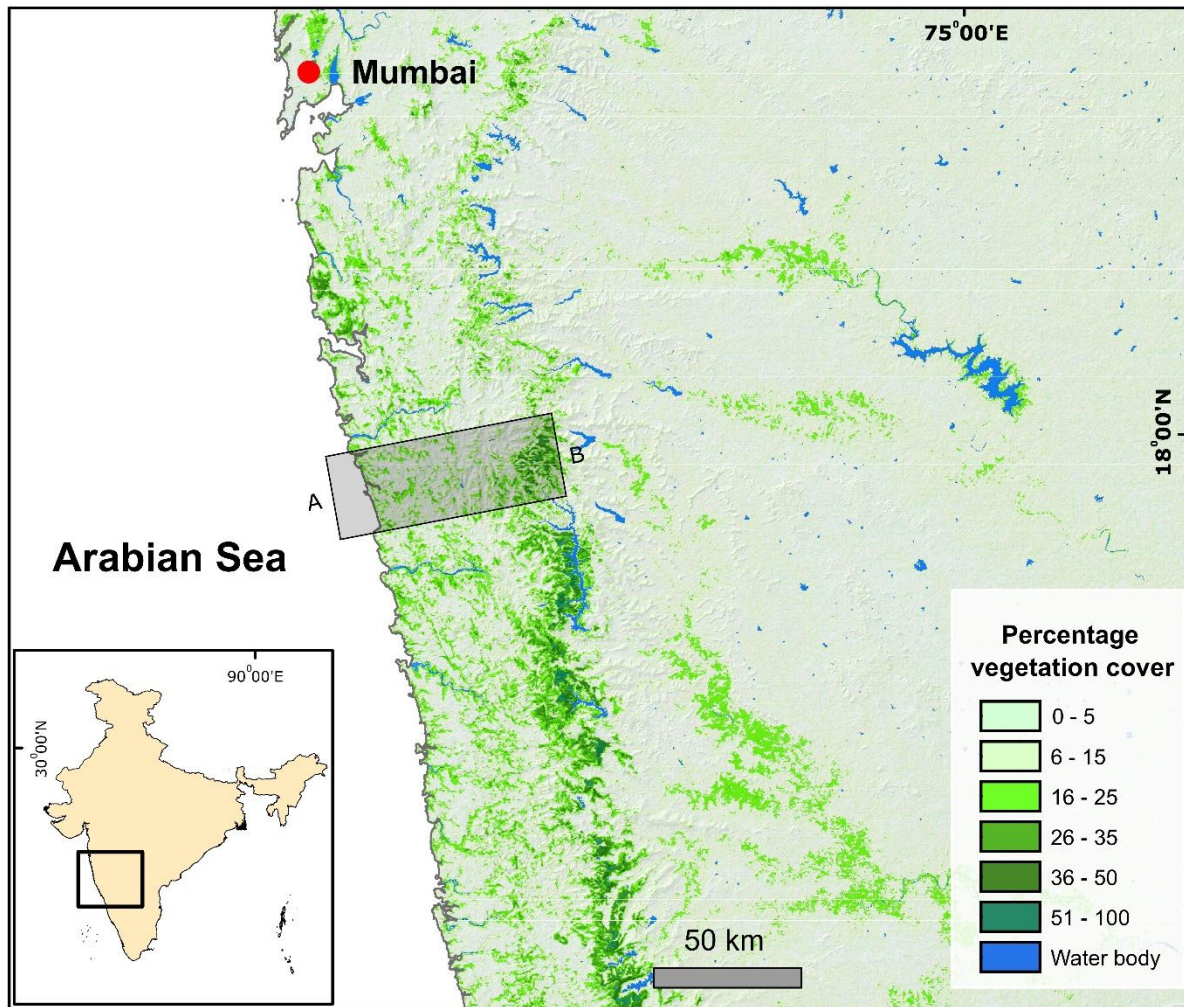
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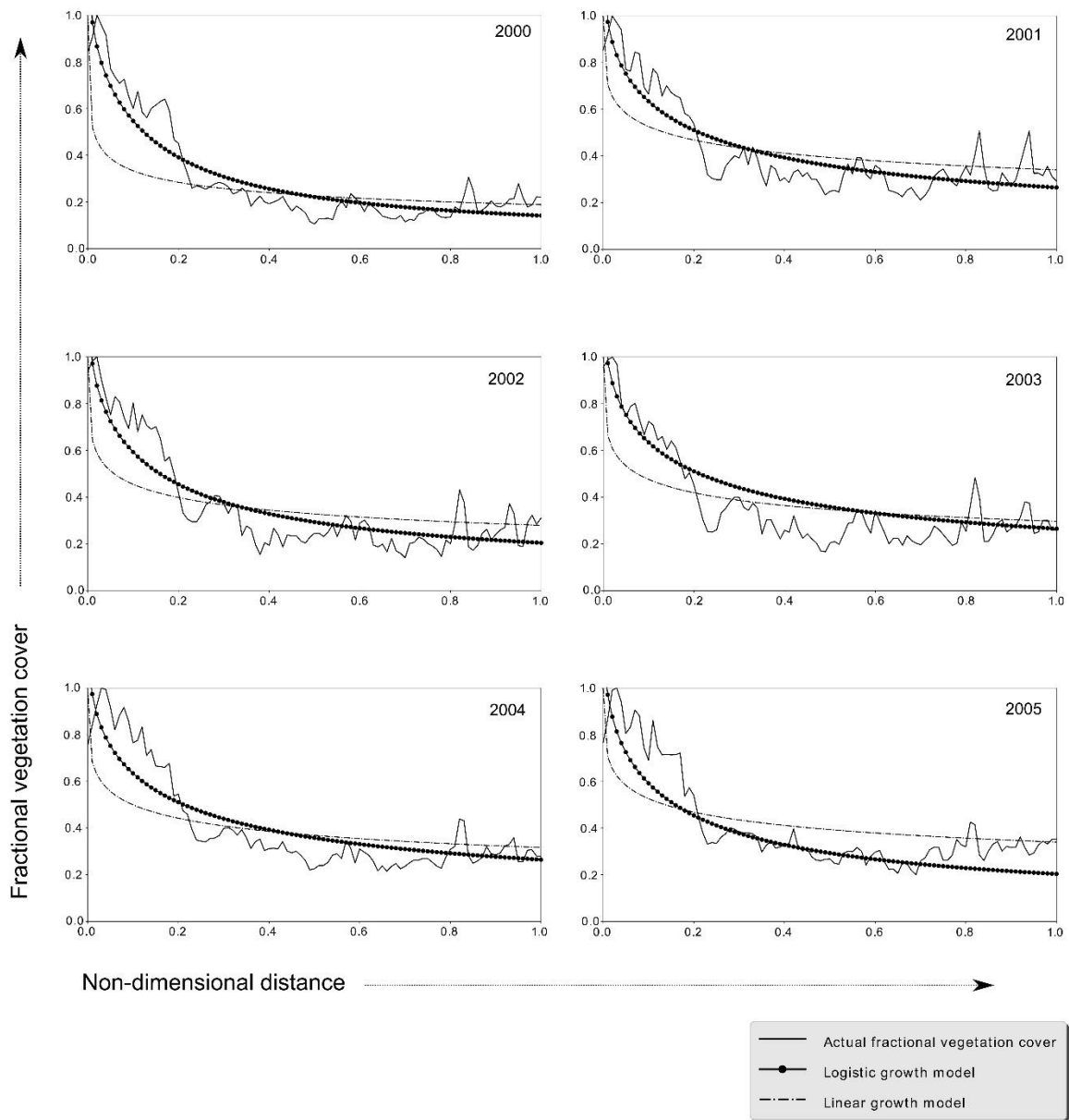
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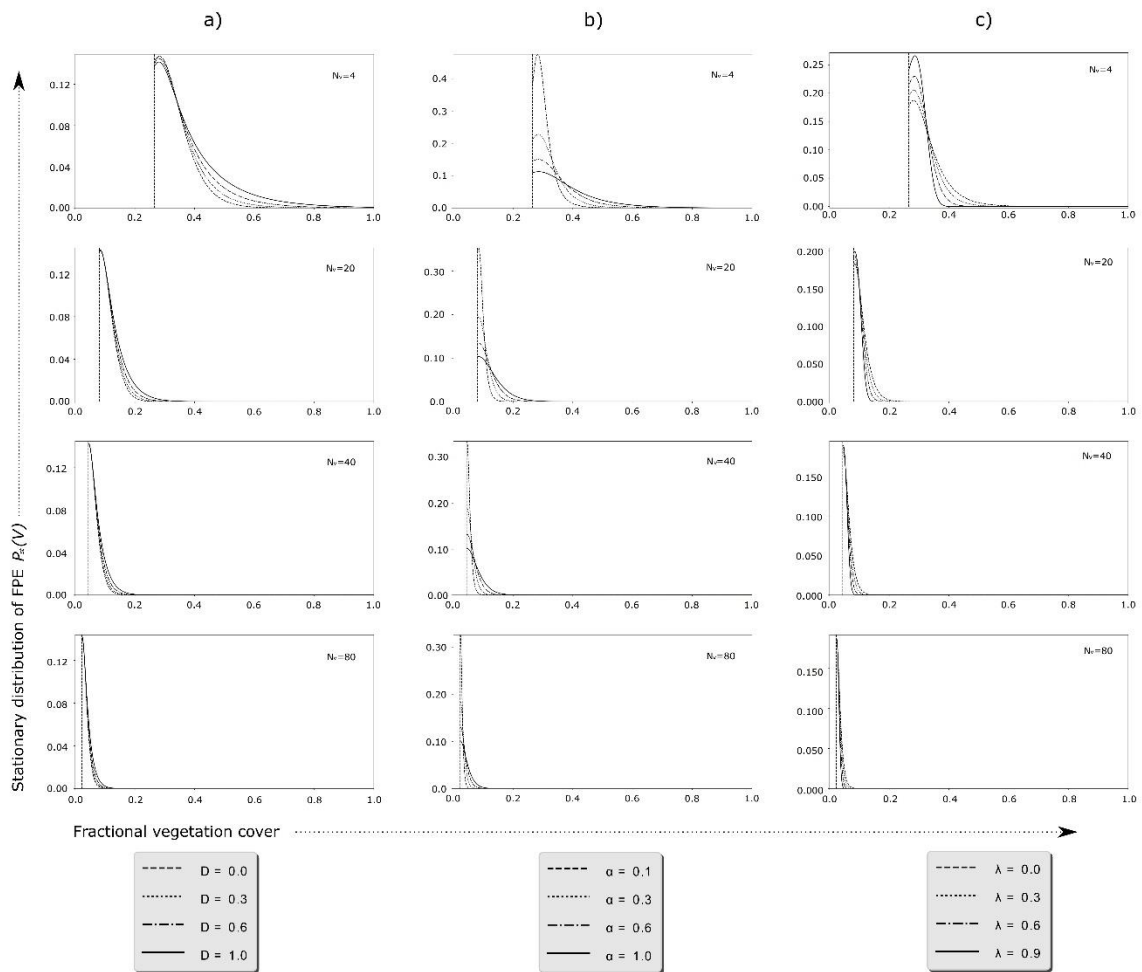
449 **Figure 1.** Six years (2000 - 2005) average of percentage vegetation cover map derived from
450 MODIS VCF dataset for the Konkan region. The transect from A to B is the reference grid
451 for the swath averaged vegetation profile that has been extracted for all six years.



453 **Figure 2.** Comparative assessment of the actual and modelled steady state vegetation profiles
 454 for six representative years. Note that the linear model consistently underpredicts the
 455 vegetation cover in the upstream part while overpredicts in the downstream part. Contrary to
 456 this, the logistic growth model serves as a better estimate all along the transect.

457

458



460 **Figure 3.** SPD distribution for the logistic growth model with respect to the vegetation cover.
 461 Panel a), b) and c) exhibits the effect of the N_v on the SPD for different noise parameters. In
 462 panel a), we fixed $\lambda = 0.1$ and $\alpha = 0.5$ to showcase the effect of multiplicative noise. The
 463 effect of additive noise is displayed in panel b) by fixing $\lambda = 0.1$ and $D = 0.4$. Panel c)
 464 illustrates the effect of correlation between the two Gaussian noises where D and α have
 465 been fixed to be 0.3.