Steady state analysis of vegetation growth models with correlated white noises

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Key Points:
- Comparison between logistic and linear vegetation growth model coupled with a surface erosion model.
- Steady state equilibrium vegetation profile along slope suggests logistic growth model is suitable.
- Stationary probability distribution shows the effect of Gaussian noises in vegetation growth.
Abstract
Vegetation community plays a pivotal role in geomorphic processes. However, the growth of vegetation intrinsically depends on the effective shear stresses exerted by the flow of material (e.g. water or soil) along the slope. We comparatively assess the growth and decay of vegetation using linear and logistic growth model coupled with a runoff erosion model. The model parameters are calibrated with normalized vegetation cover along a slope from Western Ghat escarpment. The deterministic model suggests that the logistic growth model is better predictor of vegetation profiles along a slope transect. Additionally, we propose a stochastic model to capture the role of internal or external factors in the dynamics of vegetation growth using two Gaussian noises. The steady probability distribution functions from the stochastic model provide insight about the role of different noises on the reaction of the system and suggest that bio-environmental factors are difficult to separate out.

Plain Language Summary
Earth surface is shaped by different surface processes which are controlled by the plant community. They restrict the erosion process by binding the soil. However, the vegetation community is also removed by the same processes that shape the earth’s surface while the remaining vegetation tends to grow naturally. We are trying to model the balance between the growth and decay which will eventually provide us the amount of vegetation on a slope. While this is one part of the complex interrelated processes, the other aspect deals with the randomness in growth and decay of vegetation. This randomness is primarily driven by either the environmental factors (e.g. rainfall, solar radiation or diseases leading to destruction of vegetation) or inherent to the vegetation species (sudden growth or mortality). Due to these external or internal factors the aforementioned model of vegetation growth and decay falls short. Our aim is to check, how the external or internal factors attribute to the change in vegetation growth.
1. Introduction

Vegetation community is efficient to enrich its condition through growth, decay and sustenance by virtue of inherent physico-chemical processes (Wilson & Agnew, 1992). The spatio-temporal modulation in vegetation mass is greatly influenced by the coupled amalgamation of fluvial hydrodynamic regime, hillslope configuration, climatic factors and soil cover which acts as feedback mechanism to modify the geomorphic features of earth’s surface (Tucker & Bras, 1999). In addition to this, the response of vegetation to the environmental elements affecting the geomorphic variabilities is rather complex with inherent nonlinearities and stochasticity rooted within the system.

The earliest vegetation growth model of forest cover system was elaborated by Botkin et al., (1972) where the environment was considered as carrying capacity limited. Subsequently, over the past few decades, there has been significant contribution of exploring the vegetation growth utilizing linear (Collins et al., 2004), exponential or logarithmic relationship between plant cover and biomass (Flanagan et al., 2007; Martinez et al., 2008) and predator-prey models (Kallay & Cohen, 2008; Tanner, 1975; Yoshida et al., 2003). Thornes (1985) initiated the pioneering step and introduced the concept of a coupled system for vegetation growth with a logistic growth of vegetation and slope dependent erosion model. The intricate details of the evolution of vegetation has been further explored using the CHILD numerical tool (Tucker et al., 2001) in various hydro-climatic conditions which capture certain complexities of the physical processes involved.

Deterministic models of many real-world phenomena are a difficult task owing to the fact that the various variables and parameters of the system can behave randomly within a similar environment. Therefore, in several instances, it fails to incorporate this stochasticity of coupled biophysical systems. Noise induced phenomena for vegetation growth and resilience have been widely examined by various scholars in differing hydro-climatic conditions. These studies include the feedback mechanism between soil moisture (Borgogno et al., 2007; D’Odorico et al., 2005), water table (Ridolfi et al., 2006), stream flows (Camporeale & Ridolfi, 2007) or geomorphology (Muneepeerakul et al., 2007; Vesipa et al., 2015) with riparian vegetation. Although, a significant amount of study has been undertaken, very limited understanding has been provided with calibration of model parameters using actual vegetation cover data set.

In this work, an attempt has been made to couple a logistic vegetation growth model with a wash profile model (Tucker & Bras, 1999) to evaluate the model predictions with previously available analytical solutions of linear growth model. The novelty of the present study lies in the fact of calibration methodology of the coupled model of vegetation growth using actual vegetation cover dataset. Furthermore, we have implemented a steady state stochastic model to analyse the bioenvironmental stochasticity and their effect on the steady state distribution of vegetation cover. The modelling approach and its results makes an effort to address two major issues: (1) which is a better growth model (linear or logistic) in case of a coupled system? (2) How does the noise-induced phenomena affect the steady state probability distribution of the vegetation?

2. Methods and Solution Scheme

2.1. Deterministic vegetation growth model

We follow a modelling scheme on similar lines of Tucker & Bras (1999). However, the present formulation takes into account the vegetation proliferation as a logistic growth model that considers the growth of a particular vegetation species is dependent on the existing fractional cover of vegetation (Collins & Bras, 2010).
2.1.1. Logistic growth model

Our modelling scheme utilizes the logistic vegetation growth (Collins et al., 2004) with a model of wash profile (Tucker & Bras, 1999). Unlike the linear growth models, logistic model captures the reproduction limited and resource limited condition (Thornes, 1990). This yields to the following mathematical relation

\[
\frac{dv_g}{dt} = K_{vg} V (1 - V)
\]  

(1)

\(v_g\) is vegetation growth, \(K_{vg}\) is rate of growth of vegetation on the unvegetated surface. Reciprocal of the vegetation regrowth rate implies the time taken by a plant community for regrowth.

In natural system, plant community are removed from the soil by various means. However, we consider that the loss of vegetation is primarily by virtue of the channel and riparian processes. The simplest physical process for removal of the vegetation cover will depend on the excess shear stress.

\[
\frac{dv_e}{dt} = -K_{vd} V (R_f \tau - \tau_c)^\eta
\]  

(2)

\(v_e\) denotes the vegetation erosion, \(K_{vd}\) is the species-dependent erosion parameter, \(R_f\) is the factor of friction, \(\tau\) and \(\tau_c\) are the shear stress and effective critical shear stress.

The effective critical shear stress is posed as a sum of critical shear stress for pure unvegetated surface (\(\tau_{es}\)) and critical shear stress under 100% vegetation cover (\(\tau_{cv}\)).

\[
\tau_c(V) = \tau_{es} + V \tau_{cv}
\]  

(3)

Combining the erosion and growth terms (Eq. (1) and Eq. (2)) the governing equation yields the following form:

\[
\frac{d(v_g-v_e)}{dt} = \frac{dv}{dt} = K_{vg} V (1 - V) - K_{vd} V (R_f \tau - \tau_c)^\eta
\]  

(4)

2.1.2. Steady state solutions

For simplicity purposes we assume \(\eta = 1\) i.e., the erosion law follows linear function. \(\tau_{es}\) is considered as zero as we have idealized that bare soil surface does not introduce resistive shear stress. All the physical quantities, which have been taken into account to model the vegetation growth, have been converted to non-dimensional quantities for ease of computation.

The final form of the non-dimensional steady state equation for fractional vegetation cover (VCF) is

\[
V - V^2 \{1 + N_v N_{rv} (N_{e}^q x^{q} + V)\} + N_v V^2 = 0
\]  

(5)

\(N_v\) is the vegetation number which describes the growth relative to destruction. \(N_{rv}\) signifies the friction coefficient, \(N_e\) is the erosion number that relates the shear stress with distance and \(q\) is the non-dimensional exponent that explains the non-linearity in the process involved.

Solution of Eq. (5) yields \(V = 0\) and the other two roots are

\[
V = \frac{(N_v - N_v N_{rv} N_{e}^q x^{q}) \pm \sqrt{(N_v - N_v N_{rv} N_{e}^q x^{q})^2 + 4 N_v N_{rv}}}{2 N_v N_{rv}}
\]  

(6)
The first solution \((V = 0)\) corresponds to the specific condition where there is no vegetation along the slope. The positive root among the other two roots has been considered for evaluation of the VCF for steady state logistic growth model since there is no physical significance of negative vegetation cover.

The steady state solution of linear vegetation growth model has also been evaluated for the calibration procedure. The solution is

\[
V = \frac{1}{1 + N_v N_e^q x'^q} \tag{7} \quad (\text{Tucker & Bras, 1999})
\]

2.2. Stochastic vegetation growth model

We consider two prominent sources of stochasticity in the evolution of vegetation. The inherent characteristics of the vegetation community has been coined as ‘intrinsic’ noise. On the other hand, the external factors, viz. inhomogeneity in precipitation amount, spatial variation of temperature, soil moisture retention capacity, ground-water table variability, aspect of slope etc. apparently serve as ‘extrinsic’ noise. In subsequent sections, we describe that the separation of the intrinsic and extrinsic noise is difficult owing to the fact of complex interrelationship between the external and internal factors with the system.

2.2.1. Formulation of stochastic model

The stochastic vegetation model with logistic growth is driven by two white Gaussian noises \(\epsilon(t)\) and \(\Gamma(t)\) termed as (negative) additive and multiplicative noise respectively. One-dimensional Langevin equation with two correlated Gaussian white noises \(\epsilon(t)\) and \(\Gamma(t)\) with a non-zero correlation between the multiplicative and negative additive noises leads to

\[
\frac{dV}{dt} = V + C_1 V^2 + C_2 V^3 + V\epsilon(t) - \Gamma(t) \tag{8}
\]

where

\[
C_1 = N_v - N_v K_{rv} N_v^q x'^q - 1 \tag{9}
\]

and

\[
C_2 = -N_v K_{rv} \tag{10}
\]

The Gaussian noises have zero mean and are defined as,

\[
\langle \epsilon(t)\epsilon(t') \rangle = 2D\delta(t - t') \tag{11}
\]

\[
\langle \Gamma(t)\Gamma(t') \rangle = 2\alpha\delta(t - t') \tag{12}
\]

\[
\langle \epsilon(t)\Gamma(t') \rangle = \langle \epsilon(t')\Gamma(t) \rangle = 2\lambda\sqrt{D\alpha}\delta(t - t') \tag{13}
\]

\(\lambda\) denotes the degree of correlation between the noises \(\epsilon(t)\) and \(\Gamma(t)\). \(D\) and \(\alpha\) are the strength of the noises \(\epsilon(t)\) and \(\Gamma(t)\) respectively.

2.2.2. Steady state analysis

We derive the Fokker-Planck equation (FPE) (Ai et al., 2003; Da-Jin et al., 1994; Li et al., 2015) for estimation of steady state of probability density function corresponding to Eq. (8) which is of the following form,

\[
\frac{\partial P(V,t)}{\partial t} = \frac{\partial A(V)P(V,t)}{\partial V} + \frac{\partial^2 B(V)P(V,t)}{\partial V^2} \tag{14}
\]

where \(P(x,t)\) is the probability density and
\[ A(V) = V + C_1V^2 + C_2V^3 + D V + \lambda \sqrt{D} \alpha \]  

(15)

\[ B(V) = DV^2 + 2\lambda \sqrt{D} \alpha V + \alpha \]  

(16)

The stationary probability distribution of FPE is given by

\[ P_{st}(V) = \frac{N}{B(V)} \exp \int_0^V \frac{A(V')}{B(V')} dV' \]  

(17)

where \( N \) is a normalization constant. In addition, the extrema of \( P_{st}(V) \) obeys a general equation

\[ A(V) \frac{dB(V)}{dv} = 0. \]

It leads to

\[ C_2V^3 + C_1V^2 + (1 - D)V - \lambda \sqrt{D} \alpha = 0 \]  

(18)

If \( \lambda = 0 \) then there exists no correlation between the two types of noises. This shows that there is no such dependency on (negative) additive noise at the extrema position \( V = 0 \) and

\[ V = -\frac{\sqrt{c_2^2 - 4c_1(1-D)}}{2c_2} \]

of the Stationary Probability Distribution (SPD) of FPE for zero correlation.

3. Data, Calibration and Parameterization

We use MODIS Vegetation Continuous Field (VCF) product (MOD44B) for the years 2000-2005 for calibration of the parameters for linear and logistic growth model. A small transect of Western Ghat escarpment is chosen for the current study. A swath average vegetation profile of 15 km wide and 80 km long stretch (as shown in Figure 1) has been accounted for in the present context. Observed vegetation data has been transformed into non-dimensional vegetation cover with respect to the maximum VCF value within the particular transect. Distance has been non-dimensionalized with respect to the total length of the transect. We have idealized that the linear shear stress model for vegetation erosion holds true for overland flow (Dietrich et al., 2003) as the swath average profile covers the channel as well as the valley region.

Our calibration scheme provides a simplified approach to validate the existing model outcomes with an available dataset. We have optimized the \( N_v \) value by a brute force for each model run so that the Root Mean Square Error (RMSE) between the modelled and the observed VCF is minimum. Once the optimal value of \( N_v \) is obtained, we reiterate the same scheme with variable \( K_{rv} \). The erosion number \( N_e \) is primarily a function of uniform rate of erosion (\( E \)) and coefficient of erosion (\( K \)) (Tucker & Bras, 1999). Considering homogeneity and constant critical shear stress along the slope as well as uniform and constant erosion rate (\( E \)), we have relaxed the effectiveness of erosion number, \( N_e \) and assumed that the value of \( N_e \) is 10.

The integral in Eq. (17) has been estimated numerically, with the logistic growth model and varied the noise parameters and \( N_v \). We have plotted the curves of the SPD after varying the value of one particular stochastic noise parameter among \( \lambda, D, \alpha \) and fixing the value of other two parameters. Since, in the deterministic model, the vegetation cover \( V \) is a function of the normalized position \( x \), therefore in stochastic model, SPD has been considered as an implicit function of \( V \) and \( x \). The optimal value of \( N_v \) from the calibration of the logistic model provide the stable solution in terms of the probability distribution of the vegetation cover. We have considered those numeric values of \( N_v \) which optimize the minimum error obtained from the deterministic model as discussed in earlier section of the article. Also, the range of \( V \) has been taken based on the actual vegetation cover data to plot the SPD.
4. Results and Discussion

4.1. Steady state vegetation profiles and sensitivity of deterministic model parameters

Non-dimensional actual fractional vegetation profile reveals that for most of the years, ~ 60-80% reduction of vegetation cover occurs within ~ 20-40% of the initial length of the total transect (Figure 2). This suggests a steady decrease of actual vegetation cover in upstream zone of the transect, although fluctuation of the VCF is easily observed along the entire transect. The prominent observable fact is moderate increase of the vegetation cover after ~60% of the total transect. It is worth to note that this moderate increase of vegetation cover is more than the existing vegetation cover between ~20-40% of the total transect. Therefore, a steady decrease of vegetation away from the divide does not always hold true.

Solutions for the best fit linear model exhibit that the equilibrium vegetation profile declines ~50% within less than initial ~10% of the total transect i.e. adjacent to the hilltop region (Figure 2). After ~20% of the transect length, the VCF for linear model shows a very low rate of decrease in the downstream. The logistic model describes the steady decrease of vegetation cover. We observed ~40-50% reduction of the non-dimensional vegetation cover takes within ~30-40% of the total distance of the transect (Figure 2). The prime important fact to note is that the vegetation cover decreases steadily for logistic growth model and tends to match visually more similar than the vegetation profile of linear growth model.

In order to identify the commonalities and discrepancies between model and actual vegetation cover data, we have assessed Root Means Square Error (RMSE) as a metric for error. RMSE is lower in all the years for logistic growth model when compared with the linear one. Unlike the linear model, the logistic model portrays a steady decrease of vegetation cover which can be supported by the observed dataset. The best fit $N_p$ values for the logistic growth model is always higher ($N_p = 1 – 10$) than the linear model ($N_p = 0.4 – 2$). The difference of $N_p$ values can be considered as the inherent characteristics of the model formulation and attributing the conceptualization of the modelling and solution scheme. Error is consistently minimum for $K_{rv} = 0.7$. This indicates a high coefficient of resistance offered by vegetation possibly due to higher vegetation cover in the Western Ghat.

The most interesting outcome of the present work is calibration of $N_p$ and $K_{rv}$ with the help of the real vegetation cover dataset. The main driving force of the growth of the vegetation is assessed as the availability of moisture content, slope aspect (Stephenson, 1998) or land surface temperature (Weng et al., 2004). $N_p$ is the critical parameter which controls the growth and decay of the vegetation simultaneously and therefore it includes all of the aforesaid effects ($N_p = K_{vd} \times \tau_{cv}$). Inclusion of all the effects reduced the complicated problem into a single vegetation number. In our results $N_p$ reflects a very low vegetation number in comparison to most of the model parameter values adopted in the other study (e.g., Collins et al., 2004).

4.2. Role of noise induced phenomena in vegetation distribution

We show the effect of the Gaussian noises, degree of correlation between these two noises and the vegetation number $N_p$ parameter in Figure 3. In all three cases of the noise induced system, the peak of SDP shifts left as the $N_p$ increases. This feature is universal and common, because vegetation number actually defines the ratio between decay and growth parameters. Therefore, as $N_p$ increases, the vegetation cover decreases and value of $P_{st}(V)$ peaks for small vegetation cover. In other words, the overall vegetation cover disappears for high value of $N_p$. However, the change in the strength of the noises with low $N_p$ values does not affect the position of the maxima of the SPD.
Figure 3, panel a represents the effect of multiplicative noise \((D)\) that acts as a constructive force by increasing the vegetation cover. We find \(P_{st}(V)\) is weakly affected by the strength of \(D\) when the degree of \(\lambda\) and the strength of \(\alpha\) is fixed corresponding to any value of \(N_v\). The prominent cause of the similarity of different SPD is primarily due to the normalization factor that stretches the vegetation cover between 0 to 1. SPD can be distinguished for small value of vegetation number \((N_v = 4)\) while, when it is increased to the tune of 80 the SPD are barely separable. At low vegetation cover \((<-0.35)\) \(P_{st}(V)\) decreases; on the contrary at high vegetation cover \((\sim 0.4 – 0.8)\) it increases as the strength of \(D\) is increased. As value of \(N_v\) increases, the difference in \(P_{st}(V)\) is indistinguishable as destruction of vegetation is enhanced with higher decay coefficient. This reflects the fact that, the vegetation cover along the transect is not significantly influenced by the multiplicative noise when \(N_v\) value is quite high. This high value of \(N_v\) sets the stage for a certain extinction of vegetation. One can appreciate another fact that with increasing \(N_v\), \(P_{st}(V)\) for higher vegetation cover is always high.

The role of the (negative) additive noise strength \((\alpha)\) on the SPD with the fixed \(\lambda\) and \(D\) is described in Figure 3, panel b. With increasing the strength of \(\alpha\), we observed that the peak of the \(P_{st}(V)\) reduces for any value of \(N_v\). Although the magnitude of \(P_{st}(V)\) decreases for lower vegetation cover, it is actually higher for higher vegetation cover (Figure 3, panel b). Therefore, as the strength of \(\alpha\) is increased, the magnitude of \(P_{st}(V)\) for small vegetation cover decreased while for high vegetation cover increased. This is indicative to the fact that the (negative) additive noise is actually equalizing the vegetation distribution along the profile by reducing the \(P_{st}(V)\) at small vegetation cover. Figure 3, panel c offers the effect of correlation between the two Gaussian noises on the SPD. It is evident that as the correlation strength \((\lambda)\) increases, the probability for the smaller vegetation cover values increases, then drops sharply around 30% of vegetation coverage. For smaller values of \(N_v\), \(P_{st}(V)\) increases for lower VCF \((\sim <0.35)\) and decreases when the vegetation cover is higher \((>\sim 0.4 – 0.5)\). This implies that higher values of \(\lambda\) promotes the destruction of the overall vegetation pattern. We observed that on increasing the strength of \(\lambda\) at low \(N_v\) value, position of peak of \(P_{st}(V)\) remains stationary.

4.3. Implication of the proposed model

For the first time, the current study attempts to present a direct calibration of the coupled biophysical model by extracting the model parameters with the actual vegetation cover dataset. Most of the previous studies (Collins & Bras, 2010; Collins et al., 2004; Istanbulluoglu & Bras, 2005) fall short in calibrating the model parameters for the actual vegetation, as the prime interest was to explore the effect of vegetation on the relief and drainage development. In addition to this, we asserted that the logistic growth model dictates the actual vegetation cover better than the linear growth model. Best fit models for the linear growth underestimate vegetation cover in the upslope region, however, it overestimates the vegetation cover in the downslope (Figure 2). Logistic growth model depicts the nature of VCF distribution more accurately because of its inherent property of growth in resource limited condition. Low value of the \(N_v\) suggests that integrated coefficient of vegetation mortality and shear stress is not more than 10 times of the coefficient of vegetation growth for this vegetation type.

All characteristic curves of Figure 3 indicate that the multiplicative noise does not act as a drift term unlike discussed in Ai et al., (2003), and vegetation community remains stationary with a fixed peak of \(P_{st}(V)\). However, one can consider the (negative) additive noise as a diffusive term which results in reduction of vegetation growth and flattens the peak of \(P_{st}(V)\). We also observed that the position of the maxima of \(P_{st}(V)\) is not at all affected by the
strength of noises. Therefore, we suggest that the intensity of different Gaussian noises does not effectively drive the system to an effective growth or destruction. However, these noises effectively reshape the SPD by decreasing or increasing the magnitude of $P_{st}(V)$ at a certain extremum of VCF. Segregation of the intrinsic and extrinsic factors in the evolution of the vegetation is difficult due to their complicated behaviour. When the internal factors predominantly influence the system, it results in an increase in vegetation growth and VCF. The external factors on the contrary, delay the spread of the vegetation cover.

Additionally, the value of $P_{st}(V)$ at higher vegetation cover is also higher when we increased the strength of $\alpha$. Our results are on similar lines as that of the observations reported by Ai et al. (2003). This could be attributed to the erosion model and steady distribution of the vegetation profile. In the erosion model, erosion rate increases from upstream to downstream.

Therefore, the rate of vegetation destruction is lower in the upstream region. The amount of vegetation cover is also higher in the upstream region. We suspect that the combined effect of higher vegetation cover and the lower erosion rate in the upstream region results in lower sensitivity of the vegetation destruction. This particular phenomena should be further investigated.

4.4. Revisiting modelling assumptions

The spatial resolution of our vegetation dataset is 250 m and therefore it does not distinguish between the vegetation within the channel and in the floodplains. In general, channels are devoid of vegetation owing to the fact that the fluid motion does not promote vegetation growth. The excess shear stress model, used in numerous other studies, has been previously implemented as idealized cases of transport and incision process within the channel (Baldwin et al., 2003; Whipple, 2004). However, numerical models take into consideration a single transport law for both channel as well as surface wash processes (Dietrich et al., 2003). This justifies our data preparation process elaborated in Section 3 and adaptation of the model. Idealized value of $N_e$ is another simplification of the erosion model as we do not consider substantial change in erodibility downslope. Erodibility at a regional scale varies significantly if the landscape encounters a set of different lithology or climatic condition. Similarly, we do not consider that the friction factor $k_{rv}$ changes substantially in order to retain simplification of the model. We have also kept it constant in view that the scale of the transect of vegetation profile is small enough to idealize it as a constant friction condition.

The major issue with the logistic model is that the model does not implicate $V = 1$ at $x = 0$. It is acknowledged as a small limitation of our model formulation with the logistic growth. In spite of this, our model solution present better estimates than the existing linear model. Additionally, we do not intend to present sensitivity of the growth, decay, friction, lithologic and noise parameters and this is beyond the objective of the present work. However, the sensitivity analysis can shed light on the role of noise levels on the steady distribution.

Western Ghat is considered as a biodiversity hotspot (Cincotta et al., 2000) dominated by various species of flora. Therefore, the most important simplification of the present model is the constant $N_p$ that incorporates growth, decay and shear stress. This implies that the slope is dominated only by a single particular type of vegetation and there is no inter species interaction. We have lumped the factors of multiple species into one vegetation number and did not consider model for intra or inter-species competition.

We have characterized the multiplicative noise as a positive role playing agent while the additive noise plays a negative role. One can argue about the character of these noises and may idealize them differently. Additionally, it is nearly impossible to segregate the internal and external factors that lead to environmental stochasticity. Factors such as, solar radiation, precipitation or nutrients generally augment the growth of the vegetation. However,
anomalous amount and intensity of these factors can lead to a probable destruction of vegetation cover as well. For example, increased rainfall can lead to higher runoff which can eventually result in vegetation destruction. Similarly, intrinsic character of the vegetation species can simultaneously increase or decrease the vegetation cover along a slope.

5. Conclusions

Here, we have proposed and presented the solutions for a logistic growth model of vegetation and a novel stochastic model with two Gaussian noises. We affirm that the logistic growth model predicts a better estimate of the VCF along a slope. A low vegetation number calibrated from the model needs further investigation to interpret the interaction between the growth and decay of vegetation community. Biological evolution is always regarded as a stochastic system and this gave the motivation to explore the effect of random noises in the vegetation growth along a slope profile. The Gaussian noises and their correlation parameter implicate a stable change in the SPD. Additionally, in the context of the noise level we have chosen, the vegetation growth system does not shift towards immediate sporadic growth or extinction. We observed anomalous effect of the (negative) additive noise which needs further elaboration. We conclude that the effects of different intrinsic and extrinsic noises are difficult to separate out due to complex interrelationship between the environment and biological community.

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References


Figure 1. Six years (2000 - 2005) average of percentage vegetation cover map derived from MODIS VCF dataset for the Konkan region. The transect from A to B is the reference grid for the swath averaged vegetation profile that has been extracted for all six years.
Figure 2. Comparative assessment of the actual and modelled steady state vegetation profiles for six representative years. Note that the linear model consistently underpredicts the vegetation cover in the upstream part while overpredicts in the downstream part. Contrary to this, the logistic growth model serves as a better estimate all along the transect.
Figure 3

Panel a), b) and c) exhibits the effect of the $N_v$ on the SPD for different noise parameters. In panel a), we fixed $\lambda = 0.1$ and $\alpha = 0.5$ to showcase the effect of multiplicative noise. The effect of additive noise is displayed in panel b) by fixing $\lambda = 0.1$ and $D = 0.4$. Panel c) illustrates the effect of correlation between the two Gaussian noises where $D$ and $\alpha$ have been fixed to be 0.3.

Figure 3. SPD distribution for the logistic growth model with respect to the vegetation cover.