Lunar Formation via Triple Phase Transition in the Differentiating Proto-Earth

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2026

Abstract. The origin of the Moon remains one of the unsolved problems of planetary science. The canonical giant-impact model faces growing geochemical difficulties: it does not naturally predict the near-perfect Earth–Moon isotopic identity, the crustal dichotomy, or the ≈ 300 Myr delay of the terrestrial dynamo. This work submits to examination an alternative grounded in a necessary thermodynamic observation: any Earth-mass planet emerges from accretion in a state of near-total silicate mantle melting, accretion energy exceeding total fusion energy by a factor ≈ 155 (Solomatov, 2000; Elkins-Tanton, 2012; Rubie et al., 2015). The proto-Earth is therefore a rapidly rotating magma body ($T_{\text{rot}} \approx 3.5$ h) without a stabilising satellite, whose axis wobbles within $[40°, 70°]$ in its own co-rotating frame. In this context, a single driver—progressive Fe-Ni segregation—governs three coupled transitions: a rheological transition (structuring of the Coherent Magmatic Torus, CMT), a mechanical transition (episodic hypersonic ejections governed by a double potential well and the Kramers stochastic crossing rate), and a magnetic transition (dynamo delayed by ≈ 350 Myr, with no free parameter). The central prediction—seismic interface(s) between 200 and 530 km depth, impedance contrast $|R| \in [0.01, 0.04]$—is testable by **Chang'e 7** (South Pole, 2026), **FSS** (Farside Seismic Suite), **LEMS** (Lunar Environment Monitoring Station), and **Artemis III** (2028–2029). A complementary test based on the South Pole–Aitken basin (SPA) predicts that deep mantle ejecta exposed there exhibit Fe/Si significantly higher than any terrestrial mantle rock and higher than surrounding mare basalts, testable by **Chang'e-6** (2026). All hypotheses are explicit and ranked. Limitations are acknowledged. The validation programme is defined and sequenced.

Keywords: lunar origin · Triple Phase Transition · chaotic axis wobble · Hadean proto-Earth · Bingham-Herschel · Coherent Magmatic Torus · double potential well · Kramers rate · Mathieu resonance · seismic interface · Hadean dynamo · falsifiable predictions · South Pole–Aitken.

Epistemic posture

This work submits a theory to examination by the scientific community. The formulations deliberately reflect this posture: the theory describes, predicts and constrains—it does not assert absolute certainties. All hypotheses are numbered and ranked. All limitations are acknowledged. Final validation belongs to forthcoming lunar seismic data and, in a second step, to three-dimensional numerical simulations.

1 Introduction

1.1 The Problem of Lunar Origin

The giant-impact hypothesis—collision between the proto-Earth and a Mars-sized body, Theia (Hartmann & Davis, 1975; Cameron & Ward, 1976; Canup, 2004)—has provided the reference framework for lunar formation for four decades. It qualitatively satisfies the angular momentum constraint of the Earth–Moon system (Čuk & Stewart, 2012; Canup, 2012), but faces growing geochemical difficulties: the near-perfect isotopic identity ($\Delta^{17}\text{O} < 5$ ppm, Wiechert et al. 2001; Dauphas et al. 2014; Dauphas 2017) is difficult to obtain without ad hoc adjustments, the crustal dichotomy is not naturally predicted, and the Hadean dynamo delay remains unexplained within this framework.

A recent review concludes that there is currently no unambiguous geochemical or isotopic evidence for the role of an external impactor in lunar formation (Sossi et al., 2025). This motivates the exploration of alternatives grounded in internal proto-Earth dynamics.

This work proposes such a mechanism. It does not claim to resolve the question definitively—final validation belongs to forthcoming lunar seismic data and 3D simulations. It offers a physically coherent framework, quantitative falsifiable predictions, and a clear hierarchy between what is established, what is conjectured, and what remains to be measured.

1.2 Three Discriminating Observational Constraints

Isotopic similarity. The Earth–Moon isotopic composition is quasi-identical for O, Cr and Ti, established to better than 5 ppm (Wiechert et al., 2001; Dauphas et al., 2014; Dauphas, 2017; Sossi et al., 2025). The present theory satisfies this constraint mechanically: ejected material is sampled from the contemporaneous Hadean magma at each ejection, with no exogenous isotopic source.

Iron depletion. The Moon is significantly less iron-rich than the terrestrial mantle (O'Neill, 1991; Jones & Drake, 1986). Fe-Ni segregation continually depletes the source magma between successive ejections.

Angular momentum. The angular momentum budget requires a very rapidly rotating proto-Earth at formation. The value adopted, $T_{\text{rot}} = 3.5$ h, is consistent with planetary accretion simulations (Agnor et al., 1999; Kokubo & Genda, 2010).

1.3 Architecture of the Theory

Established result

One driver, three coupled transitions. Progressive Fe-Ni segregation toward the forming core simultaneously governs:

1. a **rheological transition**: structuring of a Coherent Magmatic Torus (CMT) in the oblique Hadean intertropical band $|\phi| < 30°$;
2. a **mechanical transition**: episodic hypersonic ejections governed by a double potential well $V(U)$ and the Kramers crossing rate;
3. a **magnetic transition**: delayed emergence of the terrestrial dynamo via three independently constrained saturation steps.

2 Initial State of the Proto-Earth

2.1 Near-Total Melting: a Necessary Thermodynamic Consequence

The accretion energy in the homogeneous-sphere approximation (Safronov, 1969):

$$E_{\text{grav}} = \frac{3\,G\,M_{\oplus}^2}{5\,R_{\text{eq,proto}}} \approx 2.49 \times 10^{32}\,\text{J}. \tag{1}$$

The energy required to melt the entire silicate mantle (Stixrude & Lithgow-Bertelloni, 2014):

$$E_{\text{fus}} \approx M_{\text{mantle}} \times L_{\text{fus}} \approx 4 \times 10^{24} \times 4 \times 10^{5} \approx 1.6 \times 10^{30}\,\text{J}. \tag{2}$$

Established result

Energy ratio—a consensus result.

$$E_{\text{grav}}/E_{\text{fus}} \approx 155. \tag{3}$$

Established independently by Solomatov (2000), Elkins-Tanton (2012) and Rubie et al. (2015). This is not a hypothesis: it is the obligatory starting point of any serious theory of the Hadean period. The physically relevant question is not *how to melt the mantle*, but **how and how fast to cool it**.

Additional heat sources: Fe-Ni segregation heat ($\approx 3 \times 10^{30}$ J, Flasar & Birch 1973; Rubie et al. 2003), short-lived radioactivity (^{26}Al, ^{60}Fe, Urey 1955; Huss et al. 2009), and continuous planetesimal bombardment (Elkins-Tanton, 2011). Together they maintain surface temperatures between 2 000 and 4 000 K, preventing any stable protocrust (Solomatov, 2000).

Hypothesis – explicit status

H0—Total volumetric melting. At the onset of Fe-Ni segregation, the entire silicate mantle is above the liquidus, with no stable solid internal interface during the active window 4.568–4.4 Ga. This is a self-gravitating fluid volume, not a magma ocean resting on a substrate: the dynamics, rheology and response to perturbations are physically distinct from those of a surface ocean. Pressure waves generated by planetesimal impacts propagate throughout the entire volume.

Caveat: Nomura et al. (2011) suggest the deep lower mantle may not have been fully molten due to the increase of the solidus with pressure. The TPT requires melting only of the mass available in the Hadean intertropical band, which the factor 155 guarantees thermodynamically for the upper and mid mantle.

2.2 Initial Rotation Period: $T_{\text{rot}} \approx 3.5$ h

The initial rotation of the proto-Earth is not a free parameter. Three convergent physical mechanisms constrain it toward $T_{\text{rot}} \approx 2.5$–$3.5$ h.

1. T-Tauri magnetic torque. The young Sun, in rapid rotation (≈ 3 days; Bouvier 2014), magnetically couples the inner protoplanetary disk (Königl, 1991; Shu et al., 1994), transferring angular momentum toward inner-orbit material.

2. Angular momentum conservation during contraction. Contraction from $R_0 \approx 2\,R_\oplus$ to the final radius:

$$\frac{T_{\text{rot}}^{(f)}}{T_{\text{rot}}^{(0)}} = \left(\frac{R_\oplus}{R_0}\right)^2 \approx \frac{1}{4}. \tag{4}$$

Starting from a median $T_{\text{rot}}^{(0)} \approx 14$ h yields $T_{\text{rot}}^{(f)} \approx 3.5$ h.

3. Skater effect: Fe-Ni segregation. Fe-Ni migration toward the centre reduces the moment of inertia:

$$I_f/I_0 \approx 0.82, \qquad \Rightarrow \qquad T_{\text{rot}}^{(f)} \approx 0.82 \times T_{\text{rot}}^{(0)}. \tag{5}$$

Established result

$T_{\mathrm{rot}} \approx 3.5$ **h—consistent with the community consensus.** N-body simulations give rotation periods between 2 and 8 h, median 3–5 h (Agnor et al., 1999; Kokubo & Genda, 2010; Chambers, 2013). The value 3.5 h is the same as used by Čuk & Stewart (2012) and Canup (2012) in the giant-impact model. The remarkable emerging property is the Coriolis compensation parameter:

$$b' = \frac{2\Omega \sin\varepsilon \cdot U_{\mathrm{crit}}}{|g_{\mathrm{eff}}|} \approx 1.000 \quad \text{for } T_{\mathrm{rot}} = 3.5\,\mathrm{h},\ \varepsilon = 55^\circ. \tag{6}$$

The radial Coriolis force *exactly* compensates effective gravity at the ejection threshold. This is not postulated: it emerges from the equations. Over the full Hadean range, $b' \in [0.85, 1.35]$: qualitative Coriolis compensation is guaranteed for every plausible $(\varepsilon, T_{\mathrm{rot}})$ combination.

Robustness of b' over the Hadean range

The exact value $b' = 1.000$ is a numerical coincidence at the nominal point. The underlying physical property—radial Coriolis force of the same order as effective gravity—is robust over the entire Hadean range:

$$b'(\varepsilon, T_{\mathrm{rot}}) = \frac{2\sqrt{2}\,\Omega(T_{\mathrm{rot}})\,\sin\varepsilon\,\sqrt{R_{\mathrm{eq,proto}}}}{\sqrt{|g_{\mathrm{eff}}(\varepsilon, T_{\mathrm{rot}})|}}. \tag{7}$$

	$b'(\varepsilon, T_{\mathrm{rot}})$		
T_{rot}	$\varepsilon = 40^\circ$	$\varepsilon = 55^\circ$ (nominal)	$\varepsilon = 70^\circ$
2.5 h	0.89	1.12	1.31
3.0 h	0.94	1.05	1.16
3.5 h (nominal)	0.97	**1.000**	1.07
4.0 h	0.99	0.94	0.98

Neither $b' \ll 1$ (Coriolis negligible) nor $b' \gg 1$ (Coriolis dominant). The value $b' = 1.000$ at the nominal point is a remarkable numerical coincidence, not a necessary condition: the mechanism operates for any $b' \sim \mathcal{O}(1)$.

2.3 Proto-Earth Geometry: the Maclaurin Spheroid

A rapidly rotating fluid mass takes the shape of a Maclaurin spheroid (Chandrasekhar, 1969). The equatorial radius of the undifferentiated proto-Earth is $R_{\mathrm{eq,proto}} \approx 7.4$ Mm (vs. $R_\oplus = 6.371$ Mm today), combining three effects: reduced density (+4%), thermal expansion (+7.5%), and equatorial bulge (+7.5%). Dynamic flattening:

$$J_2 \approx \frac{5}{4} \cdot \frac{\Omega^2 R_\oplus}{g_0} \approx 0.101, \tag{8}$$

with equatorial bulge $\delta R \approx 642$ km. Effective gravity:

$$g_{\text{eff}}(\phi) = g_{\text{eff}}^{(0)}\left(1 + \tfrac{3}{2} J_2 \sin^2\phi\right) - \Omega^2\, r(\phi)\cos^2\phi. \tag{9}$$

Minimal in the band $|\phi| < 30^\circ$: the geometric attractor of the CMT.

3 Fundamental Geometric Clarification: Axis Wobble and the Hadean Intertropical Band

Geometric warning—read first

The Hadean rotation axis is not the present terrestrial axis. Today, the terrestrial axis is inclined at $23.5°$ to the ecliptic normal, stabilised at this value by the Moon. Lunar formation is precisely what this work seeks to explain: it would therefore be circular to invoke this stabilised value as an initial condition.

The range $\varepsilon \in [40°, 70°]$ adopted in the TPT does not result from a simple transposition of Laskar et al. (1993a,b), whose results concern the *present* Earth without the Moon (rotation at ≈ 24 h). These foundational works establish that the Moon shifts the terrestrial spin-axis precession frequency away from spin-orbit resonances that produce chaos. Without the Moon, the chaotic zone of the present Earth extends from $0°$ to $\approx 85°$. But this result does not apply directly to a rapidly rotating proto-Earth.

The precession constant satisfies $\alpha_{\text{prec}} \propto \omega$: **rapid rotation raises the precession frequency**, pushing the chaotic resonances identified by Laskar out of reach. Li & Batygin (2014), working with a primitive Earth (rapid rotation, absent Moon, primitive solar system architecture), conclude that without the Moon the obliquity is chaotic but *constrained between* $0°$ *and* $45°$ over billion-year timescales, and that the chaotic diffusion rate is slow enough not to preclude habitability. Néron de Surgy & Laskar (1997) and Boué et al. (2010) clarify that obliquity chaos depends on the ratio between spin precession frequencies and secular orbital frequencies, not merely on the presence or absence of a satellite.

For a proto-Earth at $T_{\text{rot}} \approx 3.5$ h, the precession is so rapid that Laskar-type chaotic resonance conditions are difficult to reach. Furthermore, during the still-active terminal accretion phase, each major planetesimal impact *abruptly resets* the spin vector: the regime is then stochastic, not secular. The range $\varepsilon \in [40°, 70°]$ therefore characterises the **normal misalignment** between the rotation axis and the principal axis of inertia, maintained by continuous perturbations on timescales $\tau_{\text{pert}} \ll \tau_{\text{relax}}$ (see §3.4).

The Hadean intertropical band $|\phi| < 30°$ has no connection with the present tropical zone at $\pm 23.5°$. It is defined perpendicularly to the *instantaneous* Hadean rotation axis—an axis that continuously sweeps all geographic latitudes in ≈ 47 h. It is therefore not geographically fixed. Over the duration of one ejection episode, it traverses the entire globe.

3.1 Definition of ε in the TPT

Throughout this work, ε denotes the instantaneous angle between the rotation vector $\boldsymbol{\Omega}$ and the principal axis of inertia of the Maclaurin spheroid, defined **in the co-rotating body frame**. It is not the astronomical obliquity in the classical orbital sense. It is a purely internal geometric angle describing the *wobble*—the free nutation—of the axis relative to the body's proper shape.

3.2 Transformation to Hadean Latitude

Let $\hat{\boldsymbol{\Omega}}_{\text{had}} = (\sin\varepsilon, 0, \cos\varepsilon)$ be the unit vector of the Hadean rotation axis in the co-rotating frame. For any particle at position $\boldsymbol{r}$, the **Hadean latitude** is:

$$\phi_{\text{had}}(\boldsymbol{r}) = \arcsin\left(\frac{x\sin\varepsilon + z\cos\varepsilon}{r}\right), \tag{10}$$

and the **Hadean longitude**:

$$\lambda_{\text{had}}(\boldsymbol{r}) = \arctan\left(\frac{\boldsymbol{r}\cdot\hat{\boldsymbol{e}}_2}{\boldsymbol{r}\cdot\hat{\boldsymbol{e}}_1}\right), \tag{11}$$

where $\hat{\boldsymbol{e}}_1 = (\cos\varepsilon, 0, -\sin\varepsilon)$ and $\hat{\boldsymbol{e}}_2 = (0, 1, 0)$.

Established result

The Hadean intertropical band is defined by $|\phi_{\text{had}}| < 30^\circ$ (Eq. 10), formulated in the Hadean co-rotating frame, not the present geographic frame. Any projection or spectral analysis must be expressed in this frame to be physically meaningful.

3.3 The Wobble as a Direct Force Source

The axis wobble implies $\dot{\boldsymbol{\Omega}} \neq \boldsymbol{0}$. Poincaré (1910) showed that this generates an Euler acceleration on each fluid particle:

$$\boldsymbol{a}_{\text{Euler}} = \dot{\boldsymbol{\Omega}} \times \boldsymbol{r}, \tag{12}$$

with radial projection:

$$\boldsymbol{a}_{\text{Euler}} \cdot \hat{r} = |\dot{\Omega}|\, r(\phi)\, \sin(\varepsilon + \phi). \tag{13}$$

This acceleration is maximal in the band $|\phi| < 30^\circ$ where $r(\phi)$ is largest (equatorial bulge): the wobble is therefore not merely context but a *direct energy source* for the CMT.

3.4 Maintenance of the Wobble in the Hadean Context

The relaxation time toward axis alignment for an isolated fluid body:

$$\tau_{\text{relax}} \sim \frac{\eta\, R^3_{\text{eq,proto}}}{G\, M^2_{\oplus}\, f} \approx 10^6\,\text{yr} \quad (\eta \sim 10^{1\text{–}3}\,\text{Pa\,s},\; f \approx 0.107). \tag{14}$$

Every Hadean perturbation (T-Tauri CMEs, planetesimal impacts, Fe-Ni segregation, giant-planet tides) acts on timescales far shorter. The misalignment $\varepsilon \in [40^\circ, 70^\circ]$ is the *normal condition* of a Hadean proto-Earth, not an exceptional state.

Perturbation	**Timescale**
T-Tauri eruptions (CME)	Days to weeks
Residual planetesimal impacts	10^3–10^5 yr
Thermal convection ($\Delta T \sim 2\,000$ K)	Continuous, $\sim 10^3$ yr
Fe-Ni segregation (inertia tensor reorganisation)	$\tau_{\text{seg}} \approx$ 30–50 Myr
Giant-planet tides	Continuous, variable

4 Complete Force Inventory

All forces, projections and equations are expressed in the instantaneous Hadean corotating body frame.

4.1 Conservative Forces—Structural

Effective gravity $g_{\text{eff}}^{(k)}$: decreasing episode by episode. At the oblique equator:

$$g_{\text{eff}}^{(0)} = g_0 - \Omega^2 R_{\text{eq,proto}} \sin\varepsilon \approx 8.51\,\text{m}\,\text{s}^{-2} \quad (T_{\text{rot}} = 3.5\,\text{h},\ \varepsilon = 55°). \tag{15}$$

At each episode k, a coupled double effect decreases it:

$$g_{\text{eff}}^{(k)} = \frac{G\,M_\oplus^{(k)}}{R_\oplus^2} - \left(\Omega^{(k)}\right)^2 R_\oplus \sin\varepsilon, \tag{16}$$

where $M_\oplus^{(k)} = M_\oplus^{(0)} - \sum_{j=1}^{k} M_{\text{ej}}^{(j)}$ and $\Omega^{(k)}$ decreases by angular momentum conservation at each ejection. Both effects act in the same direction: $g_{\text{eff}}^{(k)}$ decreases episode by episode, lowering the critical velocity and mechanically facilitating subsequent episodes.

Centrifugal force. $F_c = \Omega^2 r(\phi)\cos^2\phi$; $\approx$ 5% of g_0 at the equator. Concentrates matter in the intertropical bulge.

Rayleigh angular momentum gradient (Rayleigh, 1917). For $U \propto r^\alpha$, stability requires $\partial(rU)/\partial r > 0$. Contribution to the instability coefficient (term λ_3, Eq. 34): destabilising for $\alpha > 1$ (super-rigid profile).

Maclaurin tangential gravity gradient. $g_{\tan} \in [0.08, 0.55]\,\text{m}\,\text{s}^{-2}$ for $\phi \in [5°, 30°]$: ten to sixty times smaller than vertical g_{eff}.

4.2 Dynamic Forces—Destabilising

Radial Coriolis force.

$$f_{\text{rad}} = 2\,\Omega\,\sin\varepsilon \cdot U, \tag{17}$$

$$f_{\text{horiz}} = 2\,\Omega\,\cos\varepsilon \cdot U, \tag{18}$$

$$\frac{f_{\text{rad}}}{f_{\text{horiz}}} = \tan\varepsilon. \tag{19}$$

Established result

Geometric threshold $\varepsilon = 45°$—necessary condition. For $\varepsilon > 45°$: $\tan\varepsilon > 1$, the destabilising radial component dominates. For $\varepsilon < 45°$: the confining horizontal component dominates and the Taylor-Proudman constraint inhibits radial ejection. This threshold follows solely from Coriolis kinematics: no adjustable parameter enters. For $\varepsilon = 55°$: $f_{\text{rad}}/f_{\text{tot}} = \sin 55° \approx 0.82$.

Euler precession acceleration. Constitutes term λ_4 of the instability equation (Eq. 13): the direct dynamical signature of the axis wobble.

4.3 Taylor-Proudman Columns: Curved, Truncated, Channelling

In a rapidly rotating fluid (Ro $\ll 1$), the Taylor-Proudman theorem imposes $(\boldsymbol{\Omega} \cdot \nabla)\boldsymbol{u} = 0$, producing rigid columns parallel to $\boldsymbol{\Omega}$ in simple cylindrical geometry. This classical result does not apply as-is to the Hadean proto-Earth: five mechanisms curve and truncate these columns.

1. **Obliquity $\varepsilon \in [40°, 70°]$: $\boldsymbol{\Omega}$** is not aligned with the axis of equipotential surfaces. Columns cut obliquely through surfaces of constant effective gravity and cannot propagate indefinitely.
2. **Spherical curvature and β-effect:** $f = 2\Omega\cos\phi$ varies with latitude, curving columns along $f =$ const lines— direct analogy with Jupiter's zonal bands.
3. **Bingham-Herschel rheology:** a Taylor column in a BH fluid with yield stress τ_y is sharply truncated at the boundary where $\tau < \tau_y$. The band $|\phi| < 30°$ is precisely the zone where $\tau > \tau_y$: columns are confined there by the rheological physics itself.
4. **Wobble ($\dot{\boldsymbol{\Omega}} \neq \mathbf{0}$):** the Taylor-Proudman theorem holds for *stationary* $\boldsymbol{\Omega}$. As soon as $\dot{\boldsymbol{\Omega}} \neq \mathbf{0}$, the Euler acceleration breaks the condition $(\boldsymbol{\Omega} \cdot \nabla)\boldsymbol{u} = 0$. Columns cannot form stably.
5. **Radial density gradient:** Fe-Ni segregation creates a non-uniform profile $\rho(r, t)$. The Solberg-Høiland criterion (Solberg, 1936; Høiland, 1941)—generalising Rayleigh to the stratified case—curves columns according to the density profile.

Established result

Curved columns generate the toroidal flow. Their curvature deflects fluid in the azimuthal direction rather than inhibiting it. It is this deflection that feeds the toroidal mode $\ell = 1, m = 1$ selected by the inverse cascade. The poloidal/toroidal decomposition shows that $(\boldsymbol{\Omega} \cdot \nabla)\boldsymbol{u}_{\text{tor}} = 0$ automatically (Frigaard, 2017): the Taylor-Proudman constraint does not apply to the pure toroidal field.

4.4 Tidal Forces

Solar tide. Relative amplitude $\approx 10^{-4} g_0$. Contributes as low-frequency resonant forcing on obliquity.

Growing proto-lunar tide—Mathieu resonance. From episode 2 onward, the proto-satellite tide modulates the barrier height:

$$\Delta E^{(k)}_{\text{barr}}(t) = \Delta E^{(k,0)}_{\text{barr}} - A^{(k)}_{m}(t)\cos(n_{\text{Moon}}\, t), \tag{20}$$

giving the radial perturbation equation:

$$\ddot{\xi}_r = \left[\lambda + A^{(k)}_{m}(t)\cos(n_{\text{Moon}}\, t)\right]\xi_r, \tag{21}$$

which is Mathieu's equation (McLachlan, 1947). Parametric resonance bands exist for $n_{\text{Moon}} \approx 2\sqrt{|\lambda|}$ even when $\lambda < 0$. The growing proto-lunar tide is the **dominant trigger** for episodes 2 to N.

Giant-planet tides. Jupiter primarily: low-frequency perturbation on obliquity maintaining the wobble over 10^5–10^6 yr timescales.

4.5 Rheological Forces

Bingham-Herschel yield stress τ_y: ensures CMT cohesion and closes the TPT window irreversibly below $T^* \approx 1\,800$ K.

Kelvin-Helmholtz instability: suppressed by $\text{Bi}_{\text{KH}} \gg 1$ (Section 7).

Rayleigh-Taylor instability at the flux/atmosphere interface: suppressed by $\rho_{\text{flux}}/\rho_{\text{atm}} \approx 200$.

4.6 External Energetic Perturbations: the T-Tauri Sun

The T-Tauri Sun is not merely stellar context: it is an active coupled physical agent playing three distinct roles.

1. Rheological modulator. Surface temperature under T-Tauri irradiation (Stefan-Boltzmann law):

$$T_{\text{surf}} = \left[\frac{(1 - A_0)\, L_*}{16\pi\sigma_{\text{SB}}\, d^2} \right]^{1/4}, \tag{22}$$

with $A_0 \approx 0.1$ (magmatic albedo), giving $T_{\text{surf}} \approx 1\,800$–$2\,200$ K between flares and $2\,800$–$4\,200$ K during flares. This thermal maintenance keeps τ_y in the optimal range for ejections (10^2–10^4 Pa).

2. Impulsional CME trigger. Coronal mass ejections (10^{27}–10^{29} J per event, several per day, Airapetian et al. 2016): $\sim 10^9$–10^{10} opportunities over 10^6 yr.

3. Independent extinction mechanism. As the Sun joins the main sequence, T-Tauri activity declines, τ_y increases, and the CMT loses the required rheological accessibility.

4.7 Bulk Pressure Waves: Planetesimal Impacts

Each planetesimal impact generates a spherical pressure wave:

$$P_{\text{wave}}(r) \approx \frac{E_{\text{impact}}^{1/2}\, \rho^{1/2}}{4\pi\, r}, \tag{23}$$

propagating through the entire volume (not merely on a surface). Three roles: turbulent energy injection feeding the inverse cascade, local perturbation of $U(r)$ that may cross τ_y, and finite perturbation on ΔE_{barr} contributing to the Kramers rate.

4.8 Geostrophic β-Force

$$\beta = \frac{2\Omega\cos\varepsilon}{R_{\text{eq,proto}}} \approx 8.98 \times 10^{-11}\,\text{m}^{-1}\,\text{s}^{-1} \quad (\varepsilon = 55^\circ). \tag{24}$$

This gradient arrests the inverse cascade at the Rhines scale:

$$L_\beta = \sqrt{\frac{U_{\text{turb}}}{\beta}} \approx 3.34 \times 10^6\,\text{m} \approx 0.52\, R_\oplus, \tag{25}$$

corresponding to mode $\ell = 1$, $m = 1$.

4.9 Atmospheric Confinement Pressure

The Hadean silicate atmosphere ($T \approx 2\,000$–$4\,000$ K, $P \approx 10^5$–10^7 Pa): opposes lateral expansion of the flux during ejection, suppresses long-wavelength KH modes, and co-mobilises corotating zonal jets.

Force inventory—summary table

Force	**Rel. amp.**	**Role in TPT**
Effective gravity $g_{\text{eff}}^{(k)}$	1.000	Term λ_1 (stabilising)
Radial Coriolis	≈ 0.62	Term λ_2 (destabilising)
$L = rU$ gradient (Rayleigh)	$\sim 10^{-7}$	Term λ_3 ($\alpha > 1$)
Euler accel. (wobble)	variable	Term λ_4 (precession)
Tang. gradient (Maclaurin)	$\sim 10^{-2}$	Term λ_5
Centrifugal	≈ 0.05	Concentrates CMT
β-force	≈ 0.30	Selects $\ell = 1$
Solar tide	$\sim 10^{-4}$	Weak resonant forcing
Proto-lunar tide	variable	Mathieu resonance (ep. 2–N)
Giant-planet tides	weak	Maintains obliquity
Impacts: bulk waves	$\sim 10^{-2}$	Injection + trigger
T-Tauri CME	$\sim 10^{-10}$	First episode trigger
BH rheology τ_y	threshold	CMT cohesion
Atmospheric pressure	$\sim 10^{-3}$	Lateral confinement

5 Emergence of the Coherent Magmatic Torus (CMT)

5.1 Geostrophic Regime and Inverse Cascade

$$\mathrm{Ro} = \frac{U_{\text{turb}}}{2\,\Omega\,R_{\text{eq,proto}}} \approx 1.35 \times 10^{-4} \ll 1, \qquad \mathrm{Re} = \frac{\rho\,U_{\text{turb}}\,R_{\text{eq,proto}}}{\eta_{\text{eff}}} \approx 2.4 \times 10^{9} \gg 10^{5}. \tag{26}$$

The turbulent regime physically justifies the geostrophic inverse cascade. The cascade arrests at the Rhines scale L_β (Eq. 25), corresponding to mode $\ell = 1$, $m = 1$. The CMT is the spatial expression of this mode in the band $|\phi| < 30°$.

5.2 The Band $|\phi| < 30°$ as a Geometric Attractor

Four convergent and independent effects make the Hadean intertropical band the stable spatial attractor of the inverse cascade:

1. **Minimal β at the intertropical bulge**: $L_\beta \propto \beta^{-1/2}$ is maximal where β is minimal.

2. **Stable attractor for all** $\varepsilon \in [40°, 70°]$: $|\phi| < 30° \subset$ oblique IT zone for any Hadean ε.
3. **BH rheology breaks ergodicity**: τ_y traps energy in the band via a stress barrier at $|\phi| = 30°$.
4. **Rossby waves toward low latitudes**: $\beta > 0$ for all $\varepsilon \in [40°, 70°]$.

5.3 The *'Aā* Nature of the CMT: Intrinsic Thermal Insulator

The CMT has a biphasic structure: a mobile plastic core at high temperature, surrounded by a fragmented clinker crust (lava-type *'aā* structure). This is a direct consequence of BH rheology: the outer layer, where $\tau < \tau_y$, solidifies; the core remains plastic. The clinker crust is an intrinsic structural thermal insulator that blocks radiative energy loss from the core without requiring an external lid atmosphere.

5.4 Inter-CMT Coupling and Uniqueness of the Accretion Stratum

Physical Definition of E_{CMT}

The CMT is defined by two conditions: (1) **spatial**: $|\phi_{\text{had}}| < 30°$ (Eq. 10); (2) **spectral**: coherent kinetic energy carried by toroidal mode $\ell = 1$, $m = 1$.

The coherence factor of particle i:

$$f_{\text{coh}}(i) = \frac{|\boldsymbol{v}_i \cdot \boldsymbol{u}_{11}(\boldsymbol{r}_i)|}{|\boldsymbol{v}_i|}, \qquad f_{\text{coh}} \in [0, 1], \tag{27}$$

where $\boldsymbol{u}_{11}(\boldsymbol{r}) = \hat{\boldsymbol{\Omega}}_{\text{had}} \times \hat{\boldsymbol{r}} / |\hat{\boldsymbol{\Omega}}_{\text{had}} \times \hat{\boldsymbol{r}}|$. The CMT energy:

$$\boxed{E_{\text{CMT}} = \sum_{i\,:\,|\phi_{\text{had}}(i)|<30°} w_i \cdot \tfrac{1}{2}\rho_i\, v_i^2 \cdot f_{\text{coh}}(i).} \tag{28}$$

The condition $E_{\text{CMT}}/E_{\text{ext}} \gtrsim 5$ is the quantitative criterion for a well-established CMT, with no double-counting of turbulent energy.

Three Inter-CMT Coupling Mechanisms

1. Phase synchronisation by Rossby waves. The effective β parameter on the oblate spheroid:

$$\beta_{\text{eff}} = \frac{2\Omega\cos\varepsilon}{R_{\text{eq,proto}}}\left(1 + \tfrac{3}{2}J_2\right) \approx \beta \times 1.15. \tag{29}$$

In a Bingham-Herschel fluid, Rossby waves are confined to the band $|\phi| < 30°$ (finite-width waveguide $L_\phi \approx 0.5\,L_\beta$). The BH channel-analogue correction (Vallis, 2006, Sec. 6.5):

$$\tau_{\text{sync}}^{(\text{BH})} \approx 40 \times \tau_{\text{sync}}^{(\text{newt})} \approx 4 \times 10^4\text{–}4 \times 10^5\ \text{yr}. \tag{30}$$

2. Energetic coupling via shared inverse cascade. Multiple CMTs in the same band share the same turbulent reservoir: they are both expressions of the mode $\ell = 1$, $m = 1$ selected at the Rhines scale. This coupling is continuous and instantaneous.

3. Direct gravitational coupling. $F_{\text{grav,CMT}} \sim GM^2_{\text{CMT}}/(2R_{\text{eq,proto}})^2 \sim 10^{10}$ N; active on 10^4–10^5 yr timescales.

Established result

$\tau^{(\text{BH})}_{\text{sync}} \ll \tau_{\text{seg}}$**: CMTs synchronise before the next segregation step.** Even with the BH channel correction, $\tau^{(\text{BH})}_{\text{sync}} \approx 4 \times 10^5$ yr remains two orders of magnitude below $\tau_{\text{seg}} \approx 30$–$50$ Myr. Scale separation holds with a comfortable margin $(\times 100)$.

L8—Rossby wave group velocity in a BH fluid: channel analogue

The rigorous derivation of $c^{(\text{BH})}_{\text{Rossby}}$ for a BH fluid on an oblate spheroid ($\varepsilon \neq 0$, realistic $U(r)$ profile) remains an **open problem** (Priority 6 of the analytical programme). The channel analogue gives a factor-40 correction that strengthens rather than invalidates the conclusion.

Established result

Emergence theorem: one stratum per episode. Local complexity (multiple CMTs) vanishes in the global observable: one unique stratum per episode, regardless of n_{CMT}. Limitation L1 loses its critical character: even in the worst case $(n_{\text{CMT}} > 1)$, prediction P22 remains intact.

6 First Phase Transition: Rheological

The Bingham-Herschel (BH) law (Bingham, 1922; Herschel & Bulkley, 1926):

$$\tau = \tau_y + k\dot{\gamma}^n \quad (\tau > \tau_y), \tag{31}$$

with $n = 1.2$ (shear-thickening) for a partially crystallised magma at solid fraction $\phi \approx 0.1$–0.3 (Caricchi et al., 2007). Yield stress $\tau_y \in [10^2, 10^4]$ Pa for silicate magma at $T = 3\,000$–$3\,500$ K (Giordano et al., 2008). Arrhenius dependence: $\tau_y(T) = \tau_y^{(0)} \exp(-E_a/RT)$, $E_a \approx 150$–250 kJ mol^{-1}.

Established result

Natural thermal lock at $T^* \approx 1\,800$ K. Below this temperature, $\tau_y \gtrsim 10^6$ Pa: cohesive ejection becomes physically inaccessible. The TPT window closes irreversibly through thermodynamics alone. This extinction mechanism is a consequence of the Arrhenius law, not an additional hypothesis.

7 High-Velocity Cohesion: the $\mathrm{Bi}_{\mathrm{KH}}$ Criterion

7.1 Why the Weber Number Does Not Apply

For a free flow, $We \approx 1.4 \times 10^{15}$—implying atomising fragmentation. But the CMT is not a free flow: it is a self-gravitating collective toroidal flux confined by its toroidal symmetry, self-gravity and atmospheric pressure. The Weber number—based on surface tension vs. inertia at a free interface—is not the relevant criterion.

7.2 The Correct Criterion: $\mathrm{Bi}_{\mathrm{KH}}$

The Kelvin-Helmholtz shear stress at the interface between two layers:

$$\mathrm{Bi}_{\mathrm{KH}} = \frac{\tau_y\, k}{\rho\, \sigma_{\mathrm{KH}}\, \Delta U}. \tag{32}$$

Established result

For $\mathrm{Bi}_{\mathrm{KH}} \gg 1$: the yield threshold suppresses KH instability before any fragmentation in the confined toroidal flux. Cohesion is ensured by the fluid's own rheological physics.

Acknowledged limitation

At small scales ($\lambda_{\mathrm{KH}} \ll R_{\mathrm{torus}}$), partial fragmentation cannot be analytically excluded. However, this limitation is less critical than it appears: fragments produced by the same toroidal flux, ejected with the same velocity and angle, end up on quasi-identical orbits. They accumulate in the same co-orbital ring and accrete onto the POB: the fragmentation modifies the size of fragments, not their orbital destination. The mass budget (Eq. 40) is not affected. Quantitative validation is Prediction P11.

Three conditions independently reinforce cohesion: (1) atmospheric confinement ($P \approx 10^5$–10^7 Pa), (2) internal crystal network ($\phi \approx 0.1$–0.3, multiplying fragmentation resistance by 3–10), (3) $\rho_{\mathrm{flux}}/\rho_{\mathrm{atm}} \approx 200$ (Rayleigh-Taylor suppression).

8 Second Phase Transition: Mechanical

8.1 The Generalised Instability Equation λ

The evolution of an infinitesimal radial displacement ξ_r of a CMT parcel is governed by the Solberg-Høiland parcel-displacement equation (Solberg, 1936; Høiland, 1941):

$$\ddot{\xi}_r = \lambda\, \xi_r, \quad \lambda > 0\text{: instability; } \lambda < 0\text{: stable confinement.} \tag{33}$$

Setting $a = (1-\alpha)/r^2$, $b = 2\Omega^{(k)} \sin\varepsilon > 0$, $c = g_{\text{eff}}^{(k)} < 0$:

$$\boxed{\lambda(U,\varepsilon,\alpha,k) = \underbrace{c}_{\lambda_1<0} + \underbrace{bU}_{\lambda_2>0} + \underbrace{aU^2}_{\lambda_3} + \underbrace{\frac{|\dot{\Omega}| r(\phi)\sin(\varepsilon+\phi)}{R_{\text{eq,proto}}}}_{\lambda_4} + \underbrace{\frac{g_{\text{eff}}^{(0)} J_2 \sin(2\phi)}{R_{\text{eq,proto}}}}_{\lambda_5}.} \tag{34}$$

Physical meaning: $\lambda_1 < 0$ (effective gravity, stabilising); $\lambda_2 > 0$ (radial Coriolis, destabilising); λ_3 (angular momentum gradient, destabilising for $\alpha > 1$); λ_4 (Euler acceleration, wobble signature); λ_5 (tangential gravity gradient).

The amplification factor $\rho_0/\rho_{\text{CMT}}(t)$ monotonically amplifies all destabilising terms as the CMT lightens by Fe-Ni segregation.

8.2 Episodicity Regimes According to α

Super-rigid regime ($\alpha > 1$): instability only within a bounded interval $[U_1, U_2]$, producing spontaneous episodicity.

Sub-rigid regime ($\alpha \leq 1$): single threshold $U > U_{\text{crit}}$; episodicity driven by Mathieu resonance and the Kramers rate.

Established result

Robustness: α is a shape parameter, not an existence parameter. The discriminant $\Delta > 0$ for any α as soon as $\varepsilon > 45^\circ$. The double well exists regardless of α, given $\varepsilon > 45^\circ$. α determines the geometry of the well, not the existence of the mechanism.

8.3 Progressive CMT Lightening by Fe-Ni Segregation

$$\rho_{\text{CMT}}(t) = \rho_0 - \xi_{\text{Fe}}(t)(\rho_0 - \rho_f), \quad \rho_0 \approx 4\,500\,\text{kg}\,\text{m}^{-3}, \ \rho_f \approx 3\,200\,\text{kg}\,\text{m}^{-3}. \tag{35}$$

8.4 Double Potential Well and Kramers Crossing Rate

Integrating $\lambda(U) = -\mathrm{d}V/\mathrm{d}U$ yields:

$$V(U) = -\tfrac{a}{3}U^3 - \tfrac{b}{2}U^2 - cU + V_0, \tag{36}$$

with two wells P_1 (confined) and P_2 (ejective) separated by barrier $\Delta E_{\text{barr}}^{(k)}$, decreasing at each episode.

Ejection episodes are spontaneous transitions between metastable states, analogous to phase transitions in statistical physics: they emerge from the dynamics, not from a priori scripted thresholds.

The Kramers crossing rate (Kramers, 1940):

$$\Gamma_{\text{Kramers}} = \frac{\omega_0 \omega_b}{2\pi \gamma_{\text{diss}}} \exp\left(-\frac{\Delta E_{\text{barr}}}{k_B T_{\text{eff}}} \right). \tag{37}$$

Established result

The first episode is a quantified barrier crossing. The Kramers rate unifies: energy accumulation in P_1, metastability, stochastic triggering by impact waves or CMEs, episodicity by return to P_1 after each ejection. With $\sim 10^9$–10^{10} opportunities over 10^6 yr, the probability of no triggering event is negligible.

8.5 Critical Velocity and Maximum Ejectable Mass

$$U_{\text{crit}}^{(0)} = \sqrt{2\, g_{\text{eff}}^{(0)}\, R_{\text{eq,proto}}} \approx 11\,220\,\text{m}\,\text{s}^{-1}, \tag{38}$$

a conservative upper bound. Mach number at ejection ($c_s \approx 0.9\,\text{km}\,\text{s}^{-1}$, Mader 2013): $\text{Ma} = U_{\text{crit}}/c_s \approx 12.5$: ejection is **hypersonic**.

$$\boxed{M_{\text{ej}}^{\text{max}} \approx \frac{2\Omega^{(k)} \sin\varepsilon \cdot U_{\text{crit}}^{(k)} \cdot M_{\text{CMT}}}{g_{\text{eff}}^{(k,0)}} \approx 1.0\text{–}2.0 \times 10^{22}\,\text{kg}.} \tag{39}$$

8.6 Hypersonic Flux → Proto-Orbital Body: Three Phases

Phase I ($r < 2$–$3\,R_\oplus$). Ballistic expansion, cohesion secured by $\text{Bi}_{\text{KH}} \gg 1$. Adiabatic cooling: $\tau_{\text{cool}} \approx 12$ min $\ll$ transit time ≈ 2–3 h. The sheet arrives at orbit in a plastic but not yet solid state: favourable for sticky accretion.

Phase II (to orbit). Inherited spin $\omega_{\text{spin}} \sim 10^{-3}\,\text{rad}\,\text{s}^{-1}$ stabilises against gravitational collapse by a gyroscopic effect (inverse skater effect).

Phase III ($> R_{\text{Roche}} \approx 3.6\,R_\oplus$). Sticky collisions ($\delta v \approx 50$–$200\,\text{m}\,\text{s}^{-1} \ll v_{\text{frag}}$), plastic accretion.

L6—Flux → POB transition: qualitative status

The three-phase description is physically motivated and consistent with calculated orders of magnitude, but remains qualitative. A 3D SPH simulation with coupled radiative cooling is required (Priority 2b).

9 Proto-Orbital Body and Mass Budget

9.1 Proto-Orbital Body (POB)

The first episode places a solid-plastic body ($R_1 \approx 800$–950 km) beyond R_{Roche} at orbital distance $d_{\text{POB}} \approx 3$–$6\,R_\oplus$. The inherited tangential bias guarantees a spin inherited from the CMT by conservation of angular momentum. The Moon builds by concentric plastic accretion, each ejected sheet archiving in its internal structure the state of terrestrial Fe-Ni segregation at the time of ejection.

9.2 Mass Budget

$$M_{\text{Moon}} = N \cdot f_{\text{cap}} \cdot M_{\text{ej}}^{\text{max}} + \Delta M_{\text{late}}, \tag{40}$$

$N \approx 5$–9 (nominal 7 ± 2), $f_{\text{cap}} \in [0.65, 0.85]$ (nominal 0.70), ΔM_{late} including the SPA impactor contribution ($\approx 3.2 \times 10^{19}$ kg, Schultz et al. 2026) and late Hadean accretion (Zellner, 2019).

Established result

f_{cap} **is an increasing state variable.** $\partial f_{\text{cap}}/\partial M_{\text{Moon}} > 0$: the larger the Moon, the more efficiently it captures. This positive feedback, combined with the decrease of $g_{\text{eff}}^{(k)}$, defines a dynamic attractor that progressively reinforces the mechanism.

10 Internal Stratification and Seismological Programme

10.1 Architecture of Concentric Layers

Each ejected sheet originates from magma at a different Fe-Ni segregation stage:

$$\left(\frac{\text{Fe}}{\text{Si}}\right)_{\text{layer}\,k} > \left(\frac{\text{Fe}}{\text{Si}}\right)_{\text{layer}\,k+1}. \tag{41}$$

Density contrasts $\Delta\rho_{k,k+1} \approx 100$–$200\ \text{kg}\,\text{m}^{-3}$ (Garcia et al., 2019), producing potentially detectable seismic interfaces.

10.2 Two Stratification Scenarios

Scenario A (equal masses, $q = 1$, $N = 7$):

$$R_k^{(A)} = R_{\text{Moon}} \left(\frac{k}{N}\right)^{1/3}, \qquad d_k^{(A)} = R_{\text{Moon}} \left[1 - \left(\frac{k}{N}\right)^{1/3}\right]. \tag{42}$$

Scenario B (growing masses, $q \approx 1.122$, $M_{\text{ej}}^{(7)}/M_{\text{ej}}^{(1)} \approx 2$).

Both scenarios converge toward a preservation window between ≈ 200 and ≈ 530 km.

10.3 Preservation Window: Two Destructive Processes

From below. Ilmenite cumulate overturn (Briaud et al., 2023) may have remelted the deepest layers (episodes 1–2, > 600 km).

From above. Mare basalt volcanism, active until ≈ 2 Ga (Li et al., 2021; Che et al., 2021), remelted the shallowest layers (episodes 6–7, < 100 km).

11 Third Phase Transition: The Progressive Dynamo

The 350 Myr delay between lunar formation (≈ 4.55 Ga) and the first geological detection of the magnetic field (≈ 4.2 Ga, Tarduno et al. 2025) is not an adjusted parameter: it is the sum of three independently constrained durations:

$$\Delta t_{\text{dynamo}} = \underbrace{\Delta t_{\text{ej}}}_{\approx 60\,\text{Myr}} + \underbrace{\Delta t_{\text{cool}}}_{\approx 140\,\text{Myr}} + \underbrace{\Delta t_{\text{emer}}}_{\approx 150\,\text{Myr}} \approx 350\,\text{Myr}. \tag{43}$$

Phase 1 (≈ 60 Myr). Active Fe-Ni segregation, constrained by the hafnium-tungsten (Hf-W) chronometer (Kleine et al., 2002; Yin et al., 2002): the core grows but the thermal gradient is subadiabatic; no dynamo is possible.

Phase 2 (≈ 140 Myr). Obliquity stabilised at $23.5°$ by the growing Moon; protocrust forms near 4.4 Ga (Jack Hills zircons, Wilde et al. 2001; Valley et al. 2002).

Phase 3 (≈ 150 Myr). Compositional convection in the Fe-Ni core grows progressively until the magnetic field is strong enough to react back on the flow. The Elsasser number compares the Lorentz force $F_L \sim B^2/(\mu L)$ to the Coriolis force $F_C \sim 2\rho\Omega U$. Setting $F_C \approx F_L$:

$$2\rho\,\Omega\,U \sim \frac{B^2}{\mu\,L}, \quad \Rightarrow \quad B^2 \sim 2\mu\rho\,\Omega\,\eta, \tag{44}$$

giving the Elsasser number:

$$\Lambda = \frac{B^2}{\rho\,\mu\,\eta\,\Omega} \sim 1\text{–}2, \tag{45}$$

and the critical field:

$$B_{\text{crit}} = \sqrt{\Lambda_{\text{crit}}\,\rho\,\mu_0\,\eta\,\Omega} \approx 6\,\mu\text{T}, \qquad \sigma \approx 5 \times 10^5\,\text{S}\,\text{m}^{-1}. \tag{46}$$

Established result

The "threshold" is a progressive transition zone, not a sharp front. $\Lambda \sim 1$ is an order-of-magnitude reference, not a strict threshold. In a turbulent multi-scale fluid, the dynamics are non-linear and structures are distributed: no unique value of Λ exists where everything changes abruptly. Modern geodynamo simulations (Christensen & Aubert, 2006; Olson & Christensen, 2006) show a **progressive transition zone**: $\Lambda \ll 1$: Coriolis-dominated, aligned structures, weak or disorganised field; $\Lambda \sim 0.1$–1: Lorentz begins to react back, field organisation, Earth-like regime; $\Lambda > 1$: Lorentz strongly influences the dynamics, without abrupt rupture. *Analogy*: just as water boiling has a theoretical threshold at 100°C but manifests as bubbles, fluctuations and superheat, or laminar-turbulent transition has a critical Reynolds number but proceeds gradually, the dynamo builds over a range of Λ. The ≈ 150 Myr delay for Phase 3 represents precisely the time required to traverse this transition zone, consistent with the uncertainty ± 50 Myr (L4).

Established result

350 Myr delay with no free parameter. Δt_{ej}: Hf-W chronometer (Kleine et al., 2002; Yin et al., 2002). Δt_{cool}: Jack Hills zircons, 4.4 Ga (Wilde et al., 2001; Valley et al., 2002). Δt_{emer}: Tarduno et al. (2025), 4.2 Ga. Three independent observations—not an interpolation.

12 Falsifiable Predictions

P1—Isotopic identity $\Delta^{17}\mathrm{O} < 5$ ppm

Common Hadean magma source at each ejection. *Falsification:* $\Delta^{17}\mathrm{O} > 20$ ppm between deep lunar layers and the terrestrial mantle.

P3—Non-linear Hf-W chronology

Discrete episodes reflected in the W-isotope signature. *Falsification:* continuous uniform W-age distribution over 4.568–4.49 Ga.

P4—Bimodal inter-episode intervals

Early chaotic phase (episodes $< 30\,000$ yr apart) then Mathieu-regulated phase (widely spaced episodes). *Falsification:* unimodal distribution.

P6—Increasing radial Fe/Si gradient

Fe/Si increasing with depth in the lunar mantle. The SPA basin preferentially exposes episode-1 material (richest in Fe). *Missions:* Chang'e-6, Artemis III.

P7—$m = 1$ asymmetry of the lunar mantle

Zonal $m = 1$ structure detectable by multi-station seismic tomography. *Falsification:* spherical symmetry confirmed.

P11—Size and distribution of co-orbital fragments

Fragments produced by the same toroidal flux, ejected with the same velocity and angle, end up on quasi-identical orbits. They accumulate in the same co-orbital ring and accrete onto the POB: fragmentation modifies fragment size, not orbital destination. The mass budget is unaffected. *Falsification:* if more than 20% by mass of fragments escape the co-orbital ring.

P12—Dependence of f_{cap} on growing lunar mass

f_{cap} is an increasing state variable: $\partial f_{\text{cap}}/\partial M_{\text{Moon}} > 0$. *Falsification:* if f_{cap} is independent of M_{Moon} or decreasing.

P17—Gradual Hadean geomagnetic growth

Continuous palaeointensity growth 4.55–4.20 Ga (Tarduno et al., 2025), without abrupt onset. *Falsification:* abrupt onset detected.

P20—Growing ejected masses episode by episode

At each episode k, $g_{\text{eff}}^{(k)}$ decreases and $\Omega^{(k)}$ decreases: $M_{\text{ej}}^{(k+1)} > M_{\text{ej}}^{(k)}$. Outer strata (late episodes) should be thicker. *Falsification:* uniform or decreasing strata thicknesses.

P22—Seismic interfaces in the 200–530 km window

Concentric stratification produces impedance contrasts $|R| \in [0.01, 0.04]$, detectable as:

- a single sharp interface at $d \approx 250$–315 km ($\Rightarrow N = 2$–3);
- several distinct interfaces at $d \approx 200$–530 km ($\Rightarrow N = 5$–9: stronger confirmation).

Falsification condition: no phase conversion in the 200–530 km window after at least 3 stations deployed and at least 10 lunar events $M > 2.5$. **Missions:** Chang'e 7 (2026), FSS, LEMS, Artemis III (2028–2029). **(Priority 1 of the validation programme.)**

P22b—Interface width as CMT-multiplicity discriminant

- $n_{\text{CMT}} = 1 \Rightarrow$ sharp interfaces, seismic reflectors, width $\ll 10$ km;
- $n_{\text{CMT}} > 1$, synchronised $\Rightarrow$ progressive impedance gradient, transition width 20–50 km.

The impedance contrast $|R| \in [0.01, 0.04]$ is unchanged in both cases. Chang'e 7 and FSS can distinguish the two scenarios by waveform inversion. *Falsification:* if the measured width is intermediate (10–20 km) without a clear transition.

13 Complementary Stratification Test: the South Pole–Aitken Basin

The seismic prediction P22 is the ultimate test of the concentric stratification predicted by the TPT, but it requires a deployed station network not fully operational until 2030. The South Pole–Aitken (SPA) basin offers an **earlier and independent test**, based on the geochemistry of mantle ejecta.

13.1 Physical Principle

The SPA basin is the largest lunar impact basin, with an excavation depth estimated between 150 and 300 km (Potter et al., 2012; Melosh et al., 2017). This depth reaches the preservation window 200–530 km predicted by the TPT (Section 10). Central peaks of craters within SPA (e.g., Aitken crater) and ejecta on the basin rim therefore sample **directly** the material of the earliest ejection episodes (episodes 1 to 3 depending on the exact depth).

In the TPT, each episode ejects a layer of different composition because the source magma becomes progressively depleted in iron by Fe-Ni segregation:

$$\left(\frac{\text{Fe}}{\text{Si}}\right)_{\text{layer }k} > \left(\frac{\text{Fe}}{\text{Si}}\right)_{\text{layer }k+1}. \tag{47}$$

Material from the earliest episodes (deepest in the Moon) must therefore exhibit a Fe/Si ratio **significantly higher** than that of a typical terrestrial mantle rock (peridotite, xenolith), regardless of its age, and significantly higher than that of surrounding mare basalts (late-episode material, Fe-depleted by progressive segregation).

P23—Geochemical signature of SPA mantle ejecta

Mantle rocks excavated by the SPA basin and identifiable as such must exhibit:

1. a Fe/Si ratio **significantly higher** than that of a typical terrestrial mantle rock (peridotite, xenolith), regardless of its age — the Earth having continued its differentiation after lunar formation, the terrestrial value constitutes an intermediate reference, not a target to match;
2. a Fe/Si ratio **significantly higher** than that of surrounding mare basalts (late-episode material, Fe-depleted by progressive segregation);
3. a positive correlation between Fe/Si and estimated depth of origin (via crater density or indicator mineralogy).

Falsification: after correction for impact mixing effects, if the Fe/Si of SPA mantle ejecta is not significantly higher than a typical terrestrial mantle rock **and** not significantly higher than surrounding mare basalts (within analytical uncertainties), the prediction is falsified.

Missions: Chang'e-6 (sample return, SPA region, 2026).

13.2 Caveats and Limitations

L9—Impact mixing and partial sampling

A basin as large as SPA reworks and mixes excavated material. Ejecta are not a clean core. Prediction P23 is falsified only if:

- the predicted trend (Fe/Si higher than terrestrial reference and higher than mare basalts) is **absent** after statistical analysis of multiple samples/clasts identified as mantle-derived;
- **or** the measured difference is not statistically significant (below cumulative analytical uncertainties).

A simple absence of signal (e.g., samples too altered or too rare) is not a falsification — only a clear, reproducible, statistically significant counterexample qualifies.

L10—No existing sample constrains the deep mantle

All returned lunar samples (Apollo, Luna, lunar meteorites) come from the crust or the immediate subsurface (typical excavation depth <100 km). None sample the 200–530 km mantle window. The rheological extrapolation used in the TPT ($\tau_y(T)$ via Arrhenius law, $T \sim 3000$ K) is physically founded but experimentally unverified. Only 3D numerical simulation can validate or invalidate the postulated rheological values.

13.3 Relationship with P22 and P6

P23 is an **independent and complementary test**:

- If P23 is verified (Fe/Si anomalously high in SPA) and P22 also (seismic interfaces 200–530 km), the TPT receives strong confirmation.
- If P23 is verified but P22 is not, the stratification is real but its seismic preservation in depth is questioned — the TPT would be partially salvaged.
- If P23 is falsified (Fe/Si not higher than terrestrial reference), prediction P6 is also falsified, regardless of the outcome of P22.

P23 is therefore a *short-term test* (2026), predating full lunar seismology (2030).

14 Acknowledged Limitations

L1—Spontaneous CMT organisation

The emergence of a unique torus in the band $|\phi_{\text{had}}| < 30^{\circ}$ is physically motivated by four convergent effects (Section 5). A preliminary simplified test ($N = 500$ particles, $8\,T_{\text{rot}}$, without coupled BH rheology or full self-gravity) shows qualitative energy concentration consistent with CMT emergence; this test is not a 3D SPH simulation and its results will only be published after complete validation (code, parameters and results described in a dedicated paper). Whether a unique torus or multiple coexisting structures arise in the exact Hadean regime can only be resolved at high resolution with full BH rheology. **Priority 2** of the validation programme.

L2—Numerical value of f_{cap}

Physically motivated parametrisation (Eq. 40). The 3D SPH simulation is the ultimate arbiter on the numerical value and its dependence on M_{Moon} (Prediction P12).

L3—Small-scale cohesion: limited impact on the mass budget

The argument $\text{Bi}_{\text{KH}} \gg 1$ holds at large and intermediate scales. At very small scales ($\lambda_{\text{KH}} \ll R_{\text{torus}}$), partial fragmentation cannot be analytically excluded. However, this limitation is less critical than it appears: fragments from the same toroidal flux, ejected with the same velocity and angle, end up on quasi-identical orbits. They accumulate in the same co-orbital ring and accrete onto the POB: the fragmentation modifies fragment size, not their orbital destination. The mass budget (Eq. 40) is not affected. Quantitative validation of fragment size and distribution is Prediction P11.

L4—Dynamo delay: ± 50 Myr

Simple exponential core-growth model; associated uncertainty ± 50 Myr.

L5—Application of $\sigma_{\text{res}} = \Omega f/2$ to this context

The elliptic parametric instability growth rate (Kerswell, 2002; Lacaze et al., 2004) was established for an externally forced fluid ellipsoid in small-amplitude regime. Its application to a Maclaurin spheroid in free chaotic nutation is a physically motivated extrapolation, not a direct deduction. Level-3 hypothesis: to be validated by simulation.

L6—Flux → POB transition: qualitative status (see §8.6)

The three-phase description is physically motivated and consistent with calculated orders of magnitude, but remains qualitative. A 3D SPH simulation with coupled radiative cooling is required (Priority 2b).

L7—Crustal dichotomy: qualitative $m = 1$ asymmetry

The formation mechanism of the lunar crustal dichotomy (near side ≈ 30 km vs. far side ≈ 60 km crustal thickness, Wieczorek et al. 2013) is described qualitatively in the TPT but not yet derived analytically.

Proposed mechanism (status: qualitative). After the first episode, the POB has an inherited spin ($\omega_{\text{spin}} \sim 10^{-3}$ rad s^{-1}, Section 8.6) at orbital distance ≈ 3–$6\,R_\oplus$. The tidal locking time:

$$\tau_{\text{lock}} \sim \frac{\omega_{\text{spin}}\, a^6\, Q_{\text{Moon}}}{k_2\, G\, M_\oplus^2\, R_{\text{Moon}}^5} \approx 10^4\text{–}10^5\ \text{yr}, \tag{48}$$

for $Q \approx 100$ and $k_2 \approx 0.02$: *very short* on a geological timescale. The POB is quasi-instantaneously tidally locked while still hot ($T \approx 2\,000$–$2\,500$ K).
From the second episode onward, the growing proto-lunar tide generates a forced precession of the proto-Moon's rotation axis. This precession, coupled to the thermal asymmetry between the near side (kept warmer by terrestrial and T-Tauri fluxes) and the far side (free radiative cooling), produces differential crystallisation:

- **Near side**: deeper lunar magma ocean, later crystallisation, thinner anorthositic crust;
- **Far side**: faster cooling, earlier crystallisation, thicker crust.

The asymmetric accretion of the sheets from episodes $N-1$ and N (when forced precession is strongest) reinforces this nascent dichotomy. The $\approx 2:1$ thickness ratio could emerge from this coupling, but the coupled differential thermal calculation has not yet been derived analytically.

Status: **qualitative prediction** (P7—$m = 1$ asymmetry). The physical mechanism is consistent with the TPT and requires no exogenous ingredient; it awaits confirmation or refutation by Artemis III missions and dedicated thermal models.

L8—Rossby wave group velocity in a BH fluid: channel analogue

See Section 5.4.

15 Discussion

15.1 Comparison with the Giant-Impact Model

Constraint	Giant impact	TPT
Isotopic identity	Difficult; very specific conditions required	Satisfied mechanically
Fe depletion	Requires differentiated Theia	Direct consequence of driver
Crustal dichotomy	Not naturally predicted	Qualitative mechanism (forced precession + thermal asymmetry, L7)
Dynamo delay	Not predicted	Predicted: 350 Myr, no free parameter
Seismic predictions	None	P22 testable from 2026

15.2 Responses to Principal Objections

"The CMT is not demonstrated." It is not postulated: it is the spatial expression of mode $\ell = 1$, $m = 1$ from the geostrophic inverse cascade at the Rhines scale in a spherical oblate system in rapid rotation with obliquity $> 45°$. The qualitative proof of concept ($N = 500$, $E_{\text{CMT}}/E_{\text{ext}} \approx 10$) is encouraging. Quantitative validation is Priority 2.

"Taylor-Proudman prevents the CMT." Columns exist but are curved by five mechanisms (Section 4). Their curvature generates the toroidal flow. $(\boldsymbol{\Omega} \cdot \nabla)\boldsymbol{u}_{\text{tor}} = 0$ automatically (Frigaard, 2017).

"$We \approx 10^{15}$: cohesion is impossible." The Weber number applies to free flows. The relevant criterion is $\text{Bi}_{\text{KH}} \gg 1$ (Section 7).

"The dynamo delay is arbitrary." It is the sum of three durations constrained by three independent observations (Section 11).

15.3 Sequenced Validation Programme

1. **Priority 1—Lunar seismology** (2026–2030): Chang'e 7, FSS, LEMS, Artemis III. Direct test of P22.
2. **Priority 2—3D SPH simulation**: honest measurement of whether $\lambda(U) = 0$ is reached, measurement of $\hat{\alpha}$, CMT validation.
3. **Priorities 3–5**: hypersonic cohesion (P11), Mathieu resonance, magma-atmosphere coupling.

16 Conclusion

This work proposes that the formation of the Moon may be the product of a Triple Phase Transition in a fully molten proto-Earth rotating at ≈ 3.5 h, whose axis wobbles within $[40°, 70°]$ in its own co-rotating frame. A single driver—progressive Fe-Ni segregation— simultaneously governs three transitions: rheological, mechanical and magnetic. The result is a Moon built layer by layer, archiving in its internal structure the history of terrestrial differentiation.

The central prediction stands: one or more seismic interfaces in the 200–530 km window, testable by Chang'e 7 from 2026 and by the FSS/LEMS/ Artemis III network by 2030. If Chang'e 7 detects an interface with $|R| > 0.02$ in this window, concentric stratification receives its first observational support prior to any SPH simulation. Multiple interfaces constitute stronger evidence than a single one. If no interface is detected under these conditions, prediction P22 is seriously challenged and the theory will need to be revised—which the author accepts explicitly.

This work invites the community to evaluate the theory on its own terms: equations are numbered, derivations are explicit, hypotheses are ranked, limitations are acknowledged, and predictions are falsifiable.

A Derivation of the Elsasser Number and Critical Field B_{crit}

Two competing forces in the core

Coriolis force.

$$F_C \sim 2\rho\,\Omega\,U, \tag{49}$$

where ρ is the fluid density, Ω the angular rotation velocity and U the characteristic fluid velocity.

Lorentz force.

$$F_L \sim \frac{B^2}{\mu\,L}, \tag{50}$$

where B is the magnetic field intensity, μ the magnetic permeability and L a typical length (core thickness).

Coriolis-Lorentz balance

Setting $F_C \approx F_L$:

$$2\rho\,\Omega\,U \sim \frac{B^2}{\mu\,L}. \tag{51}$$

In turbulent convective regime, $\eta \sim UL$ (magnetic Reynolds number of order unity at the dynamo threshold). Substituting:

$$B^2 \sim 2\mu\rho\,\Omega\,\eta, \qquad \Rightarrow \qquad \Lambda \equiv \frac{B^2}{\rho\,\mu\,\eta\,\Omega} \sim 2, \tag{52}$$

i.e. $\Lambda \sim \mathcal{O}(1)$—standard result.

Critical field

Using $\eta = (\mu_0\sigma)^{-1}$ with $\sigma \approx 5 \times 10^5\ \mathrm{S\,m^{-1}}$, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$, $\rho \approx 10^4\ \mathrm{kg\,m^{-3}}$ and $\Omega \approx 4.99 \times 10^{-4}\ \mathrm{rad\,s^{-1}}$:

$$B_{\text{crit}} = \sqrt{\Lambda_{\text{crit}}\,\rho\,\mu_0\,\eta\,\Omega} \approx 6\,\mu\mathrm{T}. \tag{53}$$

Progressive nature of the transition

$\Lambda \sim 1$ is an order-of-magnitude reference, not a sharp front. In modern geodynamo simulations (Christensen & Aubert, 2006; Olson & Christensen, 2006), the transition between Coriolis-dominated ($\Lambda \ll 1$) and magnetostrophic regimes ($\Lambda \sim$ 1–10) is progressive, probabilistic, parameter-dependent, and spread over a range. Certain changes become qualitative (appearance of a stable dipolar field, transition to multipolar regime), but without abrupt rupture. The 350 Myr delay is consistent with this gradual transition.

B Complete Derivation of the λ Coefficient

Solberg-Høiland parcel-displacement formalism (Solberg, 1936; Høiland, 1941) adapted to an oblate sphere with obliquity ε. Angular momentum conservation ($\tau_{\text{relax}}/\tau_{\text{dyn}} \approx 3 \times 10^9 \gg 1$):

$$U_{\text{parcel}}(r+\xi_r) \approx U(r)\left(1 - \frac{\xi_r}{r}\right), \qquad U_{\text{background}}(r+\xi_r) \approx U(r)\left(1 - \alpha\frac{\xi_r}{r}\right). \tag{54}$$

Velocity difference: $\delta U = U(\alpha-1)\xi_r/r$. The net force sums effective gravity ($-g_{\text{eff}}^{(k)}\xi_r$), radial Coriolis ($+2\Omega^{(k)}\sin\varepsilon \cdot U\xi_r$) and the Rayleigh term ($U^2(1-\alpha)\xi_r/r^2$). Equation (34) follows.

C Derivation of $\tau_{\text{Stokes}} \approx 257$ s

Stokes sedimentation velocity for a Fe-Ni droplet of radius $r \approx 5$ cm in molten silicate:

$$v_{\text{Stokes}} = \frac{2}{9}\frac{(\rho_{\text{Fe}} - \rho_{\text{sil}})\, g_{\text{eff}}^{(0)}\, r^2}{\eta} \approx \frac{2}{9}\frac{3\,300 \times 8.51 \times (0.05)^2}{0.1} \approx 156\,\mathrm{m\,s^{-1}}. \tag{55}$$

Over a characteristic distance $\ell \approx 30$ km: $\tau_{\text{Stokes}} = \ell / v_{\text{Stokes}} \approx 192$ s $(\sim 200\ \mathrm{s})$. Robust over $r \in [1, 10]$ cm and $\eta \in [0.05, 1]$ Pa s.

D Adiabatic Temperature Profile

With $T_{\text{surf}} \approx 3\,500$ K and adiabatic gradient $\mathrm{d}T/\mathrm{d}r \approx -0.3\ \mathrm{K\,km^{-1}}$, central temperature reaches $\approx 5\,000$ K. The peridotite solidus at 135 GPa is $\approx 4\,300$ K (Stixrude & Lithgow-Bertelloni, 2014): melting is thermodynamically stable throughout the entire mantle, confirming H0.

E List of Abbreviations

BH	Bingham-Herschel: rheological law describing a yield-stress fluid.
BiKH	Bingham-Kelvin-Helmholtz number: cohesion criterion of the toroidal flux at high velocity.
CMT	Coherent Magmatic Torus: self-organised toroidal structure in the Hadean equatorial band, driving episodic ejections.
CME	Coronal Mass Ejection: energetic eruption from the T-Tauri Sun.
FSS	Farside Seismic Suite: future seismometer network deployed on the lunar far side by Chang'e 7 (2026) and Artemis missions.
Hf-W	Hafnium-tungsten chronometer: isotopic reference tool for dating metal-silicate segregation.
LHB	Late Heavy Bombardment: period of intense meteoritic bombardment, $\approx$ 3.9 Ga.
LEMS	Lunar Environment Monitoring Station: geophysical station of the Artemis programme.
POB	Proto-Orbital Body: first solid-plastic body placed in orbit by the first ejection episode.

SPA	South Pole-Aitken: largest lunar impact basin, exposing deep mantle layers.
SPH	Smoothed Particle Hydrodynamics: numerical simulation method by particles, standard for astrophysical fluid dynamics.
TPT	Triple Phase Transition: name given to the theory presented in this work.

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