Lunar Formation via Triple Phase Transition in the Differentiating Proto-Earth

Michel Debailleul
Geophysicist — Université libre de Bruxelles (ULB)
ORCID: 0009-0003-1222-1433
michel.debailleul@yahoo.fr

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Abstract. The origin of the Moon remains one of the unsolved problems of planetary science. The canonical giant-impact model faces growing geochemical difficulties: it does not naturally predict the near-perfect Earth–Moon isotopic identity, the crustal dichotomy, or the ≈ 350 Myr delay of the terrestrial dynamo. This work proposes an alternative grounded in a necessary thermodynamic observation: any Earth-mass planet emerges from accretion in a state of near-total silicate mantle melting, accretion energy exceeding total fusion energy by a factor ≈ 155 (Solomatov, 2000; Elkins-Tanton, 2012; Rubie et al., 2015). The proto-Earth is therefore a rapidly rotating magma body ($T_{\mathrm{rot}} \approx 3.5$ h) without a stabilising satellite, whose axis wobbles within $[40°, 70°]$ in its own co-rotating frame — a range justified by five independent convergent arguments, foremost among which is the complete absence of any tidal stabiliser. In this context, a single driver — progressive Fe-Ni segregation — governs three coupled transitions: a rheological transition (structuring of the Coherent Magmatic Torus, CMT), a mechanical transition (two to three episodic hypersonic ejections governed by a double potential well and the Kramers stochastic crossing rate), and a magnetic transition (terrestrial dynamo delayed by ≈ 350 Myr, with no free parameter).

Epistemic note: The TPT makes testable observational predictions that precede and do not depend on full numerical validation. The absence of a 3D SPH simulation does not constitute a falsification; the theory will be primarily judged by Chang'e-7 seismic data (August 2026) and Artemis III samples (2028–2029).

The mass budget is consistent: the target lunar mass to be produced by the TPT mechanism is *less* than the present observed lunar mass, since the South Pole–Aitken impactor ($\approx 3.2 \times 10^{19}$ kg, Wakita et al. 2026) and documented late accretion (Zellner, 2019) contributed after the ejection episodes. The central prediction — seismic interface(s) between 200 and 530 km depth, impedance contrast $|R| \in [0.01, 0.04]$ — is testable by **Chang'e 7** (South Pole, August 2026), **FSS** (Farside Seismic Suite), **LEMS** (Lunar Environment Monitoring Station), and **Artemis III** (2028–2029). Preliminary observational support is provided by Chang'e-6 samples (Yue et al., 2026; Xu et al., 2025): norites dated at 4247 ± 5 Ma (consistent with Episode 1) and anomalously high Fe/Mn ratios in deep olivines, both consistent with predictions P6 and P23 of this work. All hypotheses are explicit and ranked. Limitations are acknowledged. The validation programme is defined and sequenced.

Keywords: lunar origin · Triple Phase Transition · chaotic axis wobble · Hadean proto-Earth · Bingham-Herschel · Coherent Magmatic Torus · double potential well · Kramers rate · Mathieu resonance · seismic interface · Hadean dynamo · falsifiable predictions · South Pole–Aitken · Chang'e-6.

Epistemic posture

This work proposes a theory and submits it to examination by the scientific community. The formulations deliberately reflect this posture: the theory describes, predicts and constrains — it does not assert absolute certainties. All hypotheses are numbered and ranked. All limitations are acknowledged.

On numerical simulations: A full 3D SPH validation with complete Bingham-Herschel rheology, radiative cooling, Fe-Ni segregation, and $\varepsilon \neq 0$ geometry is beyond the capacity of a single independent researcher and properly belongs to an institutional team with HPC resources. The TPT does not require such a simulation to be evaluated; it stands or falls primarily on its observational predictions (Sections 9 and 10). The final arbitration belongs to forthcoming lunar seismic data (Chang'e 7, August 2026) and, in a second step, to three-dimensional numerical simulations when they become available.

1 The Hadean Inferno: Physical Context

Before entering the physics of the Triple Phase Transition, it is essential to understand the setting in which it operates. This setting is not a quiet void: it is one of the most energetically violent periods in the history of the Solar System.

1.1 The T-Tauri Sun: Active Perturbator of Obliquity and Rheology

In its first few million years, the Sun was in its T-Tauri phase: stellar winds 100 to 1 000 times more intense than today (Airapetian et al., 2016), coronal mass ejections (CMEs) delivering 10^{27}–10^{29} J per event at a rate of several per day, and extreme X-ray and UV radiation maintaining surface temperatures above 3 000 K. The T-Tauri Sun is not a background detail in the TPT: it plays three physically distinct roles.

Role 1 — Rheological modulator. By maintaining surface temperatures at $T_{\text{surf}} \approx$ 1 800–2 200 K between flares and 2 800–4 200 K during flares (Stefan-Boltzmann law, albedo $A_0 \approx 0.1$), it keeps the yield stress τ_y of the silicate melt in the range 10^2–10^4 Pa (Giordano et al., 2008), the optimal window for cohesive CMT ejections. Below $T^* \approx$ 1 800 K, τ_y rises above 10^6 Pa and cohesive ejection becomes physically inaccessible.

Role 2 — CME trigger. With $\sim 10^9$–10^{10} individual CME opportunities over 10^6 yr, the T-Tauri Sun provides ample stochastic perturbations capable of pushing the CMT over the potential barrier separating the confined state from the ejective state (Section 5.2.2).

Role 3 — Obliquity perturbator. CME impacts on the proto-atmosphere and magnetosphere deliver impulsive torques on the spin vector, contributing to the maintenance of the axial wobble $\varepsilon \in [40°, 70°]$ (Section 1.5). The T-Tauri phase also eventually provides an independent extinction mechanism: as the Sun joins the main sequence, irradiation declines, τ_y rises, and the TPT window closes.

1.2 The Silicate Atmosphere: Thermal Cocoon and Lateral Confinement

At $t = 0$, the proto-Earth is enveloped in a dense silicate atmosphere — rock vapour, SiO, gaseous Mg and Fe — at temperatures between 2 000 and 4 000 K and pressures of 10^5–10^7 Pa. This cocoon plays two roles in the TPT: (1) it blocks radiative surface cooling, extending the rheological window beyond what the accretion energy budget alone would predict; (2) its confinement pressure $P \approx 10^5$–10^7 Pa opposes lateral expansion of the CMT flux during ejection, contributing to the cohesion of the ejected sheet. The high sound speed in this atmosphere ($c_s \approx 1\,500$ m s^{-1} at $T \approx 3\,000$ K) also reduces the effective Mach number of the flux at the flux/atmosphere interface, improving cohesion relative to vacuum ejection.

1.3 Continuous Planetesimal Bombardment

The protoplanetary disc remains populated with planetesimals and embryos during the active TPT window (Agnor et al., 1999; Kokubo & Genda, 2010). Each impact: (1) re-injects thermal energy into the mantle, sustaining melting; (2) generates a bulk spherical pressure wave propagating throughout the entire molten volume, perturbing $U(r)$ and contributing stochastically to Kramers barrier crossings; (3) delivers an impulsive torque on the spin vector, resetting the axis wobble.

1.4 Short-Lived Radioactivity: Internal Heat Source

The isotopes ^{26}Al (half-life 0.72 Myr) and ^{60}Fe (half-life 2.6 Myr) are still active in the first few million years (Urey, 1955; Huss et al., 2009), injecting an additional $\approx 3 \times 10^{29}$ J into the mantle and contributing to sustained melting throughout the TPT window.

1.5 Axial Obliquity $\varepsilon \in [40°, 70°]$: Five Convergent Arguments

The range $\varepsilon \in [40°, 70°]$ is not a free parameter. It is constrained by five independent and convergent physical arguments. Before listing them, a fundamental geometric clarification is required.

Geometric clarification — read first

Throughout this work, ε denotes the instantaneous angle between the rotation vector $\mathbf{\Omega}$ and the principal axis of inertia of the Maclaurin spheroid, **defined in the co-rotating body frame**. It is not the astronomical obliquity relative to the ecliptic plane: that concept has no physical meaning for the Hadean proto-Earth, which has no fixed ecliptic reference and no Moon. It is a purely internal geometric angle describing the *wobble* — the free nutation — of the rotation axis relative to the body's own shape. The Hadean intertropical band $|\phi| < 30^\circ$ is defined perpendicularly to the *instantaneous* Hadean rotation axis, not relative to any fixed geographic frame.

Argument 1 — Absence of tidal stabiliser (fundamental). The proto-Earth has no Moon. Without a massive satellite, there is no lunar tidal torque to damp the precession of the spin axis and keep it near a stable equilibrium. This argument is logically prior to all others: it is precisely the Moon whose formation we seek to explain, so invoking it as a stabiliser would be circular. The absence of a tidal stabiliser is therefore a necessary, not contingent, condition of the Hadean proto-Earth.

Argument 2 — Laskar et al. (1993): chaotic zone without a satellite. Laskar et al. (1993a) demonstrated that without its Moon, the Earth's obliquity would evolve chaotically over a zone extending from nearly 0° to $\approx 85^\circ$. Although this result strictly applies to a present-day Earth rotating at ≈ 24 h, it establishes the general principle: *the Moon is the stabiliser, and without it, large obliquity excursions are the norm*. For the Hadean proto-Earth, this principle applies a fortiori.

Argument 3 — Rapid rotation at $T_{\text{rot}} = 3.5$ h. The precession constant satisfies $\alpha_{\text{prec}} \propto \omega$. At $T_{\text{rot}} = 3.5$ h, the precession frequency is $\approx 7\times$ higher than at 24 h, pushing the secular spin-orbit resonances identified by Laskar et al. (1993a) well out of reach. Li & Batygin (2014), working with a moonless Earth in rapid rotation, found that obliquity remains chaotic but *diffuses slowly* across a broad range. The rapid rotation does not stabilise the axis — it simply changes the character of the chaos from resonance-driven to diffusive.

Argument 4 — Continuous planetesimal bombardment. Each major impact during terminal accretion (Agnor et al., 1999) abruptly resets the spin vector. The relaxation time toward axis alignment for an isolated fluid body,

$$\tau_{\text{relax}} \sim \frac{\eta\, R_{\text{eq,proto}}^3}{G\, M_\oplus^2\, f} \approx 10^6\,\text{yr} \quad (\eta \sim 10^{1\text{–}3}\,\text{Pa\,s},\ f \approx 0.107), \tag{1}$$

far exceeds the inter-impact timescale (10^3–10^5 yr). The misalignment is continuously maintained: $\tau_{\text{pert}} \ll \tau_{\text{relax}}$.

Argument 5 — T-Tauri CME torques. As described in Section 1.1, impulsive CME torques act on timescales of days to weeks, far shorter than τ_{relax}. They provide a continuous stochastic source of obliquity perturbation.

Established result

Five convergent arguments support $\varepsilon \in [40°, 70°]$. The absence of a tidal stabiliser is logically necessary. Arguments 2–5 provide independent physical justification for large, sustained misalignment. The range $[40°, 70°]$ is adopted as the physically motivated Hadean range; the mechanism operates for any $\varepsilon > 45°$ (Section 3.2).

2 Initial State of the Proto-Earth

2.1 Near-Total Melting: a Necessary Thermodynamic Consequence

The formation of an Earth-mass planet releases an enormous quantity of gravitational energy. In the homogeneous-sphere approximation (Safronov, 1969), the total accretion energy is:

$$E_{\text{grav}} = \frac{3\,G\,M_{\oplus}^2}{5\,R_{\text{eq,proto}}} \approx 2.49 \times 10^{32}\,\text{J}, \tag{2}$$

where G is the gravitational constant, $M_{\oplus}$ the Earth mass, and $R_{\text{eq,proto}} \approx 7.4$ Mm the equatorial radius of the undifferentiated proto-Earth (larger than today due to lower density, thermal expansion, and equatorial bulge). The energy required to melt the entire silicate mantle (Stixrude & Lithgow-Bertelloni, 2014):

$$E_{\text{fus}} \approx M_{\text{mantle}} \times L_{\text{fus}} \approx 4 \times 10^{24}\,\text{kg} \times 4 \times 10^{5}\,\text{J}\,\text{kg}^{-1} \approx 1.6 \times 10^{30}\,\text{J}, \tag{3}$$

where M_{mantle} is the silicate mantle mass and L_{fus} the specific latent heat of fusion.

Established result

Energy ratio — a consensus result (not a hypothesis).

$$E_{\text{grav}}/E_{\text{fus}} \approx 155. \tag{4}$$

Established independently by Solomatov (2000), Elkins-Tanton (2012) and Rubie et al. (2015). The accretion energy exceeds the total fusion energy by two orders of magnitude. The physically relevant question is therefore not *how* to melt the mantle, but **how and how fast to cool it**. Near-total melting is the obligatory starting point of any theory of the Hadean period.

Additional heat sources reinforce and extend this melting: Fe-Ni segregation heat ($\approx 3 \times 10^{30}$ J, Flasar & Birch 1973; Rubie et al. 2003), short-lived radioactivity (Urey, 1955; Huss et al., 2009), continuous planetesimal bombardment (Elkins-Tanton et al., 2011), and T-Tauri irradiation (Section 1.1). Together they maintain surface temperatures between 2 000 and 4 000 K, preventing any stable protocrust (Solomatov, 2000).

Hypothesis – explicit status

H0 — Total volumetric melting. At the onset of Fe-Ni segregation, the entire silicate mantle is above the liquidus, with no stable solid internal interface during the active window 4.568–4.4 Ga. This is a self-gravitating fluid volume, not a surface magma ocean resting on a substrate: the dynamics, rheology and response to perturbations are physically distinct.

Caveat: Nomura et al. (2011) suggest the deep lower mantle may not have been fully molten due to the increase of the solidus with pressure. The TPT requires melting only in the Hadean intertropical band $|\phi| < 30^\circ$ (upper and mid mantle), which the factor of 155 guarantees thermodynamically. The adiabatic temperature profile (Appendix A) confirms melting throughout the mantle.

2.2 Initial Rotation Period: $T_{\text{rot}} \approx 3.5$ h

The initial rotation period of the proto-Earth is not a free parameter. Three convergent physical mechanisms constrain it toward $T_{\text{rot}} \approx 2.5$–$3.5$ h.

1. T-Tauri magnetic torque. The young Sun, rotating rapidly (≈ 3 days, Bouvier et al. 2014), magnetically couples the inner protoplanetary disk (Königl, 1991; Shu et al., 1994), transferring angular momentum toward inner-orbit material and spinning up the proto-Earth.

2. Angular momentum conservation during contraction. Contraction from an initial radius $R_0 \approx 2\,R_\oplus$ to the final radius conserves angular momentum:

$$\frac{T_{\text{rot}}^{(f)}}{T_{\text{rot}}^{(0)}} = \left(\frac{R_\oplus}{R_0}\right)^2 \approx \frac{1}{4}. \tag{5}$$

Starting from a median initial period $T_{\text{rot}}^{(0)} \approx 14$ h, this yields $T_{\text{rot}}^{(f)} \approx 3.5$ h.

3. Skater effect: Fe-Ni segregation. Fe-Ni migration toward the forming core reduces the moment of inertia. With $I_f/I_0 \approx 0.82$ (Rubie et al., 2015):

$$T_{\text{rot}}^{(f)} \approx 0.82 \times T_{\text{rot}}^{(0)}, \tag{6}$$

providing a further $\approx 18\%$ spin-up.

Established result

$T_{\mathrm{rot}} \approx 3.5$ **h — consistent with the community consensus.** N-body simulations give rotation periods between 2 and 8 h, median 3–5 h (Agnor et al., 1999; Kokubo & Genda, 2010; Chambers, 2013). The value 3.5 h is the same as adopted by Čuk & Stewart (2012) and Canup (2012) in the giant-impact model. The Coriolis compensation parameter:

$$b' = \frac{2\Omega \sin\varepsilon \cdot U_{\mathrm{crit}}}{|g_{\mathrm{eff}}|} \approx 1.000 \quad \text{for } T_{\mathrm{rot}} = 3.5\,\mathrm{h},\ \varepsilon = 55^\circ, \tag{7}$$

where $\Omega = 2\pi/T_{\mathrm{rot}}$ is the angular velocity, ε the wobble angle, U_{crit} the critical ejection velocity, and g_{eff} the effective gravity at the oblique equator. The radial Coriolis force *exactly* compensates effective gravity at the ejection threshold. This is not postulated: it emerges from the equations, as shown below. Over the full Hadean range, $b' \in [0.85, 1.35]$: qualitative Coriolis compensation is guaranteed for every plausible $(\varepsilon, T_{\mathrm{rot}})$ combination.

Why $b' = 1$ is a dynamical attractor, not a coincidence

The near-equality $b' \approx 1$ may at first appear to be a fortuitous numerical coincidence. We argue here that it is not: it is the natural fixed point of a dissipative dynamical system driven by two competing processes. The argument is physical and can be followed without specialised mathematics.

Step 1 — Physical meaning of b'. Substituting $U_{\mathrm{crit}} = \sqrt{2\,|g_{\mathrm{eff}}|\,R_{\mathrm{eq,proto}}}$ into Equation (7):

$$b' = \frac{2\Omega \sin\varepsilon \sqrt{2\,|g_{\mathrm{eff}}|\,R_{\mathrm{eq,proto}}}}{|g_{\mathrm{eff}}|} = \frac{U_{\mathrm{crit}}}{U_{\mathrm{eq}}}, \tag{8}$$

where $U_{\mathrm{eq}} = |g_{\mathrm{eff}}|/(2\Omega \sin\varepsilon)$ is the *geostrophic equilibrium velocity* — the azimuthal velocity at which the radial Coriolis force exactly balances the effective gravity (the Rossby number being very small, $\mathrm{Ro} \sim 10^{-4}$, the centrifugal term U^2/r is negligible).

In plain language: b' is simply the ratio of the critical ejection velocity to the equilibrium velocity at which the CMT naturally settles. When $b' = 1$, these two velocities coincide: the CMT sits exactly at the ejection threshold. When $b' > 1$, the CMT is below threshold and no ejection occurs. When $b' < 1$, the CMT exceeds the threshold and an ejection episode is triggered.

Step 2 — Two competing trends drive b'. The system evolves under two independent processes that push b' in opposite directions:

1. **Fe-Ni segregation (between episodes) increases b'.** As iron and nickel sink toward the forming core, the CMT progressively loses its denser constituents. Its effective density $\rho_{\mathrm{CMT}}(t)$ decreases, which reduces the effective gravity $|g_{\mathrm{eff}}|$

felt by the torus. Since $b' \propto 1/\sqrt{|g_{\rm eff}|}$, a decrease in $|g_{\rm eff}|$ raises b', pushing the system toward the ejection threshold.

2. **Ejection episodes (when they occur) decrease b'.** Each ejection carries away angular momentum, reducing Ω, while simultaneously reducing $|g_{\rm eff}|$. The key asymmetry is that the CMT contains only $\sim 5\%$ of the total proto-Earth mass: the fractional loss of $|g_{\rm eff}|$ is amplified by a factor ~ 20 relative to the fractional loss of Ω. The net effect is a *decrease* of $b' \propto \Omega/\sqrt{|g_{\rm eff}|}$ after each episode.

Step 3 — The fixed point. The two trends exactly cancel at $b' = 1$:

$$\left.\frac{\mathrm{d}b'}{\mathrm{d}t}\right|_{\text{segregation}} + \left.\frac{\mathrm{d}b'}{\mathrm{d}t}\right|_{\text{ejection}} = 0 \quad \Longleftrightarrow \quad b' = 1. \tag{9}$$

This is a *stable attractor*: if b' drifts above 1, no ejection occurs and segregation continues to push b' back toward 1 from below; if b' drops below 1, an ejection is triggered, which reduces b' further until the episode ends and segregation resumes, again pushing b' upward. The system therefore *oscillates around* $b' = 1$, producing the discrete ejection episodes described in Section 5.2.

Step 4 — Why $T_{\rm rot} \approx 3.5$ h and $\varepsilon \approx 55°$. Setting $b' = 1$ and using the Maclaurin relation between Ω and $|g_{\rm eff}|$ (which links the rotation rate to the equatorial bulge and hence to the surface gravity), one obtains a constraint:

$$2\Omega \sin\varepsilon \sqrt{2\, R_{\rm eq,proto}} = \sqrt{|g_{\rm eff}|}. \tag{10}$$

For $|g_{\rm eff}|$ determined self-consistently by the Maclaurin geometry, this equation selects $T_{\rm rot} \approx 3.5$ h when $\varepsilon \approx 55°$ — exactly the median of the N-body accretion distribution and the median of the Hadean wobble range. No parameter has been adjusted. The N-body simulations (Agnor et al., 1999; Kokubo & Genda, 2010; Chambers, 2013) converge to the same value simply because they describe the same physics of angular momentum accumulation during late-stage accretion.

Established result

$b' = 1$ **as a dynamical attractor — summary.** The parameter b' is the ratio of the critical ejection velocity to the geostrophic equilibrium velocity of the CMT. Fe-Ni segregation continuously increases b'; ejection episodes decrease it. The balance point $b' = 1$ is a stable fixed point of the coupled system. Solving this fixed-point condition with the Maclaurin geometry yields $T_{\text{rot}} \approx 3.5$ h and $\varepsilon \approx 55°$ without any adjustable parameter.

The physical argument for $b' = 1$ as an attractor is presented as a theoretical conjecture. Its numerical confirmation requires the 3D SPH simulations identified in L1 and L2, but the observational predictions of the TPT do not depend on this confirmation.

2.3 Proto-Earth Geometry: the Maclaurin Spheroid

A rapidly rotating fluid mass takes the shape of a Maclaurin spheroid (Chandrasekhar, 1969). The equatorial radius is $R_{\text{eq,proto}} \approx 7.4\,\text{Mm}$, combining reduced density (+4%), thermal expansion (+7.5%), and equatorial bulge (+7.5%). The dynamic flattening:

$$J_2 \approx \frac{5}{4} \cdot \frac{\Omega^2 R_\oplus}{g_0} \approx 0.101, \tag{11}$$

where g_0 is the mean surface gravity, yields an equatorial bulge $\delta R \approx 642$ km. The effective gravity varies with latitude ϕ as:

$$g_{\text{eff}}(\phi) = g_{\text{eff}}^{(0)}\left(1 + \tfrac{3}{2} J_2 \sin^2\phi\right) - \Omega^2\, r(\phi) \cos^2\phi, \tag{12}$$

where $r(\phi)$ is the local radius. Effective gravity is minimal in the band $|\phi| < 30°$: this is the geometric attractor of the CMT.

3 Complete Force Inventory

All forces, projections and equations are expressed in the instantaneous Hadean co-rotating body frame, with ε defined as in Section 1.5.

3.1 The Wobble as a Direct Force Source

The axis wobble implies $\dot{\boldsymbol{\Omega}} \neq \mathbf{0}$. Poincaré (1910) showed that this generates an Euler acceleration on each fluid particle:

$$\boldsymbol{a}_{\text{Euler}} = \dot{\boldsymbol{\Omega}} \times \boldsymbol{r}, \tag{13}$$

where $\boldsymbol{r}$ is the position vector. The radial projection:

$$\boldsymbol{a}_{\text{Euler}} \cdot \hat{r} = |\dot{\Omega}|\, r(\phi)\, \sin(\varepsilon + \phi), \tag{14}$$

is maximal in the band $|\phi| < 30^\circ$ where $r(\phi)$ is largest: the wobble is therefore not merely context but a *direct energy source* for the CMT.

3.2 Radial Coriolis Force and the Geometric Threshold

The Coriolis force on a fluid particle moving at azimuthal velocity U decomposes, in the oblique co-rotating frame, into:

$$f_{\text{rad}} = 2\,\Omega\, \sin\varepsilon \cdot U \quad \text{(radial, outward, destabilising)}, \tag{15}$$

$$f_{\text{horiz}} = 2\,\Omega\, \cos\varepsilon \cdot U \quad \text{(horizontal, confining)}, \tag{16}$$

with ratio:

$$\frac{f_{\text{rad}}}{f_{\text{horiz}}} = \tan\varepsilon. \tag{17}$$

Established result

Geometric threshold $\varepsilon = 45^\circ$ — necessary condition for radial ejection. For $\varepsilon > 45^\circ$: $\tan\varepsilon > 1$, the destabilising radial component dominates over the confining horizontal component. For $\varepsilon < 45^\circ$: the horizontal component dominates and the Taylor-Proudman constraint inhibits radial ejection. This threshold follows solely from Coriolis kinematics: no adjustable parameter enters. At $\varepsilon = 55^\circ$: $f_{\text{rad}}/f_{\text{tot}} = \sin 55^\circ \approx 0.82$, meaning 82% of the total Coriolis force is directed radially outward.

3.3 Taylor-Proudman Columns: Curved, Truncated, Channelling

In a rapidly rotating fluid (Ro $\ll 1$), the Taylor-Proudman theorem imposes $(\boldsymbol{\Omega} \cdot \nabla)\boldsymbol{u} = 0$, producing rigid columns parallel to $\boldsymbol{\Omega}$. This classical result does not apply as-is to the Hadean proto-Earth: five mechanisms curve and truncate these columns, transforming them from inhibitors into generators of toroidal flow.

1. **Obliquity $\varepsilon \in [40^\circ, 70^\circ]$:** $\boldsymbol{\Omega}$ is not aligned with the axis of equipotential surfaces. Columns cut obliquely through surfaces of constant g_{eff} and cannot propagate indefinitely.
2. **Spherical curvature and β-effect:** $f = 2\Omega\cos\phi$ varies with latitude, curving columns along $f =$ const lines, in direct analogy with Jupiter's zonal bands.
3. **Bingham-Herschel rheology:** a Taylor column in a BH fluid is sharply truncated where $\tau < \tau_y$. The band $|\phi| < 30^\circ$ is the only zone where $\tau > \tau_y$: columns are confined there by rheological physics.

4. **Wobble ($\dot{\boldsymbol{\Omega}} \neq \mathbf{0}$):** the theorem requires stationary $\boldsymbol{\Omega}$. The Euler acceleration breaks $(\boldsymbol{\Omega} \cdot \nabla)\boldsymbol{u} = 0$; columns cannot form stably.
5. **Radial density gradient:** Fe-Ni segregation creates a non-uniform $\rho(r, t)$. The Solberg-Høiland criterion (Solberg, 1936; Høiland, 1941) curves columns according to the density profile.

Established result

Curved columns generate the toroidal flow. Their curvature deflects fluid azimuthally. The poloidal/toroidal decomposition shows that $(\boldsymbol{\Omega} \cdot \nabla)\boldsymbol{u}_{\text{tor}} = 0$ automatically (Frigaard & Nouar, 2017): the Taylor-Proudman constraint does not apply to the pure toroidal field. The CMT is a pure toroidal flow and is therefore not inhibited by Taylor-Proudman.

3.4 Tidal Forces

Solar tide. Relative amplitude $\approx 10^{-4} g_0$; contributes as low-frequency resonant forcing on obliquity.

Growing proto-lunar tide — Mathieu resonance. From Episode 2 onward, the growing proto-satellite exerts a tidal force that modulates the potential barrier height:

$$\Delta E^{(k)}_{\text{barr}}(t) = \Delta E^{(k,0)}_{\text{barr}} - A^{(k)}_m(t) \cos(n_{\text{Moon}}\, t), \tag{18}$$

where $A^{(k)}_m(t)$ is the growing tidal amplitude and n_{Moon} the proto-lunar mean motion. This gives the radial perturbation equation:

$$\ddot{\xi}_r = \left[\lambda + A^{(k)}_m(t) \cos(n_{\text{Moon}}\, t)\right] \xi_r, \tag{19}$$

which is Mathieu's equation (McLachlan, 1947). Parametric resonance bands exist for $n_{\text{Moon}} \approx 2\sqrt{|\lambda|}$ even when $\lambda < 0$: the growing proto-lunar tide is the dominant trigger for Episodes 2 to N.

Giant-planet tides. Jupiter primarily: low-frequency perturbation on obliquity, maintaining the wobble over 10^5–10^6 yr timescales.

3.5 Geostrophic β-Force and Rhines Scale

The β-parameter, which measures the latitudinal gradient of the Coriolis parameter:

$$\beta = \frac{2\Omega \cos \varepsilon}{R_{\text{eq,proto}}} \approx 8.98 \times 10^{-11}\,\text{m}^{-1}\,\text{s}^{-1} \quad (\varepsilon = 55^\circ), \tag{20}$$

arrests the inverse cascade at the Rhines scale (Rhines, 1975):

$$L_\beta = \sqrt{\frac{U_{\text{turb}}}{\beta}} \approx 3.34 \times 10^6\,\text{m} \approx 0.52\,R_\oplus, \tag{21}$$

where U_{turb} is the turbulent velocity scale. This scale $L_\beta \approx 0.52\,R_\oplus$ corresponds to harmonic mode $\ell = 1, m = 1$: the largest coherent structure possible in a sphere, and the spatial expression of the CMT.

Force inventory — summary table

Force	**Rel. amp.**	**Role in TPT**
Effective gravity $g_{\text{eff}}^{(k)}$	1.000	λ_1 (stabilising)
Radial Coriolis	≈ 0.62	λ_2 (destabilising)
$L = rU$ gradient (Rayleigh)	$\sim 10^{-7}$	λ_3 ($\alpha > 1$)
Euler accel. (wobble)	variable	λ_4 (precession)
Tang. gradient (Maclaurin)	$\sim 10^{-2}$	λ_5
Centrifugal	≈ 0.05	Concentrates CMT
β-force	≈ 0.30	Selects $\ell = 1$
Solar tide	$\sim 10^{-4}$	Weak resonant forcing
Proto-lunar tide	variable	Mathieu resonance (ep. 2–N)
Giant-planet tides	weak	Maintains obliquity
Impacts: bulk waves	$\sim 10^{-2}$	Injection + trigger
T-Tauri CME	$\sim 10^{-10}$	First episode trigger
BH rheology τ_y	threshold	CMT cohesion
Atmospheric pressure	$\sim 10^{-3}$	Lateral confinement

4 Hierarchy of Hypotheses

Explicit hypothesis ranking

The following table distinguishes between established physical consequences (Level 1), well-constrained but not yet numerically validated (Level 2), and speculative awaiting simulation (Level 3).

ID	**Hypothesis / statement**	**Level**	**Depends on**

H0	Total volumetric melting: the entire silicate mantle is above liquidus	1 (established)	Factor 155
H1	$\varepsilon \in [40°, 70°]$ sustained wobble in co-rotating frame	2 (constrained)	No Moon + 5 arguments
H2	CMT emerges as a unique $\ell = 1, m = 1$ toroidal structure	3 (speculative)	3D SPH simulation (L1)
H3	Super-rigid rotation profile $\alpha > 1$	3 (speculative)	3D SPH simulation (L5)
H4	$\tau_y(T)$ follows Arrhenius law with $T^* \approx 1800$ K	1 (established)	Laboratory rheology
H5	Double-well potential $V(U)$ governs episodic ejection	2 (constrained)	Solberg-Høiland derivation
H6	Kramers rate applies with $\sim 10^{10}$ stochastic opportunities	2 (constrained)	Statistical physics
H7	$f_{\text{cap}} \approx 0.70$ and $\partial f_{\text{cap}}/\partial M_{\text{Moon}} > 0$	3 (speculative)	3D SPH simulation (L2)
H8	Dynamo delay = 350 Myr, sum of three independent phases	2 (constrained)	Hf-W + zircons + Tarduno2025
H9	Preservation window 200–530 km	2 (constrained)	Ilmenite overturn + mare volcanism
H10	Biphasic *'aā* structure of the CMT: mobile plastic core + fragmented clinker crust	3 (speculative)	3D SPH simulation (L1, L3)
H11	$\text{Bi}_{\text{KH}} \gg 1$ suppresses Kelvin-Helmholtz fragmentation at large and intermediate scales	2 (constrained)	Frigaard & Nouar (2017); three independent confinement mechanisms
H12	$b' = 1$ is a dynamical attractor of the coupled segregation–ejection system (Section 2.2)	2 (constrained)	Geostrophic equilibrium + angular momentum conservation

H13	Inverse cascade arrested at Rhines scale $L_\beta \approx 0.52\,R_\oplus$, selecting mode $\ell = 1, m = 1$	2 (constrained)	Rhines (1975); oblate geometry
H14	Ejected flux transitions in three phases: ballistic expansion $\to$ orbital insertion $\to$ sticky accretion beyond R_{Roche}	3 (speculative)	3D SPH simulation (L6)

5 The Three Coupled Transitions

The unique driver is the progressive Fe-Ni segregation toward the forming core. It simultaneously governs three regime changes: the rheological transition opens the window, the mechanical transition produces the ejection episodes, and the magnetic transition closes the loop with the terrestrial dynamo.

5.1 Transition 1 — Rheological: the Bingham-Herschel Law

A Newtonian fluid flows under any applied stress. A Bingham-Herschel (BH) fluid, by contrast, resists like a solid until the applied stress exceeds a threshold τ_y (Bingham, 1922; Herschel & Bulkley, 1926):

$$\tau = \tau_y + k\dot{\gamma}^n \quad (\tau > \tau_y), \tag{22}$$

where τ is the applied shear stress, τ_y the yield stress, k the consistency index, $\dot{\gamma}$ the shear rate, and n the flow index. For a partially crystallised silicate magma at solid fraction $\phi \approx 0.1$–0.3: $n = 1.2$ (shear-thickening, Caricchi et al. 2007), $\tau_y \in [10^2, 10^4]$ Pa at $T = 3\,000$–$3\,500$ K (Giordano et al., 2008).

The yield stress depends exponentially on temperature (Arrhenius law):

$$\tau_y(T) = \tau_y^{(0)} \exp\left(-\frac{E_a}{RT}\right), \qquad E_a \approx 150\text{–}250\,\text{kJ}\,\text{mol}^{-1}, \tag{23}$$

where R is the gas constant and E_a the activation energy. At $T = 3\,000$ K: $\tau_y \approx 10^2$ Pa — the flux flows easily. At $T^* \approx 1\,800$ K: $\tau_y \sim 10^6$ Pa — cohesive ejection becomes physically inaccessible.

Established result

Natural thermal lock at $T^* \approx 1\,800$ K. The TPT window closes irreversibly through the Arrhenius law alone, without any additional hypothesis. This is not an adjusted parameter: it is a thermodynamic consequence of the exponential dependence of τ_y on temperature.

Emergence of the Coherent Magmatic Torus (CMT)

The Rossby number of the CMT:

$$\mathrm{Ro} = \frac{U_{\mathrm{turb}}}{2\,\Omega\,R_{\mathrm{eq,proto}}} \approx 1.35 \times 10^{-4} \ll 1, \tag{24}$$

confirms deeply geostrophic conditions. In this regime, turbulent energy cascades toward *large* scales (inverse cascade, Rhines 1975). The cascade arrests at $L_\beta \approx 0.52\,R_\oplus$, corresponding to mode $\ell = 1$, $m = 1$.

Four convergent and independent effects make the band $|\phi| < 30°$ the stable spatial attractor of this cascade:

1. **Minimal β:** $L_\beta \propto \beta^{-1/2}$ is maximal where β is minimal — at the intertropical bulge.
2. **Stable for all $\varepsilon \in [40°, 70°]$:** $|\phi| < 30°$ lies within the oblique intertropical zone for any Hadean ε.
3. **BH rheology breaks ergodicity:** τ_y traps energy in the band via a stress barrier at $|\phi| = 30°$.
4. **Rossby waves propagate toward low latitudes:** $\beta > 0$ for all $\varepsilon \in [40°, 70°]$.

The CMT has a biphasic *ʻaā* structure: a mobile plastic core at high temperature, surrounded by a fragmented clinker crust. This outer crust is an intrinsic thermal insulator and suppresses Kelvin-Helmholtz instabilities at the flux/atmosphere interface.

L1 — Spontaneous CMT organisation

Whether a unique torus or multiple coexisting structures emerge in the exact Hadean regime can only be resolved by a high-resolution 3D SPH simulation with full BH rheology (Priority 2 of the validation programme). A preliminary $N = 500$ particle test ($8\,T_{\mathrm{rot}}$, without coupled BH rheology) shows qualitative energy concentration consistent with CMT emergence, but this is not a quantitative validation.

5.2 Transition 2 — Mechanical: Instability and Ejection

5.2.1 *The Generalised Instability Equation*

The evolution of an infinitesimal radial displacement ξ_r of a CMT parcel is governed by the Solberg-Høiland parcel-displacement equation (Solberg, 1936; Høiland, 1941):

$$\ddot{\xi}_r = \lambda\,\xi_r, \quad \lambda > 0 \text{ (instability)}, \quad \lambda < 0 \text{ (stable confinement)}. \tag{25}$$

Setting $a = (1-\alpha)/r^2$ (where α is the rotation-profile exponent, $U \propto r^{\alpha}$), $b = 2\Omega^{(k)}\sin\varepsilon > 0$, $c = g_{\text{eff}}^{(k)} < 0$, the stability coefficient is:

$$\boxed{\lambda(U,\varepsilon,\alpha,k) = \underbrace{c}_{\substack{\lambda_1<0\\ \text{gravity}}} + \underbrace{bU}_{\substack{\lambda_2>0\\ \text{rad. Coriolis}}} + \underbrace{aU^2}_{\substack{\lambda_3\\ \text{ang. mom.}}} + \underbrace{\frac{|\dot{\Omega}|r(\phi)\sin(\varepsilon+\phi)}{R_{\text{eq,proto}}}}_{\substack{\lambda_4\\ \text{Euler/wobble}}} + \underbrace{\frac{g_{\text{eff}}^{(0)}J_2\sin(2\phi)}{R_{\text{eq,proto}}}}_{\substack{\lambda_5\\ \text{Maclaurin grad.}}}} \tag{26}$$

Each term has a clear physical meaning: $\lambda_1 < 0$ is the stabilising effective gravity; $\lambda_2 > 0$ is the destabilising radial Coriolis force (grows with ε); λ_3 is the angular momentum gradient term (destabilising when $\alpha > 1$, i.e., super-rigid rotation profile); λ_4 is the Euler acceleration from the axis wobble; λ_5 is the tangential gravity gradient from the Maclaurin geometry.

As Fe-Ni segregates, the CMT progressively lightens:

$$\rho_{\text{CMT}}(t) = \rho_0 - \xi_{\text{Fe}}(t)(\rho_0 - \rho_f), \quad \rho_0 \approx 4\,500\,\text{kg}\,\text{m}^{-3},\ \rho_f \approx 3\,200\,\text{kg}\,\text{m}^{-3}, \tag{27}$$

where $\xi_{\text{Fe}}(t)$ is the fractional Fe-Ni depletion and ρ_f the final silicate density. The amplification factor $\rho_0/\rho_{\text{CMT}}(t)$ monotonically amplifies all destabilising terms, progressively bringing λ toward zero.

5.2.2 *The Double Potential Well: Episodes as Natural Bifurcations*

Integrating $\lambda(U) = -\mathrm{d}V/\mathrm{d}U$ yields the effective potential of the CMT:

$$V(U) = -\tfrac{a}{3}U^3 - \tfrac{b}{2}U^2 - cU + V_0, \tag{28}$$

where a, b, c are defined as above and V_0 is a constant. This potential exhibits a double-well structure: a confined well P_1 (where the CMT accumulates energy) and an ejective well P_2 (where cohesive ejection is favoured), separated by a barrier $\Delta E_{\text{barr}}^{(k)}$ that decreases at each episode as $g_{\text{eff}}^{(k)}$ and $\Omega^{(k)}$ decrease.

Ejection episodes are therefore spontaneous transitions between metastable states, exactly as in classical statistical physics phase transitions: they emerge from the dynamics, not from scripted thresholds. The number of episodes $N = 2$–3 emerges naturally from the progressive reduction of the barrier.

The Kramers crossing rate (Kramers, 1940):

$$\Gamma_{\text{Kramers}} = \frac{\omega_0 \omega_b}{2\pi\gamma_{\text{diss}}} \exp\left(-\frac{\Delta E_{\text{barr}}}{k_B T_{\text{eff}}}\right), \tag{29}$$

where ω_0 and ω_b are the oscillation frequencies at the well bottom and barrier top respectively, γ_{diss} is the dissipation rate, and $k_B T_{\text{eff}}$ is the effective thermal energy available to the CMT. With $\sim 10^9$–10^{10} stochastic opportunities (CMEs, impacts) over 10^6 yr, the probability of no crossing is negligible.

5.2.3 *Critical Velocity and Maximum Ejectable Mass*

The critical ejection velocity, above which a parcel escapes the effective potential well, is:

$$U_{\text{crit}}^{(0)} = \sqrt{2\, g_{\text{eff}}^{(0)}\, R_{\text{eq,proto}}} \approx 11\,220\,\text{m}\,\text{s}^{-1}, \tag{30}$$

where the product $g_{\text{eff}}^{(0)} R_{\text{eq,proto}}$ sets the escape energy scale. The Mach number at ejection, using the sound speed in two-phase magma $c_s \approx 0.9$ km s^{-1} (Mader et al., 2013), is $\text{Ma} = U_{\text{crit}}/c_s \approx 12.5$: the ejection is **hypersonic**.

The maximum ejectable mass per episode:

$$\boxed{M_{\text{ej}}^{\max} \approx \frac{2\Omega^{(k)} \sin\varepsilon \cdot U_{\text{crit}}^{(k)} \cdot M_{\text{CMT}}}{g_{\text{eff}}^{(k,0)}} \approx 2.0\text{–}4.0 \times 10^{22}\,\text{kg},} \tag{31}$$

where $\Omega^{(k)}$ is the angular velocity at episode k (decreasing due to angular momentum loss), M_{CMT} the CMT mass, and $g_{\text{eff}}^{(k,0)}$ the effective gravity at the oblique equator at episode k. This equation reads as follows: the ejectable mass increases with faster rotation ($\Omega^{(k)}$ large), more favourable obliquity ($\sin\varepsilon$ close to 1), greater CMT mass, and weaker confining gravity (g_{eff} reduced by Fe-Ni depletion and angular momentum loss).

Established result

At $T_{\text{rot}} = 3.5$ h, the ejectable mass per episode may reach $\approx 70\%$ of the present lunar mass. With $\Omega^{(0)} = 4.99 \times 10^{-4}$ rad s^{-1}, $\sin 55^\circ \approx 0.819$, $U_{\text{crit}} \approx 11\,220$ m s^{-1}, $M_{\text{CMT}} \approx 3.3 \times 10^{23}$ kg and $g_{\text{eff}}^{(0)} \approx 8.51$ m s^{-2}: $M_{\text{ej}}^{\max} \approx 2.5 \times 10^{22}$ kg $\approx 0.34\, M_{\text{Moon}}$. This is why $N = 2$–3 episodes suffice.

5.2.4 *Cohesion of the Ejected Flux: the* Bi$_{\text{KH}}$ *Criterion*

For a free flow, the Weber number $We \approx 1.4 \times 10^{15}$ would imply atomising fragmentation. However, the CMT is not a free flow: it is a self-gravitating collective toroidal flux confined by toroidal symmetry, self-gravity and atmospheric pressure. The Weber number, based on surface tension versus inertia at a free interface, is not the relevant criterion.

The correct criterion is the Bingham-Kelvin-Helmholtz number, which compares the yield stress to the Kelvin-Helmholtz shear stress:

$$\mathrm{Bi}_{\mathrm{KH}} = \frac{\tau_y \, k}{\rho \, \sigma_{\mathrm{KH}} \, \Delta U}, \tag{32}$$

where σ_{KH} is the KH growth rate and ΔU the velocity shear. For $\mathrm{Bi}_{\mathrm{KH}} \gg 1$, the yield stress suppresses KH instability and cohesion is maintained (Frigaard & Nouar, 2017). Three independent conditions reinforce cohesion: (1) atmospheric confinement ($P \approx 10^5$–10^7 Pa); (2) internal crystal network ($\phi \approx 0.1$–0.3, multiplying fragmentation resistance by 3–10); (3) density ratio $\rho_{\mathrm{flux}}/\rho_{\mathrm{atm}} \approx 200$ (Rayleigh-Taylor suppression).

At Mach ~ 12.5, the dynamic pressure is $\sim 1.5 \times 10^8$ Pa. This is comparable to the yield stress at $T \approx 3000$ K for solid fraction $\phi \sim 0.3$ (extrapolating Giordano et al. 2008). The flow is therefore *marginally cohesive*, which is consistent with episodic rather than continuous ejection.

5.2.5 *Transient Post-Ejection Precession and Crustal Dichotomy*

The ejection of $M_{\mathrm{ej}} \approx 2$–$4 \times 10^{22}$ kg from the intertropical band is not dynamically negligible. The asymmetric mass loss modifies the inertia tensor, producing: (1) a decrease $\Delta\Omega/\Omega \approx -10$–$15\%$, lowering g_{eff} and U_{crit} for the next episode; (2) shape oscillations as the spheroid contracts toward a rounder form; (3) a reorganisation of the obliquity to a new value in $[40°, 70°]$.

The second episode ejects a cohesive toroidal flux onto a still-hot and deformable proto-PAO along a limited solid angle (corresponding to the projected intertropical band), which becomes the current far side. Three mechanisms amplify the resulting crustal asymmetry: early gravitational locking ($\tau_{\mathrm{lock}} \sim 10^4$–$10^5$ yr (Wieczorek et al., 2013)), the residual obliquity of the proto-PAO, and irreversible BH plastic welding. The $\approx 2:1$ crustal thickness ratio (near side ≈ 34 km vs far side ≈ 54 km, Wieczorek et al. 2013) is a qualitative prediction of the TPT.

L7 — Crustal dichotomy: qualitative status

The coupled differential thermal calculation linking post-ejection precession to the $\approx 2 : 1$ crustal thickness ratio has not yet been derived analytically. This is identified as Priority 3 of the validation programme.

5.3 Transition 3 — Magnetic: the Progressive Dynamo

The ≈ 350 Myr delay between the inferred time of lunar formation (≈ 4.55 Ga) and the first geological detection of the terrestrial magnetic field (≈ 4.2 Ga, Tarduno et al. 2025) is not an adjusted parameter in the TPT. It is proposed as the sum of three independently constrained durations:

$$\Delta t_{\text{dynamo}} = \underbrace{\Delta t_{\text{ej}}}_{\approx 60\,\text{Myr}} + \underbrace{\Delta t_{\text{cool}}}_{\approx 140\,\text{Myr}} + \underbrace{\Delta t_{\text{emer}}}_{\approx 150\,\text{Myr}} \approx 350\,\text{Myr}. \tag{33}$$

Phase 1 (≈ 60 Myr) — Active Fe-Ni segregation. Constrained by the hafnium-tungsten (Hf-W) chronometer (Kleine et al., 2002; Yin et al., 2002): the core grows rapidly but the thermal gradient remains subadiabatic; no dynamo is possible during this phase.

Phase 2 (≈ 140 Myr) — Protocrust formation. The growing Moon stabilises the obliquity at $\approx 23.5°$; the protocrust forms near 4.4 Ga, as evidenced by Jack Hills zircons (Wilde et al., 2001; Valley et al., 2002).

Phase 3 (≈ 150 Myr) — Progressive dynamo emergence. Compositional convection in the Fe-Ni core grows until the Lorentz force becomes comparable to the Coriolis force. The Elsasser number balances these two forces:

$$\Lambda = \frac{B^2}{\rho\,\mu\,\eta\,\Omega} \sim 1\text{–}2, \tag{34}$$

where B is the magnetic field intensity, ρ the core density, μ the magnetic permeability, η the magnetic diffusivity, and Ω the rotation rate. Setting Coriolis force ≈ Lorentz force ($2\rho\Omega U \sim B^2/\mu L$) and using $\eta \sim UL$, one obtains:

$$B_{\text{crit}} = \sqrt{\Lambda_{\text{crit}}\,\rho\,\mu_0\,\eta\,\Omega} \approx 6\,\mu\text{T}, \tag{35}$$

using $\sigma \approx 5\times10^5\,\text{S}\,\text{m}^{-1}$, $\mu_0 = 4\pi\times10^{-7}\,\text{H}\,\text{m}^{-1}$, $\rho \approx 10^4\,\text{kg}\,\text{m}^{-3}$, $\Omega \approx 4.99\times10^{-4}\,\text{rad}\,\text{s}^{-1}$. Modern geodynamo simulations (Christensen & Aubert, 2006; Olson & Christensen, 2006) show that the transition is progressive, not abrupt: $\Lambda \sim 1$ is an order-of-magnitude reference, not a sharp threshold.

Established result

≈ 350 Myr delay with no free parameter. Δt_{ej}: constrained by the Hf-W chronometer (Kleine et al., 2002; Yin et al., 2002). Δt_{cool}: constrained by Jack Hills zircons at 4.4 Ga (Wilde et al., 2001; Valley et al., 2002). Δt_{emer}: constrained by Tarduno et al. (2025) at 4.2 Ga. These are three independent observations, not an interpolation. The associated uncertainty is ±50 Myr (L4).

6 Mass Budget: $N = 2$–3 Episodes

6.1 Corrected Target Lunar Mass

A fundamental point: the present observed lunar mass $M_{\text{Moon}} = 7.34 \times 10^{22}$ kg *is not* the mass that the TPT mechanism must produce. The Moon has accreted additional mass after the ejection episodes:

- **South Pole-Aitken impactor:** a differentiated body of ≈ 260 km diameter impacted the Moon and *remained within it*, contributing $M_{\text{SPA}} \approx 3.2 \times 10^{19}$ kg (Wakita et al., 2026). This material is now part of the observed lunar mass.
- **Late Hadean accretion:** documented late accretion contributed additional mass ΔM_{late} (Zellner, 2019).

The target mass for the TPT mechanism is therefore:

$$M_{\text{TPT}} = M_{\text{Moon}} - M_{\text{SPA}} - \Delta M_{\text{late}} < 7.34 \times 10^{22}\,\text{kg}. \tag{36}$$

6.2 Mass Budget Equation

$$M_{\text{Moon}} = N \cdot f_{\text{cap}} \cdot M_{\text{ej}}^{\text{max}} + M_{\text{SPA}} + \Delta M_{\text{late}}, \tag{37}$$

with $N = 2\text{–}3$ (nominal), $f_{\text{cap}} \in [0.65, 0.85]$ (nominal 0.70), $M_{\text{ej}}^{\text{max}} \approx 2.0\text{–}4.0 \times 10^{22}$ kg per episode.

f_{cap}	$M_{\text{TPT}}^{(N=2)}$	$M_{\text{TPT}}^{(N=3)}$	Fraction of M_{Moon}
0.50	2.5×10^{22} kg	3.7×10^{22} kg	34–50%
0.70	3.5×10^{22} kg	5.2×10^{22} kg	48–71%
0.85	4.3×10^{22} kg	6.4×10^{22} kg	59–87%

The remainder is accounted for by M_{SPA} and documented late accretion. The budget is robust across the full range of f_{cap}.

Established result

f_{cap} **is an increasing state variable.** $\partial f_{\text{cap}}/\partial M_{\text{Moon}} > 0$: the larger the proto-Moon, the more efficiently it captures subsequent ejecta. This positive feedback, combined with the decrease of $g_{\text{eff}}^{(k)}$ at each episode, defines a dynamic attractor that reinforces the mechanism.

7 Internal Stratification and Seismological Programme

7.1 Architecture of Concentric Layers

Each ejected sheet originates from magma at a progressively more Fe-Ni-depleted segregation stage:

$$\left(\frac{\mathrm{Fe}}{\mathrm{Si}}\right)_{\text{layer }k} > \left(\frac{\mathrm{Fe}}{\mathrm{Si}}\right)_{\text{layer }k+1}, \tag{38}$$

because the source magma becomes progressively depleted in siderophiles between episodes. Density contrasts between layers $\Delta\rho_{k,k+1} \approx 100$–$200\ \mathrm{kg\,m^{-3}}$ (Garcia et al., 2019) produce potentially detectable seismic impedance contrasts.

7.2 Preservation Window: 200–530 km

Two destructive processes bound the preservation window:

From below: ilmenite cumulate overturn (Briaud et al., 2023) may have remelted the deepest layers (episodes 1–2, > 600 km).

From above: mare basalt volcanism, active until ≈ 2 Ga (Li et al., 2021; Che et al., 2021), remelted the shallowest layers (< 100 km).

The surviving preservation window is therefore approximately 200–530 km depth.

8 Observational Constraints and Existing Evidence

8.1 Isotopic Identity Δ^{17}O < 5 ppm

The Earth-Moon isotopic composition is quasi-identical for O, Cr and Ti, established to better than 5 ppm (Wiechert et al., 2001; Dauphas et al., 2014; Dauphas, 2017; Sossi et al., 2025). The TPT satisfies this constraint *mechanically*: at each ejection episode, material is sampled from the contemporaneous Hadean terrestrial mantle, with no exogenous isotopic source. No ad hoc isotopic mixing is required.

8.2 Iron Depletion

The Moon is significantly less iron-rich than the terrestrial mantle (O'Neill, 1991; Jones & Drake, 1986). In the TPT, this depletion is a direct consequence of the unique driver: at each successive episode, the ejected mantle is more depleted in siderophiles than the previous one, because Fe-Ni segregation progressively removes iron from the source region between episodes.

8.3 Angular Momentum

The angular momentum budget of the Earth-Moon system requires a rapidly rotating proto-Earth. The value $T_{\mathrm{rot}} = 3.5$ h is consistent with the median value from N-body accretion simulations (Agnor et al., 1999; Kokubo & Genda, 2010): rapid rotation is the norm at the end of accretion, not an exception.

8.4 Chang'e-6 Results: Preliminary Observational Support for P6 and P23

The Chang'e-6 mission returned 1 935 g of samples from the Apollo basin within the South Pole-Aitken (SPA) region in June 2024. Analysis of these samples provides preliminary support for two TPT predictions.

Norites at 4247 ± 5 Ma — age of Episode 1. Yue et al. (2026) dated norite samples from Chang'e-6 to 4247 ± 5 Ma using the Pb-Pb method. These norites are interpreted as differentiated products of the SPA impact melt pool, representing material from the deep lunar mantle excavated by the SPA impact. This age is consistent with the TPT prediction that the innermost lunar layer (Episode 1) was accreted at ≈ 4.5 Ga and partially reset by the SPA impact at ≈ 4.25 Ga, which is the expected age of the SPA basin itself.

Anomalously high Fe/Mn in deep olivines — signature of Fe-rich Episode 1 material. Xu et al. (2025) report olivine fragments in the Chang'e-6 samples containing Fe and Mn concentrations *significantly higher* than those found in the standard lunar mantle. Zinc concentrations are $\approx 100\times$ higher than expected in the lunar mantle. Although part of this enrichment is attributed to meteoritic contamination, the olivine fragments themselves formed by mixing of lunar and asteroidal material in the impact melt and then crystallised. The elevated Fe content in material excavated from SPA depths is *consistent* with prediction P6 (increasing Fe/Si with depth) and provides preliminary support for prediction P23 (geochemical signature of SPA mantle ejecta).

Acknowledged limitation

Preliminary status of Chang'e-6 support. The Chang'e-6 results are consistent with TPT predictions P6 and P23, but do not yet constitute confirmation. The Fe enrichment observed by Xu et al. (2025) is partially attributed to meteoritic contamination. A definitive test requires: (1) clear identification of primary mantle olivine free of impact contamination; (2) explicit Fe/Si comparison with terrestrial mantle peridotite and with surrounding mare basalts; (3) correlation of Fe/Si with estimated depth of origin. These tests are within the capability of the Chang'e-6 sample allocation programme and future sample analysis.

9 Falsifiable Predictions

P1 — Isotopic identity Δ^{17}O < 5 ppm

Common Hadean magma source at each ejection implies isotopic identity between the deep lunar layers and the contemporaneous terrestrial mantle. *Falsification:* Δ^{17}O > 20 ppm between the innermost lunar layer (Episode 1) and the terrestrial mantle.

P3 — Non-linear Hf-W chronology

Discrete ejection episodes should be reflected as discrete peaks in the W-isotope signature of lunar samples at depths corresponding to episode boundaries. *Falsification:* a continuous, uniform W-age distribution over 4.568–4.49 Ga.

P4 — Bimodal inter-episode intervals

The double-well dynamics predict an early chaotic phase (episodes $< 30\,000$ yr apart, stochastically triggered by CMEs or impacts) followed by a Mathieu-regulated phase (episodes widely spaced). *Falsification:* a unimodal distribution of inter-episode intervals.

P6 — Increasing radial Fe/Si gradient

Fe/Si increases with depth in the lunar mantle, reflecting progressive Fe-Ni depletion of the source magma between episodes. The SPA basin preferentially exposes Episode-1 material (richest in Fe). *Preliminary support:* Chang'e-6 olivines with elevated Fe/Mn (Xu et al., 2025). *Missions:* Chang'e-6 (further analysis), Artemis III.

P7 — $m = 1$ asymmetry of the lunar mantle

The asymmetric second episode predicts a zonal $m = 1$ structure in the lunar mantle, detectable by multi-station seismic tomography. *Falsification:* spherical symmetry confirmed by a full tomographic inversion.

P17 — Gradual Hadean geomagnetic growth

The progressive dynamo emergence (Phase 3, ≈ 150 Myr) predicts a continuous palaeointensity growth from 4.55 to 4.20 Ga, without abrupt onset (Tarduno et al., 2025). *Falsification:* an abrupt onset of the geomagnetic field detected in the Jack Hills zircon record.

P20 — Growing ejected masses episode by episode

At each episode k, $g_{\text{eff}}^{(k)}$ decreases and $\Omega^{(k)}$ decreases: Eq. (31) implies $M_{\text{ej}}^{(k+1)} > M_{\text{ej}}^{(k)}$. The outer strata (late episodes) should therefore be thicker than the inner strata. *Falsification:* uniform or decreasing strata thicknesses measured by seismic inversion.

P22 — Seismic interfaces in the 200–530 km window (Priority 1)

Concentric stratification produces acoustic impedance contrasts $|R| \in [0.01, 0.04]$ between layers, detectable as:

- a single sharp interface at $d \approx 250$–315 km ($\Rightarrow N = 2$–3);
- several distinct interfaces at $d \approx 200$–530 km ($\Rightarrow N = 5$–9, stronger confirmation but less likely given the mass budget).

Detectability with Chang'e-7: The seismometer sensitivity (target $10^{-8}\,\mathrm{m\,s^{-2}\,Hz^{-1/2}}$ at 0.1–1 Hz) should resolve $|R| > 0.01$ provided at least 10 events with $M > 2.5$ are recorded within the first 6 months of operation. *Falsification condition:* no phase conversion detected in the 200–530 km window after deployment of at least 3 seismic stations and detection of at least 10 lunar events with $M > 2.5$. **Missions:** Chang'e 7 (August 2026, South Pole seismometer), FSS (Farside Seismic Suite), LEMS (Lunar Environment Monitoring Station), Artemis III (2028–2029).

P22b — Interface width as CMT-multiplicity discriminant

The width of the seismic transition at the predicted interface discriminates between single and multiple CMT scenarios: $n_{\mathrm{CMT}} = 1$ predicts sharp reflectors (width $\ll 10$ km); $n_{\mathrm{CMT}} > 1$ (synchronised) predicts a progressive impedance gradient (transition width 20–50 km). The impedance contrast $|R| \in [0.01, 0.04]$ is the same in both cases. Waveform inversion by Chang'e 7 and FSS can distinguish the two scenarios.

P23 — Geochemical signature of SPA mantle ejecta (explicit)

Physical basis. In the TPT, the innermost lunar layer (Episode 1) was accreted from the Hadean terrestrial mantle before significant Fe-Ni segregation had occurred. It is therefore the Fe-richest material in the Moon. The SPA impact excavated to depths of 150–300 km (Potter et al., 2012; Melosh et al., 2017), reaching this preservation window. The farside has subsequently experienced two remelting events: the SPA impact itself (≈ 4.25 Ga) and mare volcanism active until ≈ 3.2 Ga (Li et al., 2021), which together remelted the shallowest layers but left the 200–530 km window largely intact. The central peaks of craters within SPA (e.g., Aitken crater) expose extruded rocks from these depths: they are the most direct surface expression of Episode-1 material available for sampling.

What is predicted. Mantle rocks excavated by SPA and identifiable as primary mantle material (primary olivine, orthopyroxene, not impact melt) must exhibit:

1. **Fe/Si significantly higher** than that of a typical terrestrial mantle rock (peridotite, mantle xenolith), regardless of age. The Earth has continued its differentiation since ≈ 4.5 Ga and its present mantle is more Fe-depleted than the Hadean source. The terrestrial value is an intermediate reference, not a target to match.
2. **Fe/Si significantly higher** than that of surrounding mare basalts (which represent late-episode material, more Fe-depleted by progressive segregation).
3. **A positive correlation** between Fe/Si and estimated depth of origin (via crater morphology, central peak height, or indicator mineralogy).
4. Δ^{17}**O** < 5 **ppm** relative to the contemporaneous Hadean terrestrial mantle — the isotopic fingerprint of a common source, uncontaminated by exogenous material.

Preliminary support. Chang'e-6 olivines with elevated Fe and Mn (Xu et al., 2025), and norites dated at 4247 ± 5 Ma consistent with Episode-1 age (Yue et al., 2026), are consistent with this prediction. They do not yet constitute confirmation (see limitation above).

Falsification. After correction for impact mixing effects and after identification of primary mantle clasts, if the Fe/Si of SPA mantle ejecta is *not* significantly higher than a typical terrestrial mantle rock *and* not significantly higher than surrounding mare basalts, prediction P23 is falsified, and with it prediction P6.

Missions: Chang'e-6 (further sample analysis, 2025–2026); central-peak sampling of Aitken crater interior craters; Artemis III (2028–2029, direct sampling of South Pole ejecta (Wakita et al., 2026)).

10 Validation Windows: Three Imminent Tests

10.1 Chang'e 7: Imminent Seismic Validation (August 2026)

The Chang'e 7 mission (CNSA, launch scheduled August 2026) will deploy the first seismometer at the lunar South Pole since Apollo 17 (1972). Its data will provide a direct test of P22. If an impedance contrast $|R| > 0.02$ is measured between 200 and 530 km depth, concentric stratigraphy receives its first observational support. If no interface is detected under the falsification conditions stated in P22, the theory requires revision of at least the number of episodes N or the inter-layer Fe/Si gradient.

10.2 South Pole-Aitken Basin: Stratigraphic Revealer

The SPA basin did not create the concentric lunar stratigraphy: it *excavated* it. The layered structure pre-existed the SPA impact at ≈ 4.25 Ga (Yue et al., 2026), encoded during TPT differentiation at ≈ 4.5 Ga. The impact is the accidental revealer of a much older internal architecture.

High-resolution 3D impact simulations (Wakita et al., 2026) demonstrate that the oblique north-to-south trajectory of the SPA impactor projected deep mantle ejecta preferentially toward the South Pole in a butterfly-shaped plume — precisely the Artemis III landing zone. The same simulations predict ≈ 350 m of mantle material on average at this site, excavated from depths greater than 90 km, consistent with the TPT preservation window.

10.3 Artemis III: Direct Sampling of Episode-1 Material

The Artemis III mission (NASA, planned 2028–2029) will land astronauts near Shackleton crater at the South Pole. According to Wakita et al. (2026), surface samples at this site may include deep mantle ejecta from the SPA impact. If so, these samples would allow direct testing of P23: their Fe/Si ratio, crystallisation age (≈ 4.5 Ga for primary Episode-1 material), and isotopic signature (Δ^{17}O < 5 ppm relative to the contemporaneous terrestrial mantle) are all quantitative predictions of the TPT.

11 Predicted Falsification Paths

What would seriously challenge the TPT

1. **No seismic interface in 200–530 km** after adequate station coverage (P22 falsified).
2. **Spherically symmetric lunar mantle** from full tomography (P7 falsified).
3. **Abrupt dynamo onset** at ≈ 4.2 Ga rather than continuous growth (P17 falsified).
4. **Fe/Si of SPA mantle ejecta not higher** than terrestrial peridotite or mare basalts after correction for impact mixing (P23 and P6 falsified).
5. **Uniform or decreasing strata thicknesses** from seismic inversion (P20 falsified).

Conversely, the TPT does **not** require:

- A specific initial rotation period other than the range 2.5–4.0 h.
- A unique value of f_{cap}; the mass budget is robust over $f_{\text{cap}} \in [0.5, 0.85]$.
- A fully intact ejected flux; partial fragmentation (L3) does not break the mechanism.
- A 3D SPH simulation prior to observational testing.

12 Discussion

12.1 Comparison with the Giant-Impact Model

Constraint	Giant impact	TPT (this work)
Isotopic identity	Requires very specific impactor composition and mixing conditions	Satisfied mechanically by construction
Fe depletion	Requires a pre-differentiated impactor (Theia)	Direct consequence of the unique driver
Crustal dichotomy	Not naturally predicted	Qualitative prediction: post-episode-2 precession (L7)
Dynamo delay	Not predicted	Proposed: ≈ 350 Myr, no free parameter
Seismic predictions	None	P22 testable by Chang'e 7 (August 2026)
Chang'e-6 Fe anomaly	Not predicted	Consistent with P6 and P23 (Xu et al., 2025; Yue et al., 2026)

A recent review concludes that there is currently no unambiguous geochemical or isotopic evidence for the role of an external impactor in lunar formation (Sossi et al., 2025). This finding motivates exploration of alternatives grounded in internal proto-Earth dynamics.

12.2 Responses to Principal Objections

"Equation (2) assumes instantaneous Earth formation." This objection misidentifies the nature of the calculation. Equation (2) is a *thermodynamic energy bound*, not a formation timescale. It asks: does the total gravitational energy released during accretion, however slowly, exceed the energy needed to melt the mantle? The answer, established by Solomatov (2000), Elkins-Tanton (2012) and Rubie et al. (2015), is yes by a factor of 155. The ≈ 30 Myr timescale of core formation (Kleine et al., 2002) is explicitly incorporated in the TPT chronology as Δt_{ej} (Section 5.3).

"ε is the astronomical obliquity; there is no radial Coriolis force." This objection conflates two distinct geometric angles. In the TPT, ε is the *body-frame* angle between $\mathbf{\Omega}$ and the principal inertia axis of the Maclaurin spheroid — an internal geometric quantity with no reference to the ecliptic plane. The decomposition of the Coriolis force into radial and horizontal components (Eqs. 15–16) follows directly from this geometry: $f_{\text{rad}} = 2\Omega \sin \varepsilon \cdot U$ is elementary mechanics in the co-rotating oblique frame.

"The CMT is not demonstrated." The CMT is not postulated: it is the spatial expression of mode $\ell = 1$, $m = 1$ from the geostrophic inverse cascade at the Rhines scale in a rapidly rotating oblate body with $\varepsilon > 45°$. Four independent physical effects converge on the band $|\phi| < 30°$ as the attractor (Section 5.1). Quantitative validation requires a 3D SPH simulation (Priority 2), but the theory does not depend on this for its observational testing.

"Taylor-Proudman prevents the CMT." Columns exist but are curved by five mechanisms (Section 3). Their curvature generates the toroidal flow rather than inhibiting it. The Taylor-Proudman constraint does not apply to the pure toroidal field (Frigaard & Nouar, 2017).

"The dynamo delay is arbitrary." It is the sum of three durations constrained by three independent observations, not a fitted parameter (Section 5.3).

13 Acknowledged Limitations

L1 — Spontaneous CMT organisation

Whether a unique torus or multiple coexisting structures emerge in the exact Hadean regime can only be resolved by high-resolution 3D SPH simulation with full BH rheology. **Priority 2.**

L2 — Numerical value of f_{cap}

The capture efficiency $f_{\text{cap}} = 0.70$ is physically motivated but not derived from first principles. The 3D SPH simulation is the ultimate arbiter on this value and its dependence on M_{Moon} (Prediction P12: $\partial f_{\text{cap}}/\partial M_{\text{Moon}} > 0$).

L3 — Small-scale cohesion

The argument $\text{Bi}_{\text{KH}} \gg 1$ holds at large and intermediate scales. At very small scales ($\lambda_{\text{KH}} \ll R_{\text{torus}}$), partial fragmentation cannot be analytically excluded. However, fragments from the same toroidal flux, ejected with the same velocity and angle, end up on quasi-identical orbits and accrete onto the POB: the mass budget is not affected by fragmentation, only the fragment size.

L4 — Dynamo delay: ± 50 Myr

The exponential core-growth model is simplified; the associated uncertainty is ± 50 Myr.

L5 — Super-rigid profile $\alpha > 1$

The super-rigid rotation profile ($U \propto r^{\alpha}$, $\alpha > 1$) is physically motivated by differential rotation in a rapidly cooling partially crystallised magma, but has not been demonstrated for these exact Hadean conditions. It is a Level-3 hypothesis, to be validated by simulation.

L6 — Flux $\rightarrow$ POB transition: qualitative status

The three-phase description of the ejected flux (ballistic expansion, orbital insertion, sticky accretion beyond R_{Roche}) is physically motivated and consistent with calculated orders of magnitude, but remains qualitative. A 3D SPH simulation with coupled radiative cooling is required. **Priority 2b.**

L7 — Crustal dichotomy: qualitative prediction

The $\approx 2:1$ crustal thickness ratio is a qualitative prediction of the TPT. The coupled differential thermal calculation has not yet been derived analytically.

Additional context. The lunar farside has experienced at least two independent remelting events that must be accounted for in any quantitative model: (1) the SPA basin-forming impact at ≈ 4.25 Ga, which excavated and partially remelted the deepest Episode-1 layer over the farside region (Wakita et al., 2026); and (2) prolonged mare volcanism extending to ≈ 3.2 Ga (Li et al., 2021), which remelted the shallowest layers (< 100 km) across much of the farside. The net effect is to deepen and reinforce the existing asymmetry rather than to erase it. The extruded rocks exposed in the central peaks of craters within the SPA basin should preserve a geochemical record of the pre-impact Episode-1 material: their composition (Fe/Si, Δ^{17}O) is predicted to match the contemporaneous Hadean terrestrial mantle (prediction P23).

L8 — Rossby wave velocity in a BH fluid

The rigorous derivation of $c_{\mathrm{Rossby}}^{(\mathrm{BH})}$ for a BH fluid on an oblate spheroid ($\varepsilon \neq 0$) remains an open problem. The channel analogue correction (factor ≈ 40) used here strengthens rather than invalidates the synchronisation argument, but awaits rigorous derivation.

14 Conclusion

This work proposes that the formation of the Moon may be the product of a Triple Phase Transition in a fully molten proto-Earth rotating at ≈ 3.5 h, whose axis wobbles within $[40°, 70°]$ in its own co-rotating frame — a range supported by five convergent arguments, foremost among which is the logically necessary absence of any tidal stabiliser. A single driver — progressive Fe-Ni segregation — simultaneously governs three transitions: rheological, mechanical and magnetic. The result is a Moon built layer by layer in two to three massive episodes, each layer archiving in its internal structure the state of Hadean differentiation at the time of its accretion.

The mass budget is consistent without strain: the target mass for the TPT mechanism is less than the present observed lunar mass, since the SPA impactor and late accretion contributed afterwards. At $T_{\text{rot}} = 3.5$ h, the ejectable mass per episode may reach $\approx 70\%$ of the present lunar mass, which is why $N = 2$–3 suffices.

Preliminary observational support comes from Chang'e-6: norites at 4247 ± 5 Ma (Yue et al., 2026) and olivines with elevated Fe/Mn (Xu et al., 2025) are consistent with predictions P6 and P23, though not yet confirmatory.

The central prediction stands: one or more seismic interfaces in the 200–530 km window, testable by Chang'e 7 in August 2026. If an interface with $|R| > 0.02$ is detected in this window, concentric stratification receives its first strong observational support prior to any SPH simulation. If no interface is detected under the stated falsification conditions, prediction P22 is seriously challenged and the theory will require revision — which this author accepts explicitly.

This work invites the planetary science community to evaluate the TPT on its own terms: equations are numbered, derivations are explicit, hypotheses are ranked, limitations are acknowledged, predictions are falsifiable, and the first observational tests are scheduled for August 2026. The TPT will be judged primarily on its observational predictions — Chang'e 7 seismic data (August 2026) and Artemis III samples (2028–2029) — before and independently of any full numerical validation.

A Adiabatic Temperature Profile

With $T_{\text{surf}} \approx 3\,500$ K and adiabatic gradient $\mathrm{d}T/\mathrm{d}r \approx -0.3\ \mathrm{K\,km^{-1}}$, the central temperature reaches $\approx 5\,000$ K. The peridotite solidus at 135 GPa is $\approx 4\,300$ K (Stixrude & Lithgow-Bertelloni, 2014): melting is thermodynamically stable throughout the entire mantle, confirming H0.

B Derivation of the Elsasser Number and B_{crit}

Balancing Coriolis force $F_C \sim 2\rho\Omega U$ and Lorentz force $F_L \sim B^2/(\mu L)$, with $\eta \sim UL$ (magnetic Reynolds number of order unity at the dynamo threshold):

$$B^2 \sim 2\mu\rho\Omega\eta, \qquad \Rightarrow \qquad \Lambda \equiv \frac{B^2}{\rho\mu\eta\Omega} \sim 2, \tag{39}$$

giving $\Lambda \sim \mathcal{O}(1)$. Using $\eta = (\mu_0\sigma)^{-1}$ with the values in Section 5.3 yields $B_{\text{crit}} \approx 6\,\mu\text{T}$.

C Complete Derivation of the λ Coefficient

Applying the Solberg-Høiland parcel-displacement formalism (Solberg, 1936; Høiland, 1941) to an oblate sphere with obliquity ε. Angular momentum conservation ($\tau_{\text{relax}}/\tau_{\text{dyn}} \approx 3 \times 10^9 \gg 1$):

$$U_{\text{parcel}}(r+\xi_r) \approx U(r)\left(1-\frac{\xi_r}{r}\right), \qquad U_{\text{background}}(r+\xi_r) \approx U(r)\left(1-\alpha\frac{\xi_r}{r}\right). \tag{40}$$

The velocity difference $\delta U = U(\alpha-1)\xi_r/r$. The net radial force sums effective gravity ($-g_{\text{eff}}^{(k)}\xi_r$), radial Coriolis ($+2\Omega^{(k)}\sin\varepsilon \cdot U\xi_r$) and the Rayleigh term ($U^2(1-\alpha)\xi_r/r^2$). Equation (26) follows.

D Fe-Ni Stokes Sedimentation Timescale

Stokes sedimentation velocity for a Fe-Ni droplet of radius $r \approx 5$ cm in molten silicate:

$$v_{\text{Stokes}} = \frac{2}{9}\frac{(\rho_{\text{Fe}}-\rho_{\text{sil}})\,g_{\text{eff}}^{(0)}\,r^2}{\eta} \approx \frac{2}{9}\frac{3\,300 \times 8.51 \times (0.05)^2}{0.1} \approx 156\,\text{m}\,\text{s}^{-1}. \tag{41}$$

Over a characteristic sedimentation distance $\ell \approx 30$ km: $\tau_{\text{Stokes}} = \ell/v_{\text{Stokes}} \approx 192$ s. Robust over $r \in [1, 10]$ cm and $\eta \in [0.05, 1]$ Pa s.

E List of Abbreviations

BH	Bingham-Herschel rheological law.
BiKH	Bingham-Kelvin-Helmholtz number.
CME	Coronal Mass Ejection.
CMT	Coherent Magmatic Torus.
FSS	Farside Seismic Suite.
Hf-W	Hafnium-tungsten chronometer.
LHB	Late Heavy Bombardment (≈ 3.9 Ga).

LEMS	Lunar Environment Monitoring Station.
POB	Proto-Orbital Body.
SPA	South Pole-Aitken basin.
SPH	Smoothed Particle Hydrodynamics.
TPT	Triple Phase Transition.

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