

Determination of the Lithosphere-Asthenosphere Boundary using program LABWA2015

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Abstract

Lithosphere-Asthenosphere Boundary (LAB) is a lower boundary of the lithospheric plate, so, it is an important tectonic boundary. We present the package of numerical program LABWA2015 developed for simple calculations of position of LAB. It assumes isostatic state and uses gravity as well as topographic data. However, program provides better results if additional geophysical data are used, e.g. seismic data about position of Moho. If position of LAB is determined by other methods, the package can be used for determining density or thermal properties of lithosphere and asthenosphere. Contrary to earlier methods, LABWA2015 uses the full equation of thermal conduction. Problem of the lack of isostasy in some regions is also discussed.

Key words: asthenosphere, lithosphere, isostasy, geoid anomaly, determination of LAB, mantle convection and isostasy.

1. INTRODUCTION

Lithosphere, asthenosphere and LAB (lithosphere-asthenosphere boundary) are popular terms in the physics of the solid Earth, however their meaning are differently understand by different specialists. In the plate tectonics theory the lithosphere is a layer divided on tectonic plates, i.e. the units which could move one in respect to another. The asthenosphere is a layer of low viscosity detaching the lithospheric plates from the mantle below. Unfortunately, the motion of

some plates is very slow and we do not know the true motion of the mantle material below the plates. The models of mantle convection have limited resolution. Therefore sometimes this definition is difficult to be used. A few other definitions of the lithosphere, asthenosphere and LAB are:

- seismic definition (asthenosphere is a layer of low velocity and high attenuation of seismic waves, lithosphere is a layer consisting of the crust and mantle above the asthenosphere),
- thermal definition (lithosphere is a thermal boundary layer of the mantle convection cell, asthenosphere is a layer where temperature is close to the temperature of solidus),
- magnetotelluric definition (asthenosphere is a high electrical conductivity layer),
- rheologic definition (the lithosphere is elastic or brittle (especially the crust), while the asthenosphere deforms viscously and could accommodate strain through plastic deformation. Of course, provided that we consider the rate of deformation corresponding to geodynamic processes).

Note also that even in the seismology a few sub-definitions exist. Jones et al. (2010) indicate a few types of LAB obtained by different seismological methods: i.e. by the receiver functions method (sLABrf) and by the determination of seismic anisotropy change (sLABa), in addition to (eLAB) obtained by magnetotellurics – e.g. Eaton et al. (2009).

Tectonics also suggests a few possibilities. Below moving plate one has an ‘active asthenosphere’ with a large vertical gradient of the horizontal velocity (this definition corresponds to the definition used in the plate tectonics). If the gradient is low but the viscosity is also low, then one can use the term: a ‘potential asthenosphere’ (i.e. it would be an active asthenosphere if the plate were in motion). Moreover, the possibility of asthenospheric layers of thermal and mechanical origin were also indicated - Czechowski and Grad (2015 a, b).

Fortunately, most of above definitions concern different properties of the same layer. Therefore, a few methods could be used independently for determination position of LAB and conditions in the asthenosphere. It is the main idea of program LABWA2015. It uses the ‘classical’ method of determination of LAB based on isostasy – e.g. Krysiński et al (2013, 2015), Grinc et al.. (2014). Moreover, it includes also full equation of heat transport (according to our best knowledge it is the first such program). It can use also seismic data in the equation of state and enables to introduce some corrections resulting from the absence of isostatic equilibrium in some regions (Czechowski 2017). Compare also with the model presented by Jones et al. (2014).??

2. LOCAL ISOSTASY

The idea of local isostasy is based on the assumption that upper layer of the Earth could be treated as series of rigid columns (blocks) that float on a liquid layer. In hydrostatic state the pressure in a given liquid depends only on the depth. It means that the total mass above some level (known as a compensation level) is the same for each column. This idea was proposed in XIX century and was used in many investigations. For some time the compensation level was placed close to the Moho (e.g. Turcotte and Schubert, 2002, p. 74). However, according to the plate tectonics theory, the asthenosphere (instead of the lithosphere just below of Moho) is a layer which could be treated as a liquid for slow tectonic motions. Hence, the compensation level should be placed inside the asthenosphere. It means that the rigid columns correspond to the lithosphere, while the ‘liquid’ layer corresponds to the asthenosphere. It is presented on Fig. 1 for the continental lithosphere (when the solid surface is above sea level) and for the oceanic lithosphere (when rocks are covered by a water layer).

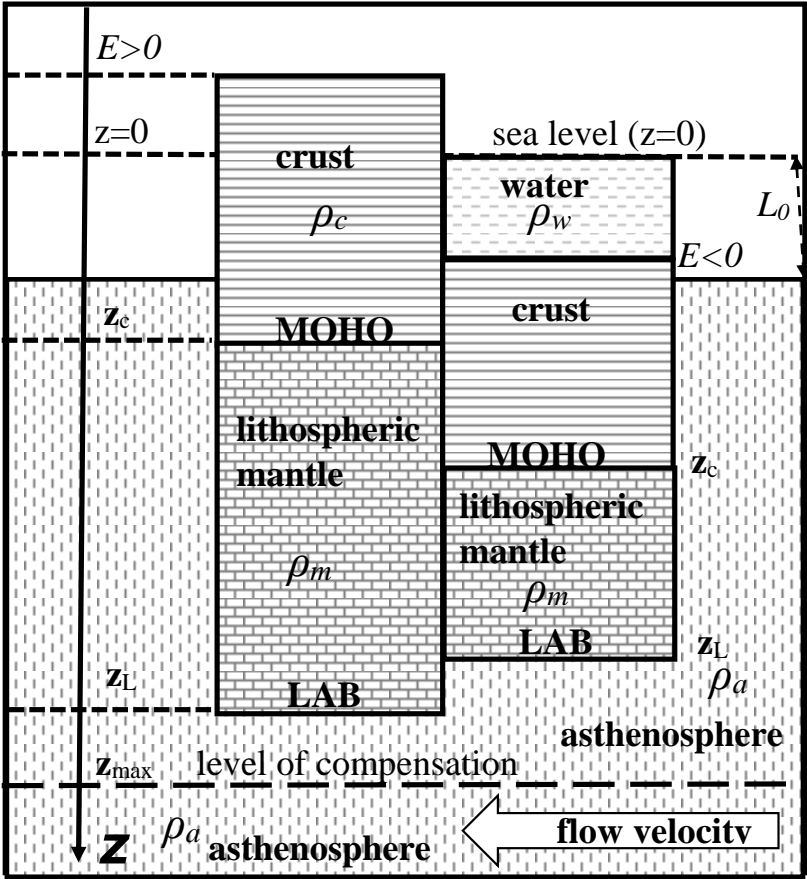


Fig. 1. Models of the lithosphere used for derivation of Eqs. (1-3). Two columns of the lithosphere are presented. Continental lithosphere is on the left hand side and oceanic lithosphere is on the right hand side. Lithosphere is composed of a few layers. The lower boundary of the crust (Moho) is at $z=z_c$ and the lower boundary of lithosphere

(LAB) is at $z=z_L$. Compensation level $z=z_{max}$ is in the asthenosphere. The large arrow indicates possibility of the flow of the material of asthenosphere. After Fullea et al. (2006), modified. Compare also with Lachenbruch and Morgan (1990).

Figure 1 presents a simple model of the lithosphere and the asthenosphere. The model contains: crust of density ρ_c [kg m^{-3}], lithospheric mantle of density ρ_m , sea water with density ρ_w , and asthenosphere with density ρ_a . E is the elevation ($E>0$ for the land and $E<0$ is for the sea floor), z_c and z_L are depths of the boundary of the crust/mantle (Moho) and the LAB, respectively, referred to the sea level. L_o is the position of the hypothetical column composed of the matter with the density of asthenosphere ρ_a . $L_o=2380$ m (below sea level) is used by Grinc et al. (2014), while Fullea et al. (2006) used $L_o=2320$ m. Assuming isostatic state, one can find that elevation E is given by (e.g. Fullea et al. 2006):

$$E = \frac{z_c(\rho_m - \rho_c) - z_L(\rho_m - \rho_a) - \rho_a L_o}{(\rho_c - \rho_w)}, \quad (1)$$

where $\rho_w=0$ is used for the land (i.e. if $E>0$). The corresponding position of the geoid anomaly is given by (e.g. Fullea et al., 2006; Grinc et al., 2014):

$$N = \frac{2\pi G}{g} \int_{sur}^{z_{max}} \Delta\rho(z)z dz, \quad (2)$$

where $g=9.81 \text{ m s}^{-2}$, $G= 6.67 \cdot 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$, and $\Delta\rho(z)$ is the density contrast in respect to a given reference column. Integration is from the surface down to the compensation level. For the presented above simplified model of the lithosphere, the geoid anomaly N is given by (Fullea et al., 2006):

$$N = -\frac{\pi G}{g} [\rho_w E^2 + \rho_c (z_c^2 - E^2) + \rho_m (z_L^2 - z_c^2) + \rho_a (z_{max}^2 - z_L^2)] - N_0, \quad (3)$$

where N_0 is an integration constant.

Topography E is known and could be taken from different databases. The geoid data for Eq. (3) are taken often from the EGM-2008 model (Pavlis et al. 2012). Note that geoid anomalies resulting from the mass below the asthenosphere should be removed. Unfortunately, the true component resulting from the deep sources is not known. In a typical approach it is assumed that all long wavelength components of geoid variations are a result of the deep sources. Therefore, for modeling the lithosphere Grinc et al. (2014) removed the harmonic terms up to degree and order 10. A little different and more complicated procedures are used by Fullea et al. (2006) and Krysinski et al. (2013, 2015).

The Eqs. (1) and (3) form a set of two equations. It could be solved if the number of unknown parameters is reduced to 2. It is the simplest application of the system (1)-(3). Fullea et al. (2006) use this system to determine positions of Moho and LAB assuming: density of the

crust $\rho_c=2780 \text{ kg m}^{-3}$, lithospheric mantle density $\rho_m=3245 \text{ kg m}^{-3}$, asthenosphere's density $\rho_a=3200 \text{ kg m}^{-3}$ and the compensation level depth $z_{\text{max}}=300 \text{ km}$. However, if Moho position is known then the system (1)-(3) could be used to determine density ρ_m or ρ_a instead of assuming their values.

3. LIMITATION OF THE METHOD

The set of equations (1) - (3) is satisfied for a lithospheric column if the isostatic equilibrium is achieved. Generally, it is possible if the column under consideration is detached from other columns enabling to move vertically without affecting neighboring columns. This detachment could be done by system of faults. However, verification of this assumption often is not possible and the system (1) - (3) (or similar) is used without such control; see for details: Fullea et al. (2006), Grinc et al. (2014), Krysinski et al. (2013, 2015).

The assumption of isostasy is justified if the crust contains many preexisting faults. These preexisting zones of weakness could be reactivated under the tectonic stresses (e.g. Turcotte and Schubert, 2002, p. 74) enabling independent motions of columns. Note also that Levander and Miller (2012) found that (in Western USA) LAB is not a continuous topographic structure (contrary to Moho in this region) but forms complex structures, which could be a result of mentioned zones of weakness. Of course, this idealized situation cannot be fully satisfied everywhere.

Another method used to avoid problem of detachment is considering the column large enough (in horizontal dimensions). For such blocks, possible effects of elastic deformation of the lithosphere could be small (see e.g. Krysinski et al. 2013, 2015), so isostasy could be reached.

The system of (1)-(3) could be written for each column (each block) of the lithosphere. If the blocks are not identical then their equations are independent (see Fig. 2). Moreover, if the column does not introduce an additional unknown (e.g. the columns 1 – 4 at Fig. 2) then we have additional constrains which can be used for determination other properties of the system. In this way several other parameters of the model could be calculated (see the next sections). If the number of equations is significantly larger than the number of unknown then better accuracy of the results could be achieved.

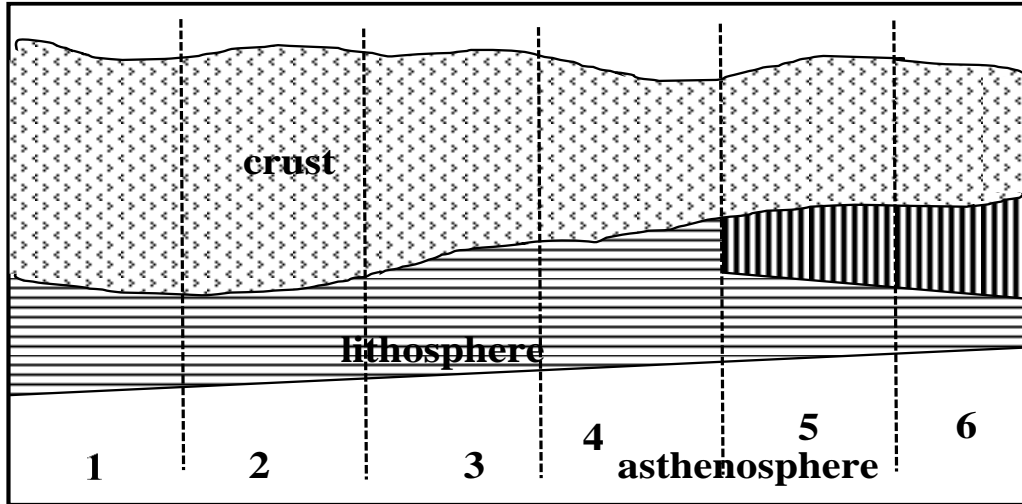


Fig. 2 An example of division of lithosphere into columns (blocks). Columns 1 – 4 give the system of 8 equations. If position of Moho is determined using seismic method then the system of 8 equations could be used for determination of position of LAB in 4 columns and to determine 4 other parameters. Note that columns 5 and 6 give equations with additional unknown parameters introduced by additional, hypothetical block with different properties.

4. ISOSTASY AND DYNAMICAL PROCESSES IN THE MANTLE

It is said above that the isostasy (isostatic equilibrium) is a state when solid blocks of the lithosphere float in the static liquid. The forces arising from the elastic deflection of the lithosphere may not allow for the attainment of the isostatic state. Other deviations from isostatic equilibrium could be a result of flow of the mantle material. This is a type of solid state convection, i.e. convection with very large Prandtl number Pr . The Prandtl number is defined as the ratio of coefficient of diffusion of velocity ν to coefficient of diffusion of temperature κ :

$$Pr = \nu/\kappa,$$

where ν [$\text{m}^2 \text{s}^{-1}$] is a kinematic viscosity of the medium, κ [$\text{m}^2 \text{s}^{-1}$] = $k/(\rho c)$ where k [$\text{W m}^{-1} \text{K}^{-1}$] is the coefficient of thermal conductivity, c [$\text{J kg}^{-1} \text{K}^{-1}$] is the specific heat at constant pressure, ρ [kg m^{-3}] is the density. The properties of solid state convection is considered in many papers (e.g. Czechowski 1993, Schubert et al. 2001, Czechowski 2014). Czechowski and Leliwa-Kopystyński (2012, 2013) and especially Czechowski (2017) discuss also some problems of isostasy in celestial body where solid state convection takes place.

Most of the mantle below the lithosphere is a subject of mantle convection. Czechowski (2017) pointed to a few types of regions where lack the isostasy can be expected. They are:

1. Subduction zones where one plate moves under another plate and sinks into the mantle. It is a place of downward convection current. The presence of thick plates and mantle flow makes the possibility of the isostasy rather doubtful.
2. Oceanic spreading centers, where the lithosphere is thin, but the upward flow of the mantle material below the lithosphere causes large vertical forces that may not allow for isostasy.
3. Hot spots. They are areas located above the rising mantle plumes that causes large vertical forces that may not allow for isostasy.
4. Some zones of horizontal flow in asthenosphere. The flow could be forced by motion the moving oceanic plates. In such a case the state of isostasy could be approximately attained. Horizontal flow in the asthenosphere could be also a result of gradient of the pressure in the asthenosphere. In such a case some modifications of the procedure in LABWA2015 are necessary (see Czechowski 2017). The simplest one is introduction of pressure gradient in the asthenosphere.

5. EXTENSION OF THE METHOD IN LABWA2015

The method based on the principle of isostasy is sensitive for the distribution of density. This distribution depends on the distributions of pressure, temperature and composition, so they could be also determined (at least to some degree).

These relationships are used in some papers. Krysiniski et al. (2015) include density gradients instead of uniform density in a given layer. Grinc et al. (2014) include a simple parameterized thermal model and thermal boundary conditions (e.g. heat flow at the LAB).

Model LABWA2015 uses a different approach. Corrections resulting from the temperature distribution are calculated using the full equation of heat conduction. Open architecture of LABWA2015 makes possible to define user's own thermal model. In the lithosphere the heat is transferred by conduction only, consequently the temperature distribution is described by the following equation of the heat transfer (e.g. Czechowski 1993; Turcotte and Schubert, 2002, Ch. 9):

$$c \rho \frac{\partial T}{\partial t} = \text{div}(k(T, p) \text{ grad } T) + Q(t), \quad (4)$$

where t [s] is the time, T [K] is the temperature, c [J kg⁻¹ K⁻¹] is the specific heat at constant pressure, ρ [kg m⁻³] is the density, k [W m⁻¹ K⁻¹] is the coefficient of thermal conductivity (it could depend on the temperature, pressure and composition), and Q [W m⁻³] is the heat generation per unit volume. For steady state the time derivative is zero, so the solution of (4) become independent of the product $c \rho$. Eventually, for simple case only two parameters are

necessary for thermal model of the layer with significant heat generation: Q and k . Moreover, heat flow density h_f at LAB (or at the compensation level) must be prescribed.

For a more complicated model more parameters are necessary. Note possibility of nonlinear effects, e.g. effects resulting from temperature dependent coefficient of thermal conductivity $k(T, p)$. Puziewicz et al. (2012) use k that is a following function of other parameters:

$$k(T, p) = k_0 \left(\frac{1+41.3 \Phi_0}{1-0.661 \Phi_0} \right) \left(\frac{1+41.3 \Phi(p)}{1-0.661 \Phi(p)} \right) \left(\frac{E+\frac{B}{350+T}}{E+\frac{B}{350+20}} \right), \quad (5)$$

where k_0 is a thermal conductivity at normal conditions and E, B, Φ_0, p_{ref} are some parameters of given rocks and $\Phi(p)=\Phi_0 \exp(-p/p_{ref})$. For details see: Puziewicz et al. (2012). Even stronger nonlinearity is introduced by processes of melting/solidification (they could be included in the way similar to that presented in Losiak et al., 2015).

The density is coupled with the temperature and pressure by the following equation of state:

$$\rho(p, T) = \rho(0, 0) (1 + p/K - \alpha (T-T_0)), \quad (6)$$

where α is the coefficient of thermal expansion and K is the bulk modulus. K is a function of density and of elastic properties of the rock. Assuming that both Lamé coefficients are equal, K could be calculated using the velocity of longitudinal waves V_P and the formula (e.g. Puziewicz et al. 2012):

$$K=(5/9) \rho V_P^2. \quad (7)$$

It means that some seismic data from the lithosphere could be also taken into account.

The number of parameters of thermal model for a given column depends on the complexity of the assumed model. It could contain several different layers if the properties of these layers are given. Unknown properties introduce unknown parameters in the numerical package. Presently, thermal model is 1D but model 3D could be incorporated into the numerical code.

In the region where position of LAB is determined by other methods (e.g. by the seismology or by the magnetotellurics), the package could be used to determine some other properties of the lithosphere and asthenosphere. The program LABWA2015 allows also to choose the unknowns in the system of equations, e.g. $k, Q, h_f, \alpha, E, B, \Phi_0$ could be treated as unknowns. The user of the program must divide the studied region into blocks to provide a sufficient number of independent equations.

6. EXAMPLES OF APPLICATION OF THE MODEL

A few examples of the results of the model are presented below.

The Fig. 3 presents a typical application of the model, i.e. using the topography, gravity data and position of Moho, the model calculates the position of LAB. The position of Moho is given by the model of Grad et al. (2009, 2014). For gravity (i.e. geoid) we use the Earth Gravitational Model EGM2008 (Pavlis et al, 2012). Flat topography is used. Note that correlations of the positions of Moho and LAB are obvious.

Figures 4-6 present different applications of the package. By using different values of some parameters of the model one can investigate the effect of this changes to position of LAB. If position of LAB is well determined, then the program could be used to determine some other parameters of the crust, lithosphere or asthenosphere. In the presented figures we check the change of the LAB position due to the difference in thermal conductivity (Fig. 4), topography (Fig. 5) and the density of the upper crust.

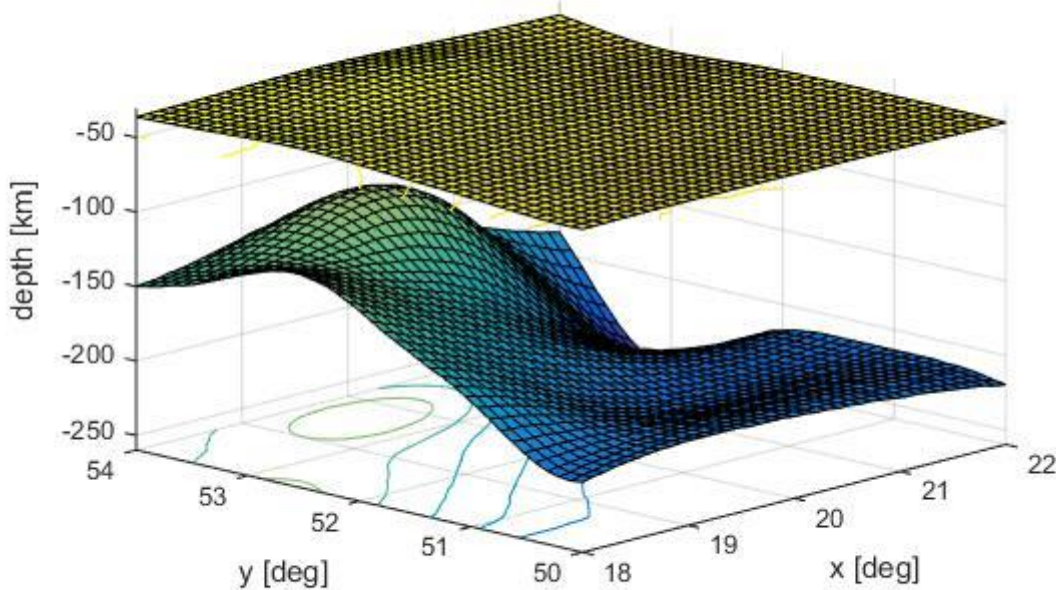


Fig. 3. Moho (the upper surface) and LAB (the lower surface). LAB is calculated using also seismic data of Moho. Geoid is from the model EGM2008. A flat topography (100 m) is used. Moreover, the radiogenic heat production in the upper crust is $2 \cdot 10^{-6} \text{ W m}^{-3}$, the density of the upper crust is 2760 kg m^{-3} , the coefficient of thermal conductivity in the upper crust is $2.6 \text{ W m}^{-1} \text{ K}^{-1}$.

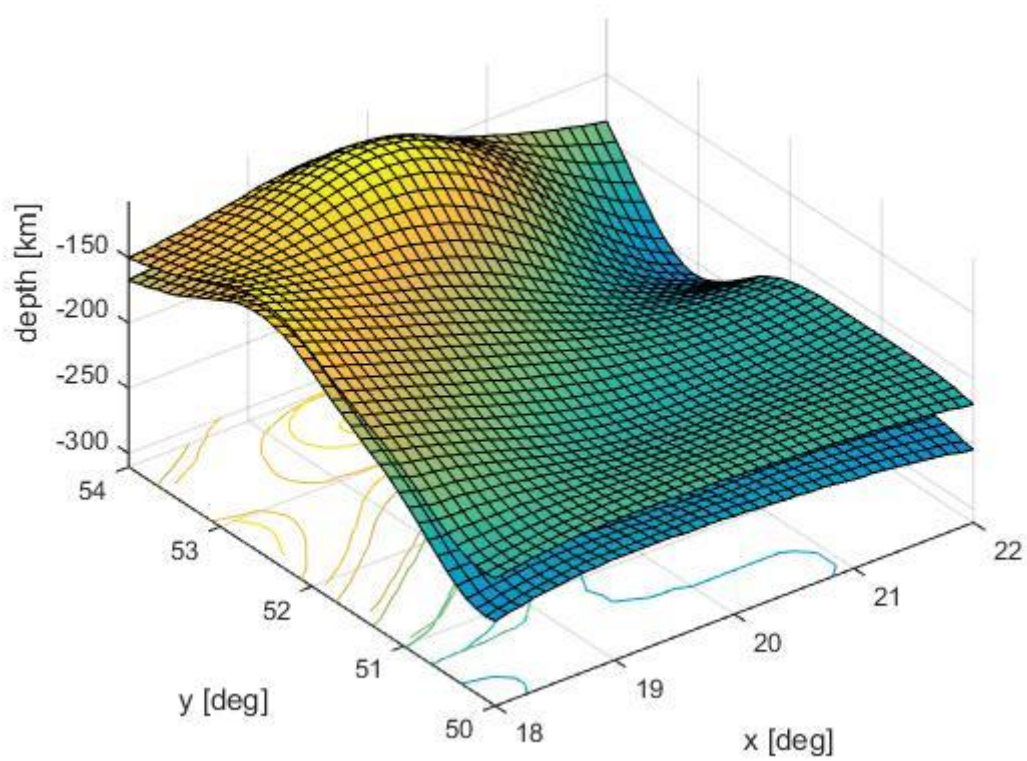


Fig. 4. Reaction of the position of LAB to the change of parameters. The upper surface shows the LAB calculated using the same parameters as in Fig. 3. The lower surface presents LAB calculated using different value of the coefficient of thermal conductivity in the upper crust ($k=1.5 \text{ W m}^{-1} \text{ K}^{-1}$). The values of other parameters are the same as in Fig. 3.

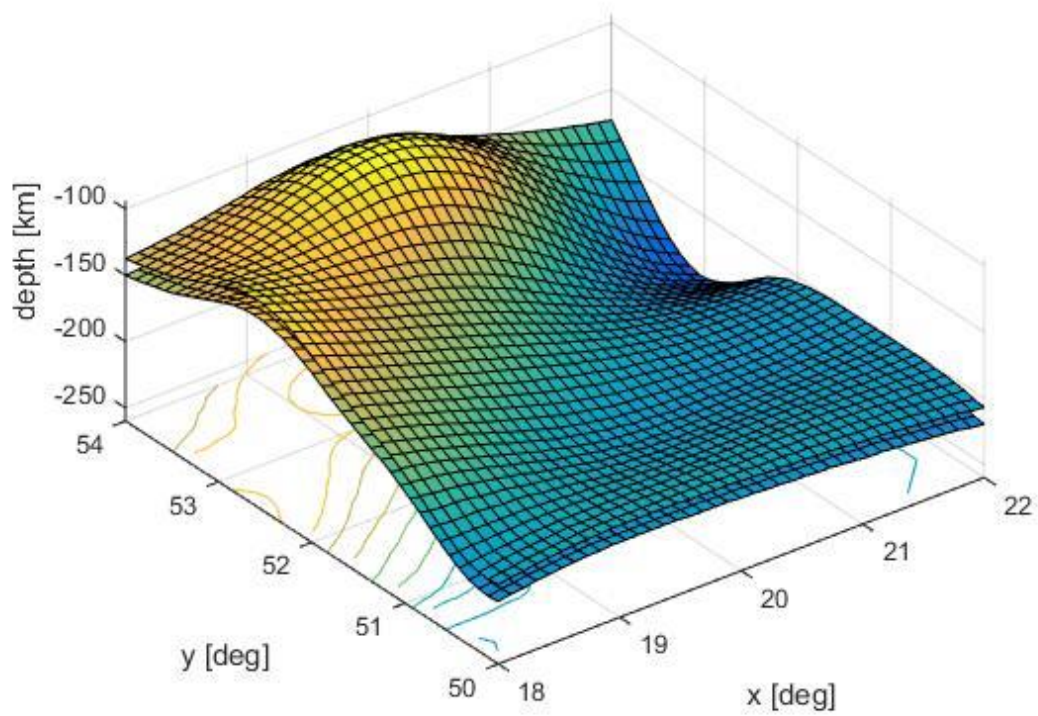


Fig. 5. Reaction of LAB for change of the parameters. The lower surface presents LAB calculated using the same parameters as in Fig. 3. The upper surface presents LAB calculated using flat topography 500 m. The values of other parameters are the same as in Fig. 3.

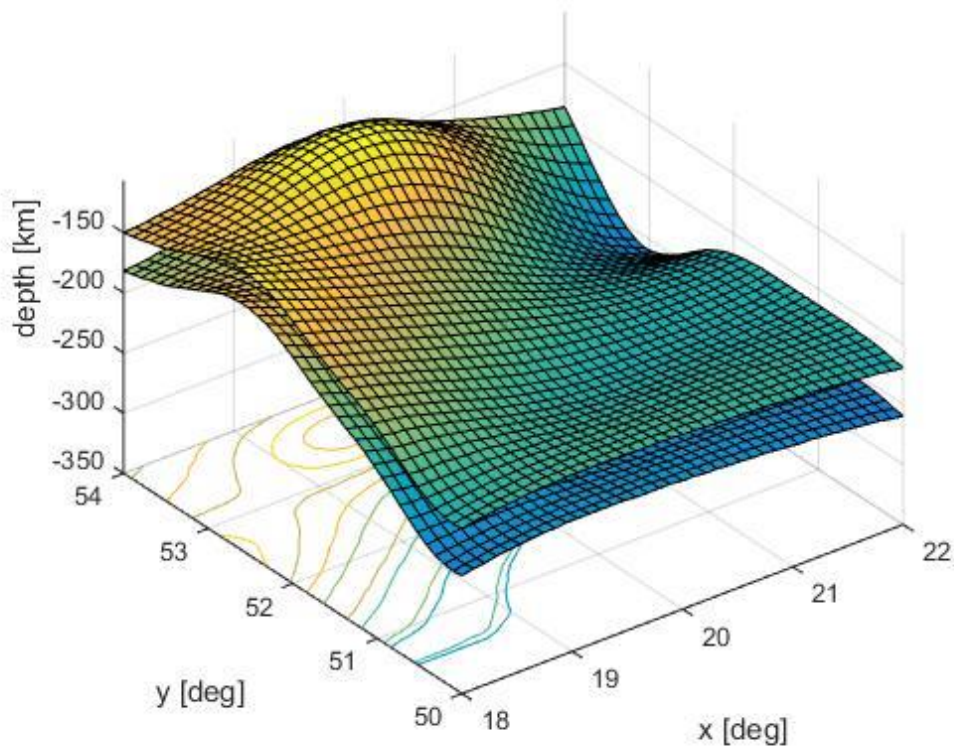


Fig. 6. Reaction of LAB to changing parameters. The lower surface presents the LAB calculated using the same parameters as in Fig. 3. The upper surface shows the LAB calculated using a higher density of the upper crust (100 kg m^{-3} higher). The values of other parameters are the same as in Fig. 3.

7. CONCLUSIONS

We presented the methods used in program LABWA2015. The basic assumption used in the methods is the assumption about isostasy. It is usually used to determine the Moho or LAB positions (e.g. Fulla et al. 2006; Grinc et al. 2014; Krysinski et al. 2015). The LABWA2015 program allows also to determine some other parameters of the crust or the lithosphere, e.g. coefficient of thermal expansion α , coefficient of thermal conduction k , rate of heat generation Q , etc. Generally, the procedure is simple. The method does not require efficient computers even for large region.

The reliability of the results depends on many factors. The chosen blocks the lithosphere should be in isostatic equilibrium, the geoid anomalies should be a result of density distribution inside the lithosphere. Moreover, the chosen models of the crust, lithosphere and asthenosphere have to ensure that the resulting system of equations could be solved.

Realistic values of the thermal and mechanical properties should be used in the model. Note, that some of these conditions could be not satisfied in some regions (e.g. the isostasy of the blocks or the position of the sources of geoid anomalies). Therefore the best way to check the reliability is to compare the results of LABWA2015 with the results of an independent method. For example, determination of LAB using seismic method in a single point of the considered region could give some insight into reliability of LABWA2015 for the whole region.

Acknowledgments

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References

1. Czechowski, L. 2017. Mantle flow and determining position of LAB using assumption about isostasy. Submitted
2. Czechowski, L. 2014. Some remarks on the early evolution of Enceladus. *Planetary and Space Science* **104**, 185-199.

3. Czechowski, L. 1993. Theoretical Approach to Mantle Convection. In: Teisseyre, R., Czechowski, L., and Leliwa-Kopystyński, J. (Eds.), Dynamics of The Earth's Evolution, vol 6 of the monographic series Physics and Evolution of the Earth's interior. R. Teisseyre (ed. of the series) Elsevier, Amsterdam, pp. 161-271, 1993.
4. Czechowski L. and Grad M., 2015 a. Two mechanisms of formation of asthenospheric layers. arXiv.1802.06843 [physics.geo-ph].
5. Czechowski L. and Grad M. 2015 b. The role of temperature and stress field for evolution of asthenospheric layers. Join Assembly of AGU, CGU, GAC, and MAC, 2015, Montreal.
6. Czechowski, L., Leliwa-Kopystyński, J., 2013. Remarks on the Iapetus' bulge and ridge. *Earth, Planets and Space*. **65**, 8, 929-934.
7. Czechowski, L., Leliwa-Kopystyński, J. 2012, Isostasy on Iapetus, *EPSC 2012*, 834, Madryt.
8. Eaton, D.W., Darbyshire, F., Evans, R. E., Grütter, H., Jones, A.G., Yuan, X., 2009. The elusive lithosphere–asthenosphere boundary (LAB) beneath cratons. *Lithos* 109, 1-2, 1-22. <https://doi.org/10.1016/j.lithos.2008.05.009>
9. Fullea, J.U., Fernández, M, Zeyen, H. , 2006. Lithospheric structure in the Atlantic-Mediterranean transition zone (southern Spain, northern Morocco): A simple approach from regional elevation and geoid data. *C.R. Geoscience* **338**, 140-151.
10. Grad, M., Timo, T., Sverker, O., Kari, K., 2014. Seismic lithosphere–asthenosphere boundary beneath the Baltic Shield. *GFF* 136, 581-598.
11. Grad, M., Tira T. and ESC Working Group., 2009. The Moho depth map of the European Plate. *Geophys. J. Int*, **176**, 279-292.
12. Grinc, M., Zeyen, H., Bielik, M., 2014. Automatic 1D integrated geophysical modelling of lithospheric discontinuities: a case study from Carpathian-Pannonian Basin region. *Contributions to Geophysics and Geodesy*, **44/2**, 115–131.
13. Jones, A. G., Plomerova, J., Korja, T., Sodoudi, F., & Spakman, W., 2010. Europe from the bottom up: a statistical examination of the Central and Northern European lithosphere–asthenosphere boundary from comparing seismological and electromagnetic observations. *Lithos*, **120**(1-2), 14-29. DOI: [10.1016/j.lithos.2010.07.013](https://doi.org/10.1016/j.lithos.2010.07.013).
14. Jones, A.G., Afonso, J.C., Fullea, J., Salajegheh, F., 2014. The lithosphere–asthenosphere system beneath Ireland from integrated geophysical–petrological modeling — I: Observations, 1D and 2D hypothesis testing and modeling. *Lithos*, 189, 28-48.
15. Krysiński, L., Grad, M., Mjelde, R., Czuba, W., Guterch A. 2013. Seismic and density structure of the lithosphere–asthenosphere system along transect Knipovich Ridge–Spitsbergen–Barents Sea – geological and petrophysical implications. *Polish Polar Research*, **34**, 111-138.

16. Krysiński, L., S. Wybraniec, and M. Grad, 2015. Lithospheric density structure study by isostatic modelling of the European geoid. *Studia Geophysica et Geodaetica*. **59**, 212-252.
17. Lachenbruch, A.H., Morgan, P., 1990. Continental extension, magmatism and elevation; formal relations and rules of thumb *Tectonophysics* **174**, 1–2, 39-62, doi.org/10.1016/0040-1951(90)90383-J
18. Levander, A., Miller, M.S., 2012. Evolutionary aspects of lithosphere discontinuity structure in the western U.S. G3, 13, *Geochem. Geophys. Geosyst.*, **13**, doi:10.1029/2012GC004056
19. Losiak, A., Czechowski, L., Velbel, M.A., 2015. Ephemeral liquid water at the surface of the Martian North Polar Residual Cap: Results of numerical modeling. *Icarus*, **262**, December 01, 2015, 131-139.
20. Pavlis, N. K., S. A. Holmes, S. C. Kenyon, and J. K. Factor, 2012. The development and evaluation of the Earth Gravitational Model 2008 (EGM2008), *J. Geophys. Res.*, **117**, B04406, doi:[10.1029/2011JB008916](https://doi.org/10.1029/2011JB008916).
21. Puziewicz, J., Czechowski, L., Krysiński, L., Majorowicz, J., Matusiak-Małek, M., 2012. Lithosphere thermal structure at the eastern margin of the Bohemian Massif: a case petrological and geophysical study of the Niedźwiedź amphibolite massif (SW Poland) *International Journal of Earth Sciences* **101** (5), 1211-1228
22. Schubert, G., Turcotte, D.L., Olson, P., 2001. *Mantle Convection in the Earth and Planets*, pp. 956. ISBN 052135367X. Cambridge, UK: Cambridge University Press, September 2001.
23. Turcotte, D.L., and Schubert, G., 2002, *Geodynamics*, Cambridge University Press, 2002. pp. 456.