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# Deep learning methods for the simulation and optimization of shallow geothermal energy systems

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## Abstract

Shallow geothermal energy (SGE) systems are crucial for decarbonizing the heating and cooling sector. Their planning, design and operation, however, rely on the simulation of heat transport in the subsurface, a task that is computationally demanding and particularly prohibitive for multi-query applications such as sensitivity analysis and optimization. Deep learning (DL) has recently emerged as a powerful means of accelerating or replacing these numerical models. This paper reviews the state of the art of DL methods for the simulation and optimization of SGE systems, with a focus on subsurface thermo-hydraulic processes rather than on above-ground system components. The existing literature is categorized by system type: closed-loop and open-loop systems; and by the applied DL methodology, the predicted quantities and the underlying numerical model. The review reveals that the field is still dominated by purely data-driven, system-level models, whereas physics-informed architectures and operator-learning approaches that explicitly resolve the subsurface remain scarce. Based on these findings, we discuss the central challenges of the field: multi-scale dynamics, well singularities, data scarcity; and outline future research directions, including physics-informed and hybrid models, neural operators for parametric problems, and generative models for uncertainty quantification, ultimately pointing toward real-time digital twins of SGE systems.

*Keywords:* Deep learning, Shallow geothermal energy, Optimization, Simulation, AI, Machine learning, Ground source heat pump

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## 1. Introduction

Shallow geothermal energy (SGE) systems are one of the key technologies to decarbonize heating and cooling sectors [1, 2], especially in the context of 4th and 5th generation of district heating (and cooling) grids [3–5]. SGE systems utilize geothermal energy within the first 400 m depth of the subsurface [6]. Due to relatively low (below 30 °C) and stable temperatures in the shallow subsurface, these systems can be used for direct heating and cooling applications throughout the year [7].

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Modeling, simulation and optimization of SGE systems, as with any other technical system, is crucial to plan, design and operate them as effectively and efficiently as possible in order to maximize their utilization [8]. As a result, shallow geothermal potential can be maximized while minimizing system costs and environmental impacts [9]. This requires the modeling of complex bidirectional interactions between such systems and the geothermal energy resource, i.e. the subsurface, such as heat and mass transport phenomena. While numerical simulations are well established for these phenomena, they are computationally intensive and impractical for the so-called multi-query problems, such as sensitivity analyses or optimization tasks [10]. On the other hand, machine learning (ML) and, in particular, deep learning (DL) methods have become increasingly important in recent years and offer a promising alternative (or hybrid solutions) to classical numerical models. These methods can generally model any type of complex phenomenon and yet remain relatively inexpensive in terms of execution time. Their applicability has been demonstrated across numerous scientific and engineering domains, ranging from climate and weather modeling to fluid dynamics, materials science and computational physics [11, 12].

The importance of ML and DL methods in the field of SGE has similarly increased, and the number of research studies dealing with this topic has grown rapidly in recent years. Several studies have investigated ML/DL methods for SGE systems in different contexts. Zhou et al. [13] reviewed research papers applying ML methods in the field of ground source heat pump (GSHP) systems, which are the most widely used type of SGE systems. The authors divided the applications into five categories: load forecasting, design data collection, geothermal heat exchangers, system performance and optimization, and system control and diagnostics. With the exception of the third category, the focus is primarily on the overall system level, including the heat pump component, and not on the underground processes. The third category lists a few studies that apply ML methods to the analysis of heat transfer performance and the design of geothermal heat exchangers.

Noye et al. [14] reviewed AI applications, including ML methods, for the optimal control of GSHP systems. They identified two main AI uses in this case: building a predictive model of the systems performance and optimizing the control decision itself. The authors concluded that the existing research body is limited and outlined future research directions. On the other hand, Ma et al. [15] and Ahmed et al. [16] presented an overview of the methods in general and data-driven ML methods, respectively, for the optimal design and control of GSHP systems. Finally, Almutairi [17] analyzed the application of ML techniques in modeling geothermal heat pumps, focusing predominantly on overall system performance and building integration rather than on the underlying fluid and heat subsurface processes.

Despite the broad scope of these existing reviews, a considerable gap remains in the literature. While previous studies have extensively cataloged purely data-driven methodologies applied to the electromechanical components of heat pumps and to above-ground building-load forecasting, they largely treat the geological subsurface as a simplified, static boundary condition. Consequently, there is a lack of comprehensive synthe-

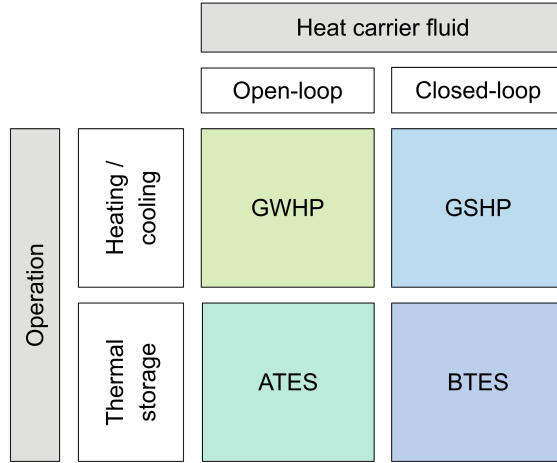


Figure 1: Overview of shallow geothermal energy systems.

sis regarding the application of advanced scientific machine learning, such as physics-informed ML and deep neural operators, to the highly complex, coupled thermo-hydraulic processes occurring within subsurface.

This paper aims to bridge that gap by providing a comprehensive, updated review of state-of-the-art deep learning methodologies utilized specifically for the simulation, surrogate modeling and optimization of shallow geothermal energy systems. By explicitly focusing on subsurface phenomena and highlighting the recent paradigm shift toward physics-guided architectures, this review establishes a contribution that is distinct from prior system-level overviews.

The remainder of this paper is structured as follows. Section 2 establishes the theoretical background, detailing the governing partial differential equations of subsurface heat and mass transport, while concurrently introducing the foundational concepts of deep learning, including physics-informed models and neural operators. Section 3 presents a review of the current state of the art, categorizing the latest literature by system type and applied deep learning methodology. Building upon this analysis, Section 4 discusses enduring disciplinary challenges, such as data scarcity, and outlines future research trajectories toward the development of real-time digital twins. Finally, Section 5 summarizes the key conclusions of this study.

## 2. Theoretical background

### 2.1. Shallow geothermal energy systems

Main types of SGE systems are summarized in Figure 1: groundwater heat pump (GWHP), ground source heat pump (GSHP), aquifer thermal energy storage (ATES) and borehole thermal energy storage (BTES) systems. Depending on how heat carrier fluid (HCF) circulates within the system, it can be differentiated between open-loop and closed-loop systems [18, 19]. The first group, encompassing GWHP and ATES systems, uses groundwater directly as a HCF to exchange heat with the subsurface (aquifer)

[20]. Groundwater is extracted at a single or multiple extraction (production) wells and, after thermal energy exchange within the heat pump unit, re-injected into the same aquifer via single or multiple injection wells [21]. On the other side, closed-loop systems use an additional HCF that circulates in enclosed pipes within geothermal heat exchangers [22]. GSHP systems, also known as ground coupled heat pump (GCHP) systems, are the most widely adopted SGE systems [23]. They can be divided into vertical and horizontal systems based on the geometry of geothermal heat exchangers [24, 25]: vertical borehole heat exchangers (BHEs) and horizontal collectors, respectively.

SGE systems can be further divided based on the operation mode / application type on those operating in a single mode, heating or cooling, and those using shallow subsurface for seasonal thermal energy storage, also known as underground thermal energy storage (UTES) systems [26]. ATES and BTES, as the main UTES types [27, 28], operate on the same physical principles as their counterparts, GWHP and vertical GSHP, respectively.

## 2.2. Governing equations

Interactions between SGE systems and the subsurface, as well as natural subsurface conditions, can be modeled using a set of partial differential equations (PDEs). The first group of PDEs describes 3D groundwater flow, which is a fully saturated porous medium, as follows [29, 30]:

$$S_0 \frac{\partial h}{\partial t} + \nabla \cdot \mathbf{q} = Q \quad (1a)$$

$$\mathbf{q} = -\mathbf{K} \cdot \nabla h \quad (1b)$$

where the unknown parameters (states) are the hydraulic head  $h$  [m] and the Darcy velocity  $\mathbf{q}$  [m/s]. The remaining parameters are summarized in Table 1. The simulation of groundwater flow is essential for the analysis of open-loop systems, since they use groundwater directly as the HCF and change the local flow conditions [31–33]. Injection and extraction wells of such systems can be modeled as idealized point sources and sinks, respectively [29]:  $Q_{hw} = -\sum_w Q_w(t)\delta(\mathbf{x} - \mathbf{x}_w)$ , where  $Q_w(t)$  is the pumping rate of the well  $w$  (injection:  $Q_w(t) < 0$ , extraction:  $Q_w(t) > 0$ ), and  $\delta(\mathbf{x} - \mathbf{x}_w)$  is the Dirac delta function associated with the well location  $\mathbf{x}_w$ . The influence of closed-loop systems on groundwater flow can be neglected since there is no mass exchange with the aquifer in this case. However, groundwater flow is also important in this case since it can influence the heat transport in the subsurface, as explained below.

The 3D heat transport in the porous media can be described by the following PDE in convective form [29, 30]:

$$\left(\varepsilon s \rho c + (1 - \varepsilon) \rho^s c^s\right) \frac{\partial T}{\partial t} + \rho c \mathbf{q} \cdot \nabla T - \nabla \cdot (\mathbf{\Lambda} \cdot \nabla T) = Q_T + Q_{Tw} - \rho c (T - T_0) Q \quad (2)$$

where the unknown parameter is the temperature of the porous medium  $T$  [K], and the remaining parameters are summarized in Table 1. The parameter  $\mathbf{\Lambda}$  represents the tensor of hydrodynamic thermodispersion,

Table 1: Parameters of the governing equations [29].

Symbol	Parameter	Unit
$S_0$	Specific storage coefficient	$\text{m}^{-1}$
$\mathbf{K}$	Tensor of hydraulic conductivity	$\text{ms}^{-1}$
$Q$	General liquid sink/source function	$\text{d}^{-1}$
$T_0$	Reference temperature	K
$\varepsilon$	Porosity	-
$s$	Saturation	-
$\rho$	Liquid (water) density	$\text{kgm}^{-3}$
$\rho^s$	Solid density	$\text{kgm}^{-3}$
$c$	Liquid specific heat capacity	$\text{Jkg}^{-1}\text{K}^{-1}$
$c^s$	Solid specific heat capacity	$\text{Jkg}^{-1}\text{K}^{-1}$
$\Lambda$	Tensor of hydrodynamic thermodispersion	$\text{Wm}^{-1}\text{K}^{-1}$
$Q_T$	Thermal supply term	$\text{Jm}^{-3}\text{d}^{-1}$
$Q_{Tw}$	Thermal well-type SPC term	$\text{Jm}^{-3}\text{d}^{-1}$
$\Lambda_0$	Tensor of thermal conductivity of liquid	$\text{Wm}^{-1}\text{K}^{-1}$
$\Lambda_0^s$	Tensor of thermal conductivity of solid	$\text{Wm}^{-1}\text{K}^{-1}$
$\mathbf{D}_{\text{mech}}$	Tensor of mechanical dispersion	$\text{m}^2\text{s}^{-1}$
$\Lambda$	Coefficient of thermal conductivity of liquid	$\text{Jm}^{-1}\text{K}^{-1}\text{s}^{-1}$
$\Lambda^s$	Coefficient of thermal conductivity of solid	$\text{Jm}^{-1}\text{K}^{-1}\text{s}^{-1}$
$\beta_L$	Longitudinal dispersivity	m
$\beta_T$	Transverse dispersivity	m
$\mathbf{I}$	Unity or identity matrix	-

defined as follows [29]:

$$\Lambda = \Lambda_0 + \Lambda_0^s + \rho c \mathbf{D}_{\text{mech}} \quad (3a)$$

$$\Lambda_0 = \varepsilon s \Lambda \mathbf{I} \quad (3b)$$

$$\Lambda_0^s = (1 - \varepsilon) \Lambda^s \mathbf{I} \quad (3c)$$

$$\mathbf{D}_{\text{mech}} = \beta_T \|\mathbf{q}\| \mathbf{I} + (\beta_L - \beta_T) \frac{\mathbf{q} \otimes \mathbf{q}}{\|\mathbf{q}\|} \quad (3d)$$

with the parameters summarized in the last part of Table 1.

In the case of simulating heat transport in groundwater, the saturation is  $s = 1$  and  $T$  corresponds to the groundwater temperature. For open-open loop systems advection term plays a crucial role in the overall heat transport, since the main heat exchange is happening within the aquifer. Only injection wells of such systems are acting as thermal sink/source terms and can be modeled as idealized single point sources [29]:  $Q_{Tw} = -\sum_w (T_w - T) \rho c Q_w(t) \delta(\mathbf{x} - \mathbf{x}_w)$ , where  $Q_w(t)$  is the pumping rate of the well  $w$  and  $T_w$  is the temperature of the injected water. On the other hand, for closed-loop systems the main part of the heat exchange is usually with the soil and advection part can be often neglected due to low groundwater flow. By introducing such conditions into the equation (2), the resulting PDE is the heat conduction (diffusion) PDE commonly-used to describe the heat transport in soil around closed-loop systems. However, it should be

noted that groundwater flow influence cannot always be neglected, especially in multi-BHE configurations [34, 35].

To accurately simulate closed-loop systems, numerical modeling of BHEs typically requires consideration of thermal dynamics in two distinct but coupled domains: the local problem, which governs heat exchange within the borehole’s internal components (tubes, circulating fluid, and grout material), and the global problem, which describes heat transfer in the surrounding subsurface [36]. To reduce the geometric complexity associated with explicitly meshing the internal BHE components within much larger geological domain, the global problem is often simplified by approximating the BHE as a thermal line source.

The governing PDEs (1)-(2) are constituting time-dependent 3D simulation models of groundwater flow and heat transport in the subsurface. These numerical models can be often simplified to steady-state and/or 2D models to reduce computational cost or overcome the lack of detailed subsurface parametrization [37]. Further simplification can be achieved by using analytical models that approximate numerical solutions [38, 39]. These models are usually used to calculate the extent of the so-called thermal plumes in groundwater [40], which are caused by open-loop systems, or the temperature change in soil induced by vertical BHEs [41]. The latter one comes from the spatial simplification of the BHE as a line source, as mentioned earlier, combined with the assumption of a homogeneous and isotropic subsurface, which allows for the derivation of established analytical solutions, such as infinite and finite line source models. While these analytical models are computationally efficient and suitable for basic sizing or steady-state estimates, they are inherently limited when it comes to, for instance, heterogeneous geological conditions, transient hydrogeological flows, or dynamic boundary conditions. Consequently, numerical methods remain essential for accurately modeling such complex, coupled thermo-hydraulic phenomena. However, numerical methods are computationally intensive and impractical for multi-query applications such as comprehensive sensitivity analysis, inverse parameter estimation, and design optimization. Therefore, this study focuses on DL methods and the question of how they can replace or complement numerical models.

### 2.3. Deep learning methods

Deep learning is a sub-category of machine learning that focuses on deep artificial neural networks (ANNs) [11, 42]. Deep ANNs contain multiple hidden layers, compared to shallow ANNs with only one single hidden layer, and thus more power to represent more complex relations [42]. However, since ANN, whether shallow or deep, are the core part of DL models, the focus of this study will be ANNs in general. ANNs can be used to map input to output parameters/values, and thus approximate well enough any continuous function [43].

The major types of ANNs, based on their architectural design, are fully connected feed-forward networks (FNN), convolutional neural networks (CNN) and recurrent neural networks (RNN) [11]. In FNNs, also known as a multi-layer perceptrons (MLP), information flows only in one direction: from the input layer to the output layer, passing through one or more hidden layers in between [44]. This means that the neurons

perform calculations sequentially through the layers. Each neuron in hidden layers performs a weighted sum of the inputs it receives, applies an activation function (such as ReLU, Sigmoid, or Tanh) to this sum, and passes the result to the next layer [44].

CNNs are type of ANNs specifically designed for images and spatial data. They normally contain convolution layers that apply filters (kernels) to the input, allowing the network to detect local patterns, and pooling layers that perform subsampling to reduce the dimensionality [11, 45]. On the other side, RNNs are type of ANNs designed for processing sequential and time-series data [46]. Compared to FNNs, RNNs have closed loops (or recurrence connections) in hidden layers that allow information to persist across time steps [47]. Major enhanced architectures of RNNs are long short-term memory (LSTM) [48] and gated recurrent unit (GRU) [49]. LSTMs have memory cells that enable to retain important information over long sequences. These cells selectively propagate relevant information throughout the temporal sequence by using special gating mechanisms (forget, input, and output gates) [42]. On the other hand, GRUs are a simplified version of LSTMs that contain less parameters and are thus easier to train.

DL methods can be used to approximate the solutions of numerical PDE models, i.e. to act as fast surrogates that map the inputs of a simulation (e.g. material parameters, boundary and initial conditions) directly to the quantities of interest (e.g. temperature or pressure fields). In this context, DL methods can be divided into purely data-driven and physics-informed methods, based on how underlying physics is embedded into the DL model [12]. Data-driven approaches rely on large amount of training data and can produce physically infeasible results due to lack of generalization and extrapolation capabilities outside of the training data set. On the other hand, physics-informed DL and ML methods enforce physical laws by integrating them in the training process and thus do not require large amount of training data. The main representative of physics-informed DL methods is a physics-informed neural network (PINN) whose general structure is depicted in Figure 2. The training (loss) function for PINNs is given as [50]:

$$\mathcal{L} = w_{PDE} \cdot \mathcal{L}_{PDE} + w_{BC} \cdot \mathcal{L}_{BC} + w_{IC} \cdot \mathcal{L}_{IC} + w_{data} \cdot \mathcal{L}_{data} \quad (4a)$$

$$\mathcal{L}_{PDE} = \frac{1}{N_{PDE}} \sum_{i=1}^{N_{PDE}} \left| \mathcal{R}(u(\mathbf{x}_i, t_i)) \right|^2 \quad (4b)$$

$$\mathcal{L}_{BC} = \frac{1}{N_{BC}} \sum_{j=1}^{N_{BC}} \left| \mathcal{B}(u(\mathbf{x}_j, t_j)) - \phi_j \right|^2 \quad (4c)$$

$$\mathcal{L}_{IC} = \frac{1}{N_{IC}} \sum_{k=1}^{N_{IC}} \left| u(\mathbf{x}_k, t_0) - \xi_k \right|^2 \quad (4d)$$

$$\mathcal{L}_{data} = \frac{1}{N_{data}} \sum_{l=1}^{N_{data}} \left| u(\mathbf{x}_l, t_l) - u_l \right|^2 \quad (4e)$$

where  $\mathcal{L}$  is the loss function that contains the terms  $\mathcal{L}_{PDE}$ ,  $\mathcal{L}_{BC}$ ,  $\mathcal{L}_{IC}$ ,  $\mathcal{L}_{data}$  corresponding to PDE, boundary conditions (BC), initial condition (IC), and data loss terms, respectively. The loss terms are

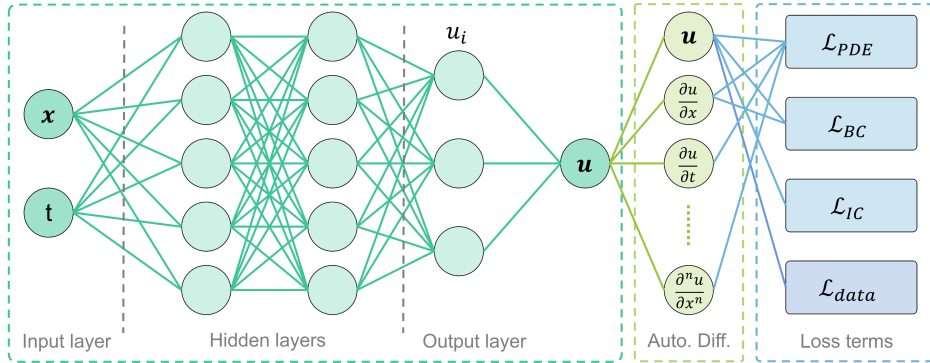


Figure 2: Physics-informed neural network.

weighted with their corresponding weights  $w$ . The PDE loss  $\mathcal{L}_{PDE}$  enforces that the underlying PDE is point-wise satisfied in strong form, i.e. that the PDE residual  $\mathcal{R}$  becomes 0, for all PDE-sampled spatio-temporal points  $(\mathbf{x}_i, t_i)$ . The residual  $\mathcal{R}$  for each predicted state  $u(\mathbf{x}_i, t_i)$  is evaluated using automatic differentiation (AD) and back-propagation techniques (see Figure 2) and does not require any labeled training data, which is one of the main PINN advantages. In the similar fashion, BCs are enforced by minimizing the  $\mathcal{L}_{BC}$  loss, which is given here as a difference between left and right hand side,  $\mathcal{B}$  and  $\phi_j$ , respectively. The operator  $\mathcal{B}$  is defined by the type of BC and can be evaluated in the same fashion as the PDE residual using AD. For example, in the case of Dirichlet's BCs, the operator becomes  $\mathcal{B}(u(\mathbf{x}_j, t_j)) = u(\mathbf{x}_j, t_j)$ . The sampling points for the BC loss term are located on the corresponding domain boundaries. The term  $\mathcal{L}_{IC}$  enforces initial condition at sampled spatial locations  $\mathbf{x}_k$ . These three loss terms (PDE, BC, IC) are sufficient for forming the loss function and training PINN without any prepared labeled training data. However, if there are some available training data (input-output training pairs), these can be also additionally included into the training function via the data loss term  $\mathcal{L}_{data}$ . This is classical type of loss function used in data-driven methods, that minimizes the discrepancy between predicted states  $u$  and data  $u_l$  for the training points  $(\mathbf{x}_l, t_l)$ . Finally, each loss term is normalized by the size of their training set  $N$ , i.e. the number of sample points in these sets.

While PINNs offer several advantages for solving PDEs, they still lack generalization and transferability, especially in the context of parametric PDEs. For instance, if some of PDE parameters or BCs changes, the PINN results would not be valid anymore and re-training is needed. This can be contributed to the nature of NNs, which can be understood as universal function approximators, i.e. mapping input to output vectors [43]. On the contrary, recently developed neural operator networks aim to approximate functional operators, i.e. mappings from one to another function [51]. This seems a promising solution for parametric PDEs, since the parameters of interest can be used as input functions to those models and thus accommodate solving PDEs with different conditions. Two main types of neural operators are deep operator networks (DeepONet)

[52] and Fourier neural operators (FNO) [53].

Here, we briefly introduce DeepONets, whose general structure is shown in Figure 3. The DeepONet architecture formulates the learning of nonlinear operators by means of two distinct sub-networks: the *branch net* and the *trunk net* [52]. The branch net ingests discrete representations of the variable input functions, such as dynamic injection-temperature profiles or heterogeneous spatial permeability fields, and encodes them into a latent representation. Simultaneously, the trunk net evaluates the continuous spatio-temporal coordinates of the target domain. The final predicted solution field is then obtained as the inner product of the outputs of both networks. For multi-query applications, such as long-term sensitivity analysis or iterative optimization, DeepONets offer a substantial computational advantage. While standard physics-informed neural networks require expensive retraining whenever the system parameters change, DeepONets learn the underlying solution operator that maps a set of initial and boundary conditions to the full spatio-temporal solution. Once trained, they act as a universal surrogate capable of evaluating entirely novel boundary conditions and operational profiles in milliseconds, which makes them powerful tools for parametric PDE learning [52].

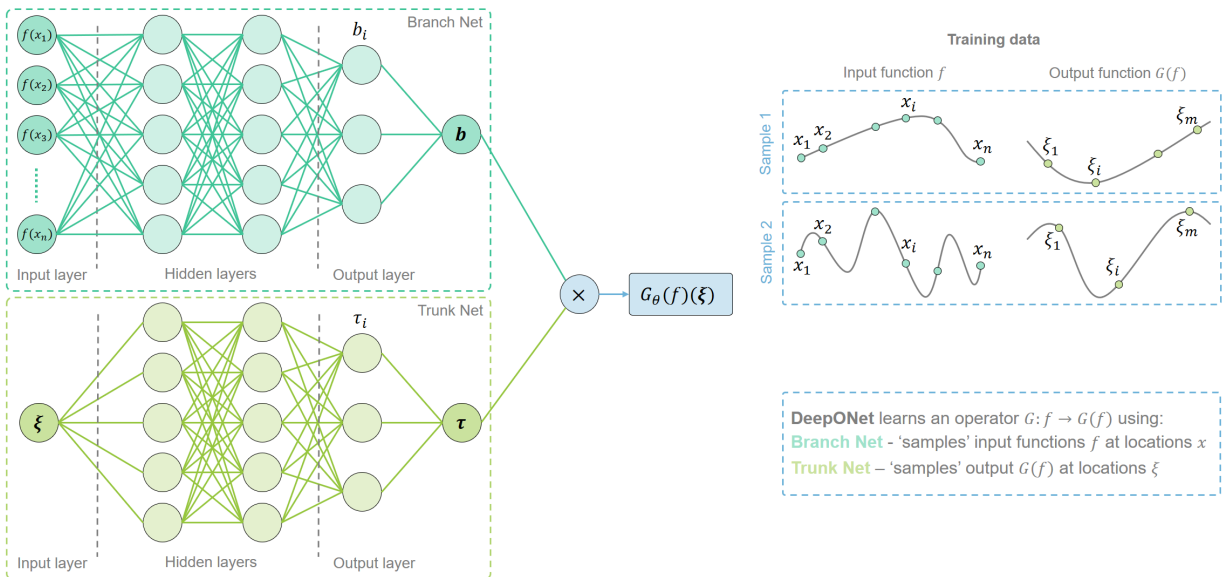


Figure 3: DeepONet architecture (left) and Example of training data (right).

A second widely used class of neural operators are Fourier neural operators, which parametrize the functional operator directly in the frequency domain and are therefore particularly efficient for problems on structured grids [53]. Beyond these two archetypes, a rapidly growing family of operator architectures and their physics-informed variants (e.g. physics-informed DeepONets and FNOs) has been proposed and applied to a wide range of parametric PDE problems [51].

### 3. Deep learning for shallow geothermal energy systems

Deep learning has a vast number of potential applications in the shallow geothermal field, be it in the analysis on the resource side (e.g. shallow geothermal potential analysis, inverse problems) or the analysis of SGE systems, i.e. their planing, design and operation. In principle, DL models can replace computationally expensive PDE numerical simulation models (see Section 2.2) in these analysis chains and speed up the entire process. While the execution of already trained DL models is much faster than a numerical simulation, DL models still require significant resources for the training phase. Therefore, one of the main payoffs for DL models is when the analysis process requires a large number of simulation runs. Typical examples of such processes include sensitivity analyses and optimization procedures, as depicted in Figure 4. In both cases, often several hundreds or more of simulation executions are required and using high-fidelity numerical simulation models becomes impractical.

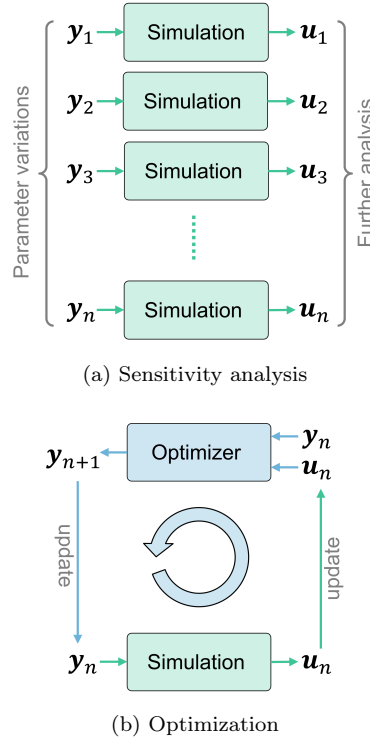


Figure 4: Many query problems.

Beyond serving as forward surrogates, DL models can support a range of further tasks along the SGE analysis chain. In inverse problems, they can estimate uncertain subsurface properties, such as hydraulic conductivity, porosity or thermal conductivity, from sparse temperature or hydraulic-head measurements. In design and operation, fast surrogates enable gradient-based or metaheuristic optimization of well placement, borehole layout and operational schedules, where each evaluation would otherwise require a full numerical

simulation. Finally, their low inference cost makes them attractive for real-time and predictive-control applications, and ultimately for digital twins that mirror the state of an SGE system during operation. The remainder of this section reviews how these opportunities have been addressed in the literature so far.

### 3.1. State of the art

Table 2 summarizes the state of the art, i.e. existing scientific publications, of deep learning for SGE systems. The table is divided in 4 sections, corresponding to 4 main SGES types and following the color scheme from Figure 1. The columns represent, from left to right: type of SGES analyzed, DL or ML method applied, predicted parameter(s), underlying underground model type and the corresponding references. For some publications, ML methods (that are not DL, marked with \*) are also included in the table for the sake of completeness. However, the main focus of this work is DL methods, and therefore ML approaches are not discussed in detail. The underground model column indicates whether the study incorporates an explicit (mostly numerical) model of the subsurface (✓) or treats the system purely from an above-ground, data-driven perspective (✗), with the type of numerical model given in parentheses where applicable.

The largest number of publications is on GSHP systems, whereas only few publications analyze other types of SGES. Furthermore, in the first group it is further distinguished between vertical BHE systems [54–80], energy piles [81–86], horizontal systems [87, 88] and general closed-loop GHEs [89]. The most common predicted parameters are BHE fluid temperatures, system’s performance, and temperature changes in the underground, whereas the most used DL method is the FNN. Several research studies analyze SGES from a system perspective, without explicitly modeling the underground processes or considering them only in an oversimplified manner [54, 55, 59–61, 75, 76, 81, 87]. The focus of these studies is not on underground processes and therefore they are not particularly relevant for the present study.

On the other hand, other research studies on GSHP systems listed in Table 2 include, usually numerical, underground model in their method. Lee et al. [56] apply an FNN to predict BHE fluid outlet temperature influenced by operational parameters and underground conditions. The study analyzes the feasibility of ANNs as a surrogate for model predictive control (MPC) of BHEs and uses finite element method (FEM) for the underlying numerical model. However, only a single BHE is considered in the underground model. Chen et al. [58] use a three-layer FNN to predict the required depth of a vertical BHE based on given design parameters. The authors simulate heat transfer in a single BHE using a 3D numerical model. Liu et al. [62] utilize a shallow FNN with 1 hidden layer to predict the long-term performance of a multiple-BHE system. The proposed model predicts ground temperatures, seasonal energy efficiency ratio (SEER), seasonal coefficient of performance (SCOP), annual electricity consumption and annual energy savings by using 19 influencing parameters (inputs). The inputs are divided into 3 groups: ground source side (ground temperature, soil thermal conductivity, depth of BHEs, BHE spacing, number of BHE rows and columns, pipe flow rates), equipment side (EER and COP coefficients), and building side group (summer cooling

Table 2: State of the art.

SGES	ML/DL method	Predicted parameter	Underground model	Reference
vBHE	LSTM, GRU, 1D-CNN, BD-LSTM <sup>∇</sup>	BHE outlet temperature	✗	[54]
	FNN	BHE outlet temperature	✗	[55]
	FNN	BHE outlet temperature	✓(FEM)	[56]
	Gaussian process regression*	BHE fluid temperature	✓(FEM)	[57]
	FNN	BHE depth	✓(FVM)	[58]
	FNN, SVM*, TEM*	performance	✗	[59]
	FNN	performance	✗	[60, 61]
	FNN	performance	✓(FEM)	[62]
	FNN	g-functions	✓	[63–65]
	FNN	soil temperature around BHE	✓(FVM)	[66, 67]
	FNN	soil temperature variations	✓(simplified)	[68]
	FNN	soil thermal properties	✓(inverse, TRT)	[69–73]
	RSM*, FNN	borehole thermal resistance	✓(FEM)	[74]
	Analytical-FNN hybrid model	heat load of BHEs	✗	[75, 76]
	LR*, PR*, FNN, RF*	heat extraction rate	✓(FVM)	[77]
LR*, NLR*, FNN	BHE heat transfer capacity	✓(experiment)	[78]	
XGBoost*	GSHP operation parameters	✓(FEM)	[79]	
FNN	hydrodynamic and heat transfer	✓(FVM)	[80]	
Energy pile	FNN	performance	✗	[81]
	MLR*	fluid temperature	✓(FEM)	[82]
	1D-CNN, LSTM, BD-LSTM <sup>∇</sup>	factored resistance	✓(FEM)	[83]
	FNN	thermally-induced displacement	✓(FEM)	[84]
	CNN-LSTM hybrid model	outlet water temperature	✓(FEM)	[85]
	KNN*, RT*, RF*, FNN	outlet water temperature	✓(FEM)	[86]
hGSHP	ARIMA*, RR*, DT*, FNN, TDNN <sup>∇</sup>	inlet fluid temperature	✗	[87]
	FNN, LR*	scale factor for design	✓(oversimplified)	[88]
GHE	CPINN <sup>△</sup>	velocity, pressure, temperature	✓(local)	[89]
BTES	FNN	emission factor, cost of energy	✗	[90]
GWHP	FNN	groundwater temperature	✓(FEM)	[91]
	CNN	thermal plume	✓(FVM)	[92, 93]
	LSTM	hydraulic drawdown in inj. well	✓(FEM)	[94]
ATES	kNN*	g-function	✓	[95]
	Transformer neural operator	temperature, pressure	✓(FEM)	[96]

\*ML, but not DL methods: SVM - Support Vector Machines, TEM - Tree Ensemble Model, RSM - Response Surface Method, LR - Linear Regression, PR - Polynomial Regression, RF - Random Forest, NLR - Nonlinear Regression, XGBoost - Extreme Gradient Boosting, MLR - Multiple Linear Regression, ARIMA - Autoregressive Integrated Moving Average, RR - Ridge Regression, DT - Decision Trees, kNN - k-Nearest Neighbors, RT - Regression Tree.

△Physics-informed methods: CPINN - Coupled Physics-Informed Neural Network

∇DL methods: BD-LSTM - Bidirectional LSTM, TDNN - Time Delay Neural Network

Underground models: FEM - Finite Element Method, FVM - Finite Volume Method

load, winter heating load, cooling/heating operation days, daily operation time). The numerical model accounts for heat transfer within the underground, including the soil, BHEs, and fluid flow within the pipes. However, homogeneous conditions in underground are assumed and groundwater flow is neglected.

In addition, multiple BHEs and their interactions are simplified by representative subsets of BHEs: corner, edge and central BHEs.

Beyond outlet-temperature and performance prediction, a substantial group of studies targets the thermal state of the ground itself. Dusseault and Pasquier [63], Pasquier et al. [64] and Shoji et al. [65] employ FNNs to approximate g-functions, i.e. the dimensionless thermal response of boreholes, enabling near-instantaneous evaluation of BHE temperature responses that conventionally require expensive numerical or semi-analytical computations. Kharseh et al. [66] and, more recently, Kharseh et al. [67] train FNNs to predict the soil temperature in the vicinity of a BHE as a function of operational and ground parameters; the latter study explicitly includes the groundwater flow velocity among its inputs and systematically optimizes the network architecture. Similarly, Zhou et al. [68] predict volume-averaged ground-temperature variations induced by GSHP operation using a TRNSYS-based dataset. A distinct line of research uses ANNs in an inverse setting to identify subsurface thermal properties from thermal response test (TRT) data: Han et al. [69], Hu et al. [70] and Zhang et al. [71, 72, 73] estimate soil thermal conductivity and related parameters, thereby replacing iterative parameter-fitting procedures. In a complementary fashion, Richter et al. [57] apply Gaussian process regression to perform global sensitivity analysis and uncertainty quantification for the design parameters of shallow geothermal systems, using an FEM model as the reference. Finally, Chaoran et al. [79] use an XGBoost surrogate, trained on an FEM model, to optimize the operational parameters of an existing GSHP system.

The last group of BHE-related studies focuses on the BHE itself, i.e. on the local problem (see [section 2.2](#)). King and Yune [74] apply response surface method and deep FNN to estimate the 3D borehole thermal resistance and total internal thermal resistance in a single BHE. The authors employ genetic algorithm (GA) to optimize the FNN architecture, including the number of hidden layers, neurons, activation functions and learning rates. The numerical model includes 1 BHE, assuming homogeneous and isotropic ground properties, and no groundwater flow. Tang et al. [77] use 3 ML methods (linear regression, polynomial regression, random forest) and FNN for predicting the heat extraction rate of a single BHE. The study employs finite volume method (FVM) to simulate 1 BHE considering homogeneous soil with groundwater flow. Similarly, Xu et al. [78] predict heat transfer capacity of a single BHE by using linear and nonlinear regression, and an FNN model. However, the training data in this study is not based on a numerical simulation model, but on experimental data from the literature and on-site measurements. Jery et al. [80] utilize an FNN to predict the Nusselt number and entropy generation, i.e. the hydrodynamic and heat transfer, of a single coaxial BHE. The study analyzes the local problem, i.e. the processes inside the BHE, and not the processes outside of BHE, such as temperature development in surrounding soil.

Several research studies apply ML and DL methods to analyze [energy pile](#) systems. Kumar and Samui [83] use CNN and LSTM models to predict the factored resistance of energy pile groups, which is relevant for structural integrity and safety of energy piles. The study focuses on structural aspects of energy piles,

not on thermo-fluid phenomena, and is therefore not relevant for the present work. Similarly, Pei et al. [84] use an FNN with 2 hidden layers to predict the long-term, thermally induced vertical displacement of energy piles. On the other hand, Zhang et al. [85] propose a hybrid CNN-LSTM model for predicting outlet water temperature of energy piles. The hybrid model captures spatial-temporal features of the surrounding soil temperature field and the results show improved prediction accuracy compared to LSTM, CNN, FNN, and simple RNN models. The study uses an FEM numerical model of a single pile and its local surrounding. More recently, Wang et al. [86] couple a 3D transient COMSOL model of a pipe-type energy pile with several supervised learning algorithms (k-nearest neighbors, regression tree, random forest and ANN) to predict the short-term outlet water temperature. The authors additionally employ Shapley additive explanations (SHAP) to quantify the relative influence of the inlet temperature, the flow velocity and the material properties on the prediction.

The studies on ML or DL methods for **horizontal GSHP** systems are limited. Baroque et al. [87] use several ML and DL methods to predict inlet fluid temperature entering horizontal collectors. However, the focus is on the overall GSHP system and the underground processes are not considered. On the other side, Jeon et al. [88] use FNN and linear regression models to predict the scale factor for designing horizontal ground heat exchangers. The authors oversimplify the underground by modeling it as a solid media and not considering convection and advection processes.

Finally, the study by Zhang and Li [89] addresses closed-loop **GHEs** in general, without explicitly restricting itself to a specific SGES type. The study uses coupled physics-informed neural networks (CPINNs) to predict velocity vector field, pressure, and temperature inside GHE (free fluid flow region) and around GHE (porous medium fluid flow region). The focus is on the interface between the pipe and porous media and, in this context, the term 'coupled' means multi-physics. The porous media region is assumed to be homogeneous and isotropic. Although the study focuses on the local problem, i.e. the interface area, it should be emphasized that this is one of the few studies to employ a physics-informed method.

When it comes to DL methods for **BTES systems**, the existing research body is rather limited, with a single study in Table 2. Hemmatabady et al. [90] use an FNN to predict emission factors and levelized costs of energy for different system layouts. The considered system does not include only BTES but also solar thermal collectors and buffer storage tanks. The study utilizes the FNN model as a surrogate in the multi-objective optimization procedure, in order to reduce computational cost and efficiently find the optimal system layout in terms of emission reduction and economic efficiency. In general, this is a promising direction for the utilization of DL methods, as previously explained in Section 3. However, the study takes the overall system's perspective, without analyzing (modeling) underground phenomena in particular.

In contrast to closed-loop systems, there are only few studies that apply ML/DL methods to analyze open-loop systems. In an early work from 2014, Lo Russo et al. [91] used a simple FNN with one hidden layer to predict groundwater temperatures around a **GWHP system**. The temperature evolution at a

given location in the aquifer is predicted based on 5 inputs describing the GWHP operation. The proposed approach requires a different NN for each location analyzed. The training data was generated using the commercial FEM simulation software FEFLOW [29, 97]. More recently, Davis et al. [92] and Pelzer and Schulte [93] applied a CNN model to predict thermal plumes in aquifers caused by GWHPs. In the former case, a CNN with a U-Net architecture is used to predict the propagation of a thermal plume from a single injection well based on Darcy velocity as input data. The latter approach is closer to numerical simulation scenarios where the input data are spatially resolved subsurface parameters (hydraulic gradient and permeability) and an arbitrary location of the injection well. Moreover, the second approach can predict not only the thermal plume caused by a single GWHP, but also that of multiple GWHPs by considering the influences of neighboring GWHPs, i.e. their thermal plumes, in a two-phase process. Although this second approach is a promising data-driven method for GWHP systems, it has certain limitations, such as considering only steady-state conditions and 2D spatial input/output data, and neglecting the influence of extraction wells on local groundwater flow conditions. Finally, Rose et al. [94] use an LSTM network with 4 hidden layers to predict changes in hydraulic head in the injection well of a single well doublet system. For the preparation of the training data, only the injection well is numerically modeled with a simplified 2D-axisymmetric FEM model. The proposed LSTM model is specific to the particular site for which it was trained and does not take into account hydraulic interactions between the wells.

Similar to BTES systems, existing research on ML/DL methods for **ATES systems** is limited to only two studies listed in Table 2. Chen et al. [95] use the k-nearest neighbors (kNN) algorithm, an ML method, to quantify the heat loss coefficient, which represents the complex thermal interactions between the aquifer and the surrounding impermeable layers. The estimated heat loss coefficient is incorporated into an analytical model (g-function) to predict the spatio-temporal temperature profile of an aquifer during the operation of a bidirectional ATES system. A recent methodological advance is provided by Ju et al. [96], who introduce the Adaptive Physics Transformer (APT), a mesh- and geometry-agnostic neural operator that combines a graph-based local encoder with a global attention mechanism, and demonstrate it, among other subsurface energy systems, on a 3D ATES benchmark that uses dynamic mesh optimization to track the moving thermal fronts. This represents one of the first applications of operator-learning and transformer-based architectures to shallow geothermal subsurface simulation, and it points toward the more general, foundation-model direction discussed in Section 4.

Figure 5 summarizes the reviewed literature quantitatively. As shown in panel (a), the GSHP category clearly dominates the field, accounting for the large majority of the compiled studies, while BTES, GWHP and ATES together represent only a small fraction. The figure further distinguishes between studies that explicitly resolve the subsurface, i.e. that include an underground model, and those that operate purely at the system level. Notably, the closed-loop GSHP literature contains a substantial share of system-level studies, whereas the few existing open-loop (GWHP, ATES) studies are almost without exception

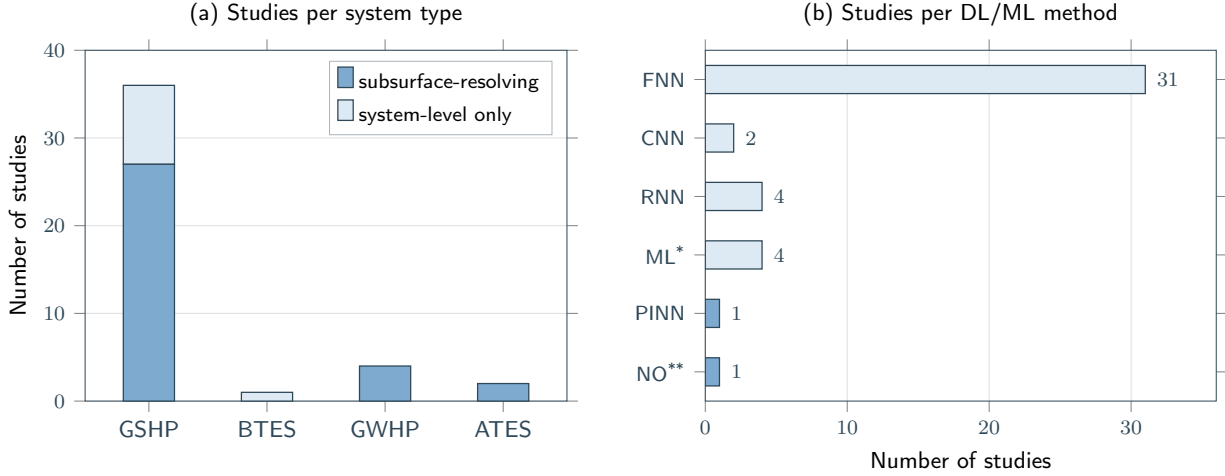


Figure 5: Quantitative overview of the reviewed literature (Table 2): (a) number of studies per SGE system type, split into subsurface-resolving studies (those including an underground model) and system-level studies; (b) number of studies per ML/DL method. \*Classical ML, non-DL methods, \*\*Neural operator.

subsurface-resolving. Panel (b) breaks the literature down by method and reveals a pronounced methodological imbalance: the FNN is by far the most common architecture, followed at a distance by CNN- and RNN-based models, while physics-informed neural networks and neural operators are each represented by only a single study. This confirms that the field remains predominantly data-driven and that physics-aware and operator-learning approaches are still in their earliest stages.

#### 4. Discussion and Outlook

The literature reviewed in Section 3 reveals several consistent trends. The overwhelming majority of existing approaches are purely data-driven and operate at the system level, predicting integral quantities such as outlet fluid temperatures or coefficients of performance, while the subsurface is either ignored or represented in a strongly simplified, often steady-state and two-dimensional manner. Studies that explicitly resolve the coupled groundwater flow and heat transport in the subsurface are comparatively rare, are dominated by feed-forward networks, and, with the exceptions of Zhang and Li [89] and Ju et al. [96], almost exclusively rely on purely data-driven formulations. This leaves a large and largely unexplored area for physics-aware and operator-based methods.

These observations are summarized conceptually in Figure 6, which positions the reviewed literature along two axes: the degree of physics integration and generalization (from purely data-driven, through physics-informed, to operator-learning and foundation-model approaches) and the fidelity with which the subsurface is resolved (from system-level, through simplified or local representations, to fully-resolved subsurface models). The bulk of the existing studies cluster in the lower-left of this map, i.e. data-driven models

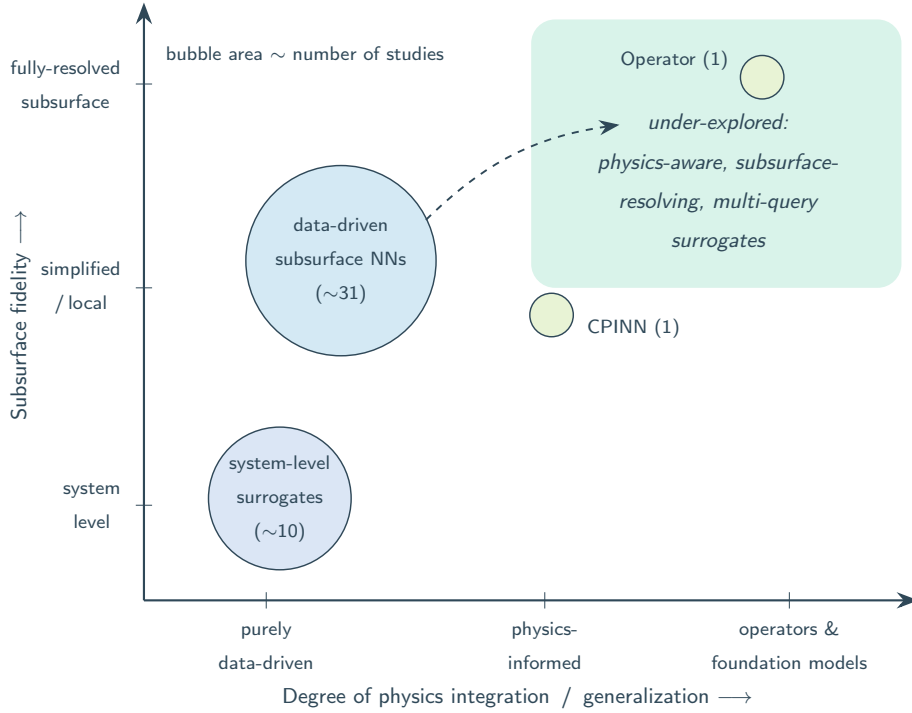


Figure 6: Conceptual map of the reviewed literature along the axes of physics integration and subsurface fidelity.

that either operate at the system level or resolve the subsurface only in a simplified manner. Physics-informed formulations (the CPINN of Zhang and Li [89]) and operator-learning approaches (the APT of Ju et al. [96]) remain isolated outliers. The upper-right region of the map, which combines strong physics integration with a fully-resolved subsurface and the multi-query capability required for sensitivity analysis, inverse modeling and optimization, is essentially unexplored and constitutes the central opportunity for future research, as indicated by the dashed arrow. However, the development of these advanced DL methods for SGE systems is linked to several challenges.

The first fundamental challenge is the wide range of spatial and temporal scales involved. Spatially, the relevant physics spans from the centimeter scale of the borehole or well to the field and aquifer scale of hundreds of meters, while temporally it ranges from the fast operational dynamics of a heat pump to the seasonal and multi-year behavior of thermal energy storage. Resolving these disparate scales simultaneously is difficult for standard neural architectures and calls for multi-scale and multi-resolution formulations.

The second challenge is the presence of singularities. Injection and extraction wells, as well as borehole heat exchangers idealized as line sources, introduce steep gradients and point-/line-singularities in the solution fields. These features are notoriously difficult to capture for both standard data-driven networks and physics-informed neural networks, and they typically require tailored collocation sampling, hard-constraint formulations, or domain-decomposition strategies.

The third, and perhaps most decisive, challenge is data scarcity. High-fidelity training data must usually be generated by computationally expensive numerical simulations, which limits the size and diversity of the available datasets and, in turn, the generalization of purely data-driven models. Physics-informed and hybrid physics-data approaches are particularly attractive in this context, as they reduce the data demand by embedding the governing equations into the training process and tend to generalize better beyond the training distribution [12, 50].

Most practical SGE tasks, such as comprehensive sensitivity analysis, inverse parameter estimation and design or operational optimization, are inherently multi-query and require hundreds or thousands of forward evaluations [36, 37]. Classical neural networks must be retrained whenever the parameters or boundary conditions change, which undermines their usefulness in such settings. Neural operators, such as DeepONets and FNOs, overcome this limitation by learning the underlying solution operator and can therefore be reused across varying parameters and conditions [51–53], making them a natural fit for parametric subsurface problems.

Finally, two emerging directions appear especially promising. Generative AI models, in particular diffusion [98] and flow-matching [99] approaches, can deliver probabilistic predictions and thus naturally incorporate the uncertainty that is inherent to heterogeneous subsurface conditions. In parallel, subsurface foundation models and PDE-transformer architectures, trained across multiple systems, meshes and physical processes, point toward general-purpose surrogates that can be adapted to new configurations with little additional data. The Adaptive Physics Transformer of Ju et al. [96] is an early example of this trend in the shallow-geothermal context. Taken together, these developments outline a clear trajectory toward fast, reliable and uncertainty-aware subsurface surrogate models and, ultimately, toward real-time digital twins of SGE systems. In addition, the successful development of such advanced DL methods will also enable their use in other applications, especially in the field of deep geothermal energy [100] as well as in other areas of thermal management and optimization (e.g. [101, 102]).

## 5. Conclusion

This paper reviews the application of deep learning methods to the simulation and optimization of shallow geothermal energy systems, with a specific focus on the underlying subsurface thermo-hydraulic processes. Starting from the governing equations of groundwater flow and heat transport, we introduced the relevant deep learning paradigms and systematically categorized the existing literature by system type, applied methodology, predicted quantities and underlying numerical model. The survey shows that research to date is concentrated on closed-loop systems, in particular vertical borehole heat exchangers, and is dominated by purely data-driven feed-forward networks used as system-level surrogates. On the other side, open-loop systems and advanced deep learning methods, including physics-informed and operator-learning approaches

that explicitly resolve the subsurface, remain comparatively underexplored. While the data-driven surrogates provide substantial speed-ups for sensitivity analyses and optimization tasks, their reliance on large and expensively generated training datasets, together with their limited generalization to unseen parameters and boundary conditions, remains a key obstacle. Physics-informed neural networks, neural operators and hybrid formulations offer promising avenues to overcome these limitations, as they reduce the data demand and naturally accommodate the parametric nature of subsurface problems. Together with emerging generative and foundation-model approaches, they pave the way toward fast, reliable and uncertainty-aware subsurface models required for the efficient simulation and optimization of shallow geothermal energy systems.

## 6. Conflict of interest

The authors declare no conflicts of interest.

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