River deltas as Multiplex networks: A framework for studying multi-process multi-scale connectivity via coupled-network theory

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Abstract

Transport of water, nutrients or energy fluxes in many natural or coupled human-natural systems occurs along different pathways that often have a wide range of transport timescales and might exchange fluxes with each other dynamically (e.g., surface-subsurface). Understanding this type of transport is key to predicting how landscapes will change under changing forcing. Here, we present a general framework for studying transport on a multi-scale coupled-connectivity system, via a multilayer network, which conceptualizes the system as a set of interacting networks, each arranged in a separate layer, and with interactions across layers acknowledged by interlayer links. We illustrate this framework by examining transport in river deltas as a dynamic interaction of flow within river channels and overland flow in the islands, when it is controlled by the flooding level. We show the potential of the framework to answer quantitatively questions related to the characteristic timescale of response in the system.

I. Introduction

Conceptualizing connectivity within a graph theoretic framework for studying processes on the Earth's surface has seen increased interest over the last decades [e.g., see reviews - *Phillips et al.*, 2015; *Heckmann et al.*, 2015; *Passalacqua*, 2017 - and references within]. In a graph or network, nodes represent physical locations or state variables and links represent the direction and strength of connectivity or interaction between nodes. In many physical systems, transport takes place by more than one mechanisms, e.g., overland flow and channel flow, and over different transport pathways, e.g., within the channel network and/or within the inter-dispersed set of islands as overland flow, with exchange occurring between these two interconnected systems depending on the magnitude of the system forcing and possibly local conditions. To represent such a process within a network framework requires conceptualizing it as a system of distinct networks, each with different transport properties, and with interaction allowed between networks to accommodate the flux exchange. It is expected that considering the overall system connectivity in this enlarged network perspective will result in emergent transport properties and dynamics not possible to decipher by analyzing each network separately, and therefore revealing key information essential to predict the system response under changing forcing.

In recent years, a new framework that generalizes the traditional representation of networks to the so-called multilayer networks was introduced [*Mucha et al.,* 2010*; De Dominico et al.*, 2013; *Boccaletti et al.*, 2014; *Kivela et al.*, 2014]. A multilayer network represents the different connectivities arising from various processes as distinct networks (layers) but allows at the same time to represent interactions between separate layers by introducing interlayer links (across process interactions). The application of this framework has spanned diverse disciplines ranging from social networks (e.g., propagation of information across different social media platforms [*Cozzo et al.*, 2013]); to transportation networks (e.g., transportation in a multiplatform system – subway and bus – [*De Domenico et al.,* 2014]) and biochemical networks (e.g., spreading of two diseases, which interact cooperatively [*Sanz et al.*, 2014]), to name a few. Despite the enormous potential for the application of this framework to the study of diverse surface or sub-surface processes, to the best of our knowledge it has not been utilized yet in the earth sciences community. In this paper, we introduce this conceptual framework and illustrate it in the case of transport in river deltas, demonstrating its potential to capture emergent overall system behavior that arises from complex smaller-scale interactions.

River deltas are depositional landforms forming downstream of major rivers when sediment-laden water slows down as it enters a body of standing water. Deltas contain nutrient-rich sediments that support agriculture, they are rich in oil and hydrocarbon deposits, and provide a variety of environmental services. However, many major deltas are losing land because of the combined effects of (i) sediment deprivation due to dams and levees construction, (ii) accelerated subsidence due to soil compaction exacerbated by groundwater and/or oil extraction, and (iii) rising sea levels [*Syvitski et al.,* 2005; *Ericson et al.,* 2006; *Blum and Roberts,* 2009, *Syvitski et al.*, 2009; *Giosan et al.*, 2014]. Deltas consist of a network of channels that tiles their surface and that are surrounded by islands (inter channel areas) that are regularly inundated by river flooding and tides. The channel networks transport water, solids (e.g., sediment) and solutes (e.g., nutrients) across the delta top, maintain biotic and abiotic diversity, create islands that trap sediment and enhance land building potential, provide corridors for the transport of goods and services, and create an orderly arrangement of water-land patches that absorb or amplify external perturbations. As such, substantial progress in understanding deltaic systems can be made by studying the structure and function of their channel networks [*Smart and Moruzzi, 1971; Morisawa 1985; Tejedor et al.,* 2015a,b; *Tejedor et al.,* 2016; *Tejedor et al.,* 2017]. However, it is well-known that water and sediment fluxes are not only confined to the delta channel network. There are event-related, seasonal and permanent (e.g., close to the delta shoreline) water and sediment exchanges between channels and islands. *Passalacqua* [2017] described this exchange of fluxes between channels and islands in deltas as a "leaky network" of channels and islands. For example, field measurements on the distal part of the Wax Lake delta in coastal Louisiana show that around 23-54% of the water flux that flows into the feeder channel enter the islands through secondary channels and subaqueous levees [*Hiatt and Passalacqua*, 2015; 2017].

Traditional single network approaches are not equipped to handle the connectivity between the channel network (channelized flow) and islands (overland flow) for sediment and water fluxes, with each network having different topologies and transport properties. In fact, the channel–island *hydrological connectivity* in some cases can significantly affect the travel time within the delta, due to the different timescales of transport occurring in the channels vs. the islands. For example, by using tracer experiments in the distal part of the Wax Lake delta, *Hiatt and Passalacqua* [2015] reported that the travel time through the islands can be three times slower than its counterpart travel time through the channel network (channels ~ 4.4 hours; islands ~ 14.3 hours). Also, the tracer particles can stay longer (~3.8 days) in the islands because of the influence of tides and winds [*Hiatt and Passalacqua,* 2015; *Sendrowski and Passalacqua,* 2017].

In this paper, we present the mathematical framework required to integrate the different connectivities into one mathematical object called multilayer network. In multilayer networks, each layer consists of a distinct network, which characterizes the connectivity of a different process acting on the system (e.g., for the channel-island dynamics in river deltas, one layer accounts for the channel connectivity and another layer represents the connectivity that arises from overland flow on islands). Since the processes occurring in each layer are not independent of each other and across process interactions are expected, connectivity across layers is accounted for by the existence of *interlayer connections* (e.g., acknowledging flux exchanges between islands and channels).

II. The mathematical framework: From single- to multi- layer connectivity

Tejedor et al. [2015a] showed that a delta channel network connecting the delta apex to the shoreline can be abstracted as a directed graph $G = (\mathcal{V}, \mathcal{E})$, defined as sets of N vertices $\mathcal{V} = \{v_i\}$, i=1,…, N (also called nodes), and E edges (or links) $\mathcal{E} = \{(uv)\}$, where $u, v \in \mathcal{V}$, and the ordered pair (uv) denotes an edge starting at vertex u (parent) and ending at vertex v (child). Specifically, for delta channel networks, the edges represent channels, and vertices correspond to the locations where one channel splits into new channels (bifurcation), or where two or more channels merge into a single channel (junction).

The connectivity structure of the delta graph can be uniquely specified by a binary square NxN matrix called *Adjacency matrix* (A), whose entry a_{uv} is 1 if there exists a (vu) edge, that is a channel from vertex v to vertex u (a_{uv}=1), and 0 otherwise. A more general version of the A matrix is called the Weighted Adjacency matrix, W, where its entries w_{uv} are non-negative numbers that quantify the strength of the connectivity between nodes v and u. For deltas, we define w_{uv} as the fraction of the flux present at node v that flows through the channel (vu) [*Tejedor et al.*, 2015a]. The advantage of this representation is that once all the connectivity information is encoded in a matrix, by simple algebraic operations we can extract important information on the structural patterns of the channel network (topology) as well as the steady state flux distribution that emerges when the weights assigned to the edges are suitably chosen to be representative of the flux partition at each bifurcation (e.g., proportional to channel widths).

From the Adjacency matrix alone, a different matrix called *Laplacian* L can be derived, which is central for the analysis of many properties of graphs. Here, we define the Laplacian as $L = S - W$, where S is the NxN diagonal matrix with diagonal entries $s_{vv} = \sum_{u=1}^{N} w_{uv}$, i.e., the sum of the weights of all the edges leaving node v. Note that we denote here by L what is generally known as the out-Laplacian [*Tejedor et al.*, 2015a].

The representation of multilayer networks requires a generalization of the previously defined *Adjacency* and *Laplacian* matrices. Specifying the connectivity of traditional single-layer networks (referred to herein as monoplex to indicate the special case of a multilayer network with only one layer), only requires two indices per edge (parent and child node), which makes matrices a suitable representation of networks. For multilayer networks two indices are not enough, since we also need to specify the layers where each of the two nodes connected by an edge belongs to. Tensors are the natural generalization of matrices when a higher dimensionality is required (in fact matrices are tensors of second order – the order of a tensor can be thought of as the number or indices needed to specify its entries). Consequently, we define the multilayer Adjacency tensor $\mathcal{M}$ whose entries $M_{uv}^{\alpha\beta}$ denote an edge starting at node *v* at layer β and ending on node *u* in layer α [*De Dominico et al.*, 2013].

There is a specific subclass of multilayer networks called multiplex networks (hereafter referred to as multiplex) [*De Dominico et al.*, 2013; *Gomez et al.,* 2013; *Tejedor et al.*, 2018], wherein each layer consists of the same set of nodes but possibly different topologies (set of edges) and layers interact with each other only via counterpart nodes in each layer (Fig. 1a). We are especially interested in the multiplex because: (1) they are relevant to networks that are embedded in space, where interactions across layers are not expected to happen between distant nodes but only between counterpart nodes in the different layers (e.g., in deltas, the exchange of fluxes between channels and islands occurs locally); (2) as we show below, the limitation of having the interlayer connectivity only among counterpart nodes makes the mathematical representation of multiplex simpler than for other multilayer networks.

Mathematically, a multiplex consisting of *P* layers, where each layer consists of a network formed by the same set of *N* nodes, is described by a tensor $\mathcal{M} = \left[M_{uv}^{\alpha\beta}\right]_{u,v=1,\ldots,N}^{\alpha,\beta=1,\ldots,P}$. Note that $M_{uv}^{\alpha\alpha}$ describes the Adjacency matrix of a monoplex in layer α, $A^{(\alpha)}$. Given the more restrictive definition of multiplex, the entries corresponding to interlayer connectivity are defined as follows: $M_{uv}^{\alpha\beta} = 0$ (different nodes *u* and *v* in different layers α and β), and $M_{uu}^{\alpha\beta} = 1$ (replica nodes *u* in different layers α and β). The simple structure of the interlayer connectivity allows us to project the Adjacency tensor in an *NP*x*NP matrix*, called *supra-Adjacency* matrix, $\mathcal{A}$. For the case of two layers, $\mathcal{A}$ takes the following form:

$$\mathcal{A} = \begin{pmatrix} A^{(C)} & I \\ I & A^{(I)} \end{pmatrix}, \quad (1)$$

where I is the NxN identity matrix. Here, we have used the notion $A^{(C)}$ and $A^{(I)}$, where the superscripts (C) and (I) denote channels and islands, respectively. Hence, the *supra-Adjacency* matrix is a block matrix, where each of the diagonal blocks encodes the intralayer connectivity of the respective layers, and the interlayer connectivity is stored in the off-diagonal blocks. Note that in $\mathcal{A}$ replica nodes are labeled to satisfy $u+kN$ for k=0,1, … P-1. The structure shown in Eq. 1 (identity matrix for the off-diagonal blocks) guarantees that only across layer interactions between replica nodes are permitted.

Equivalently, a *supra-Laplacian* matrix $\mathcal{L}$ can be defined for any multiplex. For the case of two layers, $\mathcal{L}$ is defined as [*Gomez et al.*, 2013; *Tejedor et al.*, 2018]:

$$\mathcal{L} = \begin{pmatrix} D_C L^{(c)} + D_X I & -D_X I \\ -D_X I & D_I L^{(I)} + D_X I \end{pmatrix}, \quad (2)$$

where D_C is the intralayer diffusion coefficient in the channels, D_I the intralayer diffusion coefficient in the islands, D_X is the interlayer diffusion coefficient, and $L^{(C)}$ and $L^{(I)}$ are the out-Laplacian operators of the intralayer connectivity of the respective layers as defined for the monoplex. The nomenclature of the parameters D_C, D_I and D_X as diffusion coefficients is reminiscent of the interpretation of L as the diffusive operator in networks [*Newman*, 2010]. In a more general setting, we can interpret those coefficients as scalars that allow to modify the relative celerity of the process of each layer and the interlayer processes.

This framework allows us to formally formulate questions and provide quantitative answers about the structural and dynamic connectivity of the integrated system. For instance, in a river delta system, where the timescales of transport via channels or overland flow on islands are significantly different, and where the flux exchange between these two transport mechanisms depends on variables such as river discharge, it is interesting to ask under what conditions and through which local interactions (exchanges) the overall system might exhibit accelerated transport not expected by each system alone. Quantifying the system's timescales of response as a function of the coupling (discharge) between layers and comparing this with the timescales of the forcings (e.g., the timescale at which a given water discharge is exceeded) has important implications for the understanding of many biogeomorphic processes (e.g., sediment trapping and delivery of nutrients to the delta top).

III. A Continuous Time Markov Chain (CTMC) as proxy for flux dynamics in river deltas

We use a simple CTMC model to approximate the dynamics and relative timescales for achieving steady state distributions when different values of coupling (flux exchange) are assumed between the channel and island layers. The CTMC relies on several assumptions such as: (i) a constant rate of transition, i.e., the partition of fluxes at a given bifurcation remains constant and proportional to the physical parameters of the network, e.g., channel width; and (ii) the Markovian property, i.e., the downstream direction that a given package of water or sediment particles takes at a given bifurcation depends only on the physical properties of that bifurcation, and not on the trajectory of the package in its journey from upstream. Despite these assumptions, CTMC offers a good first-order approximation of the dynamics of the system.

The negative *supra-Laplacian* -$\mathcal{L}$ (see Eq. 2) can be interpreted as the transition-rate matrix of the CTMC [*Norris,* 1997*; Masuda et al.,* 2017]. The dynamics of the corresponding Continuous Time Random Walk (CTRW) are governed by

$$\dot{\boldsymbol{p}}(t) = -\mathcal{L}\boldsymbol{p}(t), \qquad (3)$$

where the i-th component of $\boldsymbol{p}(t)$ represents the probability that the CTRW visits node i at time t.

If the directed network is strongly connected, a unique stationary distribution of probability $\boldsymbol{p_s}$, referred in the rest of the paper as steady state, exists [see *Tejedor et al.,* 2018 for further details]

$$\mathcal{L}\boldsymbol{p_s} = 0. \qquad (4)$$

The rate of convergence towards the steady state given by $\boldsymbol{p_s}$ is exponential (asymptotically) with rate Re(λ_2), where λ_2 is the eigenvalue with the smallest nonzero real part [*Lodato et al.,* 2007*; Masuda et al.,* 2017*; Tejedor et al.,* 2018]. Equivalently, the time of convergence to steady state, τ, is inversely proportional to the rate of convergence $\left(\tau \propto {}^{1}/_{Re(\Lambda_2)}\right)$. Note that the spectrum of eigenvalues of $\mathcal{L}$ is in general complex since it is not symmetric. Considering the definition of $\mathcal{L}$ (see Eq. 2), its eigenvalue spectra, and more specifically Re(λ_2), depend on the following: (1) the topology of the connectivity of layer 1 – L_1,

(2) the diffusion coefficient of layer 1 -D_1, (3) the topology of the connectivity of layer 2 – L_2, (4) the diffusion coefficient of layer 2 -D_2, and (5) the interlayer diffusion coefficient – D_X.

IV. A multiplex case study: Wax Lake delta

The Wax Lake delta is a river-dominated delta located in the coast of Louisiana, USA. Sub-aerial land developed after the 1970s flood and the delta has been rapidly prograding ever since [*Roberts et al.*, 1997; *Paola et al.*, 2011; *Shaw et al.*, 2013]. Lidar surveys have shown that 83% of the delta top experienced aggradation between 2009 and 2013 [*Wagner et al.*, 2017]. Primary channels transport water and sediment in the delta to the Atchafalaya Bay and secondary channels connect the delta channel network to the island interiors [*Shaw et al.*, 2013].

Using the channel network connectivity of Wax Lake delta – channel layer (C) - together with the island connectivity – island layer (I) – (see Fig. 1b and supplementary information for further details about the connectivity used and a brief discussion of the deltaic system), we examine the timescale of response of this coupled system. Without loss of generality, we have set the value of $D_I = 1$. The value of $D_C = 7$ has been selected in order to generate rates of convergence to steady state for the channel layer that are three times faster than those of the islands, which are compatible with data collected from field campaigns (see *Hiatt and Passalacqua* [2015] - channels ~ 4.4 hours; islands ~ 14.3 hours). The rate of water and sediment exchange between the channels and islands is controlled by hydrologic (e.g., level of water discharge) and eco-geomorphic (e.g., vegetation, existence of secondary channels connecting the channel network to the interior of the islands) attributes. The effect of vegetation is summarized into the value of D_I, i.e., more vegetated islands exhibit a higher roughness, and therefore are expected to have a lower value of the diffusion coefficient D_I. Thus, the value of

D_X is mostly controlled by the discharge level, as here other forcings such as tides and wind are ignored. Note that we assume that the value of D_X is homogeneous across the delta. This assumption is an oversimplification given the existence of secondary channels in some of the islands, gradients in vegetation and connectivity toward the distal part of the deltaic system, etc. and therefore, a spatially explicit modulation of this parameter would make the model more realistic. However, for the sake of simplicity in the presentation of the framework, we assume uniform values of D_X, showing that even in this very simplified scenario, interesting and unexpected system-wide behaviors emerge from the coupled dynamics. This simplification also allows us to demonstrate that the system response described below does not emerge from heterogeneity in the spatial patterns of D_X, but it is intrinsic to the coupled connectivity between the channel and island layers.

By analyzing the behavior of the timescale of convergence of the channel-island system to steady state, τ, as a function of the interlayer coupling, D_X, (Fig. 2a) the existence of four regimes stands out: (1) *Linear:* The dynamics in the channel network and on the islands are effectively decoupled wherein the rate of flux exchange between both layers (D_X) is the limiting factor. In this regime, the timescale of convergence to steady state (τ) decreases linearly as $2D_X$. (2) *Sublinear*: The coupling between channels and islands starts to be more significant but is limited by the slower diffusion process in the islands. Here, an increase in D_X, i.e., a larger water discharge, translates into a sublinear increase in the timescale of convergence to steady state for the overall delta. (3) *Asymptotic*: For very large values of discharge (i.e., $D_X \gg \mathrm{Re}(\lambda_2^I), \mathrm{Re}(\lambda_2^C)$) the two layers are completely coupled. This scenario can occur when the water discharge is large enough to generate sheet flow on the whole system, where the counterpart nodes in the different islands and channels are fully synchronized, behaving as single nodes. (4) *Prime*: This regime, characteristic of multiplex with directed connectivity in at least one of its layers [*Tejedor et al.*, 2018], occurs for intermediate values of coupling (discharge - D_X), wherein the rate of convergence in the overall system achieves the shortest timescale, even shorter than in the asymptotic regime. Physically, this regime can be interpreted as levels of discharge that produce rates of channel-island flux exchange similar to the rates characteristic of channel transport ($D_X \sim \mathrm{Re}(\lambda_2^c)$). Thus, the islands and channels contribute significantly to the total transport but conserving a relative degree of independence in their internal dynamics (i.e., not fully synchronized or decoupled).

It is important to notice that although the parameter that controls the flux exchange between the channel and island layers, D_X, is solely interpreted in terms of water discharge, island roughness (e.g., due to vegetation) has been shown to effectively play a fundamental role in the water exchange between islands and channels [*Hiatt and Passalacqua*, 2017]. The multiplex framework allows us to easily assess the effect of different island roughness (mediated by the value of D_I) in the system-wide response. When different scenarios of increasing island roughness ($D_I = 1$, 0.5 and 0.1) are explored for a constant $D_C = 7$ (Fig. 2b), the transition from the linear to the sublinear regime shifts to smaller values of D_X (discharge), acknowledging the fact that the increased island roughness (i.e., lower value of D_I) makes the transport on islands the limiting factor for a larger range of discharges. The immediate consequence of this shift in the transition from linear to sublinear is that the system-wide response is significantly slowed down for the same values of discharge under increased roughness scenarios. Finally, it is also interesting to note the shift in the position of the minimum timescale of convergence to steady state, τ, towards higher values of discharge (D_X) when higher values of island roughness (lower values of D_I) are explored. Thus, for example, to achieve the shortest timescale of response of the system in high roughness scenarios (D_I =0.1), a 20% increase in the discharge (D_X) is necessary when compared to the case of low roughness (D_I =1).

Depending on the discharge levels (for given values of D_I and D_C), the delta multiplex behaves as a channel-dominated system or a coupled channel-island complex. The geomorphic (e.g., island aggradation) and biogeochemical (e.g., vegetation types, nutrient nourishment and nitrogen fixation) consequences of operating in one or the other scenario are apparent. However, to fully evaluate the overall system behavior, there are three relevant timescales associated with a discharge Q (Fig. 3) that should be taken into consideration: t_d^Q - timescale associated with the duration of the forcing; i.e., the time during which the value of Q is exceeded; t_r^Q - time of recurrence of the forcing of magnitude Q; and t_S^Q - timescale of response of the channel-island delta system when both layers are coupled by a discharge level Q. The multiplex framework allows to put into perspective these three timescales. Thus, the delta as a whole would be efficient in redistributing sediments and nutrients, (i.e., it behaves as a channel-island complex), if $t_S^Q \le t_d^Q$, i.e., the time of response of the system is comparable with the duration of the forcing. Thus, a deltaic system is resilient, i.e., it exhibits aggradation rates that are fast enough to self-maintain the delta and the ecosystem services that it provides, if its overall delta connectivity has evolved to a state wherein the prime regime of transport: (1) emerges for water discharge Q (interlayer coupling) values with a recurrence time t_r^Q that is short enough to allow periodic redistribution of fluxes at the delta scale and, (2) characterized by small values of t_S^Q (i.e., comparable in magnitude with the timescale of the transport in the channels).

V. Conclusions and Perspectives

To investigate transport properties of multi-process multi-scale connected systems we introduce the framework of multilayer networks which allows to quantify properties of the system as a whole, not accessible by studying each system separately. We illustrate this framework by examining the flux dynamics in a river delta system, where channelized (within the channel network) and overland (on the islands) flows are considered. We represent the delta system as a two-layer multiplex, wherein each layer consists of the same number of nodes, but the connectivity among them is different and representative of each process. The degree of coupling among layers denotes the flux exchange in-between the two transport processes and is mostly driven by the discharge level, although a strong control is also exerted by the relative roughness of the islands (e.g., vegetation). To illustrate the potential of this framework, we investigate the timescale of convergence to the steady state flux distribution for different degrees of coupling, revealing the existence of four different regimes: linear, sublinear, prime and asymptotic. We highlight that the prime regime, where the timescale of convergence to steady state achieves its smallest value, occurs for intermediate values of coupling, i.e., not extreme values of discharge, where the redistribution of sediment and nutrients is the fastest across the delta top, enhancing the overall system aggradation and nourishment.

The application of this framework to specific systems in a more detailed manner opens up interesting research questions such as (1) what is the return period of the discharge that corresponds to the optimal coupling (1-year event, 10-year event, etc.) and how does it affect the evolution of those systems and their resilience to extreme events, (2) what specific locations of a delta might amplify across-process connectivity critically affecting the overall system transport timescales; and (3) how is the system transport timescale dependent on including more or less refined specification of across-process connectivity? For instance, by accounting for vegetation, topography, etc., more layers can be included, with islands of similar characteristics (i.e., islands that can be modeled by a similar diffusion coefficient) grouped in the same layer. Finally, we want to emphasize the broad applicability of this framework to diverse fields in the geosciences where multi-process multi-scale interactions dictate the overall system behavior. Examples include flux transport taking into account surface-subsurface exchange [*Sawyer et al.*, 2015], integrated wetland and river systems [*Hansen et al.*, 2017] and interaction types among species in ecological systems [*Pilosof et al.*, 2017], etc.

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FIGURES

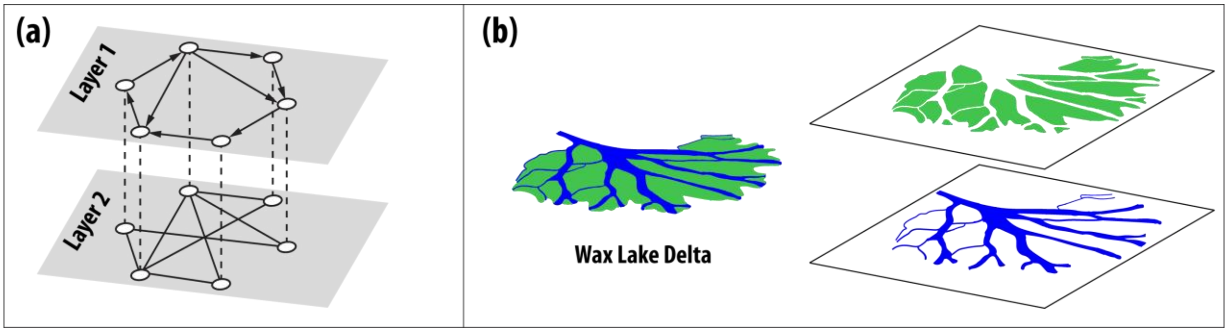

Figure 1. Delta Multiplex (a) *Illustration of a multiplex:* Multiplex are coupled multilayer networks where each layer consists of the same set of nodes but possibly different topologies (set of links) and layers interact with each other only via replica nodes in each layer (dashed lines) **(b)** *Wax Lake Delta Multiplex.* Illustration of the Wax Lake delta in the Louisiana coast (USA). The delta multiplex consists of two layers: Layer 1 (Bottom) accounts for the channel connectivity, and Layer 2 (Top) represents the connectivity that arises from overland flow on islands. For more details about the multiplex structure see supplementary materials.

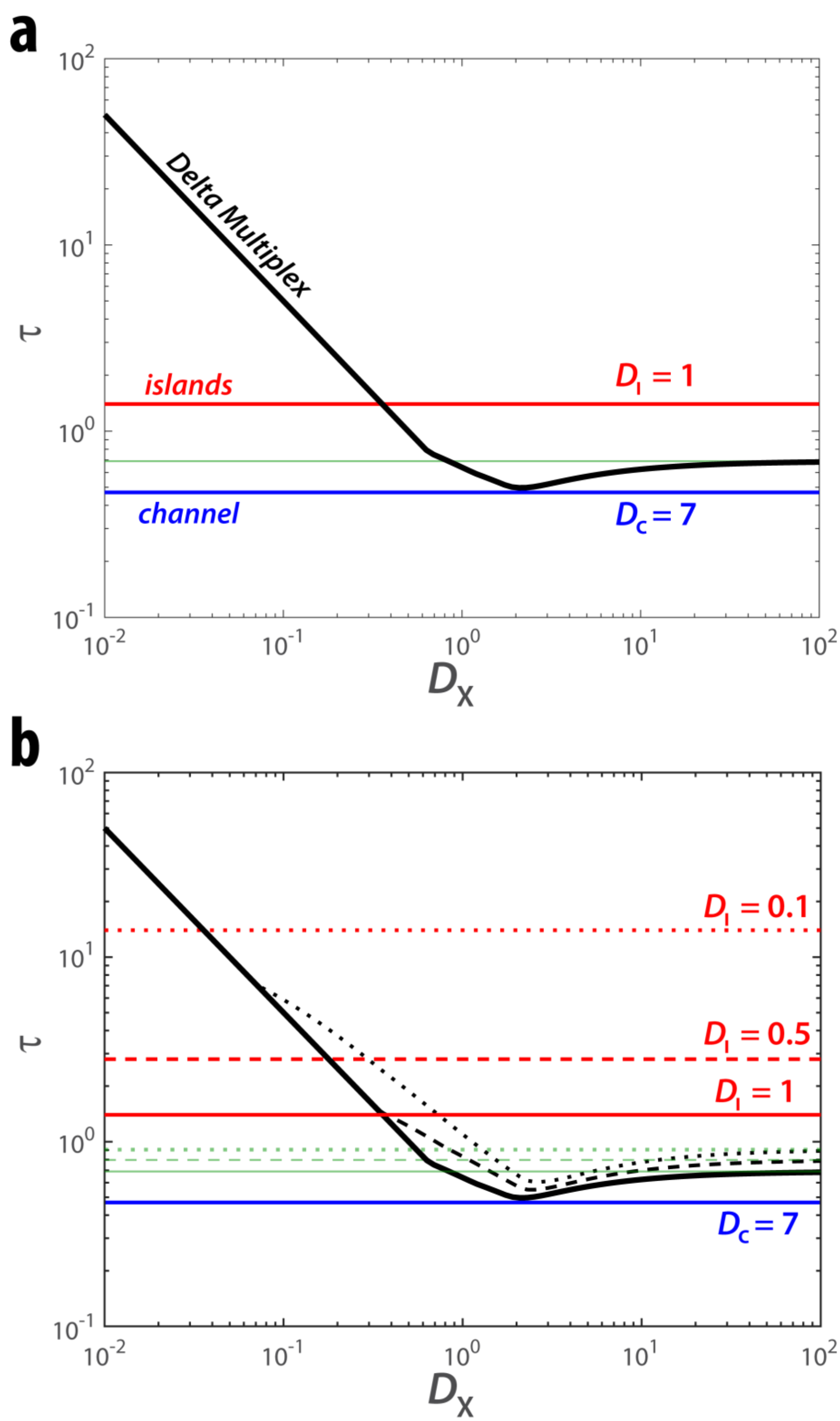

Figure 2. Flux Dynamics on the Wax Lake Delta Multiplex. We show for the Wax Lake multiplex, the timescale of convergence to steady state, τ, as a function of interlayer coupling,

D_X, which is mostly controlled by water discharge. Panel (a) shows the emergence of a non-monotonic behavior of τ as function of D_X, when the values of diffusivity of each layer are set to (D_C, D_I) =(7, 1) to reproduce the ratio of timescales of transport channel to island reported from field campaigns. Panel (b) shows the effect of island roughness in the response timescale of the delta multiplex. For intermediate values of D_X, the timescale of the delta multiplex τ decreases for higher island roughness - D_I= 1 (solid lines), 0.5 (dashed lines) and 0.1 (dotted lines) - reducing effectively the coupling between the channels and islands for the same values of D_X.

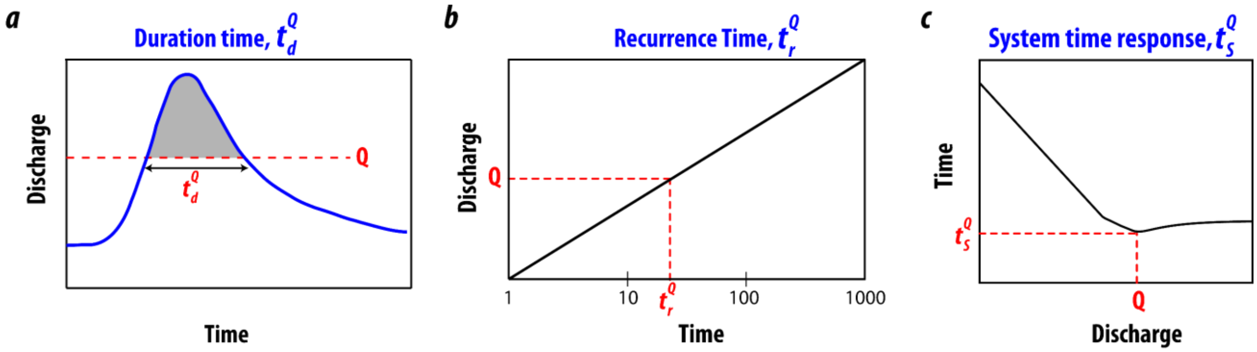

Figure 3. The timescales associated with discharge *Q*. (a) t_d^Q - timescale associated with the duration of the forcing; i.e., the time during which the value of Q is exceeded; **(b)** t_r^Q - time of recurrence of the forcing of magnitude Q; and **(c)** t_S^Q - timescale of response of the channel-island delta system (see Fig. 2) when both layers are coupled by a discharge level Q.

Supplementary Information

Description of the delta multiplex

The Wax Lake multiplex delta consists of two layers: one layer accounts for the Delta Channel Network (DCN) and another layer represents the connectivity that arises from overland flow on islands – Delta Island Network (DIN). The connectivity across layers is accounted for by the existence of *interlayer connections*, acknowledging flux exchanges between islands and channels (see Fig. 1 in main text).

Delta Channel Network (DCN)

We utilized the outline of the Wax Lake delta structure processed by *Edmonds et al.* [2011] and identified 56 nodes and 59 links (Figure S1 – left panel). All the information about the connectivity (including directionality of the links) and the relative widths of the channels of the DCN is stored in the Weighted Adjacency matrix, W^C, which is attached as a file "WeightedAdjacencyMatrixDCN.dat" in the Supplementary Materials.

Delta Island Network (DIN)

The DIN consists of the same set of nodes in the DCN (56 nodes) but has different links. An example of the DIN on one of the islands is shown in Fig. S1 (right panel). The links of the DIN are directed and oriented in the downstream direction. The weights were computed assuming that the strength of the connectivity is inversely proportional to the linear distance between nodes within an island. The Weighted Adjacency matrix for the DIN, W^I, is attached as a file "WeightedAdjacencyMatrixDIN.dat" in the Supplementary Materials. The entries, w_{uv}, of the W^I are non-negative numbers and quantify the strength of the connectivity between nodes v and u. For the DIN, w_{uv} accounts for the fraction of the flux present at node v that flows on the island (overland flow) to node *u*.

Interlayer links only exist between counterpart nodes in the DCN and DIN.

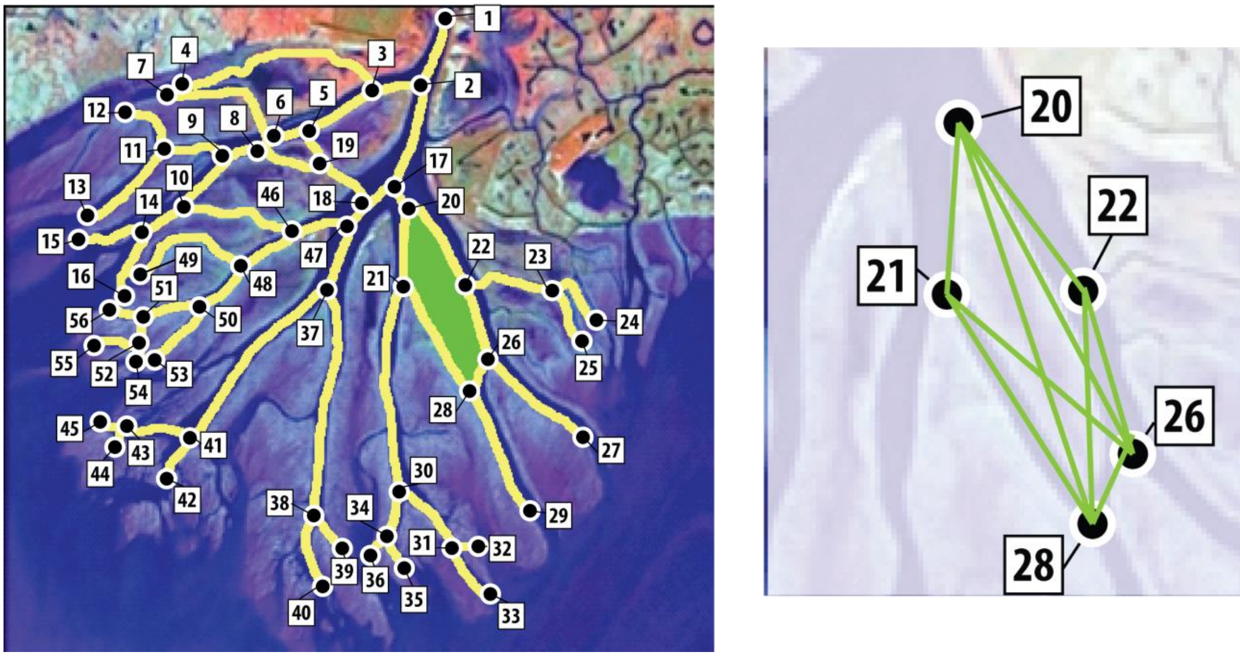

Figure S1. Wax Lake Multiplex Network. Wax Lake delta in the Louisiana coast. **(Left panel)** Delta Channel Network (DCN): The DCN (nodes as black circles and links as yellow lines) is superimposed on the aerial view of the delta (photo obtained in 2005 by the National Center of Earth-surface Dynamics, NCED). **(Right panel)** Delta Island Network (DIN): The DIN is displayed for one of the islands (island colored in green in the left panel). The islands network consists of the same set of nodes in the DCN, but the set of links (e.g., green lines) is different.

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