

1 **Transformer-based Reconstruction of Canopy Profiles from Large-Footprint**

2 **Waveform LiDAR**

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11 **Abstract**

12 Spaceborne laser scanning (SLS) presents a cost-effective means for frequent, global-
13 scale monitoring of forest ecosystem parameters. Compared to airborne laser scanning
14 (ALS), SLS offers substantially greater spatial coverage and revisit frequency, but at the
15 cost of larger footprints, sparser sampling, and attenuated return signals. These
16 constraints typically result in a loss of fine-scale vertical canopy structure in large-
17 footprint waveform LiDAR, thereby limiting the retrieval of ecologically meaningful forest
18 structural metrics. To address this challenge, we developed an encoder–decoder
19 Transformer architecture to reconstruct high-resolution vertical canopy profiles from
20 large-footprint waveform LiDAR observations. Using waveform data acquired by NASA’s
21 Land, Vegetation, and Ice Sensor (LVIS) – a high-altitude ALS instrument commonly
22 used as a proxy for spaceborne missions – we trained the model to recover fine-scale
23 canopy structure by leveraging overlapping ALS point clouds as reference data. The
24 proposed Transformer leverages long-range vertical dependencies within waveform
25 signals to infer canopy structural details that are degraded or unresolved in large-
26 footprint, high-altitude observations. Results show that the proposed approach
27 substantially improves the agreement between LVIS-derived and ALS-derived canopy
28 structural complexity metrics, increasing correlations from $R = 0.62$ to 0.84 and from $R =$
29 0.76 to 0.90 for two representative metrics. This framework is readily transferable to
30 current and future SLS missions, enabling the retrieval of super-resolved vertical
31 canopy profiles and supporting large-area assessment of ecologically meaningful
32 canopy structural metrics.

33 **Keywords** – Full-waveform LiDAR, Transformer, Canopy Structural Complexity,
34 Generative Reconstruction.

35 **1. Introduction**

36 Light detection and ranging (LiDAR) is the leading technology for accurate mapping
37 of three-dimensional (3D) canopy structure (Coops et al., 2021). In this context, aerial
38 laser scanning (ALS) provides dense, high-resolution measurements and has been
39 widely used to derive ecologically meaningful canopy structural metrics across large,
40 forested landscapes (Coops et al., 2021; Siddiqui et al., 2025). However, ALS
41 acquisitions are infrequent and spatially limited due to their high cost, thus restricting
42 their utility for routine, global-scale monitoring and change detection (Hancock et al.,
43 2021). Spaceborne laser scanning (SLS), on the other hand, offers continental to global
44 coverage with frequent revisit intervals at substantially lower cost (Hancock et al.,
45 2021), but at the expense of sparser sampling and reduced vertical resolution, which
46 limits the retrieval of detailed canopy structural metrics. To overcome these limitations,
47 prior studies have estimated canopy structural complexity (CSC) – a metric capturing
48 both horizontal and vertical variability – by statistically linking SLS waveforms from
49 NASA’s Global Ecosystem Dynamics Investigation (GEDI) (Dubayah et al., 2020) to
50 overlapping ALS-derived CSC and extrapolating these relationships to global scales (Liu
51 et al., 2024; de Conto et al., 2024). Despite their scalability, such regression-based
52 approaches have achieved only weak-to-moderate performance ($R^2 = 0.58\text{--}0.68$),
53 thereby underscoring the need for methods that more directly recover fine-scale canopy
54 structure from SLS observations.

55 The need for higher-resolution SLS observations extends beyond CSC estimation.
56 Spaceborne LiDAR measurements with finer spatial sampling and vertical resolution
57 would support a broad range of vegetation-focused applications, including precision
58 agriculture, commercial forestry, wildfire fuel and hazard assessment, biodiversity and
59 habitat mapping, and monitoring of deforestation and forest degradation (Zhao et al.,
60 2018; Donellan et al., 2021). Upcoming full-waveform satellite LiDAR missions such as
61 the Earth Dynamics Geodetic Explorer (EDGE) (UC San Diego, 2026), are poised to
62 deliver denser sampling than all previous SLS missions combined (Harding et al., 2021;
63 Yang et al., 2024), opening the door to canopy structural mapping at spatial scales
64 previously attainable only with ALS. However, recovering fine-scale vertical canopy
65 structure from SLS will remain challenging due to signal attenuation, background noise,
66 and reduced sensitivity to weak returns at orbital altitudes (Chu, 2006; Lang et al.,
67 2022), all of which can complicate the separation of canopy and ground signals within
68 large-footprint waveforms (Lang et al., 2022). To address these limitations, we designed
69 an encoder-decoder Transformer network to effectively enhance the vertical resolution
70 of large-footprint waveforms acquired by NASA's Land Vegetation and Ice Sensor
71 (LVIS) (NASA Goddard Space Flight Center, 2025a), a high-altitude ALS system
72 commonly used as an analog for satellite LiDAR missions, including calibration and
73 validation of GEDI products (NASA GSFC, 2025b; Hancock et al., 2019). LVIS operates
74 at a nominal altitude of ~10 km and carries two instruments – LVIS Facility (LVIS-F),
75 with footprint diameters of 7–10 m, and LVIS-Classic (LVIS-C), with larger footprints of
76 20–25 m (NASA GSFC, 2025a). We leveraged LVIS-F waveforms in this study because
77 this instrument's footprint size closely matches the anticipated specifications for

78 upcoming missions (Yang et al., 2024). The high operating altitude also results in
79 reduced vertical resolution relative to conventional low-altitude ALS, i.e., it closely
80 emulates SLS conditions. Densely sampled ALS point clouds overlapping LVIS-F
81 footprints are used as ground truth for training the proposed Transformer model.

82 Recent deep-learning approaches to reconstructing and super-resolving spaceborne
83 LiDAR observations have emulated orbital acquisition by synthetically downsampling
84 high-resolution airborne data (Ramirez-Jaime et al., 2024; Porras-Diaz et al., 2024;
85 Ramirez-Jaime et al., 2025). Although these studies modeled selected aspects of the
86 orbital sensing process, such as Gaussian beams and footprint aggregation,
87 retrospective degradation of low-altitude, cross-track scanning ALS point clouds cannot
88 fully reproduce the radiometric and geometric characteristics of real spaceborne
89 acquisitions. The simulated inputs remain conditioned on the sampling geometry and
90 return statistics of the source ALS data, whereas orbital waveforms are shaped by near-
91 nadir viewing geometry, substantially longer propagation distances, atmospheric
92 transmission, the transmitted pulse and receiver impulse response, detector noise, and
93 the sensitivity-dependent loss of weak canopy and ground returns (Hancock et al.,
94 2019; Lang et al., 2022). Moreover, because the models are trained and evaluated
95 using observations generated by a prescribed synthetic degradation operator, they may
96 learn to invert artifacts specific to that operator rather than the physical degradation
97 encountered by an actual spaceborne sensor. Research on deep-learning based image
98 super-resolution has shown that models trained using synthetic degradations can
99 generalize poorly when those degradations differ from real acquisition conditions (Gu et
100 al., 2019). By contrast, the LVIS waveforms used here are actual full-waveform

101 measurements acquired near the upper boundary of the mid-latitude troposphere. Their
102 two-way optical paths therefore traverse most of the tropospheric column, which
103 contains approximately 75–80% of atmospheric mass, nearly all water vapor and
104 clouds, and most atmospheric aerosols (Wei et al., 2019; Dhaka & Kumar, 2023). The
105 measured LVIS returns consequently incorporate much of the atmospheric path-
106 dependent scattering and attenuation encountered by spaceborne observations,
107 providing a more physically grounded analog of orbital acquisition than synthetically
108 downsampled low-altitude ALS. Finally, previous studies evaluated reconstruction
109 fidelity through generic similarity and error measures, canopy-height profiles, height
110 percentiles, canopy height models (CHMs), and digital terrain models (DTMs) (Ramirez-
111 Jaime et al., 2024; Porras-Diaz et al., 2024; Ramirez-Jaime et al., 2025). Such
112 evaluations do not establish whether the reconstructed data preserve higher-order
113 canopy structural metrics with demonstrated relevance to ecosystem processes and
114 applications, thereby lacking an application-specific performance baseline.

115 We addressed these gaps in prior research by deriving two ecologically relevant
116 CSC metrics from the reconstructed waveforms produced by the best-performing
117 models and assessing their agreement with corresponding metrics derived from
118 overlapping ALS data. This evaluation framework moves beyond generic similarity
119 measures and directly tests whether the proposed reconstruction approach recovers
120 fine-scale vertical canopy structure in a form that preserves ecologically meaningful
121 structural information. The main contributions of this study are as follows:

122 1) We introduce a novel encoder–decoder Transformer architecture for reconstructing
123 high-resolution vertical canopy profiles from degraded, high-altitude full-waveform

124 LiDAR measurements. When applied footprint by footprint to the dense,
125 overlapping sampling patterns anticipated for upcoming spaceborne missions, the
126 reconstructed profiles could be spatially integrated to generate contiguous three-
127 dimensional representations with substantially finer structural detail than the
128 original waveforms.

129 2) We evaluate reconstruction fidelity using two established CSC metrics derived
130 from the reconstructed profiles and their ALS reference counterparts, thereby
131 determining whether the recovered vertical structure preserves higher-order
132 canopy characteristics relevant to ecosystem monitoring rather than merely
133 reproducing the overall waveform shape.

134 3) We compare the proposed reconstruction framework with conventional waveform
135 deconvolution methods, testing whether it more effectively resolves fine-scale
136 vertical canopy features and improves the agreement of derived CSC metrics with
137 those obtained from ALS reference data.

138 **2. Materials and methods**

139 **2.1 Study site and dataset**

140 LVIS and the National Ecological Observatory Network's (NEON) Airborne
141 Observation Platform (AOP) (Kampe et al., 2010) acquired co-located LiDAR data over
142 the Smithsonian Environmental Research Center (SERC), Maryland, USA, in August
143 2021, providing an ideal dataset for evaluating waveform-based canopy profile
144 reconstruction against dense ALS reference data. SERC is particularly well suited for
145 this study because it is a temperate, humid Mid-Atlantic site dominated by deciduous

146 forests, with a mean canopy height of approximately 38 m (NEON, n.d.). The site's tall,
147 closed-canopy hardwood stands, including dominant species such as tulip poplar,
148 American beech, and American sweetgum, provide vertically complex forest structure
149 for testing whether fine-scale canopy information can be recovered from high-altitude
150 waveform measurements (NEON, n.d.).

151 NASA's Gulfstream-V aircraft carried both the LVIS-C and (~25 m footprint) and
152 LVIS-F (~10 m footprint) instruments over SERC on August 5, 2021, as part of a
153 campaign to collect calibration and validation data for GEDI vegetation products over
154 dense forested sites (NASA GSFC, 2025b). The aircraft collected 15 cm range-
155 resolution full-waveform LiDAR data at a nominal altitude of 12 km, covering an
156 approximately 2 km swath and producing an average footprint spacing of ~10 m along
157 and across track (Cornejo et al., 2024). For each laser shot, the latitude and longitude of
158 the corresponding footprint were geolocated with decimeter-level accuracy (NASA
159 GSFC, 2025a) and the elevation at each sampling bin were referenced to the WGS-84
160 ellipsoid (Blair and Hofton, 2020a).

161 Small-footprint ALS data were acquired over SERC in August 2021 during NEON
162 AOP flight campaigns using an Optech Galaxy Prime discrete-return LiDAR scanner
163 (Teledyne Optech, n.d.). Flights were conducted at a nominal altitude of 1000 m above
164 ground level (AGL) in flight lines with 37% overlap (product DP1.30003.001; NEON,
165 2026). The transmitted laser pulses had a beam divergence of 0.25 mrad ($1/e^2$),
166 corresponding to an approximate nadir footprint diameter of 0.25 m at 1000 m AGL. The
167 average pulse density was 38.5 pulses/m², and up to five discrete returns were
168 recorded per pulse. The resulting point clouds had an average density of 56 points/m²,

169 with each point geolocated with an absolute horizontal accuracy of ~ 10 cm (1σ)
170 (Teledyne Optech, n.d.) and an absolute vertical accuracy of less than 0.15 m root
171 mean square error (*RMSE*) (NSF NEON, 2026). The discrete-return LiDAR data were
172 distributed in LAS 1.3 format, with horizontal coordinates referenced to a UTM map
173 projection and ITRF00 datum and elevations referenced to Geoid12A. The point clouds
174 were processed and classified using LAStools v. 230901 (Isenburg, 2023) and provided
175 as classified 1×1 km tiles.

176 **2.1.1 LVIS data processing**

177 LVIS-F shots were filtered to retain only those overlapping forested 30 m pixels in
178 the National Land Cover Database (NLCD) 2021 land-cover product (Dewitz, 2023) in
179 order to restrict the analysis to forested areas only. The raw received LVIS-F waveforms
180 were preprocessed to remove noise and isolate the signal portion as follows:

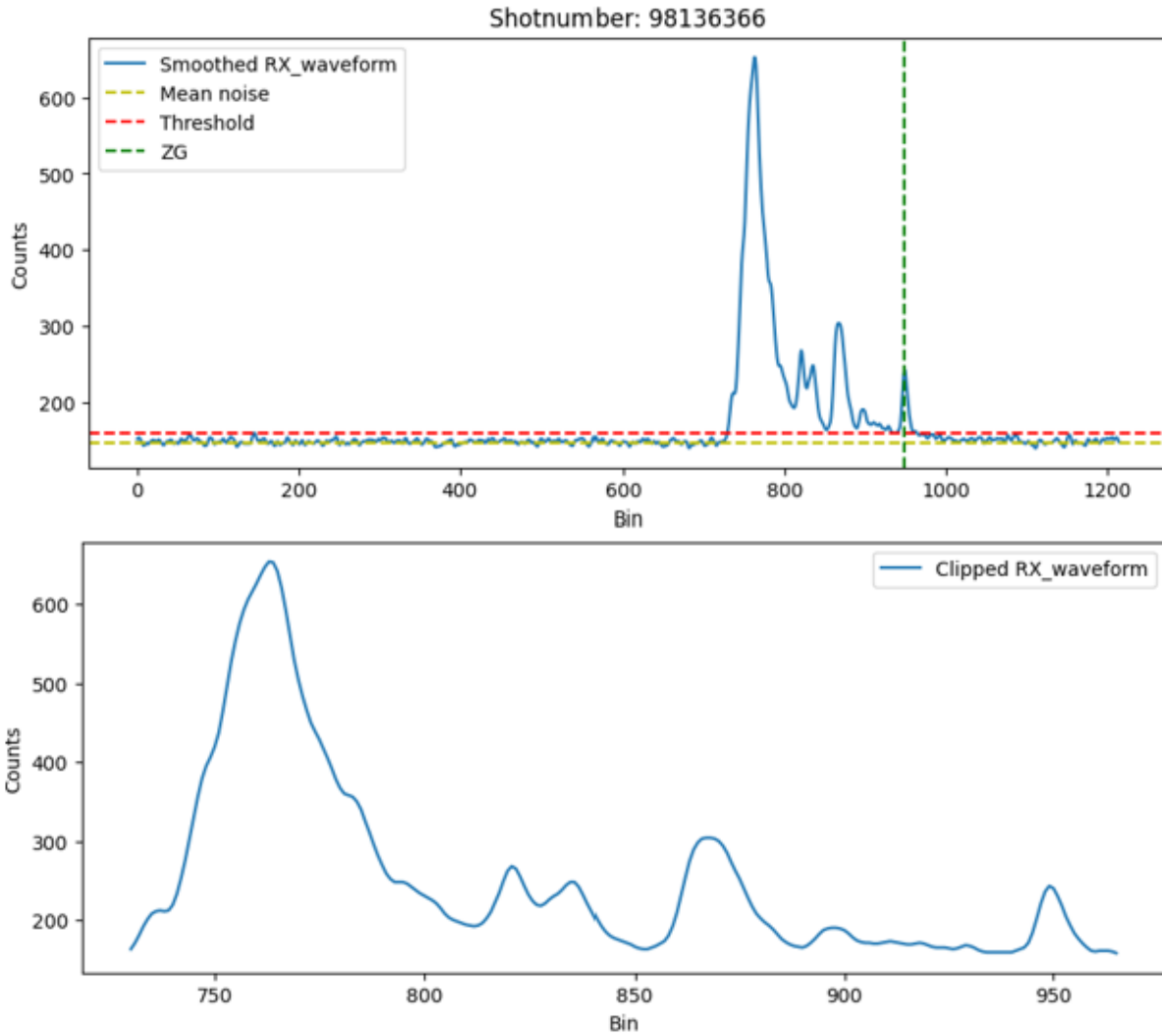
- 181 1) Each waveform was smoothed using a Gaussian kernel to suppress high-
182 frequency noise (Blair and Hofton, 2020b).
- 183 2) The mean noise level and standard deviation, provided within the LVIS-F L1B
184 data product (Blair and Hofton, 2020a), were used to compute a signal threshold
185 according to:

$$186 \quad T_{signal} = \mu_{noise} + 4 * \sigma_{noise}$$

187 where T_{signal} is the signal threshold, μ_{noise} is the signal threshold, and σ_{noise} is the
188 noise standard deviation.

- 189 3) The smoothed waveform was then clipped to retain the waveform segment
190 exceeding T_{signal} , thereby isolating the canopy and ground return signal from

191 empty, noise-dominated bins (Fig. 1). These processed waveforms served as the
192 input profiles for the proposed reconstruction framework.

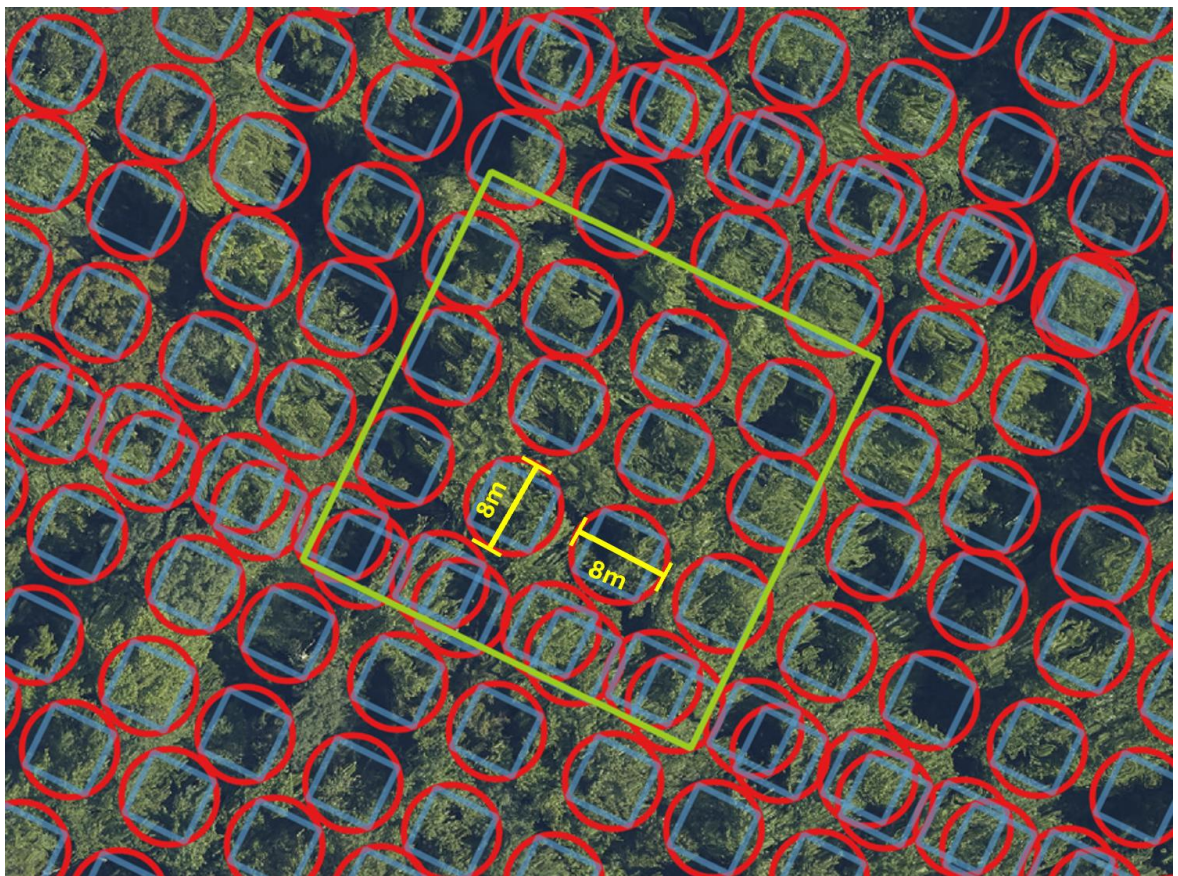


193
194 **Fig. 1. Top:** Smoothed LVIS-F received waveform. The mean noise level and assigned
195 ground elevation (ZG) are provided with the data product. **Bottom:** Corresponding
196 clipped waveform after applying the signal threshold.

197 2.1.2 ALS data processing

198 Classified ALS point clouds were clipped into square vertical columns spatially
199 coincident with the filtered LVIS-F shots, with each column centered on the geolocated

200 ground coordinates of the corresponding LVIS-F footprint (Fig. 2). Square columns were
201 used to provide a canopy structural representation consistent with prior work deriving
202 ecologically meaningful canopy structural complexity metrics from LiDAR return
203 columns (Atkins et al., 2018; Gough et al., 2019; LaRue et al., 2019; Siddiqui et al.,
204 2025). Because LVIS-F footprints range from approximately 7–10 m in diameter, 7 × 7
205 m columns were extracted to prevent the inclusion of ALS returns outside the LVIS-F
206 footprint extent. Clipping was performed using the clip_roi function in the lidR package
207 (Roussel et al., 2020).



208

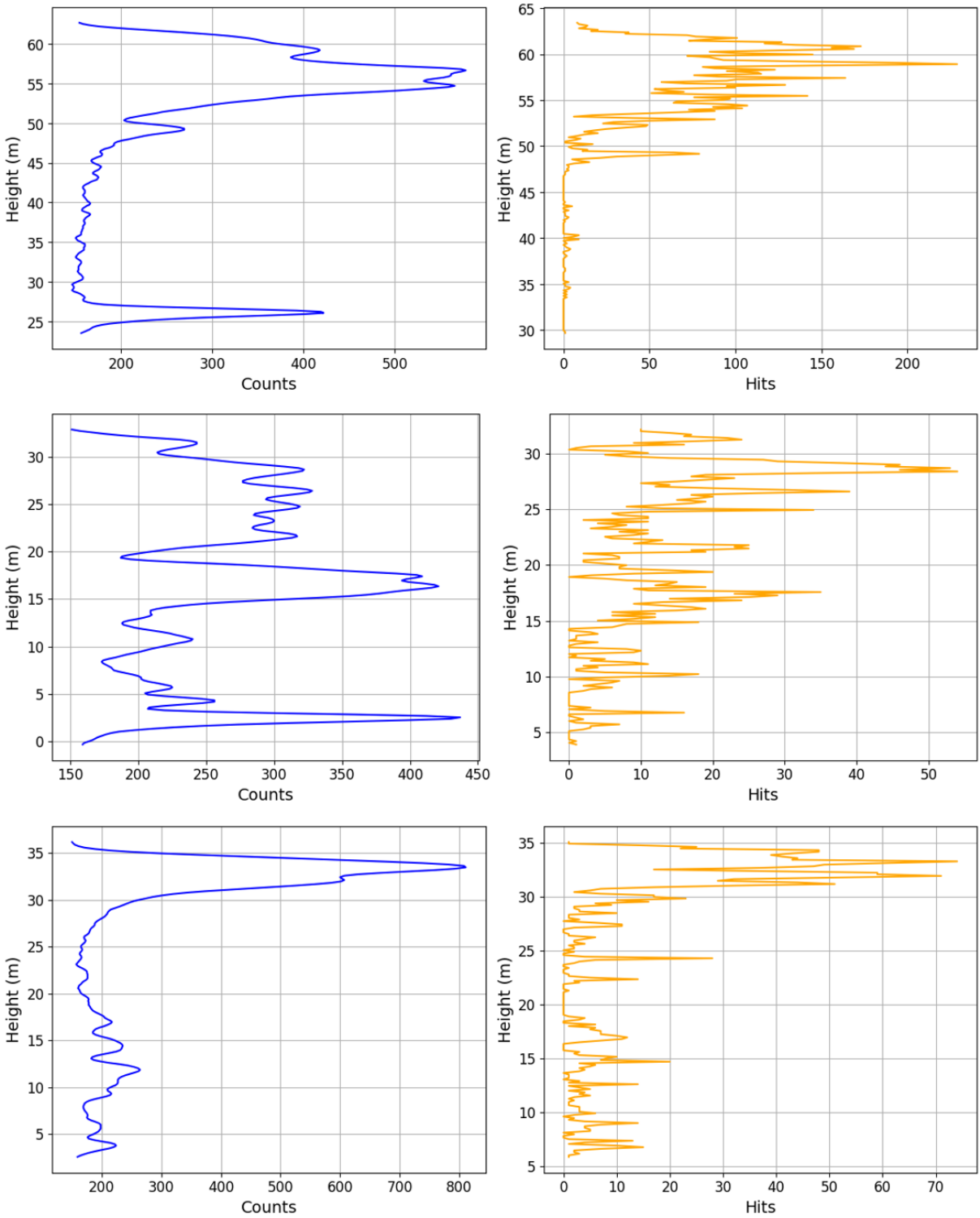
209 **Fig. 2.** LVIS-F shots (red circle) and overlapping AOP point cloud columns (blue square)
210 at a forested area in SERC.

211 A series of filtering steps was then applied to the ALS columns to retain high-quality
212 forest canopy samples:

- 213 1) Columns containing points outside the ground, vegetation, or noise classes were
214 excluded.
- 215 2) Columns for which the combined number of medium- and high-vegetation returns
216 did not exceed the number of low-vegetation points were removed to eliminate
217 non-forest or sparsely vegetated samples.
- 218 3) Points classified as ground, low vegetation, or noise were removed, leaving only
219 medium- and high-vegetation returns for canopy-profile reconstruction.

220 For applications requiring joint reconstruction of ground and canopy structure, this
221 final point-level filtering step can be omitted. After these sequential filtering steps,
222 101,110 ALS columns with corresponding LVIS-F waveforms remained for analysis. The
223 large number of co-located LVIS–ALS profile pairs is advantageous for the proposed
224 Transformer-based approach, providing sufficient samples for model training, validation,
225 and independent testing.

226 Finally, the ALS returns within each 7×7 m column were aggregated into 0.15 m
227 vertical bins to match the vertical sampling interval of the LVIS-F waveforms. These
228 vertically binned ALS return-count profiles served as the reference canopy profiles for
229 training and evaluating the waveform reconstruction models. Figure 3 shows three
230 representative LVIS–ALS waveform pairs from the final dataset.



231

232 **Fig. 3.** Co-located LVIS-F waveforms (left) and corresponding ALS-derived canopy
 233 profiles (right). In the top pair, the LVIS-F waveform contains a pronounced ground-

234 return peak at approximately 26 m, which is absent from the ALS canopy profile
235 because ground returns were removed during preprocessing. In the lower two pairs, the
236 ALS profiles reveal finer within-canopy and subcanopy structure between approximately
237 5 and 25 m that is not resolved as clearly in the corresponding LVIS-F waveforms.

238 **2.2 Transformer architecture for canopy profile reconstruction**

239 We frame canopy profile reconstruction as a cross-sensor sequence-to-sequence
240 learning problem in which a large-footprint LVIS-F waveform is mapped to a high-
241 density ALS-derived canopy profile. The input consists of time-varying LVIS-F return
242 amplitudes at 0.15 m intervals and includes canopy, near-ground, and ground signals.
243 The target consists of canopy-only ALS return counts aggregated at the same vertical
244 interval but containing substantially finer structural detail. Thus, although the input and
245 target share a common vertical coordinate system, they represent measurements with
246 markedly different sensing characteristics and information content. This formulation is
247 analogous to one-dimensional signal super-resolution (Iversen et al., 2025) and
248 sequence-transduction problems such as machine translation, in which an input
249 sequence is transformed into a distinct output representation (Vaswani et al., 2017).

250 An encoder-decoder Transformer is well suited to this problem for three primary
251 reasons. First, the encoder can accommodate waveforms of varying length through
252 padding and attention masking, allowing the model to process differences in the vertical
253 extent of the measured signal. Second, self-attention (Vaswani et al., 2017) enables the
254 encoder to capture dependencies among vertically separated LVIS-F waveform bins,
255 while decoder self-attention models relationships among output height positions within
256 the reconstructed canopy profile. Together, these mechanisms allow the model to
257 interpret each input and output position in the context of the full vertical sequence.

258 Third, decoder cross-attention (Vaswani et al., 2017) allows each output height query to
 259 condition its prediction on the most informative portions of the encoded LVIS-F
 260 waveform. This mechanism enables the model to infer canopy structure that may be
 261 attenuated, blended, or unresolved in the original large-footprint observation. The
 262 principal components of the proposed architecture are described in the following
 263 subsections.

264 **2.2.1 Input embedding**

265 The input embedding module maps each bin of a variable-length LVIS-F waveform
 266 to a fixed-dimensional token representation suitable for Transformer processing. Each
 267 token incorporates both the height of the bin and the corresponding waveform
 268 amplitude. Height values are encoded using learnable Fourier features, inspired by
 269 Tancik et al., 2020, with $K = 32$ frequency components. For a height value h (in meters),
 270 the encoding is:

$$271 \quad \gamma(h) = [\sin(\boldsymbol{\omega}h) \parallel \cos(\boldsymbol{\omega}h) \parallel \tilde{h}]^T \in \mathbb{R}^{2K+1}$$

272 where $\boldsymbol{\omega} \in \mathbb{R}^K$ contains log-spaced angular frequencies corresponding to wavelengths
 273 from 0.3 to 50 m, enabling the model to capture both fine-scale vertical variation and
 274 broader canopy-scale patterns, and $\tilde{h} = h/50$ is the normalized height. These features
 275 are projected through a multi-layer perceptron (MLP) to obtain the height embedding:

$$276 \quad \mathbf{e}_h = \text{MLP}(\gamma(h)) \in \mathbb{R}^d.$$

277 The LVIS-F digital counts, which represent the recorded return-signal amplitude,
 278 were encoded using three complementary features. For a waveform with counts $\mathbf{c} =$
 279 $[c_1, \dots, c_N]^T$ at N height bins, the features for each bin i are:

$$280 \quad \phi_i^{\text{shape}} = \frac{c_i}{\max_{j=1}^N(c_j)}, \quad \phi_i^{\text{intensity}} = \frac{\log(c_i + 1)}{\log(C_{\max} + 1)}, \quad \phi_i^{\text{energy}} = \frac{\log(\sum_{i=1}^N c_i + 1)}{\log(S_{\max} + 1)}$$

281 where C_{\max} and S_{\max} are the global maximum count and maximum waveform sum
 282 across the training dataset, respectively. The shape feature represents the relative
 283 waveform shape, the log-scaled intensity feature preserves absolute count magnitudes
 284 at each bin while compressing the wide dynamic range of LiDAR returns, and the
 285 energy feature represents the total waveform brightness. The first two features vary
 286 across height bins, whereas the waveform energy feature was shared across all bins in
 287 a given waveform. Additionally, two optional bin-wise features may be included: the local
 288 gradient, which helps identify peaks, slopes, and plateaus; and the distance from peak,
 289 which encodes the relative position of each bin with respect to the waveform's
 290 maximum. Descriptions and equations for these features are provided in the Appendix.
 291 These features are concatenated and projected through an MLP to produce the count
 292 embedding $\mathbf{e}_c^{(i)} \in \mathbb{R}^d$. The resulting count and height embeddings are then combined via
 293 element-wise addition, followed by layer normalization (Xiong et al., 2020), yielding the
 294 final input embedding:

$$295 \quad \mathbf{z}_i = \text{LayerNorm}(\mathbf{e}_h^{(i)} + \mathbf{e}_c^{(i)})$$

296 **2.2.2 Encoder**

297 The encoder processes variable-length LVIS full-waveform inputs and produces
 298 contextualized representations that capture both local signal characteristics and long-
 299 range structural patterns. It comprises four stacked encoder layers, each containing a
 300 multi-head self-attention module with four attention heads, followed by a position-wise

301 feed-forward network (FFN). The embedding dimension is $d = 128$, and the FFN hidden
302 dimension is $d_{ff} = 512$. Pre-normalization is employed, wherein layer normalization is
303 applied before each sub-layer, to improve optimization stability during training (Xiong et
304 al., 2020). Dropout regularization is applied after the attention and feed-forward
305 operations, and residual connections bypass each sub-layer. Padding masks ensured
306 that padded bins introduced during batching did not contribute to the attention
307 calculations. For a given input sequence $\mathbf{Z} = [\mathbf{z}_1, \dots, \mathbf{z}_N]^T \in \mathbb{R}^{N \times d}$, each encoder layer
308 computes:

$$309 \quad \mathbf{Z}' = \mathbf{Z} + \text{Dropout}(\text{SelfAttn}(\text{LayerNorm}(\mathbf{Z})))$$

$$310 \quad \mathbf{Z}'' = \mathbf{Z}' + \text{Dropout}(\text{FFN}(\text{LayerNorm}(\mathbf{Z}')))$$

311 The self-attention mechanism allows every height bin to incorporate information from
312 all other valid bins in the waveform, enabling the encoder to represent complex vertical
313 dependencies across the full return signal. For example, interception and scattering by
314 upper-canopy foliage influence the amount of laser energy reaching and returning from
315 lower canopy strata. Self-attention may therefore use information from upper-canopy
316 bins to contextualize attenuated returns at lower heights.

317 **2.2.3 ALS Output Queries and Decoder**

318 The decoder generates ALS return predictions through a set of learnable queries
319 anchored to a fixed height grid spanning the ALS vertical range. For our dataset, the
320 output grid extended from 1 to 80 m at 0.15 m intervals, yielding 526 output positions.
321 Each output query combines two components: (1) a learnable content embedding that
322 captures height-specific predictive characteristics, and (2) a height embedding produced

323 using the same Fourier-feature mapping applied to the LVIS-F input heights.
 324 Representing input and output positions within a common height-encoding space
 325 facilitates alignment between the LVIS-F waveform and the corresponding ALS canopy
 326 profile during cross-attention.

327 The decoder consists of four stacked pre-normalized Transformer layers, each with
 328 four attention heads. Each layer applies multi-head self-attention among the output
 329 queries, multi-head cross-attention to the encoded LVIS-F waveform, and a position-
 330 wise feed-forward network. Let $\mathbf{X}^{(l)} \in \mathbb{R}^{N_q \times d}$ denote the output-query representations
 331 entering decoder layer (l), where N_q is the number of output height positions, and let
 332 $\mathbf{E} \in \mathbb{R}^{N_s \times d}$ denote the encoder output for an LVIS-F waveform. The operations within
 333 each decoder layer are:

$$334 \quad \mathbf{S}^{(l)} = \mathbf{X}^{(l)} + \text{Dropout}(\text{MHSA}(\text{LN}_1(\mathbf{X}^{(l)})))$$

$$335 \quad \mathbf{C}^{(l)} = \mathbf{S}^{(l)} + \text{Dropout}(\text{MHCA}(\text{LN}_2(\mathbf{S}^{(l)}), \mathbf{E}, \mathbf{E}; \mathbf{M}))$$

$$336 \quad \mathbf{X}^{(l+1)} = \mathbf{C}^{(l)} + \text{FFN}(\text{LN}_3(\mathbf{C}^{(l)}))$$

337 where *MHSA* denotes multi-head self-attention, *MHCA* denotes multi-head cross-
 338 attention, *LN* denotes layer normalization, and \mathbf{M} is the encoder padding mask. In the
 339 self-attention operation, the normalized output-query representations serve as the
 340 queries, keys, and values, allowing information to be exchanged among all output
 341 heights. During cross-attention, the normalized decoder representations serve as
 342 queries, whereas the encoded LVIS-F representations serve as keys and values. This
 343 allows each output height query to selectively combine information from the observed
 344 waveform while preventing padded LVIS-F positions from contributing to the

345 reconstruction. After the final decoder layer, an additional layer-normalization operation
346 produces the decoder output, $\mathbf{D} = \text{LN}_{\text{final}}(\mathbf{X}^{(L)})$.

347 Because reconstruction at a given height can depend on canopy structure both
348 above and below that position, non-causal (bidirectional) attention was used in the
349 encoder and decoder. The final decoder representations are subsequently passed to
350 the prediction head, which maps the contextualized embedding at each height position
351 to the parameters used to estimate the ALS return distribution.

352 **2.2.4 Count prediction head**

353 The decoder output representations are mapped to discrete ALS return count
354 predictions using a two-layer feed-forward network. Instead of directly regressing
355 counts, we model the output as a Negative Binomial (NB) (Ross & Preece, 1985)
356 distribution to account for the overdispersion characteristic of ALS return counts, where
357 variance frequently exceeds the mean due to the sparse, peaky nature of discrete-
358 return waveforms. For a decoder output vector $h \in \mathbb{R}^d$ at a given height bin, a hidden
359 representation is first computed through two fully connected layers with GELU
360 activations and dropout:

$$361 \quad \mathbf{f}_1 = \text{Dropout}(\text{GELU}(\mathbf{W}_1 \mathbf{h} + \mathbf{b}_1))$$

$$362 \quad \mathbf{f}_2 = \text{Dropout}(\text{GELU}(\mathbf{W}_2 \mathbf{f}_1 + \mathbf{b}_2))$$

363 The NB distribution parameters, mean μ and dispersion r , are then predicted as:

$$364 \quad \mu = \text{softplus}(\mathbf{w}_\mu^\top \mathbf{f} + b_\mu), \quad r = \text{softplus}(\mathbf{w}_r^\top \mathbf{f} + b_r)$$

365 Here, $\mathbf{W}_1 \in \mathbb{R}^{(d \times d/2)}$, $\mathbf{W}_2 \in \mathbb{R}^{(d/2 \times d/2)}$, and $\mathbf{w}_\mu, \mathbf{w}_r \in \mathbb{R}^{(d/2)}$ are learnable parameters, ,
 366 where $d_h = d/2$ is the hidden dimension. GELU denotes the Gaussian Error Linear Unit
 367 activation (Hendrycks & Gimpel, 2016, and the softplus function, $\text{softplus}(x) = \log(1 +$
 368 $e^x)$, ensures positivity of both parameters. Under this parameterization, the predicted
 369 variance is $\text{Var}(y) = \mu + \mu^2/r$, allowing the model to capture count distributions ranging
 370 from near-Poisson (large r) to highly overdispersed (small r). The predicted ALS return
 371 count at each height bin is given by μ .

372 2.2.5 Loss functions

373 Training is guided by a composite objective designed to optimize three
 374 complementary aspects of canopy-profile reconstruction: return count fidelity, similarity
 375 of the overall profile shape, and suppression of spurious predictions outside the height
 376 range of the reference ALS profile. For each sample, the valid region is defined as the
 377 set of output-grid positions whose heights fell within the vertical range of the
 378 corresponding reference ALS profile. The zero region comprised all remaining positions
 379 in the fixed output grid outside this range. The three loss functions are:

- 380 1. **Negative Binomial Negative Log-Likelihood:** The primary loss term maximizes
 381 the likelihood of observed ALS counts under the predicted NB distribution. For
 382 target count y with predicted parameters μ and r , this loss is:

$$383 \mathcal{L}_{\text{NB}} = \log\Gamma(r) + \log\Gamma(y + 1) - \log\Gamma(y + r) + r \log\left(\frac{r + \mu}{r}\right) + y \log\left(\frac{r + \mu}{\mu}\right)$$

384 where $\log\Gamma(\cdot)$ denotes the natural logarithm of the gamma function. The loss is
 385 averaged over all bins in the valid regions of the training batch.

386 2. **Shape Similarity Loss:** To encourage recovery of the relative vertical distribution
387 of ALS returns independently of their absolute magnitude, cosine distance is
388 computed between the predicted mean profile and the reference profile:

$$389 \mathcal{L}_{\text{shape}} = 1 - \frac{\mu^T y}{\|\mu\| \|y\|}$$

390 All output positions contributed to this loss, but positions within the valid profile
391 range were assigned a weight of 1.0, whereas positions outside this range were
392 assigned a reduced weight of 0.1. This weighting allows predictions in the empty
393 regions to influence the profile shape loss without dominating it, while the
394 separate zero-region penalty provides the primary mechanism for enforcing near-
395 zero predictions outside the valid range. The loss was calculated independently
396 for each profile and then averaged across the batch.

397 3. **Zero-Region Penalty:** To suppress spurious predictions above and below the
398 height range of the reference ALS profile, an ℓ_1 penalty is applied to the
399 predicted mean counts in the zero region:

$$400 \mathcal{L}_{\text{zero}} = \frac{1}{|\mathcal{Z}|} \sum_{i \in \mathcal{Z}} |\mu_i|$$

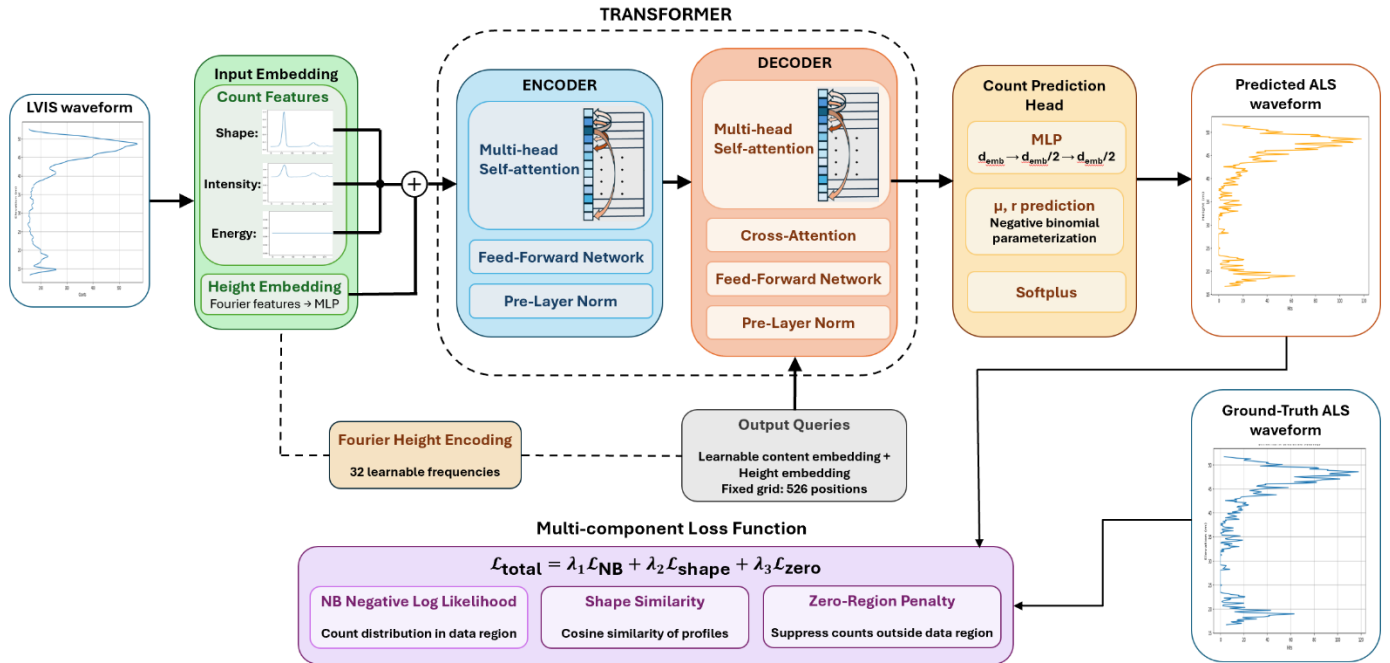
401 where \mathcal{Z} denotes the set of output positions outside the valid target region.

402 Separating the losses over valid and zero regions allows the model to optimize
403 count fidelity and profile shape within the reference canopy profile range while
404 independently penalizing unsupported predictions elsewhere in the output grid.

405 The total training objective is given by the weighted sum of these losses:

$$406 \mathcal{L}_{\text{total}} = \lambda_1 \mathcal{L}_{\text{NB}} + \lambda_2 \mathcal{L}_{\text{shape}} + \lambda_3 \mathcal{L}_{\text{zero}}$$

407 where λ_1 , λ_2 , and λ_3 control the relative contributions of the three loss terms. Figure 4
 408 presents a schematic overview of the complete Transformer architecture.



409

410 **Fig. 4.** Transformer-based canopy profile reconstruction architecture. Example input
 411 LVIS and ground-truth ALS waveforms are shown from a real paired observation. The
 412 predicted ALS waveform is illustrated by recoloring the ground truth for visualization
 413 purposes.

414 2.3 Training setup

415 The paired LVIS–ALS dataset was partitioned randomly into training, validation, and
 416 test sets using an 80/10/10 split ($n = 80,888 / 10,111 / 10,111$ waveform pairs). The
 417 model architecture employed an embedding dimension of 128, four encoder and
 418 decoder layers, four attention heads per layer, and a feed-forward hidden dimension of
 419 512. Loss weights were set to $\lambda_1 = 1.2$ (NB loss), $\lambda_2 = 0.4$ (shape similarity), $\lambda_3 = 1.0$
 420 (zero-region penalty) in the initial training run and were tuned in subsequent runs based
 421 on validation set performance. Models were trained for a maximum of 100 epochs with

422 a batch size of 32, and early stopping was triggered after 15 consecutive epochs
423 without improvement in the validation correlation between the predicted and reference
424 ALS profiles. Optimization was performed using Adam with decoupled weight decay
425 regularization (Loshchilov and Hutter, 2017). The initial learning rate was set to 1×10^{-4} ,
426 and a cosine-annealing-with-warm-restarts schedule was used to periodically reduce
427 and restart the learning rate during training (Loshchilov and Hutter, 2017). This schedule
428 was selected to promote stable convergence while allowing the optimizer to move away
429 from suboptimal local solutions following each restart. Gradient clipping with a maximum
430 norm of 1.0 was applied to ensure training stability.

431 The following dataset-level statistics are required to construct the input height and
432 waveform amplitude features and create the output height grid: LVIS-F input height
433 range, ALS reference height range, maximum LVIS-F waveform amplitude, and
434 maximum LVIS-F cumulative waveform energy. Training was conducted on an NVIDIA
435 A100 GPU with 40 GB memory using computing resources provided by the Rochester
436 Institute of Technology (Rochester Institute of Technology, 2026). A typical training run
437 required approximately four hours. The complete codebase, including the model
438 architecture, training scripts, and data preprocessing pipelines, is available in the
439 repository provided in the data availability statement to facilitate replication and future
440 extension of this work.

441 **2.4 Evaluation**

442 Reconstruction performance was evaluated using two complementary pooled
443 correlation measures. Correspondence in absolute return count distributions was
444 quantified by concatenating all predicted canopy profiles within an evaluation subset

445 into a single vector and similarly concatenating the corresponding ALS reference
446 profiles. Pearson's correlation coefficient (R) was then calculated between the pooled
447 vectors. Profile shape correspondence was assessed using max-normalized correlation
448 (R_n). Each predicted profile and its corresponding ALS reference profile were
449 independently divided by their respective maximum values before concatenation, after
450 which Pearson's correlation was calculated between the resulting normalized vectors.
451 Thus, R reflects correlation between the unnormalized vertical return distributions and
452 remains sensitive to variation in return magnitude among samples, whereas R_n
453 emphasizes similarity in profile shape independently of absolute return counts.

454 During model development, validation set R was used as the sole criterion for
455 hyperparameter tuning and checkpoint selection. Within each training run, the
456 checkpoint attaining the highest validation set R was retained. The hyperparameter
457 configuration achieving the highest overall validation set R was designated as the best-
458 performing model and applied to the held-out test subset for final evaluation. The pooled
459 R , R_n , and the canopy structural complexity metrics described below were then
460 calculated on the test predictions.

461 **2.4.1 Evaluation of CSC metrics**

462 To assess whether the best-performing model preserved ecologically meaningful
463 canopy structure, foliage height diversity (FHD) (Clawges et al., 2008) and vertical
464 canopy rugosity (VCR) (Siddiqui et al., 2025) were independently derived from the
465 predicted and ALS reference profiles for every sample in the test subset. The same
466 metrics were also derived from the corresponding LVIS-F input waveforms to provide a
467 baseline performance without reconstruction. For each metric, agreement with the ALS-

468 derived values was quantified using Pearson's R , and prediction error was measured
469 using $RMSE$. Performance of the reconstructed profiles was then compared with the
470 LVIS-F baseline to determine whether the proposed model improved recovery of ALS-
471 derived canopy structural complexity.

472 This evaluation extends beyond pooled profile similarity by directly testing whether
473 the reconstructed vertical return distributions preserve higher-order structural attributes
474 relevant to ecological applications. Previous research has shown that the spatial
475 variability of FHD and VCR metrics is strongly predictive of plot-level primary production
476 in temperate forests (Siddiqui et al., 2025). FHD has also been widely used to
477 characterize habitat complexity and biodiversity (Clawges et al., 2008; Simonson et al.,
478 2014) and to assess changes in vertical forest structure during succession (van Ewijk et
479 al., 2011).

480 **2.4.2 Metric-aware training using FHD and VCR losses**

481 After the best-performing model had been selected using validation set R , additional
482 experiments were conducted to determine whether directly incorporating FHD and VCR
483 into the training objective could improve the reconstruction of these metrics.

484 Differentiable FHD and VCR loss terms were implemented following their established
485 formulations (Clawges et al., 2008; Siddiqui et al., 2025) (see Appendix).

486 The architecture and training hyperparameters of the best-performing model were
487 held constant, while the inclusion and weights of the metric-specific loss terms were
488 varied. Three metric-aware objectives were investigated: (1) the baseline objective
489 supplemented with an FHD loss, (2) the baseline objective supplemented with a VCR

490 loss, and (3) the baseline objective supplemented with both FHD and VCR losses.
491 Within each training run, the checkpoint achieving the highest validation set R was
492 retained. The metric-aware configuration achieving the highest validation set R was
493 then evaluated on test subset using pooled R , R_n , together with Pearson's R and $RMSE$
494 for the predicted FHD and VCR values. These results were compared with those of the
495 baseline model to determine whether metric-aware optimization improved the recovery
496 of ecologically relevant canopy structure without degrading overall profile
497 reconstruction.

498 **2.4.3 Comparison with waveform deconvolution algorithms**

499 Deconvolution techniques have been widely applied to LiDAR waveforms to recover
500 fine-scale vertical features by reducing system and environmental contributions that
501 broaden or obscure the received signal (Wu et al., 2011). Among these techniques,
502 Richardson–Lucy (RL) and Gold have demonstrated strong performance for recovering
503 vegetation cross sections, detecting unresolved echoes, and deriving forest structural
504 parameters relative to alternative methods such as Wiener filtering and nonnegative
505 least squares (Wu et al., 2011; Zhou et al., 2017). Gold and RL were therefore selected
506 as representative conventional deconvolution benchmarks for comparison with the
507 proposed reconstruction framework. Their implementation was adapted from Bhatta et
508 al. (2026), who optimized both algorithms for LVIS waveforms, and the corresponding
509 code is available through the repository provided in the data availability statement.

510 Because LVIS-F waveforms and ALS reference profiles represent different sensing
511 domains, profile-level performance was compared using the max-normalized pooled
512 correlation, R_n , defined above, together with the corresponding pooled root mean

513 square error ($RMSE_n$). Each unprocessed (input) LVIS-F waveform, Transformer-
514 reconstructed profile, deconvolved waveform, and ALS reference profile was
515 independently divided by its maximum value before concatenation across the held-out
516 test subset. The following comparisons were then conducted: (1) unprocessed LVIS-F
517 waveforms versus ALS reference profiles, which served as the input baseline; (2)
518 Transformer-reconstructed profiles versus ALS reference profiles; (3) Gold-deconvolved
519 waveforms versus ALS reference profiles; and (4) RL-deconvolved waveforms versus
520 ALS reference profiles. This shape-based comparison reduces the influence of
521 differences in absolute magnitude between LVIS waveform amplitudes and ALS return
522 counts and is consistent with the FHD and VCR analyses, which derive structural
523 information from the normalized vertical distribution of returns.

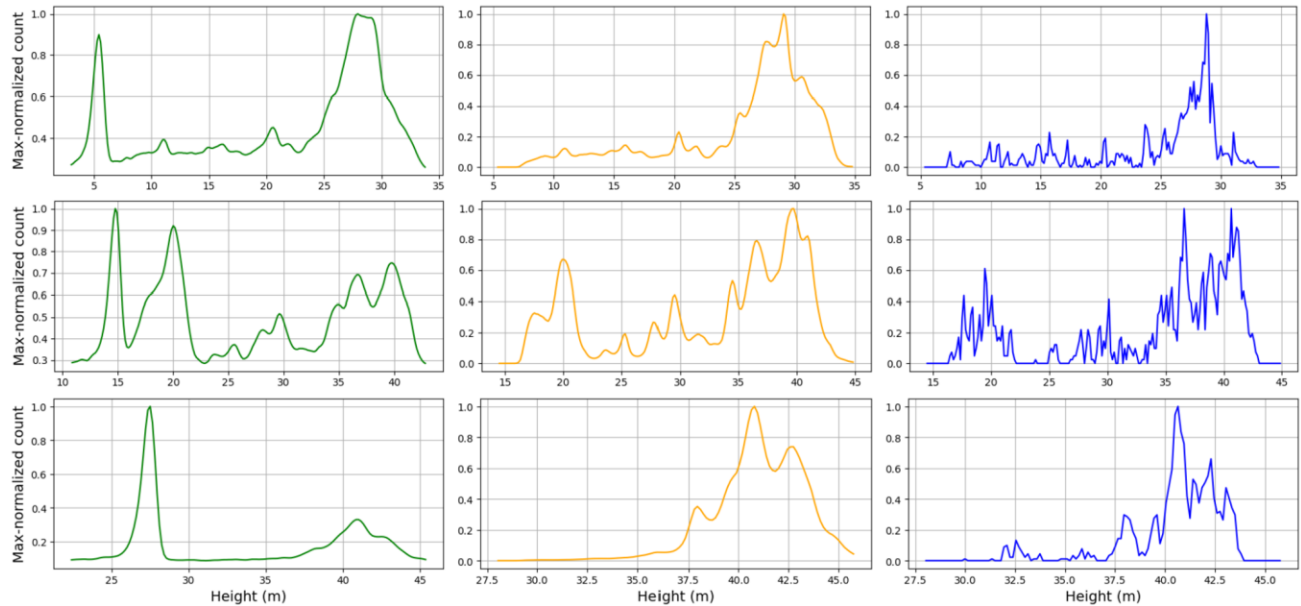
524 Recovery of canopy structural complexity was evaluated using the same four profile
525 sources. FHD and VCR were independently derived from the unprocessed LVIS-F
526 waveforms, Transformer predictions, Gold-deconvolved waveforms, and RL-
527 deconvolved waveforms for each test sample. The resulting values were compared with
528 the corresponding ALS-derived FHD and VCR values using Pearson's R and $RMSE$.

529 An important distinction among the methods is that the ALS reference profiles used
530 in this study contain only canopy returns, whereas both the original and deconvolved
531 LVIS-F waveforms retain canopy and ground components. Conventional deconvolution
532 can sharpen blurred waveform features but, without an additional ground-identification
533 and removal procedure, does not transform a canopy-plus-ground waveform into a
534 canopy-only profile. In contrast, the proposed supervised model is explicitly trained to
535 perform this cross-domain transformation and is therefore expected to correspond more

536 closely with the canopy-only ALS references. Consequently, the deconvolution results
537 should be interpreted as conventional waveform-processing baselines rather than
538 strictly equivalent canopy-profile reconstruction methods. Applications requiring both
539 canopy and ground information could repeat this comparison using ALS reference
540 profiles that retain ground returns.

541 **3. Results**

542 The best-performing model achieved pooled correlations of $R = 0.67$ and $R_n = 0.80$
543 on the test set. The normalized profile shape correlation therefore increased by 0.07
544 relative to the unprocessed LVIS-F baseline ($R_n = 0.73$). The reconstructed profiles
545 generally recovered the dominant vertical canopy return patterns observed in the ALS
546 references while suppressing the prominent low-elevation ground returns present in the
547 LVIS-F inputs. Several alternative hyperparameter configurations produced comparable
548 validation performance, indicating that similar reconstruction accuracy was attainable
549 across multiple model settings. The complete hyperparameter configuration of the
550 selected model is provided in the Appendix.



551

552 **Fig. 5.** LVIS-F input waveform (left), Transformer-reconstructed canopy profiles (center),
 553 and corresponding ALS reference profiles (right) for three random test samples,
 554 displayed by row. Each profile was independently normalized by its maximum value.

555 **3.1 CSC metric derivation performance**

556 CSC metrics derived from the Transformer-reconstructed profiles showed strong
 557 agreement with those derived from the ALS reference profiles. FHD values achieved R
 558 = 0.90 and $RMSE = 0.20$, while VCR values achieved $R = 0.84$ and $RMSE = 15.58$ m.
 559 These results represented substantial improvements over metrics derived directly from
 560 the unprocessed LVIS-F waveforms, which achieved $R = 0.74$ and $RMSE = 0.65$ for
 561 FHD and $R = 0.62$ and $RMSE = 49.85$ m for VCR. Thus, reconstruction increased the
 562 correlations with ALS-derived VCR and FHD by 0.22 and 0.16, respectively, while
 563 reducing the corresponding RMSE values by approximately 69%.

564 **3.2 Metric-aware training results**

565 Table 1 summarizes the test-set performance of the metric-aware training
 566 experiments. Adding only the FHD loss produced results similar to those of the baseline
 567 model. The max-normalized pooled correlation decreased slightly from $R_n = 0.80$ to $R_n =$
 568 0.79 , while FHD recovery improved marginally from $R = 0.90$ and $RMSE = 0.20$ to
 569 $R=0.91$ and $RMSE = 0.16$. VCR performance remained nearly unchanged.

570 In contrast, objectives containing the VCR loss produced markedly lower profile-
 571 level performance. The highest R_n values were 0.51 for the VCR-only objective and 0.56
 572 for the combined FHD-and-VCR objective, compared with 0.80 for the baseline.
 573 However, direct optimization of VCR produced modest improvements in VCR recovery.
 574 The VCR-only and combined objectives both achieved $R = 0.86$, with $RMSE$ values of
 575 14.65 m and 14.60 m, respectively, compared with $R = 0.84$ and $RMSE = 15.58$ m for
 576 the baseline. These improvements were accompanied by lower FHD accuracy and
 577 substantial degradation in overall profile correspondence. The results therefore indicate
 578 that directly optimizing VCR improved recovery of that metric slightly, but at the cost of
 579 broader canopy-profile reconstruction fidelity.

580 Table 1. Test set performance of the metric-aware training objectives. For each
 581 objective, results are reported for the configuration attaining the highest validation set R .

Training objective	Pooled R ; $RMSE$ (counts)	Pooled R_n ; $RMSE_n$ (%)	FHD R ; $RMSE$	VCR R ; $RMSE$ (m)
Baseline	0.67; 19.66	0.80; 0.18	0.90; 0.20	0.84; 15.58
FHD only	0.66; 19.83	0.79; 0.18	0.91; 0.16	0.84; 15.78
VCR only	0.43; 27.49	0.51; 0.38	0.83; 0.29	0.86; 14.65
FHD + VCR	0.47; 27.47	0.56; 0.36	0.84; 0.28	0.86; 14.60

582

583 3.3 Comparison with waveform deconvolution algorithms

584 Table 2 compares the proposed Transformer reconstruction with the unprocessed
585 LVIS-F baseline and the Gold- and Richardson–Lucy-deconvolved waveforms. The
586 Transformer-reconstructed profiles achieved the highest max-normalized pooled
587 correlation ($R_n = 0.80$), exceeding the unprocessed LVIS-F ($R_n = 0.73$), Gold ($R_n = 0.68$),
588 and RL ($R_n = 0.72$) results. The Transformer also produced the strongest agreement
589 with ALS-derived CSC metrics, achieving $R = 0.90$ for FHD and $R = 0.84$ for VCR.

590 Neither deconvolution algorithm improved CSC metric correlations relative to the
591 unprocessed LVIS-F baseline. Gold produced FHD and VCR correlations of 0.73 and
592 0.62, respectively, while RL produced corresponding correlations of 0.74 and 0.62.
593 These values were nearly identical to, or slightly lower than, those obtained from the
594 unprocessed LVIS-F waveforms. Although Gold and RL achieved lower $RMSE_n$ values
595 than the Transformer, their lower R_n values indicate weaker correspondence in the
596 pooled variation of profile shape across the test subset. Overall, the Transformer
597 provided the clearest improvement in both normalized profile correspondence and
598 recovery of ALS-derived canopy structural complexity.

599 Table 2. Correlation and CSC metric derivation results between ALS reference profiles
600 and unprocessed LVIS-F waveforms, Transformer-reconstructed profiles (baseline
601 model), and deconvolved waveforms on the test set.

Profile source	Pooled R_n ; $RMSE_n$ (%)	FHD R ; $RMSE$	VCR R ; $RMSE$ (m)
----------------	-----------------------------	------------------	-------------------------

Unprocessed LVIS-F waveforms	0.73; 0.18	0.74; 0.65	0.62; 49.85
Transformer-reconstructed profiles	0.80; 0.18	0.90; 0.20	0.84; 15.58
Gold-deconvolved waveforms	0.68; 0.12	0.73; 0.57	0.62; 50.66
RL-deconvolved waveforms	0.72; 0.14	0.74; 0.58	0.62; 49.25

602

603 **4. Discussion**

604 This study demonstrates that a Transformer-based reconstruction framework can
605 substantially improve footprint-level canopy-profile reconstruction from high-altitude full-
606 waveform LiDAR. Compared with unprocessed LVIS-F waveforms and conventional
607 deconvolution baselines, the proposed model produced stronger correspondence with
608 ALS reference profiles and markedly improved the derivation of CSC metrics. The
609 improvement was especially clear for FHD and VCR, for which correlations with ALS-
610 derived values increased substantially relative to both the unprocessed LVIS-F baseline
611 and the Gold- and RL-deconvolved waveforms. These results indicate that the proposed
612 architecture recovers ecologically meaningful vertical structure that is not preserved by
613 direct waveform use or by conventional deconvolution alone.

614 **4.1 Comparison with waveform deconvolution algorithms**

615 The comparison with deconvolution methods is particularly important because Gold
616 and Richardson–Lucy represent strong conventional waveform-processing baselines for
617 vegetation applications. Deconvolution can sharpen waveform features by reducing the
618 influence of the transmitted pulse, receiver impulse response, and other convolutional
619 effects, but it does not, by itself, transform a large-footprint canopy-plus-ground

620 waveform into a fine-scale canopy-only profile. In contrast, the proposed supervised
621 framework learns this cross-domain mapping directly from co-located LVIS-F and ALS
622 observations. This distinction explains why the Transformer-reconstructed profiles
623 showed stronger agreement with canopy-only ALS references and better preservation of
624 ALS-derived CSC metrics than the deconvolved LVIS-F waveforms. The results
625 therefore suggest that learning-based reconstruction provides an important complement
626 to physically motivated waveform-processing approaches when the objective is not
627 merely signal sharpening, but recovery of application-relevant canopy structure.

628 **4.2 Metric-aware training performance**

629 The metric-aware training experiments suggest that FHD and VCR losses should not
630 be included in the final model objective in their current form. Adding FHD loss produced
631 only negligible improvement, while objectives containing VCR loss slightly improved
632 VCR recovery but substantially degraded overall profile reconstruction. This behavior is
633 consistent with known challenges in multi-objective and multi-task learning. Weighted
634 combinations of losses can induce trade-offs among competing objectives, and
635 improvements in one objective may come at the expense of another when gradients are
636 poorly aligned (Sener and Koltun, 2018; Liu et al., 2021). In this study, the baseline
637 objective already recovered FHD and VCR with strong accuracy, leaving limited room
638 for metric-specific losses to improve performance. The additional metric-aware terms
639 therefore likely introduced optimization conflict and training instability rather than useful
640 complementary supervision. Future work could revisit metric-aware optimization using
641 gradient-surgery methods (Yu et al., 2020) or Pareto-based optimization (Sener and

642 Koltun, 2018), but the present results support using the baseline reconstruction
643 objective as the preferred training strategy.

644 **4.3 Implications for future spaceborne missions**

645 The strong agreement between CSC metrics derived from reconstructed profiles and
646 those derived from dense ALS reference data highlights the potential of this approach
647 for future spaceborne laser scanning (SLS) missions. Current and past SLS systems,
648 such as GEDI and ICESat-2, provide valuable global observations but remain limited by
649 discontinuous sampling and sparse along-track or multi-beam coverage. Future
650 missions and technology demonstrations, including the Concurrent Artificially Intelligent
651 Spectrometry and Adaptive Lidar System (CASALS) under development at NASA
652 Goddard, are expected to provide denser sampling through adaptive scanning and
653 overlapping footprints (Harding et al., 2021; Yang et al., 2023). If applied to each
654 waveform in such dense sampling configurations, the proposed reconstruction
655 framework could enable ALS-like gridded or voxelized three-dimensional
656 representations of forest structure. These products would support derivation of higher-
657 order structural metrics from 3D forest plots rather than isolated footprints alone.

658 A key advantage of the proposed framework is its potential scalability. By training
659 Transformers on co-located SLS waveforms and ALS data from representative
660 calibration regions, the learned relationship between high-altitude waveform structure
661 and fine-scale canopy profiles could be extrapolated to larger areas sampled only by
662 SLS. In this way, strategically collected ALS data could serve as high-fidelity reference
663 information for extending ALS-level structural characterization across broader spatial
664 domains. Such transfer will require careful sampling of calibration sites across canopy

665 types, biomes, terrain conditions, phenological states, and sensor geometries, since
666 model performance may degrade when applied outside the structural and radiometric
667 conditions represented in the training data. Nevertheless, the approach provides a
668 practical pathway for combining the spatial coverage of SLS with the structural detail
669 traditionally associated with ALS.

670 **4.4 Limitations and future directions**

671 Several limitations should be noted. First, although the proposed model improved
672 normalized profile correspondence and CSC metric recovery, the unnormalized pooled
673 correlation remained moderate and did not improve as strongly as the normalized
674 correlation. This indicates that the model is more effective at recovering relative profile
675 shape than absolute return-count magnitude. Consequently, the present framework may
676 be less appropriate for applications requiring raw count fidelity. This limitation is less
677 problematic for many ecosystem applications, including the CSC metrics evaluated
678 here, because such metrics are commonly derived from normalized return distributions
679 rather than raw return counts. Nevertheless, future work should investigate whether
680 losses that explicitly constrain total predicted return counts or separate modules that
681 estimate a sample-specific count-scaling factor can improve absolute count
682 reconstruction without degrading profile shape fidelity.

683 A second limitation is that the LVIS-F data used here predominantly consist of non-
684 overlapping or weakly overlapping footprints (see Fig. 2) relative to the dense
685 overlapping patterns planned for CASALS. As a result, we could evaluate footprint-level
686 canopy-profile reconstruction and footprint-level CSC metric recovery, but we could not
687 test whether aggregating reconstructed overlapping waveforms produces accurate plot-

688 level 3D structural metrics relative to ALS point clouds. Once CASALS or comparable
689 dense SLS waveform data become available, future work should evaluate whether 3D
690 representations generated from aggregating reconstructed overlapping waveforms yield
691 accurate plot-level metrics relative to ALS point clouds.

692 Future work should also link reconstructed canopy profiles directly to field-measured
693 ecosystem variables. In this study, evaluation focused on ALS-derived structural metrics
694 because ALS provides dense, spatially coincident reference data at the footprint scale.
695 The next step is to determine whether reconstructed profiles improve prediction of field-
696 measured forest attributes such as productivity, biomass, canopy fuels, and biodiversity
697 indicators. The framework is also flexible with respect to the target representation.
698 Although the present study trained the model to predict canopy-only profiles,
699 applications requiring ground and canopy information could retain ground returns in the
700 ALS reference profiles and train the Transformer to reconstruct joint canopy-ground
701 structure. Such an extension would support derivation of plot-level canopy cover,
702 canopy base height, fuel load metrics, and terrain-related variables that depend on both
703 vegetation and ground returns.

704 **5. Conclusions**

705 This study developed and evaluated an encoder–decoder Transformer framework
706 for reconstructing high-resolution canopy profiles from high-altitude LVIS-F full-
707 waveform LiDAR. Using co-located ALS canopy profiles as reference data, the
708 proposed model improved footprint-level profile reconstruction relative to unprocessed
709 LVIS-F waveforms and conventional waveform deconvolution baselines. The best-

710 performing model achieved a pooled correlation of $R = 0.67$ and a max-normalized
711 pooled correlation of $R_n = 0.80$ on the test subset, improving upon the normalized LVIS-
712 F baseline of $R_n = 0.73$.

713 The reconstructed profiles also substantially improved the derivation of ecologically
714 relevant canopy structural complexity metrics. FHD derived from reconstructed profiles
715 achieved $R = 0.90$ and $RMSE = 0.20$ relative to ALS-derived FHD, compared with $R =$
716 0.74 and $RMSE = 0.65$ for metrics derived directly from unprocessed LVIS-F
717 waveforms. Similarly, VCR derived from reconstructed profiles achieved $R = 0.84$ and
718 $RMSE = 15.58$ m, compared with $R = 0.62$ and $RMSE = 49.85$ m for the LVIS-F
719 baseline. These results demonstrate that the proposed approach recovers vertical
720 canopy structure in a form that better preserves higher-order structural metrics relevant
721 to ecosystem monitoring.

722 The Transformer framework also outperformed Gold- and Richardson–Lucy-
723 deconvolved waveforms in normalized profile correspondence and CSC metric
724 recovery. Gold and RL deconvolution achieved $R_n = 0.68$ and $R_n = 0.72$, respectively,
725 and neither approach improved FHD or VCR correlations relative to the unprocessed
726 LVIS-F baseline. These findings indicate that conventional deconvolution can sharpen
727 waveform features but is not sufficient, by itself, to transform large-footprint canopy-
728 plus-ground waveforms into fine-scale canopy-only profiles.

729 Overall, the proposed Transformer-based reconstruction framework provides a
730 promising pathway for recovering ecologically meaningful canopy structure from high-
731 altitude and future spaceborne waveform LiDAR observations. When paired with dense
732 sampling configurations from upcoming missions and representative ALS calibration

733 data, this approach could help extend ALS-like structural characterization to broader
734 spatial extents sampled by spaceborne laser scanning.

735 **Data and code availability**

736 The complete codebase for training and implementing the proposed encoder–decoder
737 Transformer, including the best-performing model checkpoint, is available at
738 <https://github.com/tahriribraq/Transformer-waveform-reconstruction>. The preprocessed
739 LVIS waveforms and corresponding ALS reference profiles used in the study are
740 available at <https://doi.org/10.5281/zenodo.21154804>.

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909 Appendix

910 Auxiliary count features

- 911 1. **Local gradient:** Captures the rate of change in counts between adjacent height
912 bins, helping identify structural features such as peaks (where gradient crosses
913 zero), slopes (positive or negative gradient), and plateaus (near-zero gradient).
914 For bin i , the gradient is computed as the first-order difference normalized by the
915 waveform maximum:

$$\phi_i^{\text{gradient}} = \frac{c_i - c_{i-1}}{N \max_{j=1} c_j}, i = 2, \dots, N \quad \text{Eq. (A.1)}$$

916

917 with $\phi_1^{\text{gradient}} = 0$ for the first bin and N is the sequence length.

918 **2. Distance from peak:** Encodes each bin's relative position with respect to the
919 waveform's maximum count, providing structural context within the vertical
920 profile. For bin i , the distance is computed as:

$$\phi_i^{\text{dist}} = \frac{i - i^*}{N} \quad \text{Eq. (A.2)}$$

921

922 where $i^* = \text{argmax}_{j=1}^N (c_j)$ is the index of the bin with maximum count. This
923 feature is negative for bins below the peak and positive for bins above, with
924 values normalized to approximately $[-1, 1]$.

925 **FHD and VCR loss implementations**

926 **Foliage Height Diversity (FHD) loss.**

927 The FHD loss directly optimizes the Shannon diversity index of the vertical return
928 distribution. Waveform counts are first aggregated into 1-meter height bins, then
929 normalized to obtain proportions. For a waveform with counts aggregated into M height
930 bins, let p_m denote the proportion of total counts in bin m :

$$931 \quad p_m = \frac{\sum_{i \in \text{bin}_m} c_i}{\sum_{i=1}^N c_i}$$

932 The FHD is computed as the Shannon entropy:

$$933 \quad \text{FHD} = - \sum_{m=1}^M p_m \log(p_m), \text{ for } p_m > 0$$

934 The FHD loss is the mean squared error between predicted and target FHD values:

935
$$\mathcal{L}_{\text{FHD}} = \frac{1}{B} \sum_{b=1}^B \left(\text{FHD}_{\text{pred}}^{(b)} - \text{FHD}_{\text{target}}^{(b)} \right)^2$$

936 where B is the batch size.

937 **Vertical Canopy Rugosity (VCR) loss.**

938 The VCR loss optimizes the weighted variance of heights, which measures vertical

939 structural complexity. For a waveform with counts $\mathbf{c} = [c_1, \dots, c_N]^T$ at heights $\mathbf{h} =$

940 $[h_1, \dots, h_N]^T$, the normalized counts are:

941
$$p_i = \frac{c_i}{\sum_{j=1}^N c_j}$$

942 The weighted mean height is:

944
$$\bar{h} = \sum_{i=1}^N p_i \cdot h_i$$

943 The VCR is computed as the weighted variance:

946
$$\text{VCR} = \sum_{i=1}^N p_i \cdot (h_i - \bar{h})^2$$

945 The VCR loss is the mean squared error between predicted and target VCR values:

947
$$\mathcal{L}_{\text{VCR}} = \frac{1}{B} \sum_{b=1}^B \left(\text{VCR}_{\text{pred}}^{(b)} - \text{VCR}_{\text{target}}^{(b)} \right)^2$$

948 **Hyperparameter configuration of best-performing model**

949 The hyperparameter configuration of the best-performing model is provided below
950 using the exact parameter names defined in the source code. Hyperparameters not
951 listed here were set to their default values in the released codebase. Readers are
952 referred to the source code and accompanying configuration files for definitions and
953 implementation-specific behavior associated with each parameter.

```
954 # Model architecture
955 embed_dim = 128
956 use_auxiliary_features = True
957 use_cls_token = False
958 als_query_fusion = "Add"
959 encoder_layers = 4
960 decoder_layers = 4
961 num_heads = 4
962 ffn_dim = 512
963 dropout = 0.2
964
965 # Data statistics
966 lvis_height_range = (-15.0, 82.0)
967 als_height_range = (1.0, 80.0)
968 height_resolution = 0.15
969 global_max_count = 4093.0
970 global_max_sum = 80684.0
971
```

```
972     # Loss weights
973     lambda_count = 0.6
974     lambda_zero_penalty = 0.4
975     lambda_shape = 1.5
976     lambda_fhd = 0.0
977     lambda_vcr = 0.0
978
979     # Training parameters
980     batch_size = 32
981     learning_rate = 1e-4
982     weight_decay = 0.05
983     num_epochs = 100
984     scheduler = "cosine"
985     early_stopping_patience = 15
986
987     # Data configuration
988     train_split = 0.8
989     val_split = 0.1
990     augment_train = False
991     boundary_buffer = 2.0
```