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The fossil-water debt of agricultural exports: satellite gravimetry and a climate–yield re-coupling index, demonstrated for Tunisian dates

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Abstract

Fossil groundwater — recharging on geological, not human, timescales — irrigates a growing share of the world's agricultural exports. Drawing it down is capital consumption, not income, yet the depletion embedded in the traded crop is priced nowhere, because the extraction driving it is largely unrecorded. We present a satellite-supported, commodity-level accounting framework that quantifies this ecological debt without relying on those records. It bounds the fossil extraction behind an exported crop between a secure consumptive floor (blue evapotranspiration, from satellite-derived cultivated area and published crop-water requirements) and a conditional gross-extraction ceiling; uses satellite gravimetry (GRACE/GRACE-FO) to confirm, independently, that the aquifer is being drawn down while official records undercount the extraction; and tracks a climate–yield re-coupling index as a qualitative diagnostic of how the buffer that protects the crop is thinning. Valuing the permanently consumed water at replacement cost, the embedded fossil-water debt is of the same order as export revenue — a central estimate near half ($\sim$53 %; about one dollar of unpriced fossil-water debt in every kilogram of dates sold for about two dollars), at least a sixth under the most conservative assumptions, and rising to parity and above ($\sim$106 %) on the gross

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ceiling; we report the range, with consumptive the secure floor and gross the conditional ceiling. Tracking the ratio of revenue to this debt — a Sovereign Return Ratio — shows the trade books aquifer drawdown as ordinary income rather than as the depletion of a capital asset. Per-tonne water intensity fell as yields rose even as total extraction climbed: relative, not absolute, decoupling. Demonstrated for Tunisian date exports (2002–2024), the framework is transferable to fossil-aquifer export systems meeting comparable data and crop-dominance conditions (§5.1).

Keywords: virtual water, fossil groundwater, GRACE gravimetry, ecologically unequal exchange, water footprint, agricultural trade, date palm, North-Western Sahara Aquifer System

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1. Introduction

A growing share of internationally traded food is grown with water that will not come back. In the world's arid export basins — the North-Western Sahara Aquifer System beneath Tunisia and Algeria, the aquifers of the Arabian Peninsula, the Ogallala beneath the North American High Plains, the North China Plain, the Indus — irrigation increasingly draws on confined "fossil" groundwater that recharges over millennia, if at all. When the crop is exported, the water is not: what leaves is the commodity and its revenue, while the extraction and its permanent consequences remain with the producing region. This is a transfer of natural capital, priced nowhere on the invoice.

Natural-capital accounting gives the transfer a precise name. Drawing down a non-renewable stock is capital consumption, not income: the user-cost method of El Serafy (1989) partitions the receipts from a depletable resource into a true-income component and a capital-consumption component that must be reinvested to yield a perpetual income stream, and the Hartwick (1977) rule holds that constant consumption across generations is sustainable only if resource rents are set aside as reproducible capital rather than spent. Booking aquifer drawdown as current export revenue without setting aside that user cost is, in these terms, at odds with the Hartwick rule — the producing economy consumes its natural capital and records the proceeds as earnings (whether it amounts to a formal Hartwick failure depends on economy-wide reinvestment we do not measure; §5). What has been missing is a way to measure the user cost at the level of the traded commodity itself.

Virtual-water and water-footprint accounting (Allan, 1998; Hoekstra and Mekonnen, 2012) revolutionized the visibility of water in trade, and later work isolated the non-renewable fraction: Dalin et al. (2017) showed that roughly 11 % of global non-renewable groundwater used for irrigation is embedded in international food trade. In parallel, the ecologically-unequal-exchange tradition has quantified the net biophysical appropriation embedded in North–South trade at the macro scale — most comprehensively by Dorninger et al. (2021), whose multi-regional input-output accounting measures the materials, energy, land, and labour that flow, under-compensated, from lower- to higher-income economies. That work establishes the phenomenon in aggregate but stops at economy-wide totals derived from national accounts. The present paper extends it in the opposite direction: to a single commodity, a single non-renewable resource, and a debt built bottom-up from satellite-derived cultivated area and corroborated by orbital gravimetry rather than inferred from input-output tables — a satellite-supported, commodity-specific instance of the unequal exchange that tradition describes.

Three gaps stand between the existing literature and such a measure, and they compound in exactly the systems where the stakes are highest.

First, an **accounting gap**. Standard footprints treat a cubic metre of rapidly recharging river water as equivalent to a cubic metre of Pleistocene-age confined groundwater; flow-based scarcity indices such as AWaRe (Boulay et al., 2018) measure the fraction of *renewable* water consumed and do not price the destruction of a non-renewable stock. The closest prior stock-based metric, the groundwater footprint (Gleeson et al., 2012), flags where abstraction exceeds recharge but assigns no monetary value and does not carry the debt on a traded commodity; and while groundwater-depletion externalities have been monetized at farm and basin scale (Kovacs and West, 2016), no method converts irreversible aquifer depletion into a monetized debt carried by the *traded product*. Virtual-water accounting, moreover, has argued that trade in water is trade in *value and services*, not merely volume (Reimer, 2012) — precisely the step from volume to price that a fossil-water debt requires.

Second, a **measurement gap**. Every method that starts from reported extraction inherits the incompleteness of those reports. In arid agricultural regions, a large fraction of pumping is informal or unauthorized and simply absent from the books; calibrating a footprint against such records builds the undercount into the answer.

Third, a **dynamics gap**. Irrigation buffering is treated as binary —

a crop is either climate-coupled (rainfed, exposed) or decoupled (irrigated, buffered). In reality the buffer is a depleting asset. As it thins, the same tonne of exported crop costs progressively more in ecological terms, because extraction that was once agronomically invisible begins to bite. No accounting framework tracks this rising marginal cost.

Contribution. We propose a transferable framework that addresses all three gaps, built from three satellite-era instruments:

1. **A bounded mining footprint.** Rather than a single point estimate, the fossil extraction behind an exported crop is bounded between a consumptive floor (blue evapotranspiration, from satellite-derived area and published crop-water requirements) and a gross-extraction band (the floor divided by irrigation efficiency). We report the range rather than a spurious point estimate.
2. **Gravimetric corroboration where records fail.** Satellite gravimetry (GRACE/GRACE-FO) is used not to attribute a commodity-specific footprint, nor to test its magnitude, but to confirm independently that the wider aquifer system is undergoing net depletion. The undercount claim is then evaluated through the bottom-up footprint's comparison with administrative extraction records.
3. **A climate–yield re-coupling index as a buffer-thinning diagnostic.** A rolling measure of how tightly per-hectare yield tracks atmospheric water demand captures the state of the irrigation buffer. It does not enter the volumetric calculation; it is a qualitative diagnostic of whether the buffer that stabilizes yields is weakening over time.

Valued at replacement cost and divided into export revenue, these yield a **Sovereign Return Ratio** — export earnings per unit of mined-water debt — whose level measures how much an export economy earns against the irreversible depletion that underwrites it.

We demonstrate the framework on Tunisian date exports, 2002–2024. Tunisia is a proof-of-concept for a general method: dates are a high-value export (Tunisia is among the world's leading date exporters by value), roughly 39–56 % of the crop is exported, the oases sit on well-characterized fossil aquifers, and a 22-year satellite record for the four producing governorates (Tozeur, Kébili, Gafsa, Gabès) provides the yield and storage series the method needs. The case is unusually clean; the method is transferable under the conditions specified in §5.1 (Fig. 1).

Our objective is to specify the framework, demonstrate that each instrument is supported by data that exist for many export basins, and report what the demonstration reveals, while remaining as explicit about what the data cannot support as about what it can.

What this paper does not claim. Because the framework combines instruments that are each easy to over-read, we state its limits at the outset. We do *not* attribute the observed aquifer depletion to date cultivation via GRACE — the gravimetric signal is basin-scale and used only to confirm that drawdown is occurring, not to apportion it to a crop (§2.2). We do *not* estimate a welfare-optimal Hotelling rent for the water, nor claim that the replacement cost we use is that rent; ours is a replacement-cost accounting price, not a market or willingness-to-pay price for irrigation water (§2.4). We do *not* observe whether the exporting economy reinvests the depletion rents through other channels, and therefore do *not* claim a formal Hartwick failure (§5). And we do *not* recover which governorate's dates reached which destination — the joint origin×destination cell is not identifiable from the data (§4.4). The claim is deliberately narrower than any of these: the replacement-cost value of the fossil groundwater consumed in producing exported Tunisian dates is large relative to the revenue those dates earn.

2. A framework for the fossil-water debt of agricultural exports

Before turning to any Tunisian value (§3–§4), we set out the method in general terms. It takes three inputs that exist for a wide class of arid export basins — a satellite-derived cultivated-area trajectory, a satellite terrestrial-water-storage series, and a sub-national yield record — together with published, locally appropriate crop-water and economic coefficients.

2.1. The bounded mining footprint

Any accounting of the water a crop takes from an aquifer must begin with the water the crop actually consumes. The consumptive blue-water use of an irrigated crop is the actual evapotranspiration less effective precipitation,

$$ET_{blue} = ET_a - P_{eff}, \tag{1}$$

the water biologically consumed by the canopy that did not come from rain. Retrieved from satellite energy-balance models (SEBAL, Bastiaanssen

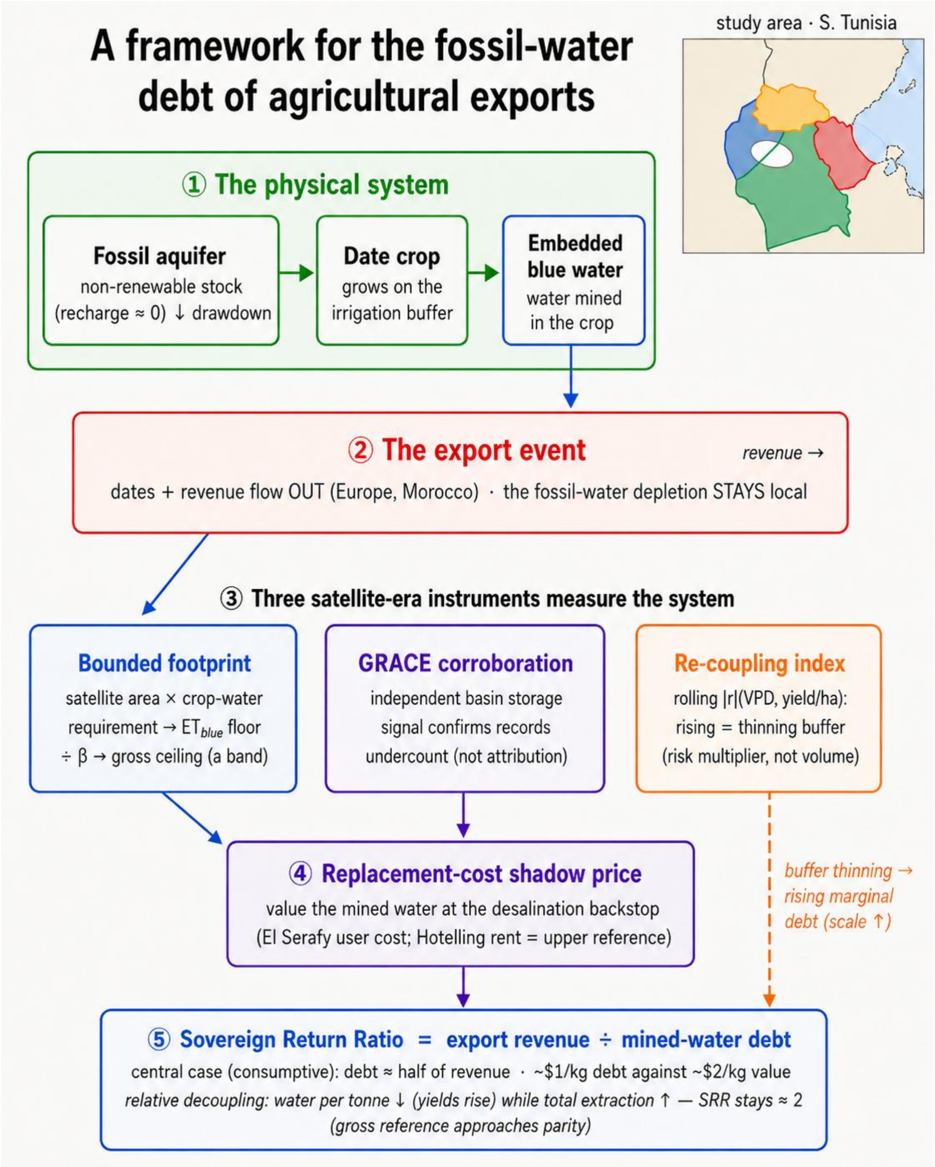


Figure 1: **The framework and its demonstration setting.** Schematic of the three-instrument framework — bounded mining footprint, gravimetric corroboration, and re-coupling index, combined through a replacement-cost shadow price into the Sovereign Return Ratio — with a study-area locator inset showing the four date-producing governorates (Tozeur, Kébili, Gafsa, Gabès) on the North-Western Sahara Aquifer System and the export gateway toward Europe and Morocco. Schematic; boundaries not to scale.

et al., 1998; METRIC, Allen et al., 2007; OpenET, Melton et al., 2022; WaPOR, FAO) or from published crop-water requirements applied to a satellite-derived area (the FAO-56 Penman–Monteith basis of CROPWAT; Allen et al., 1998), ET_{blue} is a defensible *floor* on the water attributable to the crop.

It is only a floor. In traditional surface- and flood-irrigated systems the water *extracted from the aquifer* substantially exceeds the water *consumed by the crop*, because a large fraction is lost to deep percolation, non-beneficial soil evaporation, and canal seepage. In a closed fossil system with a falling water table these losses do not return to the confined reserve, so the quantity mined from the fossil stock is the gross extraction, not the consumptive fraction alone. Gross extraction scales as the consumptive floor divided by the irrigation efficiency β,

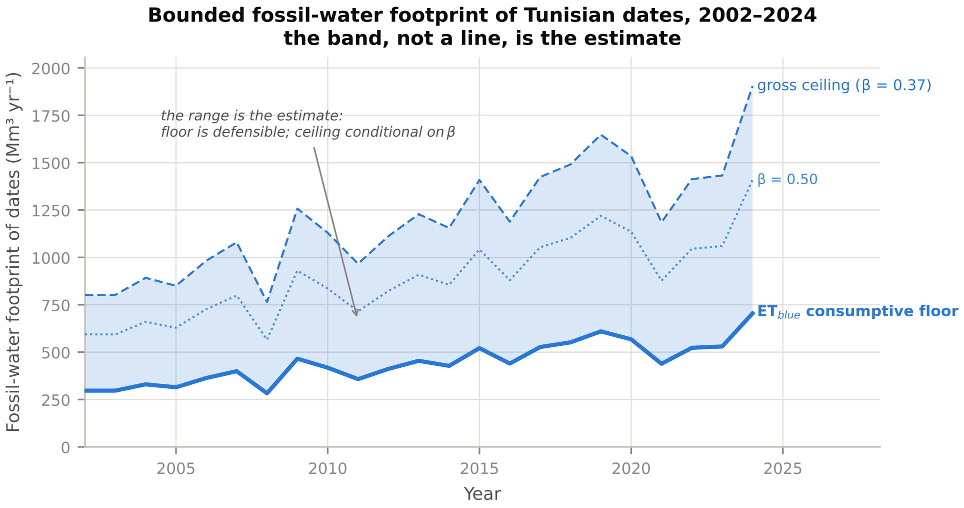
$$E_{gross} = ET_{blue}/\beta. \tag{2}$$

Because β is uncertain and site-specific, gross extraction is reported as a **band** across a defensible efficiency range, not a point. This gives the framework two bounds and reports the range between them as the estimate: the consumptive floor ET_{blue} is a secure lower bound on the water the crop takes, and gross extraction is the conditional upper bound whose width is set by the efficiency range. The floor is a defensible lower bound; the ceiling is explicitly conditional on β. This bounded footprint is the methodological centerpiece of the framework, and Fig. 2 is its visual statement: the fossil-water footprint of the crop drawn as a shaded band from the ET_{blue} consumptive floor (solid lower edge) up to the gross-extraction ceiling at the least-efficient β (dashed upper edge), widening over time as cultivated area grows.

2.2. GRACE as corroboration where records fail

The central measurement problem in arid agricultural accounting is that recorded extraction is systematically incomplete: informal and unauthorized wells are absent from administrative registers, so any footprint calibrated against those registers inherits their undercount.

Satellite gravimetry offers an *independent* physical signal. GRACE and GRACE-FO (JPL mascon solutions; Watkins et al., 2015; Wiese et al., 2016) measure terrestrial water storage (TWS) change from the Earth's time-varying gravity field, the technique that established groundwater depletion across the world's major aquifers (Rodell et al., 2009; Famiglietti, 2014).



Bounded fossil-water footprint of Tunisian dates, 2002–2024
the band, not a line, is the estimate

the range is the estimate:
floor is defensible; ceiling conditional on β

Figure 2: **The bounded fossil-water footprint of Tunisian dates, 2002–2024 (methodological centerpiece).** National footprint drawn as a shaded band: the lower edge is the ET_{blue} consumptive floor (satellite-derived palm area × published net crop-water requirement, summed across governorates); the upper edge is gross extraction at the least-efficient irrigation coefficient ($\beta = 0.37$), with the $\beta = 0.50$ mid-case shown inside. Vertical axis, $\mathrm{Mm}^3\ \mathrm{yr}^{-1}$; horizontal axis, year. The width of the band, not any single line, is the estimate — the floor is secure, the ceiling explicitly conditional on β.

Their spatial resolution — mascons of order 3° (∼300 km) — is far too coarse to attribute storage change to a specific commodity or oasis, and we do **not** use it for that. We use it for one thing only: to confirm **independently that the aquifer is being drawn down** — the basin storage decline (-16.6 cm over 2002–2024, with a 2007 break; Gasmi et al., 2026) is real and governance-independent, which is what makes the records-undercount premise consequential rather than a bookkeeping quibble.

The load-bearing evidence that records undercount is **administrative and bottom-up, not gravimetric**: the region's *own* books report Kébili at 228–230 % of renewable resources, and the modelled gross date footprint alone overshoots total recorded extraction across the plausible efficiency range (§4.1). GRACE does not corroborate the *magnitude* of the date footprint, and the "net $<$ gross" ordering is not a test (it holds by construction). The one genuinely falsifiable comparison is bottom-up modelled extraction versus *recorded* extraction (§4.1): had the modelled footprint fallen below the recorded books, the undercount premise would be undercut; it does not.

2.3. The re-coupling index as a buffer-thinning diagnostic

Irrigation buffers a crop against atmospheric water demand: when the buffer is intact, per-hectare yield is insensitive to interannual variation in vapour-pressure deficit (VPD) and related demand variables. As a fossil buffer is depleted — water tables fall, pumping costs rise, deficits open at critical phenological stages — that insensitivity erodes, and yield begins to *re-couple* to atmospheric demand.

We summarize this with a **temporal re-coupling index**: the trend in the rolling-window correlation between a detrended per-hectare yield anomaly and a water-demand variable (VPD). A positive trend is consistent with a thinning buffer. The index is a governorate-level annual quantity, not a pixel-level or volumetric one, and it does **not** enter the debt calculation. It is a **qualitative supporting diagnostic**, not a demonstrated causal mechanism: a rising index is consistent with the interpretation that the irrigation buffer is weakening — extraction that was once "ecologically cheap," with no detectable agronomic consequence, becoming costly as the buffer thins — but the index is an observational correlation and we do not treat the buffer-thinning reading as proven by it. Its role is interpretive, read as a risk indicator alongside the debt, never as a component of its volume.

Rolling-window statistics carry well-documented pitfalls, so any inference on the index trend must pass a gate of overlapping-window-aware tests:

heteroskedasticity-and-autocorrelation-consistent (HAC) corrected slopes (Newey and West, 1987, bandwidth per Andrews, 1991) with reported effective degrees of freedom; a block bootstrap (block length equal to the window length; Künsch, 1989) as a second inferential check; a leakage-free detrending alternative; and sensitivity to linear versus quadratic detrending. Detrending must be leakage-free, because a centred rolling window imports future information and spuriously tightens end-of-series correlations. For a rolling index the natural leakage-free choice is *within-window* detrending — each window's anomalies are computed from only that window's own data, so no structure from outside the window is imposed on the local correlation; full-window linear detrending serves as a sensitivity check. The HAC slope is the primary test for a temporal-trend claim; the block bootstrap answers a different question (significance under randomised block order) and, by destroying the temporal ordering the claim is about, its confidence interval can span zero even for a real trend — so the two are reported together and read accordingly, not as a single verdict.

2.4. Shadow-priced debt and the Sovereign Return Ratio

The volumetric footprint becomes an *economic* quantity through a shadow price that reflects the true cost of the resource, not its subsidized tariff. Our conceptual motivation is the natural-capital-accounting principle that drawing down a non-renewable stock is capital consumption, not income (El Serafy, 1989; Hartwick, 1977): a depleting aquifer's contribution to current receipts is in part a liquidation of a fixed asset. We do **not** compute a literal El Serafy user-cost partition — that method partitions the receipts *from selling the depleting resource itself*, whereas here the resource (water) is embedded and a different commodity (dates) is sold, and for a long-lived stock the partition fraction is in any case small. We take from El Serafy only the framing that the depletion carries a cost that ordinary income accounting ignores, and we measure that cost directly.

We measure it as **replacement cost**: the cost of the marginal backstop technology (brackish-water or seawater reverse-osmosis desalination) that would be needed to supply the water once the aquifer is exhausted (Ghaffour et al., 2013, the backstop-price logic of Nordhaus, 1973). This is a **replacement-cost accounting price** — a depletion shadow price for what supplying the service will actually cost when the stock is gone — and we are explicit about what it is *not*: it is not a welfare or willingness-to-pay measure for irrigation water, and it is not a claim to the Hotelling (1931) in-situ optimal rent, which for a stock with a long remaining life sits *below* the backstop. We therefore use replacement cost as a transparent, bounded accounting valuation and report the Hotelling rent only as a conceptual upper reference near exhaustion, not as an ordering we assert to hold today. Because replacement cost spans a real range, the debt is reported as a band, never a point.

Pricing water at replacement cost is legitimate on the resource's own terms. Replacement (substitute) cost is a valid valuation when the environmental asset is a *productive input* whose loss must be made up by a marketed substitute (Barbier, 2000), disciplined by three classic conditions: the substitute delivers an equivalent service, is the least-cost substitute, and is one society will actually incur the cost of rather than forgo the service (Shabman and Batie, 1978). Desalination is *not* the least-cost substitute for *irrigation* water — deficit irrigation, demand management, and imported virtual water are all cheaper — which is precisely why the anchoring service is not the export crop but the **domestic and municipal water supply that the same aquifer depletion ultimately threatens**, for which there is no alternative to eventual desalination and which society cannot forgo. The link is a common-stock opportunity cost: the aquifer is a single fixed stock, so a cubic metre consumed today growing dates for export is a cubic metre no longer available to the future municipal backstop. The price is therefore not a willingness-to-pay for date irrigation; it is a depletion-accounting price for the opportunity cost imposed on future non-discretionary water supply. Because we price only the consumptive quantity (§4.2), it is a conservative, not an inflating, choice.

Dividing export revenue by the debt gives the **Sovereign Return Ratio (SRR)** — a diagnostic ratio of export earnings to the replacement-cost value of the fossil water embedded in those exports, not an El Serafy income partition. We use "sovereign" in a specific, limited sense: it denotes the exporting economy's national natural-capital exposure — the stock of nonrenewable groundwater that belongs to the country and is being liquidated through trade — not fiscal revenue, sovereign debt in the financial sense, or a national-welfare measure. The ratio is:

$$SRR_t = \frac{\text{export revenue}_t}{\text{replacement-cost value of consumptive mined-water}_t}. \tag{3}$$

A low SRR indicates an export economy earning little per unit of irreversibly mined water; because the shadow price enters as a fixed 2024 band, the ratio is robust to the deflation choice within each year (numerator and denominator are in the same-year terms).

The ratio must be decomposed, or it will be misread. Writing it in logs separates three channels,

$$\Delta \ln SRR = \Delta \ln(\text{revenue}) - \Delta \ln(\text{price of water}) - \Delta \ln(\text{volume extracted}), \tag{4}$$

a **revenue** channel (real export price per tonne), an **intensity** channel (water per tonne), and a **scale** channel (total water extracted). The decomposition guards against a specific error: in a system where yields are rising, water *per tonne* can fall even as total extraction climbs, so that reporting debt-per-tonne alone would show apparent progress while the aquifer is being drained faster than ever. It makes visible that total water extracted can rise even as water per tonne falls, and it links the scale channel to the re-coupling index of §2.3: as the buffer thins, more land and more pumping are drawn in to sustain output.

3. Data

The demonstration combines governorate-level agronomic and hydrological records for Tunisia's four date-producing governorates (Tozeur, Kébili, Gafsa, Gabès) over 2002–2024, national date-trade statistics, and published crop-water and economic coefficients. All inputs derive from public sources, described below by what they measure and where they originate. A full input audit — source, spatial scale, coverage, transformation, and principal uncertainty for every quantity entering the debt calculation — is provided in Supplementary Table S1.

Date production by governorate and year is from ONAGRI (Tunisia's national agricultural observatory). Cultivated oasis area is derived from a satellite persistent-NDVI trajectory with palm-fraction correction (Tozeur and Kébili $\approx$ 1.0; Gabès 0.096; Gafsa 0.040). Terrestrial water storage is from NASA JPL GRACE and GRACE-FO mascon solutions (RL06.3 V4, monthly), aggregated per governorate, with an interpolation flag for the 2017–2018 gap between the two missions. Aquifer extraction volumes, exploitation rates, and well counts are from the regional agricultural authorities (CRDA). Climate variables, including vapour-pressure deficit and precipitation, are from ERA5-Land reanalysis.

National monthly date-export volume and value for 2018 onward are from Tunisia's national open-data DataStore (INS), with complete twelve-month coverage 2018–2025. Annual export volume and value for 2002–2017, together with the destination-country breakdown, are from UN Comtrade (HS 080410, reporter Tunisia). Tunisia reports to Comtrade intermittently — direct records exist for 2002, 2005, 2009, 2010, 2013, and 2017 — and the roughly ten non-reported years are filled by log-linear interpolation between the directly-reported anchor years. Interpolation supplies no information beyond those anchors, so the pre-2018 series is best read as a smoothed rendering of six directly-reported observations rather than an independent reconstruction; this carries little weight for the paper's result, because the Sovereign Return Ratio's level is anchored on the directly-observed post-2018 period and, as §4.2 shows, holds on the directly-reported Comtrade anchor years alone.

Two quantities are estimated. ET_{blue} is derived from published CROPWAT and field crop-water requirements specific to these oases, not from reference evapotranspiration (ET0), which is an atmospheric-demand proxy rather than actual or blue evapotranspiration; conflating the two would be a category error. The governorate of origin of exports, which is recorded nowhere, is estimated from production shares (§4.2, §4.4). A variety split (Deglet Nour versus common) is available only from ministerial bulletins and is used qualitatively.

Table 1 consolidates the date-palm water-use estimates and the yield-era reconciliation (§4.2).

Net consumptive requirements follow Tiba et al. (2025) and Haj-Amor et al. (2020). The production-weighted national consumptive intensity of $\approx$1,500 $\mathrm{m}^3\,\mathrm{t}^{-1}$ reflects current per-hectare yields; the older national blue-water footprint of 3,270 $\mathrm{m}^3\,\mathrm{t}^{-1}$ (Chouchane et al., 2015) corresponds to the $\sim$3.5 $\mathrm{t\,ha}^{-1}$ yields of 1996–2005 and is retained only as a historical upper reference (§4.2). Per-tonne consumptive water for Kébili fell from $\sim$3,040 $\mathrm{m}^3\,\mathrm{t}^{-1}$ in 2002 to $\sim$1,620 $\mathrm{m}^3\,\mathrm{t}^{-1}$ in 2024 on single-year yields as yield roughly doubled against a fixed per-hectare requirement. Gafsa's and Gabès's higher per-tonne figures reflect their low palm-fraction-corrected yields; their small production shares limit their weight in the national figure, and their palm-fraction estimates (0.040 and 0.096) carry snapshot uncertainty.

Table 1: **Date-palm water-use reconciliation.** Bottom-up consumptive water intensity (m^3 per tonne of dates) by governorate, formed as the published net crop-water requirement divided by yield per hectare. Yield is the 2020–2024 period-mean (a five-year trailing average that suppresses alternate-bearing noise); the consumptive column is therefore the exact quotient of the two columns to its left. The production-weighted national figure is the primary, current-condition water intensity used throughout the debt calculation; Chouchane et al. (2015) is shown for historical comparison only.

Governorate	Net crop-water req. ($m^3\,ha^{-1}\,yr^{-1}$)	Period-mean yield 2020–2024 ($t\,ha^{-1}$)	Bottom-up consump. ($m^3\,t^{-1}$)
Kébili	11,500	8.24	1,396
Tozeur	12,671	8.28	1,530
Gafsa	10,446	4.08	2,560
Gabès	10,446	4.73	2,208
National (prod.-weighted)	—	—	**≈1,504**
Chouchane et al. (2015), hist. ref.	—	∼3.5 (1996–2005)	3,270 (blue WF)

4. Application to Tunisian dates

4.1. Administrative records already declare unsustainable extraction

The framework's premise — that administrative extraction records undercount, making independent corroboration necessary — is directly visible in the demonstration case, and the strongest evidence comes from the records themselves.

The official books already declare mining. Kébili's recorded deep-aquifer exploitation rate is **228–230 % of renewable resources** (CRDA, 2022–2023): the administration itself reports extraction at more than twice recharge, before any modelling enters. Over the satellite record, GRACE shows a regional terrestrial-water-storage loss of **−16.6 cm across 22 years**, with a formally identified structural break in 2007 (post-break slope $-1.12\,\mathrm{cm\,yr^{-1}}$); this depletion signal, its break year, and the well-proliferation and re-coupling results drawn on below are established in our companion study of these oases (Gasmi et al., 2026), which the present framework extends from monitoring the aquifer's degradation to pricing the debt that degradation embeds in the exported crop. Kébili's private well count rose from 3,733 to 10,632 between 2008 and 2023 (+185 %), and an estimated two-thirds of deep agricultural wells are unauthorized — that is, absent from the extraction registers (CRDA well censuses, reported in Gasmi et al., 2026).

Modelled date extraction overshoots the entire recorded budget. The consumptive floor for date cultivation across the four governorates — satellite-derived corrected palm area (≈61,200 ha in 2024) times published net crop-water requirements — is **≈703 $\mathbf{Mm^3\,yr^{-1}}$**, and dividing it by irrigation efficiency lifts it to a gross-extraction band that *exceeds* total recorded CRDA extraction (≈967 $\mathrm{Mm^3\,yr^{-1}}$) across the plausible efficiency range: the gross date footprint alone runs from 1,082 to 1,900 $\mathrm{Mm^3\,yr^{-1}}$ ($\beta = 0.65$ to 0.37), 1.1× to 2.0× the entire recorded budget — for a single crop, before every other use. Together with the government's own 228–230 % exploitation rate, this is the undercount, stated by the records themselves: the recorded books cannot simultaneously be complete and be exceeded by one crop's modelled extraction. (As a corroborating cross-check, even the consumptive *floor* of ≈703 $\mathrm{Mm^3\,yr^{-1}}$ already reaches ∼73 % of the recorded budget before losses and before every other crop; we note it as supporting, not load-bearing, because it only implies undercount if dates are presumed not to dominate regional water use — whereas the 228 % self-declaration and the gross overshoot require no such presumption.) The consumptive floor is the lower edge of the bounded extraction band that is the framework's methodological centerpiece (Fig. 2); the gross band used in the overshoot is the same quantity retained only as an upper reference in the valuation of §4.2, never as the headline debt.

Gross extraction overshoots the records across the plausible efficiency range. The overshoot is a *range*, driven entirely by the efficiency assumption (Fig. 3):

Irrigation efficiency β	Gross date extraction ($\mathrm{Mm^3\,yr^{-1}}$)	vs. recorded CRDA ($967\,\mathrm{Mm^3}$)
0.37 (traditional surface)	∼1,900	1.96×
0.50 (mid)	∼1,406	1.45×
0.65	∼1,082	1.12×
≥0.73	≤967	overshoot vanishes

We deliberately do **not** headline a single "2×." The load-bearing, assumption-free statement is that official exploitation rates already exceed 200 % and that modelled date consumption alone approaches the entire recorded extraction budget; the gross overshoot corroborates the undercount across a 1.1×–2.0× range. That the records demonstrably undercount is why an *independent* signal that the aquifer is being drawn down matters: GRACE (§2.2; Fig. 3) supplies exactly that — a governance-independent confirmation of basin depletion.

4.2. The debt: floor, central case, and range

Before any figure, one choice governs the result: *which water quantity is priced?* We state it as an auditable rule (Table 2) rather than leaving it in prose, because the headline turns on it and a reader must be able to see at a glance that the reported number is the conservative one.

Water-quantity reconciliation (Table 1). Two published water-intensity figures exist for Tunisian dates and they must not be used opportunistically. The **primary** figure — and the quantity the headline debt prices — is the bottom-up, per-governorate **consumptive** requirement: published net crop-water requirements (differing by oasis) divided by *current* per-hectare yields, giving a production-weighted $\approx\mathbf{1{,}504\,m^3\,t^{-1}}$. An older national water-footprint figure of $\mathbf{3{,}270\,m^3\,t^{-1}}$ is retained only as a **historical upper reference**, explicitly attributable to the ∼$3.5\,\mathrm{t\,ha^{-1}}$ yields of 1996–2005; per-tonne water has roughly halved as yields rose (for Kébili, from ∼$3{,}042\,\mathrm{m^3\,t^{-1}}$ in 2002 to ∼$1{,}620\,\mathrm{m^3\,t^{-1}}$ in 2024). The headline never floats on whichever figure is larger.

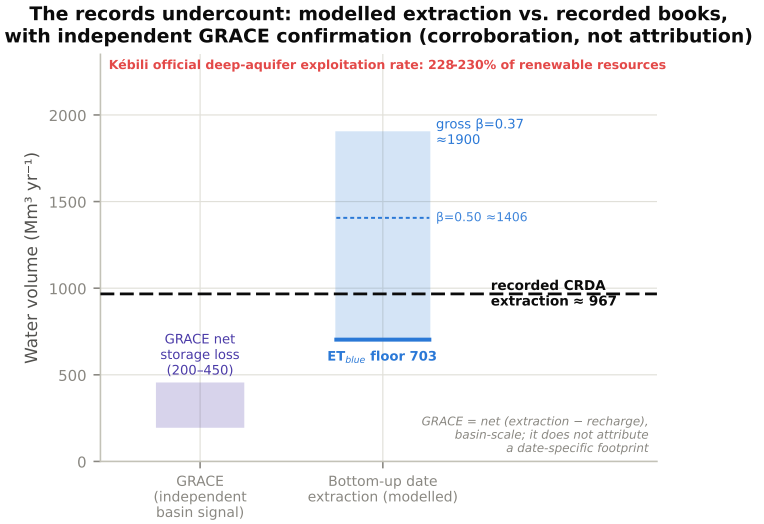


Figure 3: **The records undercount: administrative and modelled evidence, with independent GRACE confirmation of basin depletion.** The modelled bottom-up date-extraction band (ET_{blue} floor ≈703 Mm^3 yr^{-1} to gross ≈1,900 Mm^3 yr^{-1} at $\beta = 0.37$, with $\beta = 0.50 \approx 1,406$ Mm^3 yr^{-1}) shown against recorded CRDA extraction (≈967 Mm^3 yr^{-1}, dashed level); Kébili's official 228–230 % exploitation rate is annotated. **Note the differing spatial domains:** the date-extraction band and the CRDA total are for the four-governorate oasis area, whereas the GRACE net-storage-loss band (≈200–450 Mm^3 yr^{-1}) is the -1.12 cm yr^{-1} post-2007 storage change integrated over the ∼18,000–40,000 km^2 SASS basin footprint, not the ∼600 km^2 palm area (over the palm area alone the same slope is only order 10 Mm^3 yr^{-1}). The GRACE band is shown to confirm — independently and at basin scale — that depletion is real; it is **not** a like-for-like comparison with the date footprint, and the "net < gross" ordering holds by construction and is not treated as a test. GRACE does not attribute a date-specific footprint.

Table 2: **Which water quantity is priced?** The three candidate quantities and their status in the debt calculation. Only the consumptive floor is headlined; the other two are references.

Water quantity	2024 volume basis	Status	Reason
Consumptive ET_{blue} (net crop-water requirement × area)	≈1,504 $\mathrm{m}^3\,\mathrm{t}^{-1}$; ≈703 $\mathrm{Mm}^3\,\mathrm{yr}^{-1}$	**Headlined — the debt**	Permanent evapotranspiration loss; the secure lower bound on what the crop takes from the aquifer. For the confined fossil stock this *understates* depletion, so it is the conservative choice.
Gross extraction (consumptive $\div\beta$)	≈3,010 $\mathrm{m}^3\,\mathrm{t}^{-1}$ at $\beta = 0.50$	Upper reference only	The quantity actually mined from the fossil reserve (§2.1), but its treatment as permanent loss is conditional on β; carried as the ceiling of the reported range, never as the headline.
Historical blue-water footprint (Chouchane et al., 2015)	3,270 $\mathrm{m}^3\,\mathrm{t}^{-1}$	Historical reference only	Reflects the ∼3.5 $\mathrm{t\,ha}^{-1}$ yields of 1996–2005, not current yields; boxed separately to prevent mixing a historical intensity with a current-price basis.

The stacked-conservative floor. Combining every adverse assumption simultaneously — the bottom-up consumptive quantity, the cheapest cited brackish-water replacement cost (∼\$0.24 m^{-3}, Djerba), and an origin mix skewed toward the least water-intensive governorate (Kébili) — the replacement-cost debt is **∼17 % of export value** (2024 export value ≈ \$305 M, converting the reported 955 M TND at the 2024 annual-average rate of 3.13 TND/USD; the World Bank period-average of 3.11 shifts this by well under one per cent). This is the abstract's floor: the most defensive reading that remains defensible (Table 3, row 1). It sits a hair below the national-intensity price-floor of ∼18 % (national consumptive × \$0.24) that forms the lower edge of the reported band in Fig. 5: the ∼17 % figure additionally skews the origin mix toward Kébili, the least water-intensive governorate, while the ∼18 % edge uses the national production-weighted intensity at the same floor price — two near-identical floors with distinct constructions, distinguished here so they are not read as one number.

The justified central case (consumptive basis). The central case values the **consumptive** water — the permanently-lost evapotranspiration, the secure lower bound on what the crop takes from the aquifer — at the midpoint replacement price (\$0.70 m^{-3}, mid of the \$0.24 brackish–\$1.10 SWRO-high band), with the realistic Deglet-Nour origin mix (+2 %, not the +35 % commodity-grade skew). We headline the consumptive floor rather than the gross ceiling **for conservatism**: consumptive ET is water that is unambiguously gone, whereas the gross figure adds conveyance and percolation losses whose permanence is conditional on the efficiency assumption. The consumptive figure is the secure floor, the gross figure the conditional ceiling, and the range between them is the result — we headline the floor because it is the claim we can defend without reservation, not because the gross losses are somehow recovered (for the confined fossil stock, they are not; §2.1). Under this central set the debt is **≈\$161 M ≈ 53 % of the \$305 M export value**: the replacement-cost debt of the fossil water embedded in exported dates is **of the same order as export revenue — roughly half of it**. That figure rises toward parity and beyond on the gross upper reference or at a higher replacement price; the range, not the point, is the result.

Rows 1–4 are the primary consumptive range (floor → central → high price); rows 5–6 are the labelled gross ceiling; the boxed historical row is retained only to show why we do *not* lead with a ∼480 % figure (that arises from pairing the historical 3,270 $\mathrm{m}^3\,\mathrm{t}^{-1}$ intensity with the gross-adverse basis

Table 3: **2024 debt under every scenario, on one auditable sheet.** Each row is a fully specified scenario; the debt (constant-2024 US$ M), its share of the $305 M export value, and the implied Sovereign Return Ratio (revenue ÷ debt) are the exact product of the water basis, shadow price, and origin/yield choices to its left. Only the **central** row is the headline; the *gross* rows are the conditional upper reference (§2.1); the historical row is boxed. The purpose of the sheet is that $161 M (central), $322 M (gross reference), and $684 M (gross adverse) can never be confused, and that no row is cherry-picked out of view.

#	Scenario	Water basis ($m^3\,t^{-1}$)	Shadow price ($\$\,m^{-3}$)	Origin/yield	Debt ($M)	% rev.	SRR
1	**Stacked-conservative floor**	consumptive, Kébili-skew (1,396)	0.24 (brackish)	Kébili-skew	**51**	**17 %**	5.95
2	National price-floor edge	consumptive (1,504)	0.24 (brackish)	national	55	18 %	5.53
3	**Central case (headline)**	consumptive (1,504)	0.70 (mid)	Deglet-Nour (+2 %)	**161**	**53 %**	**1.90**
4	High-price consumptive	consumptive (1,504)	1.10 (SWRO-high)	national	253	83 %	1.21
5	*Gross upper reference ($\beta = 0.50$)*	*gross (3,010)*	*0.70 (mid)*	*national*	*322*	*106 %*	*0.95*
6	*Gross adverse extreme ($\beta = 0.37$)*	*gross (4,064)*	*1.10 (SWRO-high)*	*national*	*684*	*224 %*	*0.45*
—	*[boxed] Historical footprint*	*Chouchane 3,270*	*0.70 (mid)*	*national*	*350*	*115 %*	*0.87*

— an inconsistent mixing of bases). The central case is a genuine midpoint on each axis: the consumptive quantity (not gross), the mid price (not the SWRO-high), and the empirical Deglet-Nour origin mix (not the commodity-grade skew).

Per kilogram, the same result reads tangibly. Against a 2024 export unit value of **∼$2.00 kg^{-1}**, the central (consumptive) embedded fossil-water debt is **∼$1.05 kg^{-1}** — every kilogram of dates exported for about two dollars carries roughly one dollar of unpriced fossil-water debt, rising toward two dollars on the gross reference. The per-kilogram debt differs by governorate with water intensity: **∼$0.98 kg^{-1} for Kébili and ∼$1.07 kg^{-1} for Tozeur** (the palm-fraction ≈ 1.0 governorates, and the source of nearly all exports), rising to ∼$1.79 kg^{-1} for Gafsa and ∼$1.55 kg^{-1} for Gabès, whose higher figures reflect low palm-fraction-corrected yields and carry the corresponding snapshot uncertainty. Across the price band the per-kilogram figure runs from ∼$0.36 kg^{-1} (floor price) to ∼$1.65 kg^{-1} (upper price), and to ∼$4.47 kg^{-1} on the gross $\beta = 0.37$ upper reference.

A note on the water-intensity basis. Two choices sit behind every debt figure, and we make the conservative one on each. First, *quantity*: we headline the **consumptive** requirement (permanent ET loss) as the secure floor, not gross extraction — the gross figure adds conveyance and percolation losses whose treatment as permanent loss is conditional on the efficiency assumption, so it is carried as the upper reference rather than the headline. This is a conservatism choice, not a claim that the gross losses are recoverable from the fossil stock: for the confined reserve they are not (§2.1), so the consumptive floor genuinely *understates* the mining of that stock, which is exactly why headlining it is the defensible move. Second, *yield smoothing*: the ≈1,504 m^3 t^{-1} national intensity uses per-hectare yield averaged over the preceding five years (a trailing mean, no look-ahead) to suppress alternate-bearing noise; single-year 2024 yields would raise it to ≈1,810 m^3 t^{-1}, which we do not use because one alternate-bearing year is not a reliable divisor. The debt series, per-kilogram figures, decomposition, and Figs. 4–6 are all on this one consumptive, period-mean basis, so their dollars and percentages agree by construction.

The full range. On the consumptive basis the primary range runs from the **∼17 % stacked-conservative floor**, through the **∼53 % central case**, to an **∼83 % upper-price case** (consumptive × $1.10 m^{-3}). Counting gross extraction as permanent loss lifts the reference to **∼106 %** (gross $\beta = 0.50\times$

\$0.70) and, at the adverse extreme, ∼**224 %** (gross $\beta = 0.37\times$ \$1.10) — reported as an explicitly-labelled upper reference, not the headline. A ∼**480 %** figure exists but is **boxed separately**, because it depends on the 3,270 $\mathrm{m}^3\,\mathrm{t}^{-1}$ historical water intensity of Chouchane et al. (2015); leading with it would pair a bottom-up floor with a historical ceiling — an inconsistent mixing of bases this reconciliation exists to prevent. We report a range rather than a point estimate, and show it as it evolves across the record (Fig. 5), with the 2024 band marked.

The trajectory, 2002–2024. Extending the calculation across the full record — per-year export value deflated to constant-2024 USD, and per-year water volumes from the area- and yield-driven consumptive intensities — gives the Sovereign Return Ratio as an annual band (Fig. 4) and the ecological debt as a band tracking below export revenue (Fig. 5). **The load-bearing finding is the *level*: across the whole record the central consumptive debt sits at roughly half of export revenue (SRR ≈ 1.7–2.2), and on the gross upper reference it approaches parity.** The *trajectory* is a secondary observation and we do not lean on it: the central SRR is broadly flat-to-rising over the full and directly-reported windows, and the recent softening (2022–2024) is driven mainly by a real export-price fall (∼\$2.83 kg^{-1} in 2018 to ∼\$2.00 kg^{-1} in 2024, about −\$0.18 $\mathrm{kg}^{-1}\,\mathrm{yr}^{-1}$) together with a 2024 volume spike, not by accelerating depletion. This price-driven, few-year feature may be transient; we therefore report the level as the result and the trend only as context. (On the gross upper reference the same central line dips to and just below unity in 2022–2024, which is why we keep that reference visible — but it is not the headline.)

The level is robust to the pre-2018 interpolation. The central debt-to-revenue ratio is of the same order whether measured over the full 2002–2024 series (mean ≈52 %), over the six directly-reported Comtrade anchor years alone (2002, 2005, 2009, 2010, 2013, 2017; mean ≈53 %), or over the directly-observed post-2018 period (mean ≈49 %); the central SRR is near 2 in every one of these windows and never falls below 1 (full-window minimum ≈1.7). The interpolated years contribute only visual continuity between the reported anchors (Figs. 4–5); they carry no weight the result depends on, and dropping them shifts the central level by a few percentage points, not across the SRR = 1 line.

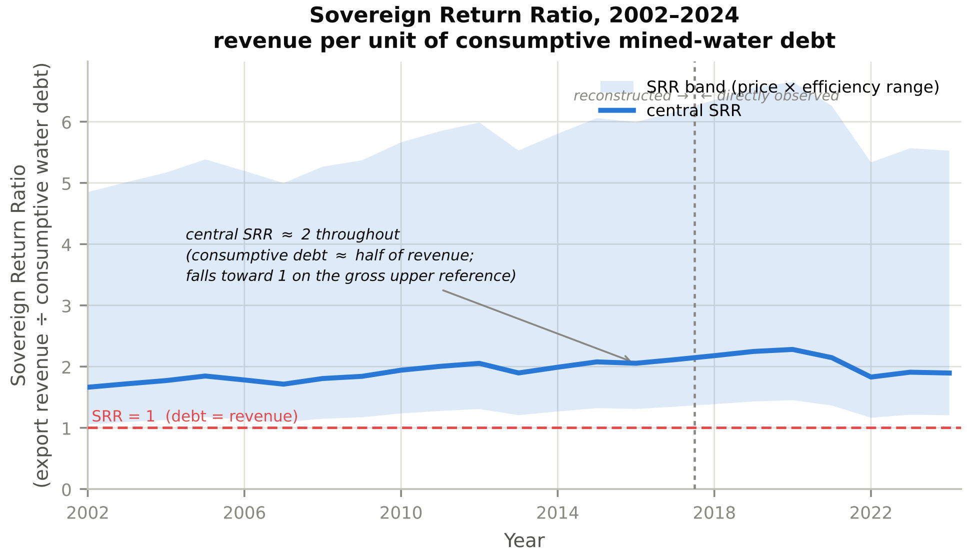


Figure 4: **The Sovereign Return Ratio, 2002–2024 (consumptive basis).** Export revenue divided by the replacement-cost value of the *consumptive* mined water, plotted as a central line (consumptive × $\$0.70\,\mathrm{m}^{-3}$) within a band spanning the price × efficiency range, on the canonical period-mean water-intensity basis. The SRR = 1 line marks where debt equals revenue; the central ratio stays near 2 across the record (consumptive debt ≈ half of revenue) and never falls below 1 — the finding is this stable *level*, not a decline. The band's lower edge approaches 1 only under the gross upper reference. The dashed vertical divider separates the interpolated pre-2018 years from the directly-observed post-2018 period. Constant-2024 US$.

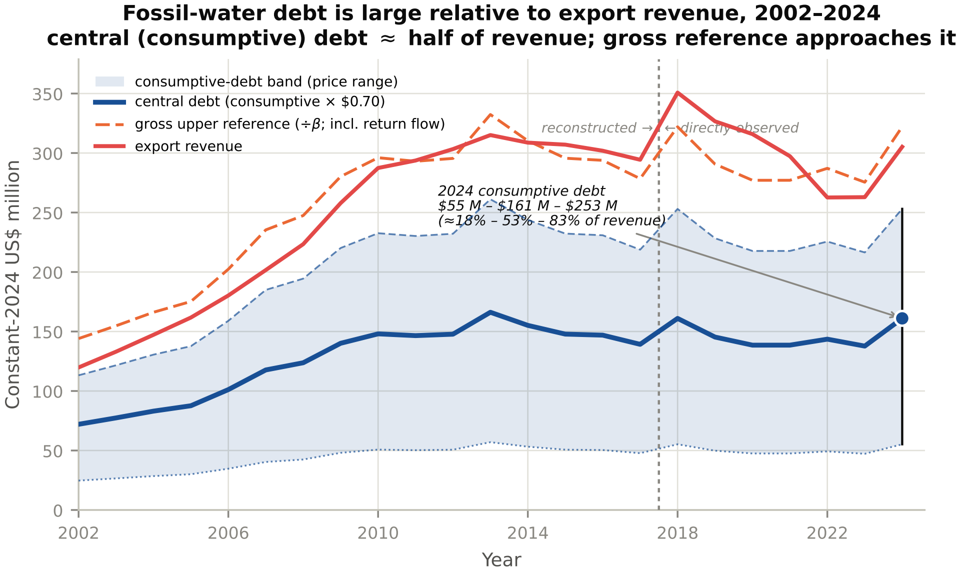


Figure 5: **The fossil-water debt tracks below export revenue, 2002–2024.** The primary (consumptive) ecological-debt band across the price range, with export revenue overlaid and the gross central figure shown as a distinct, clearly-labelled upper reference (dashed), in constant-2024 US$ million. The 2024 consumptive band is marked ($55 M–$161 M–$253 M, ≈ 18 %–53 %–83 % of that year's revenue); the gross upper reference reaches ≈ $322 M (≈ 106 %). Dollars and percentages are read from the same consumptive, period-mean series, so they agree by construction. The dashed vertical divider marks the interpolated / directly-observed boundary at 2017.5.

4.3. Relative decoupling: rising extraction behind falling intensity

Reporting the debt per tonne alone would **invert the paper's message**. Water intensity per tonne *fell* over the study period — a yield artefact, since rising yields divide a large per-hectare water use across more output (Kébili: ∼3,042 → ∼1,620 $\mathrm{m}^3\,\mathrm{t}^{-1}$, 2002→2024). Read naïvely, falling debt-per-tonne looks like improving sustainability. It is the opposite: per-tonne water fell while total extraction rose.

This is **relative decoupling without absolute decoupling** — output-per-unit-resource improving while total resource use climbs (Alcott, 2005, the relative-vs-absolute decoupling distinction). The mechanism here is a *denominator effect*: per-hectare yield rose, dividing a roughly fixed per-hectare requirement across more output, while area and total pumping expanded. We flag the kinship with the **irrigation efficiency paradox** of Grafton et al. (2018) — efficiency gains failing to conserve water at basin scale — but note the difference explicitly: Grafton's is a *supply-side rebound* in which water freed by more efficient delivery is reallocated to more land, whereas our case involves **no efficiency investment reallocating saved water**. Extraction rose because area and application rose as the buffer thinned, not because saved water was redeployed. The finding is therefore relative decoupling with a buffer-thinning driver, for which Grafton is a related water-sector illustration rather than the operative mechanism.

The pattern is visible directly on a common timeline (Fig. 6a): consumptive water per tonne falls while total extraction and total debt climb. The three-channel decomposition of the SRR (§2.4) separates the drivers:

Channel	Direction	Evidence
Revenue (real export price/t)	**falling**	real unit price declines ∼\$0.18 $\mathrm{kg}^{-1}\,\mathrm{yr}^{-1}$ post-2018; peak 2010
Intensity (water per tonne)	**falling**	yield gains roughly halve per-tonne water
Scale (total water extracted)	**rising**	oasis area and per-hectare application both climb

The relationship is governed by **rising total extraction meeting (recently) falling real export prices, even as per-tonne intensity improves** — relative, not absolute, decoupling. The re-coupling index (§2.3; Fig. 6b) is a qualitative supporting diagnostic consistent with this scale channel: a rising index is consistent with the interpretation that as the irrigation buffer thins, per-hectare yield re-couples to atmospheric water demand and more land and more pumping are drawn in to sustain output. The index is read as parallel, supporting evidence; it is **not** fitted into, and does not enter, the debt or SRR calculation.

Under leakage-free within-window detrending — the principled primary specification for a rolling index, since it uses only each window's own data and imports no structure from outside the window — the climate–yield re-coupling index (Gasmi et al., 2026) rises in Tozeur ($+0.042\,\mathrm{yr}^{-1}$, HAC $p = 0.002$, $n_{\mathrm{eff}} \approx 3$) and Kébili ($+0.037\,\mathrm{yr}^{-1}$, HAC $p < 0.001$, $n_{\mathrm{eff}} \approx 9$) — the two governorates where per-hectare yield is physically unambiguous (palm fraction ≈ 1.0). We note that this within-window specification is also the specification under which Kébili reaches its highest significance: under the full-window sensitivity Kébili's slope attenuates to $+0.017\,\mathrm{yr}^{-1}$ ($p \approx 0.07$), so its trend is genuine and directionally robust but specification-dependent, and we report the full-window $p \approx 0.07$ alongside the primary figure wherever the Kébili result is cited. The within-window choice has an independent, principled justification (leakage-free locality), but the coincidence with maximal significance is stated rather than left implicit. Tozeur is the sole specification-invariant case (its slope is essentially unchanged across specifications), though its $n_{\mathrm{eff}} \approx 3$ means even its significance rests on very few independent observations. In Gafsa the trend is positive but attenuates substantially under quadratic detrending and is reported only tentatively; in Gabès no significant trend is present ($p = 0.12$). These trends rest on overlapping windows with a small effective sample ($n_{\mathrm{eff}} \approx 3$–$13$; Supplementary Table S2), so they are suggestive rather than decisive; taken together with the documented aquifer drawdown, the structural break in terrestrial water storage, and the proliferation of unauthorized wells, they are consistent with a thinning irrigation buffer that re-couples yield to atmospheric water demand. Read this way, per-tonne efficiency improvement is not the reassurance it appears — the appearance of doing more with less while the stock that underwrites the whole system is drawn down faster.

The three-channel decomposition of §2.4 makes the pattern explicit (Fig. 6a). Over 2002–2024 the revenue channel is +0.935 in logs (real export value, \$120 M → \$305 M), but it is almost entirely offset by a volume-of-water channel of +0.805. That volume channel is itself the sum of a falling intensity term (−0.491, consumptive water per tonne down from ∼2,460 to $\sim 1{,}500\,\mathrm{m}^3\,\mathrm{t}^{-1}$ as yields rose) and a rising scale term (+1.295, export tonnage roughly tripled). Netting revenue against volume in the identity leaves $\Delta \ln SRR = +0.131$:

the return ratio barely moved over twenty-two years despite revenue nearly tripling, because the scale of water extraction grew almost as fast as revenue and outran the efficiency gain from rising yields. The growth in export earnings was underwritten by mining far more fossil water, not by mining it more valuably. (The decomposition is basis-invariant: dividing both endpoints by a constant efficiency β cancels in the log-ratio, so these channels are identical whether stated on the consumptive or the gross intensity; only the reported volumes differ — $103 \to 230\,\mathrm{Mm}^3$ consumptive.)

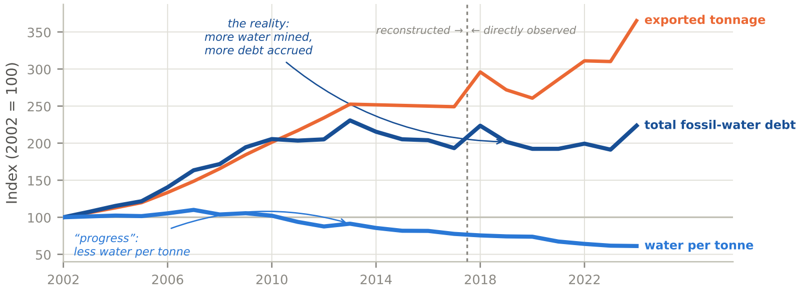
The price-of-water channel is held at zero *by construction* — replacement cost enters as a fixed 2024 shadow-price band rather than a time series, so this flat channel is a modelling choice, not an empirical finding, and Fig. 6a is annotated accordingly. The direction of that choice is worth stating. Desalination costs fell in real terms over the study period, so a time-varying replacement cost would assign the earlier years a *higher* price than the constant-2024 figure we use — raising the early-period debt. Holding the price constant is therefore the conservative treatment, not a convenience. We emphasise, consistent with §4.2, that the decomposition explains the *level* relationship between debt and revenue rather than certifying a secular decline: on the consumptive basis the central Sovereign Return Ratio stays near 2 across the record.

4.4. Destinations and the trade dimension

The exported water is attributed to destinations from Comtrade partner data. In 2017 (a directly-reported year), the largest destinations of Tunisian dates by value were Morocco (∼\$49 M; also the largest single destination by volume), followed by an EU bloc — Germany, France, Spain, Italy, the Netherlands, Belgium — that is the largest destination *group* by aggregate value, with further volumes to Malaysia, Indonesia, the United States, Russia, and Turkey. This confirms a "mostly Europe and Morocco" export geography and identifies the EU importers to whom emerging supply-chain due-diligence obligations would attach (§5).

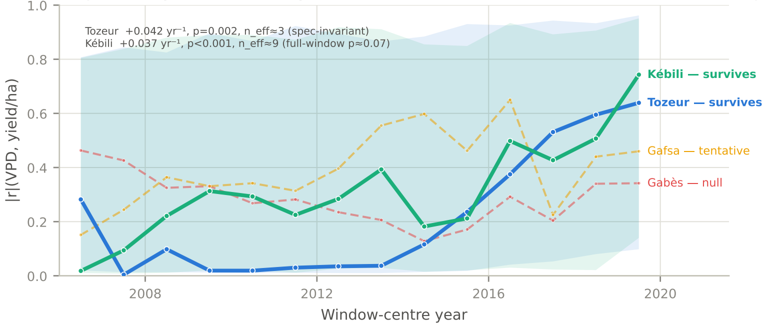
The destination mix complicates a simple North–South reading. The export geography sorts into three distinct channels, and the ecologically-unequal-exchange framing (§5) applies to them unequally. The EU bloc (∼44 % of value) is the classic **North–South** case: fossil natural capital liquidated in a lower-income producing region to supply higher-income consumers, the transfer the unequal-exchange tradition describes. Morocco — the **single largest**

(a) Relative decoupling: per-tonne water falls while total extraction and debt climb



Δln SRR decomposition (2002→2024): revenue +0.935, intensity -0.491, scale +1.295 → Δln SRR +0.131.

(b) Supporting diagnostic: yield sensitivity to atmospheric demand rises in the main oasis governorates



Within-window detrending; shaded = per-window bootstrap 95% band (survivors). Wide ribbons: the trend, not any single window, carries the signal.

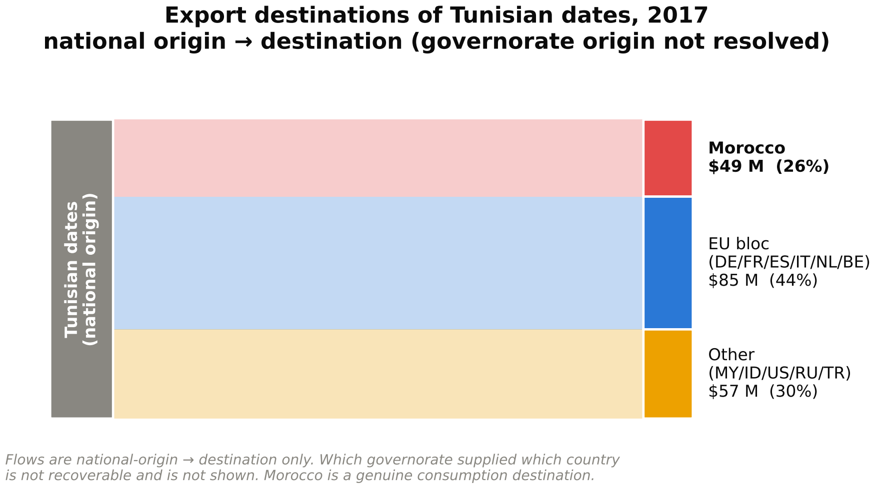
Figure 6: **Relative decoupling and a supporting diagnostic.** *(a)* Indexed (2002 = 100) time series showing consumptive water per tonne falling while exported tonnage and total consumptive debt climb — relative, not absolute, decoupling; the $\Delta \ln SRR$ decomposition (revenue +0.935, intensity -0.491, scale +1.295 $\rightarrow$ $\Delta \ln SRR$ +0.131; basis-invariant) is annotated, with the price-of-water channel zero by construction. *(b)* **Supporting diagnostic: yield sensitivity to atmospheric demand rises in the main oasis governorates.** The climate–yield re-coupling index (Gasmi et al., 2026): rolling within-window $|r|$(VPD, yield/ha) per governorate, with per-window bootstrap 95 % bands. Tozeur and Kébili (palm fraction ≈ 1.0) are foregrounded and their HAC slopes, p-values, and effective sample sizes annotated (Kébili's within-window $p < 0.001$ attenuates to $p \approx 0.07$ full-window, $n_{\text{eff}} \approx 9$; Tozeur $n_{\text{eff}} \approx 3$); Gafsa is tentative and Gabès null. The wide ribbons are the honest message — the trend across windows, not any single window, carries the signal, and n_{eff} is small throughout (Supplementary Table S2). The index is a qualitative supporting diagnostic consistent with the rising-scale channel; it is not an input to the debt calculation.

destination (∼\$49 M, ∼26 % in 2017) — is a **South–South** flow to a fellow water-stressed, middle-income country, which does not fit the North–South mold: the depletion is still externalized, but not toward a rich importing core. The remaining ∼30 % is spread across middle- and high-income destinations. What unifies the three is **general externalization**: whatever the importer's income, the permanent loss of the aquifer stays with the producing region while the commodity and its revenue leave. The accounting result — that the embedded fossil-water debt is of the same order as revenue — is invariant to which channel the dates travel; it is the *unequal-exchange interpretation* that is channel-specific, strongest for the EU bloc and weaker (though not absent) for the South–South flow to Morocco.

Governorate resolution stops here, by design. We attribute exported water to destinations at the **national-origin** level only. The origin split (which governorate grew the exported crop) is estimable from production shares; the destination split (which country received it) is from Comtrade; but the **joint origin×destination cell** — which governorate's dates went to which country — is *not recoverable* without a proportional-mixing assumption the data cannot support. We therefore make **no governorate-by-destination claims**, and the flow diagram (Fig. 7) is national-origin → destination. Morocco's share is treated as genuine consumption; we do not construct a re-export sub-story, which the data cannot substantiate.

5. Discussion

The central result has a plain economic meaning. The replacement-cost value of the fossil water embedded in exported dates is of the same order as export revenue — a central estimate near half on the conservative consumptive floor, rising to parity and above on the gross ceiling. The range between the secure floor and the conditional ceiling is the result, and both ends carry the same economic message. In natural-capital-accounting terms (El Serafy, 1989; Hartwick, 1977, the genuine-savings literature, Hamilton and Clemens, 1999), a depletion cost of this magnitude is one that ordinary income accounting records nowhere: conventional trade statistics book the proceeds of aquifer drawdown as export earnings rather than as the drawing-down of a capital asset that does not regenerate. We do not claim to compute the El Serafy set-aside itself, nor to observe whether the economy reinvests these rents through other channels; our contribution is narrower and firmer — to


Export destinations of Tunisian dates, 2017
national origin → destination (governorate origin not resolved)

Flows are national-origin → destination only. Which governorate supplied which country is not recoverable and is not shown. Morocco is a genuine consumption destination.

Figure 7: **Export destinations of Tunisian dates, 2017.** National-origin → destination Sankey (value, M USD): Morocco (\$49 M, 26 %), an EU bloc of Germany/France/Spain/Italy/Netherlands/Belgium (\$85 M, 44 %), and other destinations (\$57 M, 30 %). Flows are national-origin → destination only; which governorate supplied which country is not recoverable from the data and is not shown. Morocco is treated as a genuine consumption destination, not a re-export waypoint.

price the depletion at the commodity level and show it is large relative to what the trade earns. Whether that constitutes a Hartwick failure depends on reinvestment we do not measure; what we establish is that the unpriced depletion is of the same order as the revenue, which is the quantity a genuine-savings correction would need. The ecologically-unequal-exchange tradition (Hornborg, 1998; Martínez-Alier, 2002; Dorninger et al., 2021) describes such appropriation in economy-wide aggregates; the contribution here is to price it, from satellite evidence, for a single commodity and a single non-renewable stock. That said, the unequal-exchange reading must be applied with care to this trade: as §4.4 sets out, the single largest destination is Morocco — a water-stressed, middle-income country — so the North–South appropriation framing fits cleanly only the $\sim$44 % EU bloc, while the Moroccan flow is South–South and the accounting result itself is invariant to destination income. We therefore lean on the natural-capital-accounting claim (unpriced depletion of the same order as revenue), which holds regardless of destination, and treat the unequal-exchange interpretation as channel-specific rather than a blanket North–South story.

The contribution should be located precisely against the tools it recombines. Virtual-water and water-footprint accounting (Allan, 1998; Hoekstra and Mekonnen, 2012) made water in trade visible but treats renewable and non-renewable water alike and stops at volume; Dalin et al. (2017) isolated the non-renewable groundwater *fraction* embedded in food trade globally but did not price it or carry it on a commodity; the groundwater footprint (Gleeson et al., 2012) flags overdraft without monetizing it; SEEA and genuine-savings accounting (Hamilton and Clemens, 1999) book depletion at the *national* scale, not per traded product; and replacement-cost valuation (Barbier, 2000; Shabman and Batie, 1978) is a general method not previously applied to a fossil-water export debt. What is new here is none of these instruments individually but their assembly into a **commodity-level, depletion-adjusted export diagnostic that works under incomplete extraction records** — pricing the permanent loss of a specific non-renewable stock, carrying it on the specific traded product, and using gravimetry only to establish that the depletion is real where the books are not to be trusted.

5.1. Applicability to other fossil-aquifer export systems

We demonstrate the framework for Tunisian dates and do not claim a second worked case; what we can show is that the *specific* data each instrument requires already exists, by named source, for a class of fossil-aquifer export systems. The three instruments need three retrievable inputs — a satellite terrestrial-water-storage series, a satellite-derived cultivated-area trajectory, and a sub-national yield/production record — plus published local crop-water and replacement-cost coefficients. Table 4 names the actual instantiating source for each input across five candidate systems, rather than asserting global availability in the abstract; the point is specificity, and where a source is weak or missing we say so.

The satellite inputs are the easy ones: the JPL mascon TWS product and MODIS/Sentinel area retrievals are the *same* products used here, available for all five systems. The binding constraint is the sub-national yield/production record — strongest for the Ogallala (USDA NASS county data are complete and annual), workable but coarser for the others. This is why we present transferability as a demonstrated-input-availability claim, not a demonstrated result: the Ogallala is the obvious next application because its yield record is the strongest, but computing its debt is future work. Two conditions scope validity: a single crop should dominate regional water use (so the storage signal is not confounded), and the local replacement-cost backstop must be identifiable.

The transfer is not automatic in a second respect: in mixed agricultural basins where no single crop dominates regional water use, the records-undercount comparison of §2.2 weakens (a bottom-up single-crop footprint no longer approaches the regional budget) and the re-coupling index must be interpreted per-crop. Scoping the method to systems with a dominant water-consuming crop is thus intrinsic to its validity.

We therefore state the transferability claim precisely: the framework is **transferable to fossil-aquifer export systems that meet a specific set of data and dominance conditions**, not to any such system in the abstract. Those conditions are (i) a single crop dominating regional water use, so the storage signal is not confounded and the single-crop footprint can meaningfully approach the regional budget; (ii) published, locally appropriate crop-water requirements for that crop; (iii) a sub-national yield/production record; (iv) an identifiable local replacement-cost backstop; and (v) basin-scale depletion evidence (GRACE or equivalent). Tunisian dates meet all five, which is why the case is unusually clean. Finally, we are explicit about the division of labour with our companion study: the hydrological and agronomic backbone this demonstration draws on — the $-16.6\,\text{cm}$ TWS loss and its 2007 structural break, the well-proliferation counts, and the

Table 4: **Named data sources instantiating the three method inputs across candidate fossil-aquifer export systems.** Entries name the specific product/record that would supply each input. The table records that the inputs *exist and are identifiable*, not that the debt has been computed — each system requires its own per-basin analysis, and the corroboration logic is strongest where a single crop dominates regional water use.

System / crop	Overdraft status (source)	TWS series	Cultivated-area source	Sub-national yield record
NWSAS, Algeria — dates	Continental Intercalaire / Complexe Terminal overdraft (OSS 2008 SASS assessment)	JPL GRACE/GRACE-FO mascon RL06.3 (Watkins et al., 2015; Wiese et al., 2016)	MODIS NDVI (MOD13Q1) + Sentinel-2; same persistent-NDVI method used here	ONS Algérie / MADR wilaya-level; annual only
Saq & Wajid, Saudi Arabia — wheat, alfalfa	GRACE-documented rapid depletion (Famiglietti, 2014; Rodell et al., 2009)	JPL mascon RL06.3 (as above)	MODIS/Sentinel + center-pivot mapping	GASTAT regional crop production; region-level
Ogallala / High Plains, USA — maize, cotton, wheat	USGS High Plains water-table monitoring (long-term decline)	JPL mascon RL06.3 (as above)	USDA NASS Cropland Data Layer (30 m, annual) — strongest of the five	USDA NASS county-level yield (annual, complete) — strongest of the five
North China Plain, China — wheat, maize	Severe GRACE-documented depletion (Rodell et al., 2009)	JPL mascon RL06.3 (as above)	MODIS/Sentinel + published NCP cropland maps	Provincial/prefecture yearbooks; access varies
Indus basin, Pakistan/India — rice, wheat, cotton	Among the most stressed transboundary aquifers (MacDonald et al., 2016)	JPL mascon RL06.3 (as above)	MODIS/Sentinel; canal-command cropland maps	District-level (PBS / India DES); uneven, transboundary split complicates attribution

re-coupling index itself — is established in Gasmi et al. (2026); the present paper's own contribution is the valuation layer that converts that established degradation into a shadow-priced, commodity-level export debt. Applying the framework to a second system would require its own backbone, whether imported from a companion analysis or built anew.

5.2. Limitations and scope

The analysis is constrained in several respects, each of which bounds what the demonstration establishes.

The valuation headlines the consumptive quantity rather than gross extraction, and this is a conservatism choice, not a claim about return flow. The consumptive ET is the secure floor — water unambiguously and permanently lost — while the gross figure adds conveyance and percolation losses whose treatment as permanent loss is conditional on the efficiency assumption. For the confined fossil reserve those losses do not return to the mined stock (§2.1), so headlining the consumptive floor in fact *understates* the depletion of that stock; we headline it because it is the bound we can defend without reservation, and carry the gross figure as a labelled upper reference (Table 3, rows 5–6). Pricing gross extraction as permanent loss would raise the figure from $\sim$53 % to $\sim$106 % of export revenue: the consumptive floor and the gross ceiling are the two ends of the reported range, and the range is the result.

The GRACE signal is coarse: at mascon resolution of order 3° it integrates the wider basin rather than the oasis footprint, so it cannot attribute a date-specific footprint, and we do not use it to. Its role is confined to independent confirmation, at basin scale, that depletion is occurring; the evidence that records undercount is administrative and bottom-up (§4.1), and the resolution limit is therefore immaterial to the role GRACE actually plays.

The replacement-cost valuation could be read as inflating the debt. It is a valid input valuation (Barbier, 2000) disciplined by the Shabman and Batie (1978) conditions, and its willingness-to-pay is anchored to the domestic and municipal water the depletion threatens rather than to the export crop (§2.4); results are reported across the full \$0.24–\$1.10 m^{-3} price band rather than at a single point.

The governorate of export origin is unrecorded and time-varying. We therefore test the debt-to-revenue ratio under differential per-governorate water intensity and price (§4.2): it is robust for the realistic Deglet-Nour-dominated mix (+2 %), with the residual bounded at +35 % under an un-realistic commodity-grade skew — a bounded uncertainty rather than an assumption asserted to cancel. A fossil-water debt might also be thought redundant with existing scarcity indices; it is not, since flow-based indices (Boulay et al., 2018) price the temporary scarcity of a renewable flow and the groundwater-footprint metric (Gleeson et al., 2012) flags overdraft without pricing it or carrying it on a traded commodity, whereas we price the permanent destruction of a non-renewable stock and attribute it to the exported product (Dalin et al., 2017).

The Sovereign Return Ratio is reported as a level, not a trend. The load-bearing finding is that the consumptive debt is of the same order as revenue (roughly half; SRR ≈ 2 across the record, never below unity); the modest recent softening is price-driven and may be transient (§4.2), and only on the gross upper reference does the ratio approach unity. Finally, three data constraints bound the demonstration: the pre-2018 export series is log-linear interpolation between the six directly-reported Comtrade years and adds no information beyond them, but the result rests on the directly-observed post-2018 period and holds on the reported anchor years alone; the joint origin×destination cell is not recoverable, so no governorate-to-country claims are made (§4.4); and a fully bottom-up estimate of dates' share of all regional crop water cannot be built from public data, because the southern governorates do not publish crop-area-by-vocation, so the undercount evidence of §4.1 does not require it and the share is left for future work.

5.3. Policy legibility

The framework makes embedded fossil-water depletion auditable at the commodity level, which is the unit at which emerging supply-chain regimes operate. The EU Corporate Sustainability Due Diligence Directive (Directive (EU) 2024/1760; European Union, 2024), as subsequently amended through the "Omnibus" simplification package, illustrates the type of supply-chain regime for which commodity-level water-depletion metrics may become legible: it obliges large in-scope firms to identify and account for adverse environmental impacts across their chains of activity, and the impacts it lists include excessive water consumption and resource degradation that impairs food production or access to safe drinking water. We do not claim current law requires this metric; the point is architectural. Two limits keep the claim modest. First, the regime is still being built, so the significance is legibility, not an imminent deadline. Second, such obligations attach only to EU importers — the ∼44 % EU bloc (§4.4) — and therefore cover under half the trade and miss the single largest buyer, Morocco, which sits outside the EU regime entirely. The framework's value is scientific before it is regulatory.

6. Conclusions

Fossil groundwater irrigates a rising share of traded food, and its depletion is irreversible, yet no method has quantified the ecological debt carried by the exported product where the extraction driving it goes unrecorded. We have specified such a method: satellite gravimetry to corroborate that records undercount, satellite-derived area with published crop-water requirements to bound the mining footprint as a range, and a climate–yield re-coupling index to diagnose whether the irrigation buffer that stabilizes yields is weakening — combined, through a replacement-cost shadow price, into a Sovereign Return Ratio.

Demonstrated on Tunisian date exports over 2002–2024, the replacement-cost debt of the embedded fossil water is of the same order as export revenue: on the physically conservative consumptive basis it is roughly half of revenue (central $\sim$53 %; about one dollar of unpriced fossil-water debt in every kilogram exported for about two dollars), remaining at least a sixth of it under the most conservative combination of every assumption and reaching parity and above once irrigation losses are counted as permanent or a higher replacement price is used. In the terms of natural-capital accounting, the trade books this aquifer drawdown as ordinary income rather than as the depletion of a capital asset; whether that constitutes a formal Hartwick failure depends on economy-wide reinvestment we do not measure, but the unpriced depletion is of the same order as the revenue the trade earns. The relationship reflects relative rather than absolute decoupling — per-tonne water intensity improved while total extraction rose — so apparent efficiency gains mask rising absolute extraction from a non-renewable stock. Demonstrated here for Tunisian dates, the framework is transferable to fossil-aquifer export systems that meet its data and dominance conditions — a dominant water-consuming crop, published crop-water requirements, a sub-national yield record, an identifiable replacement-cost backstop, and basin-scale depletion evidence (§5.1) — and renders embedded-water depletion legible for the supply-chain accountability regimes now taking shape.

Data availability

The compiled datasets and analysis code supporting the reported results are available at `https://github.com/tanitdata/DatePalm` (in the `ecological_debt/` directory) and archived at `https://doi.org/10.5281/zenodo.21251191`. Annual date-export volume and value for 2002–2017, together with the destination-country breakdown, are from UN Comtrade (HS 080410, reporter Tunisia); monthly export volume and value for 2018 onward are from Tunisia's National Institute of Statistics (INS) open-data DataStore. GRACE and GRACE-FO mascon solutions (JPL RL06.3M v04) are from NASA's Physical Oceanography Distributed Active Archive Center (PO.DAAC). Ground-truth production and aquifer records (ONAGRI, CRDA) are publicly available from the Tunisian agricultural open-data portal (`https://catalog.agridata.tn`); climate variables are from ERA5-Land reanalysis (Copernicus Climate Data Store, ECMWF). Tunisian open-data portals were accessed programmatically through the TanitData MCP servers for agricultural (`https://github.com/tanitdata/agridata-mcp`) and INS (`https://github.com/tanitdata/ins-tunisia-mcp`) data.

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Supplementary material

Supplementary Table S1. Inputs to the debt calculation: source, scale, coverage, transformation, and uncertainty. Every quantity entering the fossil-water debt calculation, with its provenance and the principal uncertainty a reader should carry. Satellite and reanalysis inputs are the same products used throughout the companion study (Gasmi et al., 2026); economic coefficients are from the cited literature.

Input	Source	Spatial scale	Years	Transformation applied	Principal uncertainty
Date production	ONAGRI (national agricultural observatory)	Governorate	2002–2024	Period-mean (5-yr trailing) to suppress alternate-bearing noise	Administrative; alternate-bearing noise in single years
Cultivated oasis area	Satellite persistent-NDVI trajectory (Stage-1, MODIS/Sentinel)	Governorate	2002–2024	Persistent-NDVI area × palm-fraction correction	Snapshot NDVI-to-palm mapping; largest for low-fraction governorates
Palm-fraction correction	Stage-2 random-forest classification (2024 snapshot)	Governorate	2024	Tozeur/Kébili ≈ 1.0; Gabès 0.096; Gafsa 0.040	**Highest-uncertainty input for Gabès/Gafsa** (2024 snapshot, not a time series); Tozeur/Kébili ≈ 1.0 are robust and carry the headline
2024 palm area (≈61,200 ha)	Sum of palm-fraction-corrected governorate areas	Four-governorate	2024	Direct sum	Inherits palm-fraction uncertainty (concentrated in the small Gabès/Gafsa shares)

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Input	Source	Spatial scale	Years	Transformation applied	Principal uncertainty
Net crop-water requirement	Tiba et al. (2025); Haj-Amor et al. (2020) — CROPWAT/field	Governorate	Current	Kébili 11,500; Tozeur 12,671; Gafsa/Gabès 10,446 m^3 ha^{-1} yr^{-1}	Published coefficients; oasis-specific but point values
Consumptive intensity	Net req ÷ period-mean yield	Governorate → national	2024	Production-weighted → ≈1,504 m^3 t^{-1}	Propagates yield and area uncertainty
Irrigation efficiency β	FAO ranges for surface/flood systems	Regional	—	Band 0.37 / 0.50 / 0.65 (0.73 break-even)	Site-specific; carried as a band, not a point — the reason gross is a reference
Export value & volume	INS DataStore 2018–2025; UN Comtrade (HS 080410) 2002–2017	National	2002–2024	2002–2017 log-linear interpolation between six reported anchor years; FX at 2024 annual-average 3.13 TND/USD; CPI-U deflation to constant-2024 USD	Pre-2018 interpolation (result holds on anchor years alone, §4.2); no CIF uplift
GRACE/GRACE-FO TWS	NASA JPL mascon RL06.3 V4	Mascon ∼3°; ∼18,000–40,000 km^2 SASS footprint	2002–2024	Basin storage-change slope; 2017–2018 inter-mission gap interpolated (flagged)	Coarse resolution — corroboration only, no attribution (§2.2); wide integration-area band
Recorded extraction (CRDA)	Regional agricultural authorities	Governorate	2022–2023	Regional total ≈967 Mm^3 yr^{-1}; exploitation rates	The undercount being tested; incomplete by the paper's premise

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Input	Source	Spatial scale	Years	Transformation applied	Principal uncertainty
Replacement (shadow) price	Ghaffour et al. (2013) SWRO cost review; Djerba brackish-water cost	—	2024	Band \$0.24 (brackish RO) / \$0.70 (mid) / \$1.10 (SWRO-high) m^{-3}	**Composition:** plant-gate desalinated-water unit costs (energy + capital + O&M); low edge is brackish-water RO. Excludes downstream conveyance, brine disposal, and distribution, so conservative relative to fully delivered cost

Supplementary Table S2. Re-coupling index — full per-governorate statistics and sensitivities. Rolling 10-year $|r|$(VPD$_{\text{fruit-dev}}$, yield/ha), 14 windows (centres 2006.5–2019.5). Primary specification is **within-window (WW) linear detrending** (leakage-free locality; §2.3); full-window (FW) linear and quadratic detrending are sensitivities. HAC = Newey–West (Bartlett, bandwidth = window length − 1); block bootstrap = 5,000 reps, block length = window length; n_{eff} = autocorrelation-adjusted effective sample size.

Gov.	WW slope (yr^{-1})	WW HAC p	n_{eff}	Block-boot 95 % CI	FW slope / p	Verdict
Tozeur	+0.042	0.002	≈2.6	[−0.004, +0.076]	+0.045 / 0.008	**Survives** — specification-invariant; only caveat is small n_{eff}
Kébili	+0.037	<0.001	≈9.2	[+0.015, +0.039]	**+0.017 / 0.070**	**Survives under WW; marginal under FW** — genuine, directionally robust, but significance is specification-dependent (report WW p beside FW $p \approx 0.07$)
Gafsa	+0.019	0.002	≈12.8	[+0.012, +0.041]	+0.037 / (artefact)	**Tentative** — positive but attenuates under quadratic ($\Delta|r|$ +0.197, not to zero); FW HAC p is a numerical artefact (near-monotonic series), so the WW figure is the defensible one

Gov.	WW slope (yr^{-1})	WW HAC p	n_{eff}	Block-boot 95 % CI	FW slope / p	Verdict
Gabès	−0.011	0.124	≈4.0	[−0.035, +0.004]	−0.002 / 0.90	**Null** — no trend under any specification

Reading. The within-window specification is adopted as primary because a rolling index should be a *local* measure that imports no structure from outside each window; full-window detrending injects the global 2002–2024 trend line into every local correlation, a mild look-ahead/look-behind leakage. WW simultaneously (i) gives Kébili a well-supported slope ($n_{\text{eff}} \approx 9$, $p < 0.001$) where FW made it look marginal, and (ii) cures Gafsa's degenerate FW HAC p. Kébili's dependence on the specification is stated explicitly in §4.3; the coincidence that the principled specification is also the higher-significance one is acknowledged rather than hidden. Block-bootstrap CIs that straddle zero (Tozeur, Gabès) are expected: reshuffling blocks destroys the temporal ordering the trend claim is about, so HAC — not the bootstrap — is the appropriate test for a temporal-trend claim. Effective sample sizes are small throughout ($n_{\text{eff}} \approx 3$–13), so every p-value is read as suggestive, not decisive.