Mechanical Association between Pressure Wave Propagation Time and Constitutive Parameter Damage in Porous Media: Theoretical Derivation and Sensitivity Analysis

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Abstract:

The propagation time of pressure waves in porous media carries critical information about the constitutive parameters of the medium. This study rigorously derives the mechanical relationship between the lag time of pressure wave and the hydraulic parameters of porous media based on the diffusion wave equation. The lag time is shown to be a direct functional of the hydraulic conductivity and specific yield, with sensitivity analysis demonstrating that internal erosion—through simultaneous increase of hydraulic conductivity and decrease of specific yield—causes an accelerated shortening of the lag time. Based on the Kozeny-Carman equation, connecting fine particle loss to changes in porosity and specific surface area, and subsequently to changes in hydraulic conductivity and lag time, A mechanical transfer function is established. Parametric sensitivity analysis on typical earth dam materials reveals shows that a five-percentage-point reduction in fine content can increase hydraulic conductivity by fifty to four hundred percent, and reduce the lag time by fifteen to fifty percent. To remove the interference of geometric boundary effects caused by reservoir water level fluctuations, a medium baseline function method is proposed. By constructing a geometric effect response function using lag time versus reservoir water level data from a stable baseline period, enabling the rigorous separation of the net medium effect from

geometric effects. The theoretical framework built in this study, provides a mechanical foundation for characterizing constitutive parameter damage in porous media through pressure wave propagation time, and puts forward five quantitative predictions for subsequent experimental verification.

Author Keywords: Pressure wave propagation time; Kozeny-Carman equation; Constitutive relationship; Fine particle loss; Internal erosion; Porous media mechanics; Mechanical transfer function.

Introduction

Research Background and Engineering Demand

Porous media are widely encountered in geotechnical engineering, hydrogeology, and energy extraction. During long-term service, earth-rock dams, levees, slopes, and aquifers undergo internal erosion—the migration and loss of fine particles under seepage action, representing one of the most common progressive damage forms in porous media. Statistics indicate that nearly half of earth dam failures are caused by seepage-induced internal erosion (Foster et al. 2000), and the time window from the first detection of increased seepage to dam breach may be only a few hours (Fell et al. 2003). Therefore, developing theories and methods capable of early detection of internal erosion holds significant engineering safety value.

Existing diagnostic methods for internal erosion primarily rely on seepage discharge monitoring, piezometric head analysis, or geophysical exploration. However, the core criterion of these methods is mostly "whether the water level or discharge exceeds a design threshold," belonging to steady-state analysis in the spatial domain. Their fundamental limitation lies in the fact that when internal erosion develops slowly, macroscopic water levels and discharges may not yet exhibit significant changes, while the hydraulic constitutive parameters of the medium have already undergone irreversible alteration.

Mechanical Essence of the Problem

From a mechanical perspective, this problem can be formulated as: How do the constitutive parameters (permeability K and specific yield S) of porous media evolve during the damage process? Can this evolution be sensitively captured through the characteristic time of pressure waves propagating in the medium—the lag time τ?

This problem involves two mechanical levels: at the constitutive relationship level, it is necessary to establish the quantitative relationship between fine particle loss Δf and the changes in permeability K and specific yield S; at the wave propagation level, it is necessary to establish the quantitative relationship between the medium's constitutive parameters and the pressure wave propagation characteristic time τ. Linking these two levels constitutes a complete mechanical chain: "constitutive parameter damage → pressure wave propagation time evolution."

Review of Existing Research

The Richards equation (Richards 1931) described unsaturated seepage in porous media. Van Genuchten(1980) derived a closed-form equation for describing the soil-water characteristic curve and the unsaturated hydraulic conductivity function, It has become a standard model for unsaturated seepage analysis. The propagation of the pressure head simplifies to a diffusion equation under a linearized approximation, the diffusion coefficient ($D=K/S$) is directly determined by the constitutive parameters of the medium. Ferris(1952) first utilized periodic fluctuations of groundwater levels to invert the hydraulic diffusivity of aquifers, and established a quantitative relationship between τ and D. This work laid the foundation for using natural periodic signals as a diagnostic tool. Carslaw and Jaeger(1959) further explored analytical solution methods about the heat conduction and diffusion equations. Under various boundary conditions, He provided a classical mathematical foundation for solving the diffusion wave equation. It remains the authoritative source for the mathematical techniques used in derivation. Townley (1995, 2023) unified the distinction between steady-state and dynamic behavior with the mathematical description of periodic behavior. and developed the theoretical framework for aquifer response under periodic forcing. It is essential for understanding how external hydraulic excitations translate into pressure wave responses. Depner and Rasmussen (2017) completed a monograph introducing the diffusion wave theory of groundwater under periodic forcing, covering attenuation, delay, and wave propagation characteristics in one, two, and three-dimensional flows. All the above studies have jointly established a theoretical foundation for using periodic water level fluctuations to invert the hydraulic parameters of media. However, these works treat τ as a tool for one-time parameter inversion.

The theoretical description of the permeability of porous media is given by the Kozeny-Carman(KC) equation (Kozeny 1927; Carman 1937), which expresses hydraulic conductivity K as a function of porosity n and specific surface area S_s of particles. In essence, It is a constitutive model linking microstructural properties to macroscopic hydraulic behavior. Chapuis and Aubertin(2003) verified the applicability of the KC equation in sandy and coarse-grained soils through extensive experimentation. Rehman et al. (Rehman et al. 2024), published an authoritative review in 2024, which organized the wide-ranging applications of the KC model in mining, petroleum, and geotechnical engineering. It is the most comprehensive literature survey to date for engineering applications of the model. It also confirms the KC equation's status as a standard model for permeability prediction.Carrier(2003) further demonstrated the engineering value of the KC equation in predicting the permeability of geological materials. In the aforementioned studies, the KC equation has primarily been used as a static tool to predict the current value of K.

In the field of complex systems science,Kantelhardt et al.(2002) systematically developed the detrended fluctuation analysis(DFA),in which a scaling exponent $\alpha>0.5$ characterizes the presence of long-range memory in non-stationary time series. Schreiber(2000) proposed transfer entropy as a non-parametric method to quantify the strength and direction of information transfer between two time series, without assuming any underlying model. Scheffer et al.(2009) published their seminal work on early-warning signals for critical transitions in Nature,confirming the universal principle that"a change in the response time scale of a system is a sensitive messenger of an alteration in its internal state".

In the field of internal erosion research,Skempton and Brogan(1994) revealed the controlling role of fine particle content on stress state and erosion susceptibility through classic piping effect experiments.Foster et al.(2000) systematically assessed the internal erosion resistance of dam construction materials. Fell et al.(2003) proposed a method for assessing the time progression of internal erosion development. In recent years, X-ray and CT technology have been used to track the microstructural evolution of soils during erosion processes(Cong et al. 2019), and Thepjunthra et al.(2025) observed that the lag time continuously shortened with the extension of the operation period in dam monitoring.

However, existing research has the following deficiencies: (1) the quantitative relationship between τ and the constitutive parameters of the medium is mostly implicitly embedded in numerical models, lacking rigorous analytical derivation; (2) the influence of fine particle loss on τ lacks a complete transfer function derived from the Kozeny-Carman equation; (3) geometric boundary effects and constitutive damage effects are often confused, lacking a mechanically rigorous decoupling method.

Objectives and Contributions of This Study

This study aims to establish the mechanical association between the pressure wave propagation time and constitutive parameter damage in porous media. The main contributions include:

(1) Rigorous derivation of the analytical expression of the τ-K-S constitutive relationship based on the diffusion wave equation, with sensitivity analysis;

(2) Establishment of a complete mechanical transfer function Δf → K → τ based on the Kozeny-Carman equation, with quantitative predictions through parametric sensitivity analysis for typical parameters;

(3) Proposal of a medium baseline function method to achieve mechanical decoupling of geometric boundary effects from constitutive damage effects.

This paper is a fundamental mechanical theoretical study. The theoretical predictions established provide directly testable quantitative hypotheses for subsequent experimental verification and engineering validation.

Theoretical Basis and Fundamental Assumptions

Diffusion Wave Equation in Porous Media

Unsaturated seepage in porous media is described by the Richards equation(Richards 1931):

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[K(h)\frac{\partial h}{\partial x}\right] \qquad (1)$$

where θ is the volumetric water content, h is the pressure head, K(h) is the unsaturated permeability function, and x is the spatial coordinate.

In the saturated zone or under the small-perturbation linearization approximation, the above equation simplifies to a diffusion equation:

$$\frac{\partial h}{\partial t} = D\frac{\partial^2 h}{\partial x^2} \qquad (2)$$

where the hydraulic diffusivity D is defined as:

$$D = \frac{K}{S} \quad (3)$$

K is the saturated hydraulic conductivity (m/s), and S is the specific yield (dimensionless), representing the volume of water that a unit volume of the medium can release or store under a unit change in head. This equation shares the same mathematical form as the heat conduction equation; its analytical solution methods can be found in the classic monograph by Carslaw and Jaeger (1959).

Fundamental Assumptions

This study adopts the following fundamental assumptions:

(1) Homogeneous medium assumption: At the analysis scale, the hydraulic parameters K and S of the medium are constant or slowly varying functions. For parameter changes caused by internal erosion, the change on an annual scale is much slower than the pressure wave propagation process, allowing a quasi-static approximation.

(2) One-dimensional propagation assumption: Pressure waves propagate primarily in the horizontal direction, and the vertical component can be neglected. This assumption is applicable to regions within the dam body at a certain distance from the upstream boundary. Depner and Rasmussen(2017) pointed out that when the horizontal propagation distance is much larger than the vertical propagation distance, the one-dimensional approximation provides a solution of sufficient accuracy.

(3) Applicability of linearization: The amplitude of reservoir water level fluctuations is small relative to the dimensions of the dam body, and the linearized diffusion equation is applicable. For large-amplitude fluctuations, they can be decomposed into the superposition of multiple small-amplitude sinusoidal waves. Townley(1995, 2023) systematically demonstrated the applicability conditions of linearization in the response of aquifers under periodic forcing.

(4) Quasi-steady-state fluctuation: The period of water level fluctuation (for instance, annual scale) is much larger than the medium response time, and the system is in a quasi-steady state at each moment.

(5) Stability of the coarse particle skeleton: During the internal erosion process, the skeleton structure formed by coarse particles remains stable, and only fine particles (with particle sizes smaller than the throat size of the skeleton pores) are carried away by seepage from the skeleton pores. This assumption is supported by the experimental results of Skempton and Brogan(1994).

Theoretical Derivation of the τ-K-S Constitutive Relationship

Analytical Solution of the Diffusion Wave Equation

Consider a one-dimensional semi-infinite domain $x \geqslant 0$, with the upstream boundary subjected to a sinusoidal water level fluctuation:

$$h(0,t) = h_0 \sin(\omega t) \tag{4}$$

where h_0 is the fluctuation amplitude, ω is the angular frequency. The far-field boundary condition is $h(\infty,t) = 0$. The diffusion equation is:

$$\frac{\partial h}{\partial t} = D\frac{\partial^2 h}{\partial x^2}, \quad x \geq 0, t \geq 0 \tag{5}$$

The complex function method is used for solving. Let $h(x, t) = \mathrm{Im}[H(x)e^{i\omega t}]$, substituting into the diffusion equation:

$$i\omega H(x) = D\frac{d^2 H}{dx^2} \tag{6}$$

The characteristic root is $k = \sqrt{i\omega/D} = \sqrt{\omega/(2D)}(1+i)$. From the far-field condition, the solution is:

$$H(x) = -ih_0 e^{-(1+i)x\sqrt{\omega/(2D)}} \tag{7}$$

Taking the imaginary part yields, the physical solution:

$$h(x,t) = h_0 e^{-x\sqrt{\omega/(2D)}} \sin\left(\omega t - x\sqrt{\frac{\omega}{2D}}\right) \tag{8}$$

This solution shares the same mathematical form as the classical solution of the heat conduction equation under periodic boundary conditions in Carslaw and Jaeger(1959), was first introduced into the field of groundwater dynamics by Ferris(1952), and was further developed and systematized by Townley (1995, 2023) and Depner and Rasmussen(2017). The physical model of diffusion wave propagation is illustrated in Fig.1.

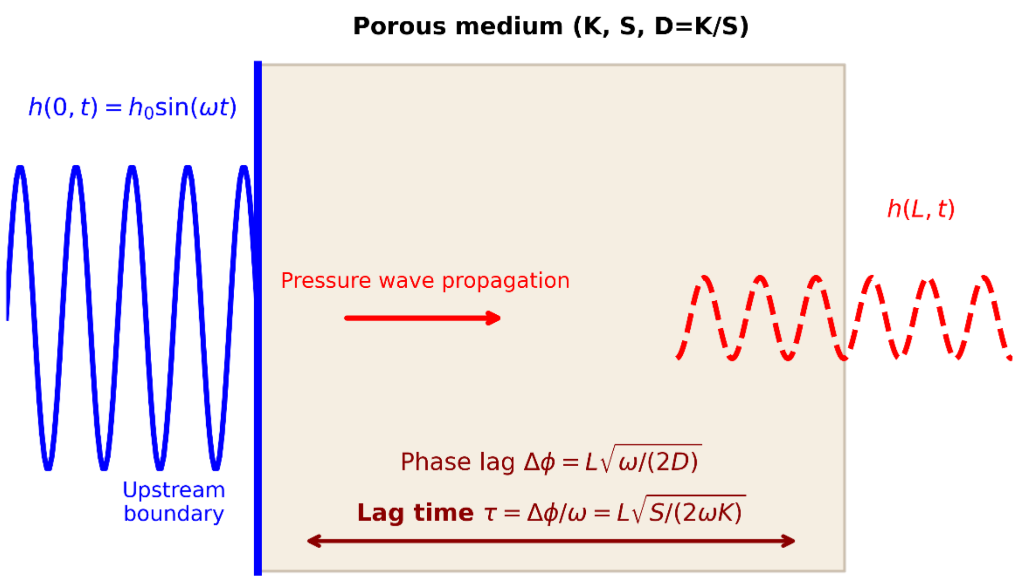

Fig. 1. Physical model of diffusion wave propagation in a semi-infinite porous medium with a sinusoidal boundary condition.

Analytical Expression for the Lag Time

From the above solution, the phase information is extracted. The phase at x is $\Delta\varphi = x\sqrt{\omega/(2D)}$, and the phase difference with respect to the boundary(x = 0) is $\Delta\varphi = x\sqrt{\omega/(2D)}$. The corresponding lag time is:

$$\tau = \frac{\Delta\varphi}{\omega} = x\sqrt{\frac{1}{2D\omega}} \tag{9}$$

Substituting D = K/S and letting L = x be the effective propagation distance, the τ-K-S constitutive relationship is obtained:

$$\tau = L\sqrt{\frac{S}{2\omega K}} \tag{10}$$

This formula indicates that τ is determined by three factors: the constitutive parameters of the medium, K and S (of which K has a greater influence , $\tau \propto 1/\sqrt{K}$); the geometric parameter L; and the external excitation parameter ω. The variation of τ with K and S is illustrated in Fig. 2.

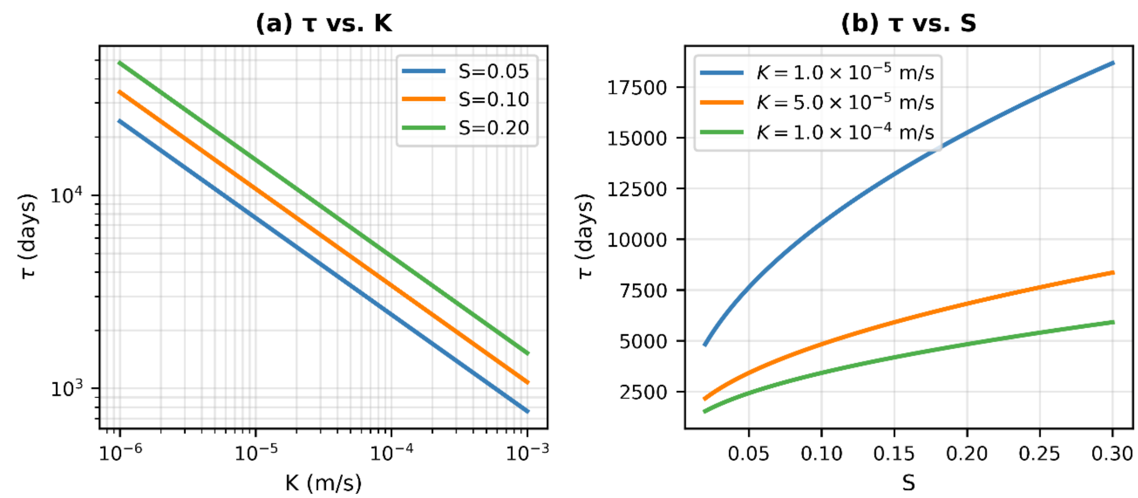

Fig. 2. τ-K-S constitutive relationship: (a) τ versus K under different S values; (b) τ versus S under different K values.

Sensitivity Analysis

Taking partial derivatives with respect to K and S respectively:

(1) Sensitivity of τ to K

$$\frac{\partial\tau}{\partial K} = L\sqrt{\frac{S}{2\omega}} \cdot \left(-\frac{1}{2}\right) K^{-3/2} = -\frac{\tau}{2K} \tag{11}$$

Relative sensitivity:

$$S_K = \frac{\partial\tau/\tau}{\partial K/K} = -\frac{1}{2} \tag{12}$$

1% increase in K leads to approximately a 0.5% shortening of τ.

(2) Sensitivity of τ to S

$$\frac{\partial \tau}{\partial S} = \frac{L}{\sqrt{2\omega K}} \cdot \frac{1}{2} S^{-1/2} = \frac{\tau}{2S} \tag{13}$$

Relative sensitivity:

$$S_S = \frac{\partial \tau / \tau}{\partial S / S} = \frac{1}{2} \tag{14}$$

1% decrease in S leads to approximately a 0.5% shortening of τ.

(3) Combined Sensitivity

When K and S change simultaneously:

$$\frac{d\tau}{\tau} = -\frac{1}{2}\frac{dK}{K} + \frac{1}{2}\frac{dS}{S} \tag{15}$$

Internal erosion simultaneously causes an increase in K (dK > 0) and a decrease in S (dS < 0). Both terms contribute negatively, and the magnitude of τ shortening is greater than that caused by any single factor. The comparison of the sensitivity of τ and h to K is illustrated in Fig. 3.

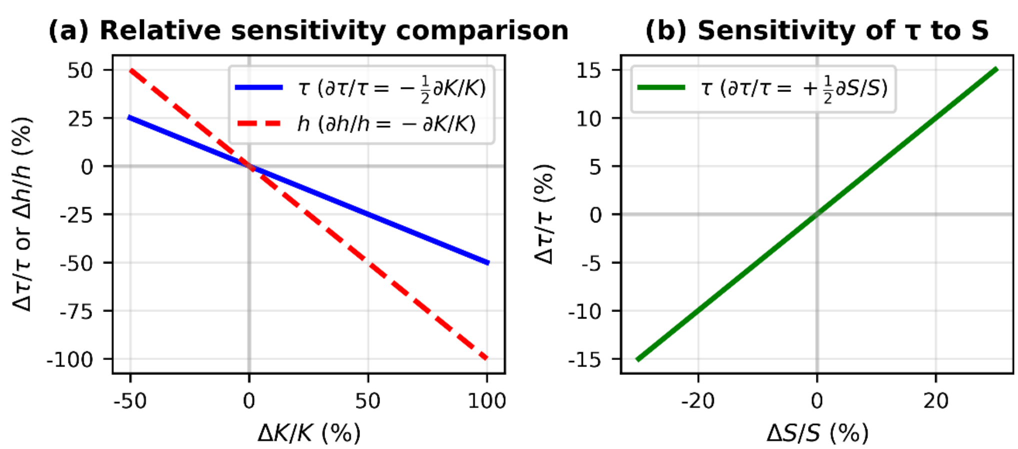

Fig. 3. Sensitivity analysis: (a) relative sensitivity of τ and h to K; (b) sensitivity of τ to S.

Discussion of Corrections under Non-Ideal Conditions

(1) Non-Sinusoidal Fluctuations

Actual water level fluctuations are non-sinusoidal and can be decomposed into multiple frequency components via Fourier decomposition:

$$h(0,t) = \sum_{n} h_n \sin(\omega_n t + \phi_n) \qquad (16)$$

The τ_n corresponding to each frequency differs as $\tau_n \propto 1/\sqrt{\omega_n}$). The τ obtained from cross-correlation analysis is the weighted average of the dominant frequency. As long as the annual cycle is the dominant frequency and remains stable, τ is relatively stable. Townley(1995, 2023) discussed in detail the method for determining the characteristic time of the periodic response under the superposition of multiple frequency components.

(2) Heterogeneous Media

For layered heterogeneous media, the effective diffusion coefficient is the harmonic mean of the diffusion coefficients of each layer:

$$\frac{1}{D_{eff}} = \sum_{i} \frac{d_i}{D_i} \qquad (17)$$

where d_i is the thickness proportion of each layer. When damage occurs only in local layers, the change in D_{eff} is smaller than the change in D_i of the damaged layer.

(3) Finite Boundary Effects

The dam body is a finite domain, and reflections from the downstream boundary can affect the fluctuation. However, when the propagation distance L is much smaller than the dam width, the semi-infinite domain solution is a good approximation. Depner and Rasmussen(2017) discussed the influence of boundary reflections on diffusion wave propagation in finite domains, which can serve as a theoretical reference for further corrections.

Mechanical Transfer Function from Fine Particle Loss to Constitutive Parameters to τ

Geometric Description of Fine Particle Loss

Let the total volume of the soil be V, the volume of solid particles be V_s, and the volume of voids be V_v. The porosity $n=V_v/V$. The solid consists of coarse particles (volume V_c) and fine particles (volume V_f), then $V_s=V_c+V_f$.

The mass content of fine particles f is defined as:

$$f = \frac{m_f}{m_s} = \frac{\rho_f V_f}{\rho_c V_c + \rho_f V_f} \tag{18}$$

When the densities of coarse and fine particles are similar ($\rho_s \approx \rho_c \approx \rho_f$), $f \approx V_f/V_s$.

Let the proportion of fine particle loss be $\alpha (0 \le \alpha \le 1)$, meaning the lost volume is αV_f. After loss: the volume of fine particles $V_f' = (1-\alpha)V_f$, the total volume of solids $V_s' = V_s - \alpha V_f$, and the volume of voids $V_v' = V_v + \alpha V_f$.

The porosity after loss:

$$n' = n_0 + \frac{\alpha V_f}{V} = n_0 + \alpha f (1 - n_0) \tag{19}$$

The fine particle content after loss:

$$f' = \frac{(1-\alpha) f}{1 - \alpha f} \tag{20}$$

The change in pore structure before and after fine particle loss is illustrated in Fig. 4.

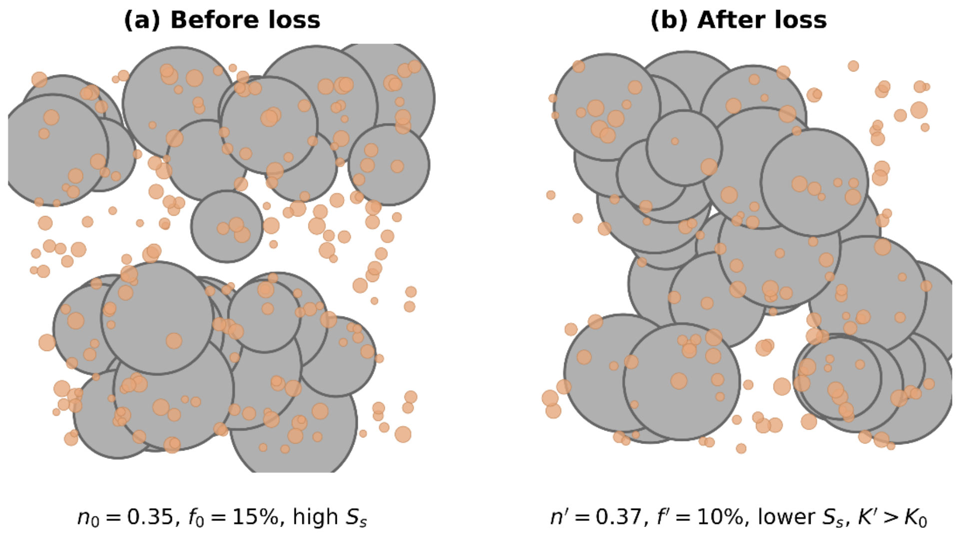

Fig. 4. Schematic illustration of pore structure change due to fine particle loss: (a) before loss; (b) after loss.

Kozeny-Carman Equation and Permeability Change

The Kozeny-Carman equation(Kozeny 1927; Carman 1937) expresses the permeability as a function of porosity and specific surface area of particles:

$$\mathrm{K} = \frac{\gamma}{\mu} \cdot \frac{1}{\mathrm{C}} \cdot \frac{1}{\mathrm{S_s^2}} \cdot \frac{\mathrm{n}^3}{(1-\mathrm{n})^2} \qquad (21)$$

where $\gamma \approx 9.81 \mathrm{KN/m^3}$ is the unit weight of water, $\mu \approx 1.0 \times 10^{-3}$ Pa·s (at 20°C), and $C \approx 4.5$–5.5 is the KC constant. Chapuis and Aubertin(2003) verified the applicability of this equation in sandy and coarse-grained soils through extensive experimentation. The review by Rehman et al.(2024) further confirmed the status of the KC equation as the standard model for permeability prediction in geotechnical engineering.

For a polydisperse system, the specific surface area S_s is related to the particle size distribution. The specific surface area of spherical particles is $S_s = 6/d$, and the weighted average is:

$$\mathrm{S_s} \propto \frac{1-\mathrm{f}}{\mathrm{d_c}} + \frac{\mathrm{f}}{\mathrm{d_f}} \qquad (22)$$

where d_c is the characteristic particle size of coarse particles and d_f is the characteristic particle size of fine particles.

Fine particle loss affects K through two independent mechanisms:

(1) Increase in porosity. $n^3/(1-n)^2$ increases monotonically with n, and an increase in n leads to an increase in K.

(2) Decrease in specific surface area. The specific surface area of fine particles is much larger than that of coarse particles ($d_c/d_f \approx 100$–1000), and the loss of fine particles significantly reducess_s. Since $K \propto 1/s_s^2$, the decrease in s_s leads to a significant increase in K.

The coupling effect of the two mechanisms:

$$\frac{K'}{K} = \left(\frac{S_s}{S_s{}'}\right)^2 \cdot \frac{n'^3/(1-n')^2}{n^3/(1-n)^2} \tag{23}$$

Treatment of the Change in Specific Yield

Fine particle loss has two opposing effects on the specific yield S: the increased proportion of large pores increases the proportion of gravitational water, tending to increase S; the decrease in adsorbed water on the surface of fine particles tends to decrease S. Conservatively, the magnitude of change in S is much smaller than that in K. In the analysis of τ change, S can be approximately taken as constant, or a fluctuation range of ±10% can be introduced.

The Δf → K → τ Transfer Function

Linking the KC equation with the constitutive relationship of τ yields the complete transfer function:

$$\frac{\tau'}{\tau} = \sqrt{\frac{S'/S}{K'/K}} \approx \sqrt{\frac{1}{K'/K}} \text{(S approximately constant)} \tag{24}$$

where K'/K is given by the equation above, and its independent variables are the initial fine particle content f, the loss proportion α, the particle size ratio d_c/d_f, and the initial porosity n_0

Sensitivity Analysis with Typical Parameters

A sensitivity analysis was conducted by selecting the parameter range of typical earth dam construction materials. The baseline parameters were: n_0= 0.35, f = 15%, d_c = 1.0 mm, d_f = 0.01 mm. The results are summarized in Table 1.

Table 1. Results of parametric sensitivity analysis under various fine particle loss conditions.

Case	f0	α	Δf	K'/K	τ'/τ	Δτ/τ0
Baseline	15%	0	0	1	1	0%
Mild loss	15%	20%	-3%	1.54	0.81	-19%
Moderate loss	15%	33%	-5%	2.18	0.68	-32%
Severe loss	15%	50%	-7.50%	3.71	0.52	-48%
Extreme loss	15%	67%	-10%	6.85	0.38	-62%
High initial fines	25%	20%	-5%	2.85	0.59	-41%
Low initial fines	10%	50%	-5%	1.89	0.73	-27%
Fine particle size*	15%	33%	-5%	4.52	0.47	-53%

* Note: For the fine particle size case, df=0.005mm, dc=1 mm, particle size ratio 200.

Main findings: (1) A five-percentage-point reduction in fine content can increase K by 50%–400% and shorten τ by 15%–50%; (2) the higher the initial fine content, the greater the change in τ for the same loss proportion; (3) the finer the fine particles (the larger the particle size ratio), the more significant the specific surface area effect, and the greater the magnitude of τ shortening; (4) the "K increase of 50%–200%" adopted in this paper corresponds to a moderately conservative estimate.

The transfer function curves from Δf to K’ and from Δf to τ are illustrated in Figs. 5, and the multi-parameter joint sensitivity is illustrated in Fig. 6.

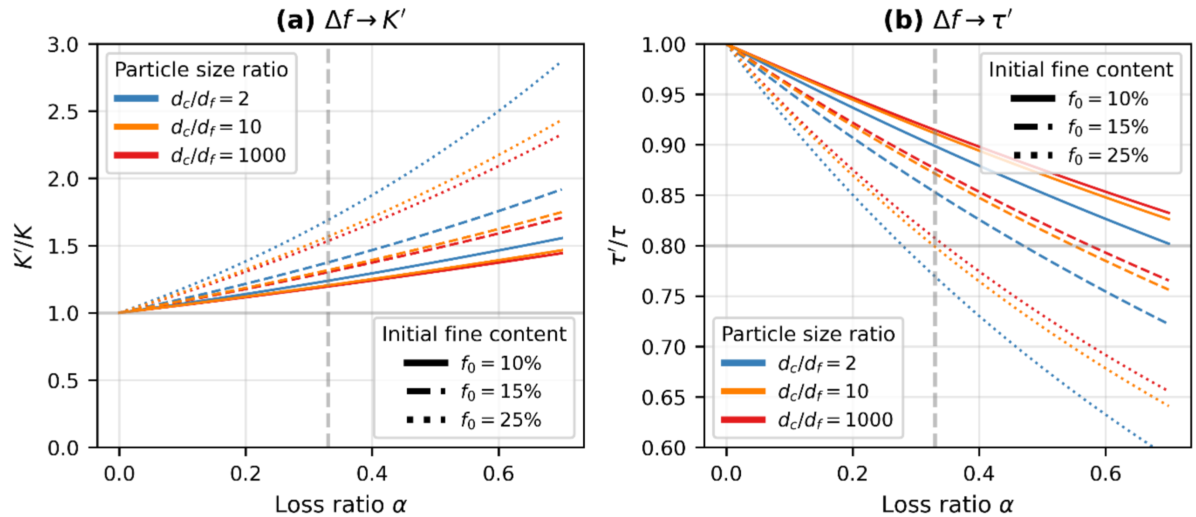

Fig. 5. Transfer functions curve of the $\Delta f \rightarrow K \rightarrow \tau$. (a) K'/K versus α under various f_0 and d_c/d_f. (b) τ'/τ versus α under various f_0 and d_c/d_f.

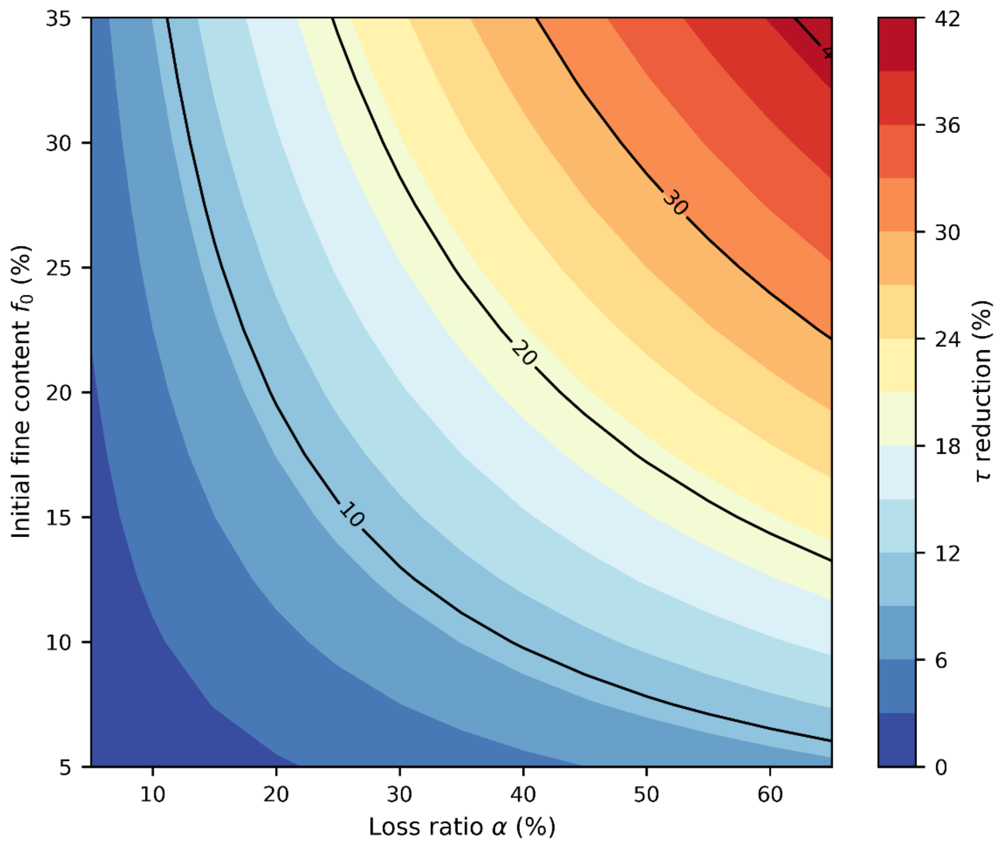

Fig. 6. Sensitivity heatmap of τ reduction as a function of initial fine content f_0 and loss ratio α ($d_c/d_f = 100$, $n_0 = 0.35$).

The above sensitivity analysis results are consistent with the parameter ranges of the KC equation in various geological materials summarized in the review by Rehman et al.(2024), confirming the reasonableness of the parameter settings in this paper.

Mechanical Decoupling of Geometric Boundary Effects

Physical Description of the Geometric Effect

For a porous medium body with an upstream boundary of slope 1:m, a change Δh in the reservoir water level causes the intersection point of the water surface with the boundary to move along the slope by m·Δh, thereby changing the effective propagation distance L of the pressure wave:

$$L_{eff} = L_0 - m \cdot \Delta h \tag{25}$$

When Δh > 0 (water level rise), the intersection point moves in the downstream direction, and L_{eff} shortens. Based on $\tau \propto L$, the relative change in τ caused by the geometric effect is:

$$\frac{\Delta\tau_{geo}}{\tau_0} = -\frac{m \cdot \Delta h}{L_0} \tag{26}$$

The physical mechanism of the geometric boundary effect is illustrated in Fig. 7.

Fig.7 shows that a water level rise (Δh > 0) shifts the water–dam intersection downstream by m·Δh, while the reduction in effective pressure wave propagation distance ΔL at each barometer station remains constant. Because the initial upstream distances L_0 differ, the relative shortening $\Delta L/L_0$ varies markedly: for the far-field piezometer C (large L_0^C) the relative change is small, so the geometric contribution to τ is minor and τ mainly reflects constitutive changes; for the near-field piezometer A (small L_0^A) the same ΔL produces a relative change several times larger, making the geometric effect the dominant driver of τ fluctuations.

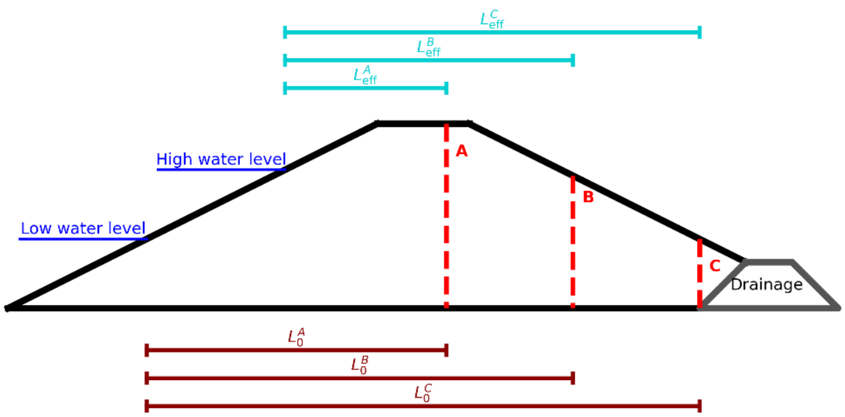

Fig. 7. Standard seepage monitoring cross-section illustrating the geometric boundary effect.

Magnitude Assessment

Taking a typical earth dam with a slope of 1:2.5 and an annual reservoir water level fluctuation of approximately 8m as an example, the magnitude assessment is summarized in Table 2.

Table 2. Magnitude assessment of geometric effect on τ for typical dam parameters.

L_0 (m)	$m \cdot \Delta h_{max}$ (m)	$\lvert \Delta\tau_{geo}/\tau_0 \rvert_{max}$
5	20	±400%
10	20	±200%
15	20	±133%
20	20	±100%
30	20	±67%
40	20	±50%

Near the boundary ($L_0 \leqslant 10$ m), the geometric effect is extremely significant, and the τ change may be dominated primarily by the geometric effect, making it unsuitable for direct diagnosis of constitutive parameter damage.

Medium Baseline Function Method

To rigorously separate the geometric effect from the constitutive damage effect from a mechanical perspective, this paper proposes the medium baseline function method.

Basic Principle: Select a baseline period during which the constitutive state of the medium is known to be stable. During this period, the variation of τ is primarily dominated by geometric effects. Using the τ-h scatter data from the baseline period, establish the response function $f_0(h)$ of τ to the reservoir water level through non-parametric regression:

$$f_0(h) = \mathrm{Spline}[\mathrm{BinMedian}\{\tau_i, h_i\}_{i \in \mathrm{Baseline}}] \quad (27)$$

This function, without relying on simplified geometric assumptions, completely captures the nonlinear pattern of the geometric effect under given constitutive parameters. For any later time, the residual of the observed τ with respect to f_0 (h) is defined as the net medium effect:

$$\Delta\tau_{medium}(t) = \tau_{obs}(t) - f_0(h(t)) \quad (28)$$

Since $f_0(h)$ has absorbed all the geometric effect information, $\Delta\tau_{medium}$ reflects only the changes in the constitutive parameters of the medium. The principle of the medium baseline function method is illustrated in Fig.8.

Scientific justification of the method: (1) It does not rely on simplified geometric assumptions; $f_0(h)$ learns the actual pattern of the geometric effect directly from measured data. (2) The net medium effect has a clear physical meaning—a positive value indicates that τ is longer relative to the same water level baseline (K decreased), and a negative value indicates that τ is shorter (K increased). (3) The sensitivity is quantifiable—the standard deviation of the baseline residuals, σ_{res}, provides an objective judgment threshold. A comparison of this method with existing methods is shown in Table 3.

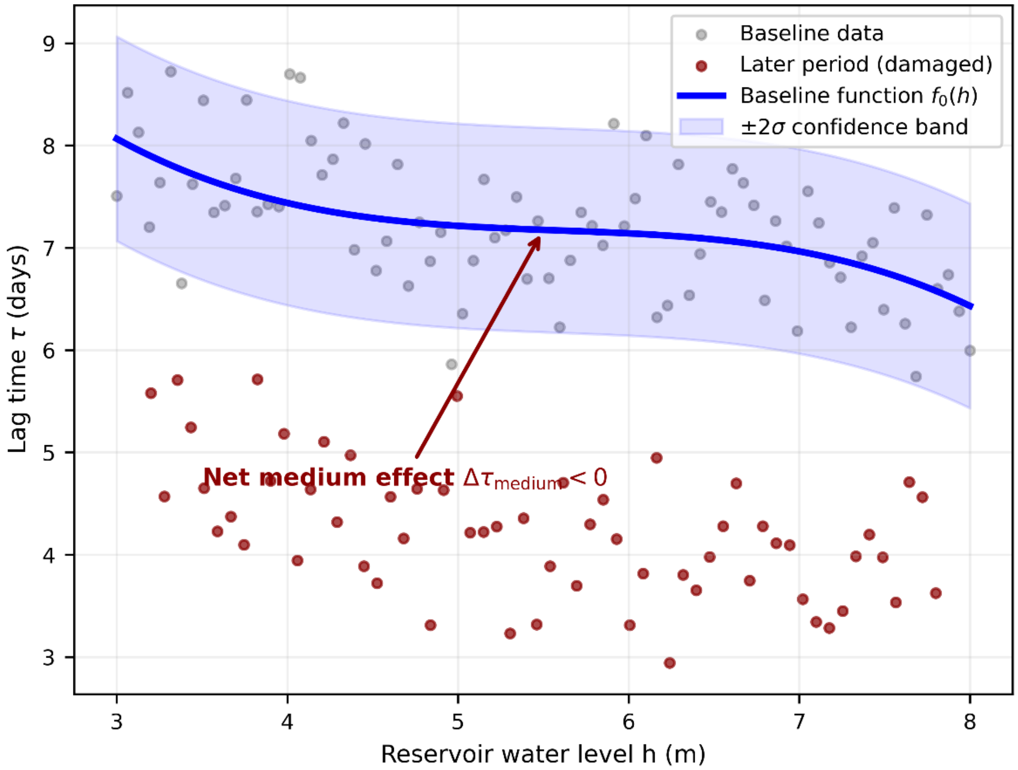

Fig. 8. Illustration of the medium baseline function method for decoupling geometric and constitutive damage effects.

Table 3. Comparison of geometric effect decoupling methods.

Method	Basic Idea	Advantages	Limitations
Mathematical correction	Directly subtract geometric effect using linear relationship ΔL vs. Δh	Simple calculation	Relies on simplified geometric assumptions; does not consider heterogeneity
Stratified comparison	Compare τ in fixed reservoir level intervals across different periods	Intuitive	Information loss from discretization; sparse samples at extreme water levels
Medium baseline function (this study)	Learn $f_0(h)$ from baseline data; calculate later residuals	Independent of geometric assumptions; nonlinear; sensitivity quantifiable	Requires sufficient baseline data

Discussion

Mechanical Connotation of τ as an Indicator of Constitutive Parameter Damage

The rigorously derived τ-K-S constitutive relationship in this paper establishes. At a mechanical level, the theoretical qualification of τ as an indicator of the constitutive parameters of the medium. τ is analogous to the "stiffness" in the stress-strain curve of solid mechanics—a decrease in stiffness indicates material damage, while a shortening of τ indicates an enhancement of the hydraulic diffusion capacity of the porous medium. Both reflect the damage state of the constitutive parameters of the medium through wave propagation characteristics (elastic wave velocity or pressure wave lag).

Boundaries of the Applicability Conditions of the Theory

The derivation in this paper is based on several simplifying assumptions. For media with extremely high fine particle content, such as cohesive soils, the applicability of the KC equation requires modification (e.g., introducing the concept of effective porosity or adopting the Hazen formula as an alternative). Rehman et al.(2024) pointed out that the applicability of the KC equation in fine-grained media such as clays and silts is limited, requiring the introduction of microstructural parameters for correction. For highly heterogeneous fractured media, the diffusion wave model needs to be adjusted to a dual-porosity model. Furthermore, assumption 5 (stability of the coarse particle skeleton) may fail under extreme erosion conditions—when the coarse particle skeleton also collapses, the changes in porosity and permeability will deviate from the predictions of the KC equation, which itself signifies that the damage has entered a new stage.

Quantitative Predictions for Subsequent Verification

The theoretical framework of this paper provides five quantitative predictions amenable to experimental verification:

(1) A five-percentage-point reduction in fine content increases the hydraulic conductivity K by 50%–400% (depending on the particle size ratio and initial porosity).

(2) When K increases by 200%, τ shortens by approximately 42% (under the assumption of constant S).

(3) The geometric effect function $f_0(h)$ should remain stable when the constitutive parameters of the medium are undamaged; i.e., τ at the same water level in different periods should fall within the baseline confidence band.

(4) An irreversible shortening of τ (after deducting the geometric effect) is an indicator of internal erosion; if τ fluctuates only within the range of the geometric effect, there is no constitutive parameter damage.

(5) The τ fluctuation of near-boundary measurement points ($L_0 \leqslant$ 10m) is dominated primarily by the geometric effect; damage diagnosis should rely mainly on measurement points with $L_0 \geqslant$ 15m.

Engineering Application Prospects and Directions for Subsequent Verification

The theoretical framework established in this paper is applicable to the following scenarios: (1) By monitoring the long-term trends of τ, we can detect changes in permeability to provide early warnings for internal erosion in dams and levees; (2) Using natural or artificial water level fluctuations, we can estimate the spatial distribution of K and S to invert aquifer parameters; (3) We can assess the seepage conditions of slopes and foundations. Future studies need to verify the reliability of these theoretical predictions both in controlled lab experiments and real-world projects. The diffusion wave theoretical framework of Depner and Rasmussen(2017) can provide a methodological reference for the extension from the laboratory scale to the field scale. In addition, the macroscopic analytical model for pressure wave propagation in variably saturated porous media proposed by Kalisman et al.(2019) provides a direct mathematical interface for the extension of this theory from the saturated zone to the unsaturated zone.

Conclusions

(1) Based on the diffusion wave equation, the constitutive relationship between the pressure wave propagation time τ and the hydraulic parameters of the medium, $\tau = L\sqrt{S/(2\omega K)}$, was rigorously derived. Sensitivity analysis showed that the relative sensitivities of τ to K and S are -1/2 and +1/2, respectively. When internal erosion simultaneously increases K and decreases S, the two effects superimpose to accelerate the shortening of τ.

(2) Based on the Kozeny-Carman equation, a complete mechanical transfer function linking fine particle loss Δf → porosity n and specific surface area S_s → hydraulic conductivity K → propagation time τ was established. Fine particle loss increases K through the dual mechanisms of increasing effective porosity and decreasing specific surface area of particles, thereby shortening τ.

(3) Typical Parametric sensitivity analysis showed that a five-percentage-point reduction in fine content can increase K by 50%–400% and shorten τ by 15%–50%. The higher the initial fine content and the larger the particle size ratio, the more sensitive τ is to fine particle loss.

(4) A medium baseline function method was proposed. By establishing the baseline period τ-h response function $f_0(h)$, a rigorous mechanical decoupling of geometric boundary effects from constitutive damage effects was achieved. This method does not rely on simplified geometric assumptions, and the net medium effect has a clear physical meaning.

(5) Five quantitative predictions amenable to subsequent experimental verification were provided, offering clear directions for the experimental validation of the theoretical framework.

Data Availability Statement

All data, models, and codes generated or used during this study that support the main findings and further perspectives appear in the published article.

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Notation

The following symbols are used in this paper:

C = Kozeny-Carman constant;

D = hydraulic diffusivity(L^2/T);

d_c = characteristic particle diameter of coarse fraction(L);

d_f = characteristic particle diameter of fine fraction(L);

f = fine particle mass content;

f' = fine particle mass content after loss;

h = pressure head(L);

h_0 = amplitude of water level fluctuation(L);

K = saturated hydraulic conductivity(L/T);

K' = saturated hydraulic conductivity after fine particle loss(L/T);

L = effective propagation distance of pressure wave(L);

L_0 = baseline effective propagation distance(L);

L_{eff} = effective propagation distance after water level change(L);

m = upstream slope ratio (1:m);

n = porosity;

n_0 = initial porosity;

n' = porosity after fine particle loss;

S = specific yield (storage coefficient);

S_s = specific surface area of solid particles;

S_s' = specific surface area after fine particle loss;

t = time;

V = total volume;

V_c = volume of coarse particles;

V_f = volume of fine particles;

V_s = total solid volume;

V_v = void volume;

x = spatial coordinate;

α = fine particle loss ratio ($0 \le \alpha \le 1$);

γ = unit weight of water;

Δf = change in fine particle content;

Δh = change in reservoir water level;

$\Delta\tau_{geometry}$ = geometric effect component of lag time change;

$\Delta\tau_{medium}$ = net medium effect;

μ = dynamic viscosity of water;

τ = pressure wave propagation time (lag time);

τ_0 = baseline memory length;

ϕ = phase;

ω = angular frequency of water level fluctuation.

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