Uncertainty in sea level rise projections due to the dependence between contributors

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Key Points:
- Dependence between sea level contributors should be taken into account for sea level projections
- The uncertainty is underestimated with the independence assumption
- The uncertainty in the dependence structure is a major uncertainty that is always neglected in projections

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Abstract

Using two process-based models to project sea level for the 21st century, it is shown that taking into account the correlation between sea level contributors is important to better quantify the uncertainty of future sea level. In these models the correlation primarily arises from global mean surface temperature that simultaneously leads to more or less ice melt and thermal expansion. Assuming that sea level contributors are independent of each other underestimates the uncertainty in sea level projections. As a result, high-end low probability events that are important for decision making are underestimated. For a probabilistic model it is shown that the 95th percentile of the total sea level rise distribution at the end of the 21st century is underestimated by 5 cm for the RCP4.5 scenario under the independent assumption. This underestimation is up to 16 cm for the 99.9th percentile of the RCP8.5 scenario. On the other hand, assuming perfect correlation overestimates the uncertainty. The strength of the dependence between contributors is difficult to constrain from observations so its uncertainty is also explored. New dependence relation between the uncertainty of dynamical processes and surface mass balance in glaciers and ice caps and in the Antarctic and Greenland ice sheets are introduced in our model. Total sea level uncertainty is found to be as sensitive to the dependence between contributors as to uncertainty in individual contributors like thermal expansion and Greenland ice sheet.

1 Introduction

Global sea level rise has accelerated in the 20th century compared to the late Holocene background rate [Gehrels and Woodworth, 2013; Church et al., 2013; Hay et al., 2015; Kopp et al., 2016; Dangendorf et al., 2017]. An acceleration has also been detected during the satellite altimetry period [Chen et al., 2017; Dieng et al., 2017; Nerem et al., 2018]. This is mainly due to anthropogenic greenhouse gas emissions [Slanger et al., 2016]. It is therefore crucial to make reliable projections of future sea level rise depending on future greenhouse gas emissions to help society make the best mitigation and adaptation decisions [Nicholls et al., 2014; Hinkel et al., 2014; Le Cozannet et al., 2017; Nauels et al., 2017]. The best way to make future projections of complex systems like the earth’s climate is generally to use numerical models that are based on a physical understanding of the relevant processes. Climate models or earth system models are used to project future temperature increase [Collins et al., 2013]. Unfortunately these models do not yet include all of the important processes driving future sea level. Glaciers and ice caps are too small to be resolved by their coarse spatial resolution. Ice sheets are large enough but their long time scale of adjustment and sensitivity to small circulation and temperature biases still make it challenging to include them in fully coupled models [Vizcaíno et al., 2010; Joughin et al., 2012; Lenaerts et al., 2015].

Until now two technics have been used to circumvent this shortcoming [Moore et al., 2013]. A semi-empirical relation can be found between sea level rise and global mean surface temperature or top of atmosphere radiative balance. It can then be used into the future using data from climate models as a forcing [Rahmstorf, 2006]. Another method called process-based tries to evaluate the magnitude of each sea level rise contributor individually using numerical models of physical processes when they are reliable and other sources of information otherwise [Church et al., 2013]. Typically thermal expansion comes from climate models, ice sheet surface mass balance comes from regional models or empirical relationship between increase precipitation and increase temperature, ice sheet dynamics comes from either ice sheet models, expert judgement or statistical projections, or from a combination of all of these. Once the probability distributions or some other uncertainty measures have been quantified for individual contributors to sea level rise, they should be combined to obtain the total future sea level rise and its uncertainty. Information about the dependence between the sea level contributors is necessary for that step [Kurowicka and Cooke, 2006; Church et al., 2013]. How this dependence influences the projection of total sea level is the subject of our paper.
This subject has received little attention in the literature until now, probably because historically the focus has mainly been on projecting the expected value or the likely range of probabilities (e.g. a range that has a probability of 66% or more, Church et al. [2013]) while it is the quantiles that are far away from the expected value that are more sensitive to the dependence between contributors. Now the probability range of interest broadens because low probability events are also important for risk-management if they have a high impact [Hinkel et al., 2015]. For example Jevrejeva et al. [2014] and Menge et al. [2016] go up to the 95th percentile, Grinsted et al. [2015], Jackson and Jevrejeva [2016] and Le Bars et al. [2017] up to the 99th percentile and Kopp et al. [2014] up to the 99.9th percentile. It is therefore time to look at the sensitivity of results from the process-based method to the dependence between contributors.

The study of dependence between sea level contributors is similar to the study of co-occurrence of storm surge, tides and fluvial transport that can lead to coastal flooding. Mathematically the problem is the same but in practice it is easier to constrain the dependence between coastal processes because observational data and more complete physical models are available [van den Hurk et al., 2015; Klerk et al., 2015]. This allowed the use of bivariate statistics tools like copulas to investigate compounding effects [Wahl et al., 2015; Moftakhari et al., 2017]. The problem of dependence of sea level contributors is also more difficult to understand because it is not about events that correlate in time, for which we have a good intuition, but about events that correlate in the ensemble of possible futures that is a more abstract concept.

In section 2 we shortly review current practices to propagate the uncertainty from individual contributors to total sea level. The two sea level rise projection models that we use in this paper are then described in section 3 and their results are analysed in section 4. The paper finishes with a discussion, a conclusion.

2 Dependence between sea level contributors: the problem and a review of current practices

Mathematically making a sea level projection using the process-based method can be seen as a sum of random variables. The random variables, which are time dependent, are the contributors to sea level rise (e.g. thermal expansion, glaciers...) and the total sea level rise is also a random variable (see appendix A: ). The expected value of the total sea level is the sum of the expected values of the contributors, it is therefore independent of the dependencies between the sea level contributors [Beaumont, 2005]. However, the distribution of the total sea level is sensitive to the dependencies. When two independent random variables are added the variance of their sum is the sum of their variances but for positive correlation the variance of the sum increases compared to the independent case and for negative correlation it decreases (see demonstration in appendix B: and also [Beaumont, 2005]). This result is obtained without any assumption on the probability distribution of the random variables and is key to understand the results described in section 4.

To compute the total sea level probability distribution it is therefore necessary to know the joint probability distribution formed by the sea level contributors. The probability distributions of each sea level contributor are then the marginal probability distributions of this joint probability distribution. This is a well known mathematical problem that has been widely discussed [Kuwowski and Cooke, 2006], but not yet in the context of sea level projections. A consequence is that the importance of the choice of dependencies between sea level contributors is not yet fully recognised in the literature.

We now give a short review of the different choices that have been made to project sea level in the literature. Kopp et al. [2014], Jackson and Jevrejeva [2016] and Kopp et al. [2017] assume independence between sea level contributors. On the other hand Slangen et al. [2012] and Slangen et al. [2014] assume complete correlation when computing the standard deviation of the sum as the sum of the standard deviations of the sea level contributors. When computing an upper limit to future sea level rise, Jevrejeva et al. [2014]
also implicitly assume a complete correlation because the upper limits of each contributors are added to give the upper limit of the sum. Similar approach is used by Hinkel et al. [2014] for land ice contribution for which the components are summed up along percentiles, which is equivalent to assuming perfect correlation. A new method was developed by Church et al. [2013] in which the Global Mean Surface Temperature (GMST) is used as a driver for some of the sea level contributors. This results in partial correlation between these contributors. The same approach was then used by Vries et al. [2014] and by Le Bars et al. [2017] who extended the temperature sensitivity to the Antarctic contribution. An approximation of the correlation structure defined by Church et al. [2013] was used by Jevrejeva et al. [2014] and Grinsted et al. [2015] in which a joint probability distribution is built using constant correlation coefficients that emulate the results from Church et al. [2013] without modelling the time dependent dependence though temperature forcing. For simpler mechanistically motivated models like as described by Wong et al. [2017] and Bakker et al. [2017] and for semi-empirical models that separate individual contributors [Mengel et al., 2016] GMST is also used as a forcing which leads to implicit dependence between contributors. Therefore even though we focus on the process-based method our conclusions also apply to these other methods to project sea level.

3 Method

Two similar models are used to project global total sea level. The process-based method as presented in the Assessment Report 5 (AR5) of the Intergovernmental Panel on Climate Change (IPCC) [Church et al., 2013] is used as a starting point (see appendix A: for a full description). For the ice sheets the Surface Mass Balance (SMB, difference between snow fall and melt/sublimation) and ice-dynamics (calving, and basal melt of ice sheet and ice shelves) are considered separately because they are generally computed from different models. Therefore seven individual contributors to sea level change are considered: thermal expansion, glaciers and ice caps, Greenland SMB, Antarctic SMB, Greenland dynamics, Antarctic dynamics and land water storage. In section 4.1 each contributor is assessed probabilistically under the assumption of a given climate scenario in the same way as Church et al. [2013]. The dependence between the sea level contributors is set indirectly through a common dependence to GMST as in Church et al. [2013]. In this model, Greenland SMB, glaciers and ice caps and Antarctic SMB are driven by GMST. Thermal expansion comes from climate models and is then assumed to be perfectly correlated to GMST. Antarctic dynamics has a small dependence on temperature because it depends only on Antarctic SMB. More surface accumulation results in more mass loss through dynamical processes. Greenland dynamics is assumed independent of GMST. See Fig. 1 for a visual summary of the dependence structure. Dependence is measured using the Pearson correlation coefficient. For each year between 2006 and 2100 each contributor’s distribution is sampled and the correlation between the samples of different contributors is then computed. This correlation is therefore not a correlation in time but an uncertainty correlation for a given year. The results obtained with this dependence structure are then compared to a case where all contributors are assumed independent and another case where contributors are assumed completely dependent. In the completely dependent case the correlation between each pair of contributors is equal to one.

A probabilistic sea level projection model is also built with three modifications to the AR5 process-based model. First, the Antarctic dynamics is modelled using response functions from three ice sheet models that have a representation of ice shelves as described in Levermann et al. [2014]. This method allows us to propagate uncertainty from GMST to the Antarctic dynamics contribution to sea level (Fig. 1). This method also has the advantage of modelling the dependence between Antarctic dynamics and other sea level contributors through GMST. Second, the standard deviation of GMST and thermal expansion that are initially computed from the Coupled Model Intercomparison Project Phase 5 (CMIP5) ensemble are multiplied by 1.64 (appendix equations A.1 and A.2),
as done by [Le Bars et al., 2017] and similar to [Kopp et al., 2014]. This step is to re-
fect the decision of the AR5 authors to give a likely probability (66% or more) to the
5th to 95th percentile range computed from the climate model ensemble. Third, the cor-
relation between thermal expansion and GMST is re-evaluated using the CMIP5 data
base. Using 28 models for RCP4.5 and 30 models for RCP8.5 we correlate the temper-
ature difference and the thermal expansion difference between the periods 2091-2100 and
1986-2006. We find a correlation of 0.3 (-0.1 to 0.6) and 0.5 (0.1 to 0.7) respectively for
the RCP4.5 and RCP8.5 scenarios. With 5 to 95th percentiles between brackets. This
shows that the simple assumption of perfect correlation made in Church et al. [2013] can
be refined. A physical understanding of these results is not in the scope of the present
paper. However, vertical mixing of heat in the ocean introduces an anti-correlation be-
tween transient temperature response to greenhouse gasses emissions and thermal expan-
sion Hansen et al. [1985]. In a transition phase when models have not yet reached the
equilibrium, models that have larger vertical mixing in the ocean have more heat uptake,
larger thermal expansion and reduced GMST. Given the uncertainty in the correlation
and the fact that we do not know of a physical mechanism that would explain why the
correlation is larger for RCP8.5 than for RCP4.5 we choose to use the central value of
0.4 for both scenarios. Using this model we assess the importance of choices about cross-
correlation between sea level contributors by defining a low and a high estimate of de-
pendence. The low estimate has a reduced correlation between GMST and thermal ex-
pansion (0.2 instead of 0.4) other dependence relations do not change. For the high es-
timate, we choose a correlation of 0.8 between GMST and thermal expansion. Additional
dependences are also introduced by, on the one hand, correlating the modelling uncer-
tainty for Greenland SMB, Antarctic SMB and Glaciers and Ice Caps and, on the other
hand, by correlating the modelling uncertainty of Antarctic and Greenland dynamics (see
Fig. 1 and appendix A.10 for the description of the implementation). The rational for
these additional dependences is that the numerical models used for these different ar-
eas are not independent because they are based on the same knowledge and that phys-
ical processes relevant for SMB or dynamics in these different regions are mostly the same.

The results are discussed for two Representative Concentration Pathways: RCP4.5
and RCP8.5.

4 Results

4.1 The IPCC AR5 process-based projections

The computations of the IPCC AR5 global process-based method are reproduced
(see “partial” columns in table 1). We focus on the 5-95th percentiles range of these dis-
tributions because they were used by Church et al. [2013] to define the likely range (prob-
ability of 66% or more) that was broadly communicated. The results that we obtain are
very close to the ranges reported by Church et al. [2013] that were 36-71 cm and 52-98
cm in 2100 respectively for RCP4.5 and RCP8.5. The small difference for some of the
percentiles might arise from the convergence of the Monte Carlo simulation or from the
final rounding of the numbers.

The correlations between GMST and each sea level contributor is computed for each
year of the projections and is shown in Fig. 2 for the RCP4.5 scenario. Contributors that
are assumed independent of GMST where not included in the figure, for these processes
the correlation is constant equal to 0. Thermal expansion is assumed to be completely
correlated to GMST so the correlation is 1 and does not change over time. Other pro-
cesses have some temperature dependence but also other sources of uncertainty, as a re-
sult the correlation with GMST is less than 1. For Antarctic SMB the correlation is neg-
ative because the increase in snow accumulation is likely to be larger than the increase
in surface runoff as Antarctica warms up [Gregory and Huybrechts, 2006]. For all pro-
cesses that depend on GMST, the correlation changes over time. The uncertainty for all
of these processes depends both on mean temperature and on temperature uncertainty.
Table 1. Global mean sea level percentiles in 2100 for RCP4.5 and RCP8.5 from IPCC AR5 (partial correlation) and computed from the same individual contributions but with two extreme choices of correlation structure: independent and completely dependent with correlation 1 between all contributors.

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>RCP4.5</th>
<th>RCP8.5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Partial</td>
<td>Independent</td>
</tr>
<tr>
<td>5.0</td>
<td>36</td>
<td>38</td>
</tr>
<tr>
<td>50.0</td>
<td>52</td>
<td>53</td>
</tr>
<tr>
<td>95.0</td>
<td>69</td>
<td>67</td>
</tr>
</tbody>
</table>

An increase in the temperature uncertainty leads to increase the correlation with the GMST but an increase in the mean temperature only leads to increase the uncertainty of the process itself which reduces the correlation with GMST. This is illustrated in the appendix A.6 using the equation for Antarctic SMB.

For most processes the evolution of the correlation over time is therefore a competition between increasing mean temperature and increasing temperature uncertainty. In the way that variables were modelled the influence of the increasing mean temperature dominates and as a result the absolute value of the correlations reduce over time. This is the case for Glaciers and ice caps, Antarctic SMB and Antarctic dynamics but Greenland SMB starts to increase again after the middle of the century. This is due to the non-linear way in which it depends on temperature (appendix A: equations A.5 and A.6).

Since GMST is not a direct contributor to sea level the correlations with GMST do not have a direct impact on the uncertainty of sea level projections. However it does have an indirect impact on the correlations between sea level contributors. Since this method to project sea level uses 7 sea level contributors, there are a total of 21 (combination of \(^\binom{7}{2}\)) correlations influencing the total sea level distribution. They all behave in a similar way so we focus on the time evolution of the correlation of Glaciers and Ice Caps with other sea level contributors (Fig. 2). As a result of decreasing correlation with GMST over time the correlation between sea level contributors also decreases over time.

To assess the impact of these dependencies on the uncertainty of total global mean sea level we compare the partial correlation structure described above with two extreme sensitivity experiments. One assuming independence between contributors and the other assuming a complete dependence with a correlation of 1 between all contributors.

Results are shown for year 2100 in table 1. We see that the 5-95th percentile ranges are sensitive to the choices of correlation between sea level contributors. The independent case gives narrower 5-95th percentile ranges while the fully dependent case gives ranges that are a lot broader. The RCP8.5 scenario is more sensitive to the dependence choices than the RCP4.5 because temperature uncertainties are larger. Also the independent assumption is a lot closer to the partial correlation used in [Church et al., 2013] than the fully dependent case. These results underline the importance of the choice of the correlation structure between sea level contributors when making projections even for the likely range.

### 4.2 A probabilistic projection

We explore here a probabilistic model in which the Antarctic dynamics is computed from the method described in Levermann et al. [2014]. With this method since the standard deviation of GMST and thermal expansion are already multiplied by 1.64, the likely range is not given by the 5th to 95th percentiles but directly by the 17th to 83rd per-
Table 2. Global mean sea level PDFs in 2100 for RCP4.5 and RCP8.5, obtained using the probabilistic method.

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>Partial</th>
<th>Low dependence</th>
<th>High dependence</th>
<th>Independent</th>
<th>Dependent</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCP4.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>33</td>
<td>35</td>
<td>31</td>
<td>38</td>
<td>15</td>
</tr>
<tr>
<td>10.0</td>
<td>38</td>
<td>39</td>
<td>36</td>
<td>41</td>
<td>22</td>
</tr>
<tr>
<td>17.0</td>
<td>42</td>
<td>42</td>
<td>40</td>
<td>44</td>
<td>30</td>
</tr>
<tr>
<td>50.0</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>55</td>
<td>53</td>
</tr>
<tr>
<td>83.0</td>
<td>70</td>
<td>70</td>
<td>72</td>
<td>68</td>
<td>82</td>
</tr>
<tr>
<td>90.0</td>
<td>77</td>
<td>76</td>
<td>79</td>
<td>73</td>
<td>94</td>
</tr>
<tr>
<td>95.0</td>
<td>85</td>
<td>84</td>
<td>88</td>
<td>80</td>
<td>108</td>
</tr>
<tr>
<td>99.0</td>
<td>106</td>
<td>104</td>
<td>109</td>
<td>98</td>
<td>144</td>
</tr>
<tr>
<td>99.9</td>
<td>141</td>
<td>139</td>
<td>146</td>
<td>132</td>
<td>202</td>
</tr>
<tr>
<td>RCP8.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>50</td>
<td>52</td>
<td>47</td>
<td>56</td>
<td>25</td>
</tr>
<tr>
<td>10.0</td>
<td>56</td>
<td>57</td>
<td>53</td>
<td>61</td>
<td>35</td>
</tr>
<tr>
<td>17.0</td>
<td>61</td>
<td>62</td>
<td>59</td>
<td>65</td>
<td>45</td>
</tr>
<tr>
<td>50.0</td>
<td>79</td>
<td>79</td>
<td>79</td>
<td>80</td>
<td>77</td>
</tr>
<tr>
<td>83.0</td>
<td>101</td>
<td>100</td>
<td>103</td>
<td>97</td>
<td>117</td>
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<tr>
<td>90.0</td>
<td>110</td>
<td>109</td>
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<td>105</td>
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</tr>
<tr>
<td>95.0</td>
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<tr>
<td>99.0</td>
<td>151</td>
<td>148</td>
<td>156</td>
<td>138</td>
<td>205</td>
</tr>
<tr>
<td>99.9</td>
<td>195</td>
<td>193</td>
<td>202</td>
<td>179</td>
<td>290</td>
</tr>
</tbody>
</table>

The distribution of future Antarctic dynamic contribution to sea level has a slightly wider likely range and the median shifts towards higher values compared to Church et al. [2013]. Most importantly for the focus of this paper, this method allows us to make an explicit dependence of the Antarctic ice sheet dynamics contribution to sea level rise on GMST. This was discussed by Le Bars et al. [2017] but using a different method. The new dependency graph is shown in Fig. 1 and the results of the total global sea level are shown in table 2.

In this model the evolution of the correlations over time is similar to the AR5 process-based model except that the magnitude of reduction over time is smaller for all processes except for Antarctic dynamics (Fig. 2). This is because the standard deviation of GMST is multiplied by 1.64 which changes the relative importance of the increase ensemble mean GMST that reduces the correlation and the increase standard deviation that increases the correlation. Also the correlation between Antarctic dynamics and GMST is a lot larger in this probabilistic model than in the AR5 model. This was to be expected because in the AR5 model the connection was only through increased Antarctic SMB that lead to small increased Antarctic mass loss due to calving [Church et al., 2013].

There is a large difference between the partial correlation case and the independent and dependent cases (table 2). The expected value of the total sea level is the sum of the expected value of the contributors, it is independent of the dependence strength between contributors [Beaumont, 2005] so since the median in these distributions is not very far from the expected value we see that dependency has little impact around the median but it becomes larger further away from the median. For example the 99th percentile is reduced by 8 cm in the independent case and increased by 38 cm in the fully dependent case compared to the partial case for the RCP4.5 scenario.
4.3 Uncertainty in the dependence between contributors for a probabilistic projection

We now turn to the problem of the uncertainty in assessing the strength of dependence between sea level contributors. We address this problem by designing two additional sensitivity experiments. One in which the dependency is reduced and another one where it is increased compared to the partial case. We use different possible links between sea level contributors instead of only GMST (Fig. 1, section 3 and appendix A.10). These two cases are considered to be the upper and lower end of a reasonable range of possible correlation strength. The uncertainty in dependence is then defined as the difference between the high and the low dependence cases. This uncertainty is compared with the uncertainty due to the main sea level contributors. To measure the importance of the uncertainty of individual sea level contributors we recompute the total sea level replacing one contributor by its expected value. The difference between the total sea level with and without including this contributor’s uncertainty gives a measure of its contribution to the total sea level uncertainty [Saltelli et al., 2008]. These results are shown for RCP4.5 in 2100 in Fig. 3a where positive (negative) values mean that a contributor leads to increase (decrease) that particular quantile. All contributors tend to increase the uncertainty of the total sea level, this can be seen by the positive (negative) values for percentiles higher (lower) than 50. Antarctica (SMB and dynamics) provides the largest uncertainty, followed by glaciers and ice caps.

We can also look at the variations in time of the relative importance of these contributors for a certain range of probability, for example the very likely range (5st to 95st percentile in this probabilistic model, Fig.3b). The relative importance of the contributors does not change much over time. The contribution of dependence uncertainty to the total uncertainty at the end of the century (around 7 cm) is similar to that of thermal expansion and Greenland ice sheet (SMB and dynamics).

5 Discussion

Our calculation of the uncertainty arising from the dependence between sea level contributors should be seen as an approximation to be refined. Ideally, the uncertainty in the dependence parameters should be included in the sea level projection model. This means that in the Monte Carlo simulation when the distributions of individual contributors are sampled, the strength of their correlation would also be sampled from a pre-defined distribution. This would increase the computational cost of the model because convergence would slow down but it would make the model more consistent.

Up to now, all probabilistic sea level projections are still conditional on future greenhouse gas concentration pathways. Therefore, the uncertainty provided do not include greenhouse gas emissions uncertainty nor carbon cycle uncertainty. For a fully probabilistic model that would propagate uncertainty all the way from emissions to sea level the issue of dependence between contributors would be even more important. This is because in such a model the GMST uncertainty would be larger and as a result the dependence between sea level contributors would increase.

The Antarctic contribution that we use here do not include the hydrofracturing of Antarctic ice shelves nor the structural collapse of tall ice cliffs [Levermann et al., 2014]. These mechanisms were shown to increase the sensitivity of Antarctic mass loss to emission scenarios because of the key role of surface melting at the surface of ice shelves [Pollard et al., 2015; Deconto and Pollard, 2016]. A model that includes these processes increases the dependence between contributors and total sea level uncertainty [Le Bars et al., 2017].

Only global sea level projections were discussed in this paper. Implementing dependence in regional projections is straightforward for ice sheets and glaciers because the dependence to GMST will not change, only fingerprints will modulate their relative contributions. A case that might become interesting and that we did not cover is the re-
duction of uncertainty close to the ice sheets due to anti-correlation between contribu-
tors. Also, while for global sea level it is reasonable to have a positive correlation between
GMST and thermal expansion, at regional scale it might not be the case, the correla-
tion should therefore be computed for the region of interest. These mechanisms could
be investigated using the CMIP climate models.

6 Conclusion

We have shown that the dependence between sea level contributors is important
to quantify the uncertainty of sea level projections. A reasonable way to include some
dependence is to include a correlation between sea level contributors and GMST [Church
et al., 2013]. The sea level projection from this approach were shown to have significantly
higher uncertainty than assuming independence and less than assuming complete depen-
dence. These two choices of independence and perfect correlation should be viewed as
extremes, that can give insightful lower and upper bound of the uncertainty. The depen-
dence choice was shown to be more important for high greenhouse gas emission scenario
and for high percentiles.

The choice of dependence between contributors is important but unfortunately it
is loosely constrained because it cannot be observed. This leads to an additional uncertain-
ty similar in magnitude to the uncertainty due to thermal expansion and Greenland
mass loss. Therefore it might be relevant to take this uncertainty into account for ap-
lications that require accurate uncertainty quantification.

Sometimes, for practical applications, mean sea level probabilistic projections are
not used on their own but together with other processes like inter-annual variability of
sea level, tides, storm surges, wave setup river discharge and rain to investigate extreme
events at coastal locations [Le Cozannet et al., 2015; Vousdoukas et al., 2017]. Devel-
oping models of dependence between these processes will improve the quantification of
the frequency of future flooding events.
A: Technical description of the method

In this section we present each process that is expected to contribute to sea level rise in the coming century and the uncertainties associated with them. The processes are evaluated in the same way as Church et al. [2013] except for the Antarctic ice sheet dynamics. The following method description builds on Church et al. [2013], Vries et al. [2014] and Le Bars et al. [2017]. We use capital letters for random variables, bold capital letters for matrices and calligraphic letters for distributions.

A.1 Global mean surface temperature

The temperature fields are derived from 21 climate models that are part of the Coupled Model Intercomparison Project Phase 5 (CMIP5). More than 21 models participated in CMIP5 but only these models provided all the necessary variables for making the sea level projections. No other selection was performed. These 21 models are forced by two different scenarios of greenhouse gas emissions: RCP4.5 for which some mitigation measures are implemented and RCP8.5 which is business as usual.

The number of models is not large enough to determine the shape of the underlying distribution of the time varying global mean surface temperature. Therefore, we assume that this distribution is normal. We represent the global annual mean surface temperature information from all models by a matrix $T$, whose first dimension is time ($t$), and second dimension are the member of the model ensemble. $N$ is a random variable following the normal distribution of mean 0 and standard deviation 1 ($N(0,1)$). Then for each time $t$ the random variable representing temperature ($T$) is computed from the mean temperature ($\bar{T}$) and a standard deviation ($\sigma(T)$) over the climate model ensemble, as:

$$T(t) = \bar{T}(t) + \sigma(T(t,.))N_1.$$  \hspace{1cm} (A.1)

The temperature is generally used as an anomaly compared to a reference period. In this case the mean temperature during the reference period has to be removed from each model time series before computing $T$. This is important because the term $\sigma(T(t,.))$ also depends on the reference period. In the following a reference temperature distribution computed with the reference period 1986-2005 will be written $T_{1986-2005}$.

A.2 Global steric expansion

Many climate models conserve volume and not mass because of the so called “Boussinesq approximation”. Therefore, in these models an increase in temperature does not lead to a global expansion of the water. This effect is computed off-line from the density fields. Because climate models have a drift in steric expansion it is necessary to diagnose this drift from each model using a control experiment that uses a constant forcing. The drift is then removed by subtracting a polynomial fit as a function of time to the control steric expansion time series. Global mean steric expansion from each model and at all time $t$ is stored in a matrix $X_{st}$. The distribution is computed in the same way as for the global mean temperature:

$$X_{st}(t) = \bar{X}_{st}(t) + \sigma(X_{st}(t,.))N_1.$$  \hspace{1cm} (A.2)

The random variable $N$ here is the same as in equation A.1. This means that the temperature and steric expansion are assumed to be completely correlated.

A.3 Land glaciers and ice caps

The contribution from land glaciers and ice caps excludes Antarctic glaciers that are included directly in the Antarctic contribution but includes Greenland glaciers. This contribution is derived from four global glacier models [Giesen and Oerlemans, 2013; Marzeion et al., 2012; Radić et al., 2014; Slangen and Van De Wal, 2011] that take into account...
local climate change and its effect on the surface mass balance and the hypsometry of individual glaciers. Each of these models computes the glacier contribution to sea level depending on a temperature pathway. Since these models where originally forced with different temperature pathways we first need to fit the time series of cumulated contribution to $f I(t)^p$, with $I(t)$ the time integral of global mean surface temperature from year 2006 to $t$. The integrated temperature needs to be used here because the cumulated sea level contribution depend on past temperatures. The fitting parameters $f$ and $p$ obtained for each model are shown in Table A.1. This method allows to apply these four models for any temperature pathway. In particular for the RCP scenarios:

$$I(t) = \int_{2006}^{t} T_{1986-2005} dt', \quad (A.3)$$

$$X_{gic}(t) = x_{gic}^0 + \frac{10}{4} N_2 \sum_{i=1}^{4} f_i I(t)^p_i \quad (A.4)$$

where $X_{gic}$ is a random variable representing the sea level change in cm and $i$ is an index looping over the four sets of parameters from the glacier models. The factor 10 is used to convert from mm to cm. The sum in the second term of the right hand side of equation A.4 shows that the average over the four glacier models is taken. The spread of the four models estimates around the mean is about 20%. This uncertainty is included with the random variable $N_2$ that follows the distribution $N(1, 0.2^2)$. The variable $N_2$ is independent from $N$ which means that glacier modelling uncertainties are not correlated with temperature. The random variable $X_{gic}$ is still partially correlated with temperature because $T_{1986-2005}$ is used to compute $I$. An additional constant ($x_{gic}^0 = 0.95$ cm) is added to include the change from 1996 to 2005.

### A.4 Greenland Ice Sheet Surface Mass Balance

The following parameterization is used for the surface mass balance tendency ($\dot{X}_{G smb}$) in terms of global temperature change [Fettweis et al., 2013]:

$$\dot{X}_{G smb}(t) = \frac{10^{-10}}{\rho_w A_{oc}} \left( 71.5 T_{1980-1999}(t) + 20.4 T_{1980-1999}^2(t) + 2.8 T_{1980-1999}^3(t) \right), \quad (A.5)$$

where the factor $10^{-10}$ is used to convert GT to kg and m to cm, $\rho_w = 1 \times 10^3$ kg m$^{-3}$ is the water density and $A_{oc} = 3.6704 \times 10^{14}$ m$^2$ is the ocean surface area. This equation is then integrated in time:

$$X_{G smb}(t) = x_{G smb}^0 + UL \int_{2006}^{t} \dot{X}_{G smb}(t') dt' \quad (A.6)$$

where $x_{G smb}^0$ is the observed contribution between 1996 and 2005. Different studies give different estimates. This uncertainty is implemented as $L$ a random variable sampled from the log-normal distribution $e^{N(0.0.4^2)}$. The log-normal distribution is used because the...
estimates of the various Greenland surface mass balance (SMB) models are positively skewed. A positive feedback between SMB and surface topography is also added. As the ice sheet loses mass its altitude decreases and the temperature at its surface increases, leading to increased melt. This is included with $U$ that is a random variable following the uniform probability distribution between 1 and 1.15.

### A.5 Greenland Ice Sheet dynamics

As in Church et al. [2013] the range of the Greenland ice sheet dynamical processes contribution for 2100 is 1.4 to 6.3 cm for all scenarios, except RCP8.5 for which it is 2 to 8.5 cm. These ranges are based on an expert assessment of the literature. The mass loss rate at the beginning of the projection is taken as half of the observed rate from 2005 to 2010 (half of 0.46–0.80 mm yr$^{-1}$), the other half being accounted for in the surface mass balance. A maximum (minimum) time series is then built starting in 2006 from the maximum (minimum) estimate of recent mass loss and ending in 2100 at the maximum (minimum) of the range for 2100 and assuming second order in time. These maximum and minimum time series are called $x_{Gdyn}^{max}$ and $x_{Gdyn}^{min}$ respectively. An additional 0.15 cm is added for the contribution before 2006 ($x_{Gdyn}^{0}$). The distribution is then taken as uniform between the maximum and minimum time series as follows:

$$X_{Gdyn}(t) = [U_2 x_{Gdyn}^{max}(t) + (1 - U_2) x_{Gdyn}^{min}(t)]$$  \hspace{1cm} (A.7)

where $U_2$ follows a uniform probability distribution between 0 and 1.

### A.6 Antarctic Ice Sheet surface mass balance

The change in Antarctic ice sheet SMB was assumed to be due solely to an increase in accumulation, e.g. possible increase in runoff is neglected. This was estimated using the results of Gregory and Huybrechts [2006] from CMIP3 AOGCMs. Accumulation was taken to increase at 5.1 ± 1.5 % per degree of warming in Antarctica. The ratio of warming in Antarctica compared to GMST was taken to be 1.1 ± 0.2. The Antarctic SMB contribution to sea level is then computed as:

$$X_{Asmb}(t) = -x_{Asmb}^{ref} (1 + M_3 M_4 T_{1986-2005}(t)),$$  \hspace{1cm} (A.8)

with $x_{Asmb}^{ref}$ the accumulation during the reference period taken to be 1923 Gt yr$^{-1}$, $M_3$ and $M_4$ uncertainties following respectively $\mathcal{N}(5.1, 1.5^2)$ and $\mathcal{N}(1.1, 0.2^2)$. A minus sign is added because this accumulation of water on Antarctica brings sea level down.

It is not directly apparent from equation A.8 how the correlation between $X_{Asmb}$ and $T_{1986-2005}$ changes over time (Fig. C.1, Fig. 2). We use equation A.1, drop the reference period to simplify the notation and write $\sigma(T(t,.))$ as $\sigma_T$ to get:

$$X_{Asmb}(t) = -x_{Asmb}^{ref} (1 + M_3 M_4 \left[ \bar{T}(t) + \sigma_T N_1 \right]).$$  \hspace{1cm} (A.9)

We see that $X_{Asmb}$ depends on three random variables: $M_3$, $M_4$ and $N_1$. Writing $E$ the expected value operator, the correlation between $X_{Asmb}$ and $T$ is then:

$$\rho_{X_{Asmb},T} = \frac{E[(X_{Asmb} - \bar{X}_{Asmb})(T - \bar{T})]}{\sigma_T \sigma_{X_{Asmb}}}$$  \hspace{1cm} (A.10)

$$= \frac{-A}{B \sigma_T^2 + C}$$  \hspace{1cm} (A.11)

with $A$, $B$ and $C$ are independent of temperature:

$$A = E[M_3 M_4 N_1^2]$$  \hspace{1cm} (A.12)
$$B = E[M_3^2 M_4^2] - E[M_3^2] E[M_4^2]$$  \hspace{1cm} (A.13)
$$C = E[M_3^2 M_4^2 N_1^2]$$  \hspace{1cm} (A.14)
It is now clear from equation A.11 that the magnitude of the correlation between Antarctic SMB and GMST decreases when $T$ increases and increases when $\sigma_T$ increases.

### A.7 Antarctic Ice Sheet dynamics

Two cases are considered in this paper. The first case is the same as IPCC AR5 with starting contribution of 0.21-0.61 mm yr$^{-1}$ reaching 2 to 18.5 cm in 2100. Numerically implemented in the same way as Greenland ice sheet dynamics. The second case makes use of the probabilistic method described by Levermann et al. [2014]. We choose to use the response functions only from the three models that explicitly represent ice shelves. These are the Pennsylvania State University 3-D ice sheet model (PenState-3D, Pollard and Deconto [2012]), the Parallel Ice Sheet Mode (PISM, Winkelmann et al. [2011]; Martin et al. [2011]) and the Simulation COde for POLythermal Ice Sheets (SICOPOLIS, Greve et al. [2011]). Noting the response functions $R_i$ and the basal melt at the Antarctic margin $\Delta b$ we have:

$$X_{Adyn}(t) = \int_{1950}^{t} \Delta b(\tau) R_i(t - \tau) d\tau.$$  

(A.15)

and modelling $\Delta b$ as a function GMST gives:

$$X_{Adyn}(t) = \int_{1950}^{t} U_3 \alpha_m T(\tau) R_i(t - \tau) d\tau,$$  

(A.16)

where $U_3$ is a continuous random variable representing basal melt sensitivity and following a uniform distribution between 7 and 16 my$^{-1}$K$^{-1}$ and $\alpha_m$ is a discrete random variable representing the scaling coefficient between GMST and subsurface ocean warming around the Antarctic ice shelves. $\alpha_m$ is selected randomly from one of 19 CMIP5 climate models (see numerical values in Levermann et al. [2014]). In the original paper Levermann et al. [2014] compares two approaches, with and without including a time delay between GMST and subsurface ocean temperature, for simplicity we chose to only present the case without time delay.

### A.8 Groundwater changes

This term is based on projections of future dam constructions and depletion of groundwater from human activities. The 5 to 95% quantiles for 2100 are −1 and 9 cm [Wada et al., 2012]. The time evolution is done with a second order polynomial starting from present observed rate estimates of (0.26, 0.49) [mm/yr] (5-95% range). A lower (upper) time series is constructed that start at the lower (upper) initial rate and end at the lower (upper) final estimate. These time series are called $x_{\text{lower}}^{\text{grw}}$ and $x_{\text{upper}}^{\text{grw}}$. A central estimate ($X_{\text{cen}}^{\text{grw}}$) is obtained as the mean of the two. The final distribution is then computed as:

$$X_{\text{grw}}(t) = x_{\text{cen}}^{\text{grw}}(t) + \sigma_{\text{grw}}(t) N_5$$  

(A.17)

where $N_5$ is sampled from $\mathcal{N}(0, 1)$ and with

$$\sigma_{\text{grw}}(t) = \left(\frac{x_{\text{upper}}^{\text{grw}}(t) - x_{\text{lower}}^{\text{grw}}(t)}{\alpha_{95} - \alpha_{05}}\right)$$  

(A.18)

and $\alpha_q$ is the quantile function for a normal distribution. The groundwater contribution is taken as independent of temperature and emission scenario.

### A.9 Final combination of contributors

Once all the contributions have been computed the total is obtained as:

$$X_{\text{total}} = X_{\text{st}} + X_{\text{gic}} + X_{\text{Gsmmb}} + X_{\text{Gdyn}} + X_{\text{Asmb}} + X_{\text{Adyn}} + X_{\text{grw}}$$  

(A.19)
A probability density function can then be constructed from $X_{\text{total}}$ for each time $t$. Practically this is performed with a Monte Carlo simulation. The distributions of individual contributors are sampled semi-randomly to retain the correlation between them. The samples are then added to construct the distribution of total sea level. The sampling is continued until convergence with an accuracy of 1 cm of the 99.9th percentile of the total sea level distribution is reached. This is found to be around $5 \times 10^5$ samplings for all cases.

### A.10 Modification of the dependence structure

Two sensitivity experiments are performed to study the impact of choices about the correlation strength between contributors on the total sea level distribution. In the low correlation experiment the correlation between GMST and thermal expansion is reduced from 0.4 to 0.2. This is performed by replacing the random variable $N_1$ in equation A.2 by $N_{\text{low}}$, defined as:

$$N_{\text{low}} = \rho N_1 + N_I \sqrt{1 - \rho^2},$$

where $N_I$ is an independent random variable with distribution $\mathcal{N}(0,1)$ and $\rho$ is the desired correlation coefficient between $N_{\text{low}}$ and $N_1$.

For the high dependence experiment the correlation between GMST and thermal expansion is 0.8. Also additional dependence is introduced between the modelling uncertainty of SMB for ice sheets and glaciers and ice caps. Practically this is implemented in the model by having a correlation of 1 between $N_2$ (equation A.4), $L$ (equation A.6) and $M_3$ (equation A.8). A dependence is also introduced between the ice sheet dynamics components by having a correlation of 1 between $U_2$ (equation A.7) and $R_i$ (equation A.16).

### B: Variance of the sum of two random variables

Let $X$ and $Y$ be random variables, $E$ the expected value operator, $\mu_X$ and $\mu_Y$ the expected values of $X$ and $Y$, $\sigma$ the standard deviation and $\rho_{XY}$ the Pearson cross-correlation between $X$ and $Y$. The variance of the sum of $X$ and $Y$ is:

\[
\begin{align*}
\sigma_{X+Y}^2 &= E[(X - \mu_X + Y - \mu_Y)^2] \\
&= E[(X - \mu_X)^2 + 2(X - \mu_X)(Y - \mu_Y) + (Y - \mu_Y)^2] \quad \text{(B.2)} \\
&= \sigma_X^2 + 2E[(X - \mu_X)(Y - \mu_Y)] + \sigma_Y^2 \quad \text{(B.3)} \\
&= \sigma_X^2 + 2\sigma_X\sigma_Y\rho_{XY} + \sigma_Y^2 \quad \text{(B.4)}
\end{align*}
\]

Since the $\sigma$ is positive we see that a positive cross-correlation between $X$ and $Y$ increases the variance of $X+Y$ and a negative cross-correlation decreases it. This demonstration is very general because it does not assume any particular distribution for $X$ and $Y$.

### C: Additional results for RCP8.5 scenario

We provide figures C.1 and C.2 equivalent to figures 2 and 3 but for the RCP8.5 scenario.

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Figure 1. Dependency graph for different sea level projections. Sea level contributors are represented in rectangular boxes while factors providing an external influence are represented in oval shapes. Arrows represent direct dependence relationship. The indirect dependences are not represented here.
Figure 2. Time evolution of Pearson correlation for RCP4.5 scenario.
Figure 3. (a) Uncertainty of total sea level in 2100 due to the uncertainty of the main sea level contributors compared to that due to the dependence between them. Result is shown for each percentile. For Greenland and Antarctica SMB and dynamics are added together. (b) Time series of the increase of the very likely range (5th to 95th percentile) of total sea level due to the uncertainty of each contributor and due to the dependence between them.
Figure C.1. Time evolution of Pearson correlation for RCP8.5 scenario.
Figure C.2.  (a) Uncertainty of total sea level in 2100 due to the uncertainty of the main sea level contributors compared to that due to the dependence between them. Result is shown for each percentile. For Greenland and Antarctica SMB and dynamics are added together. (b) Time series of the increase of the very likely range (5th to 95th percentile) of total sea level due to the uncertainty of each contributor and due to the dependence between them.