

1 **An estimate of equilibrium climate sensitivity from**  
2 **interannual variability**

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11 Main points:

- 12 1. We use interannual variability to estimate equilibrium climate sensitivity (ECS). We  
13 estimate ECS is *likely* 2.4-4.5 K (17-83% confidence interval), with a mode and median  
14 value of 2.9 and 3.3 K, respectively.
- 15 2. We see no evidence to support low ECS (values less than 2K) suggested by recent  
16 analyses.
- 17 3. This work shows the value of alternate energy balance frameworks for understanding  
18 climate change.  
19

20 **Abstract**

21 Estimating the equilibrium climate sensitivity (ECS; the equilibrium warming in response to a  
22 doubling of CO<sub>2</sub>) from observations is one of the big problems in climate science. Using  
23 observations of interannual climate variations covering the period 2000 to 2017, we estimate  
24 ECS is *likely* 2.4-4.5 K (17-83% confidence interval), with a mode and median value of 2.9 and  
25 3.3 K, respectively. Our analysis provides no support for low values of ECS (below 2 K)  
26 suggested by other analyses. The main uncertainty in our estimate is not observational  
27 uncertainty, but rather uncertainty in converting observations of short-term, mainly unforced  
28 climate variability to an estimate of the response of the climate system to long-term forced  
29 warming.

30 **Plain language summary**

31 Equilibrium climate sensitivity is the amount of warming resulting from doubling carbon  
32 dioxide. It is one of the important metrics in climate science because it is a primary determinant  
33 of how much warming we will experience in the future. Despite decades of work, this quantity  
34 remains uncertain: the last IPCC report stated a range for ECS of 1.5-4.5 deg. Celsius. Using  
35 observations of interannual climate variations covering the period 2000 to 2017, we estimate  
36 ECS is *likely* 2.4-4.5 K. Thus, our analysis provides no support for the bottom of the IPCC's  
37 range.

38

39 **Introduction**

40 The response of the climate system to the imposition of a climate forcing is frequently  
41 expressed using the linearized energy balance equation:

$$42 \quad R = F + \lambda T_s \quad (1)$$

43 where forcing  $F$  is an imposed top-of-atmosphere (TOA) energy imbalance,  $T_s$  is the global  
44 average surface temperature, and  $\lambda$  is the change in TOA flux per unit change in  $T_s$  [Sherwood  
45 *et al.*, 2014].  $R$  is the resulting TOA flux imbalance from the combined forcing and response. All  
46 quantities are deviations from an equilibrium base state, usually the pre-industrial climate.  
47 Equilibrium climate sensitivity (hereafter ECS, the equilibrium warming in response to a  
48 doubling of  $\text{CO}_2$ ) can be calculated as:

$$49 \quad \text{ECS} = -F_{2\times\text{CO}_2}/\lambda \quad (2)$$

50 where  $F_{2\times\text{CO}_2}$  is the forcing from doubled  $\text{CO}_2$ .

51 Equation 1 is a workhorse of climate science and it has been used many times to estimate  $\lambda$  and  
52 ECS. Many of these [e.g., Gregory *et al.*, 2002; Annan and Hargreaves, 2006; Otto *et al.*, 2013;  
53 Lewis and Curry, 2015; Aldrin *et al.*, 2012; Skeie *et al.*, 2014; Forster, 2016] combine Eq. 1 with  
54 estimates of  $R$ ,  $F$ , and  $T_s$  over the 19<sup>th</sup> and 20<sup>th</sup> centuries to infer  $\lambda$  and ECS. These calculations  
55 suggest  $\lambda$  is near  $-2 \text{ W/m}^2/\text{K}$  and appear to rule out an ECS larger than  $\sim 4 \text{ K}$  [Stevens *et al.*,  
56 2016]. The increased likelihood of an ECS below  $2 \text{ K}$  implied by these calculations led the IPCC  
57 Fifth Assessment Report (AR5) to extend their *likely* ECS range downward to include  $1.5 \text{ K}$   
58 [Collins *et al.*, 2013].

59 However, since AR5 a number of problems with this approach have been identified. These  
60 include questions about the impact of internal variability [e.g., Dessler *et al.*, 2018], arguments  
61 that ECS inferred from historical energy budget produces an underestimate of the true value  
62 [e.g., Armour, 2017; Gregory and Andrews, 2016; Zhou *et al.*, 2016; Andrews and Webb, 2018;  
63 Proistosescu and Huybers, 2017; Marvel *et al.*, 2018], the large and evolving uncertainty in  
64 forcing over the 20th century [e.g., Forster, 2016], different forcing efficacies of greenhouse

65 gases and aerosols [Shindell, 2014; Kummer and Dessler, 2014], and geographically incomplete  
66 or inhomogeneous observations [Richardson *et al.*, 2016].

67 For robust estimates of ECS, multiple lines of evidence are needed and care needs to be taken  
68 in relating the inferred ECS from any method to other estimates. Thus, there is great value in  
69 finding alternate ways to approach the problem. Relatively few papers have attempted use  
70 short-term interannual variability to estimate ECS [e.g., Forster, 2016; Tsushima *et al.*, 2005;  
71 Forster and Gregory, 2006; Chung *et al.*, 2010; Tsushima and Manabe, 2013; Dessler, 2013;  
72 Donohoe *et al.*, 2014]. Papers that do typically yield estimates of ECS consistent with the IPCC's  
73 canonical ECS range of 1.5-4.5°C, but their uncertainty is so large as to provide no meaningful  
74 constraint of the range. In this paper, we present a new methodology to infer ECS from these  
75 interannual fluctuations of the climate system.

## 76 **Results**

### 77 Traditional energy-balance framework

78 Per Eq. 2, ECS requires estimates of  $-F_{2\times\text{CO}_2}$  and  $\lambda$ . We obtain an estimate of  $F_{2\times\text{CO}_2}$  from fixed  
79 sea surface temperature and sea-ice experiments from ten global climate models that  
80 submitted output to the Precipitation Driver Response Model Intercomparison Project [Myhre  
81 *et al.*, 2017b]. They estimate  $F_{2\times\text{CO}_2}$  to be normally distributed with a mean of 3.69 W/m<sup>2</sup> and a  
82 standard deviation of 0.13 W/m<sup>2</sup>.

83 An estimate of  $\lambda$  can be obtained from observations of R and  $T_s$ . Observations of R come from  
84 the Clouds and the Earth's Radiant Energy System (CERES) Energy Balanced and Filled product  
85 (ed. 4) [Loeb *et al.*, 2018] and cover the period March 2000 to July 2017. Estimates of  $T_s$  come  
86 from the European Centre for Medium Range Weather Forecasts (ECMWF) Interim Re-Analysis  
87 (ERAi) [Dee *et al.*, 2011]. Given these data, we calculate  $\lambda$  two ways, both based on Eq. 1. First,  
88 we assume forcing changes linearly over this time period and account for it by detrending R and  
89  $T_s$  time series;  $\lambda$  is then the slope of the regression of these detrended time series. Second, we  
90 use estimates of forcing F over the CERES period and calculate  $\lambda$  as the slope of the regression  
91 of R-F vs.  $T_s$ . See the appendix for more details about this calculation.

92 Distributions of  $\lambda$  for the two approaches (estimated by Monte Carlo methods) are both quite  
 93 wide (Fig. 1), with 5-95% confidence intervals of -1.45 to -0.15 W/m<sup>2</sup>/K and -1.08 to +0.09  
 94 W/m<sup>2</sup>/K for the detrended and R-F calculations, respectively. This is a consequence of the weak  
 95 control  $T_S$  exerts on R in the observations [Xie *et al.*, 2016; Dessler *et al.*, 2018], and it means  
 96 that our observational estimate of  $\lambda$  is quite uncertain. The medians of the two distributions  
 97 are -0.81 and -0.50 W/m<sup>2</sup>/K, respectively.

98 The distributions of  $\lambda$  plotted in Fig. 1 are derived mainly from the response to interannual  
 99 variability, so we will refer to them hereafter as  $\lambda_{iv}$ . The  $\lambda$  in Eq. 2, however, is the climate  
 100 system's response to forcing from doubled CO<sub>2</sub> (hereafter  $\lambda_{2\times CO_2}$ ), so we cannot simply plug  $\lambda_{iv}$   
 101 into Eq. 2 to derive ECS. In fact, this disconnect between what we can measure ( $\lambda_{iv}$ ) and what is  
 102 required to calculate ECS ( $\lambda_{2\times CO_2}$ ) is one reason scientists have largely avoided using interannual  
 103 variability to infer ECS.

104 We therefore modify Eq. 2 to account for this:

$$105 \quad ECS = - \frac{F_{2\times CO_2}}{\lambda_{iv}} \frac{\lambda_{iv}}{\lambda_{2\times CO_2}} \quad (3)$$

106 where the ratio  $\lambda_{iv}/\lambda_{2\times CO_2}$  is a transfer function that converts the measured  $\lambda_{iv}$  into the required  
 107 value  $\lambda_{2\times CO_2}$ . In our ECS calculations, we estimate this transfer function using models that  
 108 submitted required output to the 5<sup>th</sup> phase of the Coupled Model Intercomparison Project  
 109 (CMIP5) [Taylor *et al.*, 2012]. The numerator  $\lambda_{iv}$  is derived from the models' control runs, in  
 110 which climate variations arise naturally from internal variability. The denominator is derived  
 111 from a forced run of the same model.

112 The CMIP5 archive does not include appropriate doubled CO<sub>2</sub> runs, but it does have abrupt  
 113 4xCO<sub>2</sub> runs from which we can estimate  $\lambda_{4\times CO_2}$ . Given that, we'll assume that  $\lambda_{2\times CO_2} \approx \lambda_{4\times CO_2}$ , so  
 114 we can re-write Eq. 3 as:

$$115 \quad ECS \approx - \frac{F_{2\times CO_2}}{\lambda_{iv}} \frac{\lambda_{iv}}{\lambda_{4\times CO_2}} \quad (4)$$

116 Recent work suggests that  $\lambda_{2xCO_2}$  is more negative (i.e., implying a lower ECS) than  $\lambda_{4xCO_2}$   
117 [Armour, 2017; Proistosescu and Huybers, 2017]. On the other hand, we use all 150 years of the  
118  $4xCO_2$  runs to estimate  $\lambda_{4xCO_2}$ , which also tends to produce values that are too negative  
119 [Andrews *et al.*, 2015; Rugenstein *et al.*, 2016; Rose and Rayborn, 2016; Armour, 2017]. These  
120 two errors tend to cancel, but how much of a bias is left — and in which direction — remains an  
121 uncertainty in this analysis. The CMIP5 ensemble distribution of  $\lambda_{iv}/\lambda_{4xCO_2}$  is plotted in Fig. 2; it  
122 has an average of 0.81 and a standard deviation of 0.34. See the appendix for details of the  
123 calculations.

124 We then use a Monte Carlo approach to estimate ECS. We produce 500,000 estimates of ECS  
125 by randomly sampling the distributions of  $F_{2xCO_2}$ ,  $\lambda_{iv}$  (Fig. 1), and  $\lambda_{iv}/\lambda_{4xCO_2}$  (Fig. 2) and plugging  
126 them into Eq. 3; negative ECS values or values greater than 10 K are viewed as implausible and  
127 thrown out. We produce two ECS distributions — one using  $\lambda_{iv}$  from the detrended calculation  
128 and one using  $\lambda_{iv}$  from the R-F calculation.

129 The ECS distributions (Fig. 3) have 17-83% confidence intervals (corresponding to the IPCC's  
130 *likely* range) of 2.0-5.7 K and 2.6-7.1 K for the detrended and R-F calculations, respectively. The  
131 modes are 2.4 and 3.3 K, while the medians are 3.2 and 4.4 K. Overall, our calculated ECS  
132 distributions overlap substantially with the IPCC's range, although our distributions are shifted  
133 to higher values: we see a 29% chance that ECS exceeds 4.5 K, while the IPCC assigns that a 17%  
134 chance. Perhaps more importantly, we see less support for low values of ECS: the chance of an  
135 ECS below 2 K is 11%, while the IPCC assigns a 17% chance it is below 1.5 K. See tables in the  
136 appendix for other relevant metrics of the distributions.

### 137 Modified energy-balance framework

138 Recently, Dessler *et al.* [2018] suggested a revision of Eq. 1, where the TOA flux is  
139 parameterized in terms of tropical atmospheric temperature, not global surface temperature:

$$140 \quad R = F + \Theta T_A \quad (5)$$

141 where  $T_A$  is the tropical average (30°N-30°S) 500-hPa temperature and  $\Theta$  converts this quantity  
 142 to TOA flux.  $R$  and  $F$  are the same global average quantities they were in equation 1. They  
 143 demonstrated that this way of describing energy balance has advantages over the conventional  
 144 approach described in Eq. 1.

145 In this framework, the equilibrium warming of the tropical atmosphere  $\Delta T_A$  is equal to  
 146  $-F_{2\times CO_2}/\Theta$ . ECS can therefore be written:

$$147 \quad ECS = -\frac{F_{2\times CO_2}}{\Theta_{iv}} \frac{\Theta_{iv}}{\Theta_{2\times CO_2}} \frac{\Delta T_S}{\Delta T_A} \approx -\frac{F_{2\times CO_2}}{\Theta_{iv}} \frac{\Theta_{iv}}{\Theta_{4\times CO_2}} \frac{\Delta T_S}{\Delta T_A} \quad (6)$$

148 where  $\Theta_{iv}$  is the analog to  $\lambda_{iv}$ ,  $\Theta_{iv}/\Theta_{2\times CO_2}$  is the transfer function that allows us to use short-term  
 149 variability to estimate ECS, and  $\Delta T_S/\Delta T_A$  is the ratio of the temperature changes at equilibrium.  
 150 As we did above, we will further assume that  $\Theta_{4\times CO_2} \approx \Theta_{2\times CO_2}$ .

151 We use the same forcing  $F_{2\times CO_2}$  that was used in the previous section. The distributions of the  
 152 scaling factor  $\Theta_{iv}/\Theta_{4\times CO_2}$  (Fig. 4a) and temperature ratio  $\Delta T_S/\Delta T_A$  (Fig. 5a) both come from the  
 153 CMIP5 ensemble (see appendix for more details), with ensemble averages and standard  
 154 deviations of  $0.99 \pm 0.40$  and  $0.86 \pm 0.10$ , respectively.

155 Just as we did for  $\lambda_{iv}$ , we calculate  $\Theta_{iv}$  from observations two ways: by regressing detrended  $R$   
 156 vs. detrended  $T_A$  and by regressing  $R-F$  vs.  $T_A$ . The 5-95% confidence intervals are -1.37 to -0.80  
 157  $W/m^2/K$  and -1.26 to -0.69  $W/m^2/K$ , respectively. The means of the two distributions are also  
 158 similar, with values of -1.09 and -0.98  $W/m^2/K$ . Because of their similarities, in the rest of this  
 159 section we will show results using the detrended  $\Theta_{iv}$  calculation, although results for both  
 160 distributions can be found in tables in the appendix.

161 As in the previous section, we use a Monte Carlo approach and produce 500,000 estimates of  
 162 ECS by randomly sampling the distributions of  $F_{2\times CO_2}$ ,  $\Theta_{iv}$ ,  $\Theta_{iv}/\Theta_{4\times CO_2}$ , and  $\Delta T_S/\Delta T_A$ , and then  
 163 plugging the values into Eq. 6. The resulting ECS distribution (Fig. 6a) shows a similar structure  
 164 to the  $\lambda$ -based distributions in Fig. 3: a broad maximum between 2 and 3 K and a tail towards  
 165 higher ECS values.

166 There is also a puzzling peak below 1°C. The only way for an ECS estimate to be close to zero is  
167 if  $\Theta_{iv}$  is very large or one of the other terms in Eq. 6 is close to zero. Analysis of the terms in Eq.  
168 6 suggests that the term causing the low ECS values is  $\Theta_{iv}/\Theta_{4xCO2}$ , whose distribution  
169 approaches zero (Fig. 4a). These low values come from the GISS models (Fig. 7a) and if they are  
170 removed from the ensemble, the bump below 1 K disappears (Fig. 6b), although the statistics of  
171 the distribution do not change much.

172 This result emphasizes that the scaling factor  $\Theta_{iv}/\Theta_{4xCO2}$  is unconstrained by observations. That  
173 doesn't mean, however, that we know nothing about it — we do have observations of  $\Theta_{iv}$  and  
174 can compare those to each model's value of  $\Theta_{iv}$ . We find that 15 of the 25 CMIP5 models  
175 produce estimates of  $\Theta_{iv}$  in agreement with the CERES observations (Fig. 7b). If we limit the  
176 distributions of  $\Theta_{iv}/\Theta_{4xCO2}$  and  $\Delta T_s/\Delta T_A$  to just those models (Figs. 4b and 5b), we obtain the ECS  
177 distribution in Fig. 6c (hereafter referred to as the “good- $\Theta$ ” distribution).

178 We consider the “good- $\Theta$ ” ECS distributions to be the best estimates of ECS from this analysis.  
179 Those ECS distributions have 17-83% confidence intervals (corresponding to the IPCC's *likely*  
180 range) of 2.4-4.4 K and 2.4-4.7 K for the detrended and R-F calculations, respectively. Averaging  
181 these gives us our single best estimate for the *likely* range, 2.4-4.5 K, and 5-95% range, 2.0-5.6  
182 K. The modes are 3.1 and 2.6 K (average 2.9 K), and the medians of both are 3.3 K.

183 These distributions suggest a 15-20% chance ECS exceeds 4.5 K and a 5% chance of an ECS  
184 below 2 K. We therefore conclude that the IPCC's upper end of the *likely* ECS range is about  
185 right, but that the low end is too low. We would conclude that, in the parlance of the IPCC, ECS  
186 is *very unlikely* to be below 2 K.

## 187 Discussion

188 There are several reasons why ECS estimated from the revised energy balance framework (Eq.  
189 6) should be considered more reliable than that estimated from the traditional framework (Eq.  
190 4). Fig. 1 shows the main advantage — that  $\Theta_{iv}$  is better constrained by observations than  $\lambda_{iv}$ .  
191 This is what leads to the narrower distributions of ECS in Fig. 6 than in Fig. 3. One particular



192 facet of the  $\lambda_{iv}$  distributions is that they have non-zero probabilities of  $\lambda_{iv}$  values close to zero;  
193 since ECS goes as  $1/\lambda_{iv}$ , this leads to the generation of a large tail towards unrealistically large  
194 values.

195 There are additional reasons that lead us to conclude that the estimates from the revised  
196 framework are superior. It has been suggested that  $\lambda_{iv}$  exhibits significant decadal variability in  
197 models [Andrews *et al.*, 2015; Gregory and Andrews, 2016; Zhou *et al.*, 2016; Dessler *et al.*,  
198 2018]. This opens the possibility that the observed  $\lambda_{iv}$ , which is based on 16 years of data, is  
199 biased with respect to the long-term average; if so, then ECS estimated from these observations  
200 would also be biased. Model simulations suggest that  $\Theta_{iv}$  exhibits smaller decadal variability  
201 [Dessler *et al.*, 2018], making  $\Theta_{iv}$  estimated from CERES data a more robust estimate of the  
202 climate system's actual long-term value. There is also evidence that  $\Theta$  changes less than  $\lambda$   
203 during transient climate change [Dessler *et al.*, 2018], meaning the assumption that  $\Theta_{2xCO_2} \approx$   
204  $\Theta_{4xCO_2}$  is a far better assumption than the assumption that  $\lambda_{4xCO_2} \approx \lambda_{2xCO_2}$ .

205 It is also worth stepping back and asking what could cause our calculation to be seriously in  
206 error. It seems unlikely that forcing from doubled CO<sub>2</sub> is wrong given our good understanding  
207 of the physics of CO<sub>2</sub> forcing [e.g., Feldman *et al.*, 2015]. Estimates of  $\lambda_{iv}$  and  $\Theta_{iv}$  are derived  
208 from observations we view to be reliable, so our judgment is that they are also unlikely to be  
209 significantly wrong. The  $\Delta T_S/\Delta T_A$  term comes from climate model simulations, but models have  
210 long been able to accurately reproduce the observed pattern of warming [e.g., Stouffer and  
211 Manabe, 2017], and we have data that can be used to validate this ratio (see appendix).

212 Thus, the transfer function  $\Theta_{iv}/\Theta_{4xCO_2}$  seems the most probable place for a significant error to  
213 occur. The ratio's distribution (Fig. 4) comes from climate model simulations; we have no way  
214 to observationally validate it, nor any theory to guide us. However, as discussed previously, we  
215 can compare observations of the numerator,  $\Theta_{iv}$ , to the models, and find that the majority of  
216 models produce values that agree with observations (Fig. 7). Thus, a large error in  $\Theta_{iv}/\Theta_{4xCO_2}$   
217 would require a large error in  $\Theta_{4xCO_2}$  that does not similarly influence  $\Theta_{iv}$ . While that cannot be  
218 ruled out, we see no reason to believe such an error exists.

219 We can also gain insight into this question by constructing an error budget to determine which  
220 term contributes most to the width of the distributions in Fig. 6. We do this by sequentially  
221 setting each term to have zero uncertainty by replacing that term's distribution in the Monte  
222 Carlo calculation with a single number, the ensemble average. This has little effect on the  
223 mean, median, or mode, but does change the width of the distribution. By comparing the  
224 widths of the resulting distributions (defined as the distance between the 17<sup>th</sup> and 83<sup>rd</sup>  
225 percentiles), we find that the biggest contributor to ECS uncertainty is the uncertainty in the  
226  $\Theta_{iv}/\Theta_{4\times CO_2}$  (Fig. 8). Eliminating the uncertainty on that reduces the 17-83% confidence interval  
227 to 2.8-4.0 K. *Thus, developing a theoretical argument for the value of this ratio would be a key*  
228 *advance in climate science.* The next most important uncertainty is the uncertainty in  $\Theta_{iv}$ ,  
229 followed by the uncertainty in  $\Delta T_S/\Delta T_A$  and then the uncertainty in  $F_{2\times CO_2}$ .

### 230 **Conclusions**

231 Estimating ECS from observations remains one of the big problems in climate science. Despite  
232 several decades of intense investigations, the uncertainty in this parameter remains stubbornly  
233 large, with the last IPCC assessment reporting a *likely* range of 1.5-4.5 K (17-83% confidence  
234 interval). Because of this, there is great value in finding alternate ways to approach the  
235 problem.

236 In this paper, we have used observations of interannual climate variations covering the period  
237 2000 to 2017 to estimate ECS. We interpret the observations using a modified energy balance  
238 framework (Eq. 5) in which the response of TOA flux is proportional to the atmospheric  
239 temperature.

240 We conclude ECS is *likely* 2.4-4.5 K (17-83% confidence interval), with a mode and median value  
241 of 2.9 and 3.3 K, respectively. Overall, our analysis suggests that the upper end of the IPCC's  
242 range is set about right, but we see little evidence to support estimates of ECS in the bottom  
243 third of the IPCC's *likely* range.

244 One of the key parts of our calculations is the use of CMIP5 climate models to convert the  
245 observations of interannual variability into an estimate of the response of the system to

246 doubled CO<sub>2</sub>. This is the main uncertainty in our analysis and future efforts to pin this transfer  
247 function down would be extremely valuable.

## 248 **Appendix**

249  $\lambda_{4xCO_2}$  and  $\Theta_{4xCO_2}$  are calculated from CMIP5 abrupt 4xCO<sub>2</sub> runs using the Gregory method  
250 [Gregory *et al.*, 2004]. In these calculations, we regress all 150 years of annual R vs. annual  
251 average temperature, and the resulting slope is  $\lambda_{4xCO_2}$  or  $\Theta_{4xCO_2}$ , depending on which  
252 temperature is used ( $T_S$  for  $\lambda$  and  $T_A$  for  $\Theta$ ). Table 1 lists values for each model.

253  $\lambda_{iv}$  and  $\Theta_{iv}$  are calculated from the CMIP5 control runs. To facilitate comparison with the  
254 observations, as well as avoid any issues with long-term drift in the control runs, we break each  
255 control run into 16-year segments and calculate monthly anomalies of  $\Delta R$ ,  $\Delta T_S$ ,  $\Delta T_A$  during each  
256 segment (anomalies are departures from the mean annual cycle). Then, we calculate  $\lambda_{iv}$  and  $\Theta_{iv}$   
257 for each segment as the slope of the regression of  $\Delta R$  vs.  $\Delta T_S$  or  $\Delta T_A$  for that segment. We  
258 average the segments' values to come up with a single value for each model. Table 1 lists  
259 values for each model.

260 Observational estimates of  $\lambda_{iv}$  and  $\Theta_{iv}$  come from measurements of R from the CERES Energy  
261 Balanced and Filled product (ed. 4) [Loeb *et al.*, 2018] and cover the period March 2000 to July  
262 2017. The CERES time series is stable to better than 0.5 W/m<sup>2</sup>/decade (stability of the  
263 shortwave is 0.3 W/m<sup>2</sup>/decade [Loeb *et al.*, 2007], and longwave is 0.15 W/m<sup>2</sup>/decade  
264 [Susskind *et al.*, 2012]). Our sign convention throughout the paper is that downward fluxes are  
265 positive, which means  $\lambda$  and  $\Theta$  must be negative for a stable climate.

266 In the detrended calculations, the time series are detrended by subtracting off the linear trend  
267 estimated using an ordinary linear least-squares regression. In the R-F calculations, we use  
268 radiative forcing estimates based on IPCC AR5 [Myhre *et al.*, 2013], updated to July 2017 using  
269 greenhouse gas data from NOAA and ECLIPSE (Evaluating the Climate and Air Quality Impacts of  
270 Short-Lived Pollutants) [Hofmann *et al.*, 2006; Stohl *et al.*, 2015]. These data include a higher  
271 methane forcing estimate [Etminan *et al.*, 2016], and updated ozone and anthropogenic aerosol  
272 forcing data [Myhre *et al.*, 2017a]. The time-series of solar irradiance data is extended using

273 data from the Solar Radiation and Climate Experiment (SORCE) experiment [Lean *et al.*, 2005]  
 274 and volcanic forcing data from NASA/GISS (<https://data.giss.nasa.gov/modelforce/strataer/>)  
 275 with a revised relationship of  $17 \text{ Wm}^{-2}$  per unit optical depth of stratospheric aerosol [Gregory  
 276 *et al.*, 2016]. Uncertainty is estimated using IPCC AR5 radiative forcing uncertainties from 2015.  
 277 We take the 5%-95% range for each of the 14 different forcing terms in 2015 and turn this into  
 278 a fractional range by dividing by the median 1750-2015 forcing estimate. This fractional  
 279 uncertainty is Monte Carlo sampled 100,000 times for each forcing term independently. These  
 280 fractions are then multiplied by the relevant forcing time series and summed to create 100,000  
 281 different realizations of the time series of total radiative forcing.

282 The distributions of  $\lambda_{iv}$  and  $\Theta_{iv}$  in Fig. 1 are estimated by randomly sampling (with replacement)  
 283 the detrended R or R-F and temperature time series. Each resampled data set is regressed and  
 284 the slope provides one estimate  $\lambda_{iv}$  or  $\Theta_{iv}$ . We adjust for autocorrelation of the time series by  
 285 reducing the number of samples taken following Santer *et al.* [2000].

286 To evaluate the accuracy of the CMIP5 ensemble's estimate of  $\Delta T_S/\Delta T_A$ , we re-write it as the  
 287 product of two terms:

$$288 \quad \frac{\Delta T_S}{\Delta T_A} = \frac{\Delta T_{S,tropics}}{\Delta T_A} \frac{\Delta T_S}{\Delta T_{S,tropics}} \quad (7)$$

289 where  $\Delta T_{S,tropics}$  is the tropical (30°N-30°S) average surface temperature change. The term  
 290  $\Delta T_{S,tropics}/\Delta T_A$  is a measure of the tropical lapse rate, which is understood to be controlled by  
 291 moist convective adjustment [Xu and Emanuel, 1989]. Fig. 9a plots monthly average anomalies  
 292 of  $\Delta T_{S,tropics}$  vs.  $\Delta T_A$  from the ERAi and, as expected, there is a clear correlation between these  
 293 variables. The slope derived from this regression is  $0.51 \pm 0.06$  (5-95% confidence interval).

294 The ERAi data set, covering 1979-2016 (37 years), contains both long-term warming and  
 295 interannual variability. Because of this, we compare the ERAi results to what we consider to be  
 296 the most analogous model period, the last 37 years of the CMIP5 ensemble's 150-year abrupt  
 297  $4xCO_2$  runs. Ensemble average  $\Delta T_{S,tropics}$  over this period is 1.07 K, similar to the warming in the

298 ERAI from 1979-2016. While a few models appear to have issues with this metric, there is  
299 generally good agreement between the models and from observations (Fig. 9b).

300 The second term on the right-hand side,  $\Delta T_S/\Delta T_{S,tropics}$ , is a measure of polar amplification in the  
301 pattern of surface warming. We estimate this by differencing the averages of the first and last  
302 decade of observations or models. The ECMWF 20<sup>th</sup> century reanalysis [Poli *et al.*, 2016]  
303 produces a value of 1.20 over the years 1900-2010 while the NOAA 20<sup>th</sup> century reanalysis  
304 project [Compo *et al.*, 2011] produces a value of 1.23 over the years 1851-2014. We estimate  
305 this ratio in each CMIP5 abrupt 4xCO<sub>2</sub> run and the ensemble agrees well with observations (Fig.  
306 11), with a CMIP5 ensemble average of 1.18 and standard deviation of 0.11. Such good  
307 agreement is not surprising — climate models have long demonstrated considerable skill in  
308 simulating the large-scale patterns of surface warming [e.g., Stouffer and Manabe, 2017].

309 Tables 2, 3, and 4 contain the numerical results from the ECS distributions using  $\lambda$  (Eq. 4),  $\Theta$  (Eq.  
310 6), and the error budget, respectively. The tables include the average, mode, median, 17-83%  
311 range, 5-95% range, and the probability of an ECS below 2 K or above 4.5 K.

312 The names contain “all” or “good” depending on whether they include all models or just the  
313 ones whose  $\lambda_{iv}$  or  $\Theta_{iv}$  agree with the observations. The names also include “-1” or “-2”. The  
314 “-1” results use  $\lambda_{iv}$  or  $\Theta_{iv}$  derived using estimates of forcing (the R-F calculations) while the “-2”  
315 use estimates from the detrended calculations.

316 In Table 3, the “noGiss” results include all models other than the two GISS models. In the  
317 “good-Theta-1-corr” result, each Monte Carlo value of ECS uses values of  $\Delta T_S/\Delta T_A$  and  
318  $\Theta_{iv}/\Theta_{4xCO_2}$  from the same model. The results are similar to other results, showing that  
319 correlation in parameters between the models has little impact on our results. In the “good-  
320 Theta-1-normal” result, we replace the distributions of  $\Delta T_S/\Delta T_A$  and  $\Theta_{iv}/\Theta_{4xCO_2}$  with normal  
321 distributions having the same mean and standard deviation. This has little effect on the  
322 resulting ECS distribution (compared to “good-Theta-1”).

323 Table 4 lists the results from the error analysis calculations. For these values, we take the  
324 “good-Theta-2” calculation and sequentially set the uncertainty in one term to zero. The  
325 “-noF”, “-noRat”, “-nodtdt”, and “-noTheta” correspond to no uncertainty in  $F_{2\times CO_2}$ ,  $\Theta_{iv}/\Theta_{4\times CO_2}$ ,  
326  $\Delta T_s/\Delta T_A$ , and  $\Theta_{iv}$ , respectively.

## 327 **References**

- 328
- 329 Aldrin, M., M. Holden, P. Guttorp, R. B. Skeie, G. Myhre, and T. K. Berntsen (2012), Bayesian  
330 estimation of climate sensitivity based on a simple climate model fitted to observations of  
331 hemispheric temperatures and global ocean heat content, *Environmetrics*, 23(3), 253-271,  
332 doi: 10.1002/env.2140.
- 333 Andrews, T., and M. J. Webb (2018), The Dependence of Global Cloud and Lapse Rate  
334 Feedbacks on the Spatial Structure of Tropical Pacific Warming, *J. Climate*, 31(2), 641-654,  
335 doi: 10.1175/jcli-d-17-0087.1.
- 336 Andrews, T., J. M. Gregory, and M. J. Webb (2015), The dependence of radiative forcing and  
337 feedback on evolving patterns of surface temperature change in climate models, *J. Climate*,  
338 28(4), 1630-1648, doi: 10.1175/JCLI-D-14-00545.1.
- 339 Annan, J. D., and J. C. Hargreaves (2006), Using multiple observationally-based constraints to  
340 estimate climate sensitivity, *Geophys. Res. Lett.*, 33(6), doi: 10.1029/2005gl025259.
- 341 Armour, K. C. (2017), Energy budget constraints on climate sensitivity in light of inconstant  
342 climate feedbacks, *Nature Clim. Change*, 7(5), 331-335, doi: 10.1038/nclimate3278.
- 343 Chung, E. S., B. J. Soden, and B. J. Sohn (2010), Revisiting the determination of climate  
344 sensitivity from relationships between surface temperature and radiative fluxes, *Geophys.*  
345 *Res. Lett.*, 37, doi: 10.1029/2010gl043051.
- 346 Collins, M., et al. (2013), Long-term climate change: Projections, commitments and  
347 irreversibility, in *Climate Change 2013: The Physical Science Basis. Contribution of*  
348 *Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate*  
349 *Change*, edited by T. F. Stocker, D. Qin, G.-K. Plattner, M. Tignor, S. K. Allen, J. Boschung,  
350 A. Nauels, Y. Xia, V. Bex and P. M. Midgley, Cambridge University Press, Cambridge,  
351 United Kingdom and New York, NY, USA.
- 352 Compo, G. P., et al. (2011), The Twentieth Century Reanalysis Project, *Q. J. R. Meteor. Soc.*,  
353 137(654), 1-28, doi: 10.1002/qj.776.

354 Dee, D. P., et al. (2011), The ERA-Interim reanalysis: Configuration and performance of the data  
355 assimilation system, *Q. J. R. Meteor. Soc.*, 137(656), 553-597, doi: 10.1002/qj.828.

356 Dessler, A. E. (2013), Observations of climate feedbacks over 2000-10 and comparisons to  
357 climate models, *J. Climate*, 26(1), 333-342, doi: 10.1175/jcli-d-11-00640.1.

358 Dessler, A. E., T. Mauritsen, and B. Stevens (2018), The influence of internal variability on  
359 Earth's energy balance framework and implications for estimating climate sensitivity, *Atmos.*  
360 *Chem. Phys. Discuss.*, 2018, 1-21, doi: 10.5194/acp-2017-1236.

361 Donohoe, A., K. C. Armour, A. G. Pendergrass, and D. S. Battisti (2014), Shortwave and  
362 longwave radiative contributions to global warming under increasing CO<sub>2</sub>, *Proc. Natl. Acad.*  
363 *Sci.*, 111(47), 16700-16705, doi: 10.1073/pnas.1412190111.

364 Etminan, M., G. Myhre, E. J. Highwood, and K. P. Shine (2016), Radiative forcing of carbon  
365 dioxide, methane, and nitrous oxide: A significant revision of the methane radiative forcing,  
366 *Geophys. Res. Lett.*, 43(24), 12,614-612,623, doi: 10.1002/2016GL071930.

367 Feldman, D. R., W. D. Collins, P. J. Gero, M. S. Torn, E. J. Mlawer, and T. R. Shippert (2015),  
368 Observational determination of surface radiative forcing by CO<sub>2</sub> from 2000 to 2010, *Nature*,  
369 519, 339, doi: 10.1038/nature14240.

370 Forster, P. M. (2016), Inference of climate sensitivity from analysis of Earth's energy budget,  
371 *Annual Review of Earth and Planetary Sciences*, 44, 85-106, doi: 10.1146/annurev-earth-  
372 060614-105156.

373 Forster, P. M. D., and J. M. Gregory (2006), The climate sensitivity and its components  
374 diagnosed from Earth Radiation Budget data, *J. Climate*, 19(1), 39-52.

375 Gregory, J. M., and T. Andrews (2016), Variation in climate sensitivity and feedback parameters  
376 during the historical period, *Geophys. Res. Lett.*, 43(8), 3911-3920, doi:  
377 10.1002/2016GL068406.

378 Gregory, J. M., R. J. Stouffer, S. C. B. Raper, P. A. Stott, and N. A. Rayner (2002), An  
379 observationally based estimate of the climate sensitivity, *J. Climate*, 15(22), 3117-3121, doi:  
380 10.1175/1520-0442(2002)015<3117:aobeot>2.0.co;2.

381 Gregory, J. M., T. Andrews, P. Good, T. Mauritsen, and P. M. Forster (2016), Small global-mean  
382 cooling due to volcanic radiative forcing, *Climate Dynamics*, 47(12), 3979-3991, doi:  
383 10.1007/s00382-016-3055-1.

384 Gregory, J. M., W. J. Ingram, M. A. Palmer, G. S. Jones, P. A. Stott, R. B. Thorpe, J. A. Lowe,  
385 T. C. Johns, and K. D. Williams (2004), A new method for diagnosing radiative forcing and  
386 climate sensitivity, *Geophys. Res. Lett.*, 31(3), doi: 10.1029/2003gl018747.

387 Hofmann, D. J., J. H. Butler, E. J. Dlugokencky, J. W. Elkins, K. Masarie, S. A. Montzka, and P.  
388 Tans (2006), The role of carbon dioxide in climate forcing from 1979 to 2004: introduction  
389 of the Annual Greenhouse Gas Index, *Tellus B*, 58(5), 614-619, doi: 10.1111/j.1600-  
390 0889.2006.00201.x.

391 Kummer, J. R., and A. E. Dessler (2014), The impact of forcing efficacy on the equilibrium  
392 climate sensitivity, *Geophys. Res. Lett.*, 41(10), 3565-3568, doi: 10.1002/2014gl060046.

393 Lean, J., G. Rottman, J. Harder, and G. Kopp (2005), *SORCE contributions to new*  
394 *understanding of global change and solar variability*, *Solar Physics*, 230(1-2), 27-53, doi:  
395 10.1007/s11207-005-1527-2.

396 Lewis, N., and J. A. Curry (2015), The implications for climate sensitivity of AR5 forcing and  
397 heat uptake estimates, *Climate Dynamics*, 45(3), 1009-1023, doi: 10.1007/s00382-014-2342-  
398 y.

399 Loeb, N. G., S. Kato, K. Loukachine, N. Manalo-Smith, and D. R. Doelling (2007), Angular  
400 distribution models for top-of-atmosphere radiative flux estimation from the Clouds and the  
401 Earth's Radiant Energy System instrument on the Terra satellite. Part II: Validation, *Journal*  
402 *of Atmospheric and Oceanic Technology*, 24(4), 564-584.

403 Loeb, N. G., D. R. Doelling, H. Wang, W. Su, C. Nguyen, J. G. Corbett, L. Liang, C. Mitrescu,  
404 F. G. Rose, and S. Kato (2018), Clouds and the Earth's Radiant Energy System (CERES)  
405 Energy Balanced and Filled (EBAF) Top-of-Atmosphere (TOA) Edition-4.0 Data Product, *J.*  
406 *Climate*, 31(2), 895-918, doi: 10.1175/jcli-d-17-0208.1.

407 Marvel, K., R. Pincus, G. A. Schmidt, and R. L. Miller (2018), Internal variability and  
408 disequilibrium confound estimates of climate sensitivity from observations, *Geophys. Res.*  
409 *Lett.*, doi: 10.1002/2017GL076468.

410 Myhre, G., et al. (2013), Anthropogenic and Natural Radiative Forcing, in *Climate Change 2013:*  
411 *The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report*  
412 *of the Intergovernmental Panel on Climate Change*, edited by T. F. Stocker, D. Qin, G.-K.  
413 Plattner, M. Tignor, S. K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P. M. Midgley,  
414 Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.



415 Myhre, G., et al. (2017a), Multi-model simulations of aerosol and ozone radiative forcing due to  
416 anthropogenic emission changes during the period 1990–2015, *Atmos. Chem. Phys.*, 17(4),  
417 2709-2720, doi: 10.5194/acp-17-2709-2017.

418 Myhre, G., et al. (2017b), PDRMIP: A Precipitation Driver and Response Model  
419 Intercomparison Project—Protocol and Preliminary Results, *Bull. Am. Met. Soc.*, 98(6),  
420 1185-1198, doi: 10.1175/bams-d-16-0019.1.

421 Otto, A., et al. (2013), Energy budget constraints on climate response, *Nature Geoscience*, 6(6),  
422 415-416, doi: 10.1038/ngeo1836.

423 Poli, P., et al. (2016), ERA-20C: An Atmospheric Reanalysis of the Twentieth Century, *J.*  
424 *Climate*, 29(11), 4083-4097, doi: 10.1175/jcli-d-15-0556.1.

425 Proistosescu, C., and P. J. Huybers (2017), Slow climate mode reconciles historical and model-  
426 based estimates of climate sensitivity, *Science Advances*, 3(7), doi: 10.1126/sciadv.1602821.

427 Richardson, M., K. Cowtan, E. Hawkins, and M. B. Stolpe (2016), Reconciled climate response  
428 estimates from climate models and the energy budget of Earth, *Nature Clim. Change*, 6(10),  
429 931-935, doi: 10.1038/nclimate3066.

430 Rose, B. E. J., and L. Rayborn (2016), The effects of ocean heat uptake on transient climate  
431 sensitivity, *Current Climate Change Reports*, 2(4), 190-201, doi: 10.1007/s40641-016-0048-  
432 4.

433 Rugenstein, M. A. A., K. Caldeira, and R. Knutti (2016), Dependence of global radiative  
434 feedbacks on evolving patterns of surface heat fluxes, *Geophys. Res. Lett.*, 43(18), 9877-  
435 9885, doi: 10.1002/2016GL070907.

436 Santer, B. D., T. M. L. Wigley, J. S. Boyle, D. J. Gaffen, J. J. Hnilo, D. Nychka, D. E. Parker,  
437 and K. E. Taylor (2000), Statistical significance of trends and trend differences in layer-  
438 average atmospheric temperature time series, *J. Geophys. Res.*, 105(D6), 7337-7356, doi:  
439 10.1029/1999jd901105.

440 Sherwood, S. C., S. Bony, O. Boucher, C. Bretherton, P. M. Forster, J. M. Gregory, and B.  
441 Stevens (2014), Adjustments in the forcing-feedback framework for understanding climate  
442 change, *Bull. Am. Met. Soc.*, 96(2), 217-228, doi: 10.1175/BAMS-D-13-00167.1.

443 Shindell, D. T. (2014), Inhomogeneous forcing and transient climate sensitivity, 4, 274, doi:  
444 10.1038/nclimate2136.

445 Skeie, R. B., T. Berntsen, M. Aldrin, M. Holden, and G. Myhre (2014), A lower and more  
446 constrained estimate of climate sensitivity using updated observations and detailed radiative  
447 forcing time series, *Earth System Dynamics*, 5(1), 139-175, doi: 10.5194/esd-5-139-2014.

448 Stevens, B., S. C. Sherwood, S. Bony, and M. J. Webb (2016), Prospects for narrowing bounds  
449 on Earth's equilibrium climate sensitivity, *Earth's Future*, 4(11), 512-522, doi:  
450 10.1002/2016EF000376.

451 Stohl, A., et al. (2015), Evaluating the climate and air quality impacts of short-lived pollutants,  
452 *Atmos. Chem. Phys.*, 15(18), 10529-10566, doi: 10.5194/acp-15-10529-2015.

453 Stouffer, R. J., and S. Manabe (2017), Assessing temperature pattern projections made in 1989,  
454 *Nature Clim. Change*, 7(3), 163-165, doi: 10.1038/nclimate3224.

455 Susskind, J., G. Molnar, L. Iredell, and N. G. Loeb (2012), Interannual variability of outgoing  
456 longwave radiation as observed by AIRS and CERES, *J. Geophys. Res.*, 117, doi:  
457 10.1029/2012jd017997.

458 Taylor, K. E., R. J. Stouffer, and G. A. Meehl (2012), An overview of CMIP5 and the  
459 experiment design, *Bull. Am. Met. Soc.*, 93(4), 485-498, doi: 10.1175/bams-d-11-00094.1.

460 Tsushima, Y., and S. Manabe (2013), Assessment of radiative feedback in climate models using  
461 satellite observations of annual flux variation, *Proc. Natl. Acad. Sci.*, 110(19), 7568-7573,  
462 doi: 10.1073/pnas.1216174110.

463 Tsushima, Y., A. Abe-Ouchi, and S. Manabe (2005), Radiative damping of annual variation in  
464 global mean surface temperature: comparison between observed and simulated feedback,  
465 *Climate Dynamics*, 24(6), 591-597, doi: 10.1007/s00382-005-0002-y.

466 Xie, S.-P., Y. Kosaka, and Y. M. Okumura (2016), Distinct energy budgets for anthropogenic  
467 and natural changes during global warming hiatus, *Nature Geoscience*, 9(1), 29-33, doi:  
468 10.1038/ngeo2581.

469 Xu, K. M., and K. A. Emanuel (1989), Is the tropical atmosphere conditionally unstable?, *Mon.*  
470 *Wea. Rev.*, 117(7), 1471-1479.

471 Zhou, C., M. D. Zelinka, and S. A. Klein (2016), Impact of decadal cloud variations on the  
472 Earth's energy budget, *Nature Geosci*, 9(12), 871-874, doi: 10.1038/ngeo2828.

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481 be found here: <http://bit.ly/2FRE83c>. All other data (CERES, CMIP5, reanalyses) are publicly  
482 available on the internet.  
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Table 1. Values for individual models

Model	$\lambda_{iv}$	$\Theta_{iv}$	$\lambda_{2xCO_2}$	$\Theta_{2xCO_2}$	$\Delta T_S/\Delta T_A$	$\Delta T_{S,tropics}/\Delta T_A$	$\Delta T_S/\Delta T_{S,tropics}$
ACCESS1-0	-0.69	-1.22	-0.75	-0.77	0.96	0.48	1.25
ACCESS1-3	-0.66	-0.86	-0.82	-0.74	0.91	0.46	1.19
BCC-CSM1-1	-0.74	-0.89	-1.21	-1.12	0.93	0.42	1.30
BCC-CSM1-1-M	-0.91	-0.94	-1.31	-1.23	0.92	0.51	1.15
CCSM4	-1.26	-1.25	-1.24	-1.26	0.99	0.58	1.26
CNRM-CM5	-1.14	-1.25	-1.11	-1.01	0.94	0.43	1.27
CNRM-CM5-2	-1.01	-1.25	-1.06	-0.94	0.89	0.41	1.21
CanESM2	-0.77	-0.73	-1.03	-0.90	0.88	0.49	1.16
FGOALS-g2	-1.55	-1.25	-0.83	-0.85	1.00	0.50	1.37
FGOALS-s2	-1.35	-1.60	-0.88	-0.77	0.87	0.47	1.20
GFDL-CM3	-0.21	-0.63	-0.75	-0.63	0.80	0.45	1.15
GFDL-ESM2G	-0.80	-1.24	-1.42	-0.98	0.68	0.43	1.02
GFDL-ESM2M	-1.41	-1.12	-1.34	-0.92	0.74	0.43	1.08
GISS-E2-H	-1.48	-0.36	-1.57	-1.36	0.91	0.26	1.35
GISS-E2-R	-1.03	-0.16	-1.70	-1.35	0.77	0.20	1.17
INMCM4	-0.65	-0.83	-1.51	-1.18	0.80	0.56	1.12
IPSL-CM5A-LR	-0.57	-0.61	-0.79	-0.54	0.71	0.49	1.04
IPSL-CM5A-MR	-0.46	-0.33	-0.81	-0.54	0.68	0.55	0.97
IPSL-CM5B-LR	-0.93	-0.94	-1.00	-0.87	0.91	0.45	1.33
MIROC5	-1.18	-0.90	-1.58	-1.13	0.84	0.57	1.18
MPI-ESM-LR	-0.78	-0.72	-1.14	-0.91	0.81	0.62	1.08
MPI-ESM-MR	-0.69	-0.76	-1.18	-0.93	0.80	0.52	1.05
MPI-ESM-P	-0.72	-0.70	-1.25	-0.98	0.80	0.58	1.09
MRI-CGCM3	-0.58	-1.29	-1.26	-1.11	0.88	0.40	1.21
NorESM1-M	-1.19	-1.13	-1.11	-1.15	1.02	0.47	1.34

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Units on  $\lambda$  and  $\Theta$  are  $W/m^2/K$ , other quantities are unitless. Methods of estimating these values are described in the methods section.  $\Delta T_S/\Delta T_A$  and  $\Delta T_S/\Delta T_{S,tropics}$  are calculated by differencing averages of the first and last decades of the abrupt  $4xCO_2$  run.  $\Delta T_{S,tropics}/\Delta T_A$  is estimated by regressing monthly values from the last 37 years of the abrupt  $4xCO_2$  run.

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492 Table 2. Values from the  $\lambda$  runs

run	mean	mode	median	5-95%	17-83%	%<2	%>4.5
all-Lambda-1	4.76	3.39	4.39	1.8-8.9	2.6-7.1	5	34
all-Lambda-2	3.79	2.31	3.30	1.4-8.0	2.0-5.7	15	26
good-Lambda-1	4.33	2.85	3.89	1.7-8.5	2.4-6.4	7	30
good-Lambda-2	3.66	2.31	3.19	1.4-7.7	1.9-5.4	17	24

493 Explanation of the run names is in the methods section.

494

495 Table 3. Values from the  $\Theta$  runs

run	mean	mode	median	5-95%	17-83%	%<2	%>4.5
all-Theta-1	3.32	2.71	3.15	0.7-6.1	2.1-4.6	15	19
all-Theta-2	2.96	2.31	2.82	0.7-5.4	1.9-4.1	20	11
all-Theta-1-corr	3.36	2.58	3.14	0.8-6.4	2.0-4.8	16	20
noGISS-Theta-1	3.56	2.71	3.29	1.9-6.3	2.3-4.7	7	20
noGISS-Theta-2	3.18	2.31	2.95	1.7-5.5	2.1-4.2	13	13
good-Theta-1	3.54	2.58	3.31	2.0-5.9	2.4-4.7	5	20
good-Theta-2	3.43	3.12	3.32	1.9-5.4	2.4-4.4	6	15
good-Theta-1-corr	3.55	2.44	3.29	1.9-6.1	2.3-4.8	6	21
good-Theta-1-normal	3.54	3.12	3.39	1.8-5.8	2.4-4.6	8	19

496 Explanation of the run names is in the methods section.

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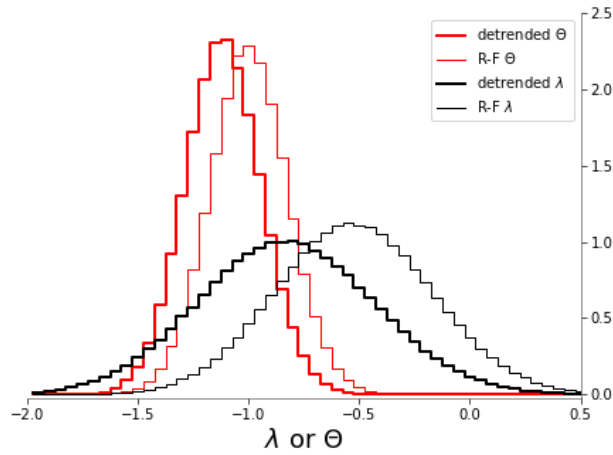
498 Table 4. Error budget calculations

run	mean	mode	median	5-95%	17-83%	%<2	%>4.5
error-goodTheta-2-noF	3.43	3.25	3.33	1.9-5.3	2.4-4.4	6	15
error-goodTheta-2-noRat	3.43	3.25	3.36	2.4-4.7	2.8-4.0	0	7
error-goodTheta-2-nodtdt	3.43	3.25	3.35	2.0-5.2	2.4-4.4	5	14
error-goodTheta-2-noTheta	3.34	3.53	3.35	2.1-4.9	2.4-4.2	4	10

499 Explanation of the run names is in the methods section.

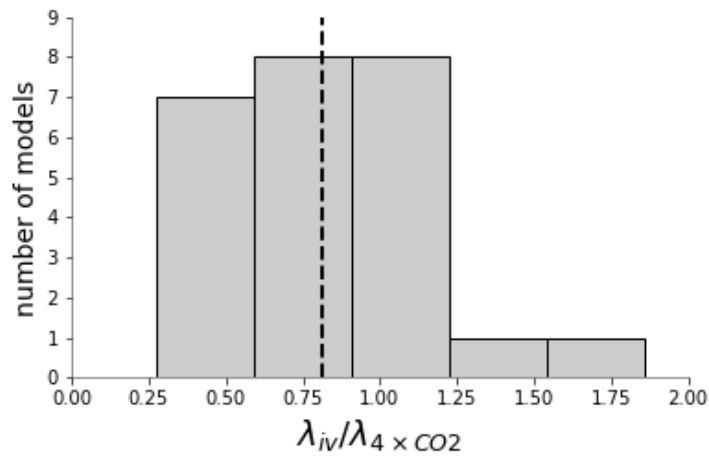
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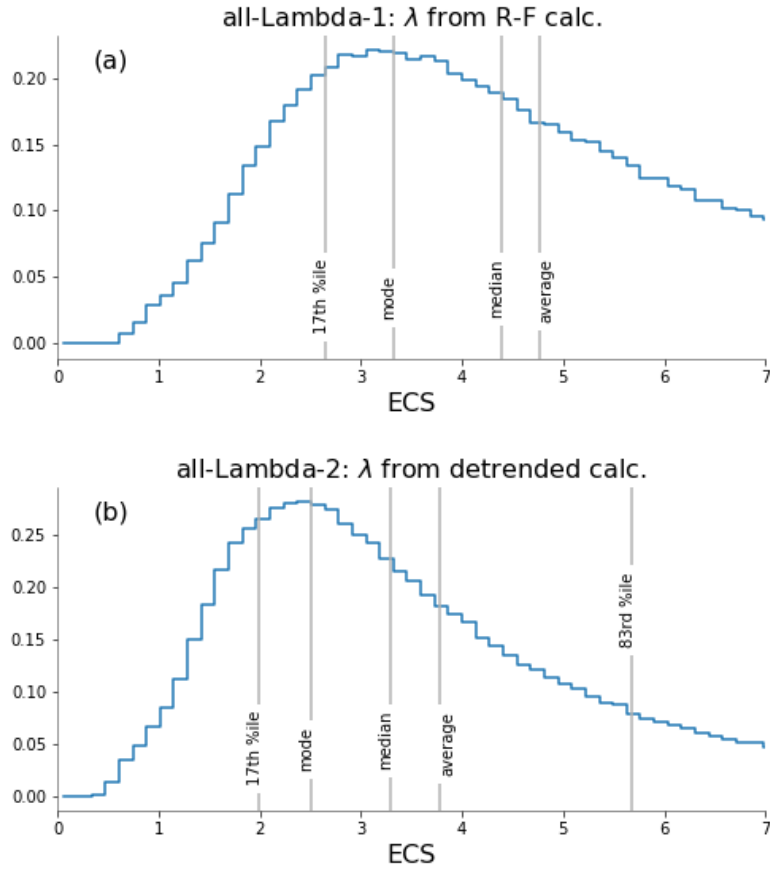
Figure 1. Distribution of  $\lambda_{iv}$  and  $\Theta_{iv}$  from observations. Distributions calculated using detrended and R-F regressions are both shown.



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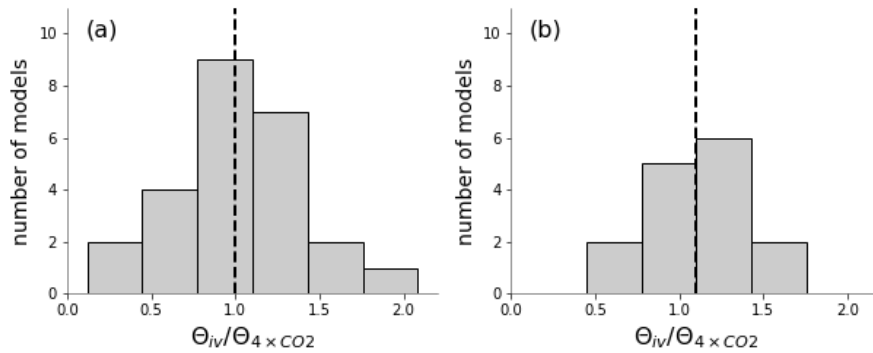
Figure 2. Distribution of  $\lambda_{iv}/\lambda_{4\times CO_2}$  from 25 CMIP5 models; the black dashed line is the mean of the distribution. See methods for description of how the value is calculated in each model.

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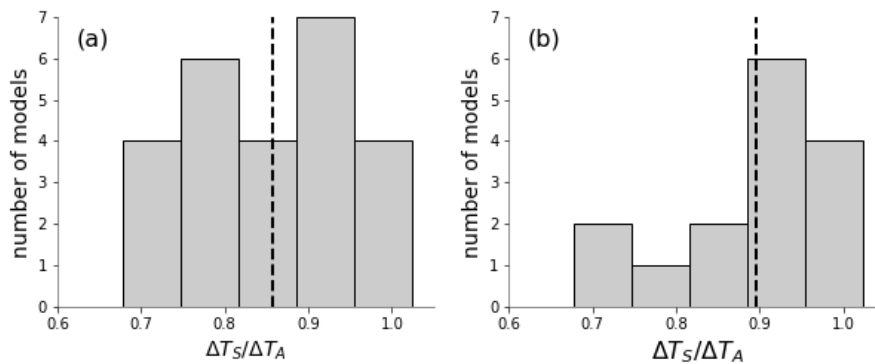
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Figure 3. Distributions of ECS using the traditional energy balance framework (Eq. 4). (a) Distribution using  $\lambda_{iv}$  from the R-F regression, (b) Distribution using  $\lambda_{iv}$  from the detrended regression. “17th %ile” and “83rd %ile” are 17<sup>th</sup> and 83<sup>rd</sup> percentile, corresponding to the IPCC’s *likely* range.



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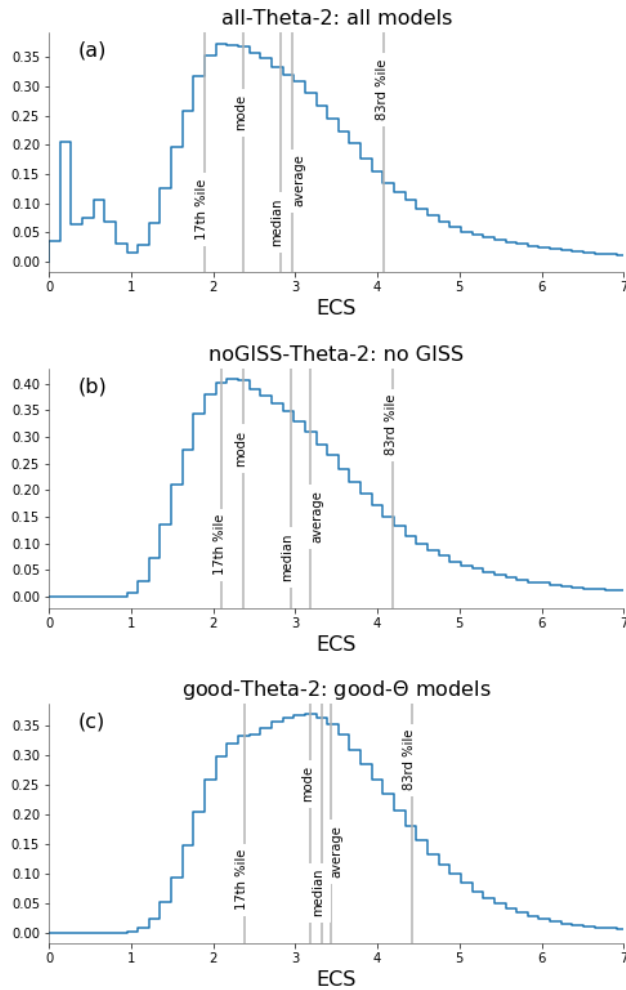
Figure 4. Distribution of  $\Theta_{iv}/\Theta_{4xCO_2}$  from (a) 25 CMIP5 models and (b) from those 15 models whose  $\Theta_{iv}$  agrees with observations. The black dashed lines are the means of the distributions. See methods for description of how this is calculated in each model.



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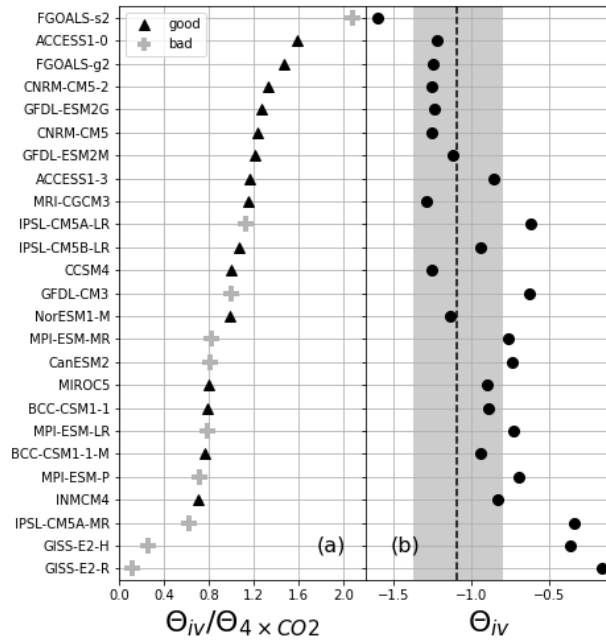
Figure 5. Distribution of  $\Delta T_S/\Delta T_A$  from (a) 25 CMIP5 models and (b) from those 15 models whose  $\Theta_{iv}$  agrees with observations. Calculated by differencing the average of the first and last decades of the CMIP5 ensemble's abrupt  $4xCO_2$  runs. The black dashed lines are the means of the distributions.





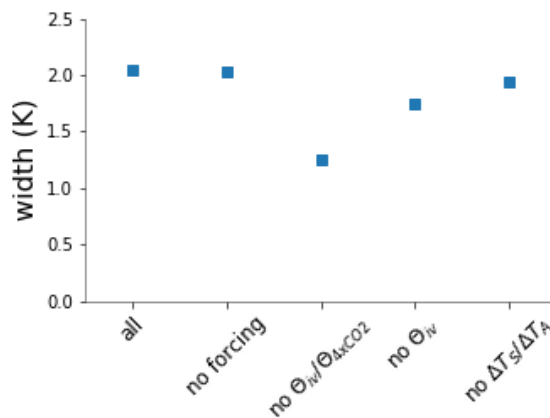
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Figure 6. Distributions of ECS using the revised energy balance framework (Eq. 6) (a) using all models, (b) using all models except for the two GISS models, (c) using 15 models whose  $\Theta_{iv}$  agrees with the value estimated from observations. All calculations use  $\Theta_{iv}$  from the detrended calculation; using  $\Theta_{iv}$  from the R-F calculation produces nearly identical results. “17th %ile” and “83rd %ile” are 17<sup>th</sup> and 83<sup>rd</sup> percentile, corresponding to the IPCC’s *likely* range.



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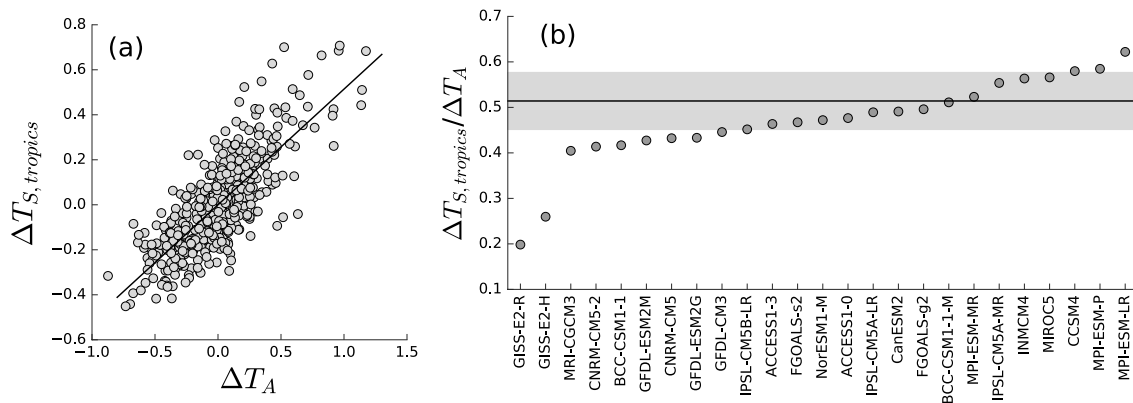
Figure 7. CMIP5 model estimates of (a)  $\Theta_{iv}/\Theta_{2\times CO_2}$  and (b)  $\Theta_{iv}$ . The gray region in panel b shows the observational range (from the CERES detrended calculation). The black triangle symbols in panel a) indicate that the model's  $\Theta_{iv}$  agrees with observations; the gray cross symbols indicate that it does not.



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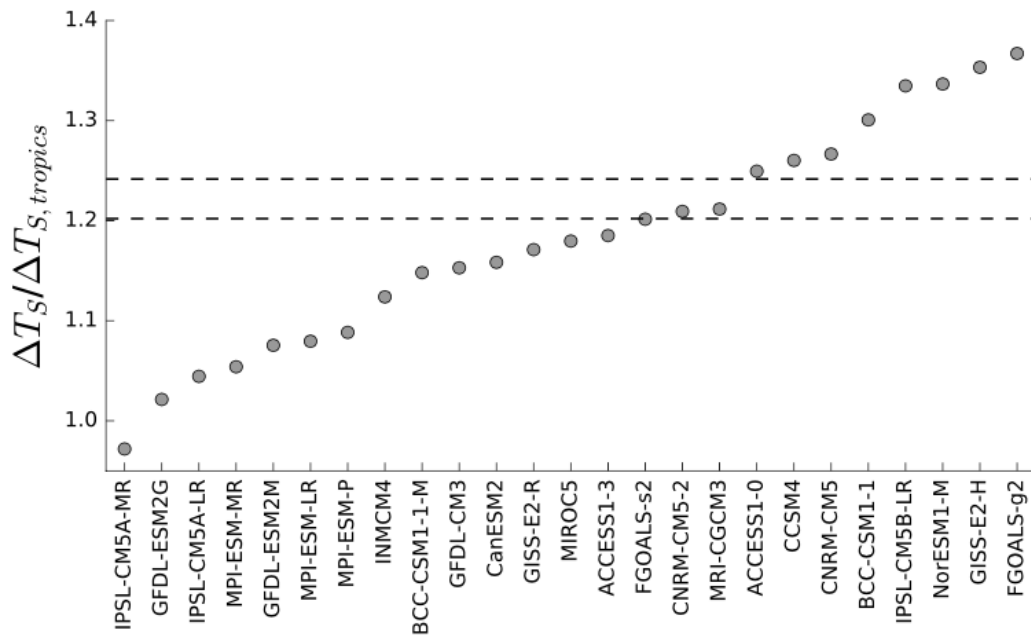
Figure 8. Error budget analysis of ECS estimates. The “all” data are the widths of the full ECS distribution from the good-Theta-2 calculation (Table 3). Then, from left to right, is the width when the uncertainty in forcing,  $\Theta_{iv}/\Theta_{4\times CO_2}$ ,  $\Theta_{iv}$ , and  $\Delta T_s/\Delta T_A$  distributions are sequentially set to zero. Dots show uncertainty as measured by the difference between the 17<sup>th</sup> and 83<sup>rd</sup> percentile of the ECS distribution.

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Figure 9. Estimates of  $\Delta T_{S, \text{tropics}} / \Delta T_A$ . (a) Scatter plot of monthly  $\Delta T_{S, \text{tropics}}$  (tropical avg. surface temperature) anomalies vs.  $\Delta T_A$  anomalies from ERAi reanalysis. The solid line is the best fit line. (b) The slope of the fit to the same regression from the last 37 years of the CMIP5 ensemble's abrupt  $4\times\text{CO}_2$  runs. The black line and gray region shows the slope and uncertainty of the fit to observations in panel a.



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Figure 10. Estimates of polar amplification in the models,  $\Delta T_S / \Delta T_{S, \text{tropics}}$ . For the CMIP5 ensemble, this is calculated by differencing the average of the first and last decades of the CMIP5 ensemble's abrupt  $4\times\text{CO}_2$  runs. The two dashed lines are observational estimates (see text).