Cover Sheet

Title: A heuristic approach to structural consistency of a horizon surface with tectonic faults based on Mallet's discrete model.

Author: Oleg L. Konovalov

Affiliation: Belarusian State University, 220030 Minsk, Belarus

ORCID: `https://orcid.org/0009-0009-6367-4655`

Corresponding author email: `konovalovol@bsu.by`

Note on provenance: This work was originally carried out in 2012–2015 as part of a PhD dissertation [1]. The present publication was prepared in 2026 and describes the approach that served as a precursor to the works [2, 3].

A heuristic approach to structural consistency of a horizon surface with tectonic faults based on Mallet's discrete model*

Oleg L. Konovalov

Belarusian State University, 220030 Minsk, Belarus

e-mail: konovalovol@bsu.by

Abstract

The paper addresses the problem of automating the consistency process between reflecting horizon surfaces and the network of tectonic faults in the construction of three-dimensional digital geological models within the explicit (surface-based) approach. The problem is formalized on the basis of Mallet's discrete model (Discrete Smooth Interpolation, DSI). It is shown that the classical formulation with dynamic weak constraints leads to instability of the gradient descent iteration process. A heuristic approach is proposed, based on restricting the class of sought surfaces to vertically co-oriented triangulated surfaces, which allows reducing the three-dimensional problem to a two-dimensional one. A parametric model of topological consistency is developed with fault amplitude as a control parameter, and an algorithm for sequential incorporation of weak control points is proposed, replacing the gradient descent with stable two-dimensional finite element interpolation. Results of testing on real geological models are presented. The work served as the basis for subsequent development of the approach within a variational free-boundary formulation [2, 3].

Keywords: structural geological model, horizon consistency, Mallet's discrete model, DSI, vertically co-oriented surfaces, finite element interpolation, fault amplitude.

1 Introduction

Construction of three-dimensional digital geological models (DGMs) is a mandatory condition for the efficient exploitation of any deposit at all stages of its development. In particular, a three-dimensional model serves as a means of improving data interpretation, as well as initial information for numerical modeling of physical phenomena occurring in the rock mass during exploitation (geodynamic, geofiltration, etc.).

There are two main approaches to constructing 3D DGMs—based on explicit and implicit representation of reflecting horizon surfaces [6]. In the English-language literature, these approaches are referred to as *surface based* (SB) and *volume based* (VB), respectively. In the SB approach, a structural geological model (SGM) is first constructed, and then a block model is built on its basis. In the VB approach, a grid block model is first constructed, and then a structural model and a rock property model are built on its basis.

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The VB approach is the main industry standard. Its main advantage is a high degree of automation: consistent horizon surfaces are constructed automatically as isosurfaces of a scalar field. However, the VB approach has a significant drawback—weak controllability of the modeling process by the user-geologist [7]. Attempts to influence the geometry of one horizon cause changes in the geometry of all others. The approach works poorly for dying-out faults and models with thin layers.

The SB approach provides direct control over the geometry and topology of each surface. However, its main drawback is the low automation of the most labor-intensive stage—consistency of horizon surfaces with the fault network (FH-cleaning), which leads to the need for a large amount of manual work and makes it inapplicable for complex large-scale models.

The present work was carried out in 2012–2015 as part of a doctoral dissertation [1] and is devoted to the automation of this stage. The main ideas—restricting the class of surfaces to vertically co-oriented triangulations, reducing the problem from $\mathbb{R}^3$ to $\mathbb{R}^2$, and replacing the unstable gradient descent procedure with stable finite element interpolation—served as a starting point for subsequent development. In [2], the approach is reformulated within a rigorous variational formulation with a free boundary (Dirichlet energy and area functional), and in [3]—generalized to systems of arbitrary, including intersecting, faults using topological cutting of a triangulation. The present article retains the original formalization based on Mallet's discrete model [4] and is published as a document recording the precursor to the aforementioned works.

The article has the following structure. Section 2 recalls the rules of structural modeling and the stages of SGM construction. Section 4 presents the formalization of the consistency problem based on Mallet's discrete model and demonstrates the difficulties of the classical approach. Section 5 contains the main content—the heuristic approach, including the parametric model of topological consistency and the algorithm for sequential incorporation of weak control points. Section 6 presents the experimental results.

2 Definition of a consistent structural model

Before the introduction of three-dimensional computer modeling into the practice of geological support for deposit development, structural models were constructed in the form of two-dimensional cross-sections. The consistency of cross-sections with each other was based, as a rule, on the intuition of a particular geologist.

The first formally described method for constructing structural models is Dahlstrom's balanced cross-section method [10]. The method represents a set of interactively repeated steps (including the construction of a restored cross-section), as a result of which a non-contradictory (balanced) cross-section is constructed. The main condition for non-contradiction (the main assumption of the method) is the preservation of the length and thickness of layers.

The transition to three-dimensional modeling required a more rigorous description of the main rules (constraints and conditions) of structural modeling. Three classes of rules can be distinguished: conditions imposed on an individual structural surface (class *Restriction Surface* (RS)); constraints imposed on the relationship between structural surfaces (class *Restriction Topology* (RT)); conditions determining the non-contradiction of the constructed model as a whole (class *Restriction Model* (RM)).

Let us begin with the conditions of the RS class. A geological surface represents a boundary between two volumes of rock with different characteristics (seismic impedance, metamorphic differences, lithological composition, etc.). Therefore, the first condition of the class is that a geological surface must be orientable. The second condition of the class is that the surface must not intersect itself, since it is assumed that the volume bounded by geological surfaces cannot overlap. The third condition of the class is that the structural surface must maximally accurately correspond to field observations. For horizons, for example, this means that their surfaces must maximally accurately correspond to seismic survey data, as well as well pick depths. For fault surfaces, the latter condition means that they must maximally accurately correspond to interpretation results.

The main problem of three-dimensional structural modeling is understanding how faults and horizons interact with each other. Therefore, RT-class constraints are the most important from the point of view of understanding the geological structure of a deposit. Historically, the first topological constraint used in constructing SGMs was proposed in [11] and consisted in the requirement that all surfaces participating in model construction must not intersect, but only touch each other along common boundaries. In fact, it was required that they form a certain geometric complex [9].

RT-class constraints for specific types of geological surfaces were formulated by Caumon [7]. The first constraint concerns the non-intersection of horizon boundaries and means that for any two horizons H_i and H_j, the surface of horizon H_i can only be on one side of the surface of horizon H_j, and vice versa. Otherwise, this would mean that the layers overlap and, consequently, are incorrectly specified (for example, due to erosion or intrusion of the underlying horizon).

For the convenience of structural modeling, the boundary of a surface can be divided into parts called *logical boundaries*, depending on their origin or role. For example, the trace of a fault on a horizon consists of two logical boundaries—for the hanging wall and the footwall of the fault.

The second RT-class constraint, called the *free boundary* constraint, can be formulated as follows: only fault surfaces can have logical boundaries that do not touch other elements of the structural model [8]. Indeed, horizon surfaces must necessarily terminate either on faults, on pinch-outs, or on the external boundaries of the model. A fault can have a logical boundary that does not touch other structural surfaces, for example, when the fault dies out between two horizons.

The class of RM rules, determining the non-contradiction of the constructed model as a whole, includes two conditions: consistency of horizon surface areas and layer thicknesses between horizons.

All approaches currently used for constructing 3D DGMs follow the rules formulated above [4, 7]. In the case of the VB approach, most rules of the RS and RT classes are satisfied automatically, since horizon surfaces are isosurfaces of the same field. At the same time, RM-class conditions are difficult to control, which is the main drawback of the approach. In the SB approach, RT-class constraints must be controlled manually, which makes it practically unacceptable for high-dimensional models. However, the user has significantly more opportunities to control RM conditions.

The goal of the present work is to develop an algorithm that would maximize the automation of the SGM construction process using the SB approach. In particular, the developed algorithm must guarantee automatic satisfaction of RS and RT rules.

3 Main stages of the construction process

Construction of a model using the SB approach consists of four sequential steps [6]: preparation of initial data (tying, filtering, digitization, etc.); construction of the fault network; consistency of horizon surfaces with the fault network; evaluation of the constructed model. Construction of the fault network and consistency of horizons are performed in accordance with the RS and RT rules. If the resulting model does not satisfy RM conditions, then steps 2–4 are repeated.

In the described scheme, the most complex and labor-intensive is the third step—consistency of horizon surfaces with the already consistent fault network. This stage of SGM construction is called "FH-cleaning" in the English-language literature and is the most labor-intensive when using the SB approach.

To automate the third step, it is necessary to formalize the problem of consistency between horizon and fault surfaces. We will use the well-known formalization proposed by the French mathematician Mallet [4].

4 Formalization of the structural consistency problem for a horizon

4.1 Discrete model of a geological object

The proposed formalization is based on the representation of a geological object in the form of a so-called n-dimensional *discrete model*—a quadruple

$$\mathscr{M} = (\Omega, \mathscr{N}, \varphi, C), \tag{1}$$

where Ω is a set of vertices, and $\mathscr{N} \subset \Omega \times \Omega$ is a binary neighborhood relation defined over this set. In fact, the specified pair defines a graph that determines the discrete topological structure of the object and is called the *topological support* of the model. The third parameter of the model is the sought m-dimensional function $\varphi : \Omega \to \mathbb{R}^m$, defined on the set of vertices Ω, which is called the *property function*. The fourth parameter is a set of constraints C specified for the function φ, divided into subsets of hard C^h and weak C^w constraints.

Each constraint $c \in C$ has a linear representation determined by the matrix A_c and the coefficient b_c:

$$A_c\,\varphi = b_c. \tag{2}$$

Accordingly, C^w are called *weak*, and C^h—*hard* constraints of the model. If a constraint relates several components of the function φ to each other, then such a constraint is called *mixed*; otherwise—*simple*.

Mallet proposed a universal method for constructing the function φ, which he called the *Discrete Smooth Interpolation Method* (DSI), which reduces the process of constructing the property function to the minimization of a special quadratic functional built on the basis of constraint coefficients:

$$E(\varphi) = \underbrace{\sum_{p\in\Omega}\sum_{q\in\mathscr{N}(p)} \omega_{pq}\,\|\varphi(p)-\varphi(q)\|^2}_{\text{smoothness}} + \lambda \underbrace{\sum_{c\in C^w} \|A_c\,\varphi - b_c\|^2}_{\text{weak constraints}}. \tag{3}$$

The first part of the functional ensures the fulfillment of differential smoothness criteria for the sought function under the condition that the hard constraints of the model are satisfied. Its second part ensures the approximate fulfillment of weak constraints. The coefficient $\lambda \geq 0$ sets the balance that determines how important the fulfillment of weak constraints is compared to hard ones.

The process of minimizing functional (3) is carried out by an iterative gradient descent algorithm.

Mallet proposed using the described formalization to implement all stages of the DGM construction process, including the solution of the problem of consistency between horizon and fault surfaces. It was assumed that the horizon surface H and the fault surface F belong to the class of piecewise-linear triangulated surfaces.

4.2 Discrete model of horizon consistency

Consider the discrete model $\mathscr{M} = (\Omega, \mathscr{N}, \varphi, C)$, whose sought property function φ corresponds to the surface of horizon H consistent with fault F. Here Ω is a set of distinct points from $\mathbb{R}^3$, and the neighborhood relation $\mathscr{N}$ is defined by triangulation T. Each component of the property function $\varphi = (\varphi_x, \varphi_y, \varphi_z)$ corresponds to the coordinate of the position of vertex $v \in \Omega$ of the modeled horizon surface. In fact, the triple $(\Omega, \mathscr{N}, \varphi)$ defines a piecewise-linear triangulated surface, within which the consistent horizon surface H is sought.

Consider the composition of constraints in the proposed model.

Hard control points

Hard constraints are specified on a subset of vertices $K \subset \Omega$, for which the value of the component φ_z is given, and φ_x, φ_y coincide with the coordinates of vertex v in Ω. The set K is called the set of *hard control points*. In practice, it corresponds to the external boundary of the horizon surface modified during the consistency process. For a given vertex $v \in K$, the corresponding hard constraint coefficients have the form:

$$\varphi_x(v) = x_v, \tag{4}$$

$$\varphi_y(v) = y_v, \tag{5}$$

$$\varphi_z(v) = z_v. \tag{6}$$

The conditions determined by coefficients (4)–(6) for all $v \in K$ will be denoted C^K and called *hard control point conditions*.

Topological consistency conditions

The weak constraints of the model specify the conditions of the RT class. In the case of a single fault, which is considered in Mallet's formalization, topological consistency means that the fault surface F must form a *cut* γ on the horizon surface (Fig. 1).

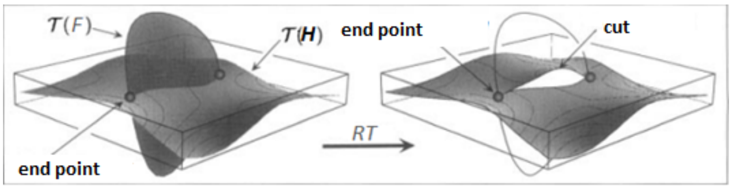

Figure 1: Example of a single cut.

Let $\Gamma \subset \Omega$ be the subset of vertices belonging to the cut γ. Obviously, for any point of the cut (except the endpoints), there exists an adjacent cut point lying on the opposite logical boundary of the cut. Denote such a pair of points by (p, p'), and the set of all such pairs by Π.

In this case, the RT constraints can be written as follows:

$$\varphi(p) = \varphi(p'), \qquad (p, p') \in \Pi. \tag{7}$$

If the conditions (7) are formulated as weak constraints of the discrete model, then, as a rule, the corresponding DSI process does not converge. Therefore, an iterative approach is used [4], in which the solution of model $\mathscr{M}$ is sought as a sequence of solutions of auxiliary discrete models:

$$\mathscr{M}^{(0)}, \mathscr{M}^{(1)}, \ldots, \mathscr{M}^{(i)}, \ldots, \tag{8}$$

where the weak constraints $C^{w(i)}$ of model $\mathscr{M}^{(i)}$ are constructed on the basis of the property function $\varphi^{(i-1)}$ of the previous model in the manner described below.

Let, in accordance with Fig. 2, the following be given:

- P—a point of the horizon surface H corresponding to vertex p at the $(i-1)$-th iteration step;
- $\vec{v}$—the projection vector of point P onto the fault surface F;
- P'—the projection of point P onto the fault surface F in the direction $\vec{v}$;
- $Q = (1-\alpha)P + \alpha P'$—a linear combination of points P and P' with coefficients $(1-\alpha)$ and α, respectively.

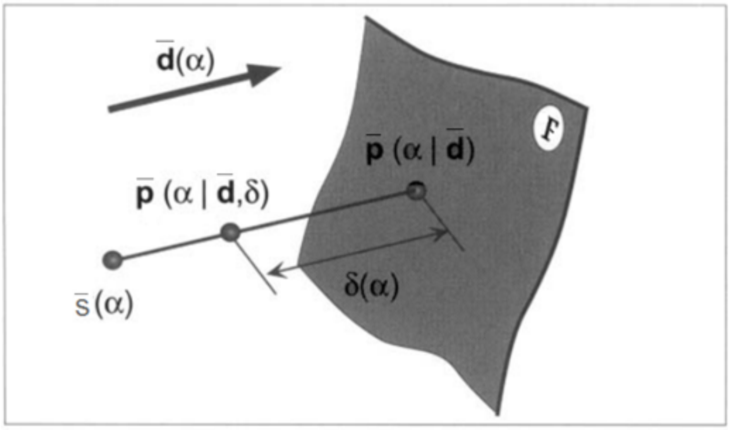

Figure 2: Projection of a horizon vertex onto the fault surface.

Then, for given vertices $p \in \Gamma$, components φ_j, and parameter α, the weak constraints of the discrete model ensuring the fulfillment of RT-class conditions have the form:

$$\varphi_j(p) = Q_j = (1-\alpha)P_j + \alpha P'_j, \qquad p \in \Gamma. \tag{9}$$

The set of weak constraints (9) for all $p \in \Gamma$ will be denoted C^T and called *topological consistency conditions*. Since these conditions only ensure an approximation to the fault surface, after each iteration they are recalculated. This way of specifying weak constraints is called *dynamic* [4].

Weak control points

The second set of weak conditions is related to the availability of a priori information about the position of the horizon surface in the vicinity of the fault, which is specified as a set of three-dimensional points $P = \{p_1, \ldots, p_n\}$, called *weak control points* of the model.

Let, in accordance with Fig. 3, the following be given:

- $\vec{v}_k$—the projection vector of control point p_k onto the horizon surface;
- p'_k—the projection of control point p_k onto the current horizon surface in the direction $\vec{v}_k$;
- $\lambda_1, \lambda_2, \lambda_3$—barycentric coordinates of the projection of control point p_k onto the triangle of the surface defined by the property function φ.

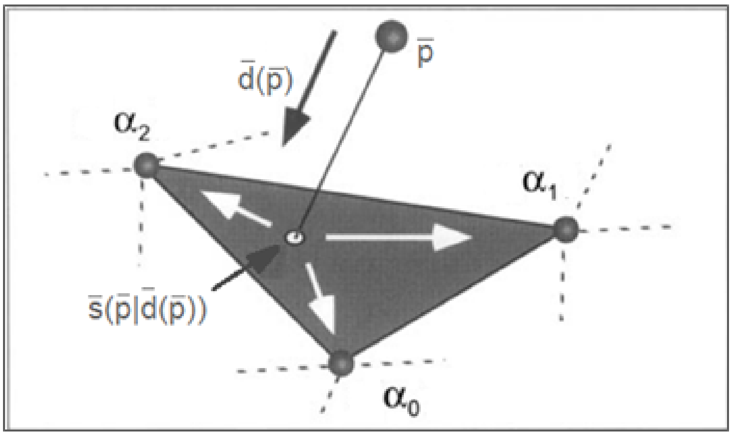

Figure 3: Weak control points.

Then, for a given component φ_j and control point p_k, the weak constraint coefficients have the form:

$$\varphi_j(v) = \sum_{l=1}^{3} \lambda_l \, \varphi_j(v_l), \tag{10}$$

where v_1, v_2, v_3 are the vertices of the triangle containing the projection p'_k. The set of such constraints for all weak control points will be denoted C^P and called *weak control point conditions*. Like the topological consistency conditions, these constraints are dynamic and require recalculation at each iteration.

Note that in the case where the projection points fall on adjacent triangles of the current horizon surface, conditions (10) do not guarantee exact passage of the surface through them at the next iteration.

Difficulties of the classical approach

Thus, the set of all constraints of the i-th model of sequence (8) can be written as the union $C^{(i)} = C^K \cup C^{T(i)} \cup C^{P(i)}$. In this case, the sequence of discrete models

$$\mathscr{M}^{(0)} \rightarrow \mathscr{M}^{(1)} \rightarrow \ldots \rightarrow \mathscr{M}^{(i)} \rightarrow \ldots \tag{11}$$

converges to the sought consistent horizon surface as $\alpha \rightarrow 1$. The rate of convergence of the sequence to the sought solution is determined by the rate of convergence of the parameter α to 1.

Nevertheless, the above scheme has not found wide practical application. This is due to the weak convergence of the method in the case of dynamic recalculation of weak constraints at each iteration step [4, 5]. In reality, the implementation of the described approach is even more complex, since at each step it is necessary to control the consistency conditions of the model itself [4]. In particular, self-intersection of the sought horizon surface may occur at the i-th iteration step. The method is especially unstable in the case of significant divergence of the cut edges, which is typical for faults with large dip angles.

5 A heuristic approach to the horizon surface consistency problem

A heuristic approach to the consistency problem is proposed, which allows, while remaining within the framework of Mallet's formalization, to efficiently solve the problem of SGM construction in geological conditions typical for most deposits of the Pripyat Trough.

We will use known techniques to increase the stability of the DSI method [4]:

- reducing the dimensionality of the optimization functional by removing mixed constraints;
- transitioning from a multidimensional problem to a set of one-dimensional problems for each individual component of the property function;
- reducing the original discrete problem to a series of discrete problems in which weak constraints are excluded.

In addition, we will narrow the class of sought structural surfaces.

5.1 Vertically co-oriented surfaces

We will assume that all structural surfaces belong to the class of *vertically orientable triangulated surfaces*. This means that the normals of all triangles of the structural surface have a positive z-component. In effect, this means that the structural surface does not change orientation along the strike and can be represented as the graph of a function $z = f(x,y)$.

This restriction leaves outside consideration, for example, fold structures with reverse dip or complex extrusions (e.g., ore bodies). However, in general, the structural surfaces of deposits of the Pripyat Trough satisfy the above restriction. Surfaces that are not vertically co-oriented (e.g., complex dislocations) can be split into several parts, each of which satisfies the introduced restriction.

Vertical co-orientation allows a significant simplification of the structural surface consistency process, reducing the dimensionality of the representation space of the sought object from $\mathbb{R}^3$ to $\mathbb{R}^{2,5}$. The introduced restriction also allows the transition from computationally unstable algorithms for three-dimensional surface consistency to planar interpolation and triangulation algorithms, which significantly increases the speed, stability, and accuracy of the method.

5.2 Discrete model of topological consistency

In order to construct a stable method for solving the consistency problem, we use the techniques described above. For this purpose, we convert the weak topological consistency conditions C^T into a subset of hard constraints, setting $\alpha = 0$. We denote the new hard constraints by $C^{T,0}$. We remove the weak constraints C^w from the model and call such a model the *topological consistency model*:

$$\mathscr{M}_{\text{topo}} = (\Omega,\ \mathscr{N},\ \varphi,\ C^K \cup C^{T,0}). \tag{12}$$

Due to the absence of weak constraints, the quadratic functional (3) will consist only of the first part, which ensures the fulfillment of differential smoothness criteria for the sought property function. Moreover, since both hard conditions are simple, DSI minimization decomposes into three independent problems for each coordinate. Since the topological support in the considered case is a planar triangulation, each of these problems can be solved by the finite element method.

Unlike (11), the constructed model (12) always has a solution. However, because each component is sought independently, the constructed surface may have self-intersection (violation of the RS condition). As a consequence, it will not be vertically oriented.

Base surface

The idea of the proposed approach is to use a certain *base surface* H_0, which is RS-correct and possesses the property of vertical orientability. The sought surface of the consistent horizon is proposed to be constructed as a certain balance between the base surface and the surface of model (12).

Such a base surface can be constructed on the basis of the initial (inconsistent) horizon surface H_{init}. For this purpose, we find the intersection of the initial surface with the fault surface F, add the resulting intersection edges to the simplicial scheme of the initial horizon surface, and then duplicate them. The constructed surface is denoted H_0. Without loss of generality, it can be assumed that the simplicial scheme of the base surface H_0 and the topological support of model (12) coincide. From a geological point of view, the base surface H_0 can be interpreted as the case of intersection of the fault and horizon with zero amplitude.

Parametric model

Let us parameterize the constraint C^T. For this purpose, we introduce a parameter $\alpha \in [0,1]$ and call it the *fault amplitude*. Then, for a given cut vertex $p \in \Gamma$ and component φ_j, the corresponding coefficients of the conditions ensuring the fulfillment of RT-class constraints have the form:

$$\varphi_j(p) = (1-\alpha)\,\varphi_j^{(0)}(p) + \alpha\,\varphi_j^{(F)}(p), \tag{13}$$

where $\varphi_j^{(0)}(p)$ is the value on the base surface H_0, and $\varphi_j^{(F)}(p)$ is the value corresponding to the projection onto the fault surface F. We denote the introduced constraints by $C^{T,\alpha}$, and the corresponding model by

$$\mathscr{M}_\alpha = (\Omega,\ \mathscr{N},\ \varphi,\ C^K \cup C^{T,\alpha}). \tag{14}$$

Consider the following iterative procedure. We construct the sought surface of model (14) at the parameter value $\alpha = 1$. If the resulting surface is not vertically oriented, then we decrease α by a small amount and repeat the previous iteration.

Obviously, the procedure will be executed in a finite number of steps, since at $\alpha = 0$ the sought surface of model (14) coincides with the base surface H_0 and, therefore, is vertically oriented by definition. The maximum α for which the surface possesses the vertical orientability property is denoted α^*.

The model obtained as a result,

$$\mathscr{M}_{\alpha^*} = (\Omega,\ \mathscr{N},\ \varphi,\ C^K \cup C^{T,\alpha^*}), \tag{15}$$

will be called the *parametric model of topological consistency*.

Thus, the problem of consistency between horizon and fault surfaces in the absence of weak constraints has been solved.

5.3 Algorithm for full consistency

Let us return to the general consistency problem, which includes the weak control point constraints C^P. Adding them to the constraints of model (15) returns it again to the variant of weak dynamic conditions and, consequently, to the situation of weak convergence of the DSI method.

A *sequential incorporation strategy* is proposed, which consists in adding conditions from C^P to the hard constraints of the model sequentially: at the i-th iteration step, the model includes only a part of the constraints on weak control points $C^{P(i)} \subseteq C^P$. In this case, the algorithm scheme takes the form:

$$\mathscr{M}_{\alpha^*}^{(0)} \to \mathscr{M}_{\alpha^*}^{(1)} \to \ldots \to \mathscr{M}_{\alpha^*}^{(i)} \to \ldots \tag{16}$$

In this case, the value of the fault amplitude parameter α^* is constructed at each i-th step and must not take values less than α^*.

Note that, as in the case of model (15), the DSI functional of model (16) will consist only of the first part, which ensures the fulfillment of differential smoothness criteria for the sought function.

Residual set

Let us refine the scheme described above. As in the case of weak constraints, the hard constraints C^P do not guarantee exact passage of the horizon surface through the weak control points. Therefore, we introduce a certain quantity $\varepsilon^{(i)}$ specifying the allowable deviation of the horizon surface from the weak control points. The subset of control points from P whose deviation from the surface $H^{(i)}$ exceeds the value $\varepsilon^{(i)}$ will be called *-residual by* $\varepsilon^{(i)}$.

Let $H^{(i)}$ be the horizon surface obtained at the i-th iteration step, and $\varepsilon^{(i)}$ be the allowable deviation at this step. The set of ε-residual vertices of set P by $\varepsilon^{(i)}$ is denoted by $R^{(i)}$:

$$R^{(i)} = \big\{\, p_k \in P \,:\, \operatorname{dist}\big(p_k, H^{(i)}\big) > \varepsilon^{(i)} \,\big\}. \tag{17}$$

The idea of the proposed algorithm is to use the set $R^{(i)}$ to construct the hard constraints $C^{P(i)}$.

For the convergence of the scheme described above, a monotonic decrease in the cardinality of the set $|R^{(i)}|$ and the value of the allowable deviation $\varepsilon^{(i)}$ at each iteration step is necessary. Indeed, if $|R^{(i)}| < |R^{(i-1)}|$ and $\varepsilon^{(i)} < \varepsilon^{(i-1)}$, then the i-th iteration of the surface takes into account a larger number of weak control points with a smaller allowable deviation. An increase in the cardinality of the residual set at the next iteration step serves as a stopping criterion for the algorithm.

Transition rule

Let us formalize the transition from the $(i-1)$-th model to the i-th one. For this purpose, it suffices to define the method for computing the value $\varepsilon^{(i)}$, i.e., to actually specify the rule for selecting points from $R^{(i-1)}$ for inclusion in the constraints $C^{P(i)}$.

Let

$$d_{\max}^{(i-1)} = \max_{p_k \in R^{(i-1)}} \operatorname{dist}\big(p_k, H^{(i-1)}\big);$$

then

$$\varepsilon^{(i)} = \mu \cdot d_{\max}^{(i-1)}, \tag{18}$$

where $\mu \in (0,1)$ is the *control coefficient* that determines the convergence rate of the algorithm. The influence of the value of μ on the convergence rate was studied experimentally; the results are presented in Section 6.

An increase in the cardinality of the residual set at the next iteration step serves as a stopping criterion for the algorithm. Fig. 4 illustrates the step-by-step surface consistency process.

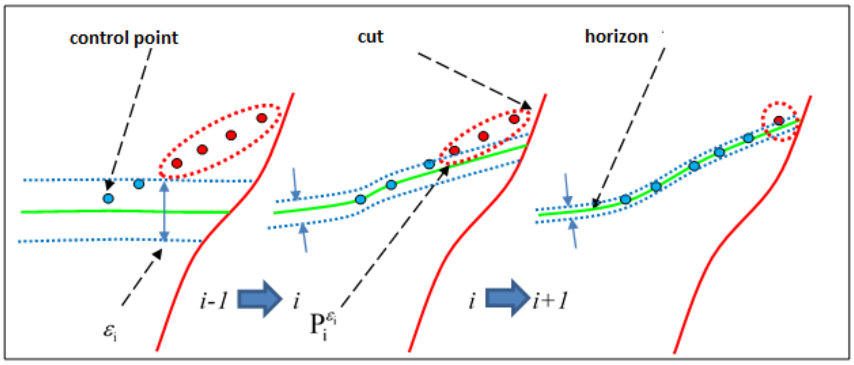

Figure 4: Step-by-step process of horizon surface consistency.

Algorithm 1 Full consistency of a horizon surface

Require: Triangulation T and the base surface H_0 defined on it; set of hard control points K; set of weak control points P; control coefficient μ

Ensure: Consistent horizon surface H

Construct the property function of model $\mathscr{M}_{\alpha^*}$ ▷ formula (15)
$i \leftarrow 1$
repeat
    Based on $H^{(i-1)}$, compute the allowable deviation $\varepsilon^{(i)}$ by rule (18)
    Construct the residual set $R^{(i-1)}$ by formula (17)
    Add points from $R^{(i-1)}$ to the hard constraints $C^{P(i)}$ of model $\mathscr{M}_{\alpha^*}$
    Construct the property function of model $\mathscr{M}_{\alpha^*}^{(i)}$ by the finite element method $\rightarrow H^{(i)}$
    $i \leftarrow i+1$
until $|R^{(i)}| \geq |R^{(i-1)}|$
return $H = H^{(i)}$

Algorithm description

Formally, the algorithm for full consistency of the horizon surface can be described as follows.

Thus, an algorithm for full consistency of horizon and fault surfaces has been described, based on the use of a special form of discrete model with hard constraints. For such a model, the property function can be efficiently constructed using the finite element method, which guarantees the stability and practical applicability of the proposed algorithm.

6 Experimental study of the algorithm

Let us study the convergence of the algorithm described above. A set of geological models was formed as test data. The set included models that contain a large number of weak control points (comparable to the number of hard control points). This property is typically possessed by geological models with large-amplitude reverse faults (so-called thrusts). As a result of consultation with specialists, several models with the indicated structure were selected (see Table 1).

All models contain several reverse faults. In the Fang model, faults do not touch each other; in the KrSloboda model, they form a complex junction including both normal and reverse faults; and in the Chicontepec model, reverse faults have large dip angles.

The structural surfaces of the KrSloboda model are shown in Fig. 5.

Table 1: Input data for testing.

Model	Horizons	Faults (reverse)	Description
Fang	4	5 (4)	Shelf (Vietnam)
Chicontepec	3	10 (5)	Shelf (Mexico)
KrSloboda	8	14 (5)	Potash horizon (Belarus)

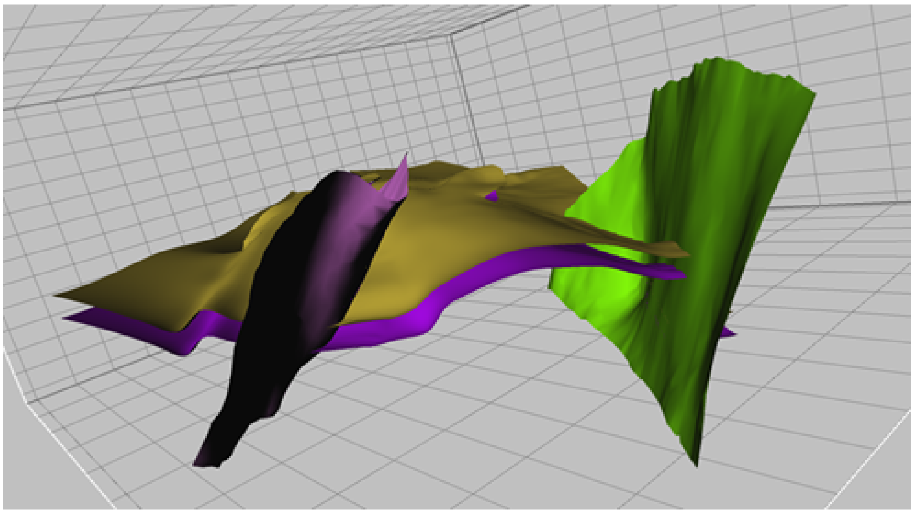

Figure 5: Structural model of the test field.

As part of the convergence analysis, the developed algorithm was applied to each horizon of the indicated models. At each iteration step, the cardinality of the residual set, the allowable deviation, and fault amplitudes were computed. The iterative process continued until the algorithm stopping criterion was met.

To visualize the convergence process, Figs. 6 and 7 show graphs of the cardinality of the residual set and the average deviation of the horizon surface from set P at each iteration step of the algorithm. For more convenient display, instead of the cardinality of the residual set, the normalized quantity $\bar{R}^{(i)} = |R^{(i)}|/|P|$ (normalized residual set size, NRSS) was used.

The experimental results confirmed the presence of correlation between the cardinality of the residual set and the average deviation of the horizon surface from the weak control points on the convergence interval.

The average convergence interval for the tested models was 7 iterations (see Table 2). For all horizons, the residual size at the stopping point exceeded 10% of the total number of weak control points, which confirms the possibility of practical use of the algorithm.

The experimental results presented above were obtained at a value of the control parameter $\mu = 1/2$. The analysis of the influence of this parameter on the convergence rate of the algorithm was carried out on a set of real geological objects of varying complexity and structure. Test objects included various situations of structural surface consistency, including: vertical and inclined faults (*thrust faults*); fault junctions; conjugations.

A series of experiments was conducted to select the optimal value of the coefficient μ. Table 3 shows the values of the normalized residual set size $\bar{R}^{(i)}$ for three values of the coefficient μ (1/3; 1/2; 3/4), obtained as a result of performing five, seven, and nine iterations, respectively.

The analysis revealed a general trend in the behavior of $\bar{R}^{(i)}$ for all models at the value of the coefficient $\mu = 1/2$. On the convergence interval, the residual set size decreases monotonically.

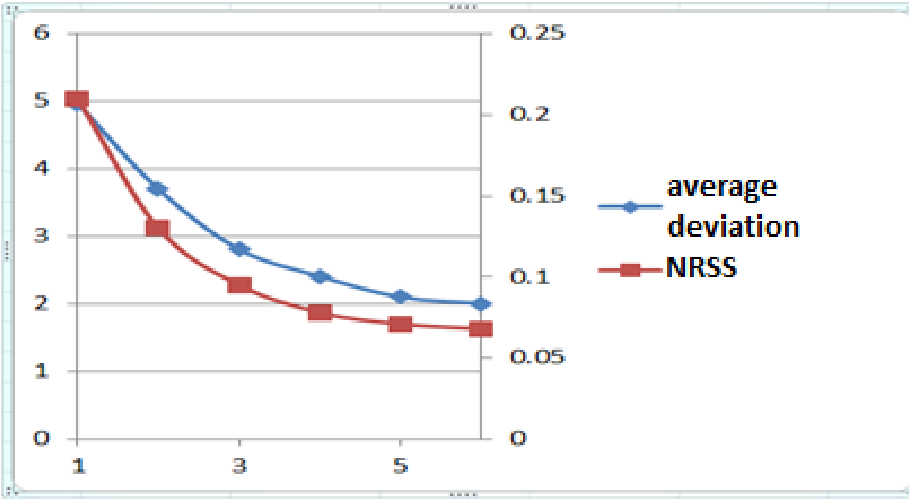

Figure 6: Graphs of $\bar{R}^{(i)}$ and average deviation for horizon T5x3.

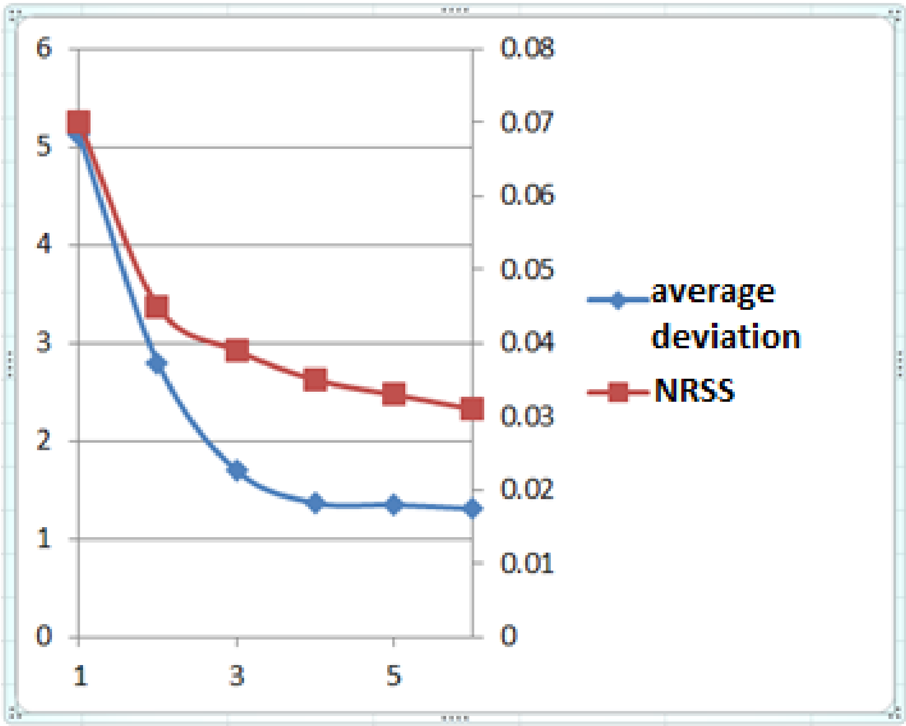

Figure 7: Graphs of $\bar{R}^{(i)}$ and average deviation for horizon HorIIa.

Table 2: Testing results.

Model	Horizon	Iterations	NRSS, %
Fang	T5x3	6	8
Fang	T8x3	7	7
Fang	T3x3_m18_20	9	5
Fang	T3x3_m18_10	5	10
KrSloboda	HorIIa	7	8
KrSloboda	HorIIIk	8	9

Table 3: Normalized residual set size vs. coefficient μ.

Object	Faults	5 iterations			7 iterations			9 iterations		
		$1/3$	$1/2$	$3/4$	$1/3$	$1/2$	$3/4$	$1/3$	$1/2$	$3/4$
Chicontepec	10	0.86	0.56	0.28	0.63	0.09	0.31	0.34	0.11	0.33
Macabu	120	0.72	0.43	0.36	0.61	0.11	0.32	0.44	0.08	0.45
Bohai	1	0.76	0.36	0.28	0.53	0.08	0.31	0.39	0.09	0.25
Krasnaya Sl.	12	0.81	0.37	0.29	0.56	0.10	0.41	0.35	0.12	0.28
Ashtart	35	0.77	0.35	0.31	0.55	0.10	0.34	0.41	0.11	0.32

On subsequent iterations, oscillations around the achieved minimum value are observed. For values of the coefficient μ significantly greater than $1/2$, the algorithm fails to converge on complex models.

At $\mu = 1/2$, for all indicated objects, the normalized residual set size $\bar{R}^{(i)}$ reaches 10% already at the 7th iteration.

Thus, the experiments confirmed the possibility of using the developed algorithm for SGM construction in conditions of plicative character of horizon occurrence.

7 Results

The following scientific results have been obtained in this work.

1. The main stages of SGM construction have been identified and the rules for consistency of structural elements of the model have been defined: conditions on an individual structural surface; constraints on the relationship between structural surfaces; conditions for model non-contradiction as a whole. It has been shown that the most complex stage is the consistency of horizon surfaces.

2. Within the formalization of the horizon surface consistency problem, the possibility of using the classical approach to modeling geological objects, based on Mallet's discrete model, has been investigated. It has been shown that the full satisfaction of consistency rules leads to a discrete model with weak dynamic constraints and, as a consequence, to instability of the procedure for obtaining the solution of the discrete problem based on the gradient descent method.

3. A modification of the classical approach has been proposed, based on restricting the class of sought structural surfaces to vertically co-oriented triangulated surfaces. This allowed reducing the SGM construction problem from a complex three-dimensional problem to a simpler two-dimensional one.

4. An algorithm for full consistency of a horizon surface has been developed, based on the strategy of sequential incorporation of weak control points, which allowed replacing the gradient descent procedure with a more computationally stable two-dimensional finite element interpolation.

5. Experimental studies have been conducted, which confirmed the convergence and stability of the developed algorithm and the efficiency of the proposed approach as a whole for SGM construction in the conditions of the Pripyat Trough.

8 Conclusion

The present work, carried out in 2012–2015 as part of a doctoral dissertation [1], laid the conceptual foundation for the subsequent development of the approach to structural modeling in the vicinity of faults.

The key idea—reducing the three-dimensional consistency problem to a two-dimensional one by restricting the class of surfaces—was realized in the present work through the requirement of vertical co-orientation of triangle normals. In the subsequent work [2], the same idea was formalized differently: through a non-parametric (functional) representation $z = u(x, y)$ with a bijective projection onto the xy-plane. Both approaches proceed from the same geological assumption—predominantly plicative occurrence of horizons.

The fault amplitude parameter α, introduced in the present work as a scalar controlling the "opening" of the cut, served as a precursor to the concept of a *free boundary* in [2, 3]. In the variational formulation [2], the "opening" is specified not by a parameter, but by iterative projection of the free boundary onto the fault surface, which naturally accounts for the fault geometry without the need for manual specification of the amplitude. In [3], this process is generalized to sub-edges between fault intersection vertices.

The strategy of sequential incorporation of weak control points with a stopping criterion based on the increase of the residual set is analogous to the diagnostic completion criteria in [3], where a distinction is made between normal convergence and three diagnostic types (stagnation, projection impossibility, and reaching an intersection vertex).

Further development of the approach also included the replacement of Mallet's formalization (DSI with hard and weak constraints) by a variational formulation—a weighted combination of the Dirichlet energy and the area functional [12] with a free boundary [13, 14], which ensured the linearity of the problem by construction thanks to the edge-based discretization of the area functional. The formalization of a single fault was generalized to systems of arbitrary, including intersecting, faults using topological cutting of a triangulation and a simplicial complex with consistency and transversality conditions [3].

Thus, the present work records the initial stage of development: a heuristic approach within the framework of Mallet's DSI formalization, which was subsequently replaced by a rigorous variational formulation with topological guarantees of correctness.

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