PAG ETR
A logical and reproducible geological analogy framework for assessing evaluability and prioritizing geochemical investigation of rare earth elements in territories with incomplete data
Application to Mato Grosso do Sul, Brazil

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Version note

This document corresponds to V20 of the PAG ETR manuscript and presents the methodological framework and its first pilot implementation in Mato Grosso do Sul. It was organized to preserve the scientific logic, the M1 to M11 rules, the cartographic results already produced in ITA ARANDU MS, the computational validation, the limitations, and the reproducibility trail. The maps are frozen historical results from the first operationalization and were not retrospectively recalculated, reclassified, or optimized during manuscript preparation. Future changes to the model library, rules, sources, or spatial criteria must generate a new explicitly identified version while preserving the result documented here.

The manuscript strictly distinguishes investigation results from mineral inference. PAG ETR does not identify deposits, resources, or reserves and does not constitute field authorization. The fundamental scientific unit is the state of a mineral model in a specified spatial unit and at a specified scale.

Abstract

Territories with uneven geoscientific coverage pose a central problem for mineral investigation because missing information can be mistaken for negative evidence, while the accumulation of proxies can yield high classifications even when necessary geological requirements remain unknown. This study presents PAG ETR as a logical, categorical, multiscale, and reproducible framework for diagnosing evaluability and organizing hypotheses for geochemical investigation of rare earth elements, accompanied by a deliberately partial spatial proof of concept in Mato Grosso do Sul, Brazil. The method organizes eleven mineralization models, M1 to M11, and distinguishes TRUE, FALSE, UNKNOWN, CONFLICT, and NA, preserving SEM_DADO and INDETERMINADO as states of unknown information rather than absence. Progression among N0, non-evaluable, N1, open model, N2, justified targeted geochemical investigation, N3, compatible local evidence, and N4, local contradiction, is governed by non-compensatory rules and by the REQ, SUP, and CONF functions. The 250, 500, and 1,000 km² grids are evaluated independently. The conceptual library contains eleven models, but the spatial proof of concept materialized only M2 and M4 and classified 49 of 1,554 cells as N1 at 250 km^2, 37 of 793 at 500 km^2, and 24 of 412 at 1,000 km^2. No cell reached N2 or N3 in the spatial snapshot. This result is not interpreted as low potential, but as a limit on the evaluability of the materialized evidence. An M4 sensitivity scenario showed that 38, 28, and 17 N1 cells remain when point phosphorus records are excluded and only polygonal phosphorite units are considered. Computational reproduction recovered the same sets of hex_id values, and 15 executable tests returned PASS. These tests demonstrate logical consistency and snapshot determinism, not predictive validity. PAG ETR does not predict the location of deposits, resources, or reserves. In this version, the product should be interpreted as a diagnosis of evaluability and knowledge gaps rather than as a regional prospectivity map. Its contribution is to make explicit when a hypothesis can be opened, tested, supported, or contradicted, and when the available information remains insufficient. Field validation, independent predictive validation, and quantitative comparison with alternative methods remain future steps.

Keywords

rare earth elements · mineral prospectivity · mineral systems · geological analogy · incomplete data · non-compensatory logic · reproducibility · Mato Grosso do Sul

1. Introduction

Rare earth elements are strategic components of technologies associated with the energy transition, advanced electronics, permanent magnets, and several industrial supply chains. Investigation of new resources, however, depends on a problem that precedes mineral discovery. It is first necessary to determine where the existing geological evidence is sufficient to retain, advance, or reject a mineral-system hypothesis. Van Gosen et al. (2019) and the U.S. Geological Survey (2026) document both the diversity of REE deposits and their relevance to modern technologies and critical-mineral supply chains.

Mineral prospectivity mapping encompasses a broad range of approaches for integrating geological, geochemical, geophysical, and remote-sensing evidence. Knowledge-driven methods, weighted overlay, fuzzy logic, weights of evidence, Bayesian approaches, and machine-learning techniques can be appropriate when the scientific question, data quality, and availability of positive examples support their use. The problem is different in territories where coverage is spatially uneven, source scales vary, and many diagnostic conditions remain simply unknown. This field ranges from knowledge-based modelling and evidential belief to Random Forest, fuzzy logic, and geodata-science approaches, as discussed by Bonham-Carter (1994), Carranza (2008), Porwal and Carranza (2015), Carranza and Hale (2003), Carranza et al. (2008), Zuo (2020), and Farahnakian et al. (2024).

Under these conditions, two errors are especially important. The first is treating absence of data as evidence of absence. The second is allowing multiple contextual or auxiliary lines of evidence to compensate for the lack of a necessary geological condition. Both can produce visually complete cartographic surfaces that are scientifically more assertive than the data allow. Creason et al. (2023) and Ford et al. (2026) show that uneven data availability and input uncertainty must be treated explicitly in regional assessments.

PAG ETR was designed to address a question that precedes continuous favorability classification. Rather than directly estimating which area is more likely to contain a deposit, the framework asks whether the available evidence allows a specific mineral hypothesis to advance in an auditable manner.

model × hexagon × scale → evidence state and decision

The architecture preserves unknown information as an explicit state, separates necessary requirements from auxiliary evidence, maintains independence among mineralization models, and records the chain linking source, evidence, rule, spatial unit, and result.

The objective of this study is to present and document PAG ETR as a reproducible methodological framework, demonstrate its first pilot implementation in Mato Grosso do Sul, make explicit the decisions that generated the frozen maps, and assess how far the available information allows REE hypotheses to be tested without converting unknown information into negative evidence. The pilot is not presented as a validated regional application or as a demonstration of predictive performance. Its role is to test operationalization, auditability, logical behavior, scale effects, and framework limitations in a real territory with heterogeneous geoscientific coverage.

The specific objectives are to define an explicit library of REE models, formalize evidence states and progression rules, prevent inappropriate compensation among lines of evidence, spatialize only hypotheses with reproducible geometry, reproduce the frozen pilot results, and quantify how coverage gaps constrain territorial evaluability.

Three hypotheses guide the methodological test. H1 predicts that explicitly preserving the unknown state prevents missing information from being converted into negative geological evidence. H2 predicts that non-compensatory rules prevent the accumulation of auxiliary proxies from substituting for necessary requirements. H3 predicts that separating documentary assessment from reproducible spatialization allows scientifically useful hypotheses to be retained without assigning cartographic precision beyond that supported by the data. These hypotheses are evaluated through logical tests, documentary controls, and spatial reproduction rather than through predictive validation of mineral discovery.

2. Geological, territorial, and information context

2.1 Mato Grosso do Sul as the application domain

Mato Grosso do Sul has an official area of approximately 357,142 km² and encompasses contrasting geological domains, from extensive Cenozoic cover of the Pantanal to Paleozoic and Mesozoic sequences of the Paraná Basin, together with Precambrian terranes exposed mainly in the western sector. The mapping of Lacerda Filho et al. (2006) shows, in this sector, the juxtaposition of Neoproterozoic, Mesoproterozoic, and Paleoproterozoic units, Mesozoic alkaline intrusions, and Phanerozoic cover. This diversity, combined with marked differences in the density and scale of available surveys, makes the state suitable for testing a method that does not assume information equivalence among all territorial units. The regional geological synthesis and reference cartography used in this study follow Lacerda Filho et al. (2006).

The frozen geometry used in the PAG ETR repository covers approximately 357,314 km². The small difference from the official area results from the geometric support used in processing and is treated as a dataset characteristic rather than a redefinition of the state's official area. Figures 1 and 2 establish the continental, statewide, and regional setting of the pilot. Figure 2 is particularly important because it preserves the cartography and unit codes of the western sector, allowing hypotheses evaluated by the framework to be related to explicitly mapped geological objects without turning the geological legend itself into evidence of mineralization.

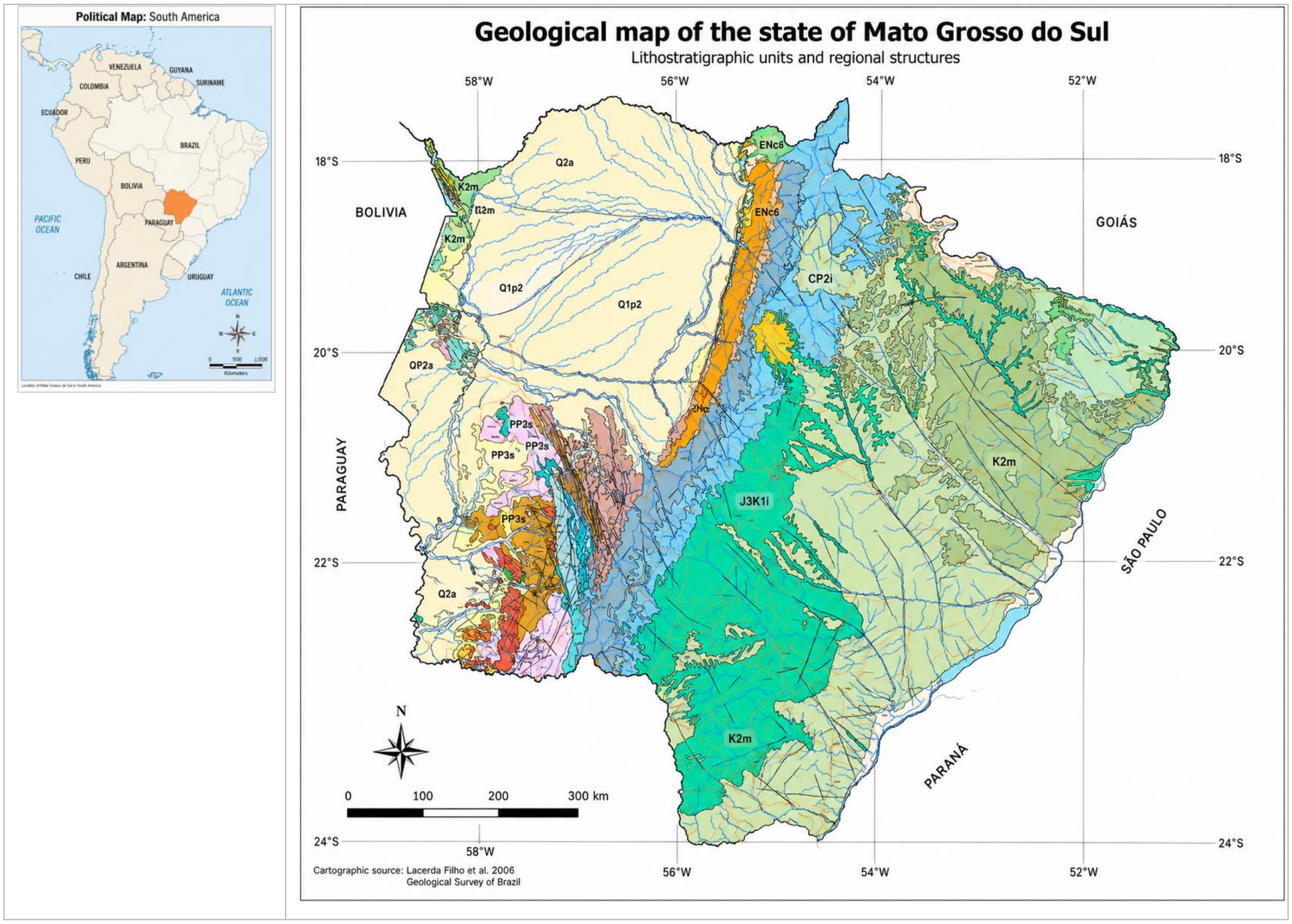

Figure 1. Study area and statewide geological context. (A) Location of Mato Grosso do Sul in South America. (B) Statewide geological context used to position the pilot within the regional distribution of lithostratigraphic units and structures. The source cartography is preserved rather than schematically redrawn. Geological source: Lacerda Filho et al. (2006), Serviço Geológico do Brasil/CPRM, scale 1:1,000,000.

2.2 Cartographic coverage

The coverage audit of the frozen dataset revealed strong inequality in the availability of more detailed mapping. The materialized 1:100,000 cartography covers approximately 21,278 km², equivalent to 5.96% of the analytical geometry. The materialized 1:250,000 cartography covers approximately 50,393 km², or 14.10%. The union, without double counting, of cartography at 1:250,000 or more detailed scales corresponds to approximately 60,150 km², or 16.83%. Thus, within the frozen dataset used in this study, 83.17% of the analytical geometry has no materialized coverage at 1:250,000 or more detailed scales.

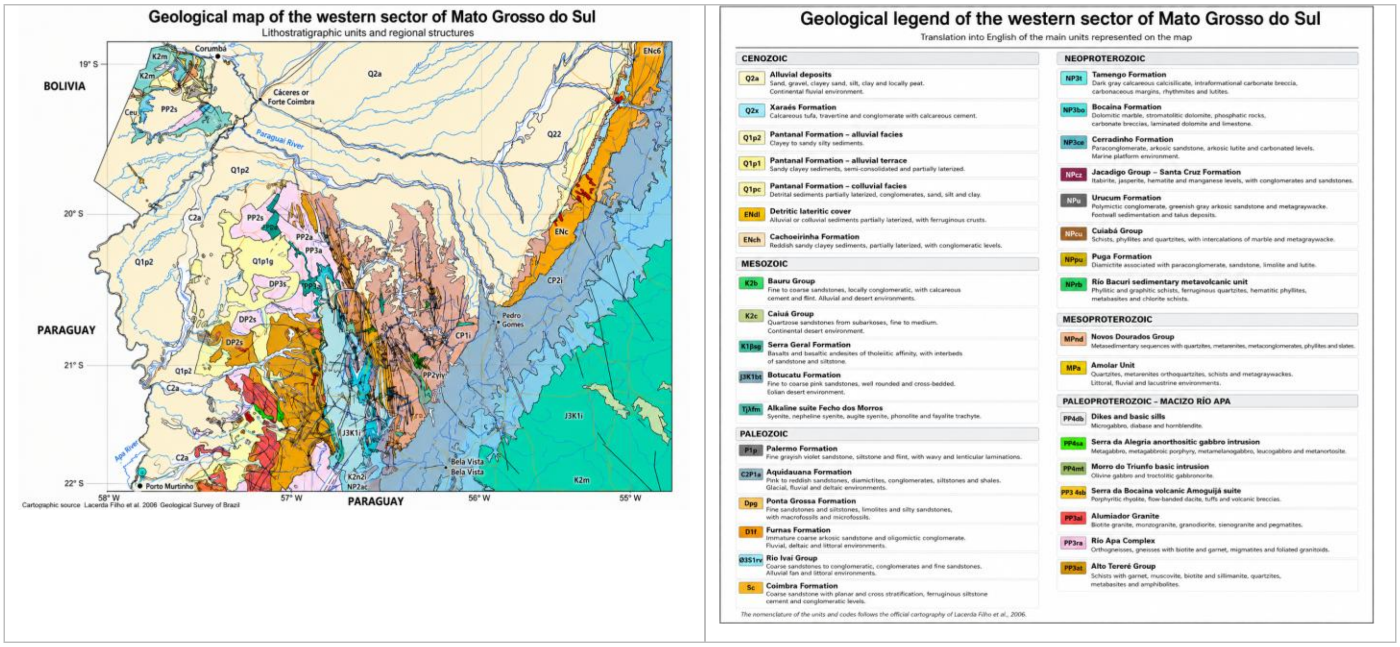

Figure 2. Geological detail of western Mato Grosso do Sul. (A) Source-derived geological map of the western sector, including the Corumbá, Serra da Bodoquena, Rio Apa and adjacent domains. (B) Complete lithostratigraphic legend corresponding to the mapped units. The map documents, among other units, the Fecho dos Morros alkaline suite, Neoproterozoic carbonate and sedimentary successions, the Amoguijá volcanic suite, Alumiador Granite and Rio Apa Complex. Original unit codes and source cartography are retained to preserve traceability. Their occurrence is treated as geological context and never, by itself, as evidence of REE mineralization. Geological source: Lacerda Filho et al. (2006), Serviço Geológico do Brasil/CPRM.

The legend of Figure 2 reinforces the need for a multimodel architecture. Within the same regional domain, Neoproterozoic carbonates, alkaline rocks, granitoids, volcanic rocks, and metamorphic complexes coexist alongside younger sedimentary cover. These lithologies may be relevant to different genetic hypotheses, but lithological relevance does not imply REE occurrence. In PAG ETR, cartography functions as contextual evidence for specific gates and must be combined with mineralogical, geochemical, structural, or geophysical requirements defined for each model before any state progression. The regional diversity represented in the cartography is documented by Lacerda Filho et al. (2006) and, for the Rio Apa Block, by Lacerda Filho (2015). Complementary regional cartography includes the Aquidauana Sheet compiled by Godoi (2001).

Indicator	Area km²	Percentage
1:100,000 cartography	21.278,16	5,96%
1:250,000 cartography	50.392,94	14,10%
1:250,000 or more detailed	60.149,67	16,83%
No materialized coverage ≤ 1:250,000	297.164,74	83,17%

Table 1. Cartographic coverage of the analytical geometry according to the materialized mapping scales.

2.3 Distribution of geoscientific observations

The distribution of observations is also uneven. In the 250 km² grid, at least one geochemical record occurs in 84 cells, equivalent to 5.41% of the total. Petrographic records occur in 37 cells, or 2.38%. Geochronological records occur in only two of 1,554 cells (0.13%). Outcrop records occur in 437 cells, or 28.12%, and mineral-resource records in 101 cells, or 6.50%. These percentages indicate the presence of records rather than effective coverage or equivalent sampling quality.

Dataset	**Records**	**250 km² cells with ≥1 record**	**Percentage**
SGB geochemistry	1.117	84	5,41%
GeoSGB petrography	339	37	2,38%
GeoSGB geochronology	5	2	0,13%
GeoSGB outcrops	7.189	437	28,12%
SGB mineral resources	222 within the MS geometry	101	6,50%

Table 2. Distribution of geoscientific observations in the 250 km² grid.

2.4 Airborne geophysical coverage

The materialized airborne geophysical projects intersect approximately 78.89% of the analytical geometry. Resolution, however, is heterogeneous. Projects with flight-line spacing of 500 m or less cover approximately 13.39% of the territory, whereas those with spacing of 1,000 m or less cover approximately 22.72%. The mere presence of an airborne geophysical project therefore does not imply equivalent resolution.

Class	**Projects**	**Area km²**	**Percentage**
All materialized projects	16	281.884,51	78,89%
Flight-line spacing ≤ 500 m	2	47.830,74	13,39%
Flight-line spacing ≤ 1,000 m	5	81.197,88	22,72%
Flight-line spacing ≤ 2,000 m	9	96.323,77	26,96%

Table 3. Coverage of materialized airborne geophysical projects according to acquisition resolution.

2.5 Documentary control cases

Three documentary cases were used to verify the logical behavior of the framework at different levels of maturity. Fecho dos Morros represents an alkaline context associated with M2 and is mapped as an alkaline suite in the western sector. Serra da Bodoquena represents a sedimentary and carbonate context associated with the M4 test. Granito Scardine represents a documentary case associated with M9. Regional cartography also records other objects relevant to the hypothesis library, such as the Suíte Amoguijá, Granito Alumiador, and Complexo Rio Apa, but their mere presence on the map does not authorize logical progression without the independent evidence required by the corresponding model. These cases do not constitute an independent predictive-validation set. Their purpose is to test whether the engine distinguishes geological context, justification for targeted investigation, and compatible local evidence. The regional alkaline context is documented by Lacerda Filho et al. (2006), the phosphate context by Justo (2000) and Godoi et al. (2001), and the more detailed granitic case by Remédio and Faleiros (2014).

2.6 Knowledge bias and evaluability

Uneven coverage introduces an unavoidable knowledge bias. Better-studied regions are more likely to satisfy criteria that depend on detailed mineralogy, geochemistry, geochronology, or geophysics. Less-studied regions may remain N0 even when they have relevant geological context. PAG ETR therefore separates evaluability from mineral potential. An area with low evaluability is not classified as unfavorable. It is recorded as an area in which the hypothesis cannot be adequately tested with the available snapshot. Separating geological potential from evidence availability is consistent with approaches that treat uncertainty and coverage as distinct products, such as Creason et al. (2023), Ford et al. (2026), and Geoscience Australia (2025).

CAPACIDADE_DE_AVALIACAO ≠ POTENCIAL_MINERAL

This separation allows gaps to be treated as a scientific product in their own right. For each model, the system can record which knowledge components are available and which are missing, including geology, geochemistry, mineralogy, geophysics, regolith, structure, and REE-specific analyses. The purpose of this output is to guide information acquisition rather than estimate favorability.

3. Data, provenance, and freezing

The reproducible package was extracted from ITA ARANDU MS to enable independent auditing of the PAG ETR pilot without dependence on the full application. The datasets are organized into frozen sources, derived evidence, grids, contextual layers, frozen results, reproduced results, methodological specification, tests, and integrity manifests.

The principal sources for the spatial pilot are the geological map of Mato Grosso do Sul and the mineral-resources map of the Serviço Geológico do Brasil. These sources were used to materialize the currently reproducible evidence for M2 and M4. Additional layers of geochemistry, petrography, geochronology, outcrops, airborne geophysics, and cartographic coverage are preserved as context and support for the discussion, but are not automatically converted into favorability. The statewide cartography derives from Lacerda Filho et al. (2006). The digital records consulted are made available by the Serviço Geológico do Brasil through GeoSGB and its environmental geochemistry and medical geology products (Serviço Geológico do Brasil, s.d.-a, s.d.-b). Bellora et al. (2019) and Ford et al. (2023b) were considered as international precedents for structured data on mineral occurrences and mineral potential.

The pilot provenance chain can be represented as

source → snapshot → evidence → rule → model → spatial intersection → hex_id → state → frozen result

Each result must be interpreted in relation to the versions that produced it.

RESULTADO = f(DATA_VERSION, RULE_VERSION, GRID_VERSION, MODEL_LIBRARY_VERSION)

The repository records SHA-256 hashes for the frozen files. The purpose of the hash is not to validate the scientific correctness of the content, but to allow verification that the file used by another researcher is exactly the same file preserved in the snapshot.

3.1 Documentary independence and duplicate control

The number of references is not used as the number of independent observations. When several publications reproduce the same primary observation, the evidence retains a shared canonical identifier. A new confirmation is treated as independent only when it derives from a distinct acquisition or observation. This rule prevents artificial multiplication of certainty through bibliographic repetition and is particularly important in regions where institutional reports are subsequently cited by multiple publications.

REFERENCIAS_DERIVADAS ≠ OBSERVACOES_INDEPENDENTES

Whenever documentation allows, the provenance record must distinguish independent evidence, derived evidence, documentary repetition, and indeterminate independence. Support counts remain descriptive and never determine N0 to N4 progression by themselves.

3.2 Canonical methodological record and map freezing

The methodological specification used to document this application was recovered from the complete PAG ETR development record and preserved separately in the reproducible compendium. The canonical block corresponds to the evidence-sufficiency protocol, formal computational specification, and adversarial testing plan defined before and during the first operationalization. This record contains the REQ, SUP, and CONF functions, the five logical states, the M1 to M11 rules, the evaluation pseudocode, provenance entities, scientific-validator invariants, and the versioning scheme.

The relationship between specification and cartographic product is deliberately historical. The rules were defined first. The available evidence was then materialized in the Atlas, and the spatial pilot produced the maps presented in Figures 5 to 9. The article was written afterward to document that process. Consequently, a methodological criticism identified during manuscript preparation does not authorize silent alteration of a map that has already been produced. Any change capable of modifying a classification requires a new RULE_VERSION, a new DATA_VERSION where applicable, a new execution, and a new result set, while preserving the version described in this study.

This separation prevents retrospective fitting of the model to its own results and distinguishes three objects that must not be conflated. The scientific specification defines what the model is intended to do. The implementation records how a specific version executed part of that specification. The frozen maps document the historical result of that implementation using the sources available at that point in time.

ESPECIFICACAO_CIENTIFICA ≠ IMPLEMENTACAO_ESPECIFICA ≠
RESULTADO_CARTOGRAFICO_CONGELADO

4. PAG ETR METHODOLOGY

4.1 General principle

Implementation terminology. Canonical variable names, rule identifiers, data fields, and computational tokens are retained in their original Portuguese form whenever they correspond directly to the frozen implementation and reproducible repository. They are treated as machine-readable identifiers rather than narrative text. Their meaning is provided in English at first occurrence or in the corresponding tables. This convention preserves one-to-one traceability between the manuscript, source code, datasets, tests, and archived outputs.

PAG ETR is structured as a spatial decision-support system for geochemical investigation based on explicit geological analogy. The scientific specification is normative and must remain separate from the code that executes it. Rules are versioned as scientific objects, allowing an implementation to be audited against the specification and ensuring that future revisions do not silently rewrite historical results.

The chain required by the specification is original source → observation → normalized evidence → model criterion → logical rule → model result → spatial relationship → synthesis in the spatial unit. The source never stores prospectivity, and the observation remains separate from the PAG ETR interpretation.

A(m,h,s) = state assigned to model m, in hexagon h, at scale s

4.2 Elementary evidence states

The canonical specification does not use only true and false. Internally, each condition can take the value TRUE, FALSE, UNKNOWN, CONFLICT, or NA. Transformation from the scientific record occurs only after coverage, method, scale, and quality have been considered.

PRESENTE → TRUE
AUSENTE_VERIFICADO → FALSE
SEM_DADO → UNKNOWN
INDETERMINADO → UNKNOWN
DADO_CONFLITANTE → CONFLICT
NAO_APLICA → NA
Invariant rule UNKNOWN must never be automatically converted into FALSE.

Symbol	**State**	**Meaning**
T	Condition demonstrated	The available evidence supports the condition
F_0	Verified absence	Adequate evidence supports absence
U	Unknown or insufficient	The required information does not exist or is insufficient
X	Conflict	Adequate lines of evidence conflict
NA	Not applicable	The condition does not apply to the case

Table 4. Formal evidence states used by PAG ETR.

$e_i \in \{T, F_0, U, X, NA\}$

The distinction between U and F_0 is mandatory. Not described, outside coverage, and insufficient scale remain semantically distinct from verified absence.

SEM_DADO ≠ AUSENTE_VERIFICADO

ABAIXO_LD ≠ ZERO

NAO_DESCRITO ≠ AUSENTE

FORA_DA_COBERTURA ≠ AUSENTE

ESCALA_INSUFICIENTE ≠ AUSENTE

4.3 Information-sufficiency gate K

K represents exclusively the input information-sufficiency gate for evaluating a model in a specified spatial unit and at a specified scale. There is no universal coverage percentage or common numerical threshold across M1-M11. In the preserved specification, K asks whether the minimum evidence required to interrogate the model rule is available, applicable to the spatial unit, and supported at an adequate scale. When this cannot be established, the condition remains UNKNOWN and the unit cannot be promoted because information is missing. For the M2-M4 pilot, K was not implemented as an independent formula based on a fixed combination of airborne geophysics, geochemistry, or coverage percentages. Operationalization used the spatially materializable presence of the corresponding opening evidence and retained additional data absences as UNKNOWN. The code is therefore not retrospectively assigned a threshold that it did not use.

$K(m,h,s) = T$ if the minimum information requirements defined to evaluate m are available and adequate in (h,s)

$K(m,h,s) = U$ if one or more minimum information requirements needed to evaluate m are unknown, insufficient, or inadequate in (h,s)

Thus, K is not an index of territorial coverage, a completeness score, or a simple function of the percentage of data present. K only determines whether a minimum information basis exists for initiating the logical evaluation of the model.

$K(m,h,s) \neq f(\%\ \text{overall territorial coverage})$

In the first spatial operationalization, K and N1 opening were not two independent measurements. For M2 and M4, the availability of spatially materializable opening evidence simultaneously served as the minimum condition for interrogating the model and as the input to N1. K therefore exerted no additional filtering power in this pilot. This is an implementation limitation, not a necessary property of the framework.

In later versions, K should be specified before execution for each model as an evaluability contract, indicating which data types must exist, at what support and scale, and at what minimum quality, without automatically using the same evidence tested at the next gate as a substitute for sufficiency. Any future quantitative threshold must be justified, versioned, and subjected to sensitivity analysis.

Accordingly, this V20 does not retrospectively introduce thresholds for geochemical density, airborne geophysical spacing, or coverage percentage that were not part of the frozen code.

4.4 Gates G1 to G4

For communication in the manuscript and figures, progression can be decomposed into gates G1 to G4. G1 represents entry into the geological context, G2 represents evidence of the mineralizing process when applicable, G3 represents satisfaction of the conditions that justify targeted investigation, and G4 represents local evidence compatible with REE. This notation is an explanatory layer. The canonical formal specification is expressed by the N1, N2, and N3 rules specific to each model, presented in Section 4.8 and Appendix A. If any terminological divergence occurs, the M1_N1 to M11_N3 expressions and the N0 to N4 states take precedence over the didactic G1 to G4 representation.

The general decision function is

N0 if K is insufficient or $G1 = U$

N4 if there is verified contradiction of a necessary requirement

N1 if $G1 = T$ and $G3 \neq T$

$$\text{N2 if G3} = \text{T and G4} \neq \text{T}$$

$$\text{N3 if G3} = \text{T and G4} = \text{T}$$

4.5 N0 to N4 states

State	Designation	Interpretation
N0	Non-evaluable	Insufficient evidence to adequately test the model
N1	Open model	The context allows the geological hypothesis to remain open
N2	Justifies geochemical investigation	The defined requirements justify targeted evidence acquisition
N3	Compatible local evidence	Local evidence compatible with the model exists, without inferring a deposit or viability
N4	Locally contradicted	A necessary condition has a verified contradiction for that model

Table 5. N0 to N4 states and their operational interpretation in PAG ETR.

4.6 Non-compensatory logic

Evidence is classified according to its logical role. REQ denotes a requirement necessary for a conclusion. SUP denotes supporting evidence, which strengthens interpretation but does not replace a requirement. CONF denotes confirmatory local evidence compatible with an REE characteristic predicted by the model. These classes are not weights. The design rule is non-compensatory and is consistent with the mineral-systems tradition and mappable criteria described by Wyborn et al. (1994), Skirrow et al. (2019), and Ford et al. (2023a).

$$\text{SUP} + \text{SUP} + \text{SUP} + \text{SUP} \neq \text{REQ}$$

$$\text{CONF from another requirement} \neq \text{missing REQ}$$

The amount of supporting evidence or confirmations belonging to another requirement does not alter the logical role of the missing requirement. A conclusion can be issued only when all REQ defined for that level are satisfied.

4.7 Conflict, independence, and sufficiency rules

State X is preserved when adequate and comparable evidence supports incompatible interpretations of a necessary condition. X is not resolved by majority of references and is not automatically converted into T or F_0. Resolution requires an audit of provenance, scale, method, quality, and independence. While the conflict persists, the condition cannot be used to promote the model.

$$X \nRightarrow T \quad e \quad X \nRightarrow F_0$$

Sufficiency for progression is model-specific. In general terms, a recommendation for investigation can be issued only when an applicable geological model, a compatible process, sufficient independent evidence, and adequate coverage for the requirements being interrogated coexist. Auxiliary evidence improves interpretation but does not replace necessary requirements.

RECOMENDACAO_PAG_ETR = MODELO ∧ PROCESSO ∧ EVIDENCIA_INDEPENDENTE_SUFICIENTE ∧ COBERTURA_ADEQUADA

RECOMENDACAO_PAG_ETR ≠ SOMA_DE_CAMADAS

4.8 M1 to M11 model library

M1 · Carbonatites

M1_N1 = MAG_CARBONATITO

M1_N2 = MAG_CARBONATITO ∧ CARBONATITO_PROCESSO_COMPATIVEL

M1_N3 = M1_N2 ∧ ETR_LOCAL_COMPATIVEL_M1

Bibliographic basis for M1. Carbonatites and related alkaline complexes associated with REE are addressed by Verplanck et al. (2014), Van Gosen et al. (2019), Foley and Ayuso (2024), and Ford et al. (2023a).

M2 · Alkaline and peralkaline intrusions

M2_N1 = MAG_ALCALINO

M2_N2 = M2_N1 ∧ ESPECIALIZACAO_ALCALINA

M2_N3 = M2_N2 ∧ ETR_LOCAL_COMPATIVEL_M2

Bibliographic basis for M2. Models of alkaline and peralkaline intrusions related to REE are described by Verplanck et al. (2014), Van Gosen et al. (2019), Aranha et al. (2022), and Ford et al. (2023a).

M3 · Hydrothermal systems

M3_N1 = HIDROTERMALISMO

M3_N2 = M3_N1 ∧ FONTE_FERTIL ∧ PATHWAY_OU_TRAP

M3_N3 = M3_N2 ∧ ETR_HIDROTERMAL

Bibliographic basis for M3. REE behavior in hydrothermal processes and the associated interpretive limitations are discussed by Lottermoser (1992). The isolated presence of hydrothermal alteration is not treated as confirmation of an REE system.

M4 · Sedimentary phosphorites

M4_N1 = SED_FOSFORITA

M4_N2 = M4_N1 ∧ APATITA_FOSFATICA ∧ ORIGEM_SEDIMENTAR

M4_N3 = M4_N2 ∧ REE_REY_FOSFATO

Bibliographic basis for M4. The potential of sedimentary phosphorites as hosts for REE and REY is discussed by Emsbo et al. (2015). For Serra da Bodoquena, the presence of phosphate and fluorapatite is documented by Justo (2000) and Godoi et al. (2001).

M5 · IOA and IOCG systems with REE

M5_N1 = IOA_CONFIRMADO ∨ IOCG_CONFIRMADO

M5_N2 = (IOA_CONFIRMADO ∧ APATITA_RELACIONADA) ∨ (IOCG_CONFIRMADO ∧ EVIDENCIA_ETR_RELEVANTE)

M5_N3 = M5_N2 ∧ ETR_LOCAL_COMPATIVEL_M5

Bibliographic basis for M5. The association among iron-oxide systems, apatite, and REE is exemplified by Watts and Mercer (2020), while regional-scale mineral-systems modelling of IOCG systems is addressed by Skirrow et al. (2019).

M6 · Placers and paleoplacers

M6_N1 = TRAP_SEDIMENTAR

M6_N2 = M6_N1 ∧ MINERAIS_PESADOS ∧ FONTE_POTENCIAL

M6_N3 = M6_N2 ∧ MONAZITA_OU_XENOTIMA_DETRITICA

Bibliographic basis for M6. The occurrence of monazite, xenotime, and other heavy minerals in detrital systems is addressed in the global synthesis of Foley and Ayuso (2024) and the occurrence inventory of Bellora et al. (2019).

M7 · Ion-adsorption regoliths

M7_N1 = PROTOLITO_POTENCIAL ∧ REGOLITO_PRESERVADO

M7_N2 = M7_N1 ∧ MOBILIDADE_REE

M7_N3 = M7_N2 ∧ FRACAO_TROCAVEL_REE

Bibliographic basis for M7. Regolith-hosted and ion-adsorption clay deposits are supported by Shah et al. (2017), Borst et al. (2020), and Li et al. (2022), who also demonstrate the need to separate regolith context, geochemical process, and direct REE evidence.

M8 · Residual lateritic enrichment

M8_N1 = PROTOLITO_FERTIL ∨ PERFIL_RESIDUAL

M8_N2 = PROTOLITO_FERTIL ∧ PERFIL_RESIDUAL_IN_SITU

M8_N3 = M8_N2 ∧ ENRIQUECIMENTO_RESIDUAL_REE

Bibliographic basis for M8. Carbonatite-derived residual enrichment is illustrated by the Mount Weld deposit discussed by Verplanck et al. (2023). In this version, that reference is used as a process precedent rather than as an automatic analogue for Mato Grosso do Sul.

M9 · Evolved granites

M9_N1 = GRANITO_INDIVIDUALIZADO

M9_N2 = M9_N1 ∧ ESPECIALIZACAO_GRANITICA

M9_N3 = M9_N2 ∧ ETR_LOCAL_COMPATIVEL_M9

Bibliographic basis for M9. The relationship between evolved igneous suites and REE is discussed by McLeod and Shaulis (2018). The local Granito Scardine case is supported by the mapping and documentation of Remédio and Faleiros (2014).

M10 · REE-bearing pegmatites

M10_N1 = PEGMATITO

M10_N2 = M10_N1 ∧ ESPECIALIZACAO_PEGMATITICA_ETR

M10_N3 = M10_N2 ∧ MINERAL_ETR_PEGMATITO

Bibliographic basis for M10. The requirement for pegmatitic specialization and REE minerals associated with pegmatite is supported by the classic synthesis of Černý (1992), which distinguishes families of rare-element pegmatites and includes systems enriched in Y and REE.

M11 · Unconformity-related HREE systems

M11_N1 = DISCORDANCIA ∧ ARQUITETURA_ESTRUTURAL

M11_N2 = M11_N1 ∧ HIDROTERMALISMO_RELACIONADO ∧ SINAL_HREE

M11_N3 = M11_N2 ∧ XENOTIMA_HIDROTERMAL

Bibliographic basis for M11. The heavy-REE model associated with unconformities and hydrothermal systems is supported by Nazari-Dehkordi et al. (2018) and the Australian national assessment of Ford et al. (2026).

4.8.1 Operational safeguards for the M1 to M11 library

The library was constructed to prevent a lithological label or an isolated proxy from producing inappropriate progression. The following safeguards summarize the most important negative conditions in the design and should be read together with the equations for each model.

Model	**Hypothesis**	**Safeguard against false positives**
M1	Carbonatite	A carbonatite label without process evidence and independent REE evidence is insufficient
M2	Alkaline/peralkaline	A generic alkaline intrusion is not equivalent to REE specialization
M3	Hydrothermal	Isolated alteration, fluorite, or fault evidence does not satisfy source + pathway/trap + REE
M4	Phosphorite	Phosphate/apatite justify analysis but do not demonstrate REE/REY enrichment
M5	IOA/IOCG	Isolated magnetite does not demonstrate an REE-bearing IOA/IOCG system
M6	Placer	A sedimentary trap without heavy minerals and a source does not advance the model
M7	Ion adsorption	Clay + granite + weathering do not replace evidence of exchangeable fraction/mobility
M8	Lateritic residual	Laterite without a fertile protolith and coherent redistribution is insufficient
M9	Evolved granite	Generic granite is not equivalent to a geochemically/mineralogically specialized facies
M10	Pegmatite	A regional pegmatite without local specialization does not generate N2
M11	HREE unconformity	An isolated unconformity, radiometric U anomaly, or fault

		does not satisfy an HREE hydrothermal system

Table 6. Operational safeguards for the M1 to M11 model library.

4.9 Multimodel architecture

The eleven models are not summed to produce a single score. The same hexagon may have different states for different mineralization systems.

$\vec{A}(h,s) = [A(M1,h,s), A(M2,h,s), ..., A(M11,h,s)]$

This vector constitutes the complete scientific profile of the spatial unit. A visual synthesis may be derived for cartographic display, but it does not replace the individual states.

$DISPLAY(h,s) = N3$ if $\exists m : A(m,h,s)=N3$

$DISPLAY(h,s) = N2$ if there is no N3 and $\exists m : A(m,h,s)=N2$

$DISPLAY(h,s) = N1$ if there is no N2 or N3 and $\exists m : A(m,h,s)=N1$

$DISPLAY(h,s) = N0$ otherwise

N4 remains model-specific and does not automatically dominate the overall synthesis.

4.10 Spatial relationships

The original geometry of the evidence is preserved whenever possible. Polygons, lines, points, contributing areas, and rasters have distinct spatial relationships with the hexagon.

$I(f,h) = area(f \cap h)$

$P\_hex(f,h) = 100 \times area(f \cap h) / area(h \cap MS)$

$P\_feature(f,h) = 100 \times area(f \cap h) / area(f)$

$L(l,h) = length(l \cap h)$

$R(p,h) = 1$ if $p \in h$ or touches its boundary, otherwise 0

$R_u(p,h) = 1$ if $U(p) \cap h \neq \emptyset$

$R(c,h) = 1$ if $area(catchment \cap h) > 0$

$V(h) = N\_pixels\_válidos(h) / N\_pixels\_totais(h)$

$COV(d,h) = area(h \cap cobertura\_d) / area(h \cap MS)$

A point observation does not semantically fill a hexagon. Intersection records a spatial relationship, not geological uniformity.

4.11 Treatment of geochemical data

Geochemical data are not automatically converted into REE evidence. Sample medium, analytical method, detection limit, campaign, unit, and context must be preserved. Values below the detection limit remain censored and are not converted to zero. The need to preserve context, data composition, and relationships among elements is reinforced by Creason et al. (2023) and Bishop and Robbins (2024).

$x < LD \rightarrow$ censored state, not $x = 0$

REE or REY sums may be calculated only when the composition of the element set and the treatment of censored values are explicitly defined. The framework does not use a generic ΣREE as a substitute for mineralogical or process evidence.

4.12 Multiscale architecture

The three scales are calculated directly from the original evidence.

$$A_{250} = F(E, Grid_{250})$$

$$A_{500} = F(E, Grid_{500})$$

$$A_{1000} = F(E, Grid_{1000})$$

$$A_{500} \neq mean(A_{250})$$

$$A_{1000} \neq mean(A_{500})$$

Comparison across scales is used to examine sensitivity to spatial support rather than as independent pseudo-replication. Sensitivity of results to cell size is a recognized issue in geochemical mapping and is discussed by Zuo (2012).

4.13 Logical and computational architecture

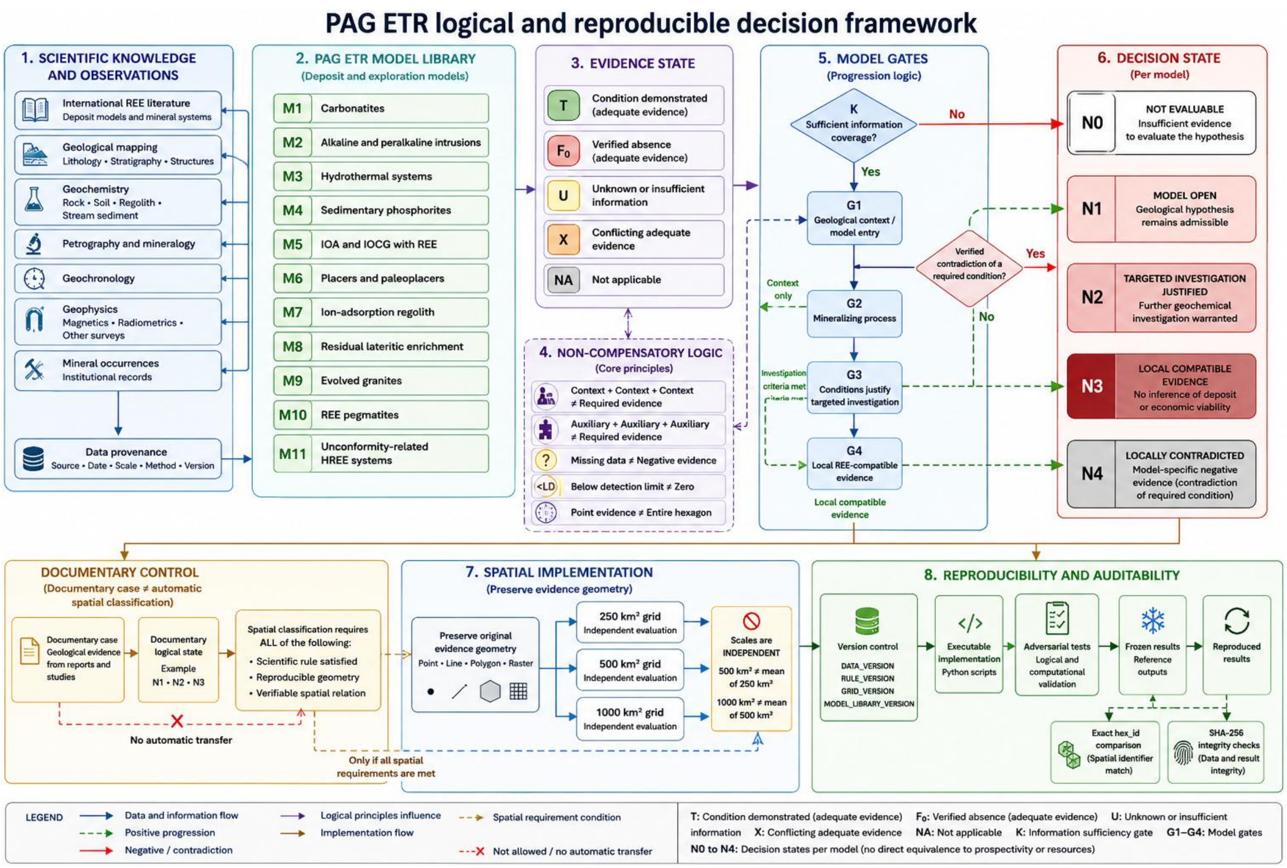

Figure 3. Conceptual and computational architecture of the PAG ETR framework. Scientific knowledge and geoscientific observations are translated into model-specific evidence states and evaluated through a non-compensatory logical architecture. Missing information is retained explicitly and is not interpreted as negative geological evidence. Documentary evaluations are not automatically transferred to spatial

units. The three spatial supports are evaluated independently and reproducibility is controlled through versioned data, rules, grids, model libraries, executable tests, frozen outputs and exact spatial identifier comparison.

4.14 Deterministic generation of investigation gaps

When a hypothesis does not progress, PAG ETR need not merely report that data are missing. The engine can record the missing requirement that blocked progression and convert it into a traceable investigation request. Conceptual examples include REE/REY analysis in apatite for M4, exchangeable fraction for M7, specialized mineralogy for M9, and heavy-mineral concentrate for M6. The recommendation derives from the rule that failed rather than from a favorability score.

LACUNA = unknown or inadequately tested necessary requirement

SOLICITACAO_DE_INVESTIGACAO = f(modelo, requisito_faltante, fonte, escala, cobertura)

This output is methodologically relevant because it converts uncertainty into an explicit data-acquisition agenda. It also preserves the interpretive boundary. Recommending an additional analysis means only that the analysis is needed to test the hypothesis.

4.15 Formal specification, pilot operationalization, and the no-retrofit rule

The M1 to M11 library represents the full scientific space of the framework, but the first spatial application did not materialize the entire library. It used only evidence that could be related to the grids reproducibly in that snapshot. Therefore, the absence of a model from the maps does not mean that the model was rejected, and the presence of cartographic evidence does not mean that all documentary requirements of the model were automatically satisfied.

A special distinction is required for M4. In the documentary specification, M4_N1 requires SED_FOSFORITA, and M4_N2 additionally requires phosphate apatite and sedimentary origin. In the spatial operationalization of the pilot, phosphorite units and documented point occurrences with phosphorus as the recorded substance were used as signals for opening the M4 hypothesis. These points were not treated as demonstrating sedimentary phosphorite throughout the cell, did not semantically fill the hexagon, and were not allowed to promote a cell to N2 or N3. Their function was conservatively limited to keeping the hypothesis open for investigation in the spatial product.

> M4_ABERTURA_ESPACIAL = materialized phosphorite unit OR documented point occurrence of phosphorus, with an operational ceiling of N1

For communication and auditing, the two opening provenance types must be distinguished even when both retain an N1 ceiling.

M4_N1_POLIGONO = intersection with a materialized polygonal phosphorite unit

M4_N1_PROXY_PONTUAL = documented point occurrence with phosphorus as the recorded substance, not verified as sedimentary phosphorite, with an N1 ceiling

M4_N1_PROXY_PONTUAL is not semantically equivalent to SED_FOSFORITA and cannot be used to satisfy APATITA_FOSFATICA, ORIGEM_SEDIMENTAR, or REE_REY_FOSFATO. The cartographic interface and derived products must preserve this provenance separately.

> M4_N2 and M4_N3 remain governed by the formal rules for SED_FOSFORITA, APATITA_FOSFATICA, ORIGEM_SEDIMENTAR, and REE_REY_FOSFATO.

5. Spatial pilot implementation in Mato Grosso do Sul

5.1 Materialized evidence

The spatial pilot already implemented in ITA ARANDU MS materialized two evidence families and is treated in this article as a frozen historical result. M2 was reconstructed from Corpo Feixe dos Morros in the preserved geological map. M4 was opened spatially from the materialized phosphorite units and documented point records with phosphorus as the recorded substance in the mineral-resources map. Inclusion of the points was an operational N1-screening decision in the first version and not a claim that each point, by itself, satisfies the complete documentary definition of SED_FOSFORITA. No point automatically promoted M4 to N2 or N3. The remaining models lacked sufficient spatialization for automatic classification in this snapshot. The local evidence derives from SGB cartographic and mineral-resource sources, with the regional basis provided by Lacerda Filho et al. (2006), Justo (2000), and Godoi et al. (2001).

The reproducible pilot chain is

SGB / GeoSGB → frozen sources → M2 and M4 → independent intersection → 250 / 500 / 1000 km² → N0 / N1 → validation by hex_id

5.2 Scientific Prototype Zero

Before spatialization, three controlled documentary cases were used to verify logical progression.

Feixe dos Morros → M2 → N1

Serra da Bodoquena → M4 → N2

Granito Scardine → M9 → N3

These documentary states were not automatically transferred to the grid. Spatialization simultaneously requires a satisfied scientific rule, reproducible geometry, and a verifiable spatial relationship.

CASO_DOCUMENTAL ≠ CLASSIFICACAO_ESPACIAL_AUTOMATICA

5.3 M4 use case

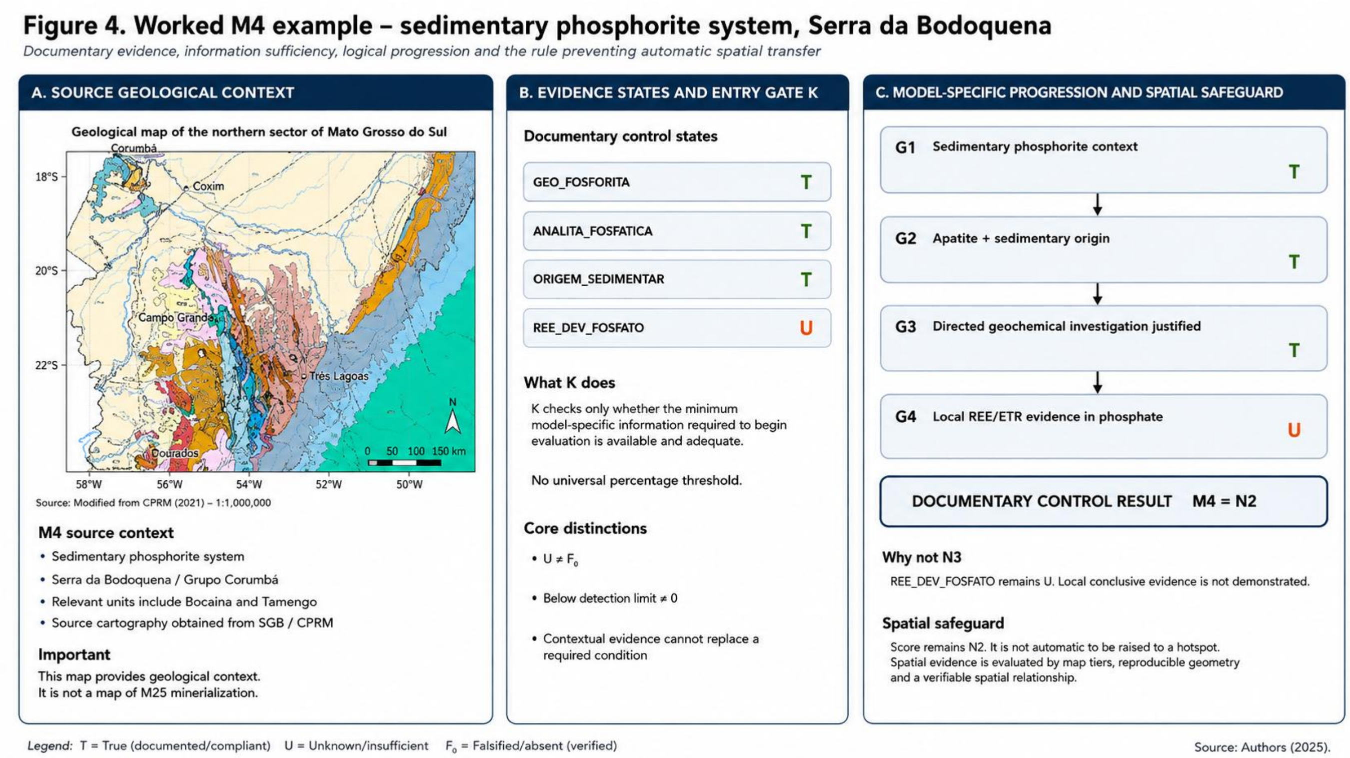

Figure 4. Worked M4 example for the sedimentary phosphorite system of Serra da Bodoquena. The figure separates the model-specific information-sufficiency gate K from subsequent progression through G1-G4. K only determines whether the minimum information needed to begin evaluating M4 is available and adequate. The documentary control reaches N2 because the sedimentary phosphorite, phosphate-apatite and sedimentary-origin conditions are satisfied, whereas REE/REY evidence in the phosphate phase remains unknown and therefore does not support N3. The documentary state is not transferred automatically to spatial cells. Spatial classification additionally requires reproducible geometry and a verifiable spatial relationship.

5.4 Pilot cartographic figures

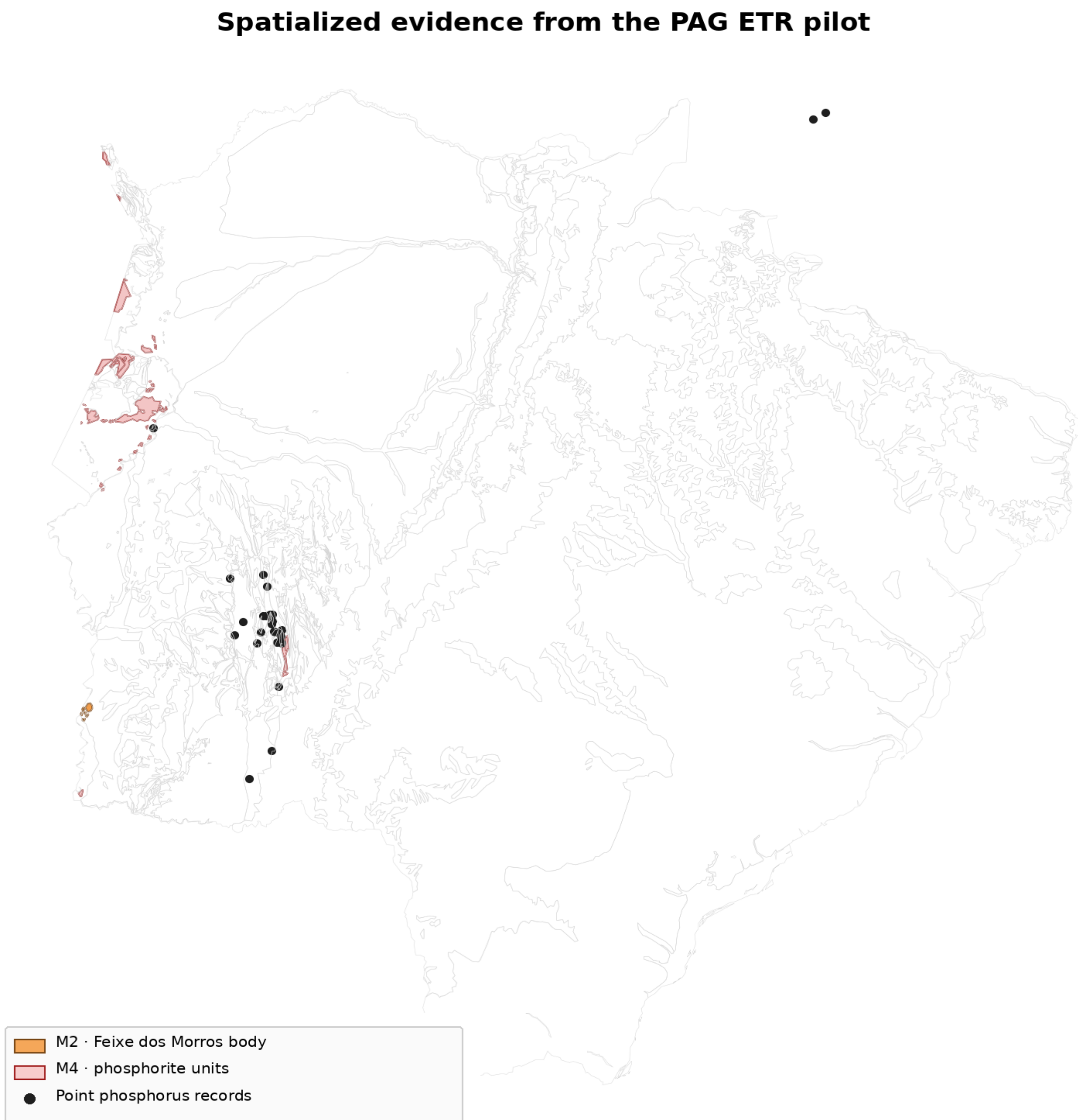

Figure 5. Spatialized evidence materialized in the PAG ETR pilot, including M2, M4 and point phosphorus records.

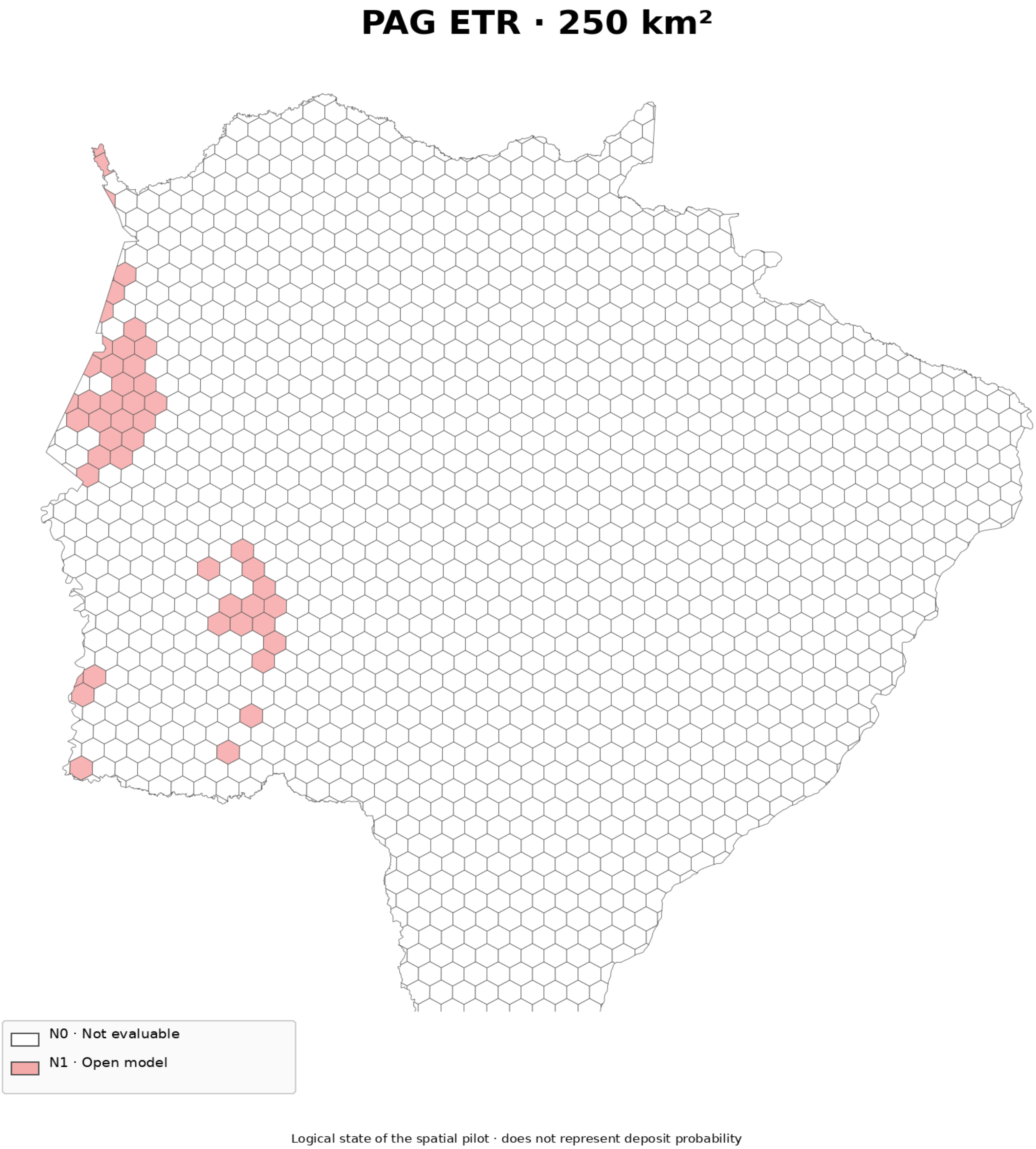

Figure 6. PAG ETR result for the nominal 250 km² grid. M4-related N1 cells may derive from M4_N1_POLIGONO or M4_N1_PROXY_PONTUAL. The frozen logical class is shown without graphical separation of these provenance types; the distinction is retained in the reproducible data and documentation.

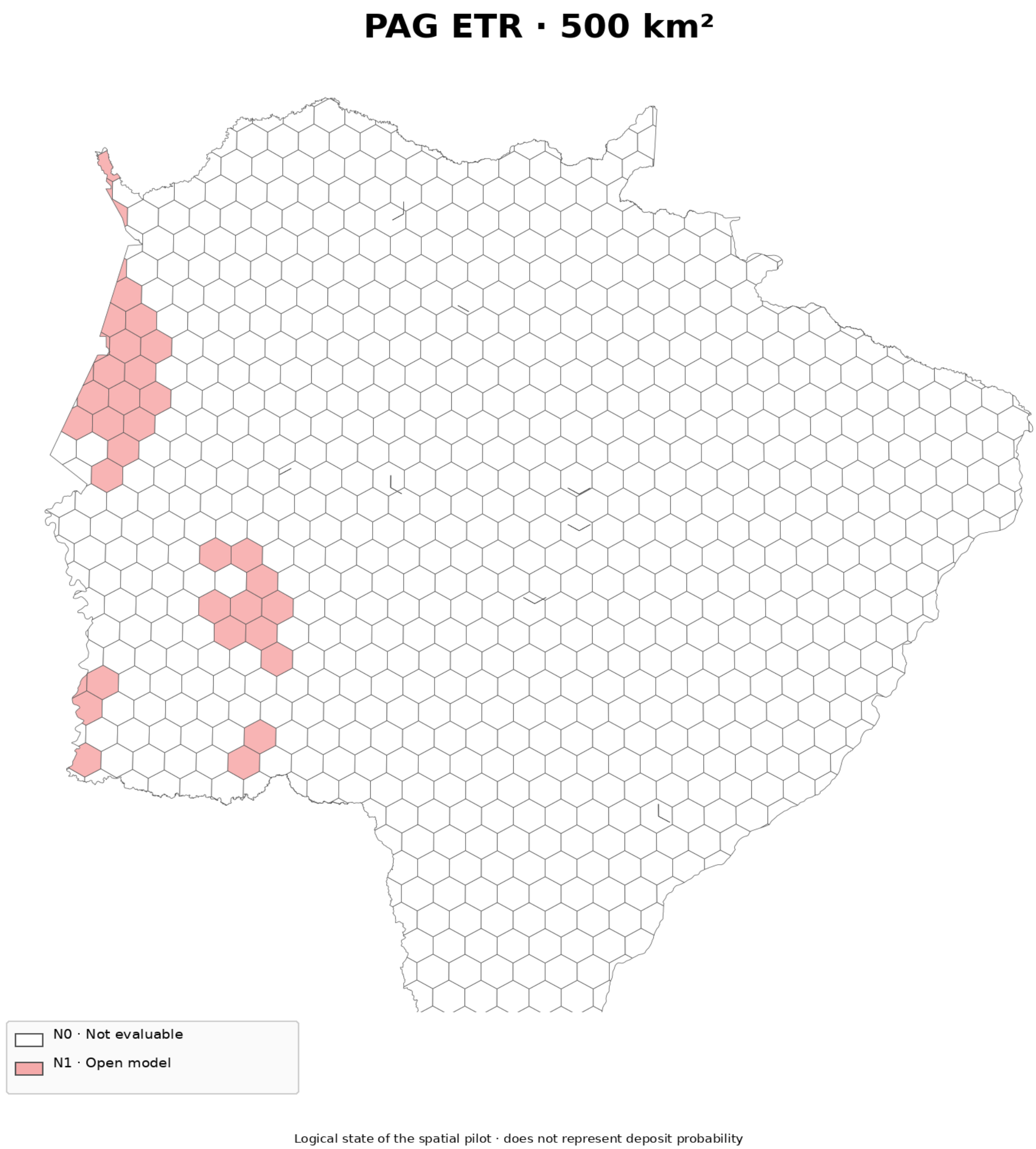

Figure 7. PAG ETR result for the nominal 500 km² grid. M4-related N1 cells may derive from M4_N1_POLIGONO or M4_N1_PROXY_PONTUAL. The frozen logical class is shown without graphical separation of these provenance types; the distinction is retained in the reproducible data and documentation.

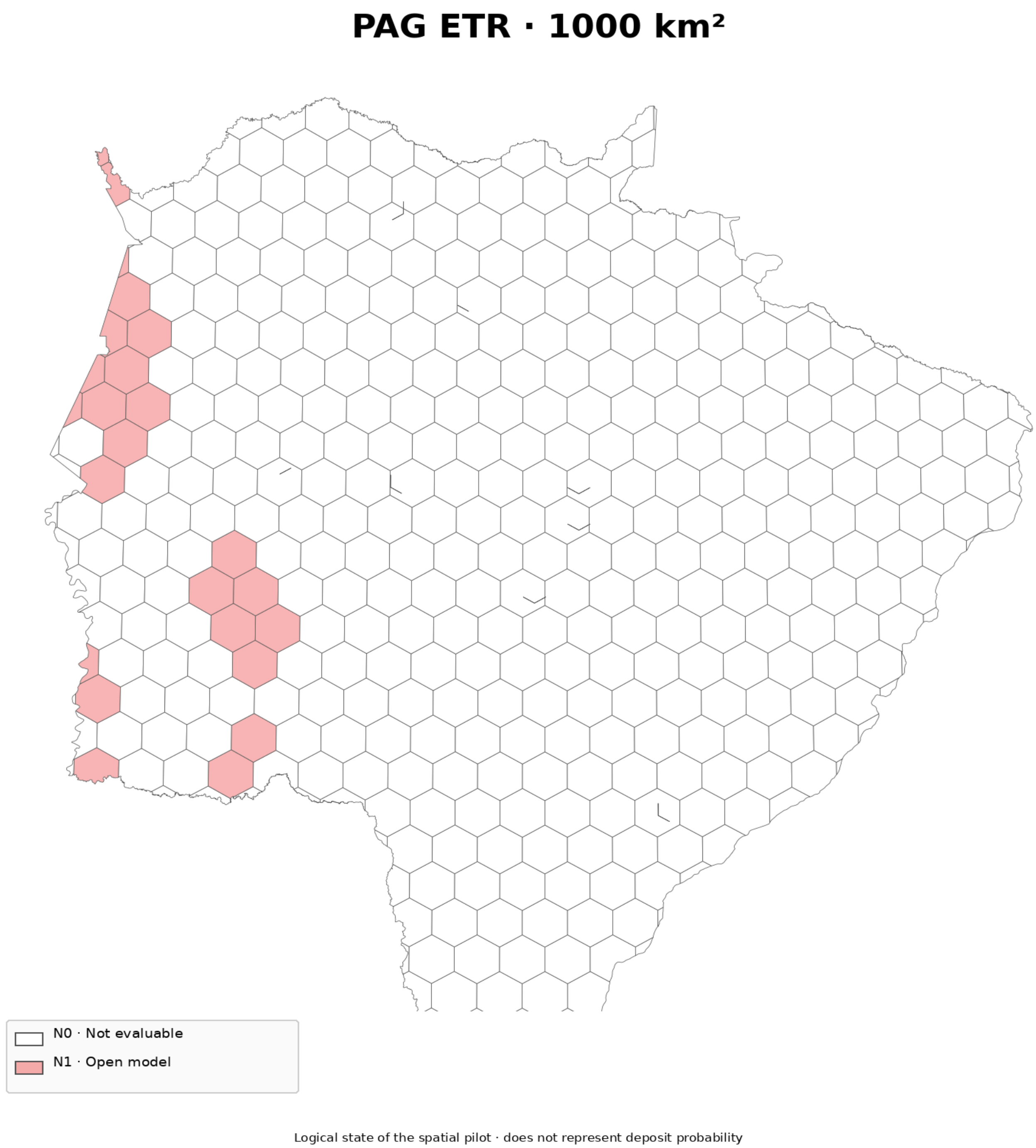

Figure 8. PAG ETR result for the nominal 1,000 km² grid. M4-related N1 cells may derive from M4_N1_POLIGONO or M4_N1_PROXY_PONTUAL. The frozen logical class is shown without graphical separation of these provenance types; the distinction is retained in the reproducible data and documentation.

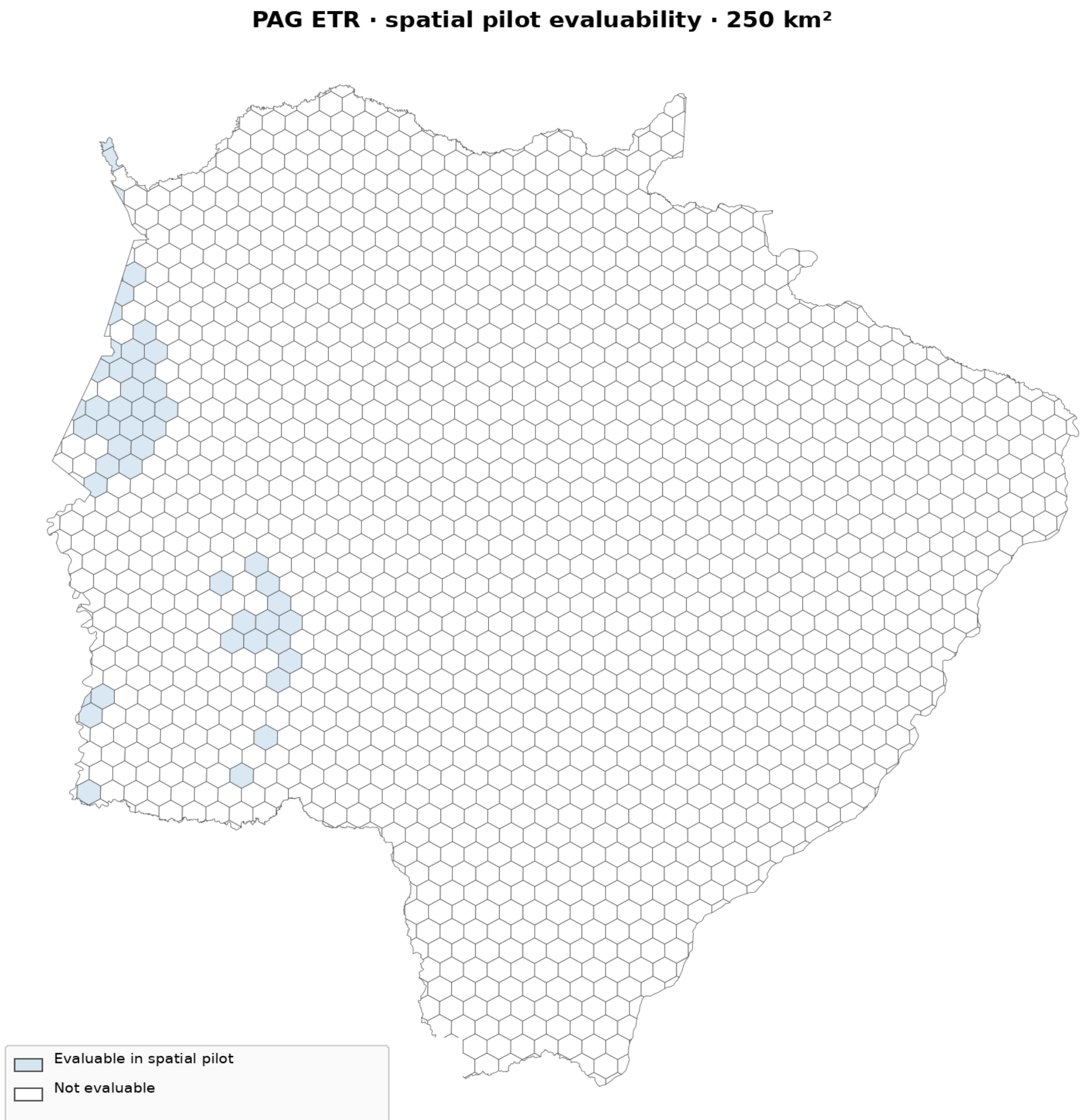

Figure 9. Evaluability of the PAG ETR spatial pilot at the 250 km² support, distinguishing non-evaluable cells from cells with an open model.

6. Results

6.1 Results of Scientific Prototype Zero

The first evaluation of the logical engine used three controlled documentary geological cases selected to represent different possible levels of progression within PAG ETR.

Feixe dos Morros → M2 → N1

Serra da Bodoquena → M4 → N2

Granito Scardine → M9 → N3

The occurrence of three distinct levels made it possible to verify whether the engine distinguished a merely open hypothesis, a hypothesis with sufficient evidence to justify targeted investigation, and a hypothesis with compatible local evidence. The documentary results were not used directly as spatial classifications.

6.2 Feixe dos Morros

For Feixe dos Morros, the available documentation allowed recognition of the alkaline-intrusion context required to open M2. The documentary result was M2 = N1, corresponding to an open model. The framework did not automatically promote the case to N2 because additional alkaline specialization was not considered sufficiently demonstrated in the documentary set used. Nor was the compatible local REE evidence required for N3 assigned. Cartographic recognition of the alkaline body derives from Lacerda Filho et al. (2006), while progression remains conservative relative to the international models of Verplanck et al. (2014) and Aranha et al. (2022).

MAG_ALCALINO = T

ESPECIALIZACAO_ALCALINA ≠ T

ETR_LOCAL_COMPATIVEL_M2 ≠ T

6.3 Serra da Bodoquena

The documentary case of Serra da Bodoquena was evaluated using M4. The documentation records phosphate in units of Grupo Corumbá and fluorapatite in phosphatic intraclasts. The methodology explicitly separates the presence of phosphate from demonstration of REE enrichment. The regional documentation is supported by Justo (2000) and Godoi et al. (2001), while the relevance of investigating REE and REY in phosphorites is supported by Emsbo et al. (2015).

M4 = N2

SED_FOSFORITA = T

APATITA_FOSFATICA = T

ORIGEM_SEDIMENTAR = T

REE_REY_FOSFATO = U

The N2 result was not interpreted as demonstrating REE mineralization.

6.4 Granito Scardine

The third documentary case was Granito Scardine. The Folha Fazenda Margarida distinguishes the unit in 1:100,000 mapping and records geochemical data and prospecting interest related to rare earth elements. In Scientific Prototype Zero, the documentary set produced M9 = N3. This N3 is a documentary model state, not automatic authorization for spatialization. The case was not transferred to grid cells because, in the frozen snapshot used by the pilot, the necessary detailed geometry had not been materialized in a way that allowed a reproducible spatial relationship under the same protocol applied to M2 and M4. Keeping M9 off the map avoids converting the strongest documentary evidence into a spatial inference with greater precision than the available data support. The local source is Remédio and Faleiros (2014), while McLeod and Shaulis (2018) provide the international context for REE in evolved igneous suites.

6.5 Spatial pilot result

Spatialization was deliberately more restrictive than documentary testing. Only evidence whose geometry had been materialized in the frozen dataset and could be reproducibly related to the grids was allowed to generate spatial states. Under this configuration, the spatial pilot primarily materialized M2 and M4.

6.6 Multiscale results

Nominal scale	Cells	N1	N0	M2	M4	Spatial N2	Spatial N3
250 km²	1.554	49	1.505	3	46	0	0
500 km²	793	37	756	3	34	0	0
1.000 km²	412	24	388	2	22	0	0

Table 7. Multiscale spatial results of the PAG ETR pilot.

The absolute number of N1 cells decreased as spatial support increased.

49 → 37 → 24

Meanwhile, the proportion of N1 cells relative to the total increased.

3,15% → 4,67% → 5,83%

This behavior is described as a geometric effect of support rather than as increased favorability.

6.7 Spatial predominance of M4

At all scales, most open cells were associated with M4. At the 250 km² scale, M4 accounts for 46 of the 49 N1 cells. At 500 km², it accounts for 34 of 37, and at 1,000 km², 22 of 24. These counts belong to the frozen spatial product of the first operationalization, in which phosphorite units and point phosphorus occurrences could keep the M4 hypothesis open at N1. The result reflects the operational rule, the distribution of the materialized geometries, and spatial support. It does not constitute a comparison of metallogenic importance and is not retrospectively reinterpreted as evidence of REE enrichment.

As a sensitivity test of the operational M4 decision, a counterfactual scenario was executed without altering the frozen maps. In this scenario, only the materialized polygonal geometries of the Bocaina and Tamengo formations were allowed to open M4_N1, and point records with phosphorus as the recorded substance were excluded from spatial opening. The procedure used the same three grids and the same intersection operation as the pilot. The aim was not to propose a new official result, but to measure how much of the M4 count depends specifically on including the point records.

The restricted scenario produced 38 M4_N1 cells at 250 km², 28 at 500 km², and 17 at 1,000 km². Compared with the frozen result of 46, 34, and 22 M4_N1 cells, respectively, the point records add 8, 6, and 5 cells. Thus, 82.61%, 82.35%, and 77.27% of the M4_N1 cells in the frozen result are already spatially supported by the polygonal phosphorite units at the three scales. The incremental contribution of the points corresponds to 17.39%, 17.65%, and 22.73% of M4_N1 cells.

The sensitivity analysis has no predictive power and does not validate M4 as a local mineralization system. It measures only the dependence of N1 counts on a spatialization decision. A cell remaining in the restricted scenario does not demonstrate REE or REY enrichment.

This test neither invalidates nor replaces the published map. It quantifies a known dependency of the first operationalization and shows that the predominance of M4 is not produced exclusively by the phosphorus points, although their contribution is material and must remain explicitly disclosed.

Scale	**Frozen M4**	**Phosphorite polygons only**	**Point-record increment**	**Polygonal base**	**Point increment**
250 km²	46	38	8	82,61%	17,39%
500 km²	34	28	6	82,35%	17,65%
1.000 km²	22	17	5	77,27%	22,73%

Table 8. Sensitivity analysis of M4 to the exclusion of point phosphorus records. The frozen result remains unchanged. The restricted scenario considers only the materialized polygonal phosphorite units.

$$\text{N\_células}(\text{M4}) > \text{N\_células}(\text{M2}) \nRightarrow \text{potencial}(\text{M4}) > \text{potencial}(\text{M2})$$

6.8 Predominance of non-evaluable cells

At the 250 km² scale, 96.85% of cells remained non-evaluable. At 500 km², 95.33% remained non-evaluable, and at 1,000 km², 94.17%. This result was not converted into low mineral potential.

$$\text{N0} \neq \text{low potential}$$

6.9 Reproducibility of results

The new pipeline execution reproduced 49, 37, and 24 N1 cells at the 250, 500, and 1,000 km² scales. Validation also compared the hex_id identifiers. The identifier sets matched at all three scales.

$$\text{HEXID\_congelado} = \text{HEXID\_reproduzido}$$

During the audit, six inherited geometries with self-intersections were identified in the 500 and 1,000 km² grids. The working geometries were corrected to permit valid operations. After correction, the counts and classified hex_id values remained identical.

6.10 Geoscientific coverage results

Coverage quantification showed that 16.83% of the analytical geometry has materialized cartography at 1:250,000 or more detailed scales. Geochemical, petrographic, and geochronological observations are even more restricted. Airborne geophysical coverage is broad when only the existence of projects is considered, but much smaller when higher resolution is required. These results support the interpretation that N0 largely records limitations in the evaluability of the frozen dataset.

6.11 Evaluability as a separate result

Combining the model library with available coverage makes it possible to distinguish evaluability without converting it into potential. At the documentary stage, M2, M4, and M9 showed greater evaluability because geological objects and sufficient information exist to test substantial portions of their respective decision trees. M3, M6, and M10 remain partially evaluable, whereas M1, M5, M7, M8, and M11 require acquisition of additional essential evidence in the audited dataset. This classification describes information maturity rather than metallogenic hierarchy.

$$\text{AVALIAVEL}(m,h,s) \nRightarrow \text{FAVORAVEL}(m,h,s)$$

7. Validation

PAG ETR validation was structured in complementary layers. The aim of this stage was not to demonstrate predictive capability for mineral discovery, but to verify logical consistency, compliance with non-compensation rules, correct treatment of missing data, preservation of geometries, spatial reproducibility, and separation between documentary assessment and territorial classification.

7.1 Logical validation

SEM_DADO ≠ AUSENTE_VERIFICADO

ABAIXO_LD ≠ ZERO

EVIDENCIA_CONTEXTUAL ⇏ CONFIRMACAO_DO_MODELO

EVIDENCIA_AUXILIAR ⇏ SUBSTITUICAO_DE_REQUISITO

EVIDENCIA_PONTUAL ⇏ PREENCHIMENTO_DO_HEXAGONO

The preserved specification also includes spatial and geological invariants such as DUPLICATED_SOURCE_DOES_NOT_CREATE_INDEPENDENT_EVIDENCE, STREAM_SAMPLE_USES_CATCHMENT_WHEN_AVAILABLE, RASTER_NODATA_DOES_NOT_BECOME_ZERO, OUTSIDE_COVERAGE_DOES_NOT_BECOME_NEGATIVE, and SCALE_INSUFFICIENT_DOES_NOT_BECOME_LOCAL_ABSENCE, together with model-specific safeguards that prevent isolated alkaline rocks, phosphorite, magnetite, alluvial sand, kaolinite, laterite, granite, pegmatite, or unconformity from producing higher levels without the corresponding requirements.

7.2 Adversarial tests

The methodological specification preserves a catalogue of scientific invariants and adversarial scenarios designed to provoke overinterpretation, false absence, inappropriate substitution of requirements, evidence duplication, and scale error. The materialized pilot implementation converted 15 of these checks into executable tests, all of which returned PASS. The number 15 therefore describes the suite actually automated in this version, not the full universe of conceptual safeguards in the model. Table 9 makes the purpose of the executable checks explicit so that the result can be audited without relying solely on the code.

ID	Verification	Failure prevented	Result
T01	Carbonatite context without independent process evidence	Prevents a carbonatite label alone from advancing the model	PASS
T04	Generic alkaline rock	Prevents generic alkalinity from being treated as REE specialization	PASS
T06	Generic hydrothermal alteration	Prevents hydrothermal context alone from satisfying an REE model	PASS
T07	Phosphorite without REE/REY evidence	Prevents phosphate occurrence from being interpreted as REE enrichment	PASS
T09	Isolated magnetite evidence	Prevents a single auxiliary proxy from replacing required geological conditions	PASS

T11	Alluvial sand without heavy-mineral evidence	Prevents a sedimentary trap alone from opening a placer model beyond its evidence	PASS
T13	Granite with kaolinite in tropical weathering	Prevents climate/weathering context from being treated as ion-adsorption REE evidence	PASS
T16	Laterite without a fertile protolith	Prevents residual weathering alone from advancing the lateritic model	PASS
T18	Generic granite	Prevents undifferentiated granite from being treated as an evolved REE-fertile granite	PASS
T21	Generic pegmatite	Prevents a pegmatite label alone from satisfying a rare-metal/REE model	PASS
T23	Isolated unconformity	Prevents a structural/stratigraphic relation alone from satisfying a mineralizing model	PASS
T25	Area without data	Verifies that missing information remains unknown/non-evaluable rather than negative evidence	PASS
P0_M2	Positive documentary control for M2 · Feixe dos Morros	Checks that the documented M2 case reproduces the expected N1 behavior	PASS
P0_M4	Positive documentary control for M4 · Serra da Bodoquena	Checks that the documented M4 case reproduces the expected N2 behavior	PASS
P0_M9	Positive documentary control for M9 · Granito Scardine	Checks that the documented M9 case reproduces the expected N3 behavior	PASS

Table 9. Executable adversarial tests and the failures each check is designed to prevent.

This result demonstrates implementation conformity with the tested specification. It does not demonstrate that the specification represents all possible geological processes.

The logic and executable code for the adversarial tests form part of the reproducible package associated with the manuscript. Table 9 serves as a human-readable index of the checks, while the scripts and execution records allow the PASS results to be audited and the tests to be adapted to other implementations without relying solely on the textual description.

7.3 Documentary validation

Feixe dos Morros: M2 → N1

Bodoquena: M4 → N2

Scardine: M9 → N3

The three cases function as behavioral tests rather than as external predictive validation.

7.4 Spatial and computational validation

The M2 and M4 evidence was independently intersected with the three grids. Reproduction regenerated the same sets of hex_id values and preserved the results after technical correction of invalid geometries.

This demonstrates operational stability of the materialized pilot. Reproduction is not presented as proof that the scientific specification is unique or definitive. It demonstrates that, given the same frozen sources, the same RULE_VERSION, and the same spatialization rules, the historical result can be reconstructed.

7.5 What was and was not validated

Validation in this version has three distinct scopes. Logical validation verifies whether the implementation respects the tested invariants and rules. Documentary validation verifies whether known cases exercise the expected progression of M2, M4, and M9. Spatial and computational validation verifies whether the same snapshot reproduces the same hex_id values across the three grids. None of these stages measures deposit-discovery capability or prospectivity performance.

No independent field tests, ROC curve, AUC, sensitivity, specificity, exploration success rate, or quantitative benchmark against fuzzy logic, weights of evidence, or machine learning were performed. PAG ETR is therefore not presented in this version as superior or equivalent to these methods in predictive performance.

Type	Performed	Evidence	Scope
Logical	Yes	Rules and invariants	Internal consistency
Documentary	Yes	Three controlled cases	Engine behavior
Spatial	Yes	M2 and M4 at three scales	Spatial consistency
Computational	Yes	Re-execution and hex_id	Reproducibility
Geometric	Yes	Grid correction	Operational stability
Independent predictive	No	Not yet available	Future step
Field	No	Not yet available	Future step

Table 10. Summary of what was validated and the scope of each validation type in this version.

VALIDACAO_COMPUTACIONAL ≠ VALIDACAO_PREDITIVA

VALIDACAO_DOCUMENTAL ≠ VALIDACAO_DE_CAMPO

7.6 Threats to validity and interpretation of the tests

The executable tests demonstrate conformity between the implementation and the tested rules, but they share the specification they are intended to verify. They are therefore not independent evidence that the geological criteria are correct. The three documentary controls cover only M2, M4, and M9 and were selected to exercise distinct levels of progression. They are not a random sample of REE systems and do not allow classification error to be estimated. Reproduction of the same hex_id values demonstrates computational determinism of the snapshot, not prospectivity validity.

Three threats remain central. The first is specification error, if a necessary requirement has been defined inadequately. The second is availability bias, because better-studied areas may progress more readily. The third is spatial-transfer error, if a local observation is generalized beyond the support it permits. The design reduces these threats through unknown states, provenance separation, original geometry, and conservative rules, but does not eliminate them.

7.7 Future validation protocol

External validation should be prospective and independent of the dataset used to formulate or materialize the rules. The minimum proposed design includes prior selection and freezing of test units, protocol-driven geochemical sampling, analytical determination of REE and REY with documented detection limits, mineralogical control of host phases, and recording of the position, support, and uncertainty of each observation. Samples used to evaluate performance should not be used to retrospectively redefine the criteria of the same model version.

When sufficient classes exist for comparison, performance should be evaluated across states and models using predefined metrics, including the ability to discriminate areas with compatible local evidence, false-positive rate, false-negative rate, and stability across scales. If the number of confirmed occurrences allows, out-of-sample or block-based spatial validation should be performed, preventing spatially proximate observations from appearing simultaneously in fitting and testing.

A future comparison with fuzzy logic, weights of evidence, or another prospectivity method should use the same domain, the same input sources, and an independent set of reference observations. The objective will not be to presume superiority of PAG ETR, but to test behavioral differences, particularly preservation of UNKNOWN, non-compensation of requirements, and transparency of the decision trail. Success criteria for PAG ETR should be defined before validation and should include not only performance, but also the absence of inappropriate promotions when necessary requirements are unknown.

Two additional designs are particularly useful. The first is a synthetic bank of logical tests in which the truth state of each requirement is controlled in advance. These tests would not estimate discovery capability, but would allow formal verification that the absence of a REQ blocks progression even when multiple SUP are present, and that UNKNOWN remains distinct from FALSE.

The second is external validation using Brazilian cases outside Mato Grosso do Sul, provided that the evidence used to test the framework is frozen before classification and kept separate from the information used to formulate the rules. Known systems such as Araxá or Catalão may be candidates for external controls, but they should not be used circularly to adjust criteria and then demonstrate that those same criteria recover the deposit.

These designs are proposed as future work. This version does not present synthetic or external results that have not actually been executed.

Illustrative logical example of the non-compensatory property

Controlled scenario: REQ1 = TRUE; REQ2 = UNKNOWN; SUP1 = TRUE; SUP2 = TRUE.

PAG ETR: progression remains blocked because a necessary requirement is still UNKNOWN.

Hypothetical compensatory scheme: summing or weighting SUP1 and SUP2 may raise an aggregate score despite the lack of confirmation of REQ2.

This example is only a conceptual demonstration of the logical property. It does not constitute an executed synthetic validation, does not produce a performance metric, and does not replace the future experimental protocol described in this section.

8. Discussion

8.1 PAG ETR in the context of mineral prospectivity mapping

PAG ETR belongs to a field in which knowledge-driven strategies, weighted overlay, fuzzy logic, Bayesian approaches, weights of evidence, and machine learning already exist. Integration of geological knowledge, treatment of uncertainty, explicit rules, and regional prioritization are not, individually, exclusive features of the framework. The contribution should therefore be framed as an operational combination of properties rather than as a claim to have invented each component. Methodological precedents include Bonham-Carter (1994), Carranza (2008), Wyborn et al. (1994), Carranza and Hale (2003), Carranza et al. (2008), Carranza and Laborte (2015a, 2015b), Yousefi and Carranza (2016, 2017), Zuo (2017, 2020), Porwal and Carranza (2015), and Farahnakian et al. (2024). For REE-specific applications in regional assessment and machine learning, see Ford et al. (2023a), Ford et al. (2026), Bishop and Robbins (2024), and Gomes et al. (2026). Formal precedents also exist for evaluating the performance of mineral-potential maps, including Agterberg and Bonham-Carter (2005).

8.2 A question distinct from continuous prospectivity

PAG ETR does not seek to answer directly which area is more prospective. Its operational question comes first.

Does the available evidence allow this hypothesis to advance?

This difference changes the nature of the output. Rather than a score or probability, the framework preserves states of hypothesis maturity.

8.3 Non-compensation

The use of conjunctive operators has precedents. The operational distinction of PAG ETR is that it maintains the restriction throughout the decision chain. A necessary condition is not subsequently converted into a compensable component of a score. Boolean operators and knowledge-based integration have precedents in Bonham-Carter (1994) and Yousefi and Carranza (2016). PAG ETR retains these operators as decision rules rather than as score components.

$$\mathrm{C} + \mathrm{C} + \mathrm{C} \neq \mathrm{F}$$

$$\text{CONF from another requirement} \neq \text{missing REQ}$$

A cell may accumulate favorable indications and remain N1 if a necessary condition for N2 is still unknown.

8.4 The meaning of N0

Approximately 94% to 97% of cells remained N0 across the three scales. N0 does not mean low potential. It indicates that the model considered lacks sufficient information for progression or that the initial gate cannot be evaluated. Given the quantified geoscientific coverage, a large extent of N0 is consistent with the observed documentary heterogeneity.

8.5 When a blank map can be a result

Cartographic systems create visual pressure to fill the entire territory. When the question is whether a geological condition has been demonstrated, filling cells without sufficient observation can obscure the difference between absence of observation and observation of absence. N0 cells identify where the hypothesis cannot be evaluated under the available rules and data.

$$\text{absence of evidence} \neq \text{evidence of absence}$$

8.6 Heterogeneity of knowledge as a scientific variable

Better-studied regions are more likely to satisfy criteria that depend on specific observations. Less-studied regions may remain N0 even when their geological context could justify investigation. This produces knowledge bias. PAG ETR reduces the negative interpretation of this bias, but does not eliminate dependence on data availability. This availability and coverage problem is consistent with the methodological concern of Creason et al. (2023) and with Australian products that separate potential from input uncertainty, such as Ford et al. (2026) and Geoscience Australia (2025).

8.7 M2 and M4 as the spatial pilot

Only M2 and M4 were spatially materialized in reproducible form in this version. This limitation reduces the coverage of the pilot but demonstrates the conservative principle. The M1 to M11 library was not used to fill the map through indirect inference.

$$\text{satisfied scientific rule} + \text{reproducible geometry} + \text{verifiable spatial relationship}$$

8.8 M4 and the risk of conflating proxy with mineralization

$$\text{phosphate} \neq \text{REE enrichment}$$

Bodoquena reached N2 documentarily because the sedimentary phosphate context justified targeted investigation. The absence of specific REE or REY evidence associated with the phosphate phase prevented N3.

8.9 M2 and alkaline specialization

$$\text{alkaline intrusion} \neq \text{REE system}$$

Feixe dos Morros remained N1 because the alkaline context opens M2, but specialization and compatible local evidence were not demonstrated in the dataset used.

8.10 Scale effect

The absolute number of N1 cells decreased from 49 to 37 and 24, while the proportion within each grid increased from 3.15% to 4.67% and 5.83% at the 250, 500, and 1,000 km² scales, respectively. This behavior does not represent increased favorability. It results from the change in spatial support and makes the influence of the modifiable areal unit problem (MAUP) visible. The three grids are evaluated independently and do not constitute hierarchical replications. The effect is treated as sensitivity to spatial support, consistent with Zuo's (2012) discussion of cell size in geochemical mapping.

The 250 km² grid is the most conservative representation for relative location because it reduces the territorial area associated with each intersection, but it is not declared universally superior. The operational choice should depend on source scale and precision, evidence support, and the investigation question. The tendency for the proportion of N1 cells to increase in larger cells reinforces that results across scales should not be compared as though they represented equivalent probabilities.

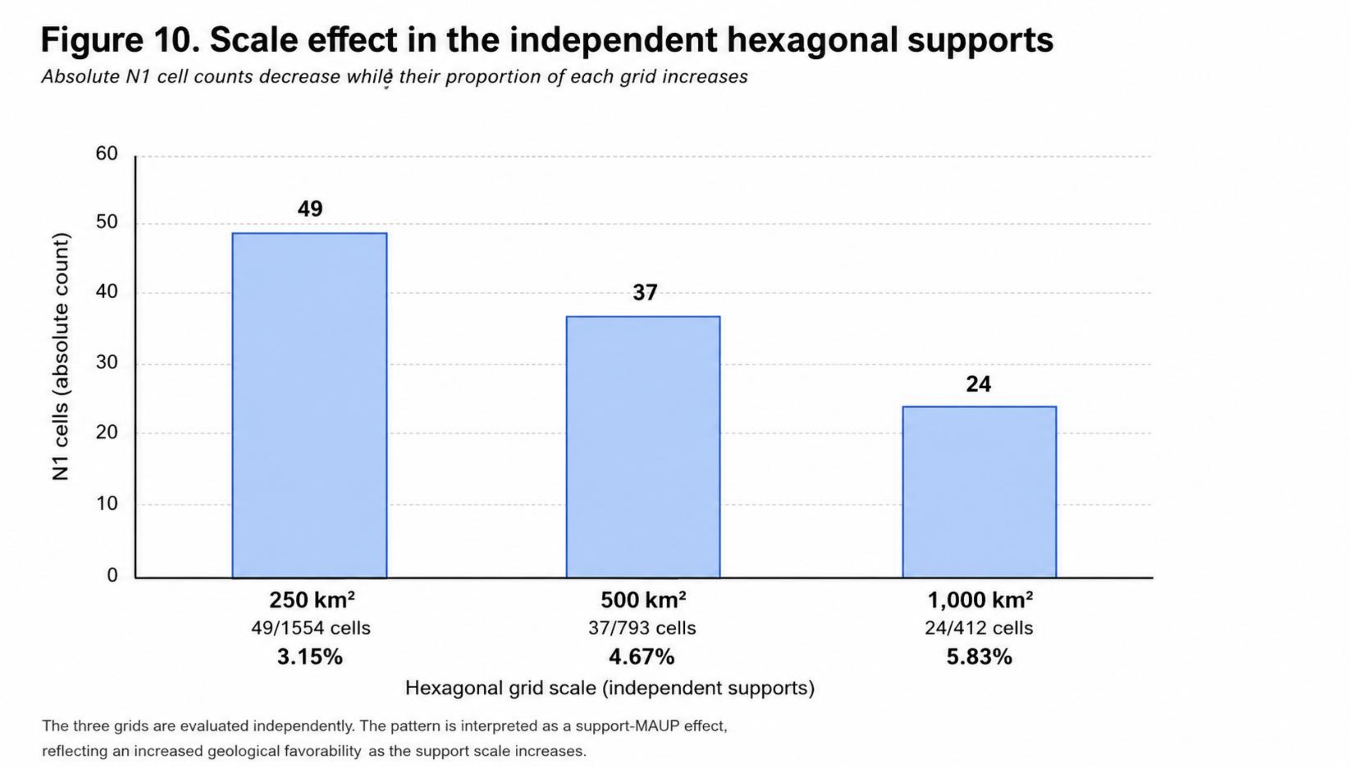

Figure 10. Scale effect in the independent hexagonal supports. Absolute N1 cell counts decrease with increasing support size, while their proportion within each grid increases. The pattern is interpreted as a support/MAUP effect and not as increased geological favorability.

As operational guidance for this proof of concept, the 250 km² grid is preferable when the objective is to plan investigation and sampling with greater spatial discrimination, provided that source resolution is compatible. The 500 and 1,000 km² grids are useful for regional interpretation and comparison of broader patterns, but the greater proportion of N1 cells at these scales should be interpreted in light of MAUP rather than as an increase in geological probability.

The N0-N4 classes are not probabilities, confidence intervals, or quantitative measures of uncertainty. Non-compensatory logic controls progression between states and preserves UNKNOWN, but does not eliminate uncertainties inherited from the sources, including cartographic scale, positional accuracy, completeness, analytical resolution, geometric generalization, and survey quality. These uncertainties must accompany interpretation of the maps and are not summarized by a single metric in this version.

8.11 Sensitivity of the operational M4 opening rule

The counterfactual analysis separates the spatial robustness of the result from its dependence on a specific operational decision. Excluding the points reduces M4_N1 by 17.39% in the 250 km² grid, 17.65% in the 500 km² grid, and 22.73% in the 1,000 km² grid. The greater relative sensitivity in the coarsest grid is consistent with the change in spatial support, because a small number of point observations can affect a larger fraction of open cells when the total number of cells is smaller.

The result should be interpreted with two cautions. First, the restricted scenario does not demonstrate that the removed cells are geologically irrelevant. It shows only that their opening depended on point phosphorus records rather than on intersection with the materialized polygonal phosphorite units. Second, the analysis does not choose between the two rules. It documents the quantitative consequence of a choice that may be revised in a future RULE_VERSION. This separation is central to preventing a sensitivity analysis from becoming retrospective recalibration.

8.12 Conceptual comparison with fuzzy logic, weighted overlay, and machine learning

Fuzzy logic is suitable for representing gradual transitions and degrees of membership. Weighted overlay allows multiple predictors to be integrated into a common surface. Data-driven and machine-learning methods can detect complex relationships when sufficient occurrences and predictor variables are available. PAG ETR adopts a different priority. When a decision depends on a necessary condition, it preserves discrete states and halts progression if the required evidence remains unknown. This choice sacrifices numerical continuity in favor of logical traceability. This version does not perform a predictive benchmark against weights of evidence, fuzzy logic, or machine learning. The omission is deliberate because the objective tested here is logical consistency and auditability under incomplete data, not maximization of predictive performance. A quantitative comparison would require an independent set of positive and negative occurrences and a common validation design across methods, conditions that are not equivalently available in the current snapshot. Any performance comparison is therefore explicitly outside the scope of this version. Examples and reviews of these method families include Yousefi and Carranza (2017), Carranza and Laborte (2015a, 2015b), Zuo (2017, 2020), Bishop and Robbins (2024), and Farahnakian et al. (2024).

8.13 Information preservation in a multimodel architecture

The fundamental decision unit is defined by the relationship among model, spatial unit, and scale. The same hexagon can have different states for different mineralization systems without contradiction.

$$\vec{A}(h,s) = [A(M1,h,s), A(M2,h,s), ..., A(M11,h,s)]$$

An N3 in a given model does not convert N0 or N1 in other models into favorable evidence. Likewise, multiple N1 states are not summed to artificially produce N2 or N3. The cartographic synthesis is a secondary visual product and does not replace the multimodel vector.

8.14 Reproducibility as part of the method

source → snapshot → evidence → rule → model → cell → state

Reproduction of the same hex_id values at three scales demonstrates that the pilot does not depend solely on the cartographic interface. Reproducibility, however, does not replace external geological validation.

8.15 False positives, false negatives, and the cost of conservatism

The non-compensatory logic was designed primarily to reduce interpretive false positives, such as promoting a generic alkaline context, phosphorite without REE/REY, isolated magnetite, or laterite without a fertile protolith. This conservatism has a cost. A real mineral system may remain N0 or N1 when diagnostic evidence exists in the field but is not documented in the snapshot. PAG ETR deliberately accepts this risk of a documentary false negative because its question is what can be supported by the available evidence, not what may exist despite the lack of such evidence.

A future quantitative comparison with fuzzy logic, weights of evidence, weighted overlay, or machine learning should use the same spatial domain, the same input evidence, independent occurrences for training and testing where applicable, predefined metrics, and explicit treatment of sampling bias. Without these conditions, comparing area percentages or cell counts would produce a methodologically uneven benchmark.

8.16 Originality and scope

It is not defensible to claim that PAG ETR individually introduces Boolean logic, mineral-systems integration, uncertainty treatment, multiscale analysis, or regional REE prioritization. Its potentially differentiating contribution lies in the explicit combination of an M1 to M11 multisystem library, states that preserve unknown information, non-compensatory necessary requirements, N0 to N4 progression, documentary-spatial separation, independently recalculated supports, and a reproducible computational chain. The absolute absence of integral equivalents should not be claimed without a broader systematic comparative review. The literature shows clear precedents for mineral systems, knowledge-based modelling, and multivariate integration, including Wyborn et al. (1994), Skirrow et al. (2019), Ford et al. (2023a), and Ford et al. (2026).

8.17 Implications for Mato Grosso do Sul

The most important result may not be the identification of 49 open cells at 250 km², but the demonstration that much of the territory cannot be evaluated at the same level of detail using the materialized dataset. PAG ETR also functions as a diagnosis of the information maturity of geological hypotheses.

8.18 Principal interpretive boundary

investigation priority ≠ deposit probability

N3 ≠ mineral resource

N1 ≠ drilling target

N0 ≠ low potential

8.19 A versionable, adaptable, and deliberately non-final model

PAG ETR is proposed as a versionable scientific specification rather than as an immutable taxonomy of REE systems. New studies may subdivide models, alter criteria, modify the role of a proxy, or demonstrate that a rule is overly permissive or restrictive. Such improvement is desirable, but it must generate a new MODEL_LIBRARY_VERSION or RULE_VERSION and a new execution. The reproducible value of the model depends precisely on not erasing the trajectory of previous versions.

Transfer to another territory should likewise not consist of copying the maps of Mato Grosso do Sul. The transferable component is the architecture of provenance, states, evidence roles, non-compensatory rules, coverage assessment, versioning, and explainability. The M1 to M11 library may be retained, expanded, or reduced according to local geology and literature, provided that every change is explicitly documented.

Thus, the maps in this study demonstrate a first implementation rather than a universal calibration of PAG ETR.

8.20 Operational implications for investigation

PAG ETR states should guide information acquisition and testing, not automatic investment decisions. N0 indicates that the hypothesis cannot be evaluated with the available information and prioritizes acquisition of the data that block evaluation. N1 indicates an open geological hypothesis and justifies cartographic refinement, field verification, or acquisition of the missing requirement, but not a targeted geochemical campaign by itself. N2 is the level at which the requirements defined by the model justify targeted geochemical investigation. N3 records compatible local evidence and requires independent mineralogical, geochemical, and spatial verification before any further inference. N4 records a documented local contradiction of the hypothesis under the conditions evaluated.

These states are not equivalent to economic categories, resources, reserves, discovery probability, or universal investment stages. The specific action depends on the model, the missing requirement, scale, acquisition cost, and environmental, territorial, legal, and geoethical constraints. In the current pilot, the absence of spatialized N2 and N3 means that the product is primarily a diagnosis of evaluability and open hypotheses rather than a drilling-target map.

In a territory dominated by N0, the most informative operational output is not to rank cells by a favorability that the system did not calculate. It is to produce an agenda for unlocking hypotheses. For each model × hexagon × scale combination, the system can record the first necessary requirement that remains UNKNOWN or inadequately tested and associate it with the type of acquisition capable of reducing that uncertainty.

This agenda allows at least three classes of action to be distinguished. First, coverage gaps, where the required source does not exist or does not cover the unit. Second, resolution or support gaps, where the data exist but are inadequate for the question. Third, diagnostic gaps, where the geological context is documented but the measurement that could promote or contradict the hypothesis is missing, such as REE/REY in the phosphate phase for M4.

A formal acquisition cost-benefit analysis is not performed in this version because comparable field, laboratory, and logistics costs are not part of the snapshot. The framework nevertheless provides the structure needed for a future version to rank acquisitions by the number of hypotheses potentially unlocked, cost, evidence independence, and territorial constraints.

9. Limitations

9.1 Heterogeneity of geoscientific coverage

The spatial distribution and resolution of the available information are heterogeneous. Different regions do not have equivalent densities of cartography, geochemistry, mineralogy, petrography, geochronology, geophysics, and mineral occurrences.

9.2 Unknown state does not represent negative evidence

Nonexistent, insufficient, or not-yet-spatialized information is not converted into zero and does not function as evidence against the model. This may produce visually incomplete maps, but it avoids false precision.

9.3 Incomplete spatial materialization of M1 to M11

The conceptual library contains eleven models, but the reproducible pilot spatializes only M2 and M4. M9 has the most advanced documentary case, Granito Scardine, which reaches N3 in the documentary control based on its cartographic delineation and the geochemical information described in the source used. However, the detailed geometry needed to transfer this documentary result to the spatial snapshot was not materialized in the package that generated the maps. M9 was therefore not projected onto the grid. An arbitrary buffer was not subsequently introduced around the granite because doing so would add a spatial rule that did not exist in the original implementation and would retrospectively alter the frozen product. A future version may incorporate the detailed unit geometry and explicitly execute M9 with a new DATA_VERSION and, where necessary, a new RULE_VERSION.

The decision not to use a 2-5 km buffer around Granito Scardine is deliberate. Such a radius would create an influence zone not documented by the source and would add to the result an inferred geometry whose distance has not been calibrated for M9. This procedure would be admissible only as a prospectively declared new spatial rule subjected to sensitivity testing, never as a retrospective repair of the snapshot.

9.4 Contextual evidence does not equal REE mineralization

Alkaline context, phosphorite, hydrothermalism, or other proxies do not independently demonstrate REE enrichment. This caution is consistent with the literature on REE deposits and processes, including Verplanck et al. (2014), Emsbo et al. (2015), Lottermoser (1992), Watts and Mercer (2020), and McLeod and Shaulis (2018).

9.5 Dependence on spatial support

The three grids are independent, but spatial-unit size influences intersection with evidence and cartographic expression. Dependence of the result on spatial support is discussed by Zuo (2012).

9.6 Spatial generalization of point evidence

The presence of a point in a hexagon does not imply uniformity of the observed property throughout the cell.

9.7 Model-dependent sufficiency criteria

K does not use a universal percentage threshold. Minimum sufficiency requirements must be defined and documented by model, preserving the distinction between insufficient information and negative geological evidence.

9.8 Absence of independent predictive validation

The documentary controls and computational reproduction do not allow sensitivity, specificity, AUC, or exploration success rate to be estimated. The absence of N2 and N3 in the spatial pilot precludes any claim that N1 cells have a higher probability of hosting REE. This version should be read as a methodological demonstration and a diagnosis of evaluability. The external validation protocol defined in Section 7.7 is future work and is not replaced by the internal tests.

9.9 Deliberate limitation of non-compensatory logic

The architecture can be conservative. If a necessary condition is poorly defined, the system may block progression excessively. The rules must remain open to scientific revision.

9.10 Dependence on the first spatial operationalization

The maps presented preserve operational decisions made in the first implementation of PAG ETR in ITA ARANDU MS. This includes the use of point phosphorus occurrences as a signal for spatial opening of M4, limited to N1. The article does not retrospectively modify this choice. In a future version, a more restrictive spatial rule or more detailed qualification of these points may be tested, but the resulting outputs must be published as a new version and explicitly compared with the present one.

The sensitivity analysis quantifies this dependence without altering the historical product. Removing the points would reduce M4_N1 from 46 to 38 cells at 250 km², from 34 to 28 at 500 km², and from 22 to 17 at 1,000 km². These values are presented as a counterfactual scenario and not as a replacement for the frozen results.

9.11 PAG ETR is not a mineral-resource assessment

No class should be interpreted as a deposit, resource, reserve, grade, tonnage, economic viability, or automatic field authorization.

9.12 Dependence on the literature and evidence independence

Interpretation of models and cases depends on published and institutional documentation. Multiple citations of the same observation are not treated as independent confirmations. Even so, the historical provenance of some data may be incomplete, which limits assessment of independence.

9.13 Selection bias in documentary controls

The Feixe dos Morros, Serra da Bodoquena, and Granito Scardine cases were selected as controls of logical behavior rather than as a representative sample of all geological environments in the state. Consequently, performance in these three cases cannot be generalized to M1 through M11.

9.14 Absence of probabilistic calibration

N0 to N4 are logical states, not ordered probabilities of deposit occurrence. In this version, there is no calibration that would allow N1, N2, or N3 to be converted into probability, odds, economic value, or expected discovery rate.

9.15 Territorial transferability not yet demonstrated

The architecture is formulated in general terms, but its application has been tested in Mato Grosso do Sul. Transferability to other territories requires reconstruction of local sources, spatial rules, coverage, and control cases, without assuming that thresholds or spatial relationships can be reused automatically.

10. Conclusions

This study presented PAG ETR as a knowledge-driven geoscientific framework for diagnosing evaluability and for organizing and explicitly testing hypotheses related to rare earth elements, accompanied by a deliberately partial spatial proof of concept in Mato Grosso do Sul. The territorial application demonstrates how the framework behaves under heterogeneous coverage, scale, and resolution, but does not constitute regional validation of prospectivity performance.

The principal contribution of PAG ETR lies in separating unknown information from verified absence and in maintaining non-compensatory necessary requirements. Favorable geological context, a compatible mineralizing process, and local REE evidence remain distinct levels of inference.

Across the three independent grids, 49 of 1,554 cells at 250 km², 37 of 793 cells at 500 km², and 24 of 412 cells at 1,000 km² reached N1. No cell automatically reached N2 or N3 in the audited snapshot. These results show the limits of the spatialized evidence and do not demonstrate low potential in the remaining areas.

Decomposition by model shows that M4 accounts for 46, 34, and 22 N1 cells at the three scales, and M2 for three, three, and two. The remaining models are non-evaluable in the spatial snapshot used.

The final product is not a prediction of deposits. It is a traceable territorial structure of hypotheses, evidence, contradictions, and gaps linked to versions of data, decision rules, and frozen results. The maps in this application should be read as the record of a first operationalization of the framework, with explicit and improvable choices, rather than as the final or universal form of PAG ETR spatialization.

In summary, PAG ETR does not seek in this version to answer where REE deposits will occur. It asks whether the available evidence is sufficient to open, test, support, or contradict an explicit mineral hypothesis and which information prevents its progression. The first implementation in Mato Grosso do Sul produced predominantly N0 and N1, which should be interpreted as a diagnosis of evaluability and knowledge gaps. The scientific value of the framework lies in making these limitations visible, reproducible, and versionable. Independent predictive validation remains necessary before any claim about exploration performance.

11. Interactive cartographic implementation and public documentation

The public, navigable PAG ETR cartographic implementation is available in Busón Buesa and Gabas (2026) at https://cbuson.github.io/pag-etr-ms/. The interface allows users to explore the 250, 500, and 1,000 km² analytical grids, evidence layers, framework states, data provenance, and the associated methodological documentation.

11.1 Interface as an auditable extension of the reproducible package

The reproducible distribution includes a cartographic interface in docs for inspecting the same spatial objects preserved in the package. The interface does not recalculate favorability or replace the scientific pipeline. Its function is to enable navigation, attribute inspection, comparison across scales, and direct access to the methodology, audit, figures, references, and reproduction materials.

11.2 Pilot layers and territorial context

The visualization explicitly distinguishes the three spatialized lines of pilot evidence, M2 Corpo Feixe dos Morros, M4 phosphorite units, and point phosphorus records, from contextual layers. The state boundary of Mato Grosso do Sul and the boundaries of its 79 municipalities are shown as territorial references. These boundaries derive from the local ITA ARANDU MS snapshot identified as IBGE Malhas territoriais de 2025 (Instituto Brasileiro de Geografia e Estatística, 2025). The received metadata record only partial provenance for these files because the capture date of the local snapshot is not documented. For this reason, in this version they are treated as navigation context rather than as new analytical evidence for the pilot.

11.3 Base grids and PAG ETR results

The interface separates the approximately 250, 500, and 1,000 km² base territorial grids from the PAG ETR results calculated on those geometries. This separation makes it possible to verify visually that the spatial unit precedes classification and avoids conflating support geometry with the logical state assigned by the model. By default, the state boundary, municipal boundaries, and the 250 km² PAG ETR result remain visible. The base grids can be activated independently for visual auditing.

11.4 Methodology, audit, authorship, and rights

The complete methodology is made available on a single navigable page and preserves the M1 to M11 library, evidence rules, the fifteen development phases, non-compensatory logic, spatial relationships, multiscale architecture, validation, and limitations. The interface also includes a dedicated authorship, rights, and provenance record. Reuse terms are defined exclusively by the license files preserved in the repository and notices concerning third-party materials. The interface does not alter the rights or licenses of the original sources.

11.5 Relationship among article, code, and visualization

The article describes the method and results, the code executes the reproducible implementation, and the interface enables inspection of the published objects. These three components are complementary but not equivalent. A visual change does not modify the scientific result, and any change capable of altering classifications requires a new version of the data, rules, grids, or model library under the versioning scheme defined in this study.

12. Data and code availability

The reproducible compendium associated with PAG ETR brings together the redistributable frozen sources, derived evidence, the three grids, frozen and reproduced results, implementation Python scripts, methodological tests, SHA-256 manifests, metadata, documentation, and figures. The package also includes the recovered canonical specification, with the M1 to M11 rules, the five logical states, provenance entities, pseudocode, validator invariants, and synthetic-test plan. The recovered structured files are not presented as a byte-for-byte copy of source code that did not exist in the original PDF, but as an auditable transcription of the rules and pseudocode readable in the source.

The public PAG ETR interface is available at https://cbuson.github.io/pag-etr-ms/, and the reproducible PAG ETR snapshot is preserved in Zenodo under DOI https://doi.org/10.5281/zenodo.22027882. The persistent record for ITA ARANDU MS, the geoscientific infrastructure from which PAG ETR was derived, is https://doi.org/10.5281/zenodo.21923101.

13. Declarations

Authorship and CRediT contributions

Carlos Busón Buesa — Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization.

Sandra Garcia Gabas — Validation, Investigation, Writing – review & editing. Her contribution included scientific reading and verification of the geological formations and units used in model interpretation.

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Conflicts of interest

The authors declare no conflicts of interest.

Data and code

Public PAG ETR interface: https://cbuson.github.io/pag-etr-ms/. Reproducible PAG ETR snapshot: https://doi.org/10.5281/zenodo.22027882. Source infrastructure ITA ARANDU MS: https://doi.org/10.5281/zenodo.21923101.

14. References

Agterberg, F. P., & Bonham-Carter, G. F. (2005). Measuring the performance of mineral-potential maps. Natural Resources Research, 14(1), 1–17. https://doi.org/10.1007/s11053-005-4674-0

Aranha, M., Porwal, A., Sundaralingam, M., González-Álvarez, I., Markan, A., & Rao, K. (2022). Rare earth elements associated with carbonatite–alkaline complexes in western Rajasthan, India: Exploration targeting at regional scale. Solid Earth, 13, 497–518. https://doi.org/10.5194/se-13-497-2022

Bellora, J. D., Burger, M. H., Van Gosen, B. S., Long, K. R., Carroll, T. R., Schmeda, G., & Giles, S. A. (2019). Rare Earth Element Occurrences in the United States (Version 4.0) [Data set]. U.S. Geological Survey. https://doi.org/10.5066/F7FN15D1

Bishop, B. A., & Robbins, L. J. (2024). Using machine learning to identify indicators of rare earth element enrichment in sedimentary strata with applications for metal prospectivity. Journal of Geochemical Exploration, 258, 107388. https://doi.org/10.1016/j.gexplo.2024.107388

Bonham-Carter, G. F. (1994). Geographic Information Systems for Geoscientists: Modelling with GIS. Pergamon.

Borst, A. M., Smith, M. P., Finch, A. A., Estrade, G., Villanova-de-Benavent, C., Nason, P., Marquis, E., Horsburgh, N. J., Goodenough, K. M., Xu, C., Kynický, J., & Geraki, K. (2020). Adsorption of rare earth elements in regolith-hosted clay deposits. Nature Communications, 11, 4386. https://doi.org/10.1038/s41467-020-17801-5

Busón Buesa, C., & Gabas, S. G. (2026). PAG ETR: framework reprodutível de analogia geológica para priorização de investigação geoquímica de elementos terras raras (Versão v1.0.0) [Software]. Zenodo. https://doi.org/10.5281/zenodo.22027882https://doi.org/10.5281/zenodo.22027882

Busón Buesa, C., & Gabas, S. G. (2026). ITA ARANDU MS: Atlas Geocientífico de Mato Grosso do Sul (Versão 2.7) [Software]. Zenodo. https://doi.org/10.5281/zenodo.21923101https://doi.org/10.5281/zenodo.21923101

Carranza, E. J. M. (2008). Geochemical Anomaly and Mineral Prospectivity Mapping in GIS. Elsevier.

Carranza, E. J. M., & Hale, M. (2003). Evidential belief functions for data-driven geologically constrained mapping of gold potential, Baguio district, Philippines. Ore Geology Reviews, 22(1–2), 117–132. https://doi.org/10.1016/S0169-1368(02)00111-7

Carranza, E. J. M., & Laborte, A. G. (2015a). Data-driven predictive mapping of gold prospectivity, Baguio district, Philippines: Application of Random Forests algorithm. Ore Geology Reviews, 71, 777–787. https://doi.org/10.1016/j.oregeorev.2014.08.010

Carranza, E. J. M., & Laborte, A. G. (2015b). Random forest predictive modeling of mineral prospectivity with small number of prospects and data with missing values in Abra, Philippines. Computers & Geosciences, 74, 60–70. https://doi.org/10.1016/j.cageo.2014.10.004

Carranza, E. J. M., Van Ruitenbeek, F. J. A., Hecker, C., Van der Meijde, M., & Van der Meer, F. D. (2008). Knowledge-guided data-driven evidential belief modeling of mineral prospectivity in Cabo de Gata, SE Spain. International Journal of Applied Earth Observation and Geoinformation, 10(3), 374–387. https://doi.org/10.1016/j.jag.2008.02.008

Černý, P. (1992). Geochemical and petrogenetic features of mineralization in rare-element granitic pegmatites in the light of current research. Applied Geochemistry, 7(5), 393–416. https://doi.org/10.1016/0883-2927(92)90002-K

Creason, C. G., Justman, D., Rose, K., Montross, S., Bean, A., Mark-Moser, M., Wingo, P., Sabbatino, M., & Thomas, R. B. (2023). A Geo-Data Science Method for Assessing Unconventional Rare-Earth Element Resources in Sedimentary Systems. Natural Resources Research, 32, 855–878. https://doi.org/10.1007/s11053-023-10163-x

Emsbo, P., McLaughlin, P. I., Breit, G. N., du Bray, E. A., & Koenig, A. E. (2015). Rare earth elements in sedimentary phosphate deposits: Solution to the global REE crisis? Gondwana Research, 27(2), 776–785. https://doi.org/10.1016/j.gr.2014.10.008

Farahnakian, F., Sheikh, J., Zelioli, L., Nidhi, D., Seppä, I., Ilo, R., Nevalainen, P., & Heikkonen, J. (2024). Addressing imbalanced data for machine learning based mineral prospectivity mapping. Ore Geology Reviews, 174, 106270. https://doi.org/10.1016/j.oregeorev.2024.106270

Foley, N. K., & Ayuso, R. A. (2024). Conventional rare earth element mineral deposits: The global landscape. In Y. V. Murty, M. A. Alvin, & J. P. Lifton (Eds.), Rare earth metals and minerals industries: Status and prospects (pp. 17–56). Springer. https://doi.org/10.1007/978-3-031-31867-2_2

Ford, A., Huston, D., Cloutier, J., Doublier, M., Schofield, A., Cheng, Y., & Beyer, E. (2023a). A national-scale mineral potential assessment for carbonatite-related rare earth element mineral systems in Australia. Ore Geology Reviews, 161, 105658. https://doi.org/10.1016/j.oregeorev.2023.105658

Ford, A., Huston, D., Cloutier, J., Doublier, M., Schofield, A., Cheng, Y., & Beyer, E. (2023b). Carbonatite-related rare earth element mineral potential maps [Data set]. Geoscience Australia. https://doi.org/10.26186/147865

Ford, A., Walsh, J., Doublier, M., Burnham, A., Cloutier, J., Fraser, G., Magee, C., & Czarnota, K. (2026). Unconformity-Related Rare Earth Element Mineral Potential of Australia. Natural Resources Research. https://doi.org/10.1007/s11053-026-10739-3

Geoscience Australia. (2025, August 18). National resource assessments. https://www.ga.gov.au/scientific-topics/resourcing-australias-prosperity/national-resource-assessments

Godoi, H. O. (Org.). (2001). Aquidauana: folha SF.21-X-A: Estado do Mato Grosso do Sul. CPRM.

Godoi, H. O., Martins, E. G., Mello, J. C. R., & Scislewski, G. (Orgs.). (2001). Corumbá: folha SE.21-Y-D, Aldeia Tomázia: folha SF.21-V-B, Porto Murtinho: folha SF.21-V-D, estado de Mato Grosso do Sul. CPRM. Escala 1:250.000.

Gomes, M. M., Castro, R. M., & Vieira, G. S. (2026). Machine Learning for Recognition and Mapping of Rare Earths in Brazil using Reflectance Spectroscopy and Hyperspectral Imagery. The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences, XLIX-B3-2026, 371–378. https://doi.org/10.5194/isprs-archives-XLIX-B3-2026-371-2026

Instituto Brasileiro de Geografia e Estatística. (2025). Malhas territoriais de 2025 [Conjunto de dados geoespaciais]. IBGE. https://www.ibge.gov.br/geociencias/organizacao-do-territorio/malhas-territoriais.html

Justo, L. J. E. C. (2000). Fosfato da Serra da Bodoquena, Mato Grosso do Sul. CPRM. Informe de Recursos Minerais, Série Insumos Minerais para Agricultura, 2.

Lacerda Filho, J. V. (2015). Bloco Rio Apa: origem e evolução tectônica [Tese de doutorado, Universidade de Brasília, Instituto de Geociências].

Lacerda Filho, J. V., Brito, R. S. C., Silva, M. G., Oliveira, C. C., Moreton, L. C., Martins, E. G., Lopes, R. C., Lima, T. M., Larizzatt, J. H., & Valente, C. R. (2006). Geologia e recursos minerais do estado de Mato Grosso do Sul: Texto explicativo dos mapas geológico e de recursos minerais do estado de Mato Grosso do Sul. CPRM. Escala 1:1.000.000.

Li, T., Zuo, R., Zhao, X., & Zhao, K. (2022). Mapping prospectivity for regolith-hosted REE deposits via convolutional neural network with generative adversarial network augmented data. Ore Geology Reviews, 142, 104693. https://doi.org/10.1016/j.oregeorev.2022.104693

Lottermoser, B. G. (1992). Rare earth elements and hydrothermal ore formation processes. Ore Geology Reviews, 7(1), 25–41. https://doi.org/10.1016/0169-1368(92)90017-F

McLeod, C. L., & Shaulis, B. J. (2018). Rare Earth Elements in Planetary Crusts: Insights from Chemically Evolved Igneous Suites on Earth and the Moon. Minerals, 8(10), 455. https://doi.org/10.3390/min8100455

Nazari-Dehkordi, T., Spandler, C., Oliver, N. H. S., & Wilson, R. (2018). Unconformity-related rare earth element deposits: A regional-scale hydrothermal mineralization type of northern Australia. Economic Geology, 113(6), 1297–1305. https://doi.org/10.5382/econgeo.2018.4592

Porwal, A., & Carranza, E. J. M. (2015). Introduction to the special issue: GIS-based mineral potential modelling and geological data analyses for mineral exploration. Ore Geology Reviews, 71, 477–483. https://doi.org/10.1016/j.oregeorev.2015.04.017

Remédio, M. J., & Faleiros, F. M. (Orgs.). (2014). Geologia e recursos minerais da folha Fazenda Margarida – SF.21-X-C-IV, estado de Mato Grosso do Sul. CPRM. Escala 1:100.000.

Serviço Geológico do Brasil. (s.d.-a). Geoquímica Ambiental e Geologia Médica. Recuperado em 13 de agosto de 2026, de https://www.sgb.gov.br/geoquimica-ambiental-saiba-mais

Serviço Geológico do Brasil. (s.d.-b). GeoSGB: Sistema de geociências do Serviço Geológico do Brasil. Recuperado em 13 de agosto de 2026, de https://geosgb.sgb.gov.br/

Shah, A. K., Bern, C. R., Van Gosen, B. S., Daniels, D. L., Benzel, W., Budahn, J. R., Ellefsen, K. J., Karst, A. T., & Davis, R. (2017). Rare earth mineral potential in the southeastern U.S. Coastal Plain from integrated geophysical, geochemical, and geological approaches. GSA Bulletin, 129(9–10), 1140–1157. https://doi.org/10.1130/B31481.1

Skirrow, R. G., Murr, J., Schofield, A., Huston, D. L., van der Wielen, S., Czarnota, K., Coghlan, R., Highet, L. M., Connolly, D., Doublier, M., & Duan, J. (2019). Mapping iron oxide Cu-Au (IOCG) mineral potential in Australia using a knowledge-driven mineral systems-based approach. Ore Geology Reviews, 113, 103011. https://doi.org/10.1016/j.oregeorev.2019.103011

U.S. Geological Survey. (2026). Mineral commodity summaries 2026 (Version 1.3). U.S. Geological Survey. https://doi.org/10.3133/mcs2026

Van Gosen, B. S., Verplanck, P. L., & Emsbo, P. (2019). Rare earth element mineral deposits in the United States (Version 1.1). U.S. Geological Survey Circular 1454. https://doi.org/10.3133/cir1454

Verplanck, P. L., Lowers, H. A., Boehlke, A., Thompson, J. M., Bhat, G., & Mercer, C. N. (2023). The Mount Weld rare earth element deposit, Western Australia: A carbonatite-derived laterite. In Proceedings of the 17th SGA Biennial Meeting (Vol. 3, pp. 236–239).

Verplanck, P. L., Van Gosen, B. S., Seal, R. R., II, & McCafferty, A. E. (2014). A deposit model for carbonatite and peralkaline intrusion-related rare earth element deposits. U.S. Geological Survey Scientific Investigations Report 2010–5070–J. https://doi.org/10.3133/sir20105070J

Watts, K. E., & Mercer, C. N. (2020). Zircon-hosted melt inclusion record of silicic magmatism in the Mesoproterozoic St. Francois Mountains terrane, Missouri: Origin of the Pea Ridge iron oxide-apatite-rare earth element deposit and implications for regional crustal pathways of mineralization. Geochimica et Cosmochimica Acta, 272, 54–77. https://doi.org/10.1016/j.gca.2019.12.032

Wyborn, L. A. I., Heinrich, C. A., & Jaques, A. L. (1994). Australian Proterozoic mineral systems: Essential ingredients and mappable criteria. In The AusIMM Annual Conference 1994 (pp. 109–115).

Yousefi, M., & Carranza, E. J. M. (2016). Data-driven index overlay and Boolean logic mineral prospectivity modeling in greenfields exploration. Natural Resources Research, 25(1), 3–18. https://doi.org/10.1007/s11053-014-9261-9

Yousefi, M., & Carranza, E. J. M. (2017). Union score and fuzzy logic mineral prospectivity mapping using discretized and continuous spatial evidence values. Journal of African Earth Sciences, 128, 47–60. https://doi.org/10.1016/j.jafrearsci.2016.04.019

Zuo, R. (2012). Exploring the effects of cell size in geochemical mapping. Journal of Geochemical Exploration, 112, 357–367. https://doi.org/10.1016/j.gexplo.2011.11.001

Zuo, R. (2017). Machine learning of mineralization-related geochemical anomalies: A review of potential methods. Natural Resources Research, 26(4), 457–464. https://doi.org/10.1007/s11053-017-9345-4

Zuo, R. (2020). Geodata science-based mineral prospectivity mapping: A review. Natural Resources Research, 29, 3415–3424. https://doi.org/10.1007/s11053-020-09700-9

Bibliographic control for version V20. The bibliography was normalized alphabetically, materially incorrect references were replaced, works by the same author and year were assigned consistent letters, specific sources for M3, M9, and M10 were incorporated, and the Geoscience Australia web reference was completed with the verified official URL.

Appendix A. Formal specification of the M1 to M11 rules

The rules below correspond to the formal specification recovered from the methodological record that originated PAG ETR. They constitute the scientific reference for auditing implementations. The term formal is preferable to executable because a specific implementation may materialize only part of the library. Any alteration of these expressions must generate a new RULE_VERSION.

M1 · Carbonatites

M1_N1 = MAG_CARBONATITO

M1_N2 = MAG_CARBONATITO ∧ CARBONATITO_PROCESSO_COMPATIVEL

M1_N3 = M1_N2 ∧ ETR_LOCAL_COMPATIVEL_M1

M2 · Alkaline and peralkaline intrusions

M2_N1 = MAG_ALCALINO

M2_N2 = M2_N1 ∧ ESPECIALIZACAO_ALCALINA

M2_N3 = M2_N2 ∧ ETR_LOCAL_COMPATIVEL_M2

M3 · Hydrothermal systems

M3_N1 = HIDROTERMALISMO

M3_N2 = M3_N1 ∧ FONTE_FERTIL ∧ PATHWAY_OU_TRAP

M3_N3 = M3_N2 ∧ ETR_HIDROTERMAL

M4 · Sedimentary phosphorites

M4_N1 = SED_FOSFORITA

M4_N2 = M4_N1 ∧ APATITA_FOSFATICA ∧ ORIGEM_SEDIMENTAR

M4_N3 = M4_N2 ∧ REE_REY_FOSFATO

M5 · IOA and IOCG systems with REE

M5_N1 = IOA_CONFIRMADO ∨ IOCG_CONFIRMADO

M5_N2 = (IOA_CONFIRMADO ∧ APATITA_RELACIONADA) ∨ (IOCG_CONFIRMADO ∧ EVIDENCIA_ETR_RELEVANTE)

M5_N3 = M5_N2 ∧ ETR_LOCAL_COMPATIVEL_M5

M6 · Placers and paleoplacers

M6_N1 = TRAP_SEDIMENTAR

M6_N2 = M6_N1 ∧ MINERAIS_PESADOS ∧ FONTE_POTENCIAL

M6_N3 = M6_N2 ∧ MONAZITA_OU_XENOTIMA_DETRITICA

M7 · Ion-adsorption regoliths

M7_N1 = PROTOLITO_POTENCIAL ∧ REGOLITO_PRESERVADO

M7_N2 = M7_N1 ∧ MOBILIDADE_REE

M7_N3 = M7_N2 ∧ FRACAO_TROCAVEL_REE

M8 · Residual lateritic enrichment

M8_N1 = PROTOLITO_FERTIL ∨ PERFIL_RESIDUAL

M8_N2 = PROTOLITO_FERTIL ∧ PERFIL_RESIDUAL_IN_SITU

M8_N3 = M8_N2 ∧ ENRIQUECIMENTO_RESIDUAL_REE

M9 · Evolved granites

M9_N1 = GRANITO_INDIVIDUALIZADO

M9_N2 = M9_N1 ∧ ESPECIALIZACAO_GRANITICA

M9_N3 = M9_N2 ∧ ETR_LOCAL_COMPATIVEL_M9

M10 · REE-bearing pegmatites

M10_N1 = PEGMATITO

M10_N2 = M10_N1 ∧ ESPECIALIZACAO_PEGMATITICA_ETR

M10_N3 = M10_N2 ∧ MINERAL_ETR_PEGMATITO

M11 · Unconformity-related HREE systems

M11_N1 = DISCORDANCIA ∧ ARQUITETURA_ESTRUTURAL

M11_N2 = M11_N1 ∧ HIDROTERMALISMO_RELACIONADO ∧ SINAL_HREE

M11_N3 = M11_N2 ∧ XENOTIMA_HIDROTERMAL

APPENDIX B. PSEUDOCODE AND INVARIANTS RECOVERED FROM THE CANONICAL SPECIFICATION

The pseudocode below is transcribed from the preserved methodological specification. It documents the logical intent of the model and should be used as the reference for auditing specific implementations.

evaluate_rule(rule, evidence_set) → TRUE: SATISFIED · FALSE: NOT_SATISFIED · UNKNOWN: INSUFFICIENT_DATA · CONFLICT: CONFLICTING_DATA · NA: NOT_APPLICABLE

evaluate_model(model, spatial_unit) → insufficient coverage: NAO_AVALIAVEL · verified necessary contradiction: CONTRADITO_LOCALMENTE · N1 not satisfied: NAO_AVALIAVEL_OR_NOT_OPEN · N2 not satisfied: MODELO_ABERTO · N3 satisfied: EVIDENCIA_LOCAL_COMPATIVEL · otherwise: JUSTIFICA_PROSPECCAO_GEOQUIMICA

When N1 fails because of a verified FALSE, the output reason is not equivalent to a failure caused by UNKNOWN. The result must preserve result_reason.

Mandatory versioning DATA_VERSION · RULE_VERSION · GRID_VERSION · MODEL_LIBRARY_VERSION.

Scientific invariants recovered from the validator

SEM_DADO ≠ AUSENTE_VERIFICADO

ABAIXO_LD ≠ ZERO

EVIDENCIA_CONTEXTUAL does not confirm the model

EVIDENCIA_AUXILIAR does not replace a necessary requirement

A duplicated source does not create independent evidence

Point evidence does not semantically fill the hexagon

A stream sample uses the contributing catchment when available

Raster NoData does not become zero

Outside coverage does not become negative evidence

Insufficient scale does not become local absence

Isolated alkaline evidence does not create M2_N2

Isolated phosphorite does not create M4_N3

Isolated magnetite does not create M5

Isolated alluvial sand does not create M6_N2

Isolated kaolinite does not create M7

Isolated laterite does not create M8_N2

Isolated granite does not create M9_N2

Isolated pegmatite does not create M10_N2

Isolated unconformity does not create M11_N2

Internal audit. The complete scientific premortem matrix and multi-team review reports are project-control documentation and will be preserved in the reproducible repository rather than as part of the scientific body of the preprint.