Early warning signals of tipping points: A deep learning approach to direction and timing

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Abstract

The crossing of climate tipping points poses serious risk to life on Earth due to the impacts they cause, and their irreversibility. These systems generally exhibit critical slowing down on their approach to tipping point, or bifurcation, which can be detected by looking for changes in statistical properties of a time series from the system, such as an increasing lag-1 autocorrelation or variance. Recently, there has been interest in the use of deep learning in tipping point detection. However, there has been a strong focus on predicting the movement towards tipping only.

Here, we present results from a CNN-GRU architecture, trained on 3 normal forms of bifurcation, linearly forced either towards or away from the bifurcation point, or remaining a constant distance away. Predicting these 3 categories yields an overall accuracy of 67%, increasing to 78% for the longest time series, outperforming traditional early warning signals in a number of ROC tests. A complementary time-to-tip model predicts the distance to tipping should the linear forcing be extrapolated towards the tipping point, with a mean absolute error of 241 time steps which decreases as the time series lengthens. Using AMOC collapse output from a GCM, the categorical model is able to correctly identify which run is forced towards tipping, and which is the control, unforced run, with the time-to-tip model able to predict a decrease in the time to tip for the forced run, and a relatively stable distance in the control run. Although it overestimates tipping distance, we attribute this to struggles the model has with the saddle-node bifurcation type of which the collapse is likely to be. Attention weight analysis of the DL model shows clear, interpretable behaviour; the categorical model searches for signals at the beginning and end of the series to determine direction, whereas the time-to-tip model uses the beginning to ascertain distance from tipping. Testing with red noise data suggests there is a ceiling to improvements due to some reliance on critical slowing down indicators.

Our work presents a novel framework that adds considerations of time series moving away from tipping as part of the null category. Although this makes our model more general than those in the literature in dealing with false positives, it also highlights the limitation of classification-based DL models – a rich set of nulls are needed to help them capture more features of tipping trajectories that distinguish it from non-tipping trajectories. Together with our complementary time to tip model, these new additions have policy relevancy regarding monitoring such as intervention or planning strategies. We hope this framework can be built upon, by exploring training sets that could reduce false positives, and use output from more complex climate models to aid in the prediction of real world climate systems.

Introduction

Tipping points, where large, often irreversible changes can occur in a system, have been a topic of high importance in climate science in the last few decades (1, 2) due to the effect crossing them would have on the Earth system. Tipping points are hypothesised to be possible in a number of climate systems, including the Atlantic Meridional Overturning Circulation, AMOC (3), the Amazon rainforest (4) and ice sheet collapse (5, 6). Generally, tipping points occur because a system has passed through a bifurcation, causing the state previously occupied to become unstable, with no choice for the system but to move to a new state.

The consequences of passing climate tipping points are potentially devastating (7). For example, the collapse of the AMOC could cause winter temperatures in Europe to drop by 10C (8). The interactions between tipping elements in the climate system are also thought to generally be destabilising (9), such that if one element tips, it is likely to cause another to move closer to tipping. Because of this, early warning of the approach of a system towards tipping is of great importance, allowing the chance for mitigation or adaptation (10, 11).

As a system approaches a bifurcation, it will begin to exhibit critical slowing down (CSD; (12)) as the restoring feedbacks causing the return to equilibrium, weaken. CSD can be estimated by measuring the change in statistical properties of the system (13, 14) such as an increasing lag-1 autocorrelation AR(1) (15) or variance (16), most often measured on a moving window of a time series of the system. Commonly, a Mann-Kendall test will measure the tendency of a resulting indicator time series, with significantly positive values suggesting the system is approaching tipping (17). These signals have been found in both model output and observations from a number of different climate systems, e.g. (18-20). However, questions remain about the robustness of these indicators, such as sensitivity to detrending choices and the potential for false positives (14, 21).

Besides AR(1) and variance as indicators, there are a numerous methods for detecting the movement of a system towards tipping in climate and other fields (14), including recently, the use of deep learning (DL) models (22, 23). Layers such as convolutional neural networks (CNNs), gated recurrent units (GRUs) and long short-term memory (LSTM) are useful in their ability to recognise changes in a time series over time, making them prime candidates for use in models predicting early warning of tipping.

Various attempts at using DL models for predicting tipping target categories. For example Bury et al., use a CNN-LSTM to predict if a system is moving towards tipping, via one of 3 normal forms of bifurcation: saddle node, transcritical or Hopf, alongside a null category where the system is not moving towards tipping at all (22). Follow-on work from this has streamlined the process by focusing on predicting if a system is approaching specifically a catastrophic bifurcation (24). While a similar CNN-LSTM architecture has been used to search for the approach towards tipping in 2D Icing models (25). In each of these cases, a model assigned a probability of a time series being in each category.

In this work, we create a model which classify if a time series would tip based on the forcing rather than focus on the specific bifurcation type as the output. Our model categorises time series into those that are either moving toward or away from tipping, or remaining a constant distance away. This is particularly useful for complex climate systems which might not follow specific low-dimensional codimension one bifurcation corresponding to one forcing parameter. By adding the category of a system moving away from tipping, there is also the option to explore if intervention has any effect on the system. Furthermore, for those that are classified as approaching tipping, we have trained a complementary model that predicts how far away tipping is in time steps, assuming a continued linear forcing. This model uses the same architecture as the categorical model and is trained on time series created in a similar way. Both models have an attention pooling layer the processed timeseries goes through before it is classified. This allows us to examine the attention scores of both models, interpreting where the models are looking for signals for the movements towards or away from tipping.

The remainder of this paper is organised as follows. In the Materials and Methods section we describe how the time series used for training and testing are created, along with the architecture of the DL models. Then, in the Results sections we show how accurate and skillful the models are when tested on out of training time series, including those from AMOC GCM output. Finally, in the Discussion section we explore under what conditions the models work well and those in which they struggle, and suggest future directions for this work.

Materials and Methods

We create a training set using 3 normal forms of bifurcation: saddle node, (supercritical) pitchfork, and transcritical (Figure 1). Time series are 1500 long and are created by an algorithm that is designed to create a dataset that equally represents a range of distances from the bifurcation (which occurs at r=0 in all 3 cases), and a range of rates of change in r. For now we are only interested in linear movements in the bifurcation parameter, and equally represent time series that are forced 'towards' bifurcation, 'away' from bifurcation, and 'constant' which remain the same distance away from the bifurcation for the full 1500 length. We should note that predicting time to tip inherently requires one to make an assumption about the future of the forcing. We think time to tip in the context of linear forcing is natural as it is the same as asking when would the system tip if the current rate of change of forcing continues.

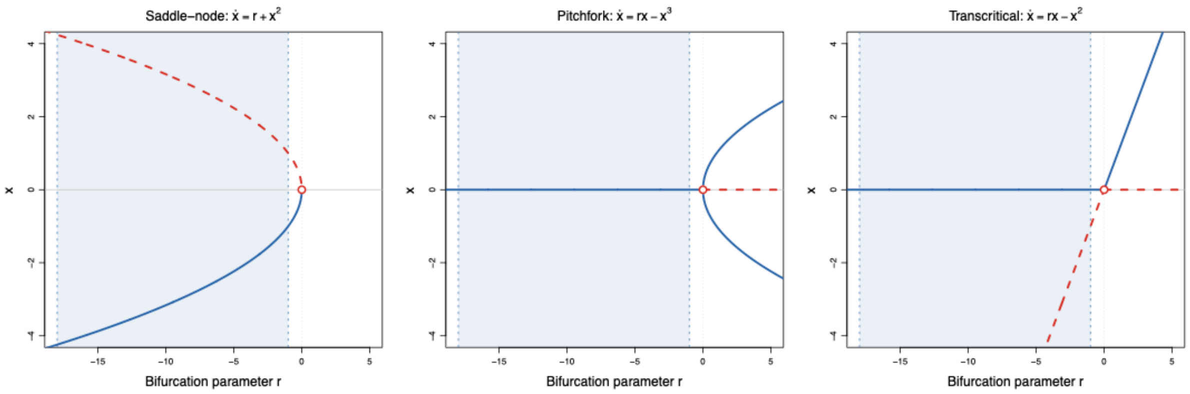


Fig. 1. Stability diagrams of the 3 normal forms of bifurcation used in the training and testing datasets. *Diagrams are shown for (a) saddle-node, (b) pitchfork and (c) transcritical bifurcations used in the training set for the DL models. Blue solid lines represent stable equilibria and red dashed lines, unstable equilibria. Start and end points for bifurcation parameter r are sampled within the shaded blue region as described in the main text, and a time series of x (of length 1500 time points) is created with r linearly increasing (decreasing) to create the movement towards (away) from the bifurcation (open circle at r=0). In 'constant' time series, the value of r is fixed.*

We begin by uniformly sampling a value of r between -10 and -1, followed by uniformly sampling a difference (or movement) in r between 0 and 8. A 1500-long time series for r is created by linearly interpolating between the initial sampled value, and this same value with the difference value subtracted. For time series moving towards bifurcation the first value of r is the most negative with an increase over the 1500 time points, but this is reversed for time series moving away. For constant time series, the initial sampled value of r is used for the full time series. Theoretically our time series of r across our full training set can vary between -18 and -1, represented by the blue shaded region in Fig. 1.

Time series are created using the Euler-Maruyama method, with a time step of $h=1/12$ to mimic monthly time series that might be observed in climate system time series. For those that are either pitchfork or transcritical, the initial starting value for time series are $x0=0$, whereas for the saddle node members, this is $x0=-\mathrm{sqrt}(-r)$, which in all cases is the stable equilibria of the system. As we are interested in predicting if a time series is moving towards tipping without observing it tip, any time series which has crossed the bifurcation point before time point 1500 is removed and a new member is rerun.

We have a training set of 54,000 runs, 18,000 of each bifurcation type and 18,000 of each direction, resulting in 6,000 of each combination (e.g. ‘towards’ X ’saddle node’. ‘Constant’ X ‘pitchfork’). 70% are used for training the DL model and 30% validation, with equal representation of each category of 6,000 (i.e. 4,200 training, 1,800 validation). The 1,500 long series are cropped to a length between 500 and 1,500 randomly along the full length of the time series, and then detrended using a lowess smoother with span equal to 0.2, before being normalised to Z-scores. Series are then left padded with zeros to create a 1,500 long time series again before being used for training. After training, we use a new set of 54,000 runs that were not used in training at all, prepared in the same way to determine how well the model predicts on data that was used out of training. The preparation of the training and testing datasets mimics how we suggest time series from systems are processed for prediction using the DL model.

We train a DL model with an CNN-GRU architecture with targets of ‘towards’, ‘constant’ and ‘away’. Initial testing showed more promising results than a CNN-LSTM architecture and was faster to train. The architecture (Fig. 2) contains 3 CNN layers, each consisting of, in that order, a convolution layer, batch normalisation and ReLU activation. The output from these is then halved in length by a MaxPool1D layer, alongside a masking timeseries that is downsampled to indicate where the real data is in a padded time series. A dropout layer follows before being passed to the GRU section of the model. There are two GRU layers with another dropout layer in between them. The output sequence from this is collapsed into a single vector in the attention pooling layer, which contains a Linear(256 -> 1) scorer and a softmax is used to produce a weighted sum. A Linear(256 -> 3) softmax head produces logits for the 3 classes (‘towards’, ’constant’, ’away’). The model is trained using cross-entropy loss and an AdamW optimiser (learning rate = 10^{-3} and weight decay = 10^{-4}), using batch size of 256 and triggering early stopping after 15 epochs of no improvement in the validation loss.

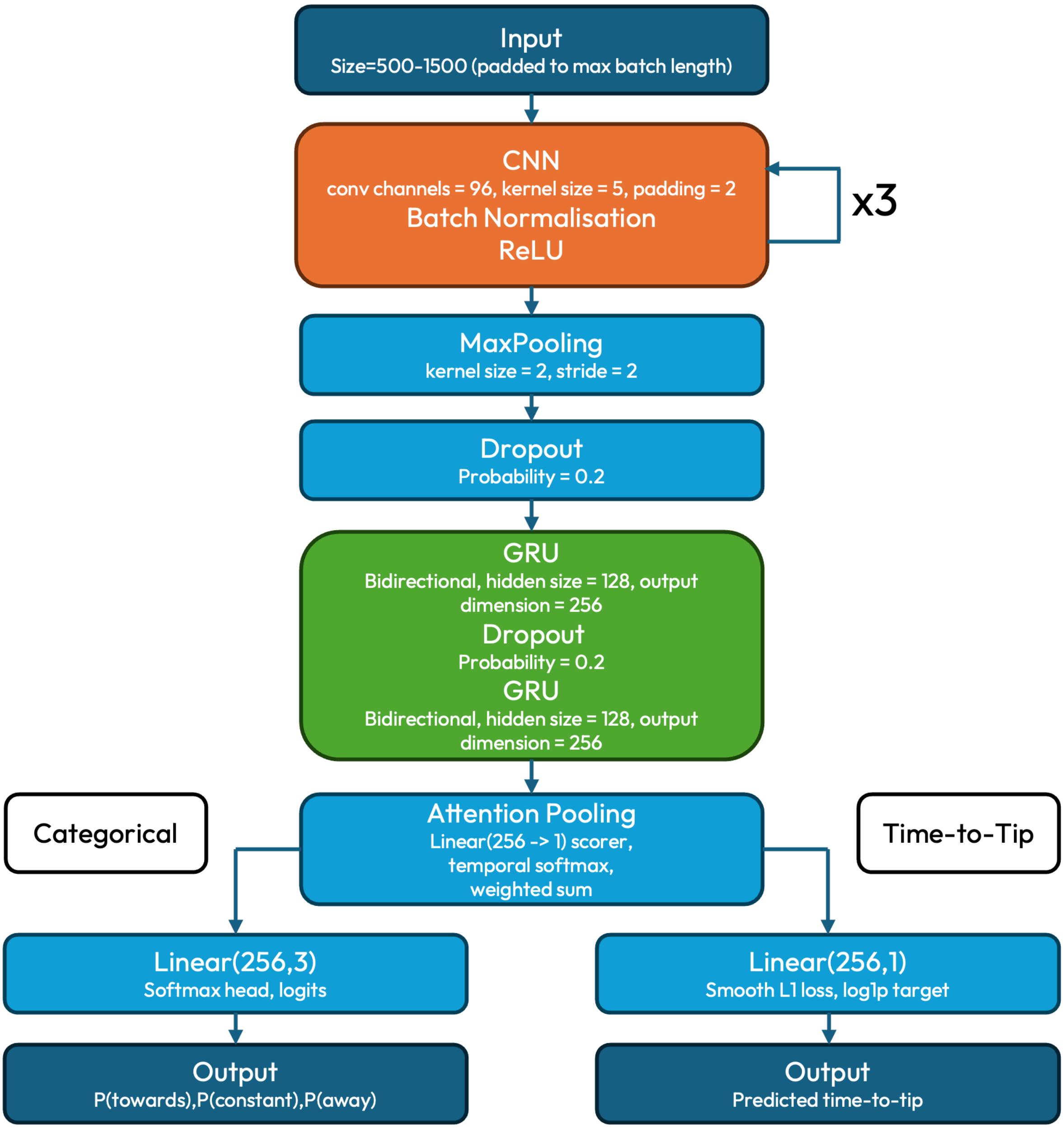


Fig. 2. Architecture of CNN-GRU categorical and time to tip models. *The architectures of both models are the same until the penultimate layer, where the (left) categorical model outputs 3 probabilities and the (right) time to tip model outputs a single scalar prediction.*

As a comparison of the predictions of categories from the DL model, we use generic EWS, specifically looking for increases in AR(1) and variance. These are calculated on the same cropped and detrended time series in the test data set, using a window length equal to half the length of the time series. Performance is calculated using receiver operating characteristic (ROC) curves (26), in 3 cases of increasing difficulty by using the Kendall's τ

correlation coefficient as an indicator of true/false from the generic EWS and probability of being in the category from the DL model: ‘towards’ vs ‘away’, ‘towards’ vs ‘not towards’, and ‘towards’ vs ‘constant’.

We also create a ‘time to tip’ DL model which predicts how far away the bifurcation is if the external forcing, r, were to continue on its linear trajectory to r=0. We adapt the previous framework of creating a training set whose targets are now the number of time steps needed for r to reach 0 if it were to be linearly extrapolated past the 1,500 time points. We have created a dataset that has equal representation across times to tipping (uniformly sampled from 100 to 1600), and ending values of r between -10 and -1. With a time to tip and the value of r known at time steps 1500, the rate of change of forcing can be calculated to work out the value of r at initialisation (i.e. at t=0) and a time series for r can be created by linearly interpolating as for the normal dataset. To ensure that the time series remain generally around the equilibrium, we resample a combination of time to tip and ending r value if the starting r value results in being less than -15 as in some abnormal cases this algorithm ended up with r value less than -100. We sample 15,000 time series per bifurcation, noting this time that all of them are moving towards tipping, resulting in 45,000 time series in total. These are also cropped and detrended as before, with the time to tip adjusted accordingly.

Our architecture for the time to tip DL model is the same as the categorical model (Fig. 2), except for outputting a continuous scalar, and trained with a smooth L1 loss against log-transformed time targets (log1p), which are then transformed back into time units afterwards. For the 45,000 time series the same split of 70% training, 30% validation is used as with the categorical model. A binning procedure such that both the training set and the validation set have equal representation of distances and length is used.

Results

Categorical Model

Overall, on the out of training test set our categorical CNN-GRU model is 66.51% accurate across the 3 categories, ranging from 50.2% for time series that are of length 500-550, up to 77.8% accurate for time series 1450-1500 long. Segregating the results per direction of movement shows it struggles slightly with the ‘constant’ category (62.1%) compared to the ‘towards’ or ‘away’ (68.3% and 69.1% respectively). We find that the accuracy of the pitchfork and transcritical normal forms (72.1% and 71.9% respectively) is higher than the saddle node normal form (55.6%). Figure 3 shows how the predictions for each normal form change with the length of time series (the crop length) the DL uses to predict. For each category, the probability of being in the correct category increases, with the probabilities of being in the other two categories decreasing, as the length of time series the DL uses to predict increases.

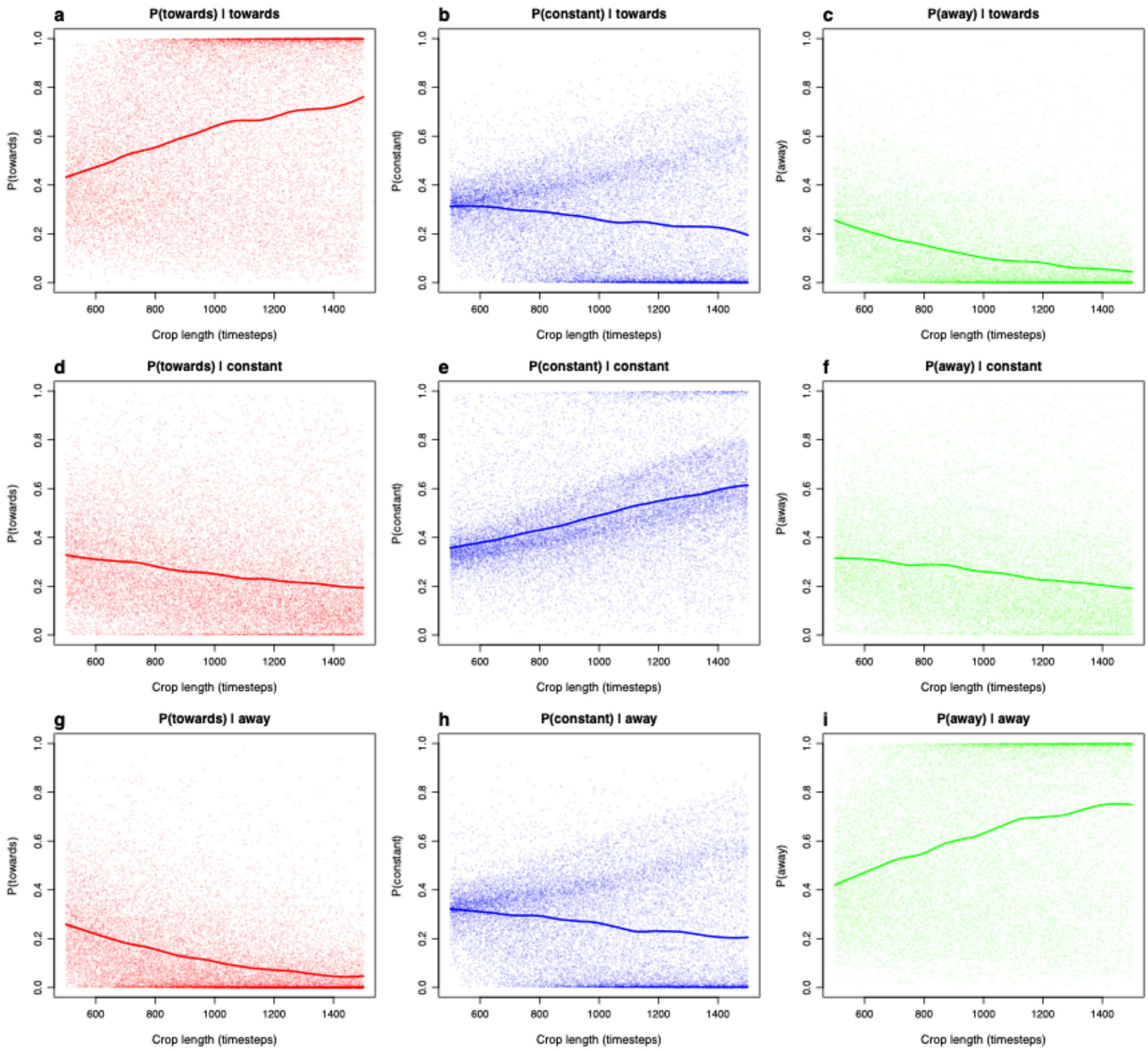


Fig. 3. The probabilities of being in each category are plotted as a function of the length of time series the model uses to predict. *Scatter plots are shown for time series (a-c) moving towards tipping, (d-f) remaining a constant distance away from tipping and (g-i) moving away from tipping, with predicted probabilities of the time series (a,d,g) moving towards tipping, (b,e,h) remaining a constant distance away from tipping (c,f,i) moving away from tipping. A lowess smoother is used to show the trend in the scatter plots.*

Furthermore, Figure 4 shows how predictions change with the rate of change (rates of change are used here because the absolute change in parameter r will vary based on the length of time series used) in the bifurcation parameter r, showing that time series that move faster towards or away from the tipping point are predicted with a higher probability to be in that category. For the constant category, histograms are shown of prediction as there is no rate of change to vary on the x-axis. These show that the mean predicted probability of 49% for those in the constant category, including a non-trivial number of predictions of ~100%, and a mean of ~25% for those in the other two categories, both positively skewed distributions with a small tail up to 100%

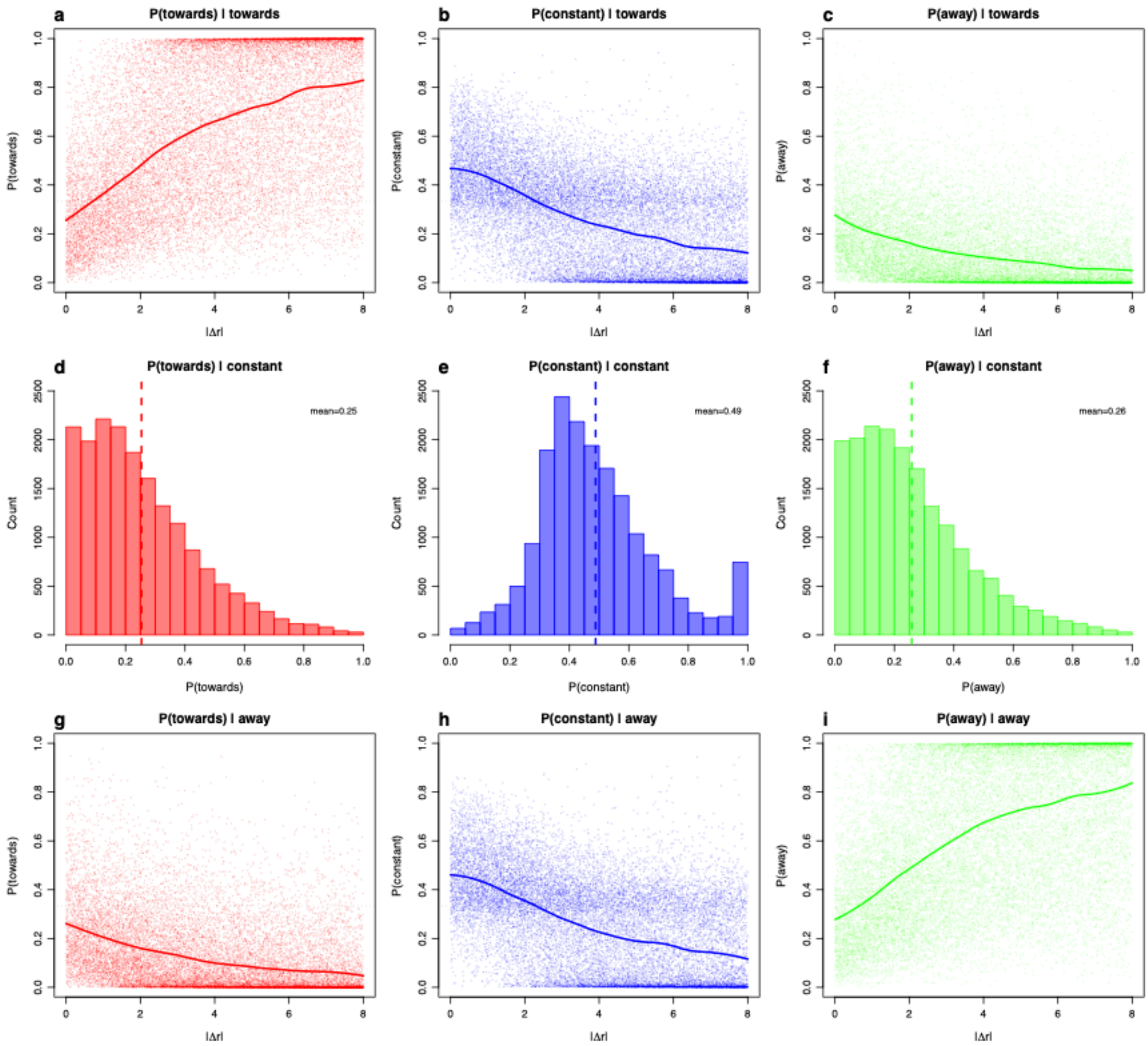
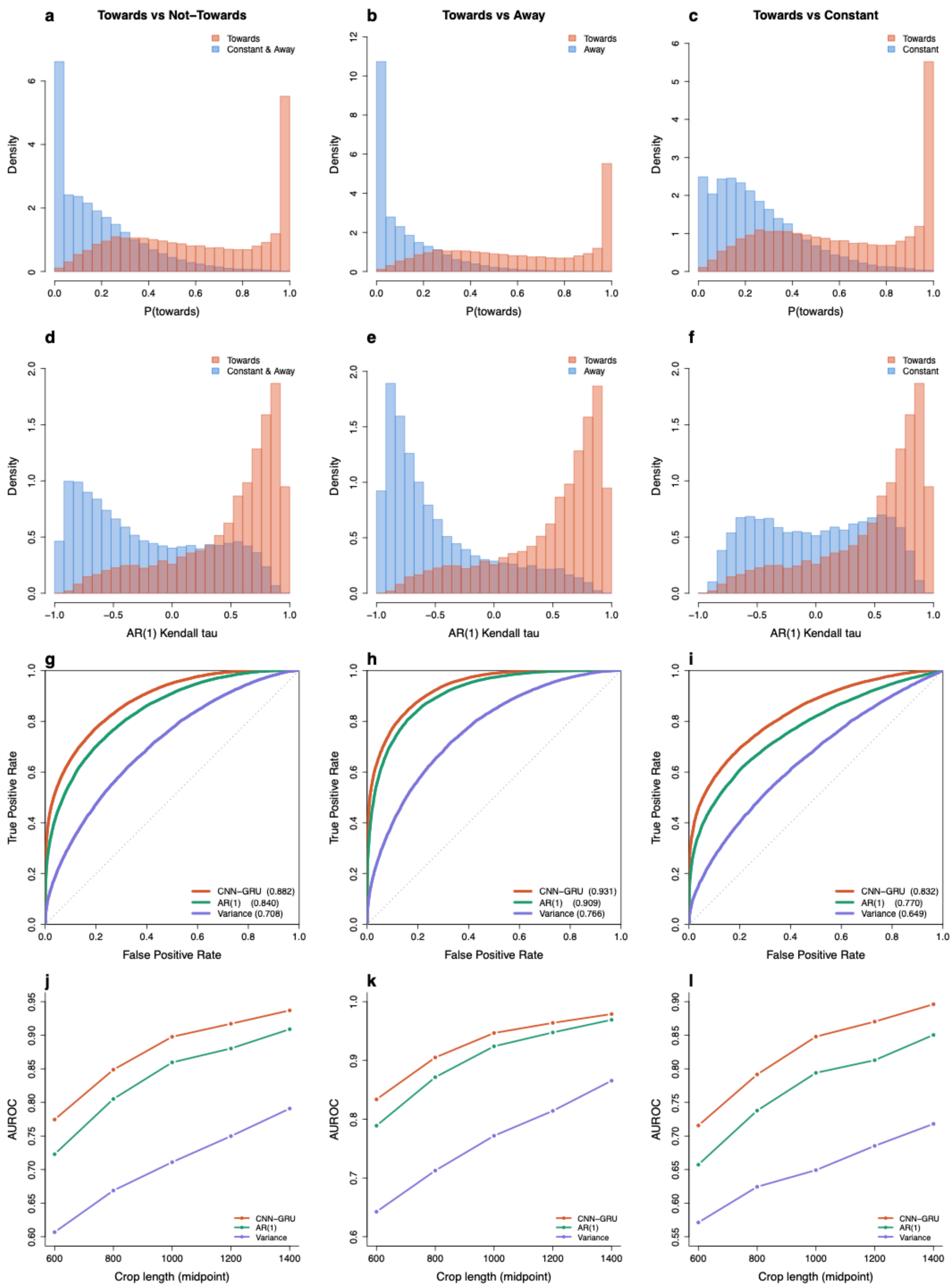


Fig. 4. The probabilities of being in each category are plotted as a function of rate of change of the bifurcation parameter r. *Plots are shown for time series (a-c) moving towards tipping, (d-f) remaining a constant distance away from tipping and (g-i) moving away from tipping, with predicted probabilities of the time series (a,d,g) moving towards tipping, (b,e,h) remaining a constant distance away from tipping (c,f,i) moving away from tipping. For*

the middle row representing the constant category, there is no rate of change and as such histograms of the predictions are shown. A lowess smoother is used to show the trend in the scatter plots.

Results from Figures 3 and 4 combined suggest that the model struggles to differentiate between time series that are being slowly forced towards or away from tipping, and the constant category. Along with predictions from longer time series are more accurate as well, it suggests that for these the model needs longer time series to differentiate. To determine the DL model's skill we compare it to the Kendall's τ values from using AR(1) and variance on the same cropped time series using ROC curve analysis (Figure 5). With the 3 categories, we compare the 'towards' vs 'not towards' (combining the constant and away categories), 'towards' vs 'away' and 'towards' vs 'constant'. These differentiations of classifications vary in their difficulty for both the DL and the traditional EWS, with towards vs away being the easiest, and towards vs constant being the most difficult.



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282 ***Fig. 5. ROC curve analysis comparing the DL model to traditional EWS.*** *Category*
283 *comparisons, 'towards' vs 'not towards', 'towards' vs 'away' and 'towards' vs 'constant' are*
284 *assessed using cropped series from Figures 3 and 4. (a-c) The probability of time series*

moving towards tipping predicted from the DL model is shown for both categories (red - towards, blue - other) each time. (d-f) The Kendall's τ values for the AR(1) time series (colours as in a-c). The histograms of Kendall's τ values from variance time series are not shown. (g-i) The ROC curves for each categorical test for (red) the DL model, (green) AR(1) and (blue) variance with the Area Under the ROC curve (AUROC) shown in the legend. (j-l) The AUROC values of the analysis shown as a function of the length of time series tested.

In each of the 3 categorical tests, the DL model outperforms the AR(1) and variance EWS, with the biggest gain in using the DL model seen when predicting towards vs constant, with a 0.062 increase in the Area Under the ROC curve (AUROC). Segregating the analysis by the length of time series the predictions are made from shows that for all 3 methods, there is an increase of skill as the length of time series increases (Fig. 5 j-l), still with the DL model being the most skillful for all combinations. Furthermore, the gradient of the DL model AUROC line as a function of time series length is shallower, suggesting its skill drops off less with shorter time series, than traditional EWS. Supplementary Figure 1 shows the ROC curve and AUC analysis for segregating by short time series (500-700 long) and long time series (1300-1500), providing a detailed exploration at the information provided by Fig. 5g-i.

To assess well the DL model predicts the movement towards tipping on complex problems, we turn to a simulated collapse of the Atlantic Meridional Overturning Circulation (AMOC) in the FAMOUS model (27), an ocean current that is believed to be approaching a tipping point due to anthropogenic climate change (2). In the FAMOUS model, an increasing freshwater forcing in the high northern latitudes causes the overturning strength of the AMOC to decrease after 800 years, accompanied by a control run with no forcing applied. We follow previous work that used AR(1) and variance to predict the movement towards tipping, focusing on 26.25°N and looking across all northern latitudes (18), by also assessing the DL model alongside this. As a caveat, we use the same detrending method as for the DL model (a Lowess smoother with a span of 0.2), rather than the Kernel smoother used in the original paper.

Figure 6 shows how the prediction of whether the AMOC is approaching tipping or not evolves in real time as more of the time series is shown to the DL model. For the forced run (Fig. 6a) for most of the time between 500 and 800 years, the model predicts that the time series is moving towards tipping, with some confusion with predicting the system is constant before the year 600. The Kendall's τ values for AR(1) and variance, using a window of 500 years to be comparable to the period of time the DL is tested on (Fig. 6b), are 0.797 ($p=0.025$) and 0.199 ($p=0.395$) respectively, suggesting critical slowing down is occurring and thus early warning of tipping is found. For the control run, the probability of the time series being constant according to the DL model is clearly higher than the other two categories (Fig. 6c), suggesting it is easily able to detect this. However, the Kendall's τ value for the AR(1) and variance are 0.499 ($p=0.215$) and 0.538 ($p=0.164$) respectively. Both of these are positive, with the variance τ being higher for the control than it is for the forced run, suggesting that the DL model is a better predictor than traditional EWS for this collapse.

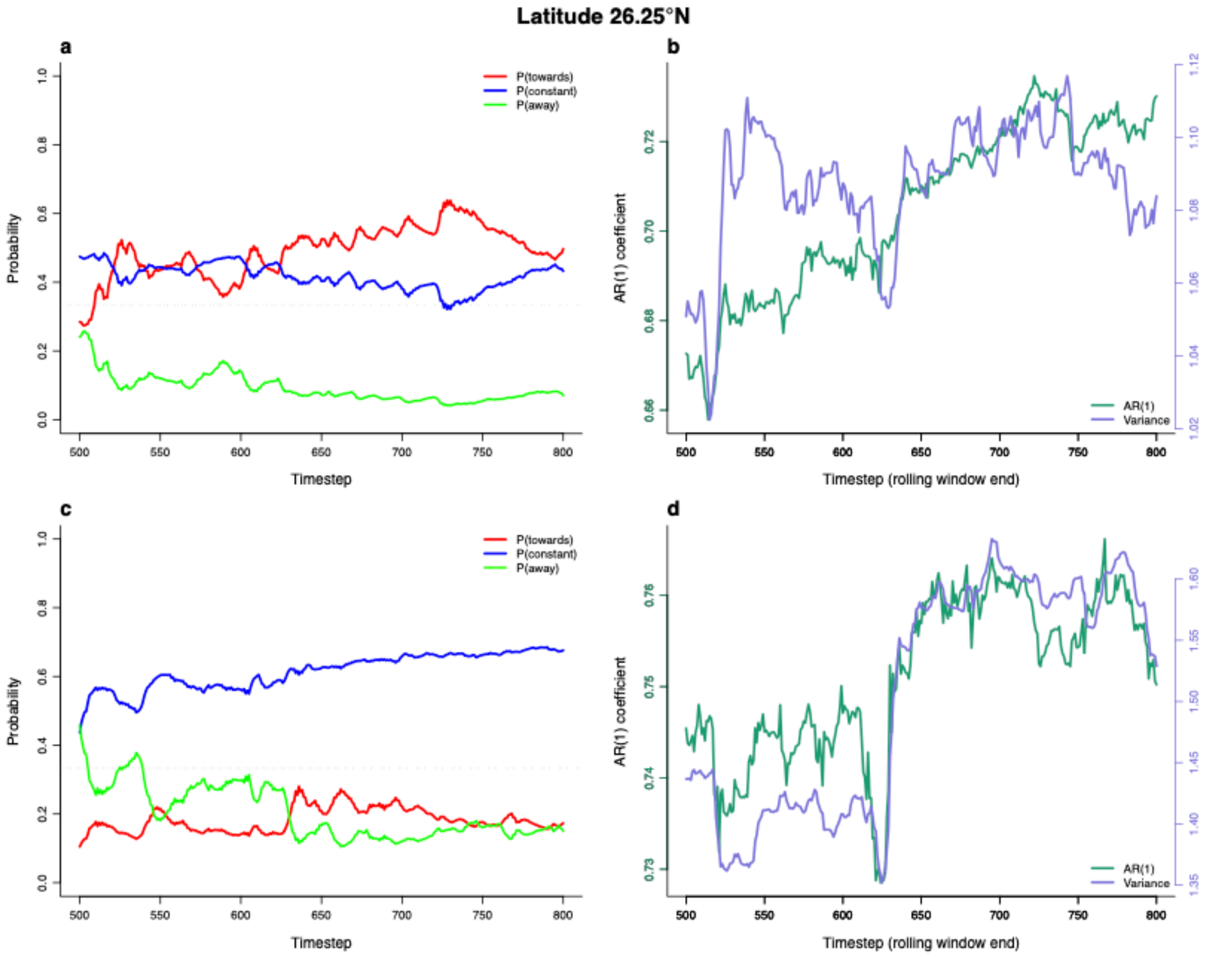

Fig. 6. Prediction from the categorical model and traditional EWS signals applied to output from FAMOUS AMOC collapse at 26.25°N. *Results are shown for the (a,b) forced and (c,d) control runs. From the (a,c) categorical model, the predicted probability of being in each category is plotted at the end of each of the expanding windows used to make the prediction, from years 1-500, up to 1-800. (b,d) Time series of AR(1) and variance are shown for comparison, using a sliding window length of 500 years.*

To look across the full model, we run the same analysis across all latitudes (Figure 7), reporting the final (at year 800) probabilities from the DL model. For the forced run (Fig. 7a), the DL model correctly predicts the system is moving towards tipping at 16 of the 38 latitudes, with correct classifications concentrated at lower latitudes where the CSD fingerprint of AMOC collapse is expected to be strongest (18). Traditional EWS detect significantly positive Kendall's τ values for AR(1) at 27 of the 38 latitudes and for variance at

17 of the 38 latitudes (Fig. 7b), suggesting that for the forced run, traditional AR(1) detects the approaching tipping signal at more latitudes than the DL model.

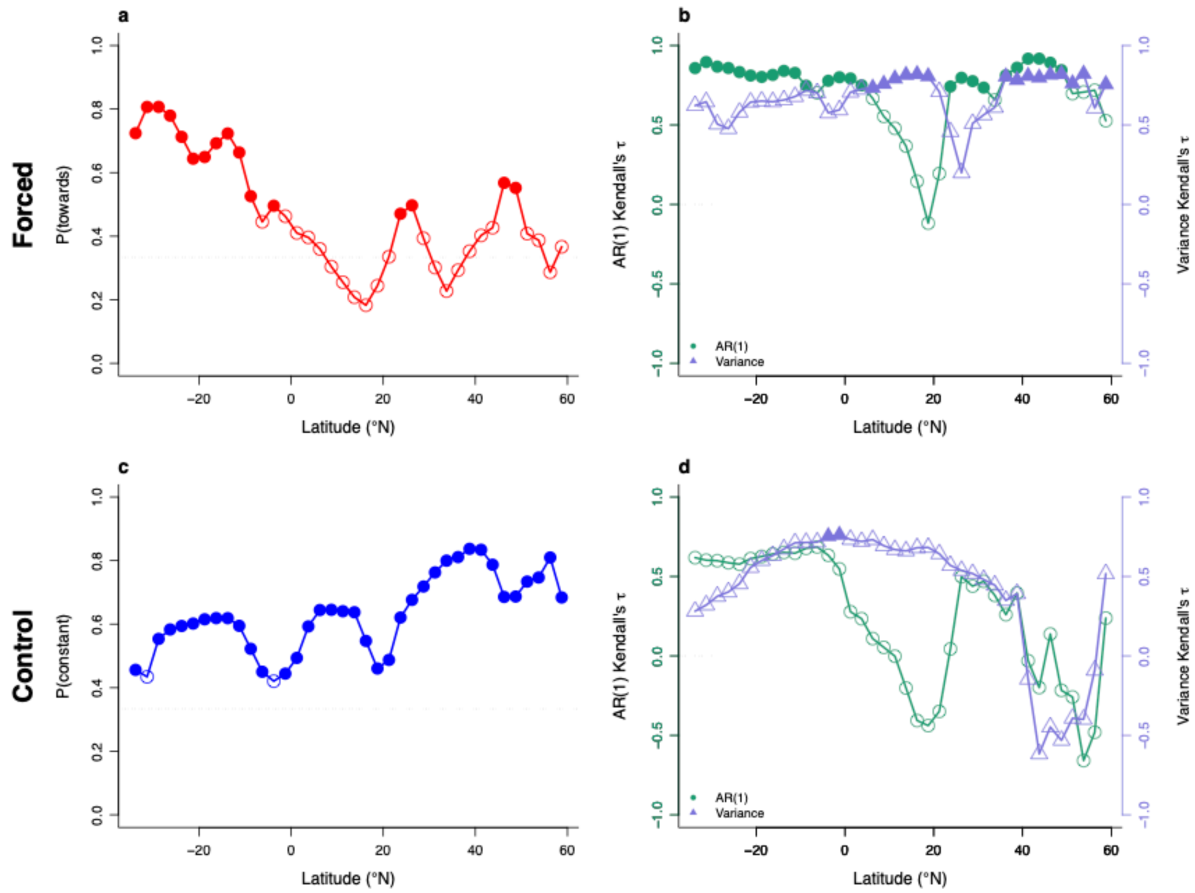

Fig. 7. Predictions using FAMOUS output from a range of latitudes. *Results shown for (a,b) the forced and (c,d) control runs. For the DL model, predictions are shown as the probability the time series is (a) moving towards tipping in the forced run, or (c) constant in the control run for the 1-800 windows. Closed circles show correct predictions, and open circles when another category had a higher predicted probability. For traditional EWS AR(1) and variance, the Kendall's τ values at time 800 are shown, for the (b) forced and (d) control runs. With closed symbols showing significant ($p<0.05$) τ values and open symbols, non-significant. Note that for the control run, significant τ values suggest early warning signals despite no movement towards tipping.*

However for the control run, both methods perform better. The DL model correctly classifies 36 of the 38 latitudes as constant (Fig. 7c), with 0 of the 38 AR(1) and only 2 of the 38 variance Kendall's τ values reaching significance (Fig. 7d). Importantly, these methods are correctly predicting the lack of movement towards tipping in different ways. The traditional EWS correctly produce non-significant τ values (an absence of signal of tipping), while the DL model makes a positive classification of stability. The stronger performance of traditional AR(1) on the forced run likely reflects both its specific design for detecting monotonic trends and the out-of-distribution nature of the DL application, which was trained on idealised normal form time series of up to 1500 timesteps and applied here without fine-tuning to the shorter length GCM output. As noted in the original paper, CSD indicators would not be expected to be detectable equally across all latitudes for this collapse (18).

Once the AMOC has collapsed in the forced run, we again test both the DL model and traditional EWS on the overturning strength time series from the year 1100 (when it has settled to a new equilibrium), up to 2383, giving a time series of length 1284 at each latitude to test on. 21 of the 38 latitudes are correctly classed as moving away from tipping by the DL model, whereas only 3 of the AR(1) and 0 of the variance Kendall's τ values are significantly decreasing (Supplementary Figure 2). This shows that the DL model is able to provide extra information about the dynamics on a more complex system that traditional methods may struggle with.

Time-To-Tip Model

After training a model on time series approaching tipping only with the architecture described in Figure 2, we now use the model to target how far away tipping would be if the forcing were to continue on a linear trajectory towards the tipping point (or in other words, when r reaches 0). With the set up of training and test time series described in the methods, our data can theoretically span the shortest time series (500 long) that tip 2600 time points into the future, and longer time series (1500 long) that only tip 100 time points away.

After training, testing using the 45,000 out of training time series shows a MAE of 240 time steps and a bias of -33. Segregated results by the length of time series the model sees shows a clear decrease in MAE as the DL model predicts from longer time series, ranging from 362 for time series that are 500-700 time series long, down to 198 for time series that are 1300-1500 long (Supplementary Figure 3). Furthermore, much like the categorical model, the accuracy of the model varies depending on normal form, with the saddle node MAE ranging from a higher 387 to 230, compared to ~350 to ~181 for transcritical and pitchfork.

Figure 8 shows how the model predicts when the test dataset is segregated by normal form as a function of how far away the tipping point is, using time series of length 1500. As suggested by the categorical model, the predictions using saddle node time series (Fig. 8a) are slightly worse than those for the transcritical and pitchfork bifurcations (Fig, 8b,c). For these latter two, predictions are accurate for time series where tipping is within 750 time points, whereas predictions of the time to tip are overestimated for the saddle node for those closer than 1000 time points. In all 3 cases there is a clear decrease in time to tip predicted as for shorter time series.

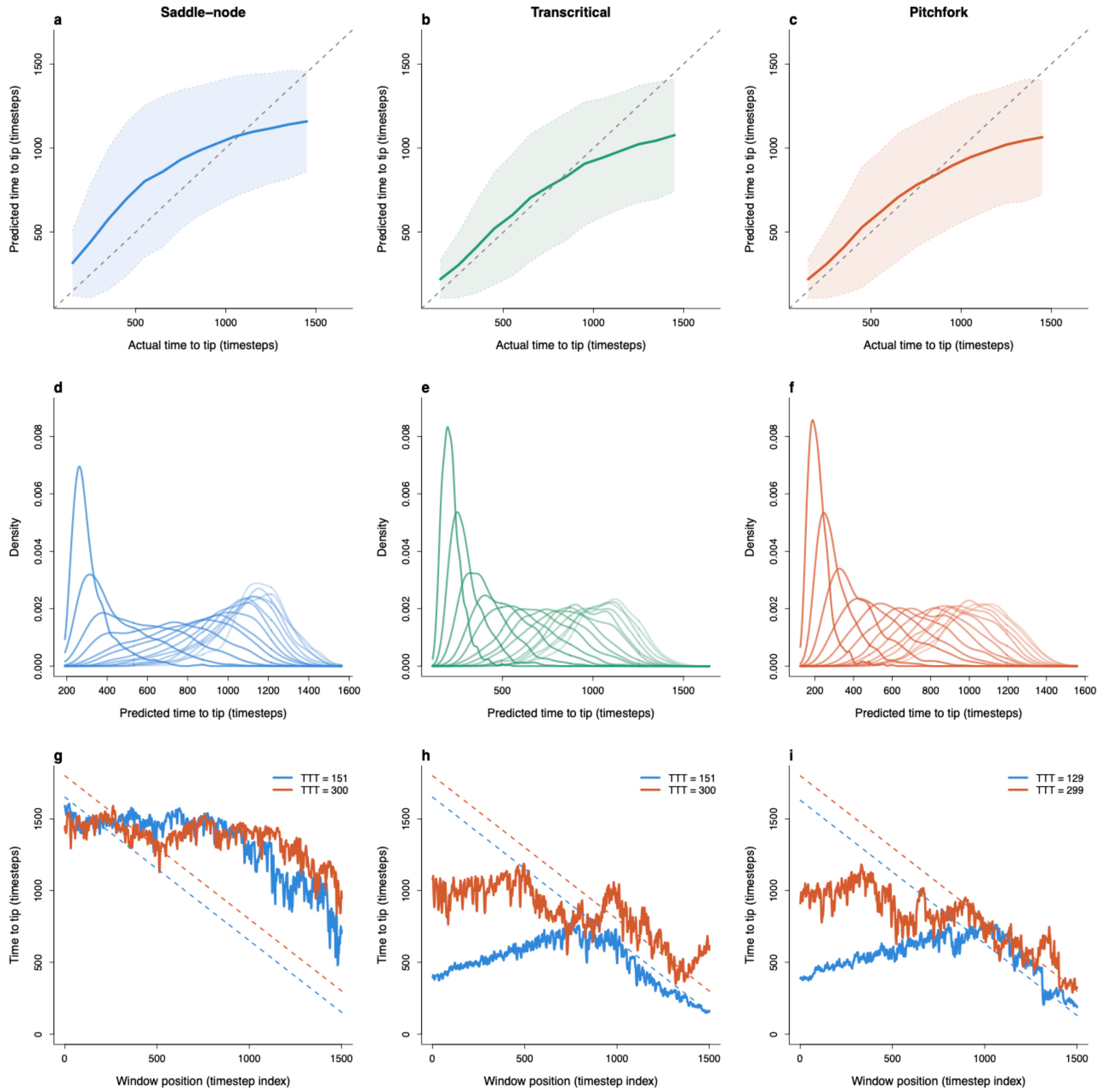

Fig. 8. The change in distribution of prediction of time to tip from a test set of time series. *(a-c) The distribution of predictions of time to tip from time series that are 1500 long for the 3 normal forms. A lowess smoother and a +/-2 standard deviation band is applied. Bottom row: (d-f) The density distribution of prediction of time to tip binned in 100 time point sections (i.e. 1600-1500, 1500-1400, 1400-1300 actual time to tipping). Darker colours are closer to tipping. (g-i) Predictions from 3000 long time series that are (blue) close to tipping and (orange) further away for each normal form. Dotted diagonal lines are the actual time to tipping of the time series, with solid lines denoting the prediction made on a sliding window of 1500 time points.*

Density profiles of the predictions made from each normal form, binned by distance to tipping (Fig. 8d-f) show that in all 3 cases the model becomes more confident of the time to tip when tipping is closer, starting as near normal distributions far from tipping, to positively skewed distributions closer to tipping. For each normal form, example predictions are shown for time series that are 3000 time points long, using a sliding window of 1500 to make the predictions (Fig. 8g-i). Two examples are shown for each normal form, with dotted lines representing how far away tipping would be if the forcing were to continue on the same trajectory. Figure 8 shows that the model's predictions overestimate the time to tip for the saddle node. However, it does get the order correct, the orange line is above with a predicted tipping time further away than the blue line, with both decreasing as tipping is approached.

The transcritical and pitchfork normal forms appear to settle down on their trajectories in both cases (Fig. 8h,i) after around 1000 time points, as expected from the envelope plots (Fig. 8b,c). Note that these are typical time series rather than chosen specifically, but despite for example, the increase in time to tip in the orange transcritical series near the end, the overall behaviour is promising.

Finally, we test how the time to tip model works for the FAMOUS AMOC collapse run at 26.25°N. Figure 9 shows how predictions of time to tip change as the model is first shown the first 500 years, and then an increasing window up to 800 years where the forced run tips. Both the forced (Fig. 9; blue) and control (orange) runs are tested. We find that there is a clear distinction between the two series, a decrease in time to tip in the forced run and a constant time to tip in the control. However, the time to tip is vastly overestimated, getting down to ~800 years as the closest estimate when tipping is only 125 years away. Analysis on this model suggests that it follows a saddle node bifurcation (18, 27) which might explain this overestimate.

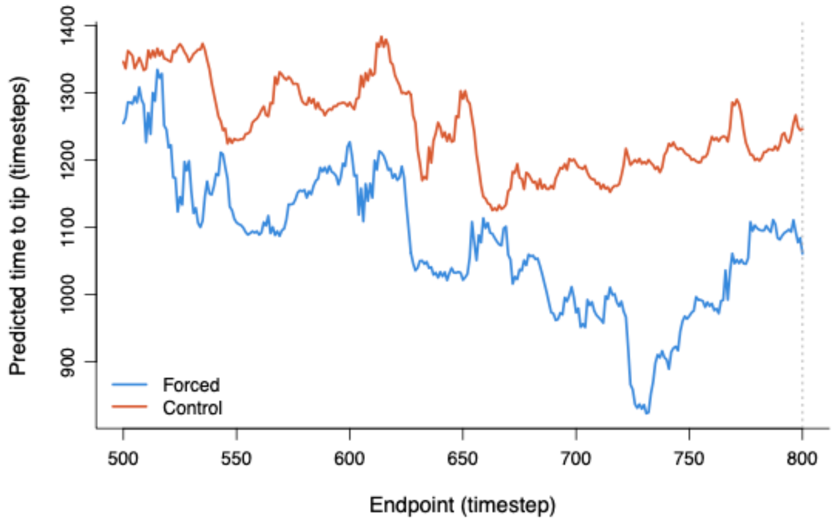

Fig. 9. Results when testing the time to tip model on the FAMOUS AMOC collapse. *Collapse occurs in the forced run just after 800 years. Prediction of time to tip at 26.25°N using an expanding window from 500 to 800 years of overturning strength for both the (blue) forced and (orange) control runs.*

Discussion

We have shown here how our CNN-GRU architecture can be used to predict the movement of time series towards or away from tipping, as well as remaining constant. It is able to clearly distinguish between series that are moving towards and those that are moving away from tipping, with most confusion coming from predicting on shorter time series, and those where the external forcing is slowly changing such that it is difficult for the model to distinguish between constant and one of the other categories. Similarly, the time to tip model is more accurate when predicting using longer time series and for those closer to tipping.

We find a distinction between the results for the transcritical and pitchfork members, compared to the saddle node ones. This most likely stems from differences in how critical slowing down indicators behave when the bifurcation is being approached. For the transcritical and pitchfork bifurcations they linearly increase as the bifurcation is approached.

However for the saddle node bifurcation the critical slowing down indicators scale as the square root of the distance to the bifurcation, such that stronger signals are seen closer to tipping and are hard to observe further away. In an attempt to combat this we trained a version of the model with an augmented training set; including an extra 12000 saddle node members per class (towards/constant/away), 70% of which were sampled with the change in bifurcation parameter being less than 2 over the 1500 time points (compared to the original uniform sampling between 0 and 8) that were added to the original training set. Using the same test dataset, this newly trained model increased accuracy on the saddle node members from 55.6% to 58.5%, whilst only reducing accuracy on both the other two normal forms by ~0.5%. It suggests that the decrease in saddle node accuracy could be partially addressed through targeted augmentation and could be a worthwhile area for future research.

We can look at the attention scores from the attention layer in the architecture of both models to see which area of the time series most of the focus is on for the DL model. For the categorical model, we look at the mean attention scores across 200, 1500 long time series for each normal form, and for moving towards and away from tipping, as well as those constant time series (Figure 10). We generally see that for those time series moving towards tipping, attention scores are higher at the start of the series, and for those moving away, scores are higher at the end. For constant time series the attention is much more uniform, with a spike at the start. Attention score profiles for transcritical and pitchfork are largely the same, with the saddle node showing a weaker version of the same pattern. Essentially, it appears as though the model looks for the behaviour of the system furthest away from tipping, suggesting it could be searching for where signals such as critical slowing down are weakest.

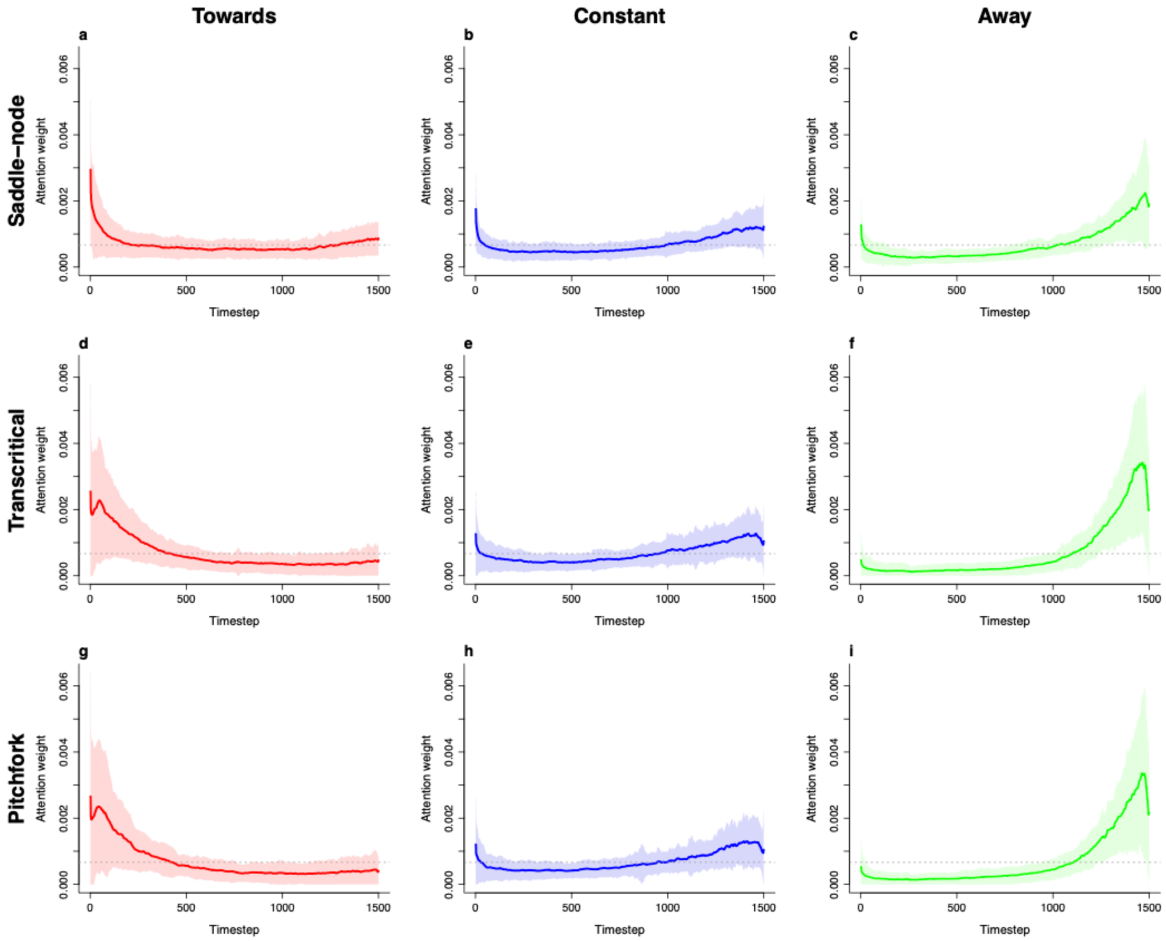

Fig. 10. Attention scores from the categorical model. *Scores are shown for the (a-c) saddle-node, (d-f) transcritical and (g-i) pitchfork normal forms for those series (a,d,g) moving towards tipping, (b,e,h) remaining a constant distance away from tipping and (c,f,i) moving away from tipping. The mean attention score from 200 of full length (1500 long) time series for normal form and category are shown. Higher attention values mean that the model is assigning more importance in the information from those time steps. Dotted lines show what a uniform attention score would look like, marking 1/1500.*

The time to tip model's attention scores are calculated for 200 time series that were near tipping (100-300 time points away), a mid-distance from tipping (600-900 time points), and far (1200-1500 time points), again segregated by normal form (Figure 11). All 3 normal forms behave in a similar way, showing that for near tipping series, there is a lot of focus on the beginning of the time series, with decreasing attention given to the start for time series that are further and far away from tipping. It appears as though the model is looking at the behaviour of the time series (e.g. autocorrelation level) near the beginning to determine how far away tipping is, which will be clearer for closer time series. Comparing the two DL models, the categorical model is comparing information from the start and end of the time series to determine direction, whereas the time to tip model only looks at the beginning to assess distance from tipping.

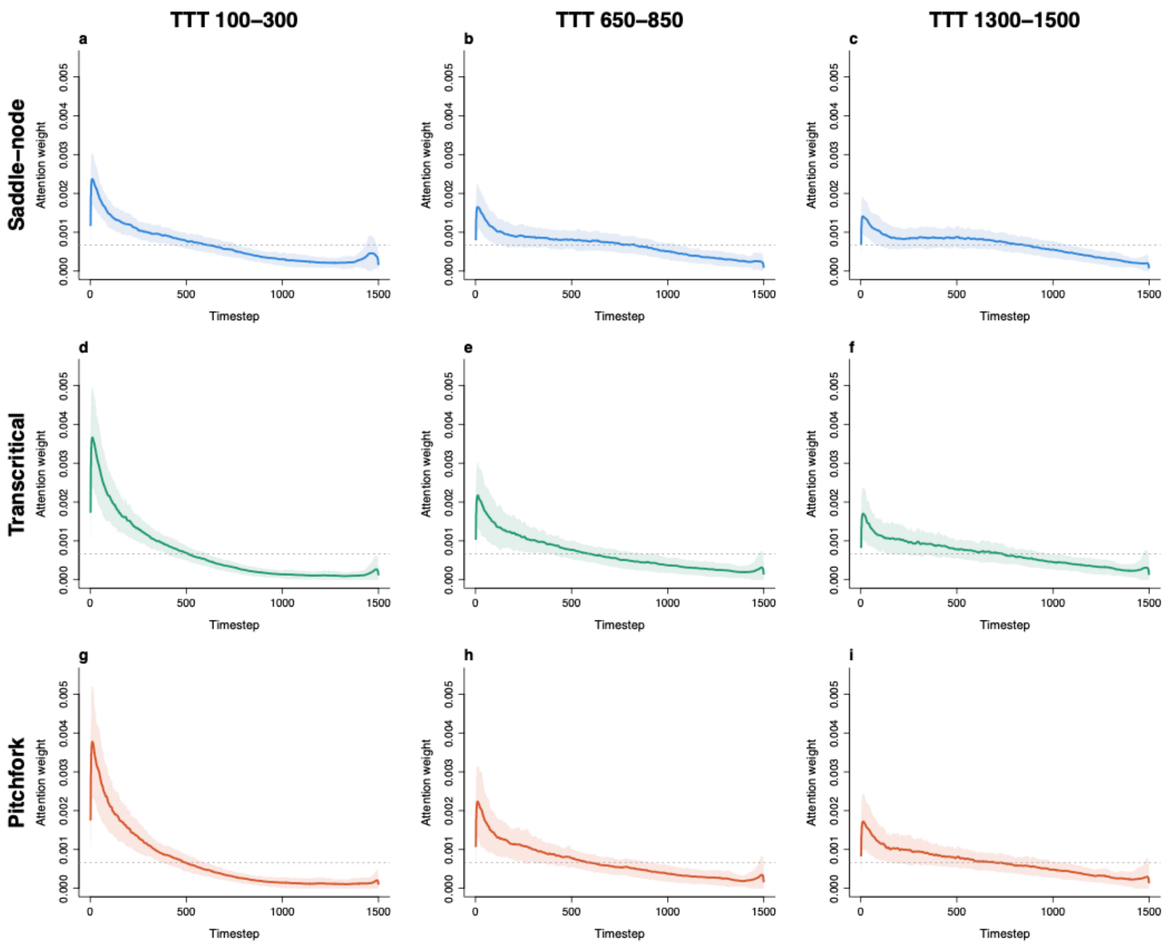

*Fig. 11. **Attention scores from the time to tip model.** Results are shown for the (a)-(c) saddle node, (d)-(f) transcritical and (g)-(i) pitchfork normal forms. The mean attention score from 200 full length (1500 long) time series (a,d,g) near or within 100-300 time steps of tipping, (b,e,h) a middle distance away or within 650-850 time steps of tipping and (c,f,i) far or within 1300-1500 time series from tipping are shown. Higher attention values mean that*

the model is assigning more importance in the information from those time steps. Dotted lines show what a uniform attention score would look like, marking 1/1500.

To determine how much the DL model relies on traditional early warning signals such as changes in AR(1), we tested the categorical model on red noise processes, time series where the memory increases or decreases but where there is only one state that remains stable. 10000 each of red noise time series where the AR(1) increases, decreases, and remains constant (30000 total), are used in the testing and the current DL model has an accuracy of 31.1%. Because of this, we also trained a categorical model using 3000 each of red noise processes that have an increase in AR(1), decrease in AR(1), and have a constant AR(1) to be targeted as 'constant' by the model, replacing 9000 of the original training set's time series. We found that the lowess detrending with a span of 0.2 caused a decrease in overall accuracy of the original test set to 63.55% (from 66.51%). Increasing the span caused an increase in the accuracy, with the best case being a lowess span of 0.65 to get back to the original accuracy (66.56%) whilst finding 44.1% of the red noise processes to be 'constant'. Increasing the span past this reduces the accuracy in both cases. This suggests there is a ceiling that the DL model cannot fully disentangle itself from critical slowing down metrics. Further work such as augmenting the training set with more 'false positive' time series and hypertuning parameters may get this accuracy even higher, however.

Moving to more complex, out of training data from the FAMOUS AMOC collapse run shows that the models work to a degree in their predictions. The categorical model is able to predict that the system is moving towards tipping in the forced run, and remaining constant in the control run with reasonable accuracy. However, a caveat here is that predictions are made across all latitudes, without consideration of where early warning signals would be expected to be found based on the physical processes of the model. Furthermore, the time to tip model can distinguish between the forced and control runs, showing a decrease in the time to tip for the forced run but without good accuracy. The decrease in time to tip looks similar to the results from the saddle node normal forms with overestimations of how far away tipping is. There is strong evidence that the FAMOUS AMOC collapse run exhibits a saddle node bifurcation (18, 27) so this is likely the reason why the time to tip model struggles. While these DL models aim to be as generalised as possible, an extra step could be added to determine what type bifurcation is being approached, e.g. (22), before models specifically trained on that specific normal form are used to predict, although as our attention analysis suggests that the model is already discriminating between different normal forms an explicit classifier may not be necessary.

Conclusions

We have created a DL model pipeline that accurately predicts if a system is moving towards or away from tipping, and for those that are approaching tipping, can predict how far away it is if the forcing were to continue on a linear trajectory. AUROC analysis shows that the categorical model improves on traditional EWS, especially for shorter time series, and for differentiating between series moving towards tipping and those remaining constant.

Our analysis of the attention scores shows the interesting mechanistic approach, the categorical model searches for what are most likely critical slowing down indicators at the beginning and end of the time series to determine direction, whereas the time to tip model focuses on the beginning of the series to see what the baseline signals are to determine distance. Preliminary work testing the framework suggests that there is a ceiling caused by a reliance on searching for traditional critical slowing down signals, despite out-performing them in out of training predictions.

The novel approach of searching for signals of movement away from tipping and predicting the time to tip provides a strong framework to be built upon, with suggestions for future work including augmenting the training set to improve how it handles false positives arising from time series that have increased memory without approaching tipping, and extending the training set to include more normal forms and output from e.g. general circulation models to aid its prediction on climate systems.

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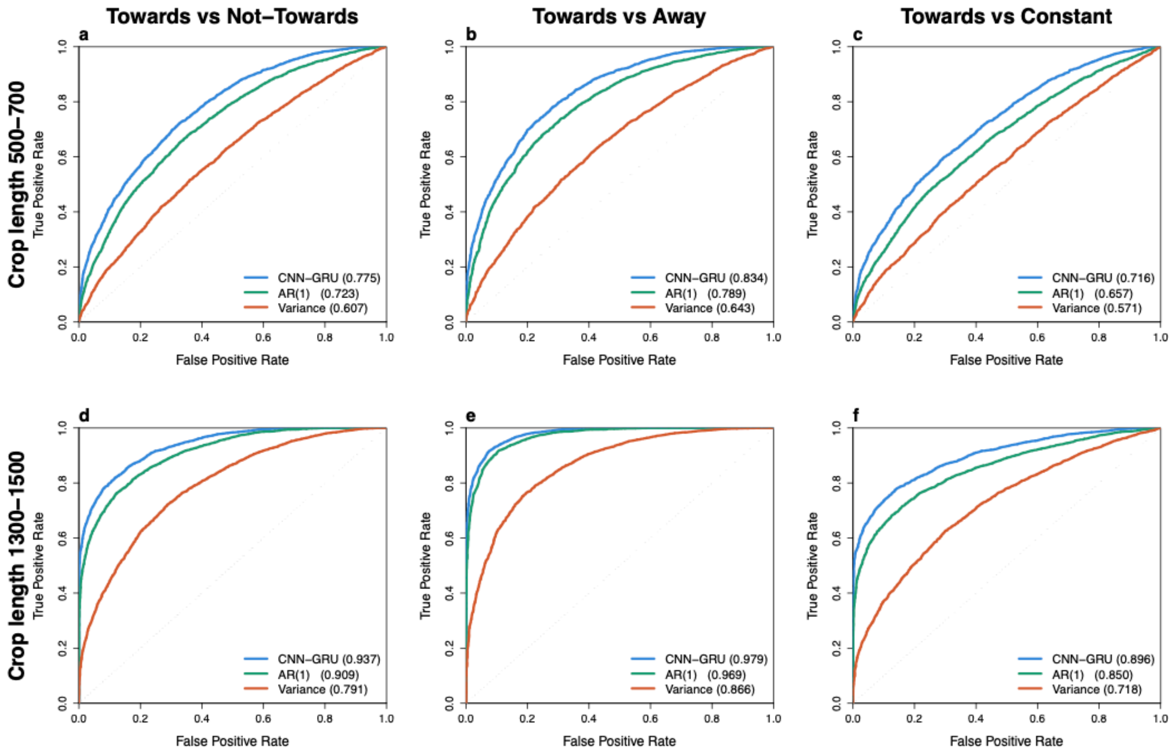
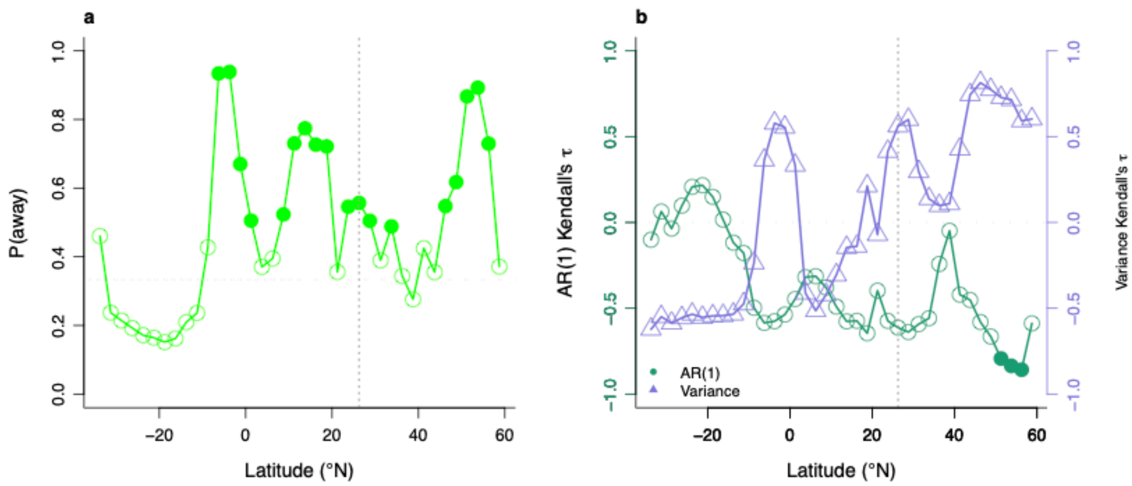
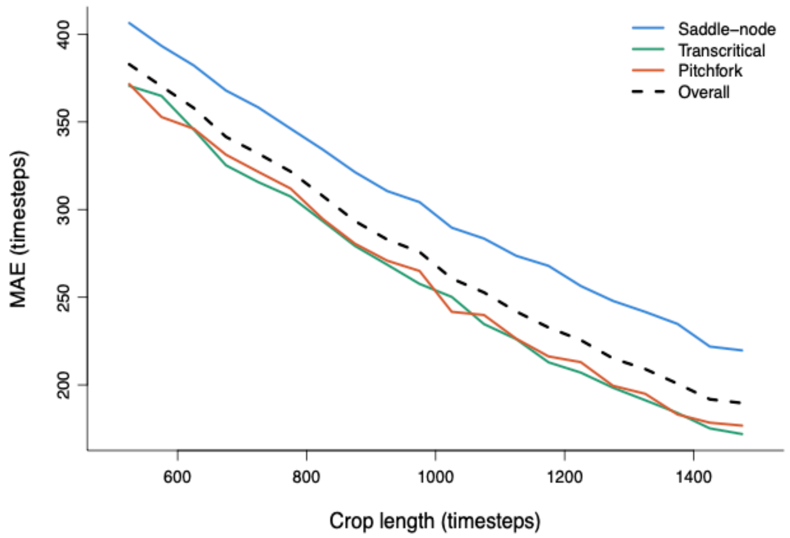
Supplementary Information



S1 Fig. ROC curve analysis comparing the DL model to traditional EWS for short and long series. *Category comparisons, 'towards' vs 'not towards', 'towards' vs 'away' and 'towards' vs 'constant' are assessed using series from Figures 3 and 4. ROC curves for each categorical test for (red) the categorical model, (green) AR(1) and (blue) variance are shown for time series which have been cropped to (a-c) 500-700 long and (d-f) 1300-1500 long. The Area Under the ROC curve (AUROC) values are shown in each legend.*



S2 Fig. Predictions using FAMOUS output from a range of latitudes for the forced and run post-tipping. *(a) For the DL model, predictions are shown as the probability the time series is moving away from tipping in the forced run over the period 1100-2383. Closed circles show correct predictions, and open circles when another category was predicted to be higher. (b) For traditional EWS AR(1) and variance, the Kendall's τ values for the time series over the same time period are shown (with window length equal to half the length of the time series), with closed circles showing significant ($p<0.05$) τ values and open circles, non-significant.*



S3 Fig. Mean absolute error (MAE) from the time-to-tip model. *MAE is presented as a function of crop length, the length of time series the prediction is made from, and separated by normal form.*