Boxology: the isobar–thickness box count as an exact advection integral, and its rendering

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Abstract. "Boxology" is the hand-analysis practice of reading thermal advection from the angle at which mean-sea-level-pressure isobars cross 1000–500 hPa thickness contours: steep crossings cut the chart into small quadrilaterals and imply strong advection, while parallel families imply none. This note shows the practice is an identity rather than a rule of thumb, and implements it as a rendered chart product. Splitting the layer-mean geostrophic wind into its bottom-surface and thermal parts removes the thermal wind exactly, since it blows along the thickness contours, so layer-mean thickness advection reduces to a Jacobian of the two plotted fields. Treating those fields as coordinates makes that Jacobian a change of variables, so the advection integrated over any quadrilateral bounded by consecutive isobars and consecutive thickness lines is a fixed quantum, independent of the box's shape — a specialization of the solenoid count in Bjerknes' circulation theorem. Box area and advection intensity are therefore reciprocal, which is what the eye is measuring. A renderer is described that labels the actual quadrilaterals by band index and fills each flat at its own quantum-over-area rate on a doubling ladder. Constants, smoothing, low-latitude tapering, and edge handling are documented.

Keywords: synoptic analysis; thermal advection; circulation theorem; objective analysis; chart rendering

1 Introduction

"Boxology" is the hand-analysis habit of reading thermal advection off a surface chart by eye: where the isobars cross the thickness (or isotherm) lines at a steep angle, the crossing cuts the plane into small quadrilaterals and the air is turning over fast; where the two families run parallel there are no boxes and nothing is advecting (Lackmann, 2011). This note derives why the technique is *exact*, not a rule of thumb, and documents a renderer that turns the same habit into a field a chart-production pipeline computes and shades rather than one left to the reader's eye.

2 The isobar-parallel wind carries the whole signal

Split the 1000–500 hPa layer-mean geostrophic wind into the bottom-surface geostrophic wind plus half the thermal wind, $\mathbf{V} = \mathbf{V}_0 + \frac{1}{2}\mathbf{V}_T$. The thermal wind blows *along* the thickness contours by construction, so $\mathbf{V}_T \cdot \nabla h \equiv 0$ and

$$-\mathbf{V}\cdot\nabla h \;=\; -\mathbf{V}_0\cdot\nabla h \tag{1}$$

identically. Only the isobar-parallel flow advects thickness — which is *why* the technique is licensed to ignore everything on the chart except the two contour families it is already reading, and why it is an identity rather than an approximation.

3 The Jacobian form

Substituting the geostrophic relation $\mathbf{V}_0 = (g/f)\,\hat{\mathbf{k}} \times \nabla Z_0$ (with Z_0 the 1000 hPa height, i.e. the isobars converted to geopotential metres) into (1) turns the advection into a Jacobian of the two *plotted* fields:

Model (geostrophic thermal advection).

$$A \;=\; -\frac{g}{f}\,J(Z_0,h) \;=\; -\frac{g}{f}\,|\nabla Z_0|\,|\nabla h|\,\sin\theta, \qquad J(Z_0,h) = \frac{\partial Z_0}{\partial x}\frac{\partial h}{\partial y} - \frac{\partial Z_0}{\partial y}\frac{\partial h}{\partial x}, \tag{2}$$

with θ the angle at which the isobar crosses the thickness line. Parallel contours ($\theta = 0$) give zero; a perpendicular crossing gives the maximum for that pair of gradient magnitudes. Nothing else in the field enters — which is the same claim (1) made, now in a form a renderer can evaluate pointwise.

4 Box counting as a change of variables

The reason box *area* is the thing a hand analyst's eye is measuring follows from treating (Z_0, h) as new coordinates over the plane. The Jacobian of that map is exactly J from (2), so $|J|\,\mathrm{d}x\,\mathrm{d}y = \mathrm{d}Z_0\,\mathrm{d}h$, and integrating A over one quadrilateral bounded by two consecutive isobars (δZ_0 apart) and two consecutive thickness lines (δh apart) gives

Identity (one box, one quantum).

$$\iint_{\text{box}} A\,\mathrm{d}A \;=\; \mp\frac{g}{f}\,\delta Z_0\,\delta h, \qquad A_{\text{box}} \;=\; \frac{\delta Z_0\,\delta h}{|J|}, \tag{3}$$

the same magnitude for *every* box regardless of its shape. This is a specialization of the solenoid count in Bjerknes' circulation theorem to a pair of level-set families rather than an arbitrary closed curve (Lackmann, 2011): box COUNT carries the area integral of A over a region (Green's theorem reduces it to a signed count around the boundary, $\iint_R A\,\mathrm{d}A = -\frac{g}{f}\oint_{\partial R} Z_0\,\mathrm{d}h$); box AREA carries the local intensity, and the two are reciprocal per (3). That reciprocity is the whole reason the hand technique works by eye, and it is why the renderer described in §6 keys its shading intensity to A (equivalently, to A_{box}^{-1}) rather than to any separately-tuned quantity.

5 Temperature form

The hypsometric relation $h = (R_d/g)\,\bar{T}\ln 2$ for the 1000–500 hPa layer converts (2) to a layer-mean temperature tendency,

$$\frac{\partial \bar{T}}{\partial t} = -\frac{g^2}{f R_d \ln 2}\,J(Z_0, h), \tag{4}$$

returned in $\mathrm{K\,d^{-1}}$. Z_0 is recovered from MSLP via the standard $8\,\mathrm{m\,hPa^{-1}}$ conversion ($p \approx 1000 + Z_{1000}/8$), so the overlay reads exactly the field the isobars on the chart were drawn from.

Quantity	Symbol / value	Note
Layer coefficient	$g^2/(R_d \ln 2) \approx 0.4833\ \mathrm{K\,s^{-1}}$	per unit J
MSLP $\to Z_0$	$8\,\mathrm{m\,hPa^{-1}}$	$p \approx 1000 + Z_{1000}/8$
Equator fade	12–20°	tapered, not cut, to 0
House contour intervals	$\delta p = 4\,\mathrm{hPa}$, $\delta h = 6\,\mathrm{dam}$	box-size diagnostic only

The equator fade exists because $f \to 0$ makes (2) blow up: the geostrophic relation itself stops holding well before the true equator, so the field is tapered smoothly through 12–20° latitude rather than masked at a hard edge, which would draw a false analysis boundary across the map.

6 Rendering: the boxes themselves

The renderer shades the *actual quadrilaterals*, not a smooth field standing in for them. A box is one connected region over which both band indices $\lfloor p/\delta p \rfloor$ and $\lfloor h/\delta h \rfloor$ are constant: the pair changes exactly where a contour is drawn, so such a region *is* the polygon bounded by the four lines around it, and the box-labeling routine recovers every box on the chart without tracing or intersecting a single contour. Connectivity is 4-way, not 8-way — two boxes touching only at a corner meet *at* an isobar/thickness crossing, which is the one place they are guaranteed to be different boxes, and 8-way connectivity welds them into a blob straddling the very feature being measured.

Each box is then filled flat, at the rate (3) assigns it: its own quantum over its own area, so a slack box is weak and a tight one intense. The shading routine bins that rate into eight bands, each edge *doubling* the one below $(3, 6, 12, \ldots, 384\,\mathrm{K\,d^{-1}})$. Because $A_{\text{box}} \propto A^{-1/2}$, a doubling ladder in A is a matched geometric ladder in box side — roughly 490 km at the floor down to a 43 km knot at the top, the same visual step at every rung — and fill opacity climbs with it, ending at 0.82 rather than 1 so the contours and any underlying field stay legible through the deepest box.

Boxes touching the frame are discarded. Those are *cut* boxes whose measured area is only the part that fits on the chart, so they would read as far tighter than they are; a hand analyst does not count a box running off the map either.

6.1 Why not a filled contour of A

Equation (2) is defined pointwise, so a filled contour of it is available and was the first implementation. It renders the same quantity and is not the technique: it smooths across the crossings instead of stopping at them, and at chart scale over the thickness ramp it read as a faint wash rather than as boxes. The per-pixel version survives as a hatched overlay option; the production path draws the discrete boxes.

6.2 Smoothing

The advection field is a product of two gradients, so it amplifies grid noise the eye never sees on the chart itself. The defaults match what the isobar and thickness fields are already smoothed to on the chart ($\sigma = 3.0$ on MSLP, thickness pre-smoothed at $\sigma = 1.5$ upstream) so the shading never disagrees with the crossing angle a reader can actually see drawn.

7 Example output

Figures 1 and 2 show the renderer running on live analyses, boxology shading drawn over the same MSLP isobars and 1000–500 hPa thickness contours the technique reads by hand.

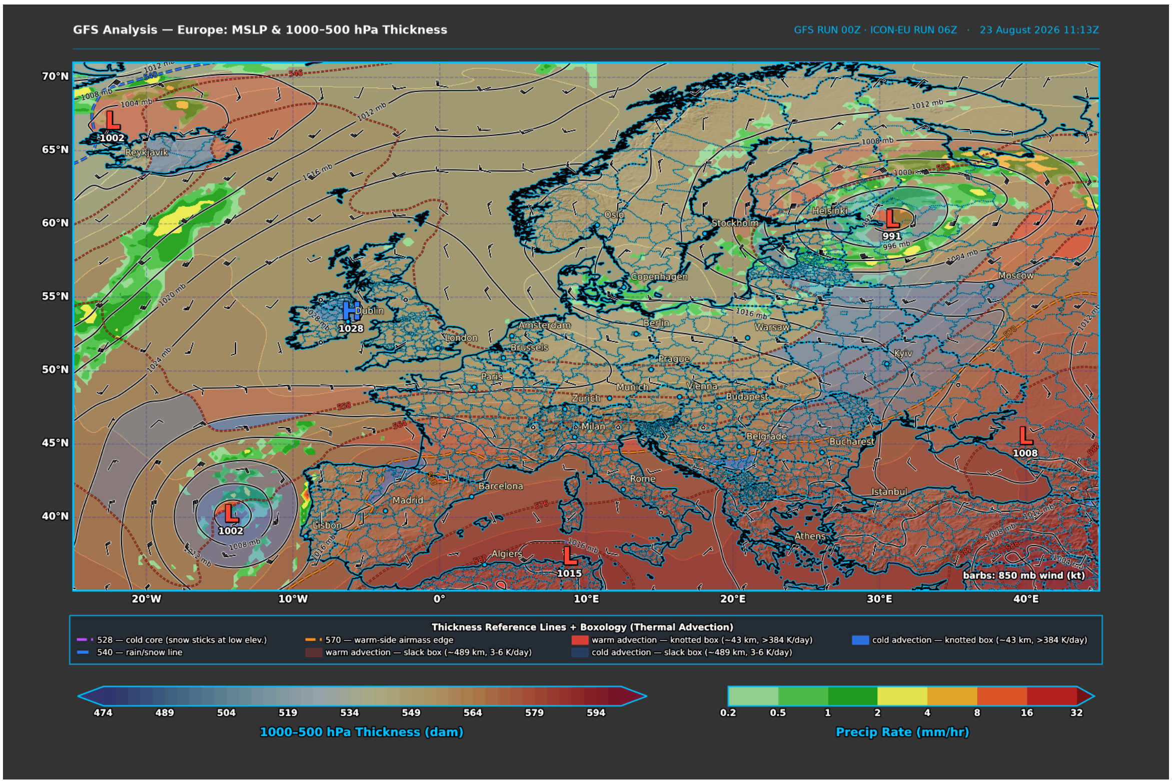

Figure 1: Europe, GFS 00Z / ICON-EU 06Z analysis valid 23 August 2026 11:13Z. The Atlantic low near Iberia and the developing low over Finland sit inside tightly packed boxes on their warm and cold sides respectively, while the broad ridge over central Europe shows no boxes at all.

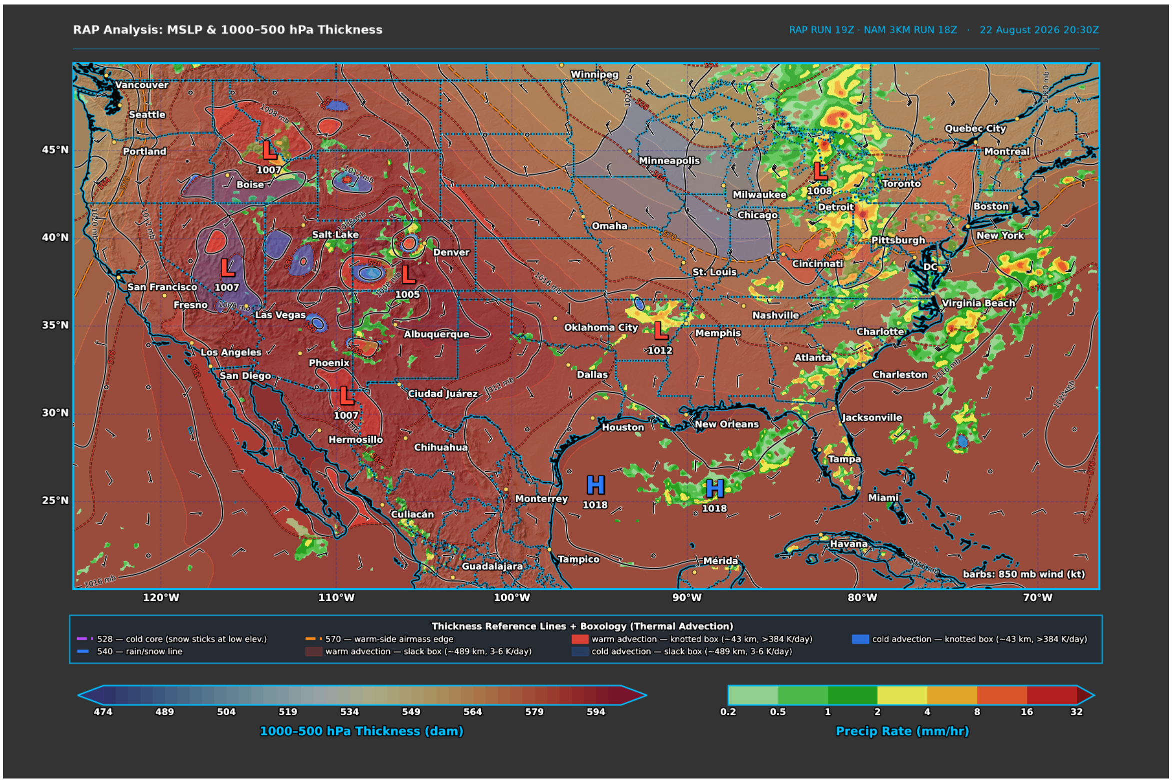

Figure 2: CONUS, RAP 19Z / NAM 3 km 18Z analysis valid 22 August 2026 20:30Z. Knotted boxes mark the sharp thermal gradients along the northern-tier frontal zone and the inter-mountain lows, against a broad slack-box warm sector over the Gulf Coast and Southeast.

Code and data availability

The renderer described here (constants, box labeling, and shading) is implemented as part of the FrostByte meteorological product suite. The governing equations and constants are documented in full, alongside the rest of the suite's physical basis, in the companion technical reference (Lane, 2026).

References

Gary Lackmann. *Midlatitude Synoptic Meteorology.* American Meteorological Society, 2011.

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