Boxology: the isobar–thickness box count as an exact advection integral, and its rendering

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This is a non-peer-reviewed preprint submitted to EarthArXiv. Version 2, September 2026.

Abstract

"Boxology" is the hand-analysis practice of reading thermal advection from the angle at which mean-sea-level-pressure isobars cross 1000–500 hPa thickness contours: steep crossings cut the chart into small quadrilaterals and imply strong advection, while parallel families imply none. This note shows the practice is an identity within geostrophic balance rather than a rule of thumb, and implements it as a rendered chart product. Splitting the layer-mean geostrophic wind into its bottom-surface and thermal parts removes the thermal wind exactly, since it blows along the thickness contours, so layer-mean thickness advection reduces to a Jacobian of the two plotted fields. Treating those fields as coordinates makes that Jacobian a change of variables, so the advection integrated over any quadrilateral bounded by consecutive isobars and consecutive thickness lines is a fixed quantum, independent of the box's shape — a specialization of the solenoid count in Bjerknes' circulation theorem. Box area and advection intensity are therefore reciprocal, which is what the eye is measuring. A renderer is described that labels the actual quadrilaterals by band index and fills each flat at its own quantum-over-area rate on a doubling ladder. Constants, smoothing, edge handling, and the limits of the geostrophic premise — low latitude, strongly curved flow, high terrain — are documented.

Keywords: synoptic analysis; thermal advection; circulation theorem; objective analysis; chart rendering

1 Introduction

"Boxology" is the hand-analysis habit of reading thermal advection off a surface chart by eye: where the isobars cross the thickness (or isotherm) lines at a steep angle, the crossing cuts the plane into small quadrilaterals and the air is turning over fast; where the two families run parallel there are no boxes and nothing is advecting (Lackmann, 2011). The name is weather-school vernacular, taught to enlisted forecasters in their first weeks; the practice it names is standard. This note derives why the technique is exact within geostrophic balance, not a rule of thumb, and documents a renderer that turns the same habit into a field a chart-production pipeline computes and shades rather than one left to the reader's eye.

None of the physics is new. The solenoid count is Bjerknes (1898); the Jacobian form of advection by a geostrophic flow is standard (Holton and Hakim, 2013), and thickness advection read from the surface chart goes back to the development literature (Sutcliffe, 1947; Petterssen, 1956). What this note adds is narrower. First, the corollary, written out for the pair of fields actually drawn on the surface chart, that every isobar–thickness quadrilateral carries the same advection integral, so that box area alone measures local intensity. Second, a renderer that shades those quadrilaterals rather than a smoothed proxy for them. Third, a statement of where the geostrophic premise fails on a routine chart, and the tapers — one implemented, two specified — that withdraw the product there. The counting argument was made redundant, for computation, the day model grids could be evaluated pointwise; it survives because it is the argument the eye is making.

2 The isobar-parallel wind carries the whole signal

Split the 1000–500 hPa layer-mean geostrophic wind into the bottom-surface geostrophic wind plus half the thermal wind, $\mathbf{V} = \mathbf{V}_0 + \frac{1}{2}\mathbf{V}_T$. The thermal wind blows along the thickness contours by construction, so $\mathbf{V}_T \cdot \nabla h \equiv 0$ and

$$-\mathbf{V} \cdot \nabla h = -\mathbf{V}_0 \cdot \nabla h \tag{1}$$

identically. Only the isobar-parallel flow advects thickness — which is why the technique is licensed to ignore everything on the chart except the two contour families it is already reading, and why it is an identity rather than an approximation. The decomposition and (1) hold pointwise for variable f, since f enters $\mathbf{V}_T$ only as a scalar factor and does not turn it off the thickness lines.

3 The Jacobian form

Substituting the geostrophic relation $\mathbf{V}_0 = (g/f)\,\hat{\mathbf{k}} \times \nabla Z_0$ (with Z_0 the 1000 hPa height, i.e. the isobars converted to geopotential meters) into (1) turns the advection into a Jacobian of the two plotted fields:

Model (geostrophic thermal advection)**.**

$$A = -\frac{g}{f}\,J(Z_0, h) = -\frac{g}{f}\,|\nabla Z_0|\,|\nabla h|\,\sin\theta, \qquad J(Z_0, h) = \frac{\partial Z_0}{\partial x}\frac{\partial h}{\partial y} - \frac{\partial Z_0}{\partial y}\frac{\partial h}{\partial x}, \tag{2}$$

with θ the angle at which the isobar crosses the thickness line. Parallel contours ($\theta = 0$) give zero; a perpendicular crossing gives the maximum for that pair of gradient magnitudes. Nothing else in the field enters — which is the same claim (1) made, now in a form a renderer can evaluate pointwise. Written this way, (2) is the familiar Jacobian form of geostrophic advection with the surface height as the streamfunction; the point of (1) is that the surface height is the *right* streamfunction for the layer-mean thickness, not merely a convenient one.

4 Box counting as a change of variables

The reason box area is the thing a hand analyst's eye is measuring follows from treating (Z_0, h) as new coordinates over the plane. The Jacobian of that map is exactly J from (2), so $|J|\,dx\,dy = dZ_0\,dh$, and integrating A over one quadrilateral bounded by two consecutive isobars (δZ_0 apart) and two consecutive thickness lines (δh apart) gives

Identity (one box, one quantum)**.**

$$\iint_{\text{box}} A\,dA = \mp\frac{g}{f}\,\delta Z_0\,\delta h, \qquad A_{\text{box}} = \frac{\delta Z_0\,\delta h}{|J|}, \tag{3}$$

the same magnitude for every box regardless of its shape. This is a specialization of the solenoid count in Bjerknes' circulation theorem to a pair of level-set families rather than an arbitrary closed curve (Bjerknes, 1898; Lackmann, 2011): box count carries the area integral of A over a region (Green's theorem reduces it to a signed count around the boundary, $\iint_R A\,dA = -\frac{g}{f}\oint_{\partial R} Z_0\,dh$); box area carries the local intensity, and the two are reciprocal per (3). That reciprocity is the whole reason the hand technique works by eye, and it is why the renderer described in §6 keys its shading intensity to A (equivalently, to A_{box}^{-1}) rather than to any separately-tuned quantity.

Equation (3) treats f as constant over the box, and the same-quantum statement is therefore exact between boxes at the same latitude. The quantum scales as $1/f = 1/(2\Omega \sin\varphi)$, so an identically spaced box at 30°N carries about 1.7 times the integral of one at 60°N. Within a single box the fractional variation of the quantum is $\cot\varphi\,\Delta y/a$ for a meridional extent Δy and Earth radius a: under 1% for the 43 km boxes at the top of the ladder in §6, and about 8% for the 490 km boxes at its floor at 45°N. Equation (3) is therefore to be read box by box, each with its own local quantum, f taken at the box's centroid. The Green's-theorem count around a large region inherits the same caveat and should be read with f at the region's centroid.

5 Temperature form

The hypsometric relation $h = (R_d/g)\,\bar{T}\ln 2$ for the 1000–500 hPa layer converts (2) to a layer-mean temperature tendency,

$$\frac{\partial \bar{T}}{\partial t} = -\frac{g^2}{f R_d \ln 2}\,J(Z_0, h), \tag{4}$$

returned in $\mathrm{K\,d^{-1}}$. Z_0 is recovered from MSLP via the standard $8\,\mathrm{m\,hPa^{-1}}$ conversion ($p \approx 1000 + Z_{1000}/8$), so the overlay reads exactly the field the isobars on the chart were drawn from.

Quantity	Symbol / value	Note
Layer coefficient	$g^2/(R_d \ln 2) \approx 0.4833\ \mathrm{K\,s^{-2}}$	divide by local f for $\mathrm{K\,s^{-1}}$ per unit J
MSLP $\to Z_0$	$8\,\mathrm{m\,hPa^{-1}}$	$p \approx 1000 + Z_{1000}/8$
Equator fade	12–20°	tapered, not cut, to 0
Curvature fade	Ro = 1–3	specified in §5.1; not yet in the renderer
Terrain fade	1500–2200 m	specified in §5.1; not yet in the renderer
House contour intervals	$\delta p = 4\,\mathrm{hPa}$, $\delta h = 6\,\mathrm{dam}$	box-size diagnostic only

5.1 Where the geostrophic premise fails

Equation (2) inherits every assumption of geostrophic balance, and three regimes on a routine chart fall outside it. Each should be tapered smoothly to zero rather than masked at a hard edge, because a hard edge draws a false analysis boundary across the map. The first taper is in the renderer; the second and third are specified here for its next version, and Figures 1 and 2 show what they will remove.

Low latitude. $f \to 0$ makes (2) blow up, and the geostrophic relation itself stops holding well before the true equator. The field is tapered through 12–20° latitude.

Strongly curved flow. In a tropical cyclone, or any tight closed low, the surface flow is in gradient rather than geostrophic balance, and geostrophy overestimates the isobar-parallel wind by a factor that grows with the curvature Rossby number

$$\mathrm{Ro} = \frac{|\mathbf{V}_0|\,|\kappa|}{f} = \frac{g}{f^2}\,|\nabla Z_0|\,|\kappa|, \qquad \kappa = \nabla\cdot\left(\frac{\nabla Z_0}{|\nabla Z_0|}\right), \tag{5}$$

where κ is the curvature of the isobars, computable from the same field as everything else. In gradient balance the cyclonic wind is $|\mathbf{V}_0|\,(\sqrt{1+4\mathrm{Ro}}-1)/(2\mathrm{Ro})$: about $0.75\,|\mathbf{V}_0|$ for $\mathrm{Ro}\approx 0.5$, typical of the center of a mature midlatitude cyclone and smaller still along its frontal zones where the boxes live, but under $0.1\,|\mathbf{V}_0|$ at the $\mathrm{Ro}\gg 10$ a tropical cyclone produces. Since only $\mathbf{V}_0$ enters (1), the box rate inherits that overestimate directly, on top of the small-f amplification a tropical system already sits in. An axisymmetric warm core would in fact give $J = 0$ — concentric isobars and thickness lines are parallel — but real cores are sheared off their pressure centers, the two families cross at small angles, and with gradients this steep even small angles pin the ladder. The field is to be tapered from $\mathrm{Ro} = 1$ to $\mathrm{Ro} = 3$, which should leave midlatitude cyclones essentially untouched and remove tropical cyclones and tight thermal lows; the thresholds are house values to be checked against cases.

High terrain. MSLP and the 1000 hPa height are extrapolations below ground over most of a continent. That is harmless at a few hundred meters and not at the elevation of the Great Basin or the Colorado Plateau, where a kilometer or more of fictitious column is being assumed and the reduced pressure gradient inherits the terrain's temperature structure. The field is to be tapered with surface elevation, taken from the model's own orography, by the same construction as the latitude taper: full weight at or below 1500 m, zero at or above 2200 m. No single elevation separates the plains east of the Rockies, where post-frontal cold advection is real and matters, from the Great Basin at similar elevation, so the ramp is a judgment call and will be reported together with what survives it.

The gradient-wind ratio is deliberately not applied as a correction. It would repair the tropical cyclones and destroy (3), since the correction varies within a box and the thermal wind no longer drops out cleanly. The identity is kept exact where its premise holds, and the display is withdrawn where it does not.

6 Rendering: the boxes themselves

The renderer shades the actual quadrilaterals, not a smooth field standing in for them. A box is one connected region over which both band indices $\lfloor p/\delta p\rfloor$ and $\lfloor h/\delta h\rfloor$ are constant: the pair changes exactly where a contour is drawn, so such a region is the polygon bounded by the four lines around it, and the box-labeling routine recovers every box on the chart without tracing or intersecting a single contour. Connectivity is 4-way, not 8-way — two boxes touching only at a corner meet at an isobar/thickness crossing, which is the one place they are guaranteed to be different boxes, and 8-way connectivity welds them into a blob straddling the very feature being measured.

Each box is then filled flat, at the rate (3) assigns it: its own quantum over its own area, so a slack box is weak and a tight one intense. The shading routine bins that rate into eight bands, each edge doubling the one below ($3, 6, 12, \ldots, 384\,\mathrm{K\,d^{-1}}$). Because $A_{\mathrm{box}} \propto A^{-1}$, box side scales as $A^{-1/2}$, so a doubling ladder in A is a $\sqrt{2}$ ladder in box side — roughly 490 km at the floor down to a 43 km knot at the top, the same visual step at every rung — and fill opacity climbs with it, ending at 0.82 rather than 1 so the contours and any underlying field stay legible through the deepest box.

Boxes touching the frame are discarded. Those are cut boxes whose measured area is only the part that fits on the chart, so they would read as far tighter than they are; a hand analyst does not count a box running off the map either.

6.1 Why not a filled contour of A

Equation (2) is defined pointwise, so a filled contour of it is available and was the first implementation. It renders the same quantity and is not the technique: it smooths across the crossings instead of stopping at them, and at chart scale over the thickness ramp it read as a faint wash rather than as boxes. The per-pixel version survives as a hatched overlay option; the production path draws the discrete boxes.

6.2 Smoothing

The advection field is a product of two gradients, so it amplifies grid noise the eye never sees on the chart itself. The defaults match what the isobar and thickness fields are already smoothed to on the chart ($\sigma = 3.0$ on MSLP, thickness pre-smoothed at $\sigma = 1.5$ upstream) so the shading never disagrees with the crossing angle a reader can actually see drawn.

7 Example output

Figures 1 and 2 show the renderer running on live analyses, boxology shading drawn over the same MSLP isobars and 1000–500 hPa thickness contours the technique reads by hand. Both are early-September cases in the Northern Hemisphere. Figure 3 shows the product on the same day in the Southern Hemisphere winter, where two deep Southern Ocean cyclones draw the cold- and warm-advection couplets the technique exists to find.

Because the product is zero wherever the flow runs along the thickness lines, it separates systems that are baroclinically active now — which draw a warm/cold couplet — from equivalent-barotropic ones, which draw nothing: tropical cyclones in a near-uniform tropical thickness field, and occluded or cut-off lows whose thermal field has wrapped up concentric with their isobars. That is a statement about current thermal advection, not about a system's structure or history, and it is meaningful only inside the validity envelope of §5.1; boxes drawn on a tropical system inside the taper bands are the failure mode, not a diagnosis. What the boxes register on a cyclone is thermal asymmetry across it: thickness lines crossing the isobars on one flank and not the other. That is the quantity the cyclone phase space of Hart (2003) measures with its parameter B, the storm-relative thickness difference between the right and left of the track, and in that sense the box field is a chart-scale proxy for B wherever the geostrophic premise holds. It is not a proxy for core type. Tropical, subtropical, and extratropical cyclones are distinguished in that framework by the sign of the thermal wind through the column — whether the core is warm or cold aloft — and a surface-to-500 hPa thickness crossing carries no information about that. A symmetric cold-core low draws nothing, and a warm-core storm sheared off its center draws boxes; neither result classifies the system. The most useful form of the distinction is therefore extratropical transition: a recurving tropical cyclone draws nothing until it reaches the baroclinic zone and acquires a thermal asymmetry, at which point the couplet appears, poleward of the tapers where the number means something. Tropical Storm Krovanh, south of Japan as this version is posted, is forecast to make that transition within days; Figure 2 is its first frame, and the sequence will follow in a later version of this note.

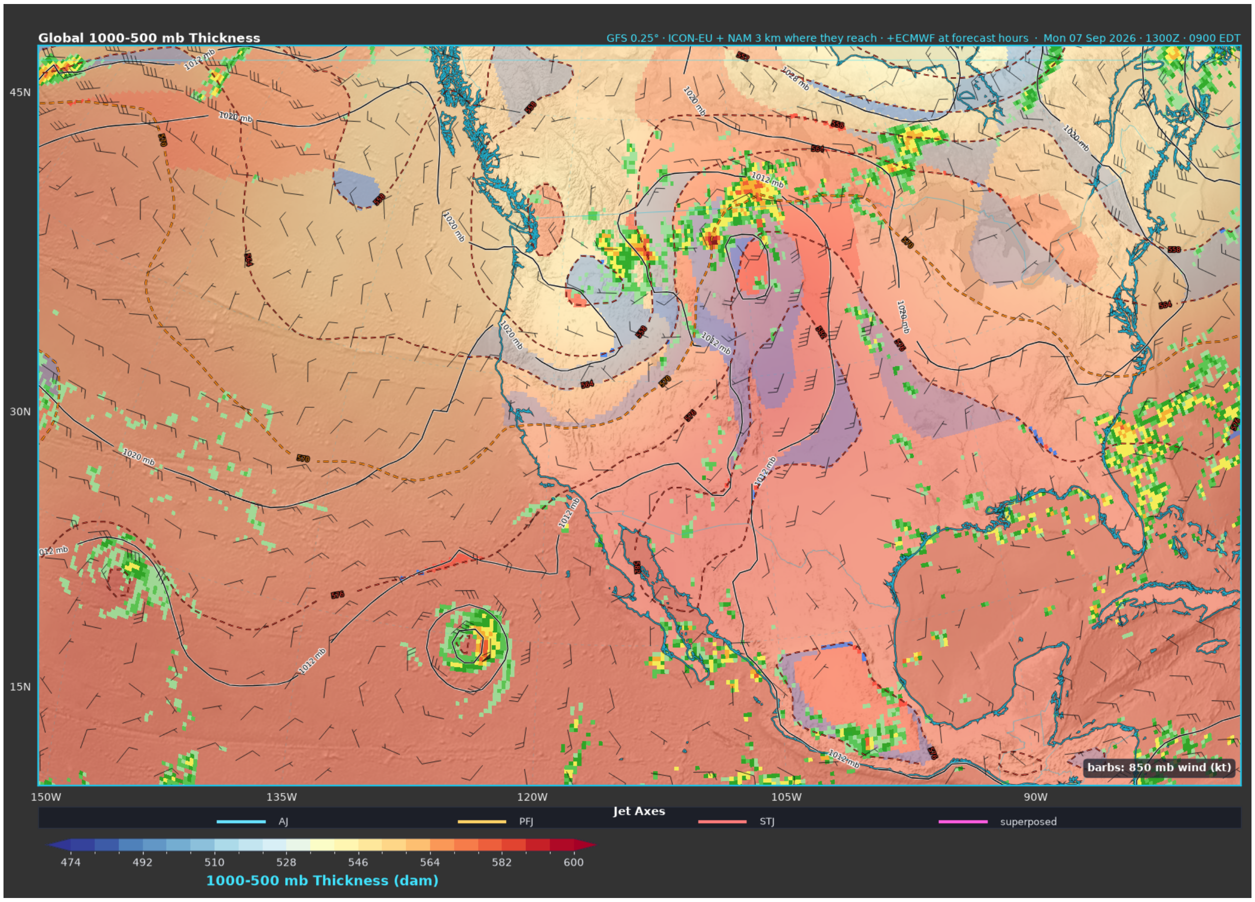

Figure 1: Western North America and the eastern Pacific, GFS 0.25° analysis (NAM 3 km where it reaches) valid 07 September 2026 1300Z. Cold-advection boxes line the eastern slope of the Rockies and the adjacent plains from roughly 40 to 28°N under northerly 850 hPa flow behind the front. The two tropical cyclones near 20°N draw no boxes: the thickness field around them is nearly uniform, and they sit at the top of the low-latitude taper. The tiles over the Great Basin and the Colorado Plateau lie over terrain high enough that the reduction to 1000 hPa is a long extrapolation, and are suspect until the terrain taper specified in §5.1 is applied.

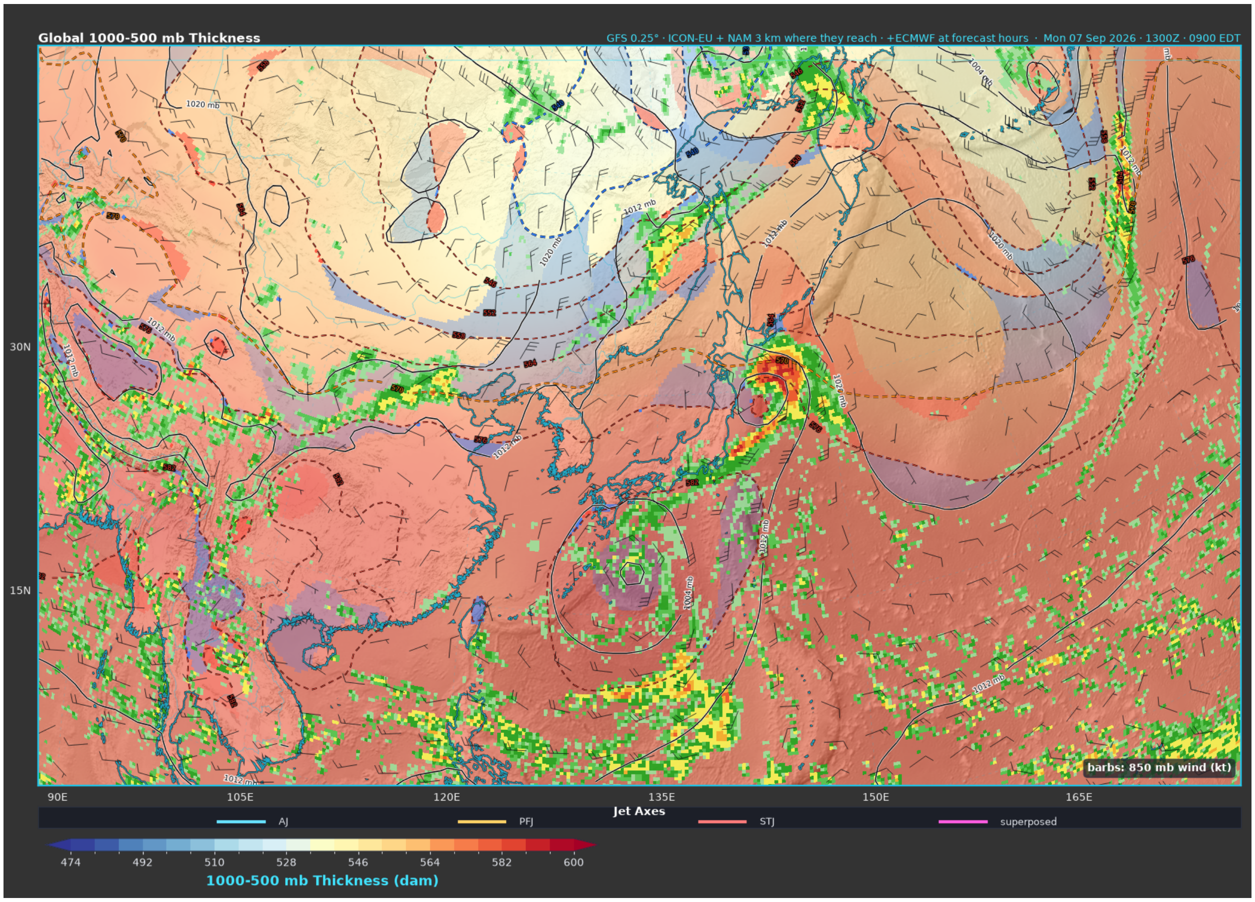

Figure 2: East Asia and the western Pacific, GFS 0.25° analysis valid 07 September 2026 1300Z. Tropical Storm Krovanh lies over the Philippine Sea south of Japan with its circulation drawn out northeastward as a long trough toward Honshu. The core, inside the curvature limit, draws little. The northeastern end of that envelope, near 29°N, 143°E, draws a couplet — cold-advection tiles on its southwestern flank, warm-advection tiles on its northeastern flank — at a latitude where the product is valid: the poleward part of the storm's circulation has reached the thickness gradient south of Japan and gone thermally asymmetric ahead of the core, which is the first signature of extratropical transition (§7). The cold-advection boxes around the tropical low near 16°N, 131°E are the failure mode the curvature taper specified in §5.1 will remove: inside the low-latitude band and at curvature Rossby numbers where geostrophy does not hold, they carry no information. The cold-advection band under the trough over northeast China and Korea is inside the premise.

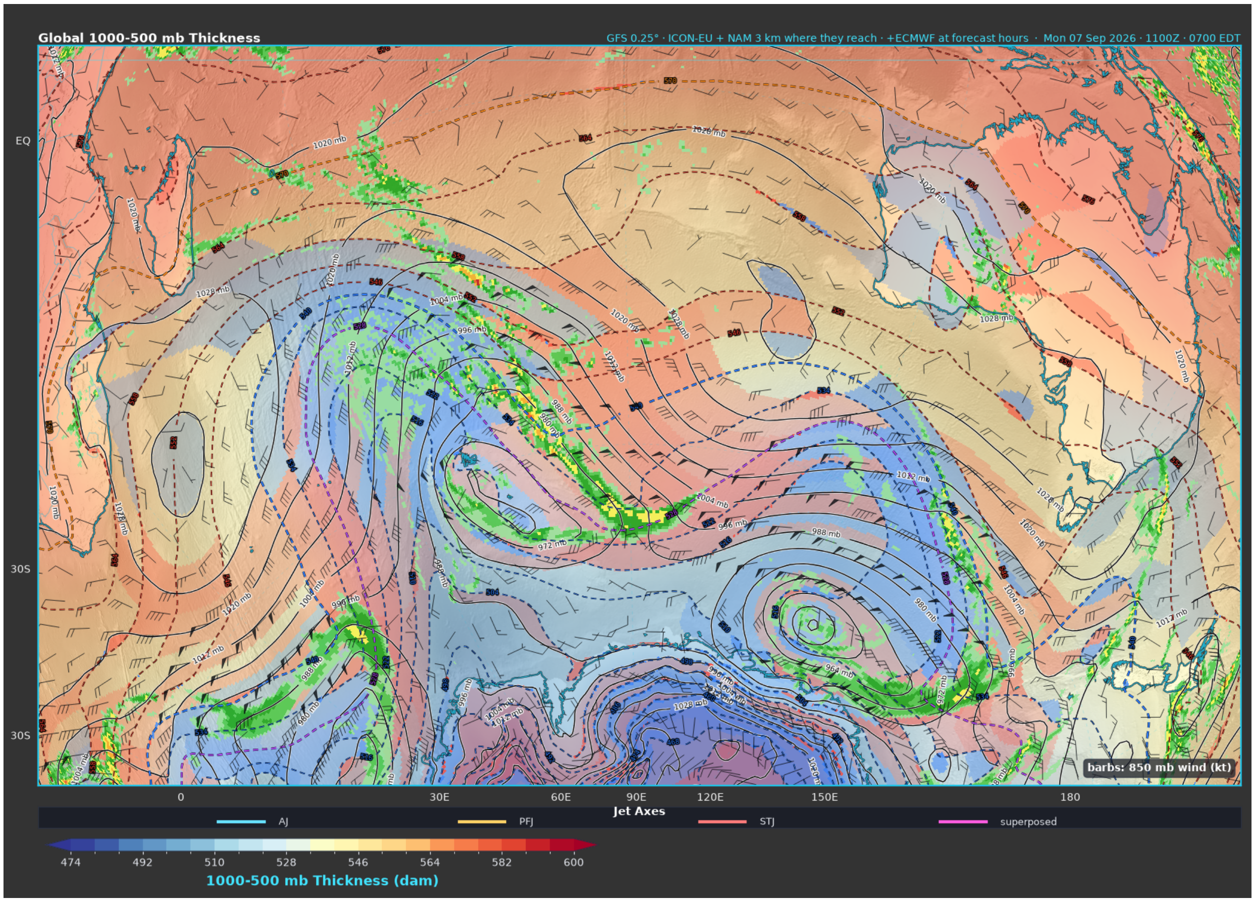

Figure 3: Southern Hemisphere, Africa to New Zealand, GFS 0.25° analysis valid 07 September 2026 1100Z: late winter south of the equator. Two deep Southern Ocean cyclones, one south of Tasmania and one in the South Indian Ocean toward Kerguelen, each draw the couplet the technique exists to find: cold-advection boxes on the western flank, where southerlies carry Antarctic air equatorward, and warm-advection boxes ahead, where northerlies push the thickness lines poleward — cold west and warm east, as it must be in this hemisphere. The ridge over Australia draws nothing, and the tropics across the top of the chart are blank under the low-latitude taper.

8 Consistency checks

Two checks are cheap on any grid and will accompany the peer-reviewed version of this note; a third checks the implementation rather than the physics.

Thermal-wind cancellation. Compute $-\mathbf{V} \cdot \nabla h$ with the full layer-mean geostrophic wind $\mathbf{V} = \mathbf{V}_0 + \frac{1}{2}\mathbf{V}_T$ and with $\mathbf{V}_0$ alone, on the same smoothed fields. By (1) the two agree to discretization error. The residual measures how far the analysis's thickness and thermal-wind fields are from mutual consistency on the grid, not any failure of (1); report its RMS and maximum as a fraction of the field's RMS.

Interval invariance. Re-render at $\delta p = 2$ and 8 hPa and $\delta h = 3$ and 12 dam. The box rate — quantum over area — is invariant to the intervals in the limit of small boxes, so the band assigned to a location should not change; report the fraction of the chart whose band moves by one rung or more.

Box rate against the pointwise field. The rate assigned to each box should equal the box-mean of the pointwise A from (2) up to the smoothing difference between the two paths. Disagreement here indicates a labeling or area error, not a physical one.

Acknowledgments

The author first met the technique, and its name, in the weather forecaster course at Keesler Air Force Base and in the Marine Corps follow-on training that succeeds it, and thanks SSgt Matthew Cline, USMC, who taught the latter and made sure it stuck. The author declares no competing interests and received no funding for this work.

Code and data availability

The renderer described here (constants, box labeling, tapers, and shading) is implemented as part of the FrostByte meteorological product suite. The governing equations and constants are documented in full, alongside the rest of the suite's physical basis, in the companion technical reference (Lane, 2026). The analyses in Figures 1 and 2 are operational GFS 0.25° fields, with NAM 3 km blended where it reaches, retrieved from their operational distribution servers at the times stated in the captions. The Southern Hemisphere analysis in Figure 3 is the same GFS 0.25° product.

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