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A Reduced-Order Dynamical Model for the Ignition of Diver-Induced Cohesive Silt-Out on Sloping Beds

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Abstract

A downward fin thrust near a cohesive seabed lofts sediment. Whether this cloud settles or organizes into a self-sustaining down-slope current determines if a diver loses visibility briefly or catastrophically. We reduce the layer-averaged balances of fluid mass, sediment mass, and momentum to a temporal slab of fixed thickness in the weakly entraining limit. This yields an autonomous planar vector field in dimensionless near-bed speed and suspended load, governed by three dimensionless groups: drag against settling, erosion strength, and near-bed concentration. The field possesses a quiescent equilibrium at the origin and, above a critical bed slope obtained in closed form as the ratio of near-bed concentration times drag-settling number to erosion-strength number, an interior saddle equilibrium. The saddle's stable manifold partitions the state space into basins of decay and unbounded growth, establishing the exact threshold between self-limiting and igniting disturbances. On a flat bed, the origin attracts globally for all admissible erosion laws. For representative silt, the ignition threshold is near a 17-degree bed angle. The closed-form structure is confirmed numerically to machine precision. A closure study demonstrates the critical slope is robust in existence but closure-dependent in value, identifying the linear erosion law as the conservative choice. The model thus isolates bed slope as the controlling parameter for ignition.

Keywords: autosuspension; cohesive sediment resuspension; diver-induced silt-out; ignition threshold; reduced-order dynamical model; saddle separatrix.

1 Introduction

The sudden loss of visibility that follows disturbance of a fine-grained bottom, known among divers as a silt-out, is a recurrent hazard of underwater work. A single downward thrust of a fin near a cohesive seabed resuspends a cloud of fine sediment that can reduce the visual range to a few centimeters within seconds, with immediate consequences for orientation, gas management, and safe ascent. The governing question is not whether a disturbance lifts sediment, which it plainly does, but whether the lofted cloud settles back to the bed once the forcing stops or instead organizes into a persistent, self-feeding down-slope flow. The two outcomes differ by orders of magnitude in duration and extent, and the distinction between them is a dynamical one: it is the same decay-against-runaway alternative that the sediment-transport literature studies under the name of autosuspension.

A sediment-laden current descending a slope can maintain its own suspension when the potential energy released by the settling load supplies the turbulent energy required to keep the grains aloft, which is the energetic criterion for autosuspension [1]. Phase-plane analysis of the coupled evolution of the current speed and its effective density separates the trajectories that decay from those that run away and furnishes a criterion for self-sustaining motion [14]. An erosive gravity current possesses an unstable ignitive equilibrium below which disturbances die out and above which they accelerate and entrain sediment toward a distinct, stable catastrophic state, so that the ignitive equilibrium itself defines the threshold for a self-sustaining flow [15]. These results rest on layer-averaged balances of fluid mass, sediment mass, and mean momentum for a turbid underflow [16, 12], closed by empirical laws for the exchange of sediment with the bed for both noncohesive and cohesive material [17, 3]. The autosuspension threshold is supported experimentally, though its precise location is sensitive to the erosion law adopted [19].

The diver-induced silt-out lies within this framework, since the resuspended cloud on a sloping bottom is a small, transient gravity current [18, 12] whose fate is governed by the same competition between slope-driven acceleration and settling that controls the ignition of turbidity currents. What the existing treatments do not supply is a minimal model at the scale of a diver disturbance that connects the resuspension event to the ignition criterion without the apparatus of a spatially resolved field solver [13], an apparatus whose turbulence closure is difficult to defend at diver-scale Reynolds numbers, whose output is a set of fields rather than a transparent

statement about the existence of a threshold, and whose layer-averaged reductions themselves remain under active scrutiny [9]. A complementary line of work has instead built compact, single-purpose idealized solvers whose numerical core is legible and whose analysis emphasizes verified structure over calibrated prediction, an approach that has been applied to shear-driven instability [6], collective animal motion [7], nonlinear dispersive waves [11], and wave attenuation in coastal vegetation [8], and in which dynamical-systems and information-theoretic diagnostics repeatedly expose structure that first-moment descriptions leave hidden. A reduced-order treatment of diver-induced silt-out belongs to that line, because the outcome of interest is precisely the qualitative one, decay against ignition, that a low-dimensional dynamical system is designed to resolve.

This paper develops and analyzes such a model. The layer-averaged balances of fluid mass, sediment mass, and momentum for a slab of resuspended cloud are reduced, under a small set of explicitly stated and justified assumptions, to an autonomous planar vector field for a dimensionless near-bed speed and a dimensionless suspended load, governed by three dimensionless groups with the bed slope as the control parameter. The equilibria of that field, their linear stability, the critical slope at which an ignitive equilibrium appears, the degenerate structure of the quiescent state, the global attraction of the flat bed, and the invariant manifold that separates decay from ignition are all obtained in closed form. The closed-form structure is then confirmed numerically, its qualitative content is shown to be robust to the erosion closure, and its physical reading is developed with explicit attention to the limitations that bound its scope. The governing equations and their reduction are derived first, the numerical and diagnostic methods are described next, the computed structure is reported in full, and the results are interpreted and delimited in the discussion.

2 Methods

2.1 Model Description

The physical setting is a diver-scale disturbance of a cohesive bottom inclined at an angle θ to the horizontal. A downward fin thrust resuspends a volume of fine sediment into a near-bed cloud; once the thrust ceases, the cloud is left to evolve under gravity, drag, settling, and its own capacity to entrain further sediment from the bed. The cloud is represented as a single layer-averaged slab of sediment-laden fluid moving along the bed, and its evolution is traced in time from the instant the external forcing stops. Figure 1 shows the physical configuration and the agencies acting on the slab. The derivation proceeds from the integral balances for the slab, introduces the closures that render the balances a closed system, reduces the system to two state variables through a justified thickness assumption, nondimensionalizes it, and analyzes the resulting planar vector field in closed form.

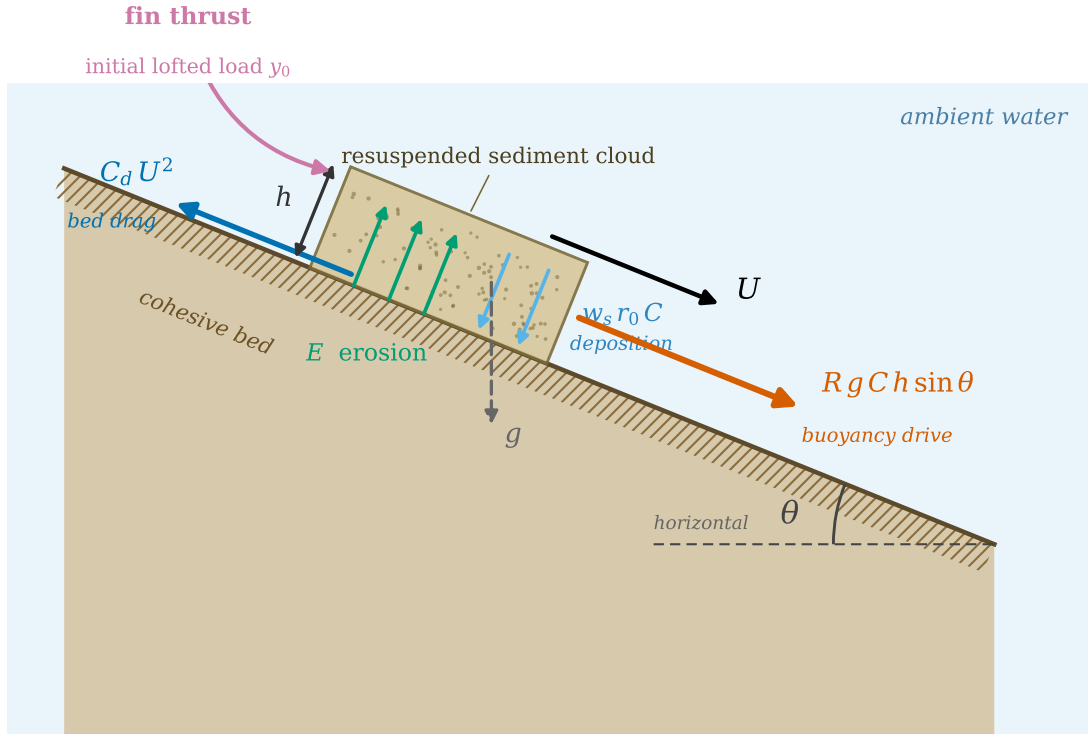


Figure 1: Physical configuration of the diver-induced silt-out. A fin thrust lofts fine cohesive sediment into a near-bed slab of thickness h that moves down a bed inclined at angle θ . The down-slope component of the reduced gravity, $R g C h \sin \theta$, drives the slab; the bed drag $C_d U^2$ retards it; the slab entrains sediment from the bed through the erosion flux E and returns it through the deposition flux $w_s r_0 C$. The initial lofted load y_0 set by the thrust is the disturbance whose fate, decay or ignition, the model determines.

Let $U(t) \geq 0$ denote the layer-averaged down-slope speed of the slab at time t , let $h(t) > 0$ denote its thickness, and let $C(t) \in [0, 1]$ denote its layer-averaged volumetric sediment concentration, each defined per unit area of bed. Let ρ_0 be the density of the clear ambient water, ρ_s the density of the sediment grains, and g the gravitational acceleration. A suspension of concentration C carries an excess density $C(\rho_s - \rho_0)$ over the ambient fluid, and the buoyant acceleration it experiences along the bed, referred to the ambient density under the Boussinesq approximation, is the reduced gravity

$$g' = g \frac{\rho_s - \rho_0}{\rho_0} C = R g C, \quad R \equiv \frac{\rho_s - \rho_0}{\rho_0}, \quad (1)$$

where R is the submerged specific gravity of the sediment, close to 1.6 for quartz grains in seawater. The three integral balances for the slab, per unit bed area, express conservation of ambient fluid, of suspended sediment, and of down-slope momentum, and follow the layer-averaged description of buoyancy-driven sediment flows [18, 12]. Ambient water is entrained across the upper interface at a rate proportional to the speed through a dimensionless entrainment coefficient e_w , which thickens the layer,

$$\frac{dh}{dt} = e_w U. \quad (2)$$

The suspended sediment per unit bed area, the load $q \equiv Ch$, changes through the difference between erosion of the bed and deposition onto it, both scaled by the settling velocity w_s ,

$$\frac{d}{dt}(Ch) = w_s (E - r_0 C), \quad (3)$$

where E is a dimensionless entrainment function of the bed and $r_0 C$ is the near-bed concentration that feeds deposition, r_0 being the ratio of the near-bed to the layer-averaged concentration. The down-slope momentum per unit bed area, $\rho_0 h U$ under the Boussinesq approximation, is driven by the down-slope component of the buoyancy force and retarded by the bed shear stress τ_b ,

$$\rho_0 \frac{d}{dt}(h U) = \rho_0 R g C h \sin \theta - \tau_b. \quad (4)$$

Equations (2)–(4) are the temporal layer-averaged balances for the slab; they are the box-model counterpart of the spatially developing three-equation model of a turbid underflow, whose formulation and range of validity are established elsewhere [16, 9].

Three closures render the balances a closed system. The bed shear stress is taken quadratic in the layer speed through a single drag coefficient C_d , which identifies the skin friction that mobilizes sediment with the form drag that retards the layer,

$$\tau_b = \rho_0 C_d U^2. \quad (5)$$

Combining (5) with a critical shear stress τ_c below which the bed does not erode defines a natural speed scale, the erosion-threshold speed U_c , at which the bed shear equals the critical stress,

$$\frac{\tau_b}{\tau_c} = \frac{\rho_0 C_d U^2}{\tau_c} = \left(\frac{U}{U_c} \right)^2, \quad U_c \equiv \sqrt{\frac{\tau_c}{\rho_0 C_d}}, \quad (6)$$

so that the shear ratio governing erosion is exactly the square of the speed measured in units of U_c . Erosion follows an excess-shear law of Partheniades type, in which sediment is entrained only when the bed shear exceeds the critical stress and at a rate proportional to the fractional excess [17, 3],

$$E = \alpha \max\left(\frac{\tau_b}{\tau_c} - 1, 0\right) = \alpha \max\left(\left(\frac{U}{U_c}\right)^2 - 1, 0\right), \quad (7)$$

where the ramp $\max(\cdot, 0)$ enforces that no erosion occurs below threshold and the dimensionless erosion magnitude α is fixed by the Partheniades erosion-rate parameter M , of dimension eroded mass per unit bed area per unit time, through $\alpha = M/(\rho_s w_s)$, so that $w_s \alpha$ is the volumetric entrainment rate at unit fractional excess [17]. The entrainment coefficient in the fluid balance closes against the bulk Richardson number $Ri = R g C h / U^2$, with e_w a decreasing function of Ri that is largest for weakly stratified, energetic currents and smallest for the strongly stratified, slow currents of interest here [16].

A diver-induced cloud is dense and slow: its near-bed concentration is high, its speed is at most comparable to the erosion-threshold speed U_c of order 10^{-1} m s^{-1} , and its bulk Richardson number is therefore large, so that the entrainment coefficient $e_w(Ri)$ is small. In the strongly stratified limit $e_w \rightarrow 0$, equation (2) gives $dh/dt = 0$, and the thickness remains at the value h_0 set by the initial disturbance,

$$h(t) \equiv h_0. \quad (8)$$

This is the assumption that makes the model minimal: it removes the thickness as a dynamical degree of freedom and retains the two variables, speed and load, whose competition controls ignition. Its physical content is that the cloud entrains ambient water slowly compared with the timescales on which its speed and load evolve, which is the appropriate limit for a slow, dense, near-bed layer and is conservative in the sense established below. Under (8) the load is $q = Ch_0$, and dividing the momentum balance (4) by ρ_0 while using (5) gives

$$h_0 \frac{dU}{dt} = R g \sin \theta q - C_d U^2, \quad (9)$$

while the sediment balance (3), using (7) and (6), becomes

$$\frac{dq}{dt} = w_s \alpha \max\left(\left(\frac{U}{U_c}\right)^2 - 1, 0\right) - w_s r_0 \frac{q}{h_0}. \quad (10)$$

Equations (9) and (10) are a closed pair of autonomous ordinary differential equations for U and q .

The pair is made dimensionless by referring the speed to U_c , the load to a scale q_c fixed below, and the time to the settling timescale of the cloud. Introduce

$$x \equiv \frac{U}{U_c}, \quad y \equiv \frac{q}{q_c}, \quad s \equiv \frac{t}{t_{\text{settle}}}, \quad t_{\text{settle}} \equiv \frac{h_0}{w_s}, \quad (11)$$

in which x is a scalar speed rather than a velocity, since the slab moves down the slope and only the magnitude enters the balances, and t_{settle} is the time for the settling velocity to clear a cloud of thickness h_0 . With $U = U_c x$, $q = q_c y$, and $d/dt = (w_s/h_0) d/ds$, the momentum balance (9) becomes

$$h_0 \frac{U_c w_s}{h_0} \frac{dx}{ds} = R g \sin \theta q_c y - C_d U_c^2 x^2, \quad (12)$$

and dividing through by $U_c w_s$ gives

$$\frac{dx}{ds} = \frac{R g \sin \theta q_c}{U_c w_s} y - \frac{C_d U_c}{w_s} x^2. \quad (13)$$

The load scale is fixed by requiring the buoyancy coefficient in (13) to reduce to the bed slope alone, which selects

$$q_c \equiv \frac{U_c w_s}{R g} \implies \frac{R g \sin \theta q_c}{U_c w_s} = \sin \theta \equiv S, \quad (14)$$

so that the momentum balance takes its final form

$$\frac{dx}{ds} = S y - P x^2, \quad P \equiv \frac{C_d U_c}{w_s}, \quad (15)$$

where S is the dimensionless bed slope and P is the drag-settling number, the ratio of the bed drag at threshold speed to the settling velocity. Applying the same substitution to the sediment balance (10), and dividing through by $q_c w_s/h_0$, gives

$$\frac{dy}{ds} = \frac{h_0}{q_c} \alpha \max(x^2 - 1, 0) - r_0 y, \quad (16)$$

and substituting the load scale (14) into the coefficient $h_0 \alpha/q_c$ yields

$$\frac{dy}{ds} = A \max(x^2 - 1, 0) - r_0 y, \quad A \equiv \frac{\alpha h_0 R g}{w_s U_c}, \quad (17)$$

where A is the erosion-strength number, the capacity of the bed to feed the suspension, and r_0 retains its meaning as the near-bed concentration ratio. Equations (15) and (17) are the reduced model: an autonomous planar vector field $\mathbf{f}(x, y) = (Sy - Px^2, A \max(x^2 - 1, 0) - r_0 y)$ on the state (x, y) , governed by the three dimensionless groups P , A , and r_0 , with the slope S as control parameter. The field is continuous everywhere and smooth away from the line $x = 1$, across which the erosion source has a corner but no jump.

The physical quadrant $Q = \{x \geq 0, y \geq 0\}$ is forward-invariant. On the edge $x = 0$ the speed derivative is $dx/ds = Sy \geq 0$, which points into Q , and on the edge $y = 0$ the load derivative is $dy/ds = A \max(x^2 - 1, 0) \geq 0$, which also points into Q ; hence a trajectory that starts with nonnegative speed and load remains in Q for all s . On a flat bed, $S = 0$, the speed equation (15) decouples and reduces to $dx/ds = -Px^2$, whose solution from any $x(0) = x_0 \geq 0$ is

$$x(s) = \frac{x_0}{1 + P x_0 s} \xrightarrow{s \rightarrow \infty} 0, \quad (18)$$

so the speed decreases monotonically and falls below the erosion threshold $x = 1$ in finite time. Once $x < 1$ the erosion source in (17) vanishes for every closure that is zero below threshold, and the load obeys $dy/ds = -r_0 y$ and decays exponentially. The origin is therefore globally attracting on a flat bed, for any disturbance and any admissible erosion law: a level cohesive bottom cannot sustain a current, and this conclusion does not depend on the form of E .

The equilibria of (15)–(17) are the intersections of the nullclines. The speed nullcline $dx/ds = 0$ is the parabola $y = Px^2/S$, and the load nullcline $dy/ds = 0$ is $y = (A/r_0) \max(x^2 - 1, 0)$, which coincides with the axis $y = 0$ for $x \leq 1$ and rises as $(A/r_0)(x^2 - 1)$ for $x > 1$. The origin $(x, y) = (0, 0)$ is always an equilibrium. In the erosion-on region $x > 1$, an interior equilibrium (x^*, y^*) satisfies both nullclines; eliminating y through $y = Px^2/S$ and substituting into $A(x^2 - 1) = r_0 y$ gives $A(x^2 - 1) = (r_0 P/S) x^2$, whence

$$(x^*)^2 = \frac{A S}{A S - r_0 P}, \quad y^* = \frac{P (x^*)^2}{S}. \quad (19)$$

This root is real, and exceeds unity so as to lie in the erosion-on region and be physically admissible, if and only if the denominator is positive, $AS > r_0 P$. That condition defines the critical slope

$$S_{\text{crit}} = \frac{r_0 P}{A}, \quad (20)$$

below which the interior equilibrium is absent and above which it exists; at $S = S_{\text{crit}}$ the ignitive speed x^* diverges, and as $S \rightarrow 1$ it decreases toward a value just above the erosion threshold. The appearance of the interior equilibrium at S_{crit} is a transcritical transition in the control parameter S [20].

The linear stability of each equilibrium follows from the Jacobian of the vector field, which in the erosion-on region $x > 1$ is

$$J(x, y) = \begin{pmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} \end{pmatrix} = \begin{pmatrix} -2Px & S \\ 2Ax & -r_0 \end{pmatrix}, \quad (21)$$

with the lower-left entry replaced by zero in the erosion-off region $x < 1$. At the interior equilibrium the trace and determinant are

$$\text{tr } J(x^*, y^*) = -2Px^* - r_0 < 0, \quad \det J(x^*, y^*) = 2x^*(r_0P - AS). \quad (22)$$

Wherever the interior equilibrium exists, $AS > r_0P$, so the determinant is negative. A planar equilibrium with negative determinant has real eigenvalues of opposite sign and is a saddle [20]; the interior equilibrium is therefore an unstable ignitive state, not a reachable steady flow. The eigenvalues are the roots of $\lambda^2 - (\text{tr } J)\lambda + \det J = 0$, one positive and one negative, and the eigenvector associated with the negative eigenvalue spans the stable direction used below to construct the separatrix.

The origin requires separate treatment because its linearization is degenerate. Evaluating (21) in the erosion-off region at $(0, 0)$ gives the matrix with rows $(0, S)$ and $(0, -r_0)$, whose eigenvalues are 0 and $-r_0$; the zero eigenvalue arises because the drag term $-Px^2$ has no linear part at $x = 0$. The nonlinear flow nonetheless attracts the origin from within Q : for any trajectory with $x < 1$ the load decays as $dy/ds = -r_0y$, and the speed obeys $dx/ds = Sy - Px^2$ with a source Sy that decays exponentially, so $x \rightarrow 0$ as well. The origin is thus an attracting quiescent state, and the reduced model is bistable above S_{crit} in the sense that trajectories tend either to the origin or to unbounded growth, the two basins being separated by the stable manifold of the saddle [20]. That manifold, the separatrix, is the exact boundary between a self-limiting disturbance and an igniting one, and its intersection with the load axis at $x = 0$ is the least lofted load, at zero initial speed, that ignites; this critical lofted load, as a function of slope, is the safe-kick envelope of the model.

2.2 Numerical Implementation

The reduced model is implemented as `siltout`, a small internal `Python` package that solves no field equations and stores no simulation archive, evaluating instead the vector field of (15)–(17), its equilibria, its Jacobian, and the diagnostics derived from them. All arithmetic is performed in IEEE 754 double precision through `NumPy` [5]; the numerical integration, root finding, and eigenvalue computations are provided by `SciPy` [21]; and the figures are rendered with `Matplotlib` [10]. Each figure and report is produced by an independent script that imports the package, so any result can be regenerated in isolation.

The vector field is evaluated directly from its definition, with the erosion source implemented as the sharp ramp $A \max(x^2 - 1, 0)$ by default. For the many-trajectory basin classifications an optional smooth surrogate replaces the ramp by the softplus of width ε ,

$$E_\varepsilon(x) = A\varepsilon \log\left(1 + e^{(x^2-1)/\varepsilon}\right), \quad E_\varepsilon(x) \xrightarrow{\varepsilon \rightarrow 0^+} A \max(x^2 - 1, 0), \quad (23)$$

which removes the corner at $x = 1$ and the attendant step-size penalty of an adaptive integrator while converging to the sharp ramp as $\varepsilon \rightarrow 0^+$; the sharp closure is used for every figure in which the separatrix and the classified samples must coincide. The Jacobian (21) is assembled analytically and, in the verification stage, is checked against a second-order central finite-difference Jacobian formed from the vector field.

Time integration uses the implicit, L-stable Radau method of order five, a member of the family of implicit Runge–Kutta schemes for stiff systems [4], applied with relative tolerance 10^{-10} and absolute tolerance 10^{-12} . The implicit scheme is chosen because the reduced system is mildly stiff near ignition, where the two Jacobian eigenvalues differ by an order of magnitude, so that an explicit method would be step-size limited by the fast stable mode; the L-stability of the Radau method integrates the system without that restriction. As an independent check, selected trajectories are re-integrated with the explicit eighth-order Dormand–Prince method [2] at the same tolerances, and the two solutions are required to agree. For the basin classifications, which require many short integrations rather than accurate trajectories, a lean adaptive Runge–Kutta pass is used with two terminal events: a runaway event that stops the integration when the speed exceeds a fixed multiple of the ignitive speed, signaling ignition, and a settling event that stops the integration when the speed and load both fall below a small threshold, signaling decay. The softplus surrogate (23) is enabled for this pass alone.

The interior equilibrium is evaluated from its closed form (19) and, independently, located as the unique root in $x > 1$ of $h(x) = A(x^2 - 1) - (r_0P/S)x^2$ by Brent’s method on the bracket $[1^+, x_{\text{max}}]$, the two determinations being required to agree. The stable manifold of the saddle is constructed by forming the Jacobian (21) at (x^*, y^*) , computing its eigenvalues and eigenvectors, selecting the eigenvector $\mathbf{v}_-$ of the negative eigenvalue as the stable direction, seeding two points $(x^*, y^*) \pm \delta \mathbf{v}_-$ at a small offset δ on either side of the equilibrium, and integrating the vector field backward in s from each seed; the union of the two backward orbits, restricted to Q , is the separatrix [20]. The critical lofted load $y_0^{\text{crit}}(S)$ that separates decay from ignition at a fixed slope is located by bisection on the classifier outcome over the initial load at zero initial speed, and the operational critical slope is

located by bisection on the classifier outcome over the slope at a fixed bounded disturbance.

2.3 Data Analysis

Five diagnostic analyses are applied to the reduced model, each implemented as an independent script that regenerates its figure and its tabulated output. The fixed-point analysis evaluates the closed-form equilibrium (19) over a set of supercritical slopes, forms the Jacobian (21) at each equilibrium and at the origin, computes the eigenvalues, classifies each equilibrium from their signs, and records the residual of the vector field at each equilibrium as a confirmation that it has been located to machine precision; the phase portrait is drawn at a representative supercritical slope with the nullclines, the equilibria, the separatrix, and two trajectories that straddle it. The bifurcation analysis evaluates the ignitive speed x^* across the slope and records the branch and its onset at S_{crit} , and the scaling study varies the critical shear τ_c and the cloud thickness h_0 in turn to record the critical bed angle as a function of each. The envelope analysis constructs the separatrix and the critical lofted load $y_0^{\text{crit}}(S)$, and validates the basins by classifying a grid of launched trajectories against the separatrix. The verification analysis performs four independent checks: a tolerance self-convergence study of the terminal state against a tightly converged reference; a cross-integrator comparison of the Radau and Dormand–Prince solutions along a common trajectory; a fixed-point residual study; and a comparison of the analytic and finite-difference Jacobian eigenvalues. The robustness analysis tests the flat-bed self-limiting result for large disturbances, compares the operational and closed-form critical slopes over a range of the near-bed ratio r_0 , records the operational critical slope as a function of the regularization width ε , and records it for the family of erosion exponents $A \max(x^2 - 1, 0)^p$ that generalizes the linear law.

3 Results

The reduced model was evaluated at a representative parameter set for a strong fin disturbance over fine cohesive silt in seawater, listed with the derived dimensionless quantities in Table 1. All values reported below are taken directly from the diagnostic output and are quoted to three decimal places; quantities are dimensionless unless a unit is stated. The submerged specific gravity is $R = 1.585$, the erosion-threshold speed is $U_c = 0.099 \text{ m s}^{-1}$, the erosion magnitude is $\alpha = 1.415 \times 10^{-4}$, and the settling timescale is $t_{\text{settle}} = h_0/w_s = 375 \text{ s}$, about 6.2 minutes. The three dimensionless groups that close the model are the drag-settling number $P = 1.235$, the erosion-strength number $A = 8.356$, and the near-bed concentration ratio $r_0 = 2$, which give the critical slope $S_{\text{crit}} = r_0 P/A = 0.296$ and the critical bed angle $\theta_{\text{crit}} = \arcsin S_{\text{crit}} = 17.189^\circ$.

Table 1: Physical parameters of the representative case and the derived dimensionless quantities. The upper block lists the dimensional inputs; the lower block lists the derived scales and the three dimensionless groups P , A , and r_0 that close the reduced model, together with the critical slope $S_{\text{crit}} = r_0 P/A$ and the critical bed angle.

Quantity	Symbol	Value
Clear-water density	ρ_0	1025 kg m^{-3}
Grain density	ρ_s	2650 kg m^{-3}
Gravitational acceleration	g	9.810 m s^{-2}
Settling velocity	w_s	$8.000 \times 10^{-4} \text{ m s}^{-1}$
Critical shear stress	τ_c	0.100 Pa
Drag coefficient	C_d	1.000×10^{-2}
Cloud thickness	h_0	0.300 m
Erosion-rate parameter	M	$3.000 \times 10^{-4} \text{ kg m}^{-2} \text{ s}^{-1}$
Near-bed concentration ratio	r_0	2.000
Submerged specific gravity	R	1.585
Kinematic critical shear	τ_c/ρ_0	$9.756 \times 10^{-5} \text{ m}^2 \text{ s}^{-2}$
Erosion-threshold speed	U_c	0.099 m s^{-1}
Erosion magnitude	α	1.415×10^{-4}
Settling timescale	t_{settle}	375.000 s
Drag-settling number	P	1.235
Erosion-strength number	A	8.356
Critical slope	S_{crit}	0.296
Critical bed angle	θ_{crit}	17.189°

The equilibrium structure is reported in Table 2. The quiescent state at the origin is present at every slope, with the degenerate eigenvalue pair $(0, -2)$, the second eigenvalue being $-r_0$; it is an attracting quiescent state as established in the derivation. No interior equilibrium exists at $S = 0$ or at $S = S_{\text{crit}} = 0.296$, consistent with the onset of the branch exactly at the critical slope. At each supercritical slope the interior equilibrium is a saddle, its eigenvalues comprising one negative and one positive real value: $(-22.256, +0.027)$ at $S = 0.300$, $(-6.663, +0.802)$ at $S = 0.500$, $(-6.695, +1.586)$ at $S = 0.800$, $(-6.820, +1.807)$ at $S = 0.900$, and

$(-6.958, +2.016)$ at $S = 1.000$. The closed-form equilibrium (19) agrees with the independent root determination to between 4.441×10^{-16} and 4.796×10^{-14} in the speed coordinate, and the residual of the vector field at each equilibrium is between 0 and 2.665×10^{-15} , at the level of machine precision. The ignitive speed decreases from $x^* = 8.192$ at $S = 0.300$, just above the critical slope where the branch diverges, to $x^* = 1.191$ at $S = 1.000$, so the ignitive state on a steep bed is a slow current poised at the edge of erosion rather than a fast one.

Table 2: Interior equilibrium and its linear stability at a sequence of supercritical bed slopes. The ignitive speed x^* and load y^* are the closed form (19); λ_1 and λ_2 are the real eigenvalues of the Jacobian (21); and the residual is the max-norm of the vector field at the equilibrium. The equilibrium is a saddle at every slope, and no interior equilibrium exists below $S_{\text{crit}} = 0.296$, where the model self-limits for every disturbance.

S	x^*	y^*	λ_1	λ_2	Residual
0.300	8.192	276.186	-22.256	+0.027	0×10^0
0.500	1.564	6.038	-6.663	+0.802	1.776×10^{-15}
0.800	1.259	2.447	-6.695	+1.586	1.776×10^{-15}
0.900	1.220	2.043	-6.820	+1.807	2.665×10^{-15}
1.000	1.191	1.753	-6.958	+2.016	4.441×10^{-16}

The phase portrait at the supercritical slope $S = 0.500$, a bed angle of 30° , is shown in Figure 2. The speed nullcline and the load nullcline intersect at the ignitive saddle $(x^*, y^*) = (1.564, 6.038)$, marked together with the quiescent rest state at the origin. The separatrix, the stable manifold of the saddle computed by backward integration, divides the physical quadrant into the basin of the rest state and the basin of unbounded growth. A trajectory launched from a lofted load below the separatrix rises briefly in speed, sheds its load to deposition, and returns to rest, whereas a trajectory launched from a lofted load above the separatrix accelerates past the erosion threshold $x = 1$ and runs away; these are the self-limiting and the igniting outcomes the model is built to distinguish.

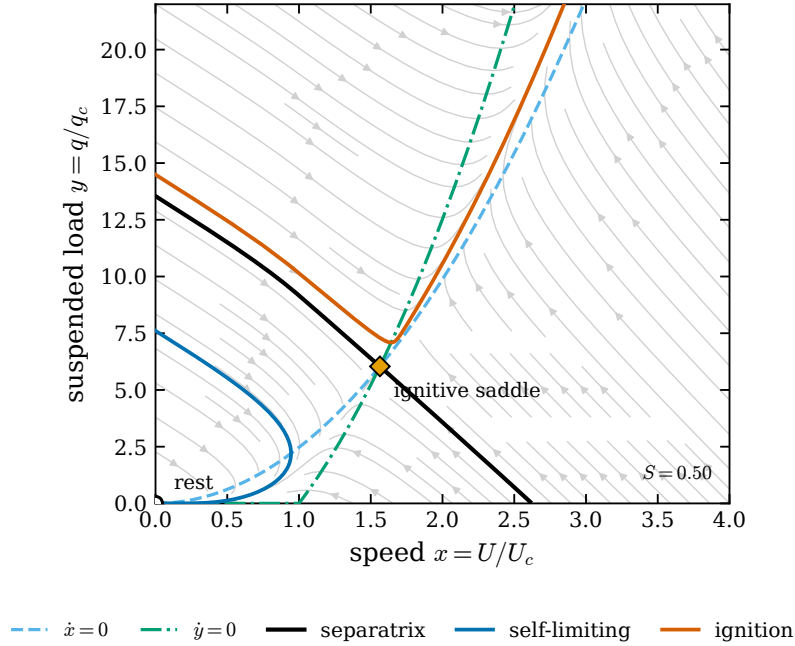


Figure 2: Phase portrait of the reduced model at the supercritical slope $S = 0.500$. The dashed curve is the speed nullcline $\dot{x} = 0$ and the dot-dashed curve is the load nullcline $\dot{y} = 0$; their intersection in the erosion-on region is the ignitive saddle, shown with the quiescent rest state at the origin. The solid black curve is the separatrix, the stable manifold of the saddle. The blue trajectory is launched below the separatrix and self-limits; the vermilion trajectory is launched above it and ignites. Gray streamlines show the vector field.

The dependence of the ignitive equilibrium on the bed slope is the transcritical appearance shown in Figure 3(a): the branch of ignitive speeds is absent below $S_{\text{crit}} = 0.296$, appears at the critical slope, diverges as the slope approaches it from above, and decreases monotonically toward the erosion threshold as the slope steepens. The branch is a saddle at every slope at which it exists, so it is the ignition threshold rather than a reachable steady state, and the shaded region below the critical slope is the parameter range in which no self-sustaining current is possible for any disturbance. Figure 3(b) reports the critical bed angle as the critical shear stress and the cloud thickness are varied in turn: a more resistant bed of higher τ_c requires a steeper slope to ignite, so the

critical angle rises with τ_c , while a thicker cloud carries more buoyant sediment per unit bed area and ignites on a gentler slope, so the critical angle falls with h_0 . The critical angle passes through its base value of 17.189° at the representative parameters and spans roughly one to six tens of degrees over the ranges examined, so the threshold is sensitive to the bed and disturbance properties but remains a moderate slope for physically reasonable values.

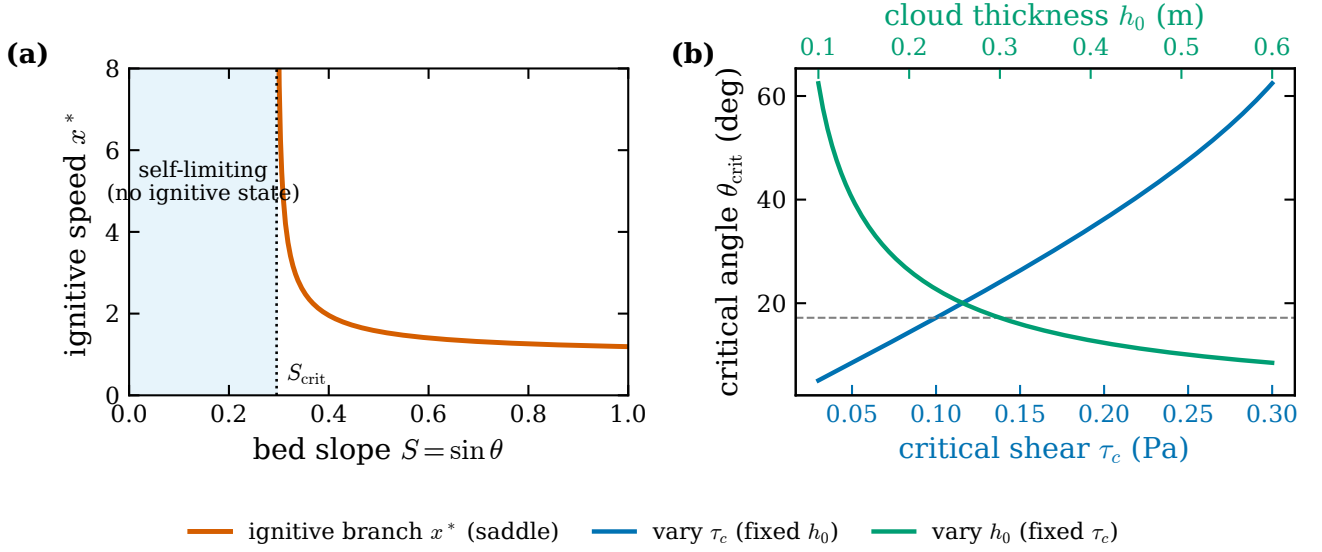


Figure 3: Ignition bifurcation and critical-slope scaling. (a) Ignitive speed x^* against bed slope S , from equation (19); the branch appears at $S_{\text{crit}} = r_0 P/A$ and is a saddle wherever it exists, and the shaded region below S_{crit} is the range in which the model self-limits for every disturbance. (b) Critical bed angle θ_{crit} as the critical shear τ_c (lower axis) and the cloud thickness h_0 (upper axis) are varied in turn, with the remaining parameters fixed; the dashed reference marks the base value 17.189° .

Above the critical slope the outcome of a disturbance depends on its magnitude relative to the separatrix, quantified by the safe-kick envelope of Figure 4(a). The critical lofted load y_0^{crit} , the least initial load at zero initial speed that ignites, diverges as the slope approaches S_{crit} from above and falls monotonically as the slope steepens: it takes the values 38.492 at $S = 0.350$ (a bed angle of 20.487°), 13.539 at $S = 0.500$ (30.000°), 8.693 at $S = 0.650$ (40.542°), 6.482 at $S = 0.800$ (53.130°), and 4.869 at $S = 1.000$ (90.000°), so a steeper bed is carried into a self-sustaining silt-out by a weaker disturbance. The region below the envelope is the self-limiting regime and the region above it the igniting regime. Figure 4(b) shows the basins of attraction in the plane of initial speed and initial load at $S = 0.500$: the separatrix divides the plane, and a grid of independently integrated trajectories, each classified by direct time integration, falls on the correct side of the separatrix at every point, including near the boundary itself, which confirms that the stable manifold computed by backward integration is the exact basin boundary.

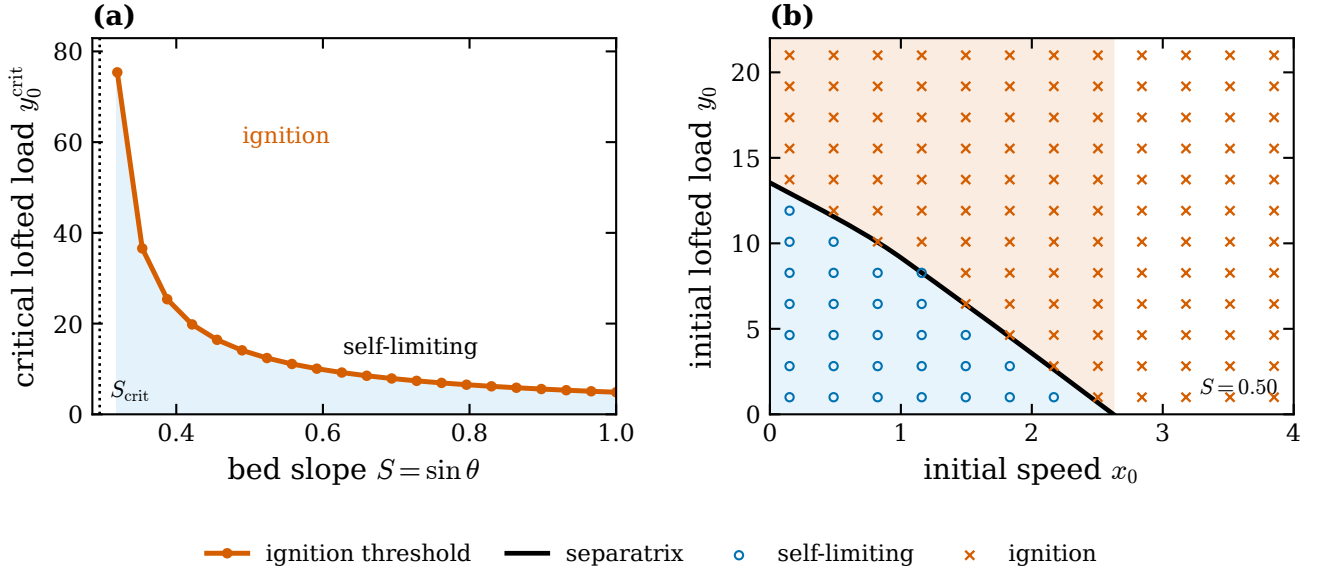


Figure 4: Safe-kick envelope and basins of attraction. (a) Critical lofted load y_0^{crit} , the least initial load at zero initial speed that ignites, against bed slope S ; it diverges at S_{crit} and falls as the slope steepens, and the shaded region below the envelope is the self-limiting regime. (b) Basins of attraction in the plane of initial speed x_0 and initial load y_0 at $S = 0.500$: the separatrix (solid) divides the plane, and independently integrated samples (open circles, self-limiting; crosses, ignition) confirm the boundary.

The evolution in time of two disturbances straddling the threshold at $S = 0.500$ is shown in Figure 5. A disturbance at ninety-five percent of the critical load decays, its load falling exponentially on the settling timescale and its speed relaxing toward zero, while a disturbance at one hundred and five percent of the critical load ignites, its load and speed growing without bound in the minimal model with the runaway reached within a few tens of dimensionless time units. The dimensionless time is referred to the settling timescale $t_{\text{settle}} = 375$ s on the secondary axis, which places the exponential clearing of a self-limiting cloud at a few minutes, consistent with the observed persistence of a diver-induced silt-out that does not ignite.

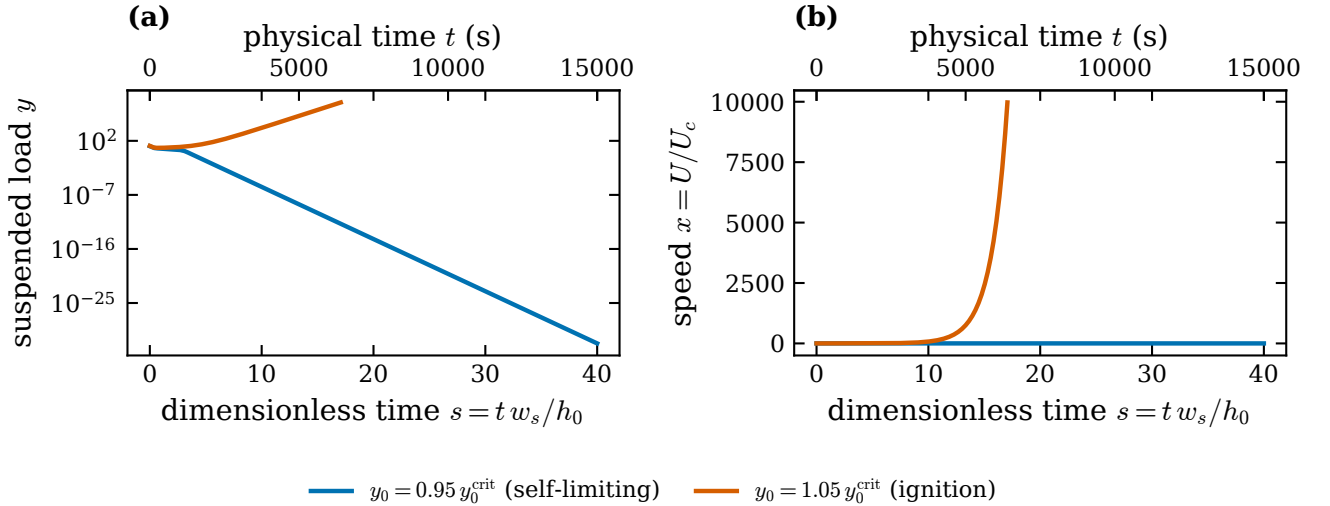


Figure 5: Representative time series straddling the ignition threshold at $S = 0.500$. (a) Suspended load y on a logarithmic ordinate for disturbances at $0.95 y_0^{\text{crit}}$ (self-limiting) and $1.05 y_0^{\text{crit}}$ (ignition). (b) Speed x for the same two disturbances. The upper axis reports physical time through the settling timescale $t_{\text{settle}} = 375$ s.

The numerical reliability of the solver is established by the four checks summarized in Table 3 and shown in Figure 6. The tolerance self-convergence study, in Figure 6(a), integrates a self-limiting trajectory at relative tolerances from 10^{-4} to 10^{-12} and measures the terminal-state error against a reference converged at relative tolerance 10^{-13} ; the error falls monotonically from 1.360×10^{-7} at 10^{-4} through 6.012×10^{-10} at 10^{-6} , 1.375×10^{-12} at 10^{-8} , and 7.185×10^{-15} at 10^{-10} , to 2.776×10^{-17} at 10^{-12} , tracking the requested tolerance down to

the double-precision floor. The cross-integrator study, in Figure 6(b), finds a maximum difference of 1.392×10^{-10} between the Radau and Dormand–Prince solutions along a common trajectory. The fixed-point residual study confirms that the closed-form equilibria annul the vector field to 1.776×10^{-15} at $S = 0.800$, 2.665×10^{-15} at $S = 0.900$, and 4.441×10^{-16} at $S = 1.000$. The Jacobian study, in Figure 6(c), finds that the eigenvalues of the analytic Jacobian at the $S = 0.500$ equilibrium, -6.663 and 0.802 , agree with those of the finite-difference Jacobian to a maximum absolute difference of 1.348×10^{-10} .

Table 3: Numerical verification summary. The tolerance self-convergence lists the terminal-state error at the loosest and tightest tolerances; the cross-integrator agreement is the maximum trajectory difference between the Radau and Dormand–Prince solutions; the fixed-point residual is the range of vector-field max-norms at the supercritical equilibria; and the Jacobian agreement is the maximum absolute difference between the analytic and finite-difference eigenvalues at $S = 0.500$.

Check	Result
Terminal error at $\text{rtol} = 10^{-4}$	1.360×10^{-7}
Terminal error at $\text{rtol} = 10^{-12}$	2.776×10^{-17}
Radau versus Dormand–Prince (max difference)	1.392×10^{-10}
Fixed-point residual (supercritical range)	4.441×10^{-16} to 2.665×10^{-15}
Analytic versus finite-difference eigenvalues	1.348×10^{-10}

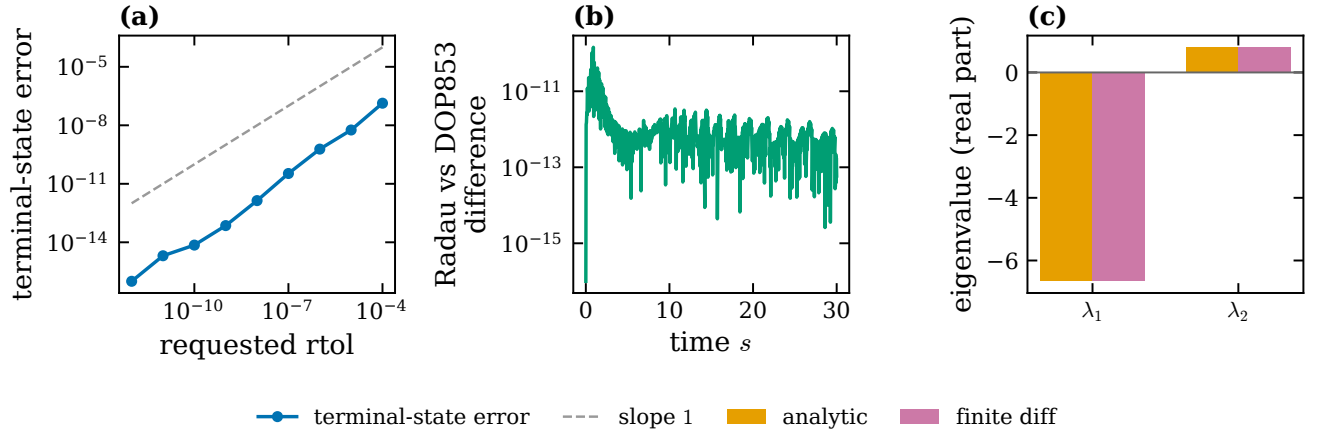


Figure 6: Numerical verification of the reduced-model solver. (a) Terminal-state error against requested relative tolerance for a self-limiting trajectory, with a reference slope of unity. (b) Difference between the implicit Radau and explicit Dormand–Prince solutions along a common trajectory. (c) Real parts of the analytic and finite-difference Jacobian eigenvalues at the interior equilibrium for $S = 0.500$.

The robustness of the ignition threshold to the erosion closure is reported in Table 4 and shown in Figure 7. The flat-bed result holds by construction and independently of the closure, as established through the explicit solution (18), and is confirmed numerically: at zero slope the model self-limits for lofted loads as large as 50, 200, and 1000. Above zero slope the operational critical slope obtained by integration, 0.298, matches the closed-form value 0.296 to within 0.002, the small excess arising from the finite disturbance cap used to detect ignition near the divergence of the envelope. Figure 7(a) shows that the operational critical slope scales linearly with the near-bed concentration ratio r_0 and coincides with the closed-form line $r_0 P/A$: the operational values are 0.224, 0.298, 0.372, and 0.446 at $r_0 = 1.5, 2.0, 2.5$, and 3.0 , against the closed-form values 0.222, 0.296, 0.369, and 0.443. Figure 7(b) shows that the operational critical slope is invariant to the softplus width, remaining at 0.298 as ε ranges over 0.0, 0.05, 0.10, 0.20, and 0.40, so the threshold is not an artifact of the corner in the sharp closure. Figure 7(c) shows the safe-kick envelope for three erosion exponents; a finite positive threshold exists for each, the envelopes converge at steep slopes, and the operational critical slope falls from 0.298 for the linear closure through 0.117 at $p = 1.25$ and 0.066 at $p = 1.50$ to 0.035 for the quadratic closure. The linear closure yields the highest critical slope and is therefore the conservative choice, while superlinear erosion lowers the slope at which a bounded disturbance ignites without removing the threshold.

Table 4: Robustness summary of the ignition threshold. The linear-closure operational critical slope is compared with the closed-form value; the scaling with the near-bed ratio r_0 and the invariance to the regularization width ε are quoted at their endpoints; and the operational critical slope is quoted for four erosion exponents p in the family $A \max(x^2 - 1, 0)^p$. The linear closure $p = 1$ is the conservative choice.

Check	Setting	Operational S_{crit}
Closed form (linear)	$r_0 P/A$	0.296
Integration (linear)	bounded disturbance	0.298
Concentration ratio	$r_0 = 1.5$	0.224
	$r_0 = 3.0$	0.446
Regularization width	$\varepsilon = 0.0$	0.298
	$\varepsilon = 0.4$	0.298
Erosion exponent	$p = 1.00$	0.298
	$p = 1.25$	0.117
	$p = 1.50$	0.066
	$p = 2.00$	0.035

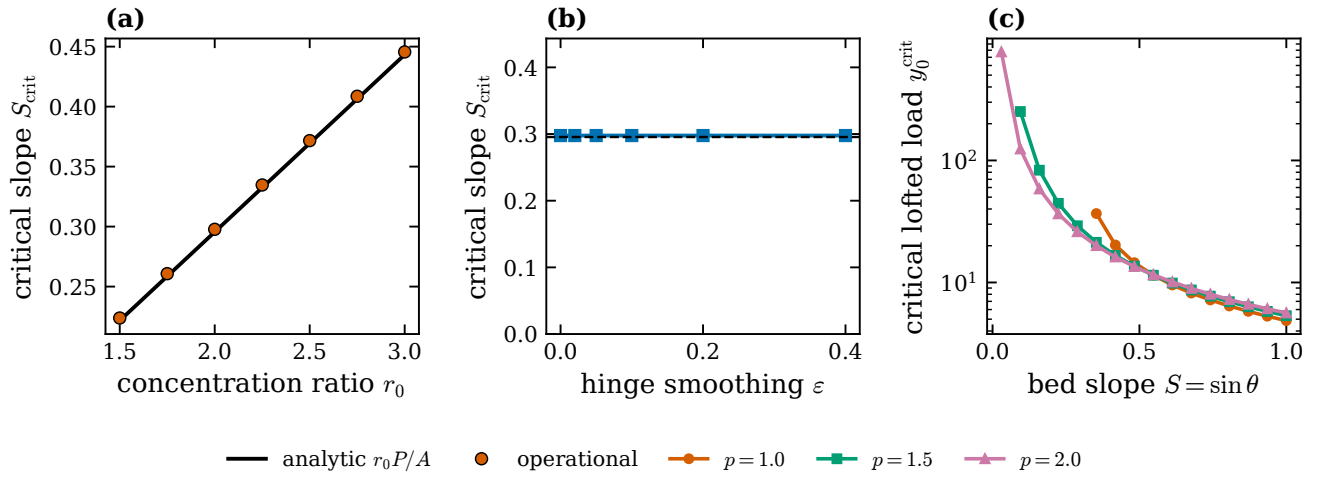


Figure 7: Robustness of the ignition threshold. (a) Critical slope against the near-bed concentration ratio r_0 : the analytic line $r_0 P/A$ and the operational values from integration coincide. (b) Operational critical slope against the softplus regularization width ε , which is flat. (c) Safe-kick envelope for erosion exponents $p = 1.0, 1.5$, and 2.0 : each has a finite positive threshold, the linear closure gives the highest threshold, and superlinear erosion lowers it.

4 Discussion

The reduced model reproduces, from a two-variable planar vector field derived at the scale of a diver disturbance, the ignition structure established for erosive turbidity currents [15]. The interior equilibrium is a saddle wherever it exists, so it is an unstable ignitive state rather than a reachable steady flow; disturbances below its stable manifold decay to the quiescent state, and disturbances above it accelerate past the erosion threshold and run away. This is the same dichotomy, between a self-limiting flow that dies out and a self-sustaining flow that ignites, that the phase-plane analysis of autosuspension delivers for turbidity currents [14, 1], now obtained in closed form for the diver-scale problem with the bed slope as the explicit control parameter. The correspondence is structural rather than incidental, since the reduced vector field descends by an explicit and justified reduction from the layer-averaged balances that underlie the three-equation turbidity-current model [16, 12], and the closed-form critical slope $S_{\text{crit}} = r_0 P/A$ is the diver-scale expression of the autosuspension threshold.

The physical reading is direct. A flat cohesive bottom cannot sustain a silt-out, a conclusion that is closure-independent because it follows from the explicit solution (18) of the speed equation at zero slope rather than from any property of the erosion law; a disturbance of any magnitude settles back on the settling timescale, which for the representative cloud is about 6.2 minutes, matching the operational experience that a silt-out over level ground clears on its own if it is not disturbed further. A self-sustaining current requires a bed steeper than the critical angle, close to 17° for the representative silt, a moderate slope of the kind encountered on reef faces, sand chutes, and drop-off approaches rather than a wall. Above that angle the outcome depends on the disturbance magnitude relative to the separatrix, and a steeper bed is crossed into ignition by a weaker disturbance, so the hazard grows with slope in two distinct ways: through the appearance of the threshold at S_{crit} and through the

fall of the safe-kick envelope above it. These readings identify the bed slope as the controlling parameter and the resuspended load as the disturbance measure, which is the qualitative guidance the model is positioned to provide.

The robustness study delimits how far these conclusions depend on the erosion closure, and its results are reported without overstatement. The existence of a finite positive critical slope is robust: it holds for the linear excess-shear law, for its smooth regularization, and for the superlinear closures, and the flat-bed self-limiting result holds for every admissible closure by an argument that does not reference the closure at all. The value of the critical slope, by contrast, is closure-dependent, and the study frames the linear closure as the conservative choice because it yields the highest threshold, with superlinear erosion lowering the slope at which a bounded disturbance ignites. The linear scaling with the near-bed ratio and the invariance to the regularization width confirm that the closed-form threshold is recovered by integration and is not an artifact of the corner in the sharp closure. The honest summary is that the model predicts the existence and the controlling parameter of the threshold firmly, and its numerical value only to within the latitude of the erosion law, a latitude that the experimental literature on autosuspension has itself emphasized [19, 3].

Several assumptions bound the scope of the model and are stated here rather than left to inference. The fixed-thickness slab of the reduction neglects the entrainment of ambient water and the growth of the layer that accompany a developing current; because entrainment dilutes the suspension and reduces the deposition flux per unit bed area, the fixed-thickness treatment overstates deposition and is conservative in the direction of a higher threshold, which justifies its use for a bounding estimate. The minimal model carries no saturation of the suspension, so an ignited current grows without bound rather than settling to the finite catastrophic state that the full theory predicts [15]; ignition is accordingly read from the crossing of the threshold rather than from the long-time asymptotics, and a saturation closure, through hindered settling or a capacity limit, would restore the stable catastrophic state and convert the picture into a complete saddle-node structure with a second attracting equilibrium at high load [20]. The single drag coefficient identifies the skin friction that mobilizes sediment with the form drag that retards the layer, whereas the two differ in a resolved boundary layer; this is a standard simplification of layer-averaged modeling, itself the subject of continuing assessment [9], and it shifts the value of the threshold through the erosion-threshold speed without affecting its existence. The erosion and deposition closures are empirical, and the settling velocity is held constant, so the model does not represent size sorting or hindered settling. Most importantly for any operational reading, the disturbance in the model is the dimensionless lofted load, and its mapping to a physical fin thrust is an order-of-magnitude correspondence rather than a calibration; the model reports dimensionless structure and the identity of the controlling parameter, and it is not a quantitative safety guideline. The reduced formulation avoids the turbulence-closure objection that attends a diver-scale field solver [13], since it resolves no turbulence and makes its idealizations explicit in the derivation, but the empirical status of its closures is the price of that transparency. Placing the model on an empirical footing would require controlled resuspension experiments over sloping cohesive beds against which the critical slope and the safe-kick envelope could be calibrated, and would admit the entrainment and saturation closures as the natural next terms, which is the direction in which the present idealized treatment extends.

5 Conclusions

This work derived a reduced-order dynamical model for the ignition of diver-induced cohesive silt-out by reducing the layer-averaged balances of fluid mass, sediment mass, and momentum, in the weakly entraining limit of a slow near-bed cloud, to a temporal slab of fixed thickness, obtaining an autonomous planar vector field in a dimensionless near-bed speed and a dimensionless suspended load governed by three dimensionless groups with the bed slope as control parameter, and it established in closed form that this field carries a quiescent equilibrium at the origin, globally attracting on a flat bed for every admissible erosion law, together with, above a critical bed slope $S_{\text{crit}} = r_0 P/A = 0.296$, an interior equilibrium whose negative Jacobian determinant marks it as a saddle, so that the model reproduces the classical ignition structure of erosive gravity currents in which disturbances below the saddle's stable manifold self-limit and disturbances above it ignite, with the stable manifold serving as the safe-kick envelope, the critical angle falling near 17.189° for representative silt, the closed-form structure confirmed against time integration to machine precision, against a second integrator to ten decimal places, and against a finite-difference Jacobian to better than 10^{-9} , and the threshold shown to be robust in existence and controlling parameter though closure-dependent in value, so that the model delivers a transparent, verified, and openly reproducible account of the slope-controlled transition between a silt-out that clears on its own and one that sustains itself, while leaving the mapping from the dimensionless disturbance to a physical thrust, and the calibration of the threshold against controlled resuspension experiments, as the natural next steps.

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Data Availability

Python source code and the analysis scripts that reproduce every figure, table, and diagnostic report in this study are available on GitHub at https://github.com/sandyherho/scuba_siltout_reduced. All supplementary outputs, comprising the diagnostic reports, the tabulated data behind each figure, and the figures in vector and raster formats, are permanently archived on the Open Science Framework at <https://doi.org/10.17605/OSF.IO/ER8K9>. The source code, the analysis scripts, and the archived outputs are released under the MIT license.

References

- [1] Bagnold, R.A. Auto-suspension of transported sediment; turbidity currents. *Proc. R. Soc. Lond. A* **1962**, *265(1322)*, 315–319. <https://doi.org/10.1098/rspa.1962.0012>.
- [2] Dormand, J.R.; Prince, P.J. A family of embedded Runge–Kutta formulae. *J. Comput. Appl. Math.* **1980**, *6(1)*, 19–26. [https://doi.org/10.1016/0771-050X\(80\)90013-3](https://doi.org/10.1016/0771-050X(80)90013-3).
- [3] García, M.; Parker, G. Entrainment of bed sediment into suspension. *J. Hydraul. Eng.* **1991**, *117(4)*, 414–435. [https://doi.org/10.1061/\(ASCE\)0733-9429\(1991\)117:4\(414\)](https://doi.org/10.1061/(ASCE)0733-9429(1991)117:4(414)).
- [4] Hairer, E.; Wanner, G. *Solving Ordinary Differential Equations II: Stiff and Differential-Algebraic Problems*, 2nd ed.; Springer Series in Computational Mathematics, Vol. 14; Springer: Berlin, Heidelberg, Germany, 1996. <https://doi.org/10.1007/978-3-642-05221-7>.
- [5] Harris, C.R.; Millman, K.J.; van der Walt, S.J.; Gommers, R.; Virtanen, P.; Cournapeau, D.; Wieser, E.; Taylor, J.; Berg, S.; Smith, N.J.; et al. Array programming with NumPy. *Nature* **2020**, *585*, 357–362. <https://doi.org/10.1038/s41586-020-2649-2>.
- [6] Herho, S.H.S.; Trilaksono, N.J.; Fajary, F.R.; Napitupulu, G.; Anwar, I.P.; Khadami, F.; Irawan, D.E. *kh2d-solver*: a Python library for idealized two-dimensional incompressible Kelvin–Helmholtz instability. *Appl. Comput. Mech.* **2025**, *19(2)*, 125–156. <https://doi.org/10.24132/acm.2025.1040>.
- [7] Herho, S.H.S.; Irawan, D.E.; Kartiko, R.D.; Pamumpuni, A.; Suwarman, R. *dewi-Kadita*: a Python library for idealized fish schooling simulation with entropy-based diagnostics. *J. Phys. Commun.* **2026**, *10(6)*, 065002. <https://doi.org/10.1088/2399-6528/ae7177>.
- [8] Herho, S.H.S.; Anwar, I.P.; Khadami, F.; Ndruru, T.R.E.B.N.; Suwarman, R.; Irawan, D.E. *wave-attenuation-1d*: An Idealized One-Dimensional Framework for Wave Attenuation through Coastal Vegetation using Numba-Accelerated Shallow Water Equations. *J. Theor. Appl. Mech.* **2026**, *56(1)*, 89–102. <https://doi.org/10.55787/jtams.2026.1.AI00236>.
- [9] Hu, P.; Pähitz, T.; He, Z. Is it appropriate to model turbidity currents with the three-equation model? *J. Geophys. Res. Earth Surf.* **2015**, *120(7)*, 1153–1170. <https://doi.org/10.1002/2015JF003474>.
- [10] Hunter, J.D. Matplotlib: A 2D Graphics Environment. *Comput. Sci. Eng.* **2007**, *9(3)*, 90–95. <https://doi.org/10.1109/MCSE.2007.55>.
- [11] Irawan, D.E.; Herho, S.H.S.; Pamumpuni, A.; Kartiko, R.D.; Khadami, F.; Anwar, I.P.; Sujatmiko, K.A.; Handayani, A.P.; Fajary, F.R.; Suwarman, R. An Open-Source Pseudo-Spectral Solver for Idealized Korteweg–de Vries Soliton Simulations. *Water* **2026**, *18(7)*, 779. <https://doi.org/10.3390/w18070779>.
- [12] Meiburg, E.; Kneller, B. Turbidity currents and their deposits. *Annu. Rev. Fluid Mech.* **2010**, *42*, 135–156. <https://doi.org/10.1146/annurev-fluid-121108-145618>.
- [13] Meiburg, E.; Radhakrishnan, S.; Nasr-Azadani, M. Modeling gravity and turbidity currents: computational approaches and challenges. *Appl. Mech. Rev.* **2015**, *67(4)*, 040802. <https://doi.org/10.1115/1.4031040>.
- [14] Pantin, H.M. Interaction between velocity and effective density in turbidity flow: phase-plane analysis, with criteria for autosuspension. *Mar. Geol.* **1979**, *31(1–2)*, 59–99. [https://doi.org/10.1016/0025-3227\(79\)90057-4](https://doi.org/10.1016/0025-3227(79)90057-4).
- [15] Parker, G. Conditions for the ignition of catastrophically erosive turbidity currents. *Mar. Geol.* **1982**, *46(3–4)*, 307–327. [https://doi.org/10.1016/0025-3227\(82\)90086-X](https://doi.org/10.1016/0025-3227(82)90086-X).
- [16] Parker, G.; Fukushima, Y.; Pantin, H.M. Self-accelerating turbidity currents. *J. Fluid Mech.* **1986**, *171*, 145–181. <https://doi.org/10.1017/S0022112086001404>.
- [17] Partheniades, E. Erosion and deposition of cohesive soils. *J. Hydraul. Div. ASCE* **1965**, *91(1)*, 105–139. <https://doi.org/10.1061/JYCEAJ.0001165>.
- [18] Simpson, J.E. Gravity currents in the laboratory, atmosphere, and ocean. *Annu. Rev. Fluid Mech.* **1982**, *14*, 213–234. <https://doi.org/10.1146/annurev.fl.14.010182.001241>.
- [19] Southard, J.B.; Mackintosh, M.E. Experimental test of autosuspension. *Earth Surf. Process. Landf.* **1981**, *6(2)*, 103–111. <https://doi.org/10.1002/esp.3290060204>.
- [20] Strogatz, S.H. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*, 2nd ed.; CRC Press: Boca Raton, FL, USA, 2018. <https://doi.org/10.1201/9780429492563>.
- [21] Virtanen, P.; Gommers, R.; Oliphant, T.E.; Haberland, M.; Reddy, T.; Cournapeau, D.; Burovski, E.; Peterson, P.; Weckesser, W.; Bright, J.; et al. SciPy 1.0: fundamental algorithms for scientific computing in

Python. *Nat. Methods* **2020**, *17*, 261–272. <https://doi.org/10.1038/s41592-019-0686-2>.