

The Apparent Stratification at the Top of Earth's Liquid Core

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Earth's magnetic field is generated by turbulent motion in its fluid outer core. Although the bulk of the outer core is vigorously convecting and well-mixed, some seismic, geomagnetic, and geodynamic evidence suggests that a global stably stratified layer exists at the top of Earth's core. Such a layer would strongly influence thermal, chemical, and momentum exchange across the core-mantle boundary (CMB) and thus have significant implications for the dynamics and evolution of the core. However, the existence of thick and strong global stratification is incompatible with the radial motions near the top of the core that are believed necessary to explain observed high-latitude magnetic flux concentrations and patches of reversed magnetic flux. Here we argue that the relevant scenario is not a global layer but regional stratification arising from the lateral variations in CMB heat flux. Based on our extensive suite of numerical simulations we expect that regional thermal inversion layers extend 100's of kilometres into the core under anomalously hot regions of the lowermost mantle. Sufficiently large and strong regional anomalies will dominate the average radial temperature profile and could be mistaken for a globally stratified layer. Dynamic links be-

21 **tween regions of thermal inversion and active convection result in radial motion everywhere**
22 **within the core, thereby avoiding any conflict with geomagnetic observations.**

23 There is little doubt that the bulk of Earth's liquid core is undergoing turbulent convection
24 and the horizontal temperature fluctuations within the adiabatically well-mixed fluid are expected
25 to be very small ($\sim 10^{-4}\text{K}$)^{1,2}. Since the thermal heterogeneity in the lowermost mantle is much
26 stronger than in the core ($\sim 10^2\text{K}$) and evolves much more slowly, the mantle imposes a laterally
27 varying pattern of heat flux across the core-mantle boundary (CMB)^{3,4}. Estimates of the lateral
28 variations in CMB heat flux⁵⁻⁷ are sufficiently large that significant regional variations in core
29 dynamics are expected⁸⁻¹², particularly near the top of the core.

30 Seismic studies¹³⁻¹⁵ have found body wave speeds near the top of the core that depart from
31 those expected if the entire core is adiabatically and chemically well-mixed; although unavoidable
32 limitations in the geographic coverage of seismic data result in under-sampling of large geographic
33 regions of the outermost core. The seismic wave speeds have been matched to a compositional
34 model for the core¹⁴ and interpreted as the signature of a global layer that is both thick ($\sim 300\text{ km}$)
35 and strongly stratified (Brunt-Väisälä periods of 1.63–3.43 hr). However, other studies^{16,17} have
36 found that the seismic structure of the core does not require global stratification.

37 A stratified layer at the top of the core would support a range of wave dynamics not found
38 in a fully convecting core¹⁸; for example, MAC waves, which involve Magnetic, Archimedean,
39 and Coriolis forces. Assuming such a layer is present, a model of MAC waves can match the
40 decadal axisymmetric velocity fluctuations in a core flow model^{19,20}, though this explanation is not

41 unique²¹. Models that match the geomagnetic variations have a stratified layer ~ 140 km thick with
42 a maximum Brunt-Väisälä frequency that is roughly diurnal^{19,20}; which is somewhat thinner and an
43 order of magnitude less strongly stratified than models derived from seismic wave speeds. Global
44 stratification would also influence the structure of the geomagnetic field^{11,22}. Non-axisymmetric
45 patches of concentrated magnetic flux at high latitudes and reversed flux in the southern Atlantic²³
46 are hard to explain without invoking radial motion²⁴ within ~ 100 km of the CMB²⁵, which is
47 incompatible with models of thick global stratification.

48 Three principle mechanisms have been invoked to explain a global non-adiabatic structure
49 at the top of the core. The first mechanism²⁶ supposes that the core has slowly cooled to a point
50 where the heat flux, q , across the CMB has fallen below the adiabatic heat flux, q_{ad} . This scenario
51 produces a wide range of thickness estimates²⁷ that rely on the poorly-known CMB heat flow
52 and much-debated core conductivity²⁸. The second mechanism invokes chemical diffusion, either
53 along the core pressure gradient²⁹ or across the CMB from the mantle³⁰, which enriches the top
54 of the core in light elements. The third possibility is emplacement of a light layer during core
55 formation³¹, which must then avoid disruption throughout the lifetime of the Earth or by the moon-
56 forming impact³². However, all of these mechanisms are motivated by the idea of a global stable
57 layer at the top of the core and therefore cannot explain the seismic and geomagnetic observations
58 that are incompatible with global stratification.

59 We argue that the relevant scenario is not global stratification but regional thermal inversion.
60 A net superadiabatic heat flow across the CMB ensures the bulk of the core remains vigorously

61 convecting and adiabatically well mixed; however, sufficiently warm regions in the lowermost
62 mantle reduce q below q_{ad} allowing regional accumulations of hot fluid at the top of the core. The
63 radial temperature gradient ($\partial T/\partial r$) within the accumulated hot fluid can be insufficient to drive
64 convection, resulting in relatively quiescent fluid volumes that we refer to as regional inversion
65 layers. The spatial extent and buoyancy anomaly of these regional inversion layers are primarily
66 set by the long-wavelength high-amplitude variations in CMB heat flux imposed on the core by
67 the mantle. Large and strong regional inversion layers can dominate the spherically averaged
68 temperature profile resulting in an apparent thermal stratification near the top of the core even
69 though much of the core's surface remains actively convecting. This scenario allows for both
70 substantial non-adiabatic structure at the top of the core, as has been inferred from seismology, and
71 areas of active upwelling, as inferred from geomagnetism, thereby resolving the apparent paradox
72 that arises in scenarios of global stratification. There is no doubt that the fundamental physical
73 mechanism that underpins our scenario, namely large lateral variations in CMB heat flux, exists
74 within the Earth; the only question is how significant its influence might be.

75 **Modelling of Regional Inversion Layers**

76 We investigate regional inversion layers in the core using a suite of numerical simulations of non-
77 magnetic rotating convection that includes two patterns and two amplitudes of CMB heat flux het-
78 erogeneity (see methods and our previous work¹²). The amplitude of CMB heat flux heterogeneity
79 in our numerical model is described by $q^* = \frac{q_{\text{max}} - q_{\text{min}}}{q_{\text{ave}}}$, where q_{max} , q_{min} , and q_{ave} are the maxi-
80 mum, minimum and horizontally averaged heat fluxes through the outer boundary, respectively. In
81 this study we consider strong lateral variations in CMB heat flux with $q^* = \{2.3, 5.0\}$. We im-

82 pose one of two patterns of CMB heat flux heterogeneity. The first pattern is derived from seismic
83 tomography³³ under the assumption that Large Low Velocity Provinces (LLVP's) are anomalously
84 warm and therefore impose a reduced CMB heat flux on the core. The second is a hemispheric
85 pattern that could represent the configuration of mantle flow during times of super-continent for-
86 mation (for the hemispheric pattern, q_{\min} is located under Null Island).

87 Numerical models of core convection can be characterised by three control parameters: the
88 Prandtl number (Pr), which is the ratio of the fluid's viscous and thermal diffusivities; the Rayleigh
89 number (\widetilde{Ra}) and the Ekman number (E), which primarily reflect balances between buoyancy,
90 rotational, and viscous forces. Our simulations consider higher \widetilde{Ra} and lower E than previous
91 models that incorporate CMB heat flux heterogeneity⁹⁻¹¹. In particular, we restrict our attention
92 to simulations for which \widetilde{Ra} is at least ten times greater than the critical Rayleigh number for the
93 onset of convection (\widetilde{Ra}_c) to ensure that we have left the weakly non-linear regime near the onset
94 of convection. Consideration based on the force balance between inertia, viscosity, and rotation
95 suggests that the dynamic regime be characterised using the Reynolds number, $Re = UL/\nu$, and
96 Rossby number, $Ro = U/2\Omega L = ReE$, where U and L are the characteristic velocity and length
97 scale of the flow, respectively, and ν is the magnetic diffusivity. Assuming a turbulent viscosity
98 value for the core, our simulations are within one to two orders of magnitude of Earth-like values
99 for these emergent parameters (Table 1); crucially, these parameters indicate that the dynamics in
100 our simulations are both turbulent (large Re) and strongly influenced by rotation (small Ro) as in
101 Earth's core. Our simulations produce small scale behaviour (e.g., plumes, jets; see supplementary
102 movies) qualitatively similar to those observed in comparable laboratory experiments^{34,35}.

103 In all of our simulations we find that convectively-stable regions of thermal inversion ($\partial T/\partial r >$
104 0) can be maintained over large lateral and radial extents, although the bulk of the core remains
105 strongly convecting and hence well mixed on short length scales (figures 1, 2). Even in regions
106 where the CMB heat flux remains superadiabatic, an inversion layer can exist a few hundred kilo-
107 metres below the CMB as azimuthal flow sweeps hot material horizontally. The lateral and depth
108 extents of the regional inversion layers are associated with the long wavelengths of the imposed
109 boundary heterogeneity rather than the small wavelengths of the convecting core fluid (figure 1,
110 supplementary figure 1). Indeed the small scales of the convective fluctuations inhibit their ability
111 to disrupt the large regions of thermal inversion^{29,36}; previous studies at low \widetilde{Ra} did not find the
112 stratification signal⁹, perhaps because the potentially stable regions were disrupted by the large
113 scale convective patterns close to onset.

114 The strength of the thermal inversion is characterised by the maximum Brunt-Väisälä fre-
115 quency (N), which we normalise relative to 2Ω (twice the planetary rotation rate). Scaling analysis
116 (see methods) shows that the strength of the inversion should vary as

$$\frac{N}{2\Omega}\Big|_{\max} \approx \left(\frac{1}{r_o^*}\right) \sqrt{\frac{\widetilde{Ra}E}{Pr} \left(\frac{q^* - 2}{2}\right)}, \quad (1)$$

117 where r_o^* is the dimensionless CMB radius. Extrapolation to the Earth must therefore account for
118 both the increase in \widetilde{Ra} and the decrease in E relative to numerical simulations (table 1). Stronger
119 boundary heterogeneity (larger q^*) implies more anomalous $\partial T/\partial r$ at the CMB and thus N is
120 proportional to q^* .

121 The value of q^* can be estimated from first-principles calculations of thermal conductiv-

122 ity coupled to seismic tomographic models⁶ that suggest heat flux across the CMB ranges from
123 roughly 0 – 140 mW/m². Much of the net radial heat flow within the core occurs due to con-
124 duction along the adiabatic temperature gradient²⁸; this contribution needs to be removed when
125 considering the relation between our Boussinesq model and the Earth. The super-adiabatic heat
126 flow across the CMB has been estimated as 0.6 TW based on a theoretical scaling between inertial
127 and buoyancy forces in rotating convection³⁷. These values suggest q^* for the Earth may be as
128 large as ~ 35 , in which case $N/2\Omega \approx 2$ is predicted for the Earth for reasonable estimates of other
129 physical parameters (supplementary table 1).

130 No theoretical scaling exists for the thickness of the regional inversion layers; they are not
131 simple boundary layers, which would thin both as \widetilde{Ra} is increased and as E is decreased towards
132 Earth-like values. Instead we find a competition between thinner layers as the Ekman number is
133 reduced but generally thicker layers as the Rayleigh number is increased for a given choice of q^*
134 and CMB heat-flux pattern (figure 2 and supplementary figures 2 and 3).

135 If the regional inversion layers are sufficiently large and strong, the horizontally-averaged
136 temperature gradient near the top of the core can become positive (figures 2, 3), an apparent global
137 stratification despite the average heat flux across the CMB being strongly superadiabatic. Regional
138 inversion layers that are both thick (several hundred kilometres) and strong ($N/2\Omega \approx \{10^{-2} - 10^0\}$)
139 are ubiquitous in our simulations and apparent global stratification signals occur in our highest \widetilde{Ra}
140 runs. As the buoyancy forcing is increased the bulk of the core becomes more isothermal, thereby
141 causing the horizontally averaged temperature gradient near the top of the core to be increasingly

142 dominated by the large gradients that exist in the regional inversion layers.

143 **Discussion**

144 Thermal variations in Earth's lowermost mantle are sufficiently strong that large areas of the CMB
145 are expected to have a subadiabatic heat flux. Such areas will inevitably inhibit convection in
146 the outermost liquid core resulting in the development of horizontally extensive regional inversion
147 layers. The strength and extent of these regions is set by the boundary heterogeneity, which is faith-
148 fully represented in our simulations; therefore, we argue that broad and thick regional inversion
149 layers should be expected in the Earth. Following our modelling results, regional inversion lay-
150 ers should be most prominent in equatorial regions and particularly under the Pacific and African
151 LLVP's. Apparent global stratification results when the regional inversions control the sign of the
152 global average radial temperature gradient, which is particularly likely at the high Rayleigh num-
153 ber conditions relevant to the Earth. Seismic studies of average structure, or with an unfortunate
154 geographic sampling, might well mistake extensive regional inversion layers for global stratifica-
155 tion.

156 Unlike our Boussinesq numerical model, the anomalous regions in Earth's core need not have
157 a strictly positive thermal gradient, they need only have a subadiabatic gradient to be dynamically
158 distinct from the bulk of the core. The lateral temperature differences expected^{1,2} within convect-
159 ing regions of the outer core are very small; however, the temperature difference between actively
160 convecting regions and the relatively stagnant regional inversion layers can be far larger because it
161 is set by the long-wavelength and large amplitude heterogeneity in heat flux boundary conditions

162 and not the internal convective dynamics. The temperature difference between the top of actively
163 convecting regions and the regional inversion layers depends on the layer thickness, q^* and net
164 superadiabatic heat flow across the CMB; assuming purely thermal convection a simple theoreti-
165 cal analysis suggests that the boundary-forced temperature variations can be orders of magnitude
166 larger than those associated with the free convection (see methods, supplementary figure 4). The
167 CMB heat flux heterogeneity will drive flows, modulated by the magnetic field³, which previous
168 studies³⁸⁻⁴⁰ have used to explain long-term non-axisymmetric features of the geomagnetic field.

169 Convection in Earth's core arises due to the release of both compositional and thermal buoy-
170 ancy as the core cools and the inner core solidifies. Positive correlation between temperature and
171 composition is expected under the codensity approximation^{2,11} and would result in the thermal
172 inversion layers also being compositionally distinct from the actively convecting region. Although
173 the core loses heat to the mantle, it is generally assumed that the light element released by inner
174 core solidification does not escape to the mantle^{2,4}. The density anomaly, and hence dynamics, of
175 the regional inversion layers will depend on the relative contributions of thermal and compositional
176 buoyancy within these regions.

177 Although radial motion would be inhibited within a strongly stratified global layer, the re-
178 gional inversion layers in our simulations are dynamically connected to the rest of the core and
179 thus radial velocity is not completely suppressed within them (figure 4). The lateral variations in
180 CMB heat flux drive thermal winds that can sweep material from these regions into the well-mixed,
181 vigorously convecting bulk and result in a broad, weak upwelling through the regional inversion

182 layers. Unlike previous studies that consider subadiabatic, or weakly superadiabatic, heat flux at
183 the CMB^{11,26,27,41} all of the simulations we consider here are strongly supercritical ($\widetilde{Ra} \geq 10\widetilde{Ra}_c$),
184 even those for which the top of the core has an apparent global thermal stratification. In all of
185 our simulations the bulk of the core is vigorously convecting and the regional inversion layers are
186 not stagnant. As a result, there is no difficulty in reconciling this scenario with both geomagnetic
187 observations that suggest upwelling near the CMB²⁵ and seismic observations of a relatively thick
188 anomalous structure¹⁴.

189 Fluid flow in inversion layers is different to that in the bulk of the core (figure 4), which
190 would result in different geomagnetic variation. Observed regional patterns in geomagnetic secular
191 variation and inferred core dynamics^{42,43} might include a signature of regional inversion layers.
192 In our model both Large Low Velocity Provinces are associated with low CMB heat flux and
193 thus regional inversion layers; however, the latitudinal and longitudinal extents of the two LLVP's
194 are quite different, which could result in differing influences on core thermal structure and hence
195 geomagnetic variation. A hemispheric difference could also arise due to differences in temperature
196 between the Pacific and African LLVP's, which might reflect differing balances between thermal
197 and chemical contributions to these LLVP's origins.

198 In contrast to distinguishing between the different mechanisms proposed for inducing global
199 stratification, the persistent regional inversion layers scenario suggested by our simulations pro-
200 vides a clear avenue to observational investigation. The geometry and strength of regional inver-
201 sion layers in the core depends on the pattern and amplitude of the imposed heat flux heterogeneity,

202 which is set by the distributions of both temperature and thermal conductivity in the lowermost
 203 mantle. Given suitable geographic coverage of seismic ray paths and sufficient understanding (for
 204 example, from 3D wave-propagation models) of how *SmKS* phases are influenced by regional in-
 205 version layers, it would be possible to test whether the average seismic structure at the top of the
 206 core is truly the result of global stratification or if it is instead the signature of large boundary-
 207 forced regional inversion layers.

208 **Methods**

209 **Governing equations and parameter regime.** We employ a numerical model of non-magnetic
 210 rotating convection of a homogeneous Boussinesq fluid confined within a rotating spherical shell⁴⁴,
 211 with fixed-flux thermal boundary conditions and no slip velocity boundary conditions. In non-
 212 dimensional form the conservation equations for momentum, energy, and mass are

$$\frac{E}{Pr} \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) + \hat{\mathbf{z}} \times \mathbf{u} = -\nabla P + \widetilde{Ra} T' \mathbf{r} + E \nabla^2 \mathbf{u}, \quad (2)$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \nabla^2 T, \quad (3)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (4)$$

215 where \mathbf{u} and T are the velocity and temperature fields, respectively, and T' are the temperature
 216 fluctuations relative to the steady-state temperature profile in the absence of flow. The pressure
 217 term, P , includes the centrifugal potential. The fluid is characterised by its constant thermal ex-
 218 pansion, α , thermal diffusivity, κ , kinematic viscosity, ν , and reference density, ρ_0 . The fluid shell
 219 is defined by its inner and outer boundaries, r_i and r_o , respectively, and rotates with a constant

220 angular velocity $\Omega = \Omega \hat{z}$. Gravity varies with radius according to $\mathbf{g} = -(g_o/r_o)\mathbf{r}$. We have non-
221 dimensionalised using the shell thickness $L = r_o - r_i$ for the length scale, the thermal diffusion
222 time $\tau = L^2/\kappa$ for the time scale, and β/L for the temperature scale, where $\beta = Q/4\pi k$, Q is the
223 total heat flow through the outer boundary, $k = \kappa\rho_0 C_p$ is the thermal conductivity and C_p the heat
224 capacity of the fluid. The resulting control parameters are the Prandtl number $Pr = \frac{\nu}{\kappa}$, Ekman
225 number $E = \frac{\nu}{2\Omega L^2}$, and modified Rayleigh number $\widetilde{Ra} = \frac{\alpha g_o \beta}{2\Omega \kappa}$. The amplitude of the heterogeneity
226 in our heat flux boundary condition is measured by $q^* = (q_{\max} - q_{\min})/q_{\text{ave}}$, where q_{\max} , q_{\min} , and
227 q_{ave} are the maximum, minimum and horizontally averaged heat fluxes through the outer boundary,
228 respectively.

229 Our previous study¹² includes a suite of 106 simulations with values of $q^* = \{0.0, 2.3, 5.0\}$,
230 $Pr = 1$, $E = \{10^{-4}, 10^{-5}, 10^{-6}\}$, and \widetilde{Ra} up to ~ 800 times the critical value for the onset of
231 convection \widetilde{Ra}_c . The critical Rayleigh number increases as the Ekman number is reduced and
232 has values of $\widetilde{Ra}_c = \{16.4, 24.7, 41.0\}$ for the three values of E that we use. Here we include
233 six additional simulations with the hemispheric boundary forcing and $E = 10^{-6}$. In this study
234 we do not include results from our simulations that have homogeneous CMB boundary heat flux
235 ($q^* = 0.0$) or that are only weakly super-critical ($\widetilde{Ra} < 10\widetilde{Ra}_c$), leaving a total of 68 simulations.

236 The pattern and amplitude of CMB heat flux variations are difficult to estimate because
237 they must be inferred from seismic tomography accounting for possible compositional effects and
238 phase changes in the lower mantle. Nevertheless, several studies⁵⁻⁷ have found a minimum heat
239 flux of $q_{\min} \approx 0 \text{ mW m}^{-2}$, while the maximum heat flux q_{\max} could rise above 200 mW m^{-2} .
240 The adiabatic gradient at the CMB is $\partial T_{\text{ad}}/\partial r = g\gamma T/\phi \approx -0.875 \pm 0.125 \text{ K km}^{-1}$ with the

241 seismic parameter ϕ and gravity g taken from PREM⁴⁵ and the Grüneisen parameter $\gamma = 1.3 - 1.5$
242 spanning the available estimates⁴⁶. Using low⁴⁷ and high²⁸ thermal conductivity values gives $q_{\text{ad}} =$
243 $-k\partial T_{\text{ad}}/\partial r = 15 - 100 \text{ mW m}^{-2}$ and therefore hot regions of the lower mantle will result in a
244 subadiabatic heat flux across the CMB. The relative strength of CMB anomalies is often measured
245 by the parameter $q^* = (q_{\text{max}} - q_{\text{min}})/(q - q_{\text{ad}})$, which can take either sign given estimates⁴⁸ of
246 $q = 30 - 110 \text{ mW m}^{-2}$. Here we are interested in the case $q^* > 0$, as q^* of at least $o\{1\}$ is expected
247 within the Earth⁶ and it could be significantly greater (indeed q^* is unbounded as $q \rightarrow q_{\text{ad}}$). If q^* is
248 large, as expected for the Earth, thermal boundary forcing should exert a significant influence on
249 core convection^{34,35}.

250 **Brunt-Väisälä frequency.** The frequency of oscillation of a radially displaced fluid parcel within
251 a layer having stable density stratification ($\partial\rho/\partial r < 0$) is known as the buoyancy or Brunt-Väisälä
252 frequency and is defined by

$$N = \sqrt{-\frac{g}{\rho_0} \frac{\partial\rho}{\partial r}} = \sqrt{g\alpha \frac{\partial T}{\partial r}}, \quad (5)$$

253 if the density anomalies arise due to purely thermal effects. Non-dimensionalising frequency by
254 2Ω , in combination with our temperature and distance scalings gives

$$\frac{N}{2\Omega} = \sqrt{\frac{g\alpha\beta}{4\Omega^2 L^2} \frac{\partial T^*}{\partial r^*}} = \sqrt{\frac{\widetilde{Ra}E}{Pr} \frac{\partial T^*}{\partial r^*}}, \quad (6)$$

255 where T^* and r^* are non-dimensional temperature and radius, respectively.

256 The steepest temperature gradient within a regional inversion layer corresponds to the max-
257 imum buoyancy frequency and we expect that the steepest gradient near the top of the core is
258 close to that set by q_{min} of the imposed CMB heat flux. However, along some radial profiles (for

259 example, Africa, figure 2) the maximum of dT/dr occurs some depth below the CMB. A simple
 260 pattern of heat flux heterogeneity would have $q_{\text{ave}} = (q_{\text{max}} + q_{\text{min}})/2$ and from the definition of
 261 our boundary conditions $q_{\text{ave}} = k\beta/r_o^2$; therefore we expect

$$\frac{N}{2\Omega} \Big|_{\text{max}} \approx \left(\frac{1}{r_o^*} \right) \sqrt{\frac{\widetilde{Ra}E}{Pr} \left(\frac{q^* - 2}{2} \right)}. \quad (7)$$

262 **Boundary-forced lateral temperature variations** In a fully convecting core an adiabatic tem-
 263 perature gradient ($\partial T_{\text{ad}}/\partial r$) will extend from the ICB to the CMB, except within thin boundary
 264 layers. Within a regional inversion layer a shallower conductive profile ($\partial T_c/\partial r$) will exist. The
 265 temperature difference at the CMB between a fully convecting region and the top of a regional
 266 inversion layer of thickness h will be approximately

$$\delta T \approx h (\partial T_{\text{ad}}/\partial r - \partial T_c/\partial r). \quad (8)$$

267 Setting the conductive temperature gradient throughout the inversion layer equal to the minimum
 268 CMB heat flux gives

$$\delta T \approx \frac{h}{k} (q_{\text{ad}} - q_{\text{min}}), \quad (9)$$

269 where k is the thermal conductivity of the core.

270 To estimate δT for the Earth, we use $q^* = (q_{\text{max}} - q_{\text{min}})/(q - q_{\text{ad}})$ and $q - q_{\text{ad}} = Q_{\text{conv}}/4\pi r_o^2$
 271 to rewrite equation (9) as

$$\delta T \approx \frac{h Q_{\text{conv}}}{4\pi r_o^2 k} (0.5q^* - 1). \quad (10)$$

272 The thickness of the regional inversion layers arises dynamically in our models and depends on
 273 both q^* and Q_{conv} . Here we assume a superadiabatic heat flow of $Q_{\text{conv}} = 0.6$ TW and a thermal

274 conductivity $k = 100 \text{ W m}^{-1} \text{ K}^{-1}$ and simply vary q^* and h to estimate the temperature difference
275 at the CMB between fully convecting and subadiabatic regions. The likely values of δT are gen-
276 erally on the order of 10's of kelvin (supplementary figure 4). The largest values corresponding
277 to particularly thick layers that will have a large Brunt-Väisälä frequency. To explain a layer with
278 $N \approx \Omega$ by purely thermal effects requires $\partial T/\partial r \approx 35 \text{ mK/km}$, with the temperature gradient
279 scaling as N^2 . Any compositional contribution would reduce the required temperature gradient for
280 a given buoyancy frequency.

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406 manuscript. JM and CD carried out the numerical modelling and analysis.

Quantity	Definition	Molecular	Turbulent	Simulations
Rayleigh	$\widetilde{Ra} = \frac{\alpha g_0 \beta}{2\Omega\kappa}$	4×10^{13}	2×10^{10}	225 – 18000
Ekman	$E = \frac{\nu}{2\Omega L^2}$	7×10^{-16}	4×10^{-11}	$10^{-6} - 10^{-4}$
Prandtl	$Pr = \frac{\nu}{\kappa}$	0.04	1	1
Reynolds	$Re = UL/\nu$	2×10^9	4×10^4	$O(10^1 - 10^3)$
Rossby	$Ro = U/2\Omega L = ReE$	1.5×10^{-6}	1.5×10^{-6}	$O(10^{-4} - 10^{-1})$

Table 1: Nondimensional numbers.

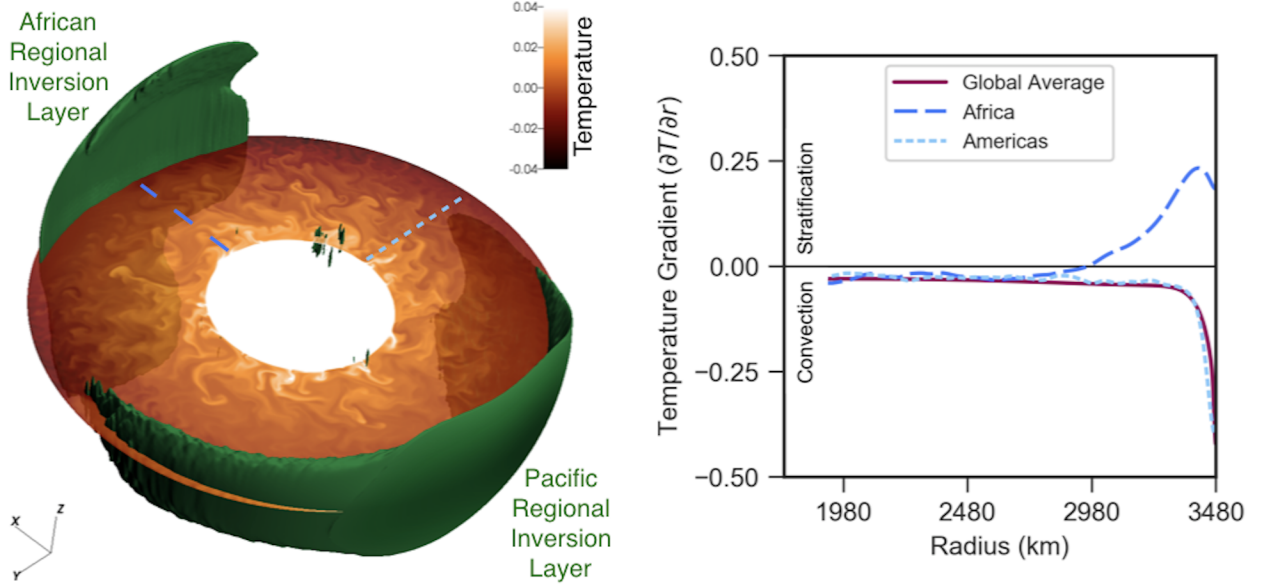


Figure 1: Thermal structure in the simulation with a tomographic CMB heat flux pattern, $q^* = 5.0$, $E = 10^{-6}$, and $\widetilde{Ra} = 1.8 \times 10^4$. Left: Green isovolumes denote the thermally stratified regional inversion layers ($\partial T/\partial r > 0$ in the time-average); equatorial slice shows instantaneous temperature anomalies at one point in time. Right: Time-averaged profiles of radial temperature gradient ($\partial T/\partial r$) in the top half of the core ($r_o/2 < r < r_o$). Regional profiles on the equator ($\theta = \pi/2$) are shown for longitudes associated with Africa ($\phi = 0$, long-dash blue line) and the Americas ($\phi = 3\pi/2$, short-dash light blue line). The horizontally-averaged profile is shown by the solid magenta line. Temperature has been non-dimensionalised as described in the methods section.

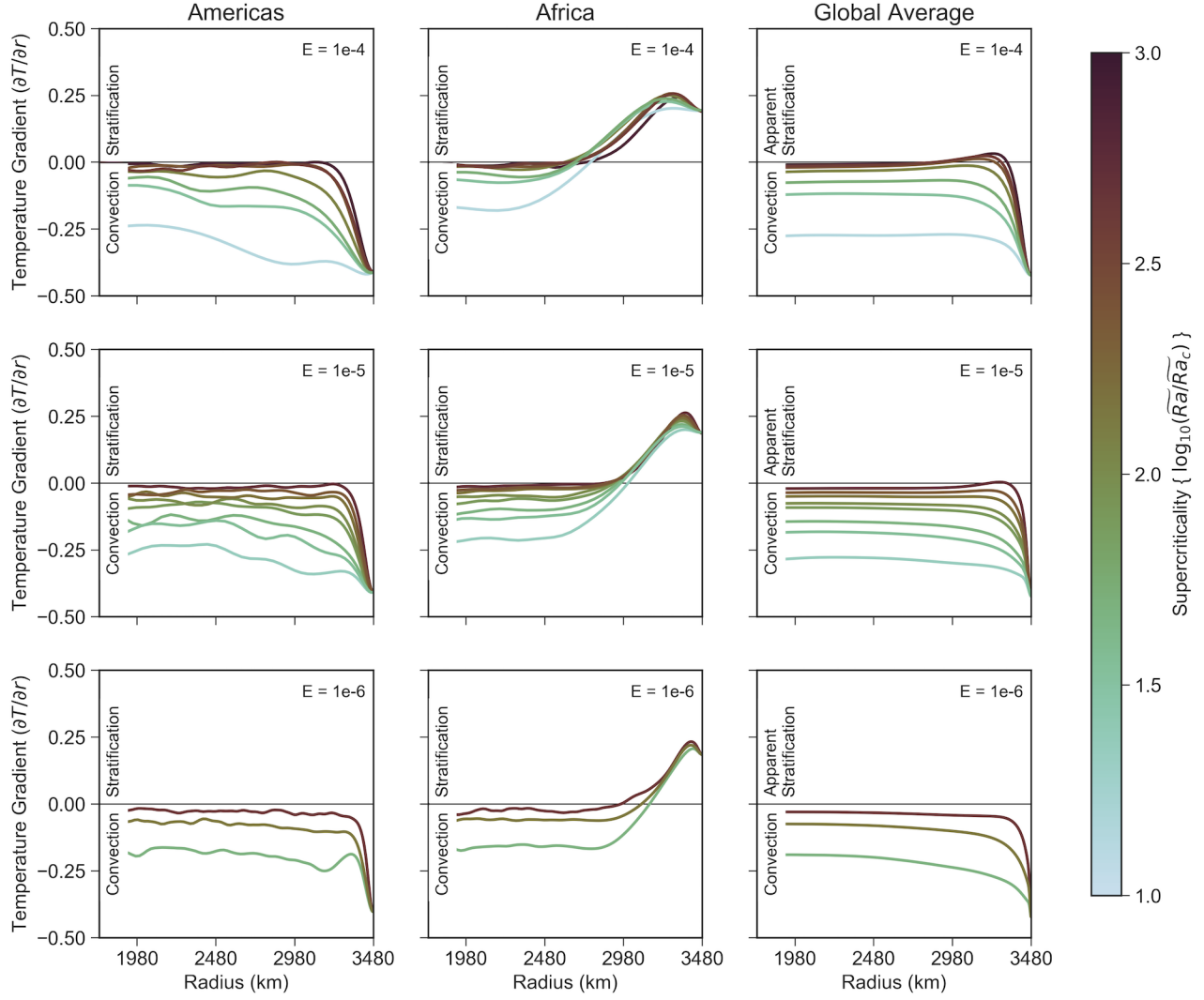


Figure 2: Profiles of time-averaged radial temperature gradient ($\partial T/\partial r$) in the top half of the core ($r_o/2 < r < r_o$). As in figure 1, we consider equatorial profiles under the Americas (left) and Africa (middle), as well as the global average (right). Simulations have a tomographic CMB heat-flux pattern, with $q^* = 5.0$, and $E = 10^{-4}$ (top), 10^{-5} (middle), or 10^{-6} (bottom). Colour of the lines indicates the super-criticality of the modified Rayleigh number from 10 times critical (light) to 1000 times critical (dark). Temperature has been non-dimensionalised as described in the methods section.

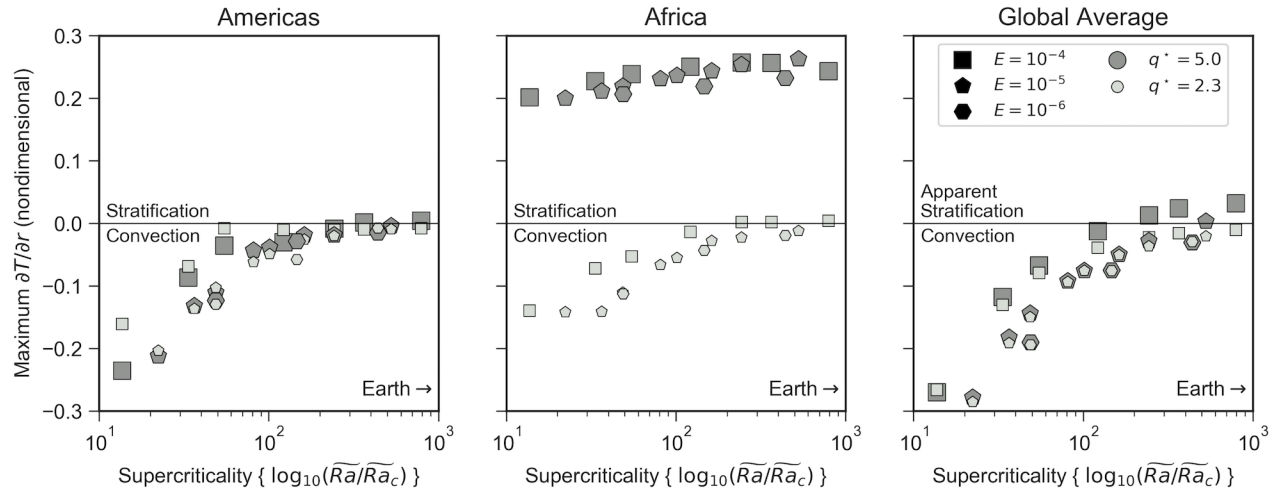


Figure 3: Maximum time-averaged radial temperature gradient ($\partial T/\partial r$) as a function of supercriticality. As in figure 2, we consider equatorial profiles under the Americas (left) and Africa (middle), as well as the global average (right). Symbol shape indicates $E = 10^{-4}$ (square), 10^{-5} (pentagon), or 10^{-6} (hexagon). Simulations have a tomographic pattern of CMB heat flux; symbol size and colour indicates $q^* = 2.3$ (small, light grey), or 5.0 (large, grey). Temperature has been non-dimensionalised as described in the methods section.

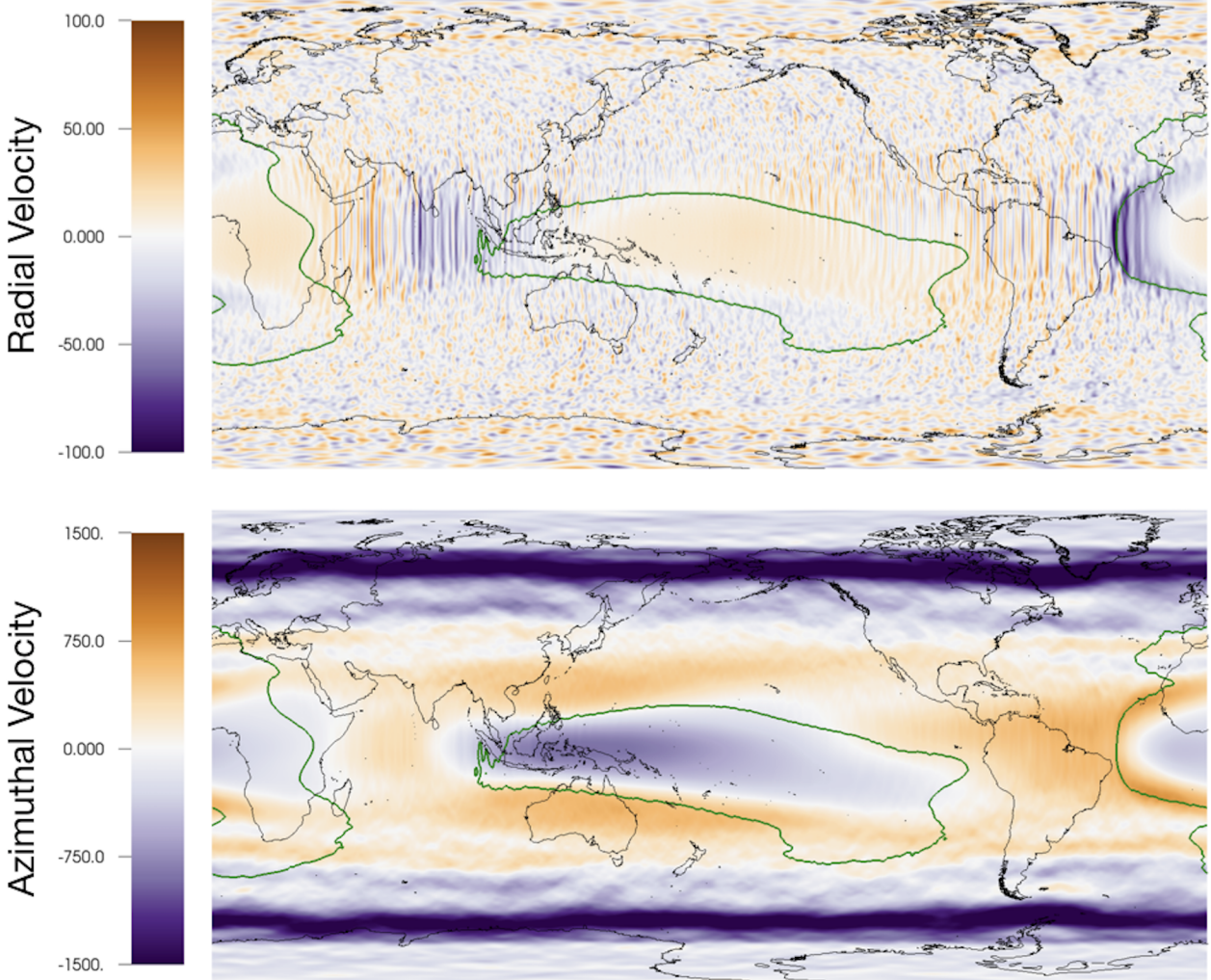


Figure 4: Flow near the top of the core for the simulation in figure 1. Time average of the radial velocity (top) and azimuthal velocity (bottom) at a depth of ~ 100 km below the CMB. The green line denotes the boundary of the regional inversion layer ($\partial T/\partial r = 0$ contour). This is the simulation with a tomographic CMB heat flux pattern, $q^* = 5.0$, $E = 10^{-6}$, and $\widetilde{Ra} = 1.8 \times 10^4$. The averaging was done over 37 advection times. The flow velocity is non-dimensionalised as described in the methods section.

Supplementary Information for “The Apparent Stratification at the Top of Earth’s Liquid Core”

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Supplemental Movie 1: Equatorial slices (viewed from above, Pacific to left, Africa to right) of thermal structure in the simulation with a tomographic pattern of CMB heat flux presented in figure 1 of the main text. Left: temperature field. Right: radial gradient of temperature. The movie spans approximately 2.6 advection time units, which is approximately 7% of the total model run.

Supplemental Movie 2: Equatorial slices (viewed from above, Pacific to left, Africa to right) of thermal structure in the simulation with a hemispheric pattern of CMB heat flux presented in supplemental figure 1. Left: temperature field. Right: radial gradient of temperature. The movie spans approximately 2.7 advection time units, which is approximately 7% of the total model run.

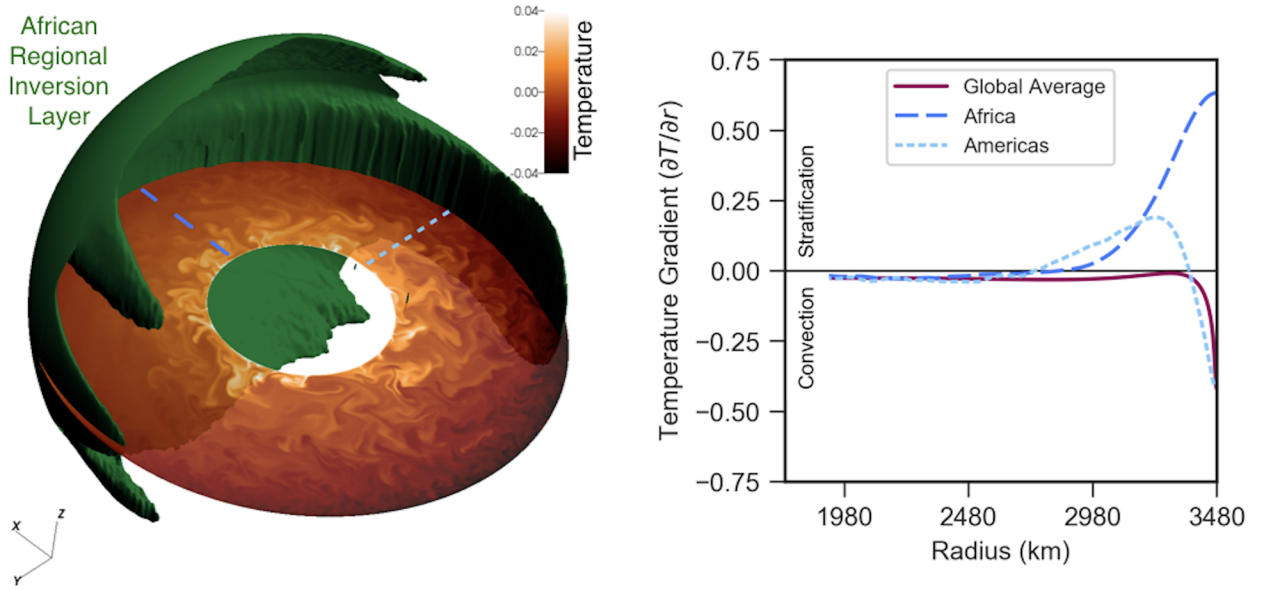


Figure 1: Thermal structure in the simulation with a hemispheric CMB heat flux pattern, $q^* = 5.0$, $E = 10^{-6}$, and $\widetilde{Ra} = 1.8 \times 10^4$. Left: Green isovolumes denote the thermally stratified regional inversion layers ($\partial T/\partial r > 0$ in the time-average); equatorial slice shows instantaneous temperature anomalies at one point in time. Right: Time-averaged profiles of temperature gradient ($\partial T/\partial r$) in the top half of the core ($r_o/2 < r < r_o$). Regional profiles on the equator ($\theta = \pi/2$) are shown for longitudes associated with Africa ($\phi = 0$, long-dash blue line) and the Americas ($\phi = 3\pi/2$, short-dash light blue line). The horizontally-averaged profile is shown by the solid magenta line. Temperature has been non-dimensionalised as described in the methods section.

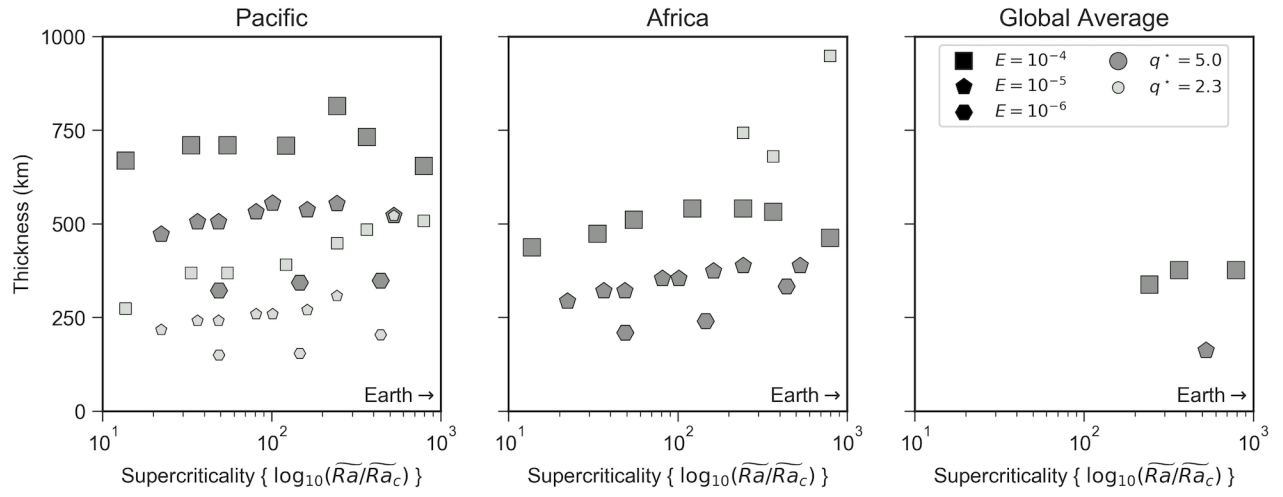


Figure 2: Thickness of the regional inversion layers under the Pacific and Africa, and the thickness of the apparent global stratification, as a function of super-criticality from all simulations with the tomographic CMB heat-flux pattern. Symbol size and colour indicates $q^* = 2.3$ (small, light grey), or 5.0 (large, grey). Symbol shape indicates $E = 10^{-4}$ (square), 10^{-5} (pentagon), or 10^{-6} (hexagon).

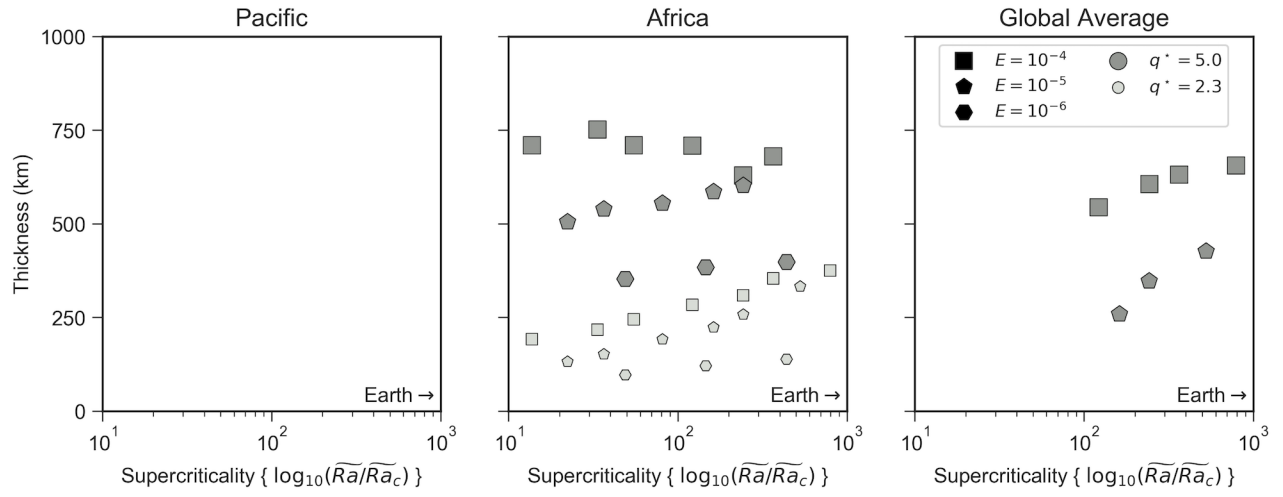


Figure 3: Thickness of the regional inversion layers under the Pacific and Africa, and the thickness of the apparent global stratification, as a function of super-criticality from all simulations with the hemispheric CMB heat-flux pattern (for this pattern no regional inversion layer forms under the Pacific). Symbol size and colour indicates $q^* = 2.3$ (small, light grey), or 5.0 (large, grey). Symbol shape indicates $E = 10^{-4}$ (square), 10^{-5} (pentagon), or 10^{-6} (hexagon).

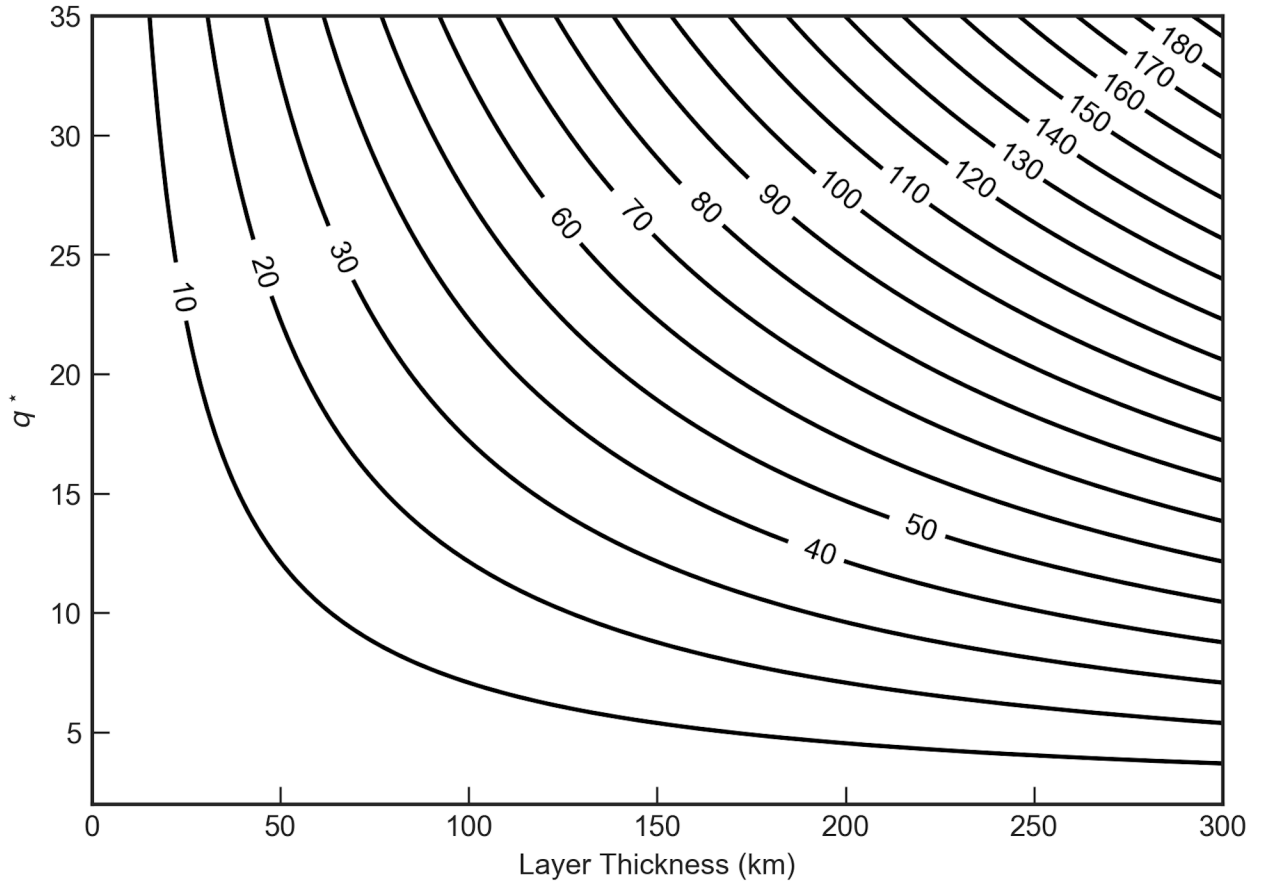


Figure 4: Excess temperature of the stratified regions. Contours of excess temperature (in kelvin) at the top of the core as a function of the layer thickness and the strength of heterogeneity, q^* . This example considers a total superadiabatic heat flow across the CMB of $Q_{\text{conv}} = 0.6$ TW, and thermal conductivity $k = 100 \text{ W m}^{-1} \text{ K}^{-1}$.

Quantity	Sybol	Value
CMB radius	r_o	$3.48 \times 10^6 \text{ m}$
ICB radius	r_i	$1.22 \times 10^6 \text{ m}$
shell thickness	$L = r_o - r_i$	$2.26 \times 10^6 \text{ m}$
gravitational acceleration at CMB	g_o	10 m s^{-2}
thermal expansivity	α	$1.5 \times 10^{-5} \text{ K}^{-1}$
rotation rate	Ω	$7.29 \times 10^{-5} \text{ s}^{-1}$
thermal diffusivity, molecular	κ_m	$1.3 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$
thermal diffusivity, turbulent	κ_t	$3 \times 10^{-2} \text{ m}^2 \text{ s}^{-1}$
kinematic viscosity, molecular	ν_m	$5 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$
kinematic viscosity, turbulent	ν_t	$3 \times 10^{-2} \text{ m}^2 \text{ s}^{-1}$
CMB superadiabatic heat flow	Q_{conv}	0.6 TW
thermal conductivity	k	$100 \text{ W m}^{-1} \text{ K}^{-1}$
thermal forcing	$\beta = Q_{\text{conv}}/(4\pi k)$	$5 \times 10^8 \text{ K m}$
characteristic flow speed	U	$5 \times 10^{-4} \text{ m s}^{-1}$

Table 1: Physical parameters.