A brief history of equifinality in hydrological modelling: from GST to GLUE to ML

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Highlights:

- Traces the 50-year evolution of equifinality from General Systems Theory and geomorphology to hydrological modelling, GLUE, limits of acceptability, and hydrological applications of machine learning.

- Synthesizes how equifinality developed from a philosophical concept into an operational framework for model evaluation, uncertainty, and hypothesis testing.
- Extends the historical arc into the AI era through neural equifinality, highlighting why predictive skill alone cannot resolve learned representation, identifiability, or robustness under change.

Abstract

Equifinality has shaped hydrological modelling for five decades. The equifinality thesis rejects the concept of an "optimal" model in favour of the possibility that different model structures or parameter sets can provide similarly acceptable representations of a system when uncertainties in the forcing and evaluation data are considered, especially when those data are subject to epistemic uncertainties and disinformation. We trace the concept from General Systems Theory and geomorphology to its adoption in hydrology, the development of GLUE, and subsequent limits-of-acceptability approaches to model evaluation. We then examine its emerging relevance to machine learning. High-dimensional and flexible ML models can achieve similar high predictive performance through different architectures and training realizations while encoding different internal representations, giving rise to a form of neural equifinality. Improved predictive skill therefore does not necessarily resolve questions of process attribution, identifiability or robustness under extrapolation and change. We argue that the ML era extends and amplifies, rather than resolves, the equifinality problem, highlighting the need for hypothesis testing, informative observations and fitness-for-purpose evaluation of alternative model representations.

Keywords: equifinality; General Systems Theory; GLUE; history of hydrology; machine learning; neural equifinality

Equifinality as a philosophy for environmental modelling

The first mention of equifinality with respect to hydrological modelling is probably in the PhD thesis of Beven (1975) where it appears with a footnote defining the meaning added at the request of the external thesis examiner, Prof. Terence O'Donnell (1927-2011), then at Imperial College, London. The footnote was literally a cut-and-paste addition, typed on a strip of paper that was glued to the bottom of the page, a necessary technique of the time when the thesis had already been bound before the examination. The note stated that: "*The term equifinality is used here to denote the possibility of obtaining equally good (but non-error free) simulations from different model formulations or different parameter values with the same model* " (footnote to page 14). At that time, Prof. O'Donnell had long been concerned with the optimization of hydrological models in respect of both parsimonious parameterisations (Dawdy and O'Donnell, 1965) and in the design of model representations with a view to avoiding difficulties in optimization (Ibbitt and O'Donnell, 1971). The philosophical approach that an optimal model would exist and could be found in model calibration was quite different from the physically-based partial differential equation approach of Beven (1975), which not only required the specification of parameters for hundreds of finite elements but, at that time, also required nearly one hour of computer time to run one hour of simulation. Given that computing requirement, the potential for equifinality in such models could not be evaluated and consequently could be only a philosophical concept.

The origins of equifinality in General Systems Theory

The conceptual roots of equifinality lie in the General Systems Theory (GST) of Ludwig von Bertalanffy (1901-1972), particularly in his book of 1968. In GST, he distinguished open systems (biological, social, geo- or ecological networks that exchange matter/energy with their environment) from closed systems and emphasized that, particularly in open systems, similar end states can be reached through different initial conditions and internal pathways (von Bertalanffy, 1951, 1968). In this formulation, equifinality is not a modelling failure but a natural property of complex open systems, analogous to later concepts of self-organisation under constraints evolving to similar outcomes. The implication for inference is immediate: observed outcomes may constrain behaviours without uniquely determining underlying causes or trajectories.

Equifinality in geomorphological explanation

The concept was later introduced into geomorphology by W.E.H. (Bill) Culling in 1957 and R. J. (Dick) Chorley (1927-2002) in 1962 (see Haines-Young and Petch, 1983). The concept in this context is used to denote the possibility of similar landforms being derived from different initial conditions in different ways by possibly different processes (e.g. Schumm, 1977). In this context, Cooke and Reeves (1976) also made a link to the method of multiple working hypotheses that had a long history in geomorphology from Grove Karl Gilbert (1843-1918) in 1886 and Thomas Chrowder Chamberlin (1843-1928) in 1897. Haines-Young and Petch

(1983) argue against this interpretation in a way that might also apply to hydrological models. They suggest that:

> *"if two or more theories cannot be distinguished on the basis of the predictions that they make about landform character then they are poor theories. To describe those features as 'equifinal' does not detract from this situation. Use of the term merely encourages the maintenance of those theories in an ad hoc and uncritical way. The aim of the geomorphologist should be to develop those theories so that they can be tested against each other. Only then, through the process of experiment and observation can the geomorphologist hope to eliminate any false conjecture."* (Haines-Young and Petch,1983, p.466)

They go on to suggest that the only valid use of the term is in the restricted sense of Culling (1957) where similar landforms might be shown to be the result of quite different initial conditions and processes, with a resonance to the dynamic equilibrium concept that had long been part of geomorphological thinking. Such a concept is, of course, difficult to apply in geomorphology when initial conditions are necessarily unknowable and, as in the case of many different hydrological models and parameter sets giving similar performance by some measure(s), the question then arises as to just how to define a test to "*eliminate false conjecture*".

Bill Culling himself later criticised the equifinality concept in the light of theories of nonlinear dynamics and chaos, suggesting that:

> "*The ubiquity of noise means that all stable systems are transient.... It is now known that transients can exhibit chaotic behaviour and that these chaotic transients may have extremely long lives (~10^6 iterations). Chaotic transients can only compound the difficulties of recognising chaotic behaviour in the landscape. Despite all these difficulties, however, it is known that chaotic motion and strange attractors enter into the heartland of physical geography for turbulent flow is irregular, intermittent, self-similar and whether we like it or not ubiquitous*."
>
> (Culling, 1988, p.68)

The period of the 1970s and 1980s was also the start of the formulation of models of landform development in geomorphology. This is a more rigorous way of testing different hypotheses about landforms by deductive reasoning, even though the process representations and what is computationally possible remained necessarily simple. The models were nonlinear and could demonstrate chaotic behaviour (Phillips 1993, 1994). Within such a model framework there may then be many combinations of initial conditions, model behaviours and parameter sets that are consistent with the limited observations available about a particular class of landform. They are then equifinal in some model sense, indeed in a very similar sense to that used by Cooke and Reeves (1976) and criticised by Haines-Young and Petch (1983) (see Beven, 1996). This is also the sense in which the concept was originally applied to hydrological modelling by Beven (1975).

Equifinality and underdetermination

The equifinality thesis as expressed in hydrology by Beven (2006) allows that multiple model structures or parameterizations might be able to reproduce the observations equally well or might not be distinguishable as hypotheses given the uncertainty in the input data and observations. Somewhat analogous concepts appear across disciplines, including non-identifiability in statistics (Bickel & Doksum, 2007), ill-posedness in inverse problems (Tarantola, 2005), degeneracy in physics (Machta et al., 2013), and underdetermination in machine learning (Goodfellow et al., 2016; Belkin et al., 2019). There are, however, some differences in philosophical approach between the concepts. The equifinality concept is firmly rooted in a rejection of the idea that a unique optimum or true model can be found. Terms such as non-identifiability, ill-posedness or underdetermination suggest an alternative philosophy that the information content of the available data is insufficient to find that optimum or true model, but that it should in principle be identifiable given more information.

However, these various concepts converge on a common insight: observations often lack sufficient independent information to uniquely constrain explanations. Recent developments in remote sensing, data assimilation, and machine learning have expanded the volume and dimensionality of both observations and model representations (Reichstein et al., 2019; Shen et al., 2023). Yet, this expansion has not eliminated the equifinality challenge; rather it shifts the problem into higher representational dimensions (D'Amour et al., 2022). The resulting increase in degrees of freedom may improve predictive skill scores while leaving fundamental questions of identifiability, process attribution and explanation unresolved, particularly in difficult-to-interpret machine learning models (see below).

This contrast in philosophies is played out in different disciplines against a background of different reliabilities of data. Hydrology is one of the inexact sciences with data that can be unreliable for simulation and sometimes disinformative for model calibration (Beven and Smith, 2015; Beven, 2019a). This can be particularly the case for estimates of catchment rainfall and snowmelt inputs, as well as flood peak discharges in times of overbank flow or very low flows in times of drought. In simulating such systems, we should expect epistemic surprises and we should not expect that a "true" model of a system will exist. The best that might be possible is to find a set of conditionally acceptable or *behavioural* models, pending further qualitative or quantitative evaluation. This is the equifinality thesis or philosophy. Ideally of course, when models might be used for a variety of predictive purposes we would like models that are *process-behavioural* as well as being acceptable on some measure of fit to data. That again raises the question of how to test hydrological models as hypothesis about system function.

Equifinality as a basis for model evaluation and uncertainty estimation in hydrological modelling

Over time, as the available computational power became greater, it became increasingly possible to evaluate the potential equifinality in hydrological models in the sense of different model realisations producing similarly acceptable or *behavioural* outcomes. This concept of differentiating between behavioural and non-behavioural models was used in the Regional or General Sensitivity Analysis (GSA) of George Hornberger, Bob Spear and Peter Young (see Hornberger and Spear, 1981), as well as in the set-theoretic approach to model calibration of Keesman and van Straten (1990) and Rose et al. (1991). In these approaches, once a simulation is defined as behavioural in some sense then it is treated as part of a predictive set with equal weights.

The first author was a colleague of George Hornberger at the University of Virginia in the period 1979-1982 at the time of the GSA papers and started to apply Monte Carlo sampling of parameter sets in applications of Topmodel (Beven and Kirkby, 1979). This showed quite clearly that many different parameter sets could achieve close to optimum values of the types of model calibration objective functions in use at that time, such as the Nash-Sutcliffe Efficiency (Nash and Sutcliffe, 1970). This work was continued when he returned to the UK and the Institute of Hydrology at Wallingford, including applications to continuous simulation flood frequency estimation (e.g. Beven, 1986, 1987). The concept of defining likelihood weights for different models based on past performance within the set of behavioural models led to the development of the Generalised Likelihood Uncertainty Estimation (GLUE) methodology, a name first coined in Virginia in about 1982 (BLUE - Best Linear Unbiased Estimator - concepts were common in statistics at the time but difficult to apply to nonlinear hydrological models with complex response surfaces). The GLUE concepts were later outlined in Beven (1989) and first published by Beven and Binley in 1992. The methodology has been widely used in applications both within and outside of hydrological modelling, with >3000 citations on Web of Science (> 6000 on Google Scholar). A 30-year review of the GLUE methodology was provided by Beven and Binley (2014).

GLUE was very much developed within a philosophy of equifinality of model structures and parameter sets in producing similar behavioural outputs (Beven and Freer, 2001; Beven, 2006). This was considered an inevitable consequence of using complex models with high dimensional parameter spaces applied with data subject to both epistemic and aleatory uncertainties. As such it was controversial. While it was Bayesian in concept, using past performance to condition the likelihood weights, it was criticized (strongly, see Mantovan and Todini, 2006) for not using formal statistical likelihoods to do so. If the strong assumptions of a formal statistical likelihood are valid, however, then a GLUE analysis based on such a likelihood definition will produce equivalent results (albeit at the expense of using more runs to define the likelihood surface depending on the sampling strategy chosen). These were defined as the ideal cases in Beven et al. (2008). Most real applications, however, are non-ideal in the sense that such strong assumptions about the statistical nature of the model residuals are not valid, and the use of a formal likelihood will lead to strong over-conditioning that is not justified. It has also been shown that some events might be disinformative for model evaluation purposes, for example where apparent event runoff coefficients are greater than 1 in the absence of snowmelt (e.g. Beven and Smith, 2015; Beven, 2016, 2019a, 2026).

Within the GLUE methodology, the behavioural ensemble of models (if it exists) can be used to provide a likelihood weighted distribution of predicted outcomes as an expression of modelling uncertainty. The expectation then is that the deviations from the evaluation data during any calibration period will have similar characteristics in model prediction. This will include all the complexities in those deviations that might arise as a result of epistemic uncertainties in the modelling process including periods of bias where the simulations do not cover the evaluation data. Such complexities will not have a simple stochastic structure since the epistemic uncertainties are those that are not easily assigned probabilities and which might then lead to surprises (Hoffman and Hammonds, 1994; Helton and Burmeister, 1996; Rougier and Beven, 2013; Beven, 2001a, 2016; Beven et al., 2018). Sources of epistemic uncertainty arise primarily from the specification of initial conditions and forcing data; model structural errors, including process representations; observation errors and potential commensurability errors between observed quantities and model simulated variables (see, for example, Beven, 2006, 2019a; Beven and Westerberg, 2011; Gupta et al., 2012; Beven and Smith, 2015; Kampf et al., 2020). The GLUE methodology has now also been used quite widely beyond hydrological modelling in disciplines where similar difficulties of epistemic uncertainties and defining likelihoods arise.

Equifinality and Limits of Acceptability

This, however, then raises a fundamental question of when a model should be considered behavioural. This is effectively the same question as that raised above as to how to test a model as a hypothesis while allowing for epistemic uncertainties. One way of looking at this question is to consider what likelihood measures or model weights might be appropriate given what might be known about potential errors in the forcing data and evaluation data for a particular application. Posing the question in another way, if all models are wrong, how can we define which ones might be considered useful?

This question was turned around in Beven and Lane (2019, 2022) to take advantage of the rejectionist nature of the behavioural/non-behavioural dichotomy and ask at what point can a model be considered an invalid or NOT fit-for-purpose representation of the system? They proposed that such an assessment might only be possible as a qualitative Turing-like test of performance because of the usual lack of knowledge about data uncertainties, since whether a model appears to be fit-for-purpose may not be a function of the model structure and parameter set alone. Whether a model can produce an acceptable simulation of the evaluation data will depend on any uncertainties associated with those data and, crucially, on uncertainties and errors in the forcing data (e.g. Heuer et al., 2026).

Such an approach also allows for the possibility that ALL the models tested might be rejected, something which had already been recorded in a number of GLUE applications (e.g. Brazier et al., 2000; Choi and Beven, 2007; Dean et al., 2009; Hollaway et al., 2018). Beven et al. (2022), for example, having specified limits of acceptability prior to making 100,000 model runs, showed that none of those runs could satisfy those limits at all time steps or, by analogy with statistical hypothesis testing, at 95% of the time steps. They also point out that the statistical analogy might not be appropriate in this type of evaluation, because the 5% failures might be those time steps (such as flood peaks) that are of most interest for a particular application.

Allowing for model rejection was one of 8 principles for model evaluation proposed by Beven and Lane (2022) as follows:

1. Definitions of ‘fitness’ for the purpose of a project should be agreed amongst the relevant stakeholders, taking expert advice as necessary, before a Turing-like Test is applied. Criteria of fitness might be both quantitative, where observed data are available for model evaluation, or qualitative.
2. Models should not be expected to perform better than the data on which runs are based and evaluated. A critical evaluation of the data for consistency and uncertainties, independent of the model being studied, should therefore be a pre-requisite for model evaluation.
3. Models should not contradict secure evidence on the nature of system response and still be considered fit-for-purpose.
4. Evaluation should be based on the science (with the aim of getting the right results for the right reasons) not on the need to make a decision.
5. Evaluation should allow for the possibility that all models might be rejected (invalidated) using criteria that allow for input and other observation uncertainties.

6. Past performance provides the only information about future performance, but the results of a Turing-like Test will always be conditional and the problem of induction and possibility of future surprise remain.
7. Achieving objective evaluation of models can be a challenge: evaluations and evaluators should themselves be evaluated in terms of their fitness-for-purpose.
8. The basis for the definition of “fitness” should be recorded in an audit trail that will allow later review of the process, including the expected sources of uncertainty.

The model invalidation approach remains, however, firmly within an equifinality philosophical framework of identifying models that might pass the Turing-like test and therefore be useful in prediction. There is then another question of whether there is enough computing resource to identify the set of behavioural models in high dimensional model structure / parameter spaces, particularly for models with long run times. This is a question of both increasing efficiency in model programming, of increasing the efficiency of search such as using the DREAM_LoA routine to search on limits of acceptability (Vrugt and Beven, 2018), and of increasing the computing resource directed at the problem, such as the use of massively parallel fast graphics processors as a modern extension of the “Transputer” and “Beowulf” parallel PC cluster systems used in some of the applications of GLUE at Lancaster University (e.g. Freer et al., 1996, 2004).

Neural equifinality in the era of machine learning

In recent years there has been an explosion of applications of machine learning and deep learning models in hydrology. Machine learning (ML) models are trained on past data with a view to making future predictions (or at least being tested on independent past data as a validation exercise). There are many studies showing that these types of models can provide better performance, as expressed in terms of objective functions, than more traditional conceptual and process-based hydrological models (Kratzert et al., 2018). Objective functions such as the Nash-Sutcliffe Efficiency are still being used in such applications (Klotz et al., 2022; Li et al., 2021) despite their known limitations (e.g. Melsen et al. 2025). Certain of these approaches have used physical constraints, such as mass balance, or even variables within conceptual models as inputs to create hybrid systems (Frame et al., 2023). The result is a class of models that can be extremely high dimensional in terms of the number of trainable parameters or weights, while being trained on data that are themselves uncertain and potentially biased.

This is not so different to the situation when the concept of equifinality as applied to hydrological models was first adopted from General Systems Theory, except that in machine learning there is no strong *a priori* expectation that the model parameters will have physical significance (though see, for example Beven, 1989, for a discussion of the limitations of physical significance of hydrological model parameters). There has also been relatively limited systematic exploration of how forcing and observational uncertainties propagate through, or are compensated by, hydrological ML models. Given their high dimensionality and flexibility, they may compensate for some consistent data uncertainties or biases while still producing high values of the objective function used in calibration. This is no bad thing of course if we could be sure that those biases might continue to hold into the future for which predictions are required. It is more of a problem if the uncertainties are epistemically non-stationary and may be different in prediction periods.

This gives rise to a form of ***neural equifinality*** as suggested by Hong et al. (2026). Neural equifinality can be viewed as an AI-era extension of the original equifinality thesis into high-dimensional neural representation spaces. More specifically, neural equifinality arises when different architectures, parameter configurations, training realizations, or input representations achieve similarly acceptable predictive performance while encoding different internal representations. Whereas many conceptual hydrological models typically involve dozens of parameter dimensions and relatively interpretable model structures, modern ML models may contain many thousands to millions of trainable weights distributed across multiple hidden layers. Different network architectures, training realizations, weight configurations, and input selections may therefore achieve similarly acceptable predictive performance while implying different internal representations of catchment behaviour or various internal compensations for uncertainty in the data (Lakshminarayanan et al., 2017; D'Amour et al., 2022), including some evidence that simple model structures might perform almost as well as complex structures (Poudel and Steinschneider, 2026).

In this sense, neural equifinality does not replace classical equifinality; rather, it amplifies it within a much larger, less interpretable, and weakly constrained representation space. From a systems analysis point of view, better performance can, in principle, lead to smaller uncertainties in fitted parameters, but this can be offset by the sheer number of parameters and their complex interactions within the model structure. ML models also cannot escape from the fact that different high-performing models can arise from different objective functions, or different training data, or different training initializations, all of which are manifestations of the model equifinality thesis. Other model structures and parameter sets may provide equally good fits to the data, in a similar way to the dotty plots used in GLUE applications to illustrate the empirical equifinality of different parameter sets. The ML era does not remove equifinality by finding better-performing models; it shifts part of the problem from non-uniqueness in parameter and model-structure space toward non-uniqueness in learned representation space.

While this increased flexibility may help explain why machine learning models can outperform traditional conceptual or process-based hydrological models under standard objective functions (Kratzert et al., 2018), improved predictive performance does not by itself resolve whether the gain reflects additional hydrological process information, more efficient use of catchment attributes and regional similarities, compensation for input and evaluation errors, or exploitation of the chosen objective function (Kratzert et al., 2019; Reichstein et al., 2019). This raises an important question: does improved predictive fit represent better extraction of hydrological information, or compensation for persistent errors in the data? Physical constraints do not necessarily improve predictive performance when the forcing or evaluation data themselves violate those constraints. For example, enforcing mass conservation can reduce performance where forcing or target data contain systematic errors that an unconstrained ML model can partially compensate for (Frame et al., 2023). Such compensation may be useful under stationary conditions but problematic when those errors are epistemically non-stationary or change during extrapolation.

However, although neural equifinality has now been articulated conceptually (Hong et al., 2026), its empirical prevalence, structural manifestations, and predictive consequences across hydrological ML models have not yet been systematically evaluated. Many of the ML models that have appeared in the literature have been based on a performance- and optimization-centred philosophy, in part because of the computing resource required in training some models but encouraged by the possibility of doing better than more traditional approaches. Although ensemble and probabilistic approaches have begun to address predictive uncertainty (e.g. Klotz et al., 2022), **predictive uncertainty and neural equifinality are not identical problems:** the implications of multiple similarly performing model realizations for internal hydrological representation remain much less explored. This is especially important if knowledge of data uncertainties is incorporated into limits of acceptability across machine learning models. One issue here is what those different models might imply about system behaviour, both in interpretation of the fitted models in terms of hydrological processes, and in the range of predictions for different purposes that might then ensue from an ensemble of such models. Interpretation, of course, remains difficult; many possible internal pathways can produce similar predictions without necessarily implying similar learned hydrologic relationships. Hence the suggestion that they might be used to indicate the right sort of functional relationships to use in prediction, without prior assumptions about model structure as in more traditional conceptual models (e.g. Beven et al., 2026).

Equifinality and hypothesis testing

The arguments for using ML models in hydrology rest primarily on empirical adequacy, a form of instrumentalist philosophy (Beven, 2002). Because they are derived from data and have structures that are not always easy to interpret, they do not represent hypotheses about catchment function in the same way as process-based hydrological models. It might therefore be interesting to consider the 50-year-old discussion about equifinality and multiple working hypotheses in geomorphology mentioned earlier (see also Beven, 2025). It was argued then that equifinality of models was just an indication that they represented poor hypotheses that could not be properly tested. We seem to be in a similar situation now. We have machine learning models of greater empirical adequacy but we do not really know why they produce better fits to hydrological data, apart from a general principle of flexibility of structure. And, if there are many models of this type that report equally good fits to the data (allowing for data uncertainties) then how can we then test or distinguish between them? We know that the rank ordering of those models in terms of fit would certainly change if a different period of data were used in training or if a different objective function were used. We also know that epistemic surprises in the data or disinformative periods of data might introduce predictive bias or mean that models that would be the most suitable for use in prediction might be rejected if such uncertainties are not explicitly considered in the calibration process in assuming, for example, that all data are precise and accurate.

In a commentary on hypothesis testing from 25 years ago, the first author proposed a number of hypotheses with a view to improving process-based hydrological models (Beven, 2001b). The final hypothesis suggested was that *"Graduate students can always think of useful new hypotheses with which to test models (rather than simply calibrating existing models and learning little)"*. Commenting on this proposal, it was suggested that:

> *"The literature does not give the impression that this hypothesis is being tested with any enthusiasm at the current time. It seems to be the case that graduates are being taught more about how to use models than about how to critically evaluate models. This is hardly surprising given the complexity and learning curves necessary for some of the models that are currently available, and being able to use models is a precursor to being able to criticize them. However, there is surely a need to confront existing models*

> *with carefully collected field data sets and new hypotheses that will lead to improvements in how we approach the modelling process."* (Beven, 2001b, p.1657)

There seems to be a resonance here with today's proliferation of ML models. Lots of calibration, occasional validation against other periods of data, but not much in the way of hypothesis testing, or whether models are getting the right results for the right reasons within the limitations of data uncertainties. There is surely more to be done here in designing observations, perturbation tests, and hypotheses capable of distinguishing or falsifying alternative machine-learning representations of catchment behaviour.

The Future of Equifinality in Hydrological Modelling

It is hard to envisage much change in the status of hydrology as an inexact science. Both ground based and remote sensing data will continue to be subject to significant uncertainties for the foreseeable future, limiting the information provided for testing models as hypotheses about catchment responses. It would therefore seem that model equifinality is likely to be ubiquitous in the application of hydrological models and that evaluations based on fitness-for-purpose in some way will continue to be necessary. This will apply equally to applications of ML models. The question of how best to define fitness-for-purpose and associated limits of acceptability before any model runs for a particular purpose remains an issue. This may be a balance of qualitative and quantitative evaluations but some observational data might be more effective for a particular purpose than others (Beven et al., 2020). This remains to be tested (e.g. Heuer et al., 2026).

It would seem to be difficult to avoid an expectation that allowing for data uncertainties there might be many models that will satisfy any definition of fitness-for-purpose, but that might only be conditionally acceptable subject to any future testing. That, however, suggests a useful way forward for any type of hydrological model: how can tests that will constrain the potential equifinality of models be best constructed and applied. One route forward is to use a wider range of informative observations in model testing. A recent theme in hydrological modelling has been that of "tracer-aided" models. These generally require additional assumptions about mixing and dispersion processes, and therefore additional parameters, but can also be applied within a limits of acceptability framework (e.g. Wu et al., 2025). There is also the increasing possibility to use spatially distributed information from remote sensing. Such datasets provide opportunities to challenge model hypotheses beyond discharge-only evaluation and to constrain the set of behavioural models. However, they also introduce additional parameter dimensions (especially for spatial patterns) and their own epistemic uncertainties in retrieval assumptions, scale mismatches, and commensurability problems when compared with model-predicted variables. Machine learning might have an advantage in that respect, in that it can be trained to reproduce the types of patterns seen in historical sequences in ways that might not be possible with traditional distributed hydrological models based on the wrong process representations (e.g. Beven, 2018, 2019b). However, that advantage will necessarily be mitigated by the problem of interpretability and the potential for neural equifinality.

Assessing neural equifinality will be computationally demanding because it requires exploring multiple architectures, training realizations, objective functions, and parameter configurations rather than evaluating a single optimized model. The question of how to conduct that evaluation also remains open, particularly when equifinality can arise from using different objective functions or, as suggested above, prior limits of acceptability to allow for uncertainties in the forcing and evaluation data. Given the good fits demonstrated in many catchments, it may also be questioned as to whether such an evaluation might be worthwhile. If we are only interested in prediction under stationary conditions, such as in short-term forecasting applications with data assimilation to correct for errors, then forecast performance based on an objective function suited for that purpose may be the only relevant criterion (e.g. Kordani et al., 2024).

More generally, however, as soon as it is allowed that neural equifinality might be an issue and there is a search for a set of behavioural models, then it will be possible to produce a range of predictions. This becomes more important when the model is being used to predict outside the range of the training data set, or under conditions of future change. A range of predictions from different behavioural models, even if they cannot be associated with any objective probability weighting for each member of the ensemble when strong statistical assumptions are difficult to justify, will still allow some choices about whether to be risk-averse or risk-accepting in decision making. The challenge in the ML era is therefore not simply to identify the best-performing model, but to determine which alternative models remain acceptable, why they differ, and where those differences matter for prediction and decision making.

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