

Hidden periodicity in LiDAR point-count rasters

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Preprint status: Non-peer reviewed EarthArXiv preprint, version 2

DOI: <https://doi.org/10.31223/X5TV40>

Keywords: LiDAR; LAS format; point density; rasterization; quantization; aliasing; spectral analysis

Abstract

LiDAR point-count rasters can contain regular stripes that are not present in the surveyed scene. They arise when the fixed step on which LAS files store coordinates, often 1 cm, interacts with the grid used to count points. We identify three mechanisms, grid mismatch, floating-point boundary rounding and uneven filling of stored positions, predict where each pattern appears and how strong it is, and name a controlled change that should weaken or move it. We tested the predictions on UAV, mobile and airborne point clouds and in PDAL, lidR and LAStools. Stripe sizes matched the predictions (median regression slope 1.00), and moving a 10 cm grid by 5 mm reduced the RMS relative count change per cell of a UAV raster from 11.6 % to 0.5 %. In an airborne tile where the stripe was too weak to see in individual cells, its spectral lines stood 4.6 to 22 times above a conservative background level. Across eight cell sizes the grid-mismatch line followed its predicted ground wavelength while a reference component already present in the finer-coordinate data stayed fixed. The patterns changed texture metrics of UAV count rasters by up to 5 %, while per-cell ratios and Voronoi-based density were almost unaffected. In a sample of 87 public files from three providers, 28 showed detectable uneven filling of stored values, mostly as weak metre-period patterns; in one redistributed file the filling produced stripes of 18 %. The effects are largest for cells below a few decimetres. Because each mechanism responds to a different controlled change, a suspicious pattern can be tested, and each mechanism has its own remedy.

1. Introduction

Figure 1a shows how rounding the coordinates of a UAV point cloud to 1 cm changed the point count of every 10 cm cell, relative to the same points at their original 0.1 mm precision. Some change is expected where rounding moves points across a cell edge, but the map shows a regular crosshatch with an RMS of 11.6 % over the 40 m crop (95 % bootstrap interval over 2 m blocks, 11.5–11.6 %). The pattern is not in the ground or the sensor. It comes from the interaction between the grid on which the file stores coordinates and the grid on which the raster counts them. Moving the counting grid by 5 mm (Fig. 1b), a change few users would expect to matter, reduces it to 0.5 %. The map RMS combines an easting stripe (6.3 %), a northing stripe (6.3 %) and a cell-level residual explained in Section 4.4. Point-count rasters are routine LiDAR products, used for density checks, ecological metrics and classifier inputs, so such patterns can enter later analyses unnoticed.

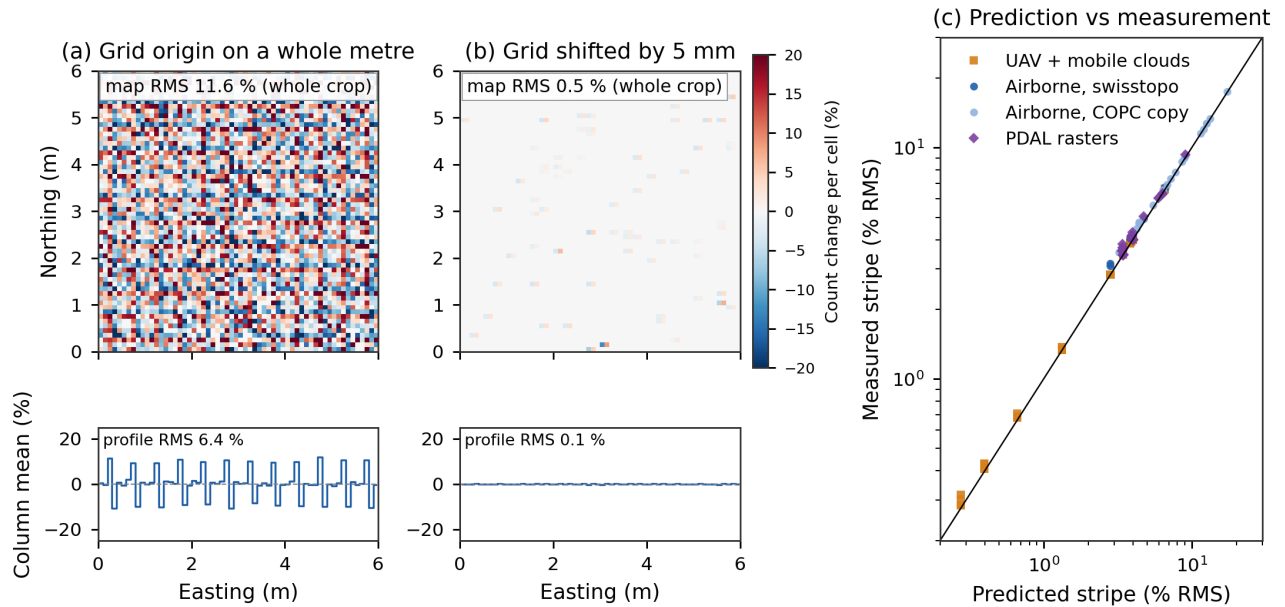


Figure 1: A regular count pattern from storage, and its removal. (a) PDAL counts in 0.1 m cells of a UAV crop stored at 1 cm, divided by the counts of the same points from their original 0.1 mm coordinates; grid origin on a whole metre. (b) The same grid shifted by 5 mm. Lower strips: means over each grid column of the window (easting profile), which isolate the easting stripe (6.4 % RMS in this 6 m window, 0.1 % in b); the map RMS also contains the northing stripe and cell-level scatter. Windows are 6 m $\times$ 6 m (south-west corner E 516,987 m, N 4,644,807 m, UTM 29N); colours clipped at ± 20 %. (c) Predicted against measured one-axis RMS stripe for real-data tests whose predicted stripe exceeds 0.2 % (UAV and mobile) or 2 % (airborne, where counting noise is near 3.5 %); airborne values are the excess over counting noise.

Density rasters are known to show artefacts. Petras et al. (2023) described moiré in airborne density rasters at 0.5 to 6 m cells and attributed it to the overlap of the sensor’s point pattern with the raster grid. The ASPRS density guidelines (ASPRS 2025)

note that point-in-cell rasters suffer aliasing and recommend a Voronoi-based density instead. Both place the cause in the sampling pattern of the sensor. Neither discusses the storage step of the file, which is the subject here. LAS files store coordinates as scaled integers (ASPRS 2019), and tools report one symptom of that: LAsTools flags “coordinate resolution fluff” when stored values use only every tenth position. Quantisation, dither and the square-root growth of signal over counting noise are standard (Widrow and Kollár 2008); counts of rounded data have a long history in statistics (Sheppard 1897; Heitjan 1989); and histogram bins that do not match a quantisation step are known from the code-density test for analog-to-digital converters (IEEE 2011). What is new is the form these take in LAS files and raster software: the storage step, floating-point cell edges in PDAL, lidR and LAsTools, uneven filling introduced by processing, their effect on derived products, and a test a user can apply to a given raster. The three mechanisms have a common structure: discrete states are aggregated into a second partition, and the result depends on whether the two partitions are commensurate, on how boundary ownership is computed in finite precision, and on how evenly the permitted states are occupied. We test that decomposition here only for LiDAR point-count rasters.

By Eq. (1) (Section 2.2) the grid-mismatch stripe is at most $q/2c$: 5 % at 10 cm but 0.5 % at 1 m for a 1 cm step. A moiré of several per cent in metre-scale cells therefore cannot come from grid mismatch. Boundary rounding (bound q/c , 1 % at 1 m) and uneven filling are not excluded by that bound, and Section 5 gives tests that separate them from sensor patterns. The effect matters mainly for cells below a few decimetres and for analyses that pool many cells, such as spectra, texture measures and, possibly, gridded inputs to classifiers.

We noticed these patterns in several point-cloud tools while developing OpenLiDARViewer (Urias 2026). Our contribution is a characterisation of how LAS storage spacing and boundary arithmetic produce count-raster patterns, with predicted size, spectral location and detectability, validated on real data in three programs, measured for its effect on derived products and its occurrence in a sample of public files, and turned into controlled changes that test whether a given pattern came from storage. We organise it around three questions: where should a false pattern appear, how strong should it be, and what controlled change should weaken or move it?

2. How counting creates a pattern

2.1 From stored positions to counts

LAS stores each coordinate as an integer X with scale s and offset o : $x = o + X \cdot s$ (ASPRS 2019). With the common $s = 0.01$ m, which PDAL’s LAS writer uses unless told otherwise, every stored easting is o plus a whole number of centimetres. We call these permitted values *storage positions* and their spacing the *storage step* q . Each storage position defines a line of constant easting (or northing) on which points can lie; to avoid confusion with raster rows and columns we call raster cells along one axis

grid columns. The storage step is a property of the file, not of sensor accuracy or of the spacing between actual points.

We use three quantities and keep them apart. The *stripe* is the periodic departure of counts from their expected value when storage positions are equally filled (the *filling model*; Sections 2.2 and 2.4); a raster can count the stored coordinates correctly and still carry one. The *count change* compares counts from 1 cm coordinates with counts of the same points from finer stored coordinates, which serve as a reference, not as ground truth; we call this pairing a *twin test* (Fig. 1). *Misassignment* is a different problem: points placed in the wrong cell by floating-point arithmetic (Section 2.3). Individual storage positions may be empty; across many cells, the unequal number of available positions produces a predictable average pattern.

2.2 Grid mismatch

With $q = 1$ cm, a 12.5 cm cell cannot contain 12.5 storage positions, so successive cells contain 12 or 13. If the positions are equally filled, counts alternate even over uniform ground (Fig. 2a). The resulting sequence 12, 13, 12, 13, ... is deterministic, so its spectrum can be predicted (Appendix A).

How large is this, typically? Write $r = c/q$ for the number of steps per cell and f for its fractional part. A fraction f of cells holds one extra storage position, and the root-mean-square (RMS) change along one axis is

$$\text{RMS} = \frac{\sqrt{f(1-f)}}{r} \leq \frac{q}{2c}. \quad (1)$$

Single cells can deviate more than the RMS: at 13.3 cm, cells hold 13 or 14 positions and deviate by -2.26% or $+5.26\%$, while the RMS is 3.45% . A two-dimensional map carries a stripe along each axis, and the two multiply: at 13.3 cm a cell holds 14×14 , 14×13 or 13×13 positions, and its count changes by $+10.8\%$, $+2.9\%$ or -4.5% , which draws a crosshatch (Fig. 3). When $f = 0$ (c a whole multiple of q) this mechanism is absent. Table 1 gives f and the bounds for common cell sizes and storage steps.

2.3 Boundary rounding

Take $c = 10$ cm, a whole multiple of q . If a boundary falls exactly on a storage position, software computes the cell index $\lfloor (x - g)/c \rfloor$ in floating point for grid origin g ; the result can land just below a whole number, and the points at that position go to the neighbouring cell (Fig. 2b). For example, with offset and grid origin both at 2,485,000 m, $s = 0.01$ m and $c = 0.1$ m, the stored integer $X = 30$ decodes to 2,485,000.3 m, and $(x - g)/c$ evaluates to 2.999999998 in float64: the point lands in cell 2 instead of 3. The integer form, $X \div 10$, gives 3. The one-axis RMS change is bounded by q/c (Appendix A).

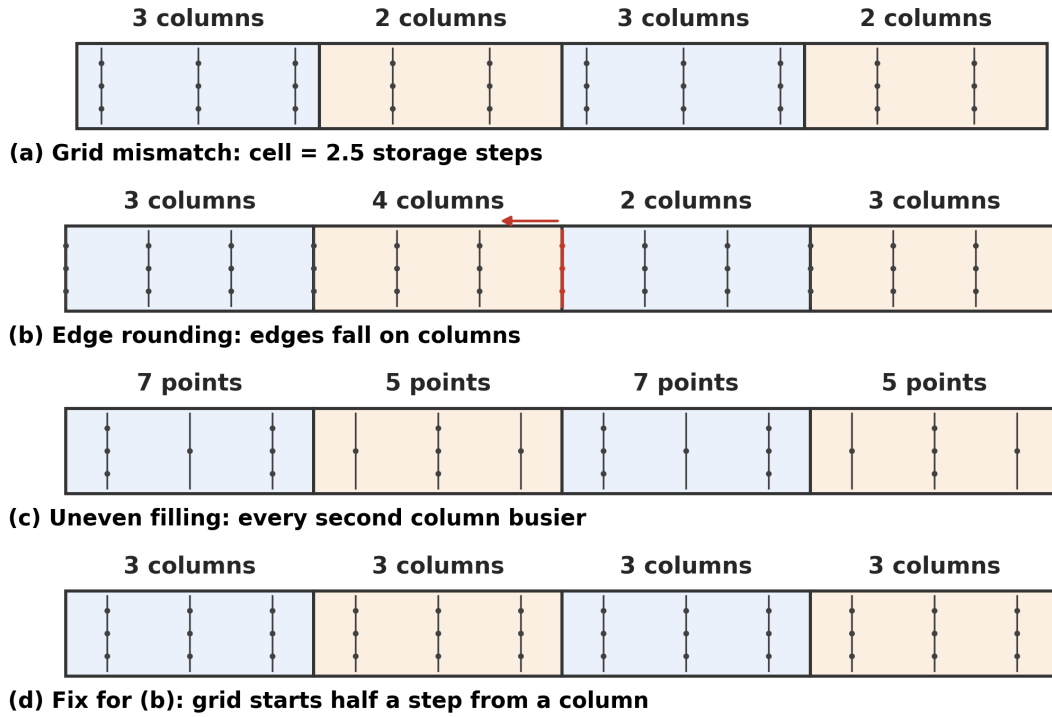


Figure 2: **Three mechanisms, and the fix for the second.** Vertical lines are storage positions, dots are points. (a) Cells 2.5 steps wide alternate between 2 and 3 positions. (b) Cells 3 steps wide with edges on storage positions: a position on an edge decodes slightly low and moves one cell left. (c) Every second position holds more points; cells of 3 positions alternately hold 2 or 1 busy position. (d) Boundaries halfway between storage positions fix (b) only. Remedies: for (a), a cell width that is a whole number of steps; for (b), the half-step origin in (d) or integer indexing; for (c), a cell width that is a multiple of the filling pattern (here 2 steps).

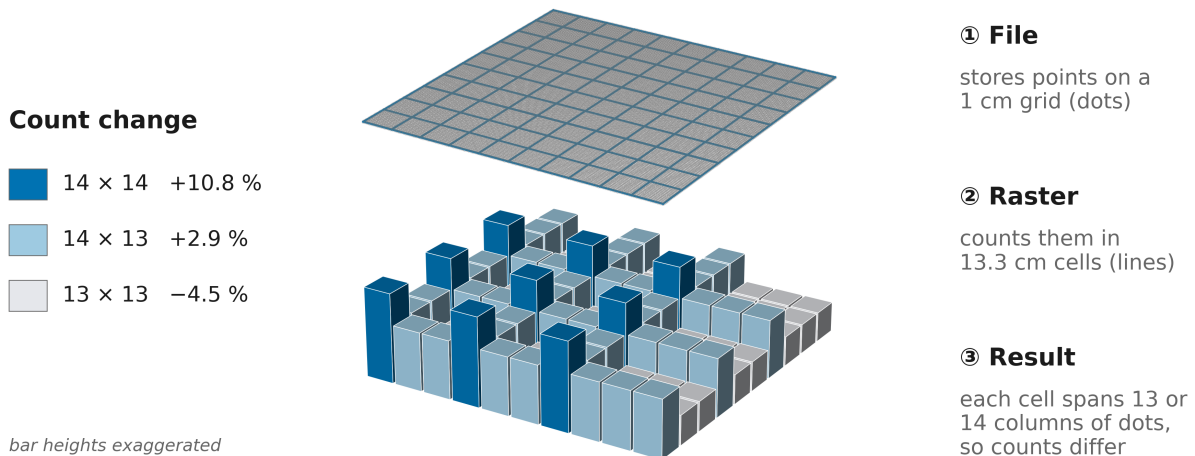


Figure 3: **Why a 13.3 cm raster of 1 cm data shows a crosshatch.** Each 13.3 cm cell spans 13 or 14 columns of 1 cm storage positions in each direction; with equally filled positions its count is the product of the two, and bars show the exact change from the mean. Code: fig_crosshatch3d.py.

When c is a whole multiple of q , the remedy is to place boundaries halfway between storage positions: $g = o + (m + \frac{1}{2})q$ for a whole number m , separately for each axis with that axis's offset and scale. This works while accumulated numerical error stays much smaller than the half-step gap: in float64 at coordinates near 10^6 m, errors are near 10^{-10} m; in float32 at 10^6 m, adjacent values are 0.0625 m apart, so coordinates must be rebased to a nearby origin before conversion to float32 (Goldberg 1991). The failure belongs to a known family in robust computational geometry, where finite-precision arithmetic can reverse a discrete decision near a boundary (Shewchuk 1997). The half-step origin does not help when c itself is not exactly representable in binary (for example 0.133 m): there float and integer indexing still disagreed for 0.4 % of UAV points. Computing cell indices from the stored integers with integer arithmetic avoids both problems. For the metric scales and grid parameters used in these tests, which are whole micrometres, the indices can be computed exactly:

```
UM = 10**6 # integer micrometres
def cell_index(X, scale, offset, origin, cell):
    # X: stored LAS integers (numpy int64); other arguments in metres
    num = round(offset*UM) + X*round(scale*UM) - round(origin*UM)
    return num // round(cell*UM)
```

X must be cast to int64 before the multiplication, and scale, offset, origin and cell must be whole micrometres; otherwise use rational or 128-bit arithmetic. Floor division rounds toward minus infinity, so negative numerators are handled correctly. With int64, micrometre values stay exact up to about 9×10^{12} m, far beyond any LAS coordinate; non-metric scales such as 0.01 ft need a common integer unit (for example 1/3937 m for US survey feet) or rational arithmetic.

2.4 Uneven filling

A file's processing history can fill storage positions unevenly, for example in a pattern repeating every M steps (Fig. 2c). Cells then hold equal numbers of positions but unequal numbers of busy ones, and a grid shift does not help. Cell widths spanning whole patterns (multiples of $M \cdot q$) remove the ideal periodic part. A histogram of the stored integers modulo a chosen number reveals such a pattern only if that number is compatible with the period; a flat histogram at one modulus does not prove even filling, and a single heavily repeated point can mimic a pattern (Section 4.3). Because the storage lattice and the raster grids studied here are aligned with easting and northing, the patterns analysed here appear as axis-aligned stripes or crosshatches (Fig. 4).

2.5 From counts to a spectrum

A spectrum measures how strongly a profile repeats at each spacing. *Where does grid mismatch appear?* Its strongest line sits at

$$\nu = \min(f, 1 - f) \text{ cycles per cell}, \quad L = c/\nu \text{ on the ground}, \quad (2)$$

with weaker lines at multiples of f , folded into the range 0 to $\frac{1}{2}$. At 13.3 cm, $\nu = 0.3$ and $L = 44$ cm; the full count sequence repeats only every 10 cells (1.33 m). For $f = 0$ there is no line. Eq. (2) needs only c and q ; lines from boundary rounding and uneven filling depend also on which edges misfire and on the filling pattern.

How strong in a spectrum? A pattern of fixed size becomes easier to detect as more points are pooled. Its line stands above the median of the spectral background by approximately

$$R \approx \frac{A\sqrt{N}}{1.67}, \quad (3)$$

where A is the line amplitude as a fraction of the mean count, N the number of points analysed, and 1.67 the rounded median of the Rayleigh-distributed noise amplitude (Eq. A4). At fixed A , four times as many points roughly doubles R . A practitioner can estimate A from Eq. (A1) and N from the data to judge whether a predicted line could stand out. Eq. (3) assumes roughly independent, Poisson-like counting noise; R is a ratio to the background, not a significance level. The number of cells does not enter the idealised noise, but enough cells are still needed to resolve the frequency. For a single map, the stripe reaches the per-cell noise level only near 20,000 points per m^2 for $q = 1$ cm (Eq. A3), so at common densities it is invisible cell by cell and visible only in pooled statistics.

What changes it? Section 5 (Table 3) lists the controlled change that tests each mechanism.

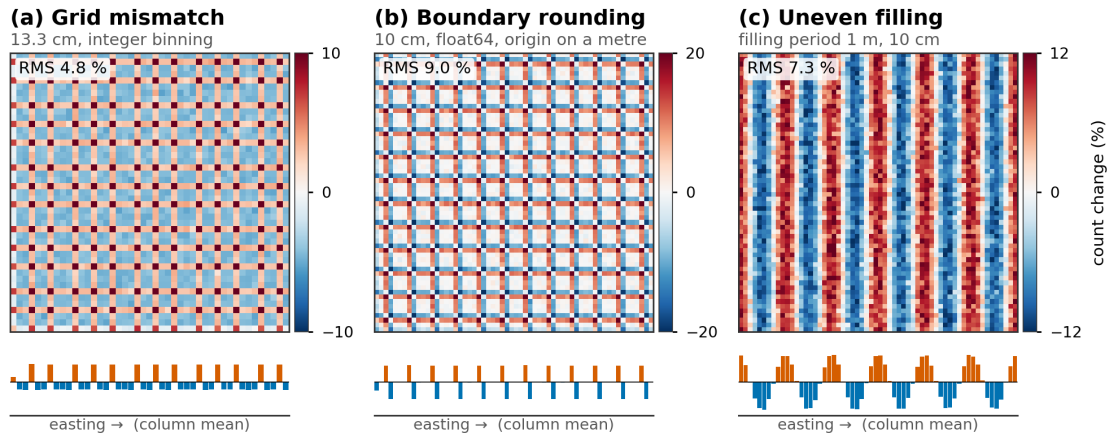


Figure 4: **Each mechanism in a count map** (synthetic; Figs. 2 and 3 show why). Count change per cell over a 6 m square with about 3,000 points per 10 cm cell, stored over unrounded counts minus one (colour scales differ); bars below each map are column means along easting. (a) 13.3 cm cells, integer binning: a crosshatch. (b) 10 cm cells, UTM-sized coordinates decoded in float64, origin on a whole metre: lines at the edges whose positions fall one cell low; which edges misfire depends on the coordinates and the origin. (c) Stored eastings weighted by $1 + 0.1 \cos(2\pi X/100)$, 10 cm cells, count over mean minus one: 1 m bands. Code: `fig_patterns.py` (seed 7).

Table 1. Grid-mismatch fraction f and stripe bounds for common cell sizes. RMS from Eq. (1); for $f = 0$, only boundary rounding can act, with bound q/c . Full table for 1 mm, 1 cm and 0.01 ft steps in the archive (results_v2_C).

Storage step	Cell	f	Grid-mismatch RMS	Rounding bound ($f = 0$)
1 cm	10 cm	0	0	10 %
1 cm	12.5 cm	1/2	4.0 %	–
1 cm	13.3 cm	3/10	3.4 %	–
1 cm	1 ft	12/25	1.6 %	–
1 cm	1 m	0	0	1 %
1 mm	1 in	2/5	1.9 %	–
0.01 ft	10 cm	308/381	1.2 %	–

3. Data and methods

Point clouds. UAV: DJI Zenmuse L1 vineyard survey (Vélez et al. 2023), 46,787,138 points stored at 0.1 mm. Mobile: mapry LA03 forest scan (Hrdina 2025), 31,566,748 points at 0.1 mm. Airborne: swisstopo swissSURFACE3D tile 2485-1109, 15,680,312 points at 1 cm. Second airborne file: a Cloud Optimized Point Cloud (COPC) file with the same tile identifier, point count and generating-software field, from a public mirror (Section 4.6); the mirror’s path labels it 2022 and the swisstopo item 2019, so we identify the two as the same tile on those three attributes.

Twin binning. We rounded the UAV and mobile clouds to 1 cm and 1 mm and counted the same points before and after, along easting and northing profiles. The ratio cancels scene variation that is smooth across a cell, but not sharp features that straddle an edge. We therefore dropped profile bins with fewer than 200 points, the two bins at each end, and bins whose count N differed from the mean of their two neighbours by at least half of N (Section 4.3 tests this choice). Grid-mismatch tests used exact integer binning, so boundary rounding could not contribute.

Held-out prediction. For the airborne files we learned the filling of storage positions from one half of the tile and predicted stripes in the other half.

Spectra. We measured the amplitude of easting stripe profiles at each predicted frequency (main line and four multiples) and compared it with the predicted profile. Two empirical references were computed from 2,000 frequencies in 0.06–0.49 cycles per cell, at least 0.01 from lines $k = 1$ –8 and filling lines: the median, for Eq. (3), and the 99.9th percentile, used to say a line stands out. For ideal Rayleigh noise the 99.9th percentile is 3.16 times the median, and the probability that any of 2,000 independent frequencies exceeds 3.91 times the median is 5 %. The observed ratios were larger (Section 4.3), so the empirical level is conservative.

Detectability and moving-line tests. We randomly thinned the northern half of the airborne tile (7,840,177 points) at four cell sizes, which changes N inside a fixed foot-

print, and recounted the UAV points along easting at eight cell sizes (10.75 to 17.45 cm, f between 0.3 and 0.75), tracking the strongest line in the 1 cm spectrum and a reference component in a fixed 3.3 to 3.9 m window (archive, METHODS.md).

Software. PDAL 2.10.2 (writers.gdal, output_type count; Butler et al. 2021; PDAL Contributors 2026) with binmode true, because the default counts all points within a radius of each cell centre, so neighbouring cells overlap. lidR 4.3.3 (pixel_metrics; Roussel et al. 2020). LAsTools lasgrid 260914 (rapidlasso 2026; official demo mode, on a UAV sub-crop below its 1.5-million-point test limit). Rasters covered a 40 m UAV crop rounded to 1 cm by PDAL (offset 0) and a 400 m crop of the swisstopo tile (offsets 2,485,000 and 1,109,000 m), at 10 cm to 1 m, with the origin on a whole metre, a whole metre plus 5 mm, or each program’s default. We regressed measured on predicted stripes bin by bin; a slope of 1 means the prediction has the right size.

Effects on products. On the UAV crop we compared count rasters from 1 cm and 0.1 mm coordinates at 10, 12.5 and 13.3 cm on the same grid, for metrics fixed before the comparison: grey-level co-occurrence (GLCM; Haralick et al. 1973) contrast, homogeneity and entropy (distance one cell, 0° and 90° , 32 levels with bin edges fixed from the reference), the mean 3×3 standard deviation of counts, and two within-cell ratios (first-return fraction, and fraction of points more than 0.5 m above a local ground surface). We also computed the ASPRS Voronoi density (per-point density 1/area of the point’s Voronoi cell, averaged per raster cell) on a 10 m UAV window and a 200 m airborne window.

Census. To see how often uneven filling occurs in distributed files, we drew 90 files at random (seed 20260926; protocol archived before download, with two logged amendments) from three open providers: USGS 3DEP (Entwine point tiles on AWS), NOAA Digital Coast (COPC) and swisstopo swissSURFACE3D. Three files with fewer than 50,000 points were excluded, leaving 87 (scale 0.01 m in 77, 0.001 m in 10). For each file we tested the stored X and Y integers at moduli 2, 5, 10 and 100 for uniformity with χ^2 at a per-test $\alpha = 0.001/8$ (Bonferroni, family-wise $\alpha = 0.001$ per file), and required a maximum relative deviation above 5 %. METHODS.md in the archive gives all parameters, seeds and versions; the archive maps each claim to its script and result file and holds the PDAL pipelines, lidR code and LAsTools command lines (Data and code availability).

4. Results

4.1 Hidden in the map, visible in the spectrum

In the airborne tile (15.7 points per m^2) a single cell has a stripe-to-noise ratio of 0.026 to 0.028 by Eq. (A3), so the stripe is far below the counting noise of any one cell. Its easting profile shows the predicted lines (Fig. 5a). Across seven cell sizes the main grid-mismatch lines of the original file reached 4.6 to 22 times the 99.9th-percentile level. Jittering each coordinate within ± 0.5 cm reduced each tested grid-mismatch line

5- to 400-fold, below that level. Lines from the file's 25 cm filling pattern remained, at up to twice the level, as expected for uneven filling.

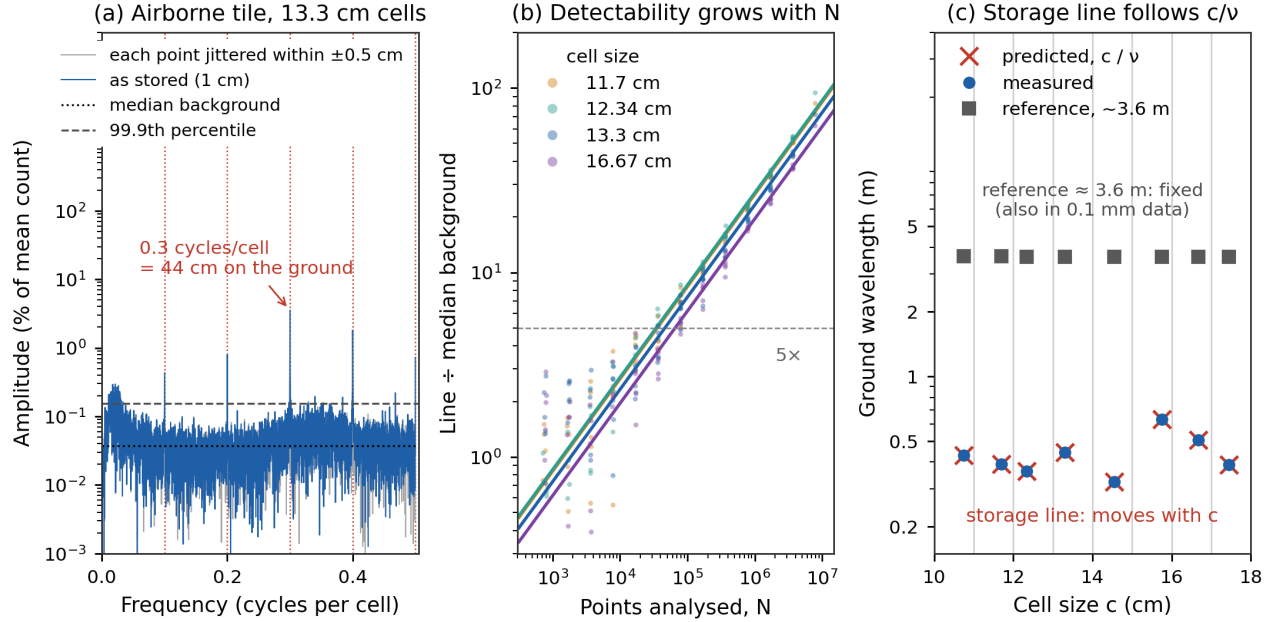


Figure 5: A pattern hidden in the map is visible in the spectrum, grows with the number of points and moves with cell size. (a) Airborne easting spectrum at 13.3 cm, as stored and jittered within ± 0.5 cm; vertical dotted lines mark multiples of $\nu = 0.3$; horizontal lines show the median and 99.9th percentile of off-line amplitudes in this spectrum. (b) Line height above the median background, R , against points analysed after random thinning, at four cell sizes; lines are Eq. (3) without fitting. (c) Ground wavelength of the strongest line in the 1 cm UAV spectrum (dots), its prediction c/ν (crosses) and a reference component tracked near 3.6 m (squares), at eight cell sizes; grey lines mark whole-centimetre cells, where the grid-mismatch line is absent.

Thinning tested Eq. (3) (Fig. 5b). For the 159 cases with predicted $R \geq 3$ (N from about 17,000 to 7.8 million), a log-log regression of measured R on predicted R gave a slope of 1.03 (expected 1), with a median measured-to-predicted ratio of 0.97. Regressing $\log R$ on $\log N$ separately for each cell size gave slopes of 0.50 to 0.53 (expected 0.5), and an independent random seed for every level gave nearly the same values (slope 1.02, median ratio 0.96). At about 7,800 points the median R stayed below 3 at all four cell sizes. The predicted $R = 5$ fell at 34,000 to 66,000 points and the thinned profiles first reached a median of 5 at 36,000 to 78,000. The median background was 0.80 to 0.90 of the Poisson prediction at full density and 0.92 to 1.28 after thinning, which suggests that airborne sampling is more regular than random; at full density Eq. (3) therefore slightly underestimates R .

4.2 The storage line follows its prediction

At all eight cell sizes the strongest line in the 1 cm UAV spectrum sat at the predicted wavelength c/ν within one frequency bin, moving between 32 and 63 cm (Fig. 5c). It measured 2.9 % to 4.5 % after rounding and 0.04 % to 0.19 % at the same frequency in the 0.1 mm data. The reference component stayed at 3.60 to 3.62 m with nearly the same amplitude in both versions (0.95 % to 1.08 %). It is present in the 0.1 mm data, so it does not come from 1 cm storage, and the dominant row structure (1.73 m, trace 1.95 m along easting) does not account for it; its physical origin was not investigated. Agreement with the predicted movement supports a storage origin for the moving line, although other peaks may still come from storage or resampling where peaks overlap. The mobile scan was not used here because a single repeated record dominates its profiles (Section 4.3).

4.3 Sizes and lines match the predictions

Exact counts of storage positions matched Eq. (1) in 1,200 synthetic grids to within 9.3×10^{-16} (Fig. 8a), and synthetic tests at $q = 1$ mm and 0.01 ft gave measured-to-predicted ratios of 0.996 to 1.002 except at very small f , where the stripe lies near the noise (1.02). Table 2 summarises the real-data tests.

Table 2. Predicted against measured stripes (n = number of conditions; twin binning at 1 cm and 1 mm).

Test	Prediction from	Slope, median		R^2 , median (range)
		n	(range)	
Twin binning, UAV and mobile	exact position counts	40	1.00 (0.99–1.02)	0.99 (0.76–1.00)
UAV twin at 1 mm and 0.01 ft	exact position counts	20	1.00 (0.99–1.01)	– (0.88–1.00)
swisstopo tile, held-out	filling of other half	12	1.00 (0.99–1.01)	0.54 (0.44–0.70)
Second public file, held-out	filling of other half	20	1.00 (0.99–1.01)	0.82 (0.56–0.93)
PDAL rasters, all origins, stripe > 2 % airborne	exact counts, float64 edges	29	1.00 (0.96–1.04)	0.99 (0.44–1.00)
lidR rasters, stripe > 0.2 %	exact counts, lidR layout	10	1.00 (0.99–1.01)	–

The n values count parameter and software conditions, not independent scenes: the rows rest on one UAV crop, one mobile file and two airborne files. R^2 is lower in the airborne rows because counting noise, not error in the prediction, dominates those profiles. Two mobile conditions (easting at 12.34 and 13.3 cm) were dominated by one

repeated record, 1,377,060 points (4.4 %) with identical X, Y and Z, falling next to a cell edge. Excluding that spike restored slopes of 1.00, and every exclusion threshold from 0.1 to 0.8 of N gave slopes between 0.99 and 1.02 (archive, results_v2_D).

We tested 79 spectral lines in 23 profiles: 75 grid-mismatch lines and four uneven-filling lines of the two airborne files at 5 cm. All 75 grid-mismatch lines exceeded the 99.9th-percentile level, and amplitudes were 0.90 to 1.06 times the predictions (Fig. 6a; slope 0.998 [0.991, 1.005] and median ratio 1.00 [1.00, 1.01] when whole profiles are resampled). The observed 99.9th-percentile level was 3.4 to 25 times the median (median 5.7, against 3.16 for Rayleigh noise), because weaker storage harmonics and scene structure fall in the background, so the expected number of false exceedances among 75 lines is below 0.08. In a two-dimensional spectrum of the UAV twin ratio, the main lines measured 3.65 % and 3.63 % against 3.59 % and 3.58 % predicted (Fig. 8c). Eq. (A3) also held in single count maps of the UAV crop, with no twin reference: the stripe was fitted to the predicted pattern and the counting noise was measured from the difference of two random halves of the points. Over densities from 46 to 4,600 points per m² at 12.5 and 13.3 cm, the observed stripe-to-noise ratio (0.05 to 0.49) was 0.97 to 1.08 times the prediction (log-log slope 0.99; Fig. 8b), and the measured noise matched the Poisson value within about 1 %.

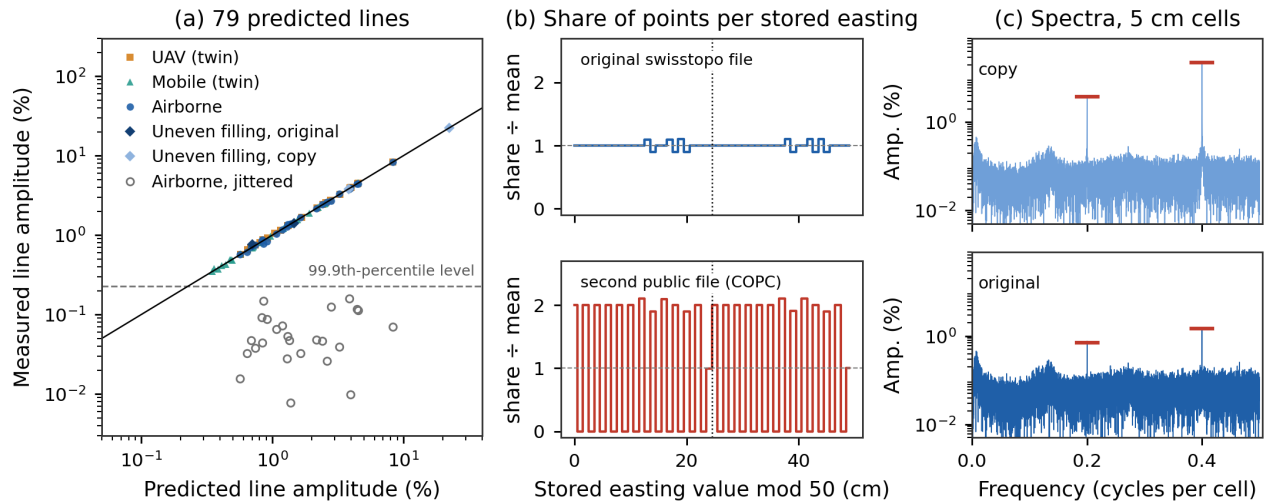


Figure 6: **Predicted and measured lines, and two public files for the same tile.** (a) Measured against predicted amplitude for 79 lines; open circles are the jittered airborne control, dashed line the median airborne 99.9th-percentile level. (b) Share of points per stored easting value, modulo 50 cm, in the swisstopo file (top) and the second public file for the same tile (bottom); 1 means even filling, and the dotted line marks the 25 cm shift of the pattern. (c) Easting spectra of both at 5 cm; red bars are predictions from the filling learned on the other half of the tile.

4.4 Boundary rounding in PDAL, lidR and LAStools

With float64 and the origin placed on a storage position at random, synthetic binning misplaced at least one position in 68 % to 71 % of configurations with 5, 10 and 20 cm cells, and in 5 % to 7 % with 25, 50 and 100 cm cells. Half-step origins removed every misplacement in 24,000 configurations. PDAL at 10 cm with the origin on a whole metre produced 6.3 % stripes on the UAV crop, against 6.3 % predicted by Eq. (A2). The misfire share and neighbour correlation in that prediction ($p = 0.20$, $\rho = -0.25$) came from evaluating the same float64 arithmetic at each on-position edge before rasterising. On the airborne crop the stripes were 6 % to 9 % above noise. The 5 mm shift brought both to noise level and left the 12.5 and 13.3 cm grid-mismatch stripes unchanged. PDAL's default easting origin fell on a UAV storage position and produced the stripe; its other defaults fell nanometres off positions and did not misfire.

Each edge that misfires moves the points of one half-step strip (5 mm wide) into the neighbouring cell. For the 10 cm UAV map, the Poisson variation of those moved counts predicts a cell-level residual of 7.6 % in the 6 m window of Fig. 1 (measured 7.7 %) and 7.2 % over the crop (measured 7.3 %). Together with the two 6.3 % stripes this accounts for the 11.6 % map RMS.

lidR builds its grid from the raster layout with cell size exactly c , columns from $x - x_{\min}$ and rows from $y_{\max} - y$. With that layout, predicted stripes matched: at 10 cm on the UAV crop the easting stripe was 6.30 % against 6.35 % predicted, and the northing stripe was 0.26 %, against a predicted 0, because counting rows from $y_{\max}$ makes every on-position edge misfire alike, which shifts the grid without striping it. All 10 lidR conditions with a predicted stripe above 0.2 % gave slopes of 0.99 to 1.01. A 5 mm shift of start removed the easting stripe.

LAStools lasgrid (demo build) at 10 cm on the UAV sub-crop reproduced the predicted northing stripe (6.4 % measured, 6.37 % predicted from float64 flooring). It wrote the requested half-step corner as a whole metre, so the 5 mm shift did not remove the stripe, and at 1 m its counts equalled integer binning. At 0.5 and 1 m (PDAL) and 1 m (lidR), no rounding stripe appeared with either origin (UAV twin stripes at most 0.09 %). On the airborne crop the 0.5 and 1 m profiles varied by 2.7 % to 4.7 %, identically for both origins, which is scene structure, not storage.

4.5 Effects on derived products

At 12.5 and 13.3 cm, the 1 cm storage changed the texture of the UAV count raster relative to the 0.1 mm reference on the same grid by 0.6 % to 4.7 % across GLCM contrast, homogeneity and entropy and the mean 3×3 standard deviation (Fig. 7b), while shifting the native data by 5 mm changed each by at most 0.2 %. At 10 cm, where $f = 0$, texture changed by at most 0.12 %. Means over the scene changed by at most 0.03 %. The two within-cell ratios changed by 0.009 to 0.010 per cell at 12.5 and 13.3 cm (absolute fraction), comparable to their sampling noise, because the stripe multiplies numerator and denominator alike. At 10 cm with the origin on a whole metre,

the edge misfires added 0.02 to per-cell ratios and 7.0 % to per-cell counts, and the half-step origin reduced both to 0.001–0.002 and 0.3 %. A distortion that multiplies numerator and denominator about equally cancels in their ratio. For ground fraction on the swisstopo tile at 12.5 cm, the predicted stripe fitted point counts with slope 1.005 but ground fraction with slope -0.007 $[-0.024, 0.010]$; the stripe appeared in absolute counts and was absent from this ratio, as it was from the two UAV ratios. Ratios whose parts are filled differently are not protected.

Voronoi density suppressed the storage line. At 13.3 cm the line at $\nu = 0.3$ measured 3.5 % in UAV counts and 0.29 % in UAV Voronoi density, the same as in the 0.1 mm data (0.32 %) and below the Voronoi background; in the airborne window it fell from 3.4 % to 0.25 %, against a background median of 0.16 % (Fig. 7c). This holds under two conditions. At 1 cm storage 45 % of UAV points share their stored XY with another point; dropping duplicates instead of giving each copy its share of the cell biased the UAV density profile by 4.7 %. And filling empty cells with zero instead of treating them as missing let the stripe back into sparse airborne rasters (2.6 %), because which cells are empty itself follows the storage pattern.

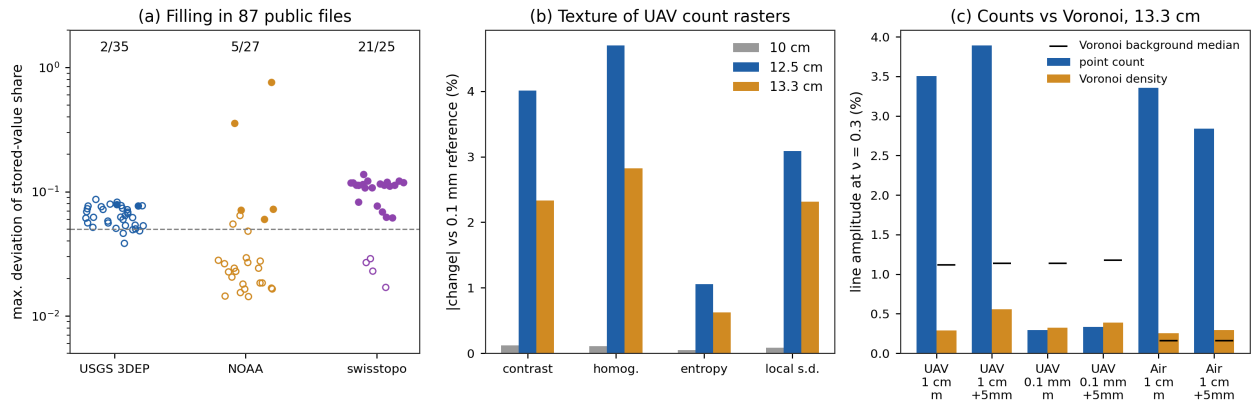


Figure 7: **Prevalence and consequences.** (a) Maximum deviation of the share of stored X or Y values (modulo 10 or 100) from even filling, in 87 randomly drawn public files; filled symbols meet the pre-specified rule (χ^2 and deviation above 5 %, dashed line). (b) Change of texture metrics of the UAV count raster from 1 cm storage, relative to the 0.1 mm reference on the same grid, at three cell sizes; 10 cm has $f = 0$. (c) Amplitude of the grid-mismatch line at 13.3 cm ($\nu = 0.3$) in point counts and in Voronoi density, with the grid origin on a whole metre (m) or shifted by 5 mm; bars for the Voronoi background median.

Removing the stripe does not change real structure. With the origin on a whole metre or shifted by 5 mm, and with 0.1 mm or 1 cm coordinates, the strongest two-dimensional peak of the 40 m crop between 1.5 and 8 m (3.57 m spacing, about twice the 1.73 m row spacing; amplitude 0.334 at 12.5 cm) and the 3.6 m easting component (1.75 % to 1.91 % on this crop) changed by less than 1 % and about 1.5 % of their value, respectively.

4.6 Uneven filling in sampled public files

In the census, 28 of 87 sampled files (32 %; 95 % Wilson interval 23–43 %) showed uneven filling detectable at the tested moduli (Fig. 7a): 2 of 35 USGS 3DEP files, 5 of 27 NOAA files and 21 of 25 swisstopo tiles. Most flags were weak and appeared only modulo 100 (a 1 m period at 1 cm storage for all but one file), with deviations of 6 % to 14 %; with moduli 2, 5 and 10 only, 2 of 87 files (2.3 %; 0.6–8.0 %) met the rule. The swisstopo tiles showed a regular comb, for example +10 % at every 25th value, which is consistent with processing rather than terrain. The strongest case, a NOAA tile, deviated by up to 76 %. No file showed coordinate resolution fluff. Provider rates differ widely, the sampling units differ between providers, and USGS and NOAA distribute re-encoded copies, so these counts describe this sample of distributed products, not public LiDAR in general.

The swisstopo file analysed above fills its storage positions within 0.90 to 1.10 of the average share, a weak filling pattern of the kind common in the census. A second public file with the same tile identifier, distributed as COPC, has a different pattern (Fig. 6b): along easting only every second centimetre is used, shifting every 25 cm; along northing one value is skipped and another doubled in each 25 cm block. The marginal distributions of both coordinates can be reproduced exactly from the original file: snapping each stored easting down to a 2 cm grid that restarts at every 25 cm block, and mapping each northing position r within a 25 cm block to $\text{round}(24r/25)$, gives sorted easting and northing values equal to those of the second file over all 15.7 million points. Under that transformation each coordinate moves by 0 or -1 cm. This matches marginals, not individual points, and does not identify the software that applied it. Converting the original file with PDAL 2.10.2 (`writers.copc`, default settings) preserved the sorted easting and northing values exactly, so COPC encoding alone did not reproduce the filling pattern. The second file shows stripes up to 18 % at 5 cm cells (17.5 % above counting noise), where the original stays within about 2 % of noise; the filling model predicted them (slopes 0.99 to 1.01) and its lines at 0.2 and 0.4 cycles per cell (3.8 % and 22.5 % measured, 3.8 % and 22.4 % predicted; Fig. 6c). At 25 and 50 cm cells, multiples of the 25 cm pattern, both files gave the same RMS (2.7 % along easting).

5. Discussion: a diagnostic and practical rules

Different responses point to different sources (Table 3), but the table is not a validated classifier and several mechanisms can coexist. A single heavily repeated record can mimic uneven filling in a histogram and a sharp feature in a profile, so the stored integers should be checked for duplicates before a pattern is attributed to processing.

Table 3. Controlled changes that test a suspected storage pattern. “Tested” marks responses observed here; the rest follow from the model.

If you suspect	Change	You should observe	You can conclude
Grid mismatch	Recount at another cell size	Peak moves to the new $L = c/v$ (tested)	Moving peak at predicted L : consistent with grid mismatch; fixed L : not grid mismatch
Grid mismatch	Jitter coordinates by up to $\pm q/2$	Grid-mismatch lines drop below background (tested)	Lines came from storage positions; a test, not a fix
Boundary rounding	Boundaries halfway between storage positions (c a multiple of q)	Stripe disappears (tested, PDAL, lidR)	Edges sat on storage positions; to keep an existing grid, index on integers
Uneven filling	Stored-integer histogram at several moduli	Comb or spike in shares (tested, 87 files)	Filling pattern and its period; check for repeated records
Uneven filling	Cells spanning whole filling periods, $M \cdot q$	Same RMS in original and refilled file (tested, 25 and 50 cm)	Pattern follows the filling; coarser cells cost resolution
Any	Compare counts with a ratio of counts, or use Voronoi density	Stripe in counts, not in ratio or Voronoi (tested)	Distortion multiplies both counts alike

The jitter test deserves a note: in signal processing, dither is a remedy for quantisation (Widrow and Kollár 2008), but survey coordinates are the data, so we use jitter only to test whether a pattern came from storage, never to alter a delivered product.

In practice, the safest workflow is to use cell widths that are whole multiples of the storage step (Table 1); where that is not possible, to compute cell ownership from the stored integers or place the origin half a step off the storage positions, checking that the software honours a fractional origin (LAStools lasgrid 260914 did not in our test); to histogram the stored integers of redistributed files and look for repeated records; and to prefer ratios of counts, or Voronoi density with duplicates weighted and empty cells left empty, when absolute counts are not needed. These rules would fit density guidance such as ASPRS (2025). Because grid mismatch and uneven filling depend only on the storage step and the grid, they can reach any program that counts stored

points into cells, from GIS tools to gridded inputs for machine learning; testing those settings with the archived scripts is an open and useful next step.

5.1 Representation and aggregation

The three mechanisms are cases of a representation–aggregation interaction: a discrete digital representation meets a later binning step, and properties of the representation appear in the measured output. Grid mismatch arises when the aggregation width is not a whole multiple of the representation step, so the two grids do not line up cell by cell; boundary rounding when ownership of states on a boundary is decided in finite precision; uneven filling when the permitted states are not equally occupied. None of these conditions is specific to LiDAR: histogram bins, voxels, image blocks and spatial tiles built from quantised values can meet them. Those cases were not tested here; what transfers is the diagnostic logic, not the measured sizes. That is the logic of Table 3: change a parameter of the representation or of the aggregation while holding the observations fixed, and predict whether a pattern should move, weaken or stay. It asks whether a structure belongs to the phenomenon being measured or to the representation used to measure it.

5.2 Limitations

The tests covered point counts along both horizontal axes, square cells of 10 cm to 1 m, three programs and storage steps of 1 cm, 1 mm and 0.01 ft. Most conditions reuse a few scenes (one UAV crop, one airborne tile, one mobile file), so the number of conditions overstates the independent evidence: the results demonstrate the mechanisms in these datasets and programs, not the general behaviour of LiDAR software. Elevation statistics, the vertical axis, voxel grids, radius-based gridding, other software and terrestrial or bathymetric scanners were not tested. The moving-line and two-dimensional tests used a single UAV survey, and the physical origin of the reference component near 3.6 m is unknown. Thinning changed N within one footprint only. Eq. (A3) was tested on one UAV crop at up to 4,600 points per m^2 , below the density of about 20,000 points per m^2 at which a single map shows the stripe at the noise level. The product effects were measured on one crop, and their size in a downstream model (for example classifier accuracy) was not tested. LAStools was tested in official demo mode on one sub-crop. The census sampled distributed, often re-encoded products from three providers, and its weak modulo-100 flags could include survey or processing structure that we did not separate. We are not aware of a published case in which a storage line was read as a real feature; we searched the PDAL, lidR and LAStools issue trackers and forums and the literature on density rasters. Whether the same decomposition predicts artefacts in voxel maps, quantised images or other discretised sensor pipelines is a testable extension of this work.

6. Conclusion

A LAS file permits coordinates only at fixed positions, and a raster counts the points in each cell. Incompatible spacing, boundary arithmetic or uneven filling turn that into regular count differences, which a spectrum can reveal even when a map cannot. The size, spectral position and detectability of these differences followed closed-form predictions in three programs and on UAV, mobile and airborne data. They changed texture metrics by a few per cent, left ratios and Voronoi density almost untouched, and uneven filling appeared in 28 of 87 sampled public files. Because each mechanism responds to a different controlled change, a suspicious pattern can be tested rather than guessed at, and any software that grids stored coordinates could build these checks in. More generally, when discrete states are re-aggregated, the representation itself can become part of the measured pattern unless lattice compatibility, boundary arithmetic and state occupancy are treated explicitly.

Declarations

Acknowledgements. I thank the developers of PDAL, lidR and LAStools, and the providers of the open data used here: S. Vélez, M. Ariza-Sentís, J. Valente, M. Hrdina, swisstopo, Flai, USGS and NOAA.

Author contributions (CRediT). A. Urias: conceptualization, methodology, software, validation, formal analysis, investigation, data curation, writing (original draft, review and editing), visualization.

Use of generative AI. The author identified the problem while developing OpenLiDARViewer (Urias 2026) and designed and ran all tests. Parts of the code and text were drafted with Claude (Anthropic) and revised by the author, who takes responsibility for the content.

Competing interests. The author develops OpenLiDARViewer, an open-source point-cloud viewer, and declares no other competing interests.

Funding. The work received no external funding.

Data and code availability

- UAV survey: CC BY 4.0, <https://doi.org/10.5281/zenodo.8113105> (20220714_FLEXIGROBOTS_L1_PRO_20M_4MS_B9.laz).
- Mobile scan: CC BY 4.0, <https://doi.org/10.5281/zenodo.15421291> (LA03mapry.laz).
- swissSURFACE3D tile 2485-1109: source Federal Office of Topography swisstopo (© swisstopo), open government data, data.geo.admin.ch (item swissurface3d_2019_2485-1109). Airborne panels in Figs. 1 and 5–7 are derived from it.

- Second public file: swissSURFACE3D tile 2485-1109 (© swisstopo), COPC copy distributed by Flai via the Registry of Open Data on AWS, https://open-lidar-data.s3.eu-central-1.amazonaws.com/data/CH/Swiss_federal_authorities/swissurface3d_2022/copc/2485_1109.copc.laz (accessed September 2026).
- Census files: URLs, access dates and per-file results in the archive (census_results.csv, census_protocol.md).
- Analysis code, pipelines and result files: Zenodo, <https://doi.org/10.5281/zenodo.22969165> (all versions; this paper uses version 2), MIT licence, separate from the OpenLiDARViewer software. Claim-to-script map in CLAIMS.md.

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Appendix A. Derivations

Line amplitudes. The number of storage positions in cell j is $n_j = \lfloor (j+1)r + \varphi \rfloor - \lfloor jr + \varphi \rfloor$, a mechanical (Sturmian) sequence (Lothaire 2002). Writing each floor as its argument minus a periodic sawtooth and expanding in sines gives lines at $\nu_k = kf \bmod 1$, folded into $[0, \frac{1}{2}]$, with amplitude

$$A_k = \frac{2 |\sin(\pi k f)|}{\pi k r}, \quad \frac{1}{2} \sum_{k \geq 1} A_k^2 = \frac{f(1-f)}{r^2}. \quad (\text{A1})$$

Frequencies are exact. Amplitudes are exact for irrational f over an infinite sequence; for $f = a/b$ the sequence repeats every b cells, lines that fold onto one frequency add as phasors, and we therefore predict each line from exact column counts. Finite windows spread each line slightly across neighbouring frequencies. At $\nu = \frac{1}{2}$ (Nyquist) we report $|Z|$ without the factor 2; for $f = \frac{1}{2}$ the counts alternate with amplitude $1/(2r)$. Uneven filling with period M adds lines at multiples of $c/(Mq)$ and their sums and differences with kf . In two dimensions the product of the axis stripes adds cross

terms at $(\pm\nu_x, \pm\nu_y)$ with amplitude $A_x A_y / 2$; for the UAV window this gives 0.064 %, against 0.07 % from exact counts and 0.08 to 0.10 % measured.

Boundary rounding. If a share p of on-column edges misfire, with correlation ρ between neighbours,

$$\text{RMS} = \frac{\sqrt{2p(1-p)(1-\rho)}}{r} \leq \frac{q}{c}. \quad (\text{A2})$$

One map. A cell expecting λc^2 points (λ = point density) has Poisson noise $1/(c\sqrt{\lambda})$. Adding the two axis stripes and neglecting their product (first order, valid while both are small) gives the stripe-to-noise ratio

$$S = q\sqrt{2\lambda f(1-f)} \leq \frac{q\sqrt{\lambda}}{\sqrt{2}}. \quad (\text{A3})$$

$S = 1$ at $\lambda = 1/[2q^2 f(1-f)]$: 20,000 points per m^2 for $q = 1$ cm at $f = \frac{1}{2}$ under this model; other f need more.

Detectability. A profile of n bins holding N points has relative Poisson noise $\sigma = \sqrt{n/N}$ per bin. At an ordinary frequency (not 0 or Nyquist) the coefficient $Z = \frac{1}{n} \sum_j d_j e^{-2\pi i \nu_j}$ of white noise has $E|Z|^2 = \sigma^2/n$, so $2|Z|$ is Rayleigh-distributed with median

$$\frac{1.665 \sigma}{\sqrt{n}} = \frac{1.665}{\sqrt{N}}, \quad R = \frac{A\sqrt{N}}{1.665}. \quad (\text{A4})$$

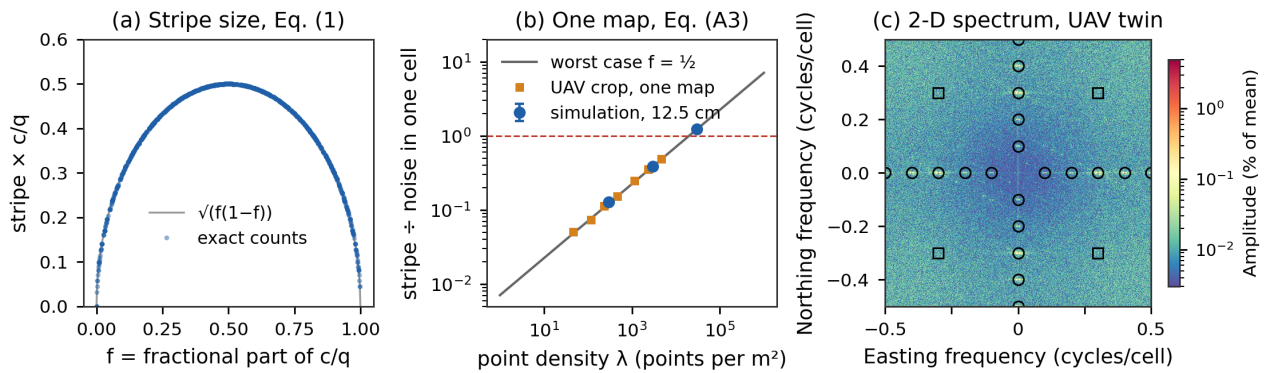


Figure 8: **Checks of the size law and its two-dimensional form.** (a) Exact counts of storage positions in 1,200 synthetic grids against Eq. (1). (b) Single-map stripe-to-noise ratio for a 1 cm step at $f = \frac{1}{2}$ (Eq. A3): Poisson simulations and single maps of the UAV crop at 12.5 cm, thinned to lower densities. (c) Two-dimensional spectrum of the UAV twin ratio at 13.3 cm; circles mark predicted axis lines, squares the weak cross terms.