

Coversheet

Title: MUSE: virtual tide gauge reference estimates and error budgets for coastal index validation at ungauged coastlines

Authors: Prasetyo Wibowo (corresponding author)

Affiliations:

1. Faculty of Data Science, Musashino University, Ariake, Koto-ku, 135-8181, Tokyo, Japan
2. Department of Informatics and Computer Engineering, Politeknik Elektronika Negeri Surabaya, Sukolilo, Surabaya, 60111, East Java, Indonesia

Email: pras@pens.ac.id

This is a non-peer reviewed preprint submitted to EarthArXiv.

MUSE: virtual tide gauge reference estimates and error budgets for coastal index validation at ungauged coastlines

Prasetyo Wibowo^{a,b,*}

^a*Faculty of Data Science Musashino University Ariake Koto-ku 135-8181 Tokyo Japan*

^b*Department of Informatics and Computer Engineering Politeknik Elektronika Negeri Surabaya Sukolilo Surabaya 60111 East Java Indonesia*

Abstract

Multi-dimensional coastal indices such as mangrove health indices aggregate tidal-range, mean higher-high water (MHHW), and tidal-anomaly sub-scores. Validating these sub-scores requires in situ tide gauges that low-data coastlines lack within 100 km. Global tide models such as EOT20 give accurate coastal tidal constituents but no tidal-anomaly metrics or per-source error terms for a composite-index budget. Such metrics would capture weather-band non-tidal variability driving coastal flooding and ecosystem exposure.

We present MUSE (MUlti-Source tidal Estimation) as a virtual tide gauge combining University of Hawai'i Sea Level Center (UHSLC) harmonic analysis from nearby stations with EOT20 constituents through a basin-friction-aware inverse-distance-weighted ensemble. MUSE adds tidal-anomaly metrics from the UHSLC residual and per-source error components that EOT20 lacks.

Seven complementary tests assess the protocol at five pilot and three development sites across four continents. They comprise leave-one-station-out cross-validation (LOO-CV), four baselines, a Diebold–Mariano test, sensitivity, subsample analysis, bootstrap confidence intervals, and adversarial tests. MUSE returns finite anomaly standard-deviation estimates while the EOT20 anomaly is zero by construction, with LOO-CV root-mean-square errors of 0.020 to 0.095 m. A worked example indicates that this error term carries 63 to 90% of an illustrative index's variance at the four coherent sites, a

*Corresponding author

Email address: pras@pens.ac.id (Prasetyo Wibowo)

component no model-only budget supplies. EOT20 alone is most accurate for range and MHHW at every site when extracted at the validation stations, with root-mean-square errors of 0.01 to 0.26 m for range and 0.01 to 0.11 m for MHHW. The independent Bitung gauge check passes two gates and fails two. Four sites pass the 0.50 m tidal-range threshold and pass/fail is consistent with a post-hoc station-coherence explanation rather than with station count, although eight sites limit confirmation. At the five sites where spatial-block cross-validation is testable both the station-level and block-level evaluations fail the 0.50 m gate. MUSE complements EOT20 by completing index error budgets rather than improving tidal-range accuracy.

Keywords: virtual tide gauge, tide model validation, tidal anomaly, uncertainty quantification, ungauged coastline, coastal index

1. Introduction

Coastal indices such as the Coastal Vulnerability Index [1] and benthic habitat indices [2] aggregate multiple environmental sub-scores into a single composite value. Each sub-score of such indices must be supported by a reference data set for the index to be credible. The tidal sub-scores of tidal range, mean higher-high water (MHHW), and tidal anomaly require in situ tide-gauge measurements as their reference. However, tide gauges are unevenly distributed and concentrated along the European and North American coasts [3]. Many coastlines with sparse data have no in situ tide gauge within 100 km [4]. The sustained financial investment and long-term operational commitment required to install and maintain tide gauges [4] preclude new deployments for validating coastal indices before construction.

Global tidal models such as EOT20 [5] and FES2014 [6] provide accurate tidal constituents for each coastal region [7, 8] and are a natural source for estimating tidal range and MHHW. However, EOT20 provides no tidal anomaly metrics because it carries no information on weather-band non-tidal variability such as storm surges and associated wind-driven setup [9]. That variability is a key sub-score for coastal index validation. EOT20 also provides no site-specific validation against in situ data at a user-selected target, and it does not assess uncertainty by source. Site-specific validation

and per-source uncertainty are both integral to a credible index error budget.

Tidal anomaly metrics quantify the weather-band variability that influences coastal flooding risk, ecosystem exposure, and sediment transport. A coastal vulnerability index that omits the anomaly standard deviation therefore misses a key mechanism by which storms raise extreme sea levels. Per-source uncertainty estimates serve a separate purpose because they let users propagate measurement and interpolation error through a composite index once the covariance between sub-scores is handled separately (Sections 3.3 and 5.5).

To address this gap, virtual tide gauges have been demonstrated for relative sea-level prediction [10], construction of navigational chart datums [11], and interpolation of tidal levels to ungauged coasts [12, 13]. Hawkins et al. [10] combined satellite altimetry, GPS, and tide gauge records in a Bayesian joint inversion to estimate multi-decadal relative sea-level trends across continental-scale regions. The reported per-site uncertainty is a trend ($\pm\sigma$) rather than a per-value uncertainty that can be propagated through a single-year composite index. Abd Rahman et al. [11, 14] combined tide-gauge and satellite-altimetry data using isotropic inverse distance weighting (IDW) to produce near-seamless tidal datums for Peninsular Malaysia, but they provide neither anomaly metrics nor per-source uncertainty. We are not aware of a study that couples University of Hawai'i Sea Level Center (UHSLC) hourly data with a global tide model to produce both tidal anomaly metrics and per-source uncertainty for coastal index validation.

We propose MUSE (MULTi-Source tidal Estimation) as a multi-source virtual tide gauge that combines harmonic analysis from at least two UHSLC stations within approximately 1500 km (the largest selected station distance is 1260 km) with EOT20 constituents through a basin-friction-aware IDW ensemble. MUSE adds two products that EOT20 cannot provide: tidal anomaly metrics derived from the UHSLC non-tidal residual, and per-source uncertainty estimates for downstream error propagation. Its basin-friction distance down-weights distant stations, so that shallow-water damping enters the interpolation (Section 3.3). We use Permanent Service for Mean Sea Level (PSMSL) monthly mean sea level only for consistency checking (Section 3.4) and exclude it from the ensemble. This study aims to build MUSE from public data, validate its four sub-scores against held-out stations and an independent gauge, and establish the

station configurations under which it is reliable.

The remainder of this paper is organized as follows. Section 2 describes the study sites. Section 3 presents the MUSE protocol. Section 4 reports the validation results. Section 5 discusses implications, limitations, and future work before Section 6 concludes.

2. Study Areas

We applied the protocol at eight sites across four continents to cover a range of tidal regimes, basin geometries, and data-availability scenarios (Table 1, Figure 1). The pilot sites where we developed the protocol are the Philippines, Raja Ampat, the North Sea, the Mediterranean, and Argentina. During protocol development, we added three further sites (Section 4.3): Iceland/Norway, the US East Coast, and the US West Coast.

The sites span mixed mainly diurnal, mixed mainly semidiurnal, and semidiurnal regimes. The Philippines is mixed mainly diurnal with ranges of 1.5 to 2.1 m. Raja Ampat is mixed mainly semidiurnal with ranges of 2.3 to 2.7 m. The US East Coast has ranges of 1.8 to 3.7 m. The North Sea at 0.2 to 3.8 m, Argentina at 1.5 to 5.1 m, and Iceland/Norway at 1.8 to 2.0 m are semidiurnal. The Mediterranean at 0.3 to 1.3 m and the US West Coast at 2.6 to 3.9 m are mixed, mainly semidiurnal. Across all sites, the tidal range spans 0.2 to 5.1 m, from microtidal in the Mediterranean to macrotidal in Argentina. The North Sea adds a strong shelf gradient, and Iceland/Norway provides a subarctic case. We sampled Raja Ampat in 2018, and the remaining sites in 2021 or 2023.

Table 1 lists per-station distances, basins, and target-year completeness, and Figure 2 maps the site configurations. Supplementary Sections S3 and S9 provide station data completeness and extended site descriptions.

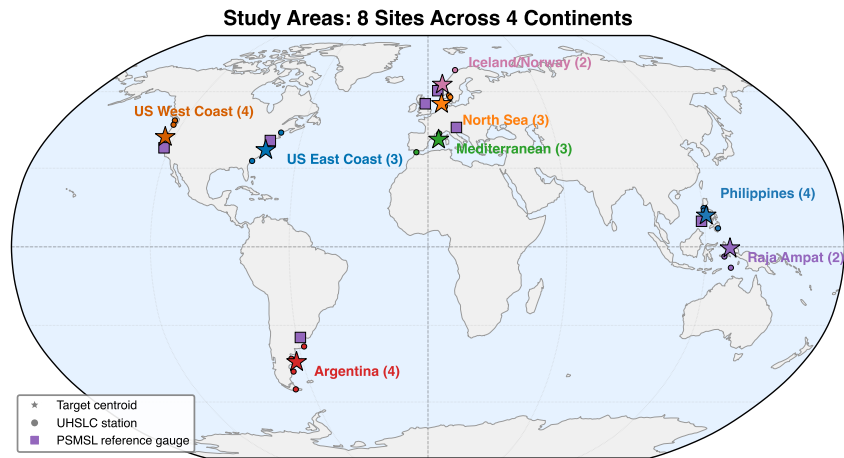


Figure 1: Global study area: five pilot and three additional sites. Stars: target centroids; circles: UHSLC stations; squares: PSMSL gauges.

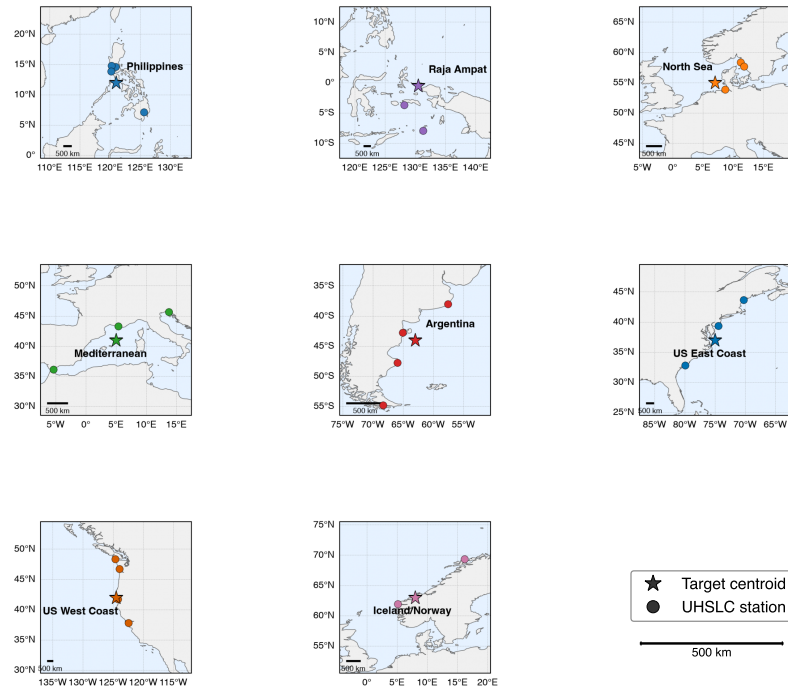


Figure 2: The eight study sites. Star: target centroid; circles: UHSLC stations; scale bar 500 km.

Table 1: UHSLC stations used, with per-station basin (UHSLC metadata), distance to target centroid, and target-year completeness.

Site	Station	ID	Lat (°N)	Lon (°E)	Basin	Year	Dist. (km)	Compl. (%)
Philippines	Manila	370	14.583	120.967	South China Sea	2023	287	97.9
	Subic Bay	382	14.765	120.252	South China Sea	2023	318	95.6
	Lubang	655	13.817	120.202	South China Sea	2023	220	93.2
	Davao	372	7.083	125.633	Davao Gulf	2023	746	99.2
Raja Ampat	Ambon	133	-3.700	128.200	Seram Sea	2018	438	95.3
	Saumlaki	420	-7.982	131.290	Arafura Sea	2018	837	100.0
North Sea	Cuxhaven	825	53.867	8.717	North Sea	2021	168	99.7
	Smögen	818	58.350	11.215	North Sea	2021	453	98.9
	Göteborg	819	57.683	11.800	North Sea	2021	420	98.9
Mediterranean	Trieste	829	45.647	13.760	Mediterranean	2023	876	100.0
	Marseille	824	43.300	5.350	Mediterranean	2023	257	96.5
	Gibraltar	289	36.117	-5.350	Mediterranean	2023	1050	12.8
Argentina	Mar del Plata	729	-38.035	-57.538	South Atlantic	2023	806	83.7
	Puerto Madryn	731	-42.763	-65.030	South Atlantic	2023	214	99.6
	Puerto Deseado	286	-47.750	-65.917	South Atlantic	2023	474	97.5
	Ushuaia	600	-54.805	-68.295	South Atlantic	2023	1260	90.5
Iceland/Norway	Måløy	802	61.933	5.117	Norwegian Sea	2021	190	100.0
	Andenes	800	69.317	16.150	Norwegian Sea	2021	791	100.0
US East Coast	Portland ME	252	43.657	-70.247	Northwest Atlantic	2021	842	100.0
	Atlantic City	264	39.355	-74.418	Northwest Atlantic	2021	267	100.0
	Charleston	261	32.782	-79.925	Northwest Atlantic	2021	649	100.0
US West Coast	San Francisco	551	37.807	-122.465	Northeast Pacific	2021	497	100.0
	Crescent City	556	41.745	-124.183	Northeast Pacific	2021	39	100.0
	Neah Bay	558	48.368	-124.617	Northeast Pacific	2021	708	100.0
	Willapa Bay	564	46.708	-123.965	Northeast Pacific	2021	525	100.0

3. Methods

The virtual tide gauge protocol builds reference estimates of four tidal sub-scores at an ungauged target site p : the tidal range (R , Eq. 2), mean higher-high-water (MHHW, Eq. 4), and the tidal anomaly mean ($\bar{a}$) and standard deviation (σ_a) of the harmonic residual (Eq. 3). These four sub-scores characterize the tidal regime at the target year listed for each site in Table 1.

The protocol consists of three computational steps: (1) harmonic analysis at the nearby UHSLC stations, (2) EOT20 constituent extraction at the target, and (3) a multi-source IDW ensemble with per-source error decomposition (Figure 3). Tidal range and

MHHW are derived from EOT20 constituents (Section 3.2) and UHSLC station data (Section 3.3). The anomaly mean and standard deviation are derived from the UHSLC non-tidal residual (Section 3.1). We use PSMSL monthly mean sea level (MSL) only for consistency checking (Section 3.4).

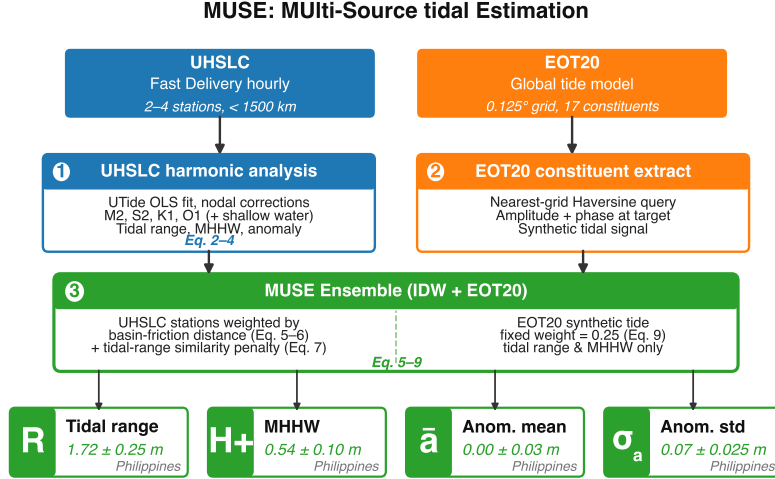


Figure 3: The MUSE protocol: UHSLC harmonic analysis (Step 1) and EOT20 extraction (Step 2) feed the basin-friction IDW ensemble (Step 3).

3.1. Step 1: Harmonic analysis at UHSLC stations

We obtained hourly sea-level records from the UHSLC Fast Delivery archive via ERDDAP (<https://uhslc.soest.hawaii.edu/erddap/>) [15]. For each site we selected stations within approximately 1500 km of the target where the largest selected station distance is 1260 km. We excluded stations in different ocean basins based on the basin field in the UHSLC metadata. That ocean-basin screening at selection is coarser than the tidal-basin labels that enter the weighting in Section 3.3. We retained only stations that provided at least 15 days of valid hourly data in the target year. Two quality-control stages filter each record before harmonic analysis. Stage 1 rejects a point whose deviation from the midpoint of its two hourly neighbors exceeds 1.0 m

as an instrument spike regardless of the local tidal slope. The filter flags it only at the local deviation maximum to preserve neighboring valid hours. Stage 2 then rejects hourly steps larger than 2 m/h as before. Table 1 summarizes station selection and data completeness.

The 15-day minimum is set by the Rayleigh criterion needed to resolve the four dominant constituents, M_2 , S_2 , K_1 , and O_1 . S_2 is the binding case and requires 14.8 days of record. Constituents are retained only when the record is long enough to separate them from their nearest neighbor: N_2 , Q_1 , Mm , and MN_4 need approximately 28 to 32 days. K_2 against S_2 , P_1 against K_1 , and Mf against MSf need approximately 183 days and are therefore absent from short records. Accordingly, confidence intervals for the recovered constituents widen. These per-station intervals propagate into σ_{uhsic} in the quadrature uncertainty bound (Eq. 10).

For each selected station s , we subjected the hourly record $\eta_s(t_k)$ to harmonic analysis using UTide [16]. Ordinary least squares estimates the amplitude and phase of each constituent, and nodal corrections account for the 18.61-year lunar nodal cycle [17]. UTide applies the Rayleigh criterion automatically, so the year-long records used here include the dominant set

$$\mathcal{F} = \{M_2, S_2, N_2, K_2, K_1, O_1, P_1, Q_1, M_4, MS_4, MN_4\}, \quad (1)$$

comprising four semidiurnal (M_2 , S_2 , N_2 , K_2) and four diurnal (K_1 , O_1 , P_1 , Q_1) astronomical constituents, together with three shallow-water quarter-diurnal compounds (M_4 , MS_4 , MN_4). Eq. 1 uses standard tidal notation in which M, S, N and K denote the lunar, solar, larger lunar elliptic and lunisolar species. O and P are the principal lunar and solar diurnal species. Q is a larger lunar elliptic diurnal species. The subscript gives the number of cycles per day. The three shallow-water compounds were verified at each station (see Supplementary Section S1). The automatic constituent selection retains between 35 and 59 constituents per station, from Gibraltar with 47 valid days to Manila with a full year. Eq. 1 lists the dominant subset common to the year-long records. The anomaly residual and the MHHW reconstruction use the full auto-selected set. At the same time, we compute the tidal range from the four dominant constituents (Eq. 2) and, for the definition-sensitivity analysis, from the Eq. 1 subset (Supplementary Table S19).

The tidal range at each station follows from the harmonic coefficients as

$$R_s = 2 \sum_{f \in \mathcal{F}_R} \hat{A}_{s,f} \quad (2)$$

where $\mathcal{F}_R = \{M_2, S_2, K_1, O_1\}$ are the four dominant constituents (a subset of the recovered set $\mathcal{F}$, Eq. 1), and $\hat{A}_{s,f}$ is the estimated amplitude of constituent f at station s . This four-constituent sum is a definition choice that affects absolute values. Extending the sum to all 11 members of the Eq. 1 subset $\mathcal{F}$ raises the UHSLC station estimates by 24 to 38%, and the mean diurnal range of the full nodal-corrected reconstruction between MHHW and MLLW sits 18 to 45% below the four-constituent sum across UHSLC stations and EOT20 targets (Supplementary Table S19). Reported tidal ranges are therefore definition-dependent, and the literature comparison in Supplementary Section S8 inherits this choice. The tidal anomaly is the residual of the harmonic fit:

$$a_s(t_k) = \eta_s(t_k) - \hat{\eta}_s(t_k) \quad (3)$$

with $\bar{a}_s$ the annual mean and $\sigma_{a,s}$ the sample standard deviation (using the $N-1$ denominator). MHHW is defined from the same reconstructed tidal series $\hat{\eta}_s(t)$ as the average of the higher of the two daily semidiurnal high waters:

$$\text{MHHW}_s = \frac{1}{D} \sum_{d=1}^D H_d, \quad H_d = \max \left(\max_{t \in \mathcal{T}_{d,1}^{\text{high}}} \hat{\eta}_s(t), \max_{t \in \mathcal{T}_{d,2}^{\text{high}}} \hat{\eta}_s(t) \right) \quad (4)$$

where D is the number of days and the two semidiurnal high waters on day d are detected via peak-finding. Peak prominence is $\max(0.02R_s, 0.005)$ m, ensuring stable peak detection across microtidal and macrotidal regimes. The minimum inter-peak distance follows from the dominant tidal period as $\text{dist} = \max(0.4T, 4)$ h, where T is the dominant tidal period estimated from the zero-crossing rate of the detided series (≈ 12 h for semidiurnal regimes, longer for mixed or diurnal regimes). Regime classification uses the same zero-crossing rate and corresponds to a Formzahl ratio threshold of $F > 3$ for diurnal dominance [17]. For diurnal regimes with only one high water per day, the formula reduces to the daily maximum. MHHW is reported relative to the same-year mean of the reconstructed tide as $\text{MHHW}_s^{\text{rel}} = \text{MHHW}_s - \bar{\eta}_s$.

Major semidiurnal and diurnal tidal constituents are effectively invariant over the 5-year span of target years 2018, 2021 and 2023 apart from the nodal amplitude modulation

quantified in Section 3.2. The anomaly metrics therefore capture the specific climate conditions of each target year. However, a single target year cannot capture interannual variability. Tides vary on multi-year time scales [18], and non-astronomical processes such as frictional modulation by seasonal river discharge also evolve between years. Because each site is sampled in only one year, weather-band residuals cannot be reliably attributed to the El Niño–Southern Oscillation (ENSO) phase.

3.2. Step 2: EOT20 constituent extraction

EOT20 [5] is a global tidal model providing 17 tidal constituents from its ocean and load atlases on a 0.125° grid of about 14 km at the equator. At each target site p , we retrieved constituents from the EOT20 NetCDF files at coordinates $(\text{lat}_p, \text{lon}_p)$ using true nearest-neighbor interpolation, in which the nearest non-NaN ocean grid cell is identified by great-circle distance computed with the Haversine formula, avoiding the search-order bias inherent in expanding-square techniques. The extraction coordinates align with the target centroid given in Supplementary Section S9 for all sites except Iceland/Norway, where extraction used 63.0°N and 5.5°E , about 130 km from the 63.0°N 8.0°E centroid.

We used the derived amplitudes and phases to calculate synthetic tidal range, MHHW, and anomaly at the target with the synthesis described below. Each target coordinate coincides with a valid EOT20 grid node, so the extraction introduces no interpolation distance. However, the grid may still represent a potential mismatch between the model’s coastal cell and the tidal conditions at the shoreline.

EOT20’s accuracy varies with water depth and basin geometry [19]. Hart-Davis et al. [5] report a root-sum-square improvement of ~ 2.2 cm in coastal areas and ~ 1.4 cm globally compared with the earlier EOT11a model. However, coastal tide-gauge errors remain the largest component [20]. We therefore evaluate EOT20 at each site by comparing its constituents with the UHSLC harmonic analysis at every station (Supplementary Table S7).

The synthesis used for constituent extraction and baseline comparisons evaluates the full astronomical equilibrium argument at absolute time so constituent phases are correct for the target year. It omits nodal modulation consistent with the mean-epoch

tidal constants of the EOT20 atlases. Tidal range is a phase-free amplitude sum and is unaffected by this choice. Nodal corrections do matter for constituent amplitudes at diurnal sites. Re-fitting the UHSLC records without nodal corrections changes the four-constituent tidal range by 7.7% at the three K_1 -dominant Philippine stations (Manila, Subic Bay, Lubang) and by at most 4.3% at the remaining 22 station-years. Single-year MHHW shifts by at most 1.3 mm. The simplification therefore remains a limitation for multi-year extensions, where the nodal amplitude factor is no longer constant.

3.3. Step 3: Multi-source IDW ensemble

3.3.1. Basin-friction IDW

Standard IDW weights each station s by the inverse of the great-circle distance $d(p, s)$ to the target as $w_s(p) = d(p, s)^{-\kappa}$, where κ is the IDW power parameter [21], set to $\kappa = 1.0$. However, tidal wave propagation along coastlines is anisotropic because shallow semi-enclosed basins add damping that grows with distance. We replace the isotropic distance with a basin-friction-aware effective distance:

$$d_{\text{eff}}(p, s) = d(p, s) \cdot \left[1 + f_p \cdot \left(\frac{d(p, s)}{D_{\text{ref}}} \right)^{0.5} \right] \quad (5)$$

where f_p is the basin-friction factor at the target site and $f_p = 1$ gives $d_{\text{eff}} = d$. The deployed values are 1.0 or 1.2, and the sensitivity sweep probes 0.8 to 1.5 times each site default. $D_{\text{ref}} = 1000$ km is a reference distance, and the term $f_p (d(p, s)/D_{\text{ref}})^{1/2}$ accounts for the cumulative frictional damping as the tidal wave propagates through the coastal basin. The square-root dependence describes the sub-linear increase of the cumulative frictional damping with distance in shallow coastal waveguides [22].

The friction assignment is a categorical site-level classification chosen a priori rather than a rule derived from depth or from the fetch argument above. The South China Sea and the Northwest Atlantic take $f_p = 1.2$, while the remaining basins take the default $f_p = 1.0$. The default $f_p = 1.0$ leaves the effective distance equal to the great-circle distance. We did not calibrate these values against independent data. The sensitivity of the output to this choice is weak (Section 4.1, Supplementary Table S2). We therefore apply f_p uniformly across all stations within a site rather than per station and classify all eight sites in Supplementary Table S8.

The IDW weight with same-basin bonus is:

$$w_s(p) = \frac{\beta \mathbb{I}[b(s) = b(p)] + \mathbb{I}[b(s) \neq b(p)]}{d_{\text{eff}}(p, s)^\kappa}, \quad W_s(p) = \frac{w_s(p)}{\sum_{s' \in \mathcal{S}_p} w_{s'}(p)} \quad (6)$$

where $d_{\text{eff}}(p, s)$ is the basin-friction-aware effective distance (Eq. 5), $b(x)$ is the tidal basin of point x , and $\mathbb{I}[\cdot]$ is the indicator function. The same-basin bonus multiplier β is disabled ($\beta = 1.0$) after sensitivity analysis (Section 4.1). Therefore, cross-basin stations are not excluded and are weighted by the same distance kernel.

Where the EOT20 range prior R_{prior} exceeds 0.01 m, the effective distance receives an additional similarity penalty that down-weights stations whose range departs from that prediction:

$$d'_{\text{eff}}(p, s) = d_{\text{eff}}(p, s) \cdot \left[1 + \alpha \cdot \frac{|R_s - R_{\text{prior}}|}{R_{\text{prior}}} \right] \quad (7)$$

where R_s is the tidal range at station s (Eq. 2), R_{prior} is the tidal range predicted by EOT20 at the target ($2 \sum_{f \in \mathcal{F}_R} A_f$), and $\alpha = 1.0$ is the similarity weight. The modified distance $d'_{\text{eff}}(p, s)$ of Eq. 7 replaces $d_{\text{eff}}(p, s)$ in Eq. 6, so stations with discrepant tidal ranges receive a smaller weight even if they are geographically close. Section 4.1 quantifies this penalty's effect on the tidal-range root-mean-square error (RMSE).

The tidal range at the target is computed directly from the weighted station-level tidal ranges:

$$\hat{R}_p^{\text{idw}} = \sum_{s \in \mathcal{S}_p} W_s(p) R_s \quad (8)$$

where R_s is the tidal range at station s (Eq. 2) and $W_s(p)$ is the IDW weight (Eq. 6). We interpolate the tidal range as a single quantity rather than the four dominant constituent amplitudes M_2 , S_2 , K_1 , and O_1 because those constituents are weighted differently at each station. In the Philippines, Davao is M_2 -dominant whereas Manila is K_1 -dominant. Interpolating the amplitudes separately would require one spatial pattern per constituent that a network of only two to four stations per site cannot resolve. Interpolating the combined range needs just one pattern, so it reproduces the observed station-to-station differences without imposing unsupported per-constituent structure.

3.3.2. Combining the sources into four sub-scores

The MUSE ensemble combines EOT20 with the UHSLC-IDW range estimate (Eq. 8) for tidal range and MHHW. It uses UHSLC-IDW alone for the anomaly mean

and standard deviation.

$$\hat{M}_p^{\text{MUSE}} = \frac{\sum_{i \in \mathcal{I}_M} w_p^i M_p^i}{\sum_{i \in \mathcal{I}_M} w_p^i} \quad (9)$$

where $\mathcal{I}_M$ is the set of sources applicable to metric M , and w_p^i is the per-source weight (Supplementary Table S9).

EOT20 enters the ensemble with a constant weight of 0.25 at all sites and therefore holds 20% of the normalized weight after aggregation with the IDW station weights. This keeps the EOT20 contribution small so that the UHSLC-IDW interpolation dominates the ensemble. We considered an amplitude-dependent rule for the EOT20 weight. However we rejected it because with two to four leave-one-station-out cross-validation (LOO-CV) folds per site it risks overfitting the weight to the same data used for validation. The sensitivity of the ensemble to this choice is reported in Section 4.2 and Supplementary Table S4.

EOT20 contributes zero to the anomaly sub-scores in Supplementary Table S9 columns 4 and 5 because its anomaly is zero by construction, and giving it weight would shrink the anomaly estimates toward zero. The ensemble (Eq. 9) therefore reduces to the UHSLC-IDW estimate for the anomaly sub-scores.

3.3.3. Per-source error decomposition

The four sources are neither fully independent nor perfectly correlated, so their combined error falls between two extreme cases. The *independence case* assumes zero cross-source covariance and is the quadrature sum of the four source variances:

$$\sigma_{\text{MUSE}}^{\text{lower}} = \sqrt{\sigma_{\text{uhslc}}^2 + \sigma_{\text{idw}}^2 + \sigma_{\text{psmsl}}^2 + \sigma_{\text{eot20}}^2} \quad (10)$$

The *perfect-correlation case* assumes the opposite extreme of perfect positive correlation, so the four source uncertainties add arithmetically:

$$\sigma_{\text{MUSE}}^{\text{upper}} = \sigma_{\text{uhslc}} + \sigma_{\text{idw}} + \sigma_{\text{psmsl}} + \sigma_{\text{eot20}} \quad (11)$$

The true combined error lies between the two cases in Eqs. 10 and 11, and the individual source uncertainties are computed per site from the actual data rather than generic defaults (Eqs. 12–15). σ_{uhslc} is the harmonic-analysis precision and equals the per-station mean of the UTide Monte Carlo 95% confidence half-width for the M_2 amplitude,

treated here as a conservative 1σ term:

$$\sigma_{\text{uhslc}} = \frac{1}{N} \sum_{s=1}^N \text{CI}_{s,M_2} \quad (12)$$

where CI_{s,M_2} is the station-level confidence interval, 0.0005 to 0.009 m across the 25 station-years. M_2 is the representative constituent because it dominates the tidal signal at most sites, and an M_2 -based interval underestimates the true harmonic-analysis error where K_1 or O_1 dominates instead. σ_{idw} is the primary validation metric and equals the LOO-CV RMSE of tidal range over the site's folds (Section 3.5):

$$\sigma_{\text{idw}} = \sqrt{\frac{1}{n} \sum_{j=1}^n (\hat{R}_j - R_j)^2} \quad (13)$$

where $\hat{R}_j$ and R_j are the ensemble-predicted and observed tidal ranges for held-out station j , and n is the number of folds. σ_{psmsl} is the long-term MSL sampling uncertainty and equals the standard error of the annual mean sea level anomalies:

$$\sigma_{\text{psmsl}} = \frac{\sigma_{\text{interannual}}}{\sqrt{n_{\text{years}}}} \quad (14)$$

where $\sigma_{\text{interannual}}$ is the standard deviation of the annual mean sea level anomalies and n_{years} is the PSMSL record length. In the reported uncertainty budget σ_{psmsl} (Eq. 14) is set to zero because we use PSMSL monthly MSL only for consistency checking and it does not contribute to the ensemble estimate (Section 3.4). σ_{eot20} is the model–station discrepancy and equals the M_2 RMSE between the EOT20 and UHSLC constituent amplitudes across the site's stations:

$$\sigma_{\text{eot20}} = \sqrt{\frac{1}{N} \sum_{s=1}^N (A_{s,M_2}^{\text{EOT20}} - A_{s,M_2}^{\text{UHSLC}})^2} \quad (15)$$

where A_{s,M_2} is the M_2 amplitude from each source at station s (Supplementary Table S7).

The operative decomposition term is the independence case $\sigma_{\text{MUSE}}^{\text{lower}}$ (Eq. 10) because the four sources are methodologically distinct: UHSLC harmonic analysis of hourly in situ data, IDW spatial interpolation derived from UHSLC, PSMSL long-term monthly mean sea level, and the satellite-assimilated EOT20 model. Three qualifications bound the interpretation of the independence case. First, σ_{idw} (Eq. 13) is estimated by LOO-CV on the same station network that constructs the ensemble. Hence, the decomposition

quantifies internal consistency within the network rather than independent predictive error at the ungauged target. The IDW term inherits the fold-count limitation of the LOO-CV across two to four folds per site. Section 4.1 quantifies the per-source shares (Figure 5). Second, IDW interpolation is derived from the UHSLC station values so σ_{uhslc} and σ_{idw} share underlying data. The independence case is not a strict lower bound under the actual dependence. Third, the IDW term dominates the decomposition at every site, accounting for at least 92.8% of the quadrature variance (Section 4.1). The composite value is therefore close to the validation RMSE, and the decomposition attributes error to physically distinct components rather than providing an independent uncertainty budget. Section 5.5 propagates those components through a worked composite index.

These bounds quantify the uncertainty of the ensemble estimate for each sub-score. For composite indices that aggregate multiple sub-scores through a non-linear formula, the covariance between sub-scores must be accounted for separately. Tidal range and MHHW, for instance, both derive from the same harmonic analysis and therefore share errors. The per-sub-score uncertainties reported here are necessary but not sufficient for a full composite error budget.

3.4. Consistency checking

We obtained MSL records from the PSMSL Revised Local Reference (RLR) dataset [23, 24]. For each target site, we selected one PSMSL station in the same oceanic basin (Supplementary Section S9). The Philippine and Raja Ampat sites share PSMSL station 207 at Puerto Princesa because the Indonesian PSMSL network is sparse. Puerto Princesa provides the closest available long-term record to both targets (~350 km from the Philippines target, ~1,700 km from Raja Ampat). The PSMSL monthly MSL anomaly for the target year was computed as:

$$\hat{\delta}_p^{\text{psmsl}} = \frac{1}{N_Y} \sum_{t_m \in Y} \eta_{t_m}^{\text{psmsl}} - \frac{1}{N_{\text{clim}}} \sum_{t_m \in \mathcal{T}_{\text{clim}}} \eta_{t_m}^{\text{psmsl}} \quad (16)$$

where Y is the target year and $\mathcal{T}_{\text{clim}}$ is the full PSMSL record.

We exclude PSMSL from the ensemble because the monthly MSL anomaly includes interannual to decadal variability from thermal expansion and climate modes such as ENSO and the Indian Ocean Dipole (IOD) [25]. The PSMSL monthly anomaly

differs from the hourly UHSLC non-tidal residual of seasonal-to-synoptic variability (Section 3.1). Including both conflates these scales and degrades the anomaly-mean estimate. We quantify that degradation by forming an equal-weight blend of the two sources and recomputing the LOO-CV anomaly-mean RMSE (Section 4.2). PSMSL also provides monthly means rather than hourly tidal constituents, so it cannot contribute to tidal range or MHHW. Figure 4 shows the full PSMSL time series with the target-year anomaly of Eq. 16 highlighted.

As a consistency check, we computed the correlation between the monthly MSL anomaly from PSMSL after subtracting the long-term monthly climatology and the monthly-mean non-tidal residual from UHSLC over the common months of the target year (Section 4.2, Supplementary Table S12). Argentina is excluded because its PSMSL record ends in 2022 and does not overlap the 2023 target year. These correlations rest on five to twelve monthly pairs, and two caveats limit their interpretation. The UHSLC residual retains its seasonal cycle whereas the PSMSL anomaly is deseasonalized. The residual’s seasonal variance is unmatched noise that attenuates the correlation. The UHSLC residual is also a site-level average over stations such as Davao that may span different basins, so it is not co-located with the PSMSL gauge. Because the two quantities are not fully independent and PSMSL provides only monthly means, PSMSL is retained as a descriptive consistency reference rather than a formal validation metric for the virtual gauge.

3.5. *Validation design*

We evaluate the protocol’s validity using LOO-CV, baseline comparisons, and statistical tests and report the results in Section 4. No single metric can establish the ensemble’s credibility at ungauged sites, so we use seven complementary tests to probe different aspects of the ensemble. Supplementary Table S10 and the Supplementary Material give full descriptions, acceptance criteria, and per-scenario results.

- (i) **LOO-CV (primary).** We hold out each UHSLC station in turn. We reconstruct the virtual gauge from the remaining stations plus EOT20, then compare the predicted sub-scores with the held-out station. We evaluate the root-mean-square error (RMSE) across folds against four a priori thresholds: 0.50 m for tidal range,

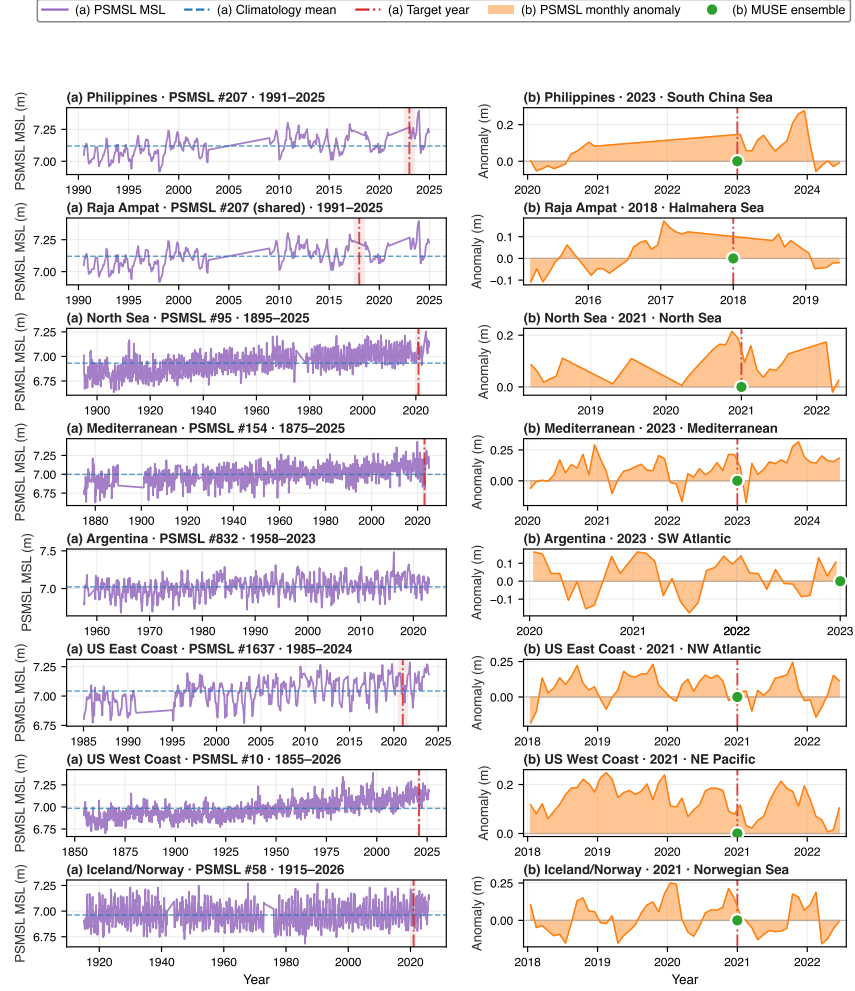


Figure 4: PSMSL monthly MSL anomalies at the eight sites, target years highlighted.

0.20 m for MHHW, 0.08 m for anomaly mean, and 0.05 m for anomaly std. Of the three per-source terms that derive these thresholds (Supplementary Table S11), σ_{UHSLC} is data-derived from the UTide Monte Carlo interval for tidal range and MHHW and takes conservative fixed values for the two anomaly sub-scores, while σ_{EOT20} is chosen conservatively from the coastal accuracy pattern of Hart-Davis et al. [5] for tidal range and MHHW. The binding term for the tidal-range threshold, $\sigma_{\text{IDW}} = 0.20$ m, is an author estimate for a 1 to 2 m reference range (10 to 20% of that range) because we found no published value for coastal IDW interpolation error. Supplementary Table S3 quantifies the resulting decision sensitivity and the US West Coast passes only for $\sigma_{\text{IDW}} \geq 0.20$ m, with the derivations in Supplementary Table S11. The anomaly-mean threshold is a sanity check on the harmonic fit rather than a discriminating accuracy gate.

- (ii) **Naïve baselines.** We evaluate four baselines (EOT20-only, single-nearest gauge, unweighted mean, and IDW-only) under the same LOO-CV.
- (iii) **Diebold–Mariano test.** An exploratory Diebold–Mariano (DM) test [26] with the Harvey–Leybourne–Newbold small-sample correction [27] uses the per-fold mean squared error across all four sub-scores as the loss. Because the tidal-range terms dominate that pooled loss, the same test is additionally run per sub-score (Supplementary Section S10.1). A Monte Carlo analysis quantifies the test’s power at two to four folds per site (Supplementary Section S10.1).
- (iv) **Sensitivity.** The three IDW parameters are power, basin friction, and same-basin bonus. We assess them with a coefficient-of-variation (CV) acceptance gate of 15%. A separate station-exclusion check requires the maximum change when removing any single station to stay below 20% (Supplementary Table S18).
- (v) **n-confound subsample analysis (n-CSA).** Applied at the six sites with at least three stations, and with the two-station sites contributing only their full-set value to Supplementary Table S14, this tests whether the RMSE degradation with fewer stations reflects a real accuracy signal or a sample-size artifact.
- (vi) **Bootstrap confidence intervals.** We obtain 95% confidence intervals on the

LOO-CV RMSE by resampling the per-fold errors (1000 replicates) [28].

- (vii) **Adversarial tests.** Five categories of noise injection, data gaps, station dropout, wrong-basin inclusion, and spike detection cover six scenarios in the acceptance gate and probe robustness to realistic data-quality failures.

In addition to the seven tests above, we report spatial-block cross-validation as a secondary diagnostic that holds out whole k-means station clusters rather than single stations [29] and tests fold independence with the same 0.50 m criterion. The virtual gauge is additionally checked against the independent Bitung gauge at Raja Ampat that is excluded from the ensemble (Section 4.3).

4. Results

For each site, we calculated the four sub-scores (R , MHHW, $\bar{a}$, σ_a) using the protocol described in Section 3 and validated them with LOO-CV, baseline comparisons, and held-out station checks.

4.1. Virtual gauge output

Table 2 lists these four sub-scores for each ungauged target site with the empirical uncertainty (LOO-CV RMSE) and the pass/fail outcome against the four a priori thresholds. The anomaly-mean gate is a sanity check rather than a discriminator. It is trivially satisfied at all eight sites (RMSE ≈ 0.000 m against a 0.08 m threshold) because the detided residual has near-zero mean by construction. A non-zero mean would indicate a systematic offset or datum error. The remaining three gates discriminate between sites, though the passing sets differ.

Anomaly standard deviation passes at five sites and fails at the North Sea, Argentina, and the US West Coast, where the ensemble does not fully resolve synoptic-scale non-tidal variability. MHHW passes at four sites and fails at the North Sea, the Mediterranean, Argentina, and the US East Coast. Tidal range passes at four sites: the Philippines, Raja Ampat, Iceland/Norway, and the US West Coast. Tidal-range failures each have a station set spanning contrasting tidal regimes rather than a low station count (Section 4.2). Both two-station sites (Raja Ampat and Iceland/Norway) pass all four gates, though

each RMSE rests on only two folds. We compare ensemble tidal-range estimates with indicative regional tidal ranges in Supplementary Figure S2.

The RMSEs in Table 2 evaluate prediction at held-out *stations*, so the table quantifies within-network interpolation skill rather than accuracy at the ungauged target centroid itself. Two checks instead constrain target-level accuracy: spatial-block cross-validation fails at all five testable sites (Section 4.2), and the Bitung check fails two of its four gates (Section 4.3). The block-level RMSE exceeds the station-level value only at the North Sea and the US West Coast, while both evaluations fail the gate at the remaining three testable sites.

EOT20 predicts zero anomaly by construction, so its anomaly standard-deviation error equals the site’s actual anomaly standard deviation (0.068 to 0.226 m across the set). MUSE returns finite estimates with RMSE 0.020 to 0.095 m. The largest gaps are at Argentina (0.064 m against 0.218 m) and Iceland/Norway (0.023 m against 0.155 m) with all values in Supplementary Table S13.

The single-nearest gauge beats the ensemble on anomaly standard deviation at only two of eight sites (the Philippines at 0.019 m against 0.020 m and the US West Coast at 0.044 m against 0.057 m). Hence, a nearby gauge is not generally a substitute for the ensemble at ungauged targets.

The physical basis for interpolating the anomaly standard deviation deserves an explicit statement because weather-band residual fields decorrelate over distances far shorter than the 39 to 1260 km station-to-target distances of Table 1. We define the between-station spread of σ_a as the station range (maximum minus minimum) relative to the maximum station value. The spread is large within every site from 13.6% at Iceland/Norway to 69.5% at the US West Coast. The low LOO-CV RMSEs do not merely reflect a flat field. Against the unweighted-mean baseline, the ensemble improves the anomaly-standard-deviation RMSE at five of the eight sites (margins 0.0004 to 0.020 m), ties both two-station sites, and is worse at the US East Coast (+0.007 m). MUSE therefore contributes a finite anomaly estimate where EOT20 has none, and its interpolation gain over the unweighted-mean baseline is modest.

The per-source error decomposition (Figure 5) indicates that IDW spatial interpolation error dominates the budget at all sites, contributing at least 92.8% of the quadrature

variance. In contrast, the UHSLC harmonic error is negligible. The UHSLC harmonic term ranges from 0.0005 to 0.009 m across the 25 station-years. The M_2 share of the four dominant amplitudes, defined as $M_2/(M_2 + S_2 + K_1 + O_1)$, is 0.30 in the Philippines, 0.44 in Raja Ampat, 0.71 in the North Sea, 0.52 in the Mediterranean, and 0.62 in Argentina. At three of the four Philippine stations, K_1 or O_1 dominates, so the M_2 -based interval underestimates the error there. The contrast within the site is large, and Davao has an M_2 share of 0.54 while Manila has a K_1 share of 0.37. The perfect-correlation case ranges from about 0.22 m at Iceland/Norway to 2.20 m at Argentina.

The ensemble is only weakly sensitive to the three IDW parameters. Adding the tidal-range similarity penalty (Section 3.3) changes the tidal-range RMSE by at most 9.7% across sites. The parameter sweeps (IDW power, basin friction, same-basin bonus) stay inside the 15% CV gate at all eight sites. The largest maxima are 12.0% at the North Sea and 11.5% at the Mediterranean (Supplementary Table S2).

Station-exclusion sensitivity is a more demanding test than the parameter sweeps, passing at only three sites (the Philippines, Raja Ampat, and Iceland/Norway, Supplementary Table S18) because removing any single station changes a sub-score by more than 20% at the other five. The largest changes occur at the Mediterranean (tidal range 146.0%, MHHW 143.2%) and the North Sea (72.5% and 71.0%).

Spatial-block cross-validation holds out whole station clusters and fails the 0.50 m tidal-range criterion at all five testable sites (Section 4.2).

4.2. Cross-validation and baseline comparison

Argentina, the US East Coast, the Mediterranean, and the North Sea fail the tidal-range gate for distinct geometric reasons. Argentina drives the largest pilot-site tidal-range RMSE of 2.155 m with a 17° latitudinal spread across fundamentally different regimes, from microtidal at the northern and southern extremes to macrotidal over the central shelf. The US East Coast (1.208 m) spans a wide latitude range, and the Mediterranean (0.730 m) spans a wide basin. The North Sea (1.975 m) is evaluated using three stations spanning the German Bight to the Skagerrak, with the closest station 168 km from the target centroid. We excluded Lerwick (ID 293) as a fourth North Sea candidate because its record did not cover the target year. Supplementary Table S1 lists

Table 2: Virtual gauge output: sub-score values, LOO-CV RMSE, thresholds, and pass/fail. The anomaly-mean gate is a sanity check rather than a discriminator (Section 4.1).

Site	Sub-score	Value (m)	LOO-CV RMSE (m)	Threshold (m)	Pass
Philippines	Tidal range	1.718	0.249	0.50	Yes
	MHHW	0.543	0.068	0.20	Yes
	Anomaly mean	0.000	0.000	0.08	Yes
	Anomaly std	0.070	0.020	0.05	Yes
Raja Ampat	Tidal range	2.253	0.353	0.50	Yes
	MHHW	0.623	0.150	0.20	Yes
	Anomaly mean	0.000	0.000	0.08	Yes
	Anomaly std	0.077	0.021	0.05	Yes
North Sea	Tidal range	1.653	1.975	0.50	No
	MHHW	0.623	0.732	0.20	No
	Anomaly mean	0.000	0.000	0.08	Yes
	Anomaly std	0.227	0.095	0.05	No
Mediterranean	Tidal range	0.374	0.730	0.50	No
	MHHW	0.118	0.220	0.20	No
	Anomaly mean	0.000	0.000	0.08	Yes
	Anomaly std	0.108	0.046	0.05	Yes
Argentina	Tidal range	3.572	2.155	0.50	No
	MHHW	1.340	0.831	0.20	No
	Anomaly mean	0.000	0.000	0.08	Yes
	Anomaly std	0.223	0.064	0.05	No
US East Coast	Tidal range	1.865	1.208	0.50	No
	MHHW	0.709	0.501	0.20	No
	Anomaly mean	0.000	0.000	0.08	Yes
	Anomaly std	0.155	0.039	0.05	Yes
US West Coast	Tidal range	3.102	0.442	0.50	Yes
	MHHW	0.915	0.152	0.20	Yes
	Anomaly mean	0.000	0.000	0.08	Yes
	Anomaly std	0.109	0.057	0.05	No
Iceland/Norway	Tidal range	1.803	0.212	0.50	Yes
	MHHW	0.635	0.092	0.20	Yes
	Anomaly mean	0.000	0.000	0.08	Yes
	Anomaly std	0.162	0.023	0.05	Yes

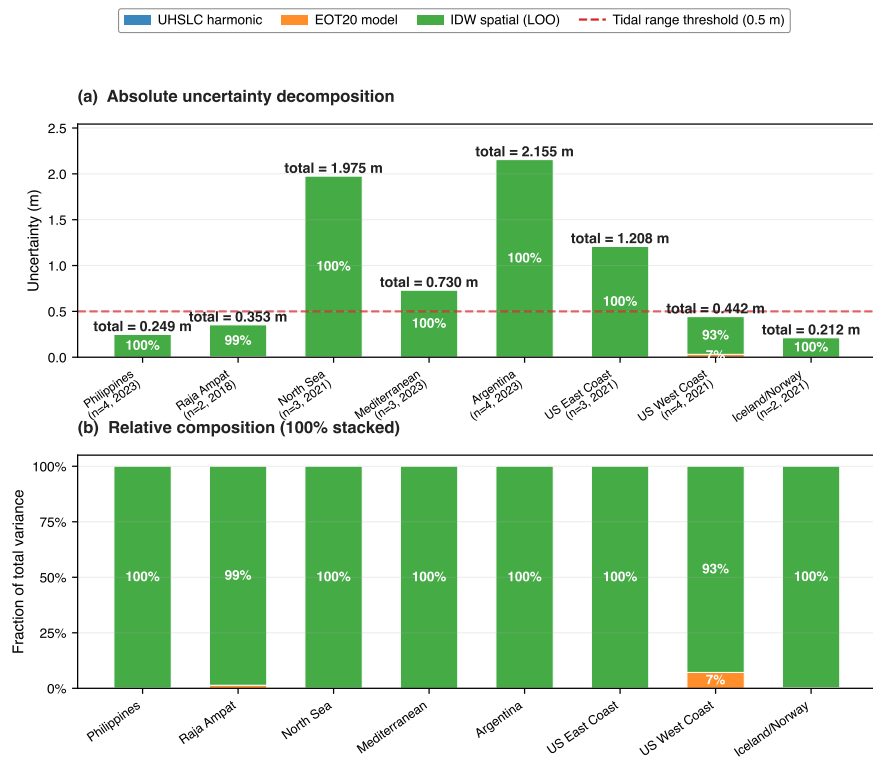


Figure 5: Tidal-range uncertainty budget: (a) absolute and (b) relative per-source decomposition. Red dashed: 0.50 m threshold.

the per-station LOO-CV biases that locate these errors at individual stations.

MHHW fails at the Mediterranean (0.220 m against a 0.20 m threshold). Station heterogeneity drives the failure because the three stations span tidal ranges of 1.31 to 0.30 m, so each held-out MHHW is predicted from stations in a different regime. At the same time, the EOT20 target contribution is anchored to a 0.268 m target range. Peak detection itself is stable there (Section 3.1), while the Mediterranean anomaly metrics are exploratory because Gibraltar contributes only 47 valid days (12.8% of the target year). Its annual anomaly mean and standard deviation are not comparable with those of full-year sites, and K_2 , P_1 , and M_f remain unresolved.

As a secondary check on fold independence, we used spatial-block cross-validation with k-means clustering in two groups. It is reported only for sites where k-means yields at least two blocks with no block larger than $\lceil n/2 \rceil$ stations, so the two-station sites (Raja Ampat and Iceland/Norway) are excluded.

The spatial-block RMSE exceeds LOO-CV at the North Sea (3.144 m against 1.975 m), where holding out an entire cluster amplifies prediction error in a heterogeneous basin. It also exceeds LOO-CV at the US West Coast (0.786 m against 0.442 m), where holding out two stations at once removes more of the IDW network. Spatial-block RMSE is lower than LOO-CV at the US East Coast (0.981 m against 1.208 m), the Mediterranean (0.505 m against 0.730 m), and Argentina (1.691 m against 2.155 m). In the Philippines, k-means produces a degenerate 3+1 split (Manila, Subic Bay, and Lubang against Davao) that exceeds the size constraint, so we skip spatial-block CV and rely on LOO-CV and the n-confound subsample analysis.

The n-confound subsample analysis (Section 3.5 and Supplementary Table S14) shows that two stations are unreliable. At two stations, the interquartile range (IQR) of LOO-CV RMSE is 0.362 m in the Philippines and 3.036 m in the North Sea, substantially wider than at three. With three stations, it narrows to 0.158 m in the Philippines, while the North Sea already uses its full set and yields no IQR. The 75th percentile of the Philippine two-station distribution (0.486 m) remains below the 0.50 m threshold, but the North Sea distribution is bimodal and highly sensitive to station selection ($p_{75} = 3.247$ m). This contrast reinforces that at least three stations are required for a stable LOO-CV uncertainty estimate, although three alone do not guarantee passing the

accuracy threshold (Section 5.2). Figure 6 visualizes these results as LOO-CV RMSE with bootstrap 95% confidence intervals for all four sub-scores, with tabular values in Supplementary Table S5.

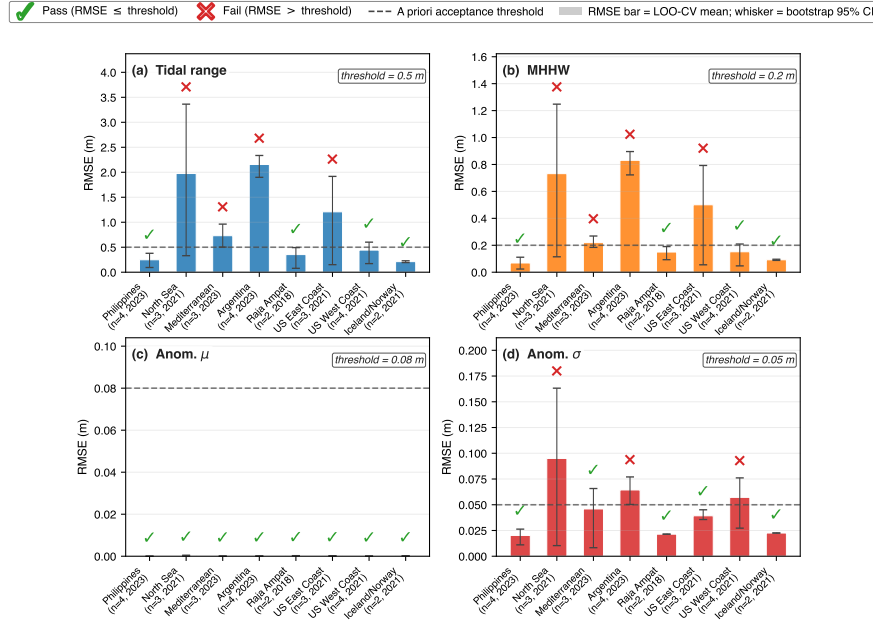


Figure 6: LOO-CV RMSE with bootstrap 95% CI: (a) tidal range, (b) MHHW, (c) anomaly mean, (d) anomaly std. Dashed: threshold.

Figure 7 relates station count to prediction accuracy, and passing the 0.50 m primary threshold reflects station coherence rather than station count. Station coherence is the ratio of the largest to the smallest station tidal range at a site. The four passing sites span ratios of 1.13 to 1.46 (Iceland/Norway 1.13, Raja Ampat 1.14, the Philippines 1.39, and the US West Coast 1.46). The four failing sites span 2.08 to 17.9 (the US East Coast 2.08, Argentina 3.30, the Mediterranean 4.38, and the North Sea 17.9). Raja Ampat and Iceland/Norway pass with only two folds and therefore reduced statistical confidence, though both remain below the 1.00 m pink reference line in Figure 7. This separation is post-hoc because, with eight sites, perfect separation by any single statistic is weak evidence. The rule has not been tested on independent sites, and coherent sites are also the small-basin, short-separation sites, so the two explanations remain confounded

(Section 5.2).

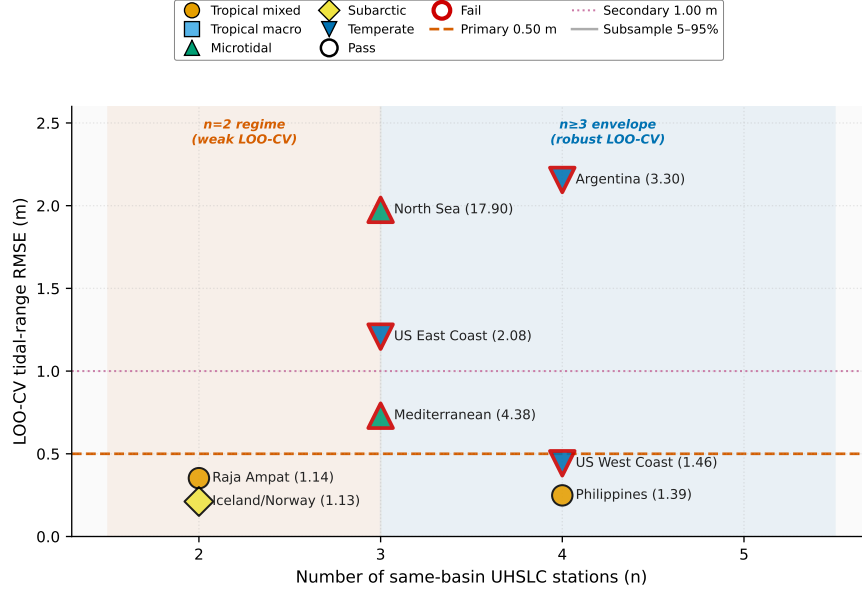


Figure 7: LOO-CV tidal-range RMSE vs station count; site labels give the tidal-range coherence ratio (largest/smallest station range). Orange dashed: 0.50 m; pink dotted: 1.00 m reference line.

Within the same LOO-CV framework, Table 3 compares the MUSE ensemble with four naive baselines across the five pilot sites, and Supplementary Table S17 gives the same comparison at the three additional sites. EOT20 alone has the lowest tidal-range RMSE at every pilot and additional site, from 0.014 m at the Mediterranean to 0.263 m at the North Sea. The ensemble beats the three UHSLC-only baselines at the Philippines (0.249 m against 0.276 to 0.310 m), the North Sea (1.975 m against 2.082 to 2.511 m), and Argentina (2.155 m against 2.228 to 2.359 m). Supplementary Figure S3 shows the Philippines site-specific output behind the first comparison. At the Mediterranean, the ensemble beats the single-nearest gauge with 0.730 against 0.893 m but performs worse than IDW-only at 0.706 m and the unweighted mean at 0.622 m. At Raja Ampat, the two-station network makes all three UHSLC-only baselines identical (0.317 m) and slightly better than the ensemble (0.353 m), though all remain below the 0.50 m

threshold. EOT20 alone also has the lowest MHHW RMSE at all eight sites, from 0.009 m at Iceland/Norway to 0.107 m at the US West Coast. Under the a priori gates, EOT20 alone passes the tidal-range and MHHW thresholds at all eight sites, while the ensemble fails four of eight on each threshold.

At the ungauged target, no in situ truth exists, so we report only the disagreement between the EOT20 and MUSE target estimates. Those estimates of tidal range agree to within 0.03 m at the US West Coast and Iceland/Norway and differ by up to 1.30 m at Argentina (Supplementary Table S20).

The DM test rarely detects accuracy differences because it has limited power at two to four folds per site (Supplementary Section S10.1). Across all eight sites and four baselines, only one of the 32 comparisons is significant. The test favors EOT20-only at Argentina ($p = 0.006$), matching the much lower tidal-range RMSE that EOT20 achieves there. The count of significant results is consistent with chance rather than indicating a systematic accuracy difference because chance alone would produce 1.6 significant results across 32 comparisons at $\alpha = 0.05$. Eight of the 32 comparisons are at the two-station sites where the test has no power at all.

The PSMSL–UHSLC monthly correlations (Section 3.4) are site-dependent and rest on 5 to 12 monthly pairs. Four of the seven sites with an overlapping record reach significance with $r = -0.80$ to $+0.82$ and $p < 0.05$: the North Sea, the Mediterranean, the US East Coast, and Iceland/Norway (Supplementary Table S12). The correlations indicate partial shared low-frequency variance between the two sources. Blending PSMSL with UHSLC at equal weight nonetheless raises the LOO-CV anomaly-mean RMSE at every site with an overlapping PSMSL record from below 0.0004 m to between 0.004 and 0.068 m, so we exclude PSMSL from the ensemble.

Varying the EOT20 ensemble weight over 0.1 to 0.5 (Supplementary Table S4) changes the tidal-range RMSE by less than 0.03 m at four of the eight sites and by up to 0.118 m at the rest, with the maximum at Raja Ampat. The set of sites passing the 0.50 m tidal-range gate is unchanged at every tested weight.

4.3. Independent validation

Supplementary Table S15 compares the virtual gauge output against the Bitung proxy reference that is excluded from the IDW ensemble using single-fold *bias* thresholds of 0.20 m for tidal range, 0.15 m for MHHW, 0.05 m for anomaly mean, and 0.03 m for anomaly std. These thresholds were fixed a priori in the validation configuration before the Bitung comparison was run. In a global surge reanalysis, 95% of validation stations have surge RMSE below 0.20 m [9], and the two anomaly thresholds are of the same order as their 1σ bounds in the threshold derivation (Supplementary Table S11). They differ from the multi-fold *RMSE* thresholds used in LOO-CV (Table 2) because bias and RMSE measure different error properties. Tidal-range bias (0.417 m) exceeds its 0.20 m threshold, and the anomaly-standard-deviation bias (0.036 m) marginally exceeds its 0.03 m threshold. The MHHW bias (0.108 m) and the anomaly-mean bias (0.000 m) pass their 0.15 m and 0.05 m thresholds. Figure 8 shows scatter plots of tidal range and MHHW with bias annotations, and Supplementary Figure S4 shows the anomaly sub-scores for the two-station ensemble. Figure 8(c) shows the leave-one-out residuals for all four sub-scores at the two ensemble stations. Figure 8(d) shows the per-source weights, with UHSLC-IDW carrying 80% of the normalized weight and EOT20 20%. PSMSL enters only as a consistency reference.

To test generality beyond the pilot sites, we added three further sites: Iceland/Norway, the US East Coast, and the US West Coast. We fixed their station sets and target years before analysis. We describe them as an *additional development set* rather than a strictly out-of-sample test because we refined some protocol parameters while preparing this manuscript, after these sites had already been inspected. Two of the three meet the a priori 0.50 m tidal-range threshold, and their results therefore support the protocol’s generality only weakly.

Iceland/Norway uses two subarctic semidiurnal stations (Måløy and Andenes) and passes the tidal-range threshold. The US West Coast passes at 0.442 m with four Pacific Northwest stations (San Francisco to Willapa Bay), but its anomaly standard deviation exceeds the 0.05 m threshold. Per-station tidal ranges span 2.6 to 3.9 m under non-uniform macrotidal conditions. The largest LOO-CV bias is at San Francisco, where IDW overestimates the smallest-range station (2.65 m) by 0.65 m because every

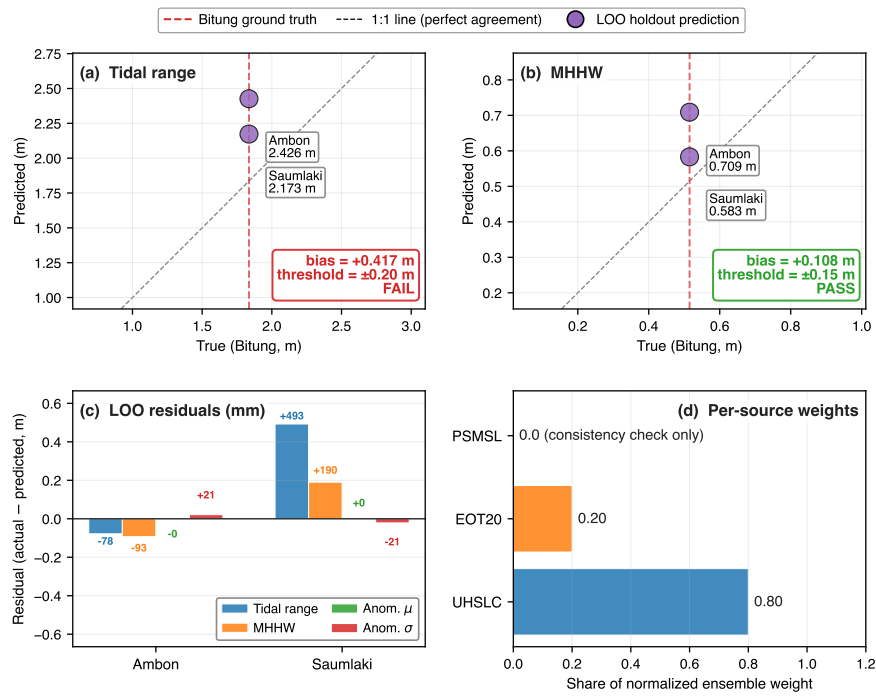


Figure 8: Raja Ampat validation against Bitung: (a–b) predicted vs observed range and MHHW; (c) LOO residuals; (d) per-source ensemble weights.

remaining station has a larger range. The US East Coast fails because its three Mid-Atlantic Bight stations from Portland, ME, to Charleston span 11° of latitude and test the IDW interpolation under a regime transition rather than a short distance. The two passing sites are consistent with the protocol’s generality. One sits in a different ocean basin (Norwegian Sea) and the other in a different climatic zone (Pacific Northwest), while the US East Coast result bounds its applicability.

4.4. Robustness and EOT20 model accuracy

Adversarial robustness tests in Supplementary Section S7 show that the protocol is generally robust to realistic failure modes. Noise injection with $\sigma = 0.1$ m passes at all eight sites, with the maximum RMSE ratio relative to the clean run below 2.0. The more extreme $\sigma = 0.5$ m stress case exceeds the criterion because the added noise inflates the anomaly standard deviation and is reported as a documented failure mode rather than as part of the acceptance gate. Data gaps of up to 25% removal pass at all eight sites, so the harmonic analysis tolerates moderate missing data. Station dropout passes only in the Philippines, where removing any single station shifts every sub-score by less than 0.15 m. At the other seven sites, losing one station shifts a sub-score by more than that bound, even when four stations remain (Supplementary Table S6). Wrong-basin exclusion passes at all eight sites, and the synthetic station receives a normalized IDW weight of 0.045 to 0.268, staying below the 0.3 criterion everywhere. The same-basin bonus is disabled with $\beta = 1.0$, so cross-basin stations are not excluded by design and the distance kernel alone down-weights the distant synthetic station regardless of its basin label. At the two-station sites, this weight is structurally larger, and wrong-basin exclusion is weaker.

Spike detection passes at all eight sites. The two-stage quality control described in Section 3.1 removes the +2 m spike hour itself before harmonic analysis at any phase of the tidal cycle. The earlier single-stage rate check passed only where the step into the spike exceeded the fixed 2 m/h threshold and otherwise flagged the following hour while the +2 m value entered the harmonic fit.

We assess EOT20 accuracy by comparing the model constituents with the UHSLC harmonic analysis at each station, and summarize the results in Supplementary Table S7.

Total RMSE ranges from 0.008 m at the Mediterranean and Iceland/Norway to 0.063 m at the US West Coast. The lowest values occur where the tidal field is nearly uniform over the station spread and the highest where four stations span tidal ranges of 2.6 to 3.9 m. In the North Sea, the dominant contributions are S_2 at 0.075 m and M_2 at 0.059 m. The mean bias is small everywhere, between -0.014 and +0.026 m, indicating no systematic over- or underestimation by EOT20.

Table 3: LOO-CV baseline comparison (RMSE, m). Bold = MUSE ensemble.

Site	Method	Tidal range	MHHW	Anomaly mean	Anomaly std
Philippines	MUSE ensemble	0.249	0.068	0.000	0.020
	IDW only	0.276	0.074	0.000	0.020
	EOT20 only	0.050	0.047	0.000	0.068
	Single nearest	0.296	0.080	0.000	0.019
	Unweighted mean	0.310	0.084	0.000	0.021
Raja Ampat	MUSE ensemble	0.353	0.150	0.000	0.021
	IDW only	0.317	0.157	0.000	0.021
	EOT20 only	0.068	0.037	0.000	0.074
	Single nearest	0.317	0.157	0.000	0.021
	Unweighted mean	0.317	0.157	0.000	0.021
North Sea	MUSE ensemble	1.975	0.732	0.000	0.095
	IDW only	2.095	0.773	0.000	0.096
	EOT20 only	0.263	0.102	0.000	0.226
	Single nearest	2.082	0.769	0.000	0.095
	Unweighted mean	2.511	0.927	0.000	0.115
Mediterranean	MUSE ensemble	0.730	0.220	0.000	0.046
	IDW only	0.706	0.206	0.000	0.054
	EOT20 only	0.014	0.017	0.000	0.109
	Single nearest	0.893	0.262	0.000	0.047
	Unweighted mean	0.622	0.180	0.000	0.061
Argentina	MUSE ensemble	2.155	0.831	0.000	0.064
	IDW only	2.265	0.874	0.000	0.064
	EOT20 only	0.180	0.093	0.000	0.218
	Single nearest	2.359	0.911	0.000	0.067
	Unweighted mean	2.228	0.860	0.000	0.074

5. Discussion

5.1. Principal findings

The MUSE protocol fills a specific gap that EOT20 alone cannot address: tidal anomaly metrics and per-source uncertainty. EOT20 alone provides the best tidal-range and MHHW estimates at all eight sites (Section 4.2). MUSE instead provides finite anomaly standard-deviation estimates of 0.070 to 0.227 m with LOO-CV RMSEs of 0.020 to 0.095 m. EOT20 is zero by construction. This is a capability gap rather than an accuracy improvement. The ensemble therefore serves as a validation cross-check for EOT20 at the four tidal-range passing sites (Section 4.2), while the four heterogeneous sites bound its applicability. The DM test has low power with only two to four folds per site, so most comparisons cannot distinguish the ensemble from the baselines. Its single significant result (Argentina, EOT20-only better, $p = 0.006$) matches the RMSE ordering. Mediterranean anomaly metrics are exploratory given Gibraltar’s 12.8% completeness.

The high EOT20 accuracy explains why EOT20 alone outperforms the ensemble for tidal range (Section 4.2). In the North Sea, the dominant EOT20 error contributions are the S_2 and M_2 RMSEs of 0.075 and 0.059 m. This is consistent with the strongly varying semidiurnal field of the southeastern North Sea amphidromic system [30]. The global tide model already captures the dominant tidal constituents at the coastal station locations used for validation, and IDW interpolation from nearby stations introduces additional spatial interpolation error without substantially improving the constituent estimates. MUSE therefore adds value not by improving tidal-range accuracy, but by providing anomaly metrics (Section 4.1) and uncertainty quantification that EOT20 alone cannot offer.

The Bitung reference lies 630 km away on the opposite side of the Halmahera Sea from the Seram and Arafura stations and indicates that the virtual gauge does not substitute for local in situ data. Its tidal-range bias (0.417 m) and anomaly-standard-deviation bias (0.036 m) exceed their thresholds, while the MHHW bias passes. The anomaly-mean bias is zero by construction and is therefore uninformative (Section 4.1).

5.2. *The operating envelope and its limits*

The coherence rule of Section 4.2 is post-hoc and not independently validated. Three or more stations alone are insufficient because four of the six such sites fail, and both two-station sites pass with only two folds each. The subsample analysis indicates that two-station networks inflate LOO-CV uncertainty (Section 4.2) but does not establish a general station-count envelope because only two sites in the validation set have two stations. The coherence explanation is further confounded with basin geometry because every coherent site also has a compact same-basin station set. Separating the coherence explanation from basin geometry would require sites that are coherent yet failing or incoherent yet passing. The present set has no such configurations, so they remain a target for future validation.

Low station count and heterogeneous station spread are the two risk factors. Raja Ampat passes with two stations but its two-fold RMSE carries wide uncertainty. Meanwhile, Argentina fails with four stations spread across contrasting regimes. At two stations, the IDW ensemble reduces to the single remaining station, and LOO-CV produces only two folds. The ensemble therefore gains no averaging benefit, and EOT20 alone is preferred there. The ensemble is most useful at coherent sites with more than two stations. The US East Coast failure additionally identifies a regime where the assumption of smooth spatial variation of tidal range breaks down across large regime transitions (11° of latitude, coherence ratio 2.08).

5.3. *Comparison with prior work and scope*

MUSE combines UHSLC and EOT20 with PSMSL consistency checking and per-source uncertainty. Prior studies of this type report trend-level uncertainty rather than per-value uncertainty, without per-station cross-validation of the interpolated tidal sub-scores (Supplementary Table S16). Our protocol adds a basin-friction distance and supplies the anomaly metrics and per-source uncertainty that these prior methods lack.

In scope, MUSE applies to semidiurnal and mixed regimes within the tested range of basin geometries and station configurations demonstrated at eight sites across four continents. MUSE is untested for purely diurnal, polar, or estuarine regimes. Performance degrades substantially in data-sparse regions such as Argentina. MUSE requires

only the globally available UHSLC stations and EOT20, while PSMSL serves as an optional consistency check.

These error bounds map directly onto coastal engineering practice because tidal range and MHHW set the datum basis for design water levels. The LOO-CV range errors translate into datum uncertainty of 0.2 to 0.4 m at the four coherent sites and 0.7 to 2.2 m at the four heterogeneous ones. The Philippines case is representative of the first group, where a 0.249 m error on a 1.72 m site range is about 14% relative. At Argentina the 2.155 m error exceeds the entire range of the microtidal Mediterranean site, so datum transfer there requires denser station coverage before operational use. The anomaly standard deviation carries the complementary message. Storm and swell raise extreme water levels above the predicted astronomical tide, and a flood-exposure assessment that uses EOT20 alone assigns zero error to that missing term. MUSE supplies the term with LOO-CV errors of 0.020 to 0.095 m, so the residual envelope can enter the calculation with a stated uncertainty rather than as an unquantified omission.

5.4. Limitations and future directions

The protocol requires at least 15 days of valid hourly data at every station and therefore cannot be applied to shorter records. Station selection further depends on the UHSLC basin field, whose coarse resolution limits how finely cross-basin stations can be distinguished. The IDW interpolation smooths local extremes, and the ensemble was not validated against extreme values. Unassessed EOT20 phase errors complete the four limitations that bound the interpretation of the validation results.

Five extensions address these limits. A multi-year baseline would capture interannual variability from ENSO and the IOD. Satellite altimetry could join as a fifth source. The interpolation itself could benefit from machine-learning methods such as Gaussian processes [31] that handle anisotropic coasts. Global Navigation Satellite System corrections could remove vertical land motion [32] for trend applications. A rate check scaled to the local tidal rate rather than the fixed 2 m/h threshold would further strengthen the two-stage pre-harmonic quality control.

To narrow the remaining tidal-range gap, the ensemble can be re-weighted per site and the EOT20 extraction corrected. A site-specific ensemble weight that may be zero

and an EOT20-only fallback at the two-station sites would let the interpolation adapt to each regime. A coastal amplification correction can improve nearest-neighbor EOT20 extraction, and per-site adaptive method selection from the LOO-CV diagnostics can pick the best configuration for each site.

5.5. Worked example: propagating MUSE errors through a composite index

This subsection demonstrates the error budget for a composite coastal index that motivates MUSE. The composite and its weights are illustrative and are not proposed as an index. The point is that the budget is *incomplete* without the anomaly term that EOT20 cannot supply.

Two dimensionless composites are evaluated per site, self-normalized by the site's station-mean tidal range R_{ref} and station-mean anomaly standard deviation s_{ref} . A perfect site-mean predictor returns $I = 1$:

$$I = w_R \frac{\hat{R}_p}{R_{\text{ref}}} + w_A \frac{\hat{\sigma}_{a,p}}{s_{\text{ref}}}, \quad I_{\text{NL}} = \frac{\hat{R}_p}{R_{\text{ref}}} \left(1 + \frac{\hat{\sigma}_{a,p}}{s_{\text{ref}}} \right) \quad (17)$$

with $w_R = w_A = 0.5$. I_{NL} adds an interaction term and is therefore non-linear in its inputs. Errors propagate through both composites via a first-order Taylor expansion, using the two extreme correlation cases from Section 3.3: $\rho = 0$ (independence) and $\rho = +1$ (perfect correlation). The error terms are the quadrature total $\sigma_{\text{MUSE}}^{\text{lower}}$ for tidal range (Eq. 10) and the LOO-CV RMSE of the anomaly standard deviation. Replacing the latter with its quadrature variant with the harmonic term changes σ_I by less than 1% at every site, so we ignore it.

The $\hat{\sigma}_a$ error share in Table 4 ranges from 9.7% of the linear-composite variance in the North Sea to 89.9% on the US West Coast. At the four coherent tidal-range-passing sites, the share is 63 to 90%, because the anomaly sub-score is the *relatively* less certain term there (σ_a is 15 to 42% of its reference scale, against 11 to 15% for the range term). At heterogeneous sites the range relative error (48 to 136%) dominates instead. The EOT20-only column shows what a model-only budget can express: a range term with a small error and *no* anomaly term, because the EOT20 anomaly is identically zero by construction. The index error budget is therefore structurally incomplete without an

in-situ-derived anomaly estimate. MUSE completes the budget and does not improve the tidal-range term that EOT20 provides more accurately at every site (Section 4.2).

Table 4: Worked propagation of MUSE errors through the illustrative composites of Eq. 17. Error share is the fraction of linear-composite variance from the LOO-CV RMSE of $\hat{\sigma}_a$. EOT20 partial I propagates the range term only.

Site	I	$\pm\sigma_I (\rho=0)$	$\pm\sigma_I (\rho=1)$	I_{NL}	$\hat{\sigma}_a$ error share (%)	EOT20 partial I
Philippines	1.030	0.169	0.225	2.058	81.6	0.585
Raja Ampat	0.976	0.162	0.217	1.846	80.9	0.293
North Sea	1.104	0.717	0.905	2.357	9.7	0.416
Mediterranean	0.752	0.492	0.663	0.921	21.0	0.161
Argentina	1.054	0.351	0.469	2.160	18.9	0.333
US East Coast	0.934	0.279	0.383	1.577	25.9	0.272
US West Coast	0.868	0.219	0.277	1.690	89.9	0.475
Iceland/Norway	1.003	0.092	0.129	1.962	62.6	0.476

6. Conclusions

We present MUSE as a local validation framework for global tide models at data-scarce coasts, adding tidal anomaly metrics and per-source uncertainty to EOT20. EOT20 alone is the primary source for tidal range and for MHHW at all eight sites under station-level cross-validation. Its tidal-range RMSE spans 0.01 to 0.26 m, and its MHHW RMSE spans 0.01 to 0.11 m. It cannot produce anomaly metrics because its anomaly is identically zero. MUSE instead provides finite standard-deviation estimates with LOO-CV RMSE as low as 0.020 m in the Philippines and 0.023 m in Iceland/Norway. The two methods differ in capability rather than in accuracy.

Whether the ensemble passes the 0.50 m tidal-range threshold reflects station coherence rather than station count. This is a post-hoc pattern on eight sites that awaits independent confirmation. The four passing sites span a tidal-range ratio of at most 1.5, while the four failing sites span ratios of two or more. Spatial-block cross-validation fails at all five testable sites, and its RMSE exceeds the station-level value at the North Sea and the US West Coast. The DM test has limited power at two to four folds per

site and rejects equal accuracy in one of 32 comparisons, consistent with the RMSE ordering. MUSE uses freely available data and produces per-source error components that composite-index error budgets can propagate once between-sub-score covariances are specified (Section 3.3), as demonstrated on an illustrative index in Section 5.5. MUSE therefore complements rather than replaces in situ gauges or EOT20.

Funding

This research received no specific grant funding from public, commercial, or not-for-profit agencies.

Conflict of Interest

The author declares no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data Availability Statement

All data used in this study are publicly available from the following sources: UHSLC Fast Delivery hourly records [15] (<https://uhslc.soest.hawaii.edu/erddap/>), PSMSL RLR monthly data (<https://www.psmsl.org/data/obtaining/>), and EOT20 tidal constituents (SEANOE, <https://www.seanoe.org/data/00683/79489/>, doi:10.17882/79489). We retrieved the UHSLC and PSMSL records in June and July 2026, and the EOT20 atlas came from SEANOE.

Code Availability

Code is available upon request from the corresponding author.

Author Contributions

Prasetyo Wibowo: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work, the author used Grammarly for copy editing and Google Translate for translation. After using these tools/services, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

References

- [1] O. A. Dada, R. Almar, P. Morand, Coastal vulnerability assessment of the West African coast to flooding and erosion, *Scientific Reports* 14 (2024) 890. doi : 10.1038/s41598-023-48612-5.
- [2] L. L. Lutzenkirchen, S. J. Duce, D. R. Bellwood, Exploring benthic habitat assessments on coral reefs: a comparison of direct field measurements versus remote sensing, *Coral Reefs* 43 (2024) 265–280. doi : 10.1007/s00338-024-02468-x.
- [3] M. Marcos, G. Wöppelmann, A. Matthews, R. M. Ponte, F. Birol, F. Ardhuin, G. Coco, A. Santamaría-Gómez, V. Ballu, L. Testut, D. Chambers, J. E. Stopa, Coastal sea level and related fields from existing observing systems, *Surveys in Geophysics* 40 (6) (2019) 1293–1317. doi : 10.1007/s10712-019-09513-3.
- [4] Intergovernmental Oceanographic Commission, Global Sea Level Observing System (GLOSS) Implementation Plan – 2012, Tech. Rep. 100, UNESCO/IOC, gOOS Report No. 194; JCOMM Technical Report No. 66 (2012).
URL <https://unesdoc.unesco.org/ark:/48223/pf0000217832>
- [5] M. G. Hart-Davis, G. Piccioni, D. Dettmering, C. Schwatke, M. Passaro, F. Seitz, EOT20: a global ocean tide model from multi-mission satellite altimetry, *Earth System Science Data* 13 (8) (2021) 3869–3884. doi : 10.5194/essd-13-3869-2021.

- [6] F. H. Lyard, D. J. Allain, M. Cancet, L. Carrère, N. Picot, FES2014 global ocean tide atlas: design and performance, *Ocean Science* 17 (3) (2021) 615–649. doi:10.5194/os-17-615-2021.
- [7] D. Stammer, R. D. Ray, O. B. Andersen, B. K. Arbic, W. Bosch, L. Carrère, et al., Accuracy assessment of global barotropic ocean tide models, *Reviews of Geophysics* 52 (3) (2014) 243–282. doi:10.1002/2014RG000450.
- [8] M. G. Hart-Davis, R. Sulzbach, S. A. Talke, et al., Tide of the Time: Global tidal characteristics observed from in-situ measurements, *Ocean Science* 22 (3) (2026) 1681–1709. doi:10.5194/os-22-1681-2026.
- [9] S. Muis, M. Verlaan, H. C. Winsemius, J. C. J. H. Aerts, P. J. Ward, A global reanalysis of storm surges and extreme sea levels, *Nature Communications* 7 (2016) 11969. doi:10.1038/ncomms11969.
- [10] R. Hawkins, L. Husson, G. Choblet, T. Bodin, J. Pfeffer, Virtual tide gauges for predicting relative sea level rise, *Journal of Geophysical Research: Solid Earth* 124 (12) (2019) 13367–13391. doi:10.1029/2019JB017943.
- [11] M. F. Abd Rahman, A. H. Md Din, et al., Quasi-continuous tidal datum for Peninsular Malaysia using tide gauge, satellite altimetry, and tide model driver (TMD) data, *Engineering, Technology & Applied Science Research* 14 (3) (2024) 14428–14436. doi:10.48084/etasr.6810.
- [12] J. F. Turner, J. C. Iliffe, M. K. Ziebart, C. Wilson, K. J. Horsburgh, Interpolation of tidal levels in the coastal zone for the creation of a hydrographic datum, *Journal of Atmospheric and Oceanic Technology* 27 (3) (2010) 605–613. doi:10.1175/2009JTECH0645.1.
- [13] K. W. Hess, Spatial interpolation of tidal data in irregularly shaped coastal regions by numerical solution of Laplace’s equation, *Estuarine, Coastal and Shelf Science* 54 (2) (2002) 175–192. doi:10.1006/ecss.2001.0838.
- [14] M. F. Abd Rahman, A. H. Md Din, M. H. Hamden, M. R. Mahmud, A. W. Rasib, Developing a near-seamless tidal datum by integrating tidal and satellite

- altimetry data with a Digital Elevation Model for maritime boundary delimitation, *International Journal of Remote Sensing* 45 (23) (2024) 8827–8858. doi:10.1080/01431161.2023.2248562.
- [15] P. C. Caldwell, M. A. Merrifield, P. R. Thompson, Sea level measured by tide gauges from global oceans – the Joint Archive for Sea Level holdings (NCEI accession 0019568), version 5.5 (2015). doi:10.7289/V5V40S7W.
- [16] D. L. Codiga, Unified tidal analysis and prediction using the UTide Matlab functions, Technical Report 2011-01, Graduate School of Oceanography, University of Rhode Island, Narragansett, RI (2011). doi:10.13140/RG.2.1.3761.2008.
- [17] D. T. Pugh, P. L. Woodworth, *Sea-Level Science: Understanding Tides, Surges, Tsunamis and Mean Sea-Level Changes*, 2nd Edition, Cambridge University Press, 2014. doi:10.1017/CB09781139235778.
- [18] I. D. Haigh, M. D. Pickering, J. A. M. Green, B. K. Arbic, A. Arns, S. Dangendorf, et al., The tides they are a-changin’: a comprehensive review of past and future nonastronomical changes in tides, their driving mechanisms, and future implications, *Reviews of Geophysics* 57 (1) (2020) e2018RG000636. doi:10.1029/2018RG000636.
- [19] M. G. Hart-Davis, G. Piccioni, D. Dettmering, C. Schwatke, M. Passaro, F. Seitz, EOT20 - A global Empirical Ocean Tide model from multi-mission satellite altimetry, *SEANOE* (2021). doi:10.17882/79489.
- [20] D. E. Gregg, N. T. Penna, C. Jones, M. A. Morales Maqueda, Accuracy assessment of recent global ocean tide models in coastal waters of the European North West Shelf, *Ocean Modelling* 192 (2024) 102448. doi:10.1016/j.ocemod.2024.102448.
- [21] D. Shepard, A two-dimensional interpolation function for irregularly-spaced data, in: *Proceedings of the 1968 23rd ACM National Conference*, 1968, pp. 517–524. doi:10.1145/800186.810616.

- [22] D. T. Pugh, *Tides, Surges and Mean Sea-Level: A Handbook for Engineers and Scientists*, John Wiley & Sons, Chichester, 1987, 472 pp.
- [23] Permanent Service for Mean Sea Level (PSMSL), PSMSL RLR monthly tide gauge data, retrieved June 2026 (2026).
URL <https://www.psmsl.org/data/obtaining/>
- [24] S. J. Holgate, A. Matthews, P. L. Woodworth, L. J. Rickards, M. E. Tamisiea, et al., New data systems and products at the Permanent Service for Mean Sea Level, *Journal of Coastal Research* 29 (3) (2013) 493–504. doi:10.2112/JCOASTRES-D-12-00175.1.
- [25] W. Han, D. Stammer, P. Thompson, T. Ezer, H. Palanisamy, X. Zhang, C. M. Domingues, L. Zhang, D. Yuan, Impacts of basin-scale climate modes on coastal sea level: a review, *Surveys in Geophysics* 40 (2019) 1493–1541. doi:10.1007/s10712-019-09562-8.
- [26] F. X. Diebold, R. S. Mariano, Comparing predictive accuracy, *Journal of Business & Economic Statistics* 13 (3) (1995) 253–263. doi:10.1080/07350015.1995.10524599.
- [27] A. C. Harvey, S. J. Leybourne, P. Newbold, Testing the equality of prediction mean squared errors, *International Journal of Forecasting* 13 (2) (1997) 281–291. doi:10.1016/S0169-2070(96)00719-4.
- [28] B. Efron, B. Narasimhan, The automatic construction of bootstrap confidence intervals, *Journal of Computational and Graphical Statistics* 29 (3) (2020) 608–619. doi:10.1080/10618600.2020.1714633.
- [29] C. Milà, J. Mateu, E. Pebesma, H. Meyer, Nearest neighbour distance matching leave-one-out cross-validation for map validation, *Methods in Ecology and Evolution* 13 (6) (2022) 1304–1316. doi:10.1111/2041-210X.13851.
- [30] L. Deyle, T. H. Badewien, O. Wurl, J. Meyerjürgens, Lagrangian surface drifter observations in the North Sea: an overview of high-resolution tidal dynamics

and surface currents, *Earth System Science Data* 16 (4) (2024) 2099–2112. doi : 10.5194/essd-16-2099-2024.

- [31] M. Sonnewald, R. Lguensat, D. C. Jones, P. D. Dueben, J. Brajard, V. Balaji, Bridging observations, theory and numerical simulation of the ocean using machine learning, *Environmental Research Letters* 16 (7) (2021) 073008. doi : 10.1088/1748-9326/ac0eb0.
- [32] J. Oelsmann, M. Passaro, L. Sánchez, D. Dettmering, C. Schwatke, F. Seitz, Bayesian modelling of piecewise trends and discontinuities to improve the estimation of coastal vertical land motion, *Journal of Geodesy* 96 (62) (2022). doi : 10.1007/s00190-022-01645-6.

Supplementary Material for: MUSE: virtual tide gauge reference estimates and error budgets for coastal index validation at ungauged coastlines

Prasetyo Wibowo

Faculty of Data Science, Musashino University, Ariake, Koto-ku, 135-8181, Tokyo, Japan

Department of Informatics and Computer Engineering

Politeknik Elektronika Negeri Surabaya, Sukolilo, Surabaya, 60111, East Java, Indonesia

`pras@pens.ac.id`

S1 Harmonic analysis verification

UTide applies a fixed Rayleigh criterion of 1.0 with no adaptive re-run. Within that setting, the shallow-water constituents M_4 , MS_4 , and MN_4 are verified post-hoc at each station. M_4 is a quarter-diurnal constituent generated by M_2 self-interaction in shallow water (< 50 m depth) and can reach 10% of M_2 amplitude in coastal basins. K_2 , P_1 , and Mf require roughly 183 days of record, about half a year, and are therefore unresolved at stations with short records such as Gibraltar. The confidence intervals of the remaining constituents widen accordingly.

S2 Applying the protocol at new sites

To apply the protocol at a new ungauged site:

1. Identify the nearest UHSLC stations within 1500 km and exclude stations in wrong basins or with fewer than 15 days of valid hourly data.
2. Extract EOT20 constituents at target coordinates.
3. Fetch PSMSL monthly record at nearest same-basin station.
4. Run harmonic analysis at each UHSLC station and verify constituent recovery and data quality.
5. Compute effective distance (Eq. 5 of the main text) and inverse distance weighting (IDW) weights (Eq. 6 of the main text).
6. Apply the tidal-range similarity penalty (Eq. 7 of the main text) if EOT20 provides a tidal range prior.
7. Interpolate tidal range at the target (Eq. 8 of the main text).
8. Build the ensemble (Eq. 9 of the main text) and report both uncertainty bounds (Eqs. 10–11 of the main text).
9. Validate via leave-one-station-out cross-validation (LOO-CV) against a held-out station.

S3 UHSLC station data completeness

Figure S1 maps the UHSLC stations used in the study, and per-station completeness is given in Table 1 of the main text. All stations exceed 83% completeness in their respective target years apart from Gibraltar where only 47 days are valid (12.8% of the year).

S4 Per-station LOO-CV biases

Table S1 shows the LOO-CV bias (tidal range, m) for each station. A positive bias means the ensemble overestimates the station’s range, and a negative bias means it underestimates. The biases are site-specific and reflect the spatial heterogeneity of the tidal field within each site.

Table S1: LOO-CV bias per station for tidal range in meters for the ensemble and IDW-only baselines. Positive values indicate overestimation.

Site	Station	Ensemble	IDW-only
Philippines	Manila	−0.039	−0.133
	Subic Bay	+0.209	+0.135
	Lubang	+0.128	+0.044
	Davao	−0.431	−0.516
Raja Ampat	Ambon	+0.078	+0.317
	Saumlaki	−0.493	−0.317
North Sea	Cuxhaven	−3.363	−3.553
	Smögen	+0.330	+0.381
	Göteborg	+0.530	+0.629
Mediterranean	Trieste	−0.962	−0.846
	Marseille	+0.649	+0.857
	Gibraltar	−0.502	−0.207
Argentina	Mar del Plata	+2.158	+2.913
	Puerto Madryn	−2.350	−1.775
	Puerto Deseado	−2.322	−1.732
	Ushuaia	+1.735	+2.427
US East Coast	Portland ME	−1.917	−1.812
	Atlantic City	+0.823	+1.327
	Charleston	−0.151	+0.272
US West Coast	San Francisco	+0.650	+0.722
	Crescent City	+0.199	+0.245
	Neah Bay	−0.141	−0.030
	Willapa Bay	−0.549	−0.494
Iceland/Norway	Måløy	+0.193	+0.234
	Andenes	−0.229	−0.234

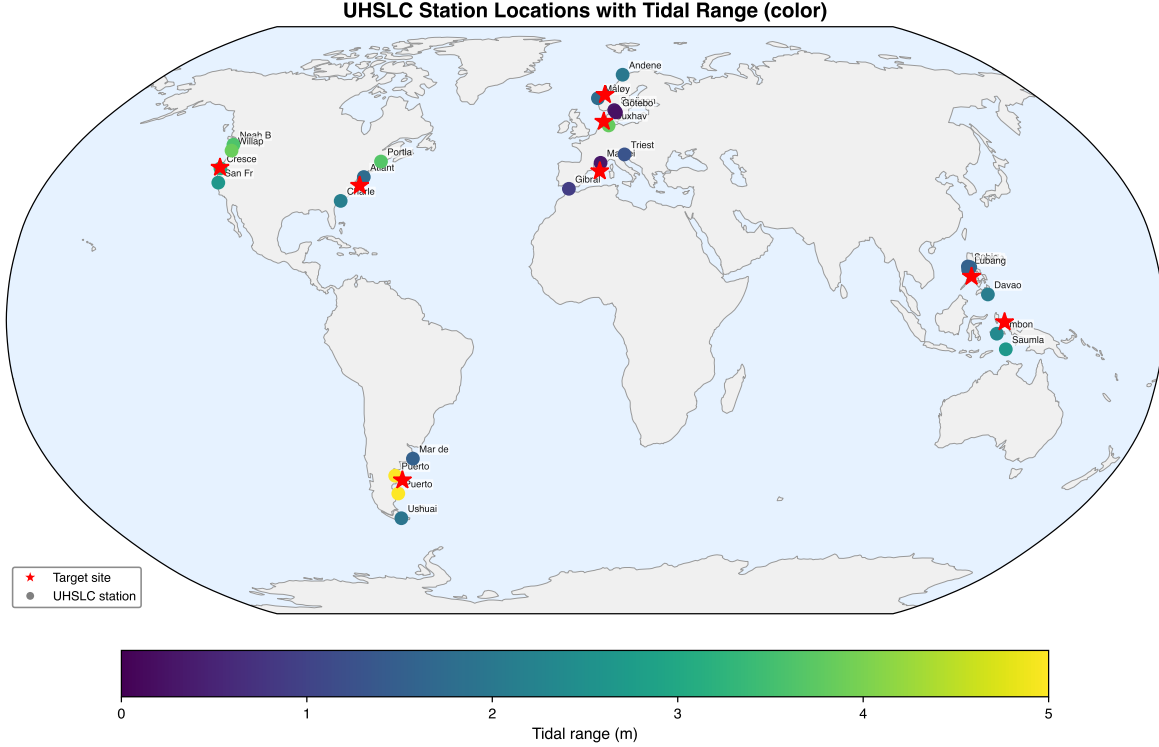


Figure S1: UHSLC station locations, colored by tidal range; red stars: target centroids.

S5 Sensitivity to IDW parameters

S5.1 IDW parameter choice

The sensitivity of the ensemble output to three parameters was tested: IDW power $\kappa \in \{1.0, 1.5, 2.0, 3.0\}$, basin-friction multiplier $\in \{0.8, 1.0, 1.2, 1.5\}$, and the same-basin bonus β set to 1.0 by default. All evaluated sub-scores of tidal range, MHHW, and anomaly std at all eight sites have a coefficient of variation (CV) $< 15\%$ across parameter values, so the ensemble is not sensitive to the choice of default parameters (Table S2). The same-basin bonus is the exception at the Philippines, the only site whose station set contains both same-basin and cross-basin members, so β changes the relative weights. Its CV is 1.08% against 0.00% at the other seven sites, where the bonus either multiplies every station (the six single-basin sites) or none of them (Raja Ampat, whose Seram and Arafura stations both lie outside the target basin), so β cancels in the normalization.

Table S2: Maximum CV of ensemble output across IDW parameter values, per site.

Site	κ CV	f_p mult. CV	β CV	Max CV	Pass ($<15\%$)
Philippines	0.92%	0.43%	1.08%	1.08%	Yes
Raja Ampat	2.58%	0.55%	0.00%	2.58%	Yes
North Sea	12.01%	4.63%	0.00%	12.01%	Yes
Mediterranean	11.45%	3.14%	0.00%	11.45%	Yes
Argentina	7.50%	2.90%	0.00%	7.50%	Yes
Iceland/Norway	1.00%	0.34%	0.00%	1.00%	Yes
US East Coast	4.87%	1.16%	0.00%	4.87%	Yes
US West Coast	2.74%	1.16%	0.00%	2.74%	Yes

S5.2 Sensitivity of the acceptance threshold to σ_{IDW}

The primary acceptance threshold of 0.50 m for tidal range was derived using $\sigma_{\text{IDW}} = 0.20$ m, an author estimate for a 1 to 2 m reference range (10 to 20% of that range) and the midpoint of the 0.10 to 0.30 m sweep. To assess the sensitivity of the pass/fail decisions to this choice, the threshold was recomputed for five values of σ_{IDW} using Eq. 1 (Table S3):

$$\text{Threshold}(\sigma_{\text{IDW}}) = 2 \times \sqrt{\sigma_{\text{UHSLC}}^2 + \sigma_{\text{IDW}}^2 + \sigma_{\text{EOT20}}^2} \quad (1)$$

with $\sigma_{\text{UHSLC}} = 0.02$ m and $\sigma_{\text{EOT20}} = 0.15$ m.

Table S3: Tidal-range threshold as a function of σ_{IDW} ($\sigma_{\text{UHSLC}} = 0.02$ m, $\sigma_{\text{EOT20}} = 0.15$ m).

σ_{IDW} (m)	0.10	0.15	0.20	0.25	0.30
Threshold (m)	0.36	0.43	0.50	0.58	0.67
Sites passing (of 8)	3	3	4	4	4
Philippines (0.249)	Pass	Pass	Pass	Pass	Pass
Raja Ampat (0.353)	Pass	Pass	Pass	Pass	Pass
Iceland/Norway (0.212)	Pass	Pass	Pass	Pass	Pass
US West Coast (0.442)	Fail	Fail	Pass	Pass	Pass
Mediterranean (0.730)	Fail	Fail	Fail	Fail	Fail
US East Coast (1.208)	Fail	Fail	Fail	Fail	Fail
North Sea (1.975)	Fail	Fail	Fail	Fail	Fail
Argentina (2.155)	Fail	Fail	Fail	Fail	Fail

At the default $\sigma_{\text{IDW}} = 0.20$ m four sites pass the tidal-range threshold, namely the Philippines at 0.249 m, Raja Ampat at 0.353 m, Iceland/Norway at 0.212 m, and US West Coast at 0.442 m. At $\sigma_{\text{IDW}} = 0.15$ m the threshold tightens to 0.43 m and three sites pass, namely the Philippines at 0.249 m, Raja Ampat at 0.353 m, and Iceland/Norway at 0.212 m. The US West Coast at 0.442 m no longer passes. The Mediterranean tidal-range RMSE of 0.730 m exceeds the threshold at all σ_{IDW} values, consistent with its wide-basin station spread (Section 4.2 of the main text).

S5.3 Sensitivity to EOT20 ensemble weight

The EOT20 ensemble weight was varied across $\{0.1, 0.15, 0.2, 0.25, 0.3, 0.5\}$ at all eight sites, and Table S4 shows the LOO-CV RMSE for tidal range at each weight. At four sites the RMSE varies by less than 0.03 m, including 0.015 m at the Philippines, 0.014 m at the US East Coast, 0.002 m at the US West Coast, and 0.026 m at Iceland/Norway, and at the other four it varies by 0.04 to 0.12 m and reaches 0.118 m at Raja Ampat. The EOT20 weight choice does not alter any pass/fail outcome. The same four sites pass at every tested weight, so the fixed weight is sufficient over the tested range.

Table S4: LOO-CV tidal-range RMSE (m) vs EOT20 ensemble weight (default 0.25).

Site	$w=0.10$	$w=0.15$	$w=0.20$	$w=0.25$	$w=0.30$	$w=0.50$	Max var.
Philippines	0.2602	0.2552	0.2514	0.2486	0.2467	0.2448	0.0154
Raja Ampat	0.3170	0.3258	0.3381	0.3527	0.3686	0.4349	0.1179
North Sea	2.0243	2.0061	1.9897	1.9748	1.9612	1.9173	0.1070
Mediterranean	0.7509	0.7431	0.7362	0.7303	0.7251	0.7100	0.0409
Argentina	2.2123	2.1907	2.1718	2.1552	2.1406	2.0968	0.1155
US East Coast	1.2202	1.2146	1.2106	1.2079	1.2063	1.2066	0.0139
US West Coast	0.4429	0.4425	0.4424	0.4424	0.4427	0.4447	0.0024
Iceland/Norway	0.2238	0.2193	0.2153	0.2116	0.2083	0.1975	0.0262

S6 Bootstrap confidence intervals

Table S5 shows the 95% bootstrap CI for the LOO-CV RMSE at each site from 1000 bootstrap replicates that resample the per-fold errors with replacement. The CI is narrow for the Philippines and Iceland/Norway because the station spread is coherent there. It is wide where stations are spread out, at the North Sea across the basin, the US East Coast across a wide latitudinal range and Argentina across a strong latitudinal gradient. That spread reflects substantial variation in ensemble accuracy. For the two-station sites (Raja Ampat and Iceland/Norway) the bootstrap CI rests on only two per-fold errors, so the resolution of the confidence interval is limited. The CI full width at Raja Ampat (0.415 m) with a 0.207 m half-width reflects the asymmetry between the two held-out biases (0.078 m and 0.493 m). At Iceland/Norway the per-fold errors are more balanced (0.193 m and 0.229 m), producing a narrower interval.

Table S5: Bootstrap 95% CI for LOO-CV RMSE (tidal range, m) and anomaly σ_a RMSE (1000 replicates).

Site	n	RMSE (m)	CI lower (m)	CI upper (m)	Anomaly σ_a RMSE (m)
Philippines	4	0.249	0.094	0.379	0.020
Raja Ampat	2	0.353	0.078	0.493	0.021
North Sea	3	1.975	0.330	3.363	0.095
Mediterranean	3	0.730	0.502	0.962	0.046
Argentina	4	2.155	1.899	2.336	0.064
US East Coast	3	1.208	0.151	1.917	0.039
US West Coast	4	0.442	0.172	0.602	0.057
Iceland/Norway	2	0.212	0.193	0.229	0.023

S7 Adversarial robustness results

Table S6 summarizes the pass/fail outcome and metric of the adversarial robustness tests for all eight sites.

Table S6: Adversarial robustness results for all eight sites. *scenarios excluded from the acceptance gate regardless of value (documented failure modes).

Site	Scenario	Metric	Value	Pass
Philippines	Noise 0.1 m	max ratio	1.744	PASS
	Noise 0.5 m	max ratio	6.462	FAIL*
	Gaps 10%	max bias (m)	0.003	PASS
	Gaps 25%	max bias (m)	0.014	PASS
	Gaps 50%	max bias (m)	0.294	FAIL*
	Station dropout	max bias (m)	0.041	PASS
	Wrong basin	IDW weight	0.077	PASS
	Spike detect.	n/a	n/a	PASS
Raja Ampat	Noise 0.1 m	max ratio	1.628	PASS
	Noise 0.5 m	max ratio	5.880	FAIL*
	Gaps 10%	max bias (m)	0.005	PASS
	Gaps 25%	max bias (m)	0.013	PASS
	Gaps 50%	max bias (m)	0.782	FAIL*
	Station dropout	max bias (m)	0.174	FAIL
	Wrong basin	IDW weight	0.268	PASS
	Spike detect.	n/a	n/a	PASS
North Sea	Noise 0.1 m	max ratio	1.102	PASS
	Noise 0.5 m	max ratio	2.221	FAIL*
	Gaps 10%	max bias (m)	0.007	PASS
	Gaps 25%	max bias (m)	0.017	PASS

Site	Scenario	Metric	Value	Pass
	Gaps 50%	max bias (m)	0.446	FAIL*
	Station dropout	max bias (m)	1.199	FAIL
	Wrong basin	IDW weight	0.131	PASS
	Spike detect.	n/a	n/a	PASS
Mediterranean	Noise 0.1 m	max ratio	1.365	PASS
	Noise 0.5 m	max ratio	4.229	FAIL*
	Gaps 10%	max bias (m)	0.001	PASS
	Gaps 25%	max bias (m)	0.030	PASS
	Gaps 50%	max bias (m)	0.106	FAIL*
	Station dropout	max bias (m)	0.546	FAIL
	Wrong basin	IDW weight	0.196	PASS
	Spike detect.	n/a	n/a	PASS
Argentina	Noise 0.1 m	max ratio	1.096	PASS
	Noise 0.5 m	max ratio	2.222	FAIL*
	Gaps 10%	max bias (m)	0.009	PASS
	Gaps 25%	max bias (m)	0.041	PASS
	Gaps 50%	max bias (m)	1.302	FAIL*
	Station dropout	max bias (m)	0.766	FAIL
	Wrong basin	IDW weight	0.140	PASS
	Spike detect.	n/a	n/a	PASS
US East Coast	Noise 0.1 m	max ratio	1.181	PASS
	Noise 0.5 m	max ratio	3.008	FAIL*
	Gaps 10%	max bias (m)	0.003	PASS
	Gaps 25%	max bias (m)	0.021	PASS
	Gaps 50%	max bias (m)	0.496	FAIL*
	Station dropout	max bias (m)	0.475	FAIL
	Wrong basin	IDW weight	0.162	PASS
	Spike detect.	n/a	n/a	PASS
US West Coast	Noise 0.1 m	max ratio	1.366	PASS
	Noise 0.5 m	max ratio	4.176	FAIL*
	Gaps 10%	max bias (m)	0.009	PASS
	Gaps 25%	max bias (m)	0.029	PASS
	Gaps 50%	max bias (m)	0.022	FAIL*
	Station dropout	max bias (m)	0.179	FAIL
	Wrong basin	IDW weight	0.045	PASS
	Spike detect.	n/a	n/a	PASS
Iceland/Norway	Noise 0.1 m	max ratio	1.178	PASS
	Noise 0.5 m	max ratio	2.951	FAIL*
	Gaps 10%	max bias (m)	0.004	PASS
	Gaps 25%	max bias (m)	0.016	PASS
	Gaps 50%	max bias (m)	0.012	FAIL*
	Station dropout	max bias (m)	0.154	FAIL
	Wrong basin	IDW weight	0.203	PASS
	Spike detect.	n/a	n/a	PASS

S8 Comparison with literature values

Figure S2 compares the MUSE ensemble tidal-range estimates against indicative regional ranges at each of the eight study sites. These ranges are author-compiled from the regional literature and are not site-specific published values. The ranges are coarse regional summaries and can differ from the site-specific station ranges in Table 1 of the main text, particularly where a site spans several sub-basins with contrasting tidal regimes,

as in the North Sea. The ensemble values fall within the stated ranges at four sites, namely the Philippines, Raja Ampat, Argentina, and the US East Coast. They fall outside at four: the North Sea at 1.65 m and the US West Coast at 3.10 m sit above their ranges, Iceland/Norway at 1.80 m by a small margin, and the Mediterranean at 0.37 m below. These comparisons are presented for regional context and not as a formal validation, and they inherit the manuscript’s four-constituent range definition: Table S19 quantifies how much the reported value depends on that definition.

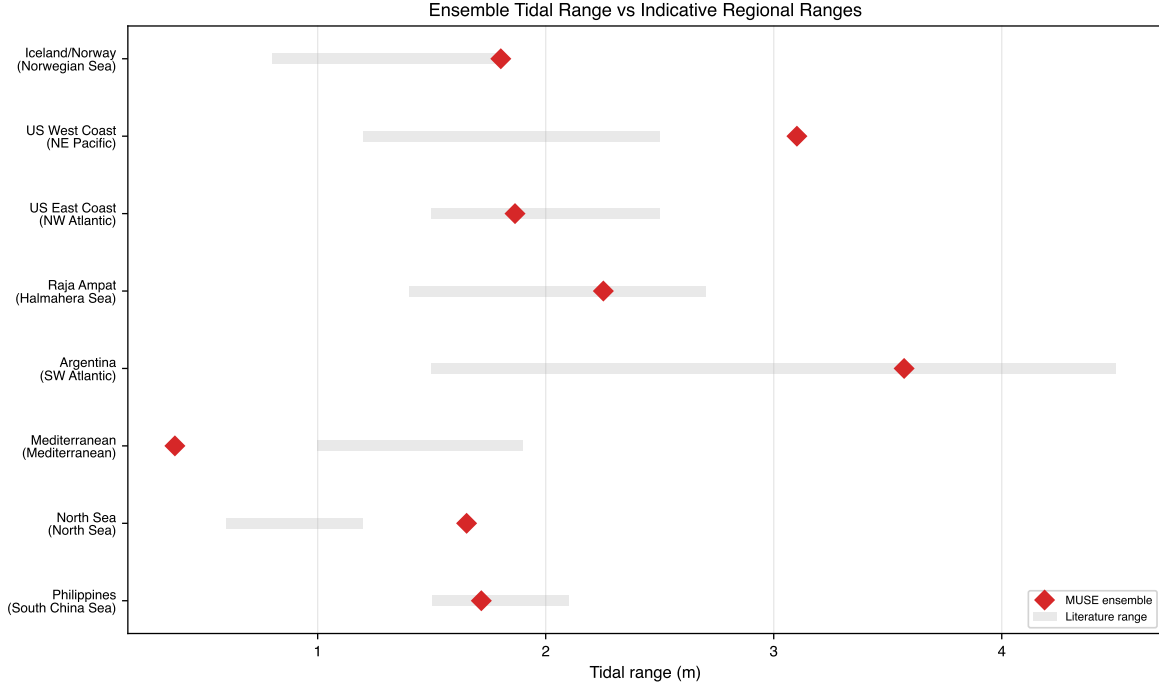


Figure S2: Ensemble tidal range vs indicative regional ranges (author-compiled coarse regional summaries, not site-specific values; station ranges are given in Table 1 of the main text).

S9 Site descriptions (extended)

The protocol was applied at eight sites across four continents (Section 2 of the main text and Table 1). Each site was selected to represent a distinct tidal regime, basin geometry, and data-availability scenario.

The Philippines target (centroid 12.0°N, 121.0°E) spans the South China Sea and Sulu Sea basins with four UHSLC stations (Table 1): Manila (ID 370), Subic Bay (ID 382), Lubang (ID 655), and Davao (ID 372). Puerto Princesa (UHSLC ID 380) was initially planned as a fifth station but its ERDDAP record ends in 2014 and was excluded. The tidal regime is mixed mainly diurnal with ranges of 1.5 to 2.1 m. The basin geometry is complex because Davao lies in the Davao Gulf where M_2 dominates with an amplitude of 0.57 m, while the remaining stations are K_1 -dominant with amplitudes of 0.28 to 0.31 m in the South China Sea. The Sulu Sea connection via the Mindoro and Balabac Straits creates inter-station tidal heterogeneity that tests the IDW interpolation under non-uniform tidal regimes over the 2023 target year (Table 1). The South China Sea and Sulu Sea basins exceed 200 m depth. The Philippine target lies in a semi-enclosed basin (Section 3.3 of the main text).

Raja Ampat (centroid 0.5° S, 130.5° E) lies in the Halmahera Sea, Seram Sea, and Arafura Sea region of eastern Indonesia. Two UHSLC stations are used in the IDW ensemble: Ambon (ID 133) and Saumlaki (ID 420), whose complete hourly coverage is 2018 rather than 2023. Major semidiurnal and diurnal constituents are effectively invariant over this timescale, so the range and MHHW comparisons are unaffected. The anomaly metrics reflect the neutral ENSO conditions of 2018 rather than the warm phase of 2023. Bitung (ID 33) serves as a *proxy reference* for protocol validation, approximately 630 km from the Raja Ampat target centroid

on the opposite side of the Halmahera Sea from the Seram and Arafura stations. The comparison therefore tests consistency with a distant reference rather than local accuracy, and Bitung is *excluded* from the IDW ensemble. The tidal regime is mixed mainly semidiurnal with ranges of 2.3 to 2.7 m.

The North Sea target (centroid 55.0° N, 7.0° E) uses three UHSLC stations: Cuxhaven (ID 825), Smögen (ID 818), and Göteborg (ID 819). The three stations span the southeastern North Sea from the German Bight to the Skagerrak. Lerwick (ID 293) was considered but excluded because its data ends in 2016. The tidal regime is semidiurnal with strong spatial gradients. Cuxhaven in the German Bight is mesotidal with a range near 3.8 m from tidal amplification in the shallowing southern North Sea, while the Swedish stations are microtidal with ranges of 0.2 to 0.3 m. The North Sea has a well-documented amphidromic system with complex M_2/S_2 interference patterns, analyzed over the 2021 target year (Table 1). The southeastern North Sea is a shallow shelf environment of 30 to 50 m depth.

The Mediterranean target (centroid 41.0° N, 5.0° E) uses three UHSLC stations: Trieste (ID 829), Marseille (ID 824), and Gibraltar (ID 289). The tidal regime is mixed mainly semidiurnal microtidal with ranges of 0.3 to 1.3 m. The three stations span the western Mediterranean from the Adriatic to the Strait of Gibraltar, introducing strong spatial heterogeneity. Gibraltar (ID 289) has only 47 days of valid hourly data in November and December, so the number of resolvable constituents is limited. Those whose separation from a stronger neighbor requires roughly 183 days of record are not recovered, namely K_2 against S_2 , P_1 against K_1 , and M_f against MS_f , while the remaining constituents have wider confidence intervals. The statistical robustness of MHHW and anomaly metrics is correspondingly reduced at this site over the 2023 target year (Table 1). The western Mediterranean is a deep basin exceeding 200 m in its central domain.

Argentina (centroid 44.0° S, 63.0° W) spans the Patagonian continental shelf in the southwestern Atlantic, a high-latitude temperate regime with strong latitudinal tidal gradients. Four UHSLC stations are available: Mar del Plata (ID 729), Puerto Madryn (ID 731), Puerto Deseado (ID 286), and Ushuaia (ID 600). The tidal regime transitions from macrotidal on the central Patagonian shelf at Puerto Madryn with a range near 5.1 m and an M_2 amplitude of 1.8 m. It reaches microtidal at both extremes with 1.9 m at Ushuaia in the south where M_2 is 0.6 m and 1.5 m at Mar del Plata in the north where M_2 is 0.4 m. This latitudinal gradient tests the IDW interpolation under non-uniform tidal conditions across a 17° latitude span with fundamentally different tidal regimes over the 2023 target year (Table 1). The Patagonian continental shelf is a broad shallow shelf of 50 to 100 m depth.

To test generality beyond the pilot sites, three further sites were added: Iceland/Norway, the US East Coast, and the US West Coast. Their station sets and target years were fixed before analysis. We describe them as an *additional development set* rather than a strictly out-of-sample test, because some protocol parameters were refined during preparation of the manuscript after these sites had already been inspected.

Iceland/Norway (centroid 63.0° N, 8.0° E) lies in the Norwegian Sea, a subarctic basin. Two UHSLC stations are available: Måløy (ID 802) and Andenes (ID 800), both on the Norwegian continental shelf. The tidal regime is semidiurnal with ranges of 1.8 to 2.0 m. This site tests protocol generality in a high-latitude, open-ocean setting with limited station coverage over the 2021 target year (Table 1). The Norwegian Sea exceeds 200 m depth along the continental shelf break.

US East Coast (centroid 37.0° N, 75.0° W) lies on the Mid-Atlantic Bight, a broad continental shelf with a mixed mainly semidiurnal regime. Three UHSLC stations were selected: Portland ME (ID 252), Atlantic City (ID 264), and Charleston (ID 261). The three stations span 11° of latitude along the shelf, analyzed over the 2021 target year (Table 1). The Mid-Atlantic Bight is a continental shelf environment of 50 to 100 m depth.

US West Coast (centroid 42.0° N, 124.5° W) lies on the Pacific Northwest shelf with a mixed mainly semidiurnal macrotidal regime. Four UHSLC stations are available: San Francisco (ID 551), Crescent City (ID 556), Neah Bay (ID 558), and Willapa Bay (ID 564). Tidal ranges span 2.6 to 3.9 m across the four stations, testing the IDW interpolation under non-uniform macrotidal conditions over the 2021 target year (Table 1). The Pacific Northwest shelf is a narrow, steep continental shelf with depths exceeding 100 m within 20 km of the coast.

Together the five pilot and three additional sites span four continents, tidal regimes from microtidal to macrotidal including mixed mainly diurnal, mixed mainly semidiurnal, and semidiurnal, and station counts of two to four per site, defining the tested operating envelope within which MUSE is validated. For consistency checking, each site uses the nearest PSMSL monthly MSL record:

PSMSL station	Site	ID	Distance (km)
Puerto Princesa	Philippines	207	350
Puerto Princesa	Raja Ampat	207	1,700
North Shields	North Sea	95	540
Trieste	Mediterranean	154	880
Bergen	Iceland/Norway	58	320
Bergen Point	US East Coast	1637	410
San Francisco	US West Coast	10	500

Distances in the table are to the target centroid, and Puerto Princesa is the closest available long-term record in the Indonesian seas, so it serves both the Philippines and Raja Ampat. No Indonesian long-term RLR record exists within 1000 km of the Raja Ampat centroid. No PSMSL record at the Argentina site overlaps the 2023 target year, so none is used and no consistency correlation is computed there (Table S12).

S10 Extended validation methods

S10.1 Testing rigor

A single validation metric is insufficient to establish the credibility of a virtual tide gauge protocol at ungauged sites so we apply seven complementary tests. The first measures primary accuracy through leave-one-station-out cross-validation, the second compares the ensemble against simpler baselines, and the third applies the Diebold–Mariano (DM) test for statistical significance. The remaining four assess parameter sensitivity, the effect of station count through the n-confound subsample analysis (n-CSA), the sampling variability of the error through bootstrap confidence intervals, and robustness to realistic failure modes through adversarial tests. These seven methods, summarized in Table S10, collectively quantify the protocol’s operating envelope.

S10.1.1 LOO-CV

LOO-CV is the primary validation metric because it directly tests the protocol’s core function of predicting tidal sub-scores at a location where no station was used in the interpolation. At each site every UHSLC station is held out in turn. The virtual gauge is reconstructed from the remaining stations together with EOT20 that constrains the IDW interpolation, and the predicted sub-scores are compared with the held-out station’s harmonic analysis. The RMSE across all held-out folds quantifies the protocol’s predictive error at ungauged locations within the site’s station network. The number of folds equals the number of stations, from two to four, so at two-station sites the RMSE rests on two comparisons. The n-CSA and bootstrap CI explicitly assess that limitation. Acceptance is determined by four a priori thresholds (Section 3.5 of the main text). Of the three per-source terms that derive them (Table S11) σ_{UHSLC} is data-derived from the UTide Monte Carlo interval for tidal range and MHHW and takes conservative fixed values for the two anomaly sub-scores, while σ_{EOT20} is chosen conservatively from the coastal accuracy pattern of Hart-Davis et al. for tidal range and MHHW. The binding term for the tidal-range threshold, $\sigma_{\text{IDW}} = 0.20$ m, is an author estimate (Section S10.1.1 below), so the thresholds are independent of the LOO-CV results they evaluate but not free of author judgment, and Table S3 quantifies the resulting decision sensitivity. The anomaly-mean threshold checks for a systematic offset in the harmonic fit rather than discriminating between sites.

The per-source uncertainties used in the main-text uncertainty equation to derive these thresholds are as follows. σ_{UHSLC} is the UTide Monte Carlo 95% confidence half-width for the M_2 amplitude, 0.0005 to 0.009 m across the 25 station-years, treated as a conservative 1σ term in the quadrature (Section 4.1 of the main text), and it propagates to 0.003 to 0.027 m for tidal range as twice the quadrature of the four dominant-constituent amplitude intervals ($R = 2 \sum A_f$; the largest single-station interval is 0.009 m) and to 0.001 to 0.014 m for the anomaly residual standard deviation (the same quadrature). For the two anomaly thresholds, Table S11 keeps the more conservative fixed values 0.03 m for the mean and 0.025 m for the standard deviation, set a priori in the validation protocol, so the anomaly gates are not limited by the harmonic half-width. MHHW uses the same value as tidal range, 0.02 m, because both derive from the same reconstructed tidal series. σ_{IDW} is an author estimate based on expected interpolation error for deterministic IDW over coastal distances of 100 to 1500 km. For a tidal range of 1 to 2 m we use $\sigma = 0.20$ m, that is 10 to 20% of that range, because no

published value for coastal IDW interpolation error was identified. Because the pass/fail structure depends on this estimate, Table S3 reports its sensitivity, where a value of 0.15 m tightens the threshold to 0.43 m and changes the set of passing sites. No separate σ_{IDW} term is used for MHHW. That term derives from the same reconstructed tidal series as tidal range where the IDW term already bounds the interpolation error, although the peak-detection non-linearity means the actual MHHW error may differ. σ_{EOT20} is an author estimate informed by the regional accuracy pattern of Hart-Davis et al., in which tide-gauge errors grow from the open ocean to the coast. We use conservative values of 0.04 m in the deep ocean, 0.08 m on the shelf, and 0.15 m at the coast. Coastal sites dominate the validation set, so tidal range uses 0.15 m while phase-based MHHW errors are lower at 0.10 m. EOT20 does not contribute to anomaly sub-scores.

The thresholds in the “ 2σ threshold” column of Table S11 represent the 95% confidence bound on the ensemble *mean* prediction. The true tidal value at the target site is expected to lie within $\pm$ threshold of the ensemble estimate at 95% confidence assuming the per-source uncertainties are uncorrelated. This is the primary validation criterion and the quantity reported as “pass/fail” in Table 2 of the main text. The single-fold bias thresholds used in the independent Bitung validation (Table S15) are of the same order as the corresponding 1σ bounds of 0.25, 0.10, 0.04, and 0.025 m. The MHHW threshold of 0.15 m is looser than its 1σ bound of 0.10 m.

S10.1.2 Naive baseline comparisons

Four naive baselines are compared against the ensemble to determine whether the multi-source combination provides any measurable improvement over simpler approaches that a practitioner might reasonably adopt. The EOT20-only baseline computes tidal range and MHHW directly from the global tide model extracted at the held-out station’s coordinates for each LOO-CV fold, with no UHSLC data and no IDW interpolation. MUSE’s own EOT20 component is instead extracted at the target so the two evaluations differ in extraction location and both are reported at the held-out stations. It represents the accuracy available from a freely available global product at a coastal location and establishes the benchmark that any multi-source ensemble must meaningfully improve upon to justify its additional complexity. Because EOT20 is a tide model that captures only astronomical forcing, its anomaly mean and anomaly standard deviation are zero by construction, providing no information about weather-band non-tidal variability.

The single-nearest-gauge baseline uses the closest UHSLC station’s values directly without any spatial interpolation or model augmentation. It serves as a lower bound on the expected interpolation quality. If the ensemble cannot outperform a single nearby gauge the spatial interpolation provides no added value. This baseline is particularly informative at sites where station density is low (two or three stations) and the nearest station may already be representative of the target’s tidal regime.

The unweighted mean baseline averages all UHSLC stations equally, removing any distance weighting, basin friction, or tidal-range penalty. It tests whether the IDW framework’s complexity (geographic distance, basin classification, and constituent similarity) improves predictions over a simple arithmetic mean.

The IDW-only baseline weights UHSLC stations by inverse distance alone, without the EOT20 prior or PSMSL consistency checking. This baseline isolates the contribution of the global tide model to the ensemble. Differences between IDW-only and the full ensemble reveal whether EOT20 adds skill, adds noise, or has negligible effect at a given site.

Each baseline is evaluated using the same LOO-CV procedure as the full ensemble, producing per-method RMSE values and DM test statistics that quantify whether the ensemble’s additional complexity yields measurably different accuracy. Significance testing is secondary given the DM test’s limited power at small fold counts.

S10.1.3 Diebold–Mariano test (exploratory)

The DM test with the Harvey–Leybourne–Newbold small-sample correction is applied as an *exploratory* assessment of whether the ensemble and each baseline have equal forecast accuracy. The test is exploratory rather than confirmatory because the per-fold loss function is the mean squared error across all four sub-scores. That pooled loss is dominated by the tidal-range sub-score whose errors are two orders of magnitude larger than the anomaly terms, so the same test is additionally run per sub-score and the per-field statistics accompany the pooled test in the analysis outputs. A significant result indicates that the two methods have measurably

different overall accuracy, but does not identify which sub-score drives the difference. The primary comparison is therefore the raw RMSE across baselines (Section 4.2). The test statistic is given by Eq. 2:

$$DM = \frac{\bar{d}}{\sqrt{\hat{V}(\bar{d})}}, \quad d_j = e_{\text{ens},j}^2 - e_{\text{base},j}^2 \quad (2)$$

where $e_{\text{ens},j}$ and $e_{\text{base},j}$ are the LOO-CV errors for the ensemble and baseline on fold j , and $\hat{V}(\bar{d})$ is the sample variance given by $\hat{V}(\bar{d}) = \frac{1}{n} \frac{1}{n-1} \sum_{j=1}^n (d_j - \bar{d})^2$.

Under H_0 (equal accuracy), the Harvey–Leybourne–Newbold (1997) small-sample correction is applied as $DM^* = DM \times \sqrt{(n-1)/n}$ and compared to a t distribution with $n-1$ degrees of freedom. Rejection at $p < 0.05$ indicates a statistically significant difference in predictive accuracy.

Given the small fold counts of two to four, the practical utility of the DM test depends on its statistical power. A Monte Carlo power simulation of 10,000 replicates with independent errors shows low power to detect a true RMSE difference of 0.14 m. Power is 0% at two folds. At three to five folds it depends on the assumed field-variance structure, reaching 6 to 16% under equal variance and 10 to 29% under the field-specific structure that matches the unequal sub-score errors observed here. It reaches about 50% only at seven to nine folds. Because independent errors maximize test power at a given variance, positively correlated fold losses would lower power further. The reported figures are upper bounds. A non-significant result at $p > 0.05$ therefore does not indicate equal accuracy between the ensemble and the baseline. It indicates only that the test lacks the sample size to discriminate. The single significant result (Argentina, EOT20-only better, $p = 0.006$) matches the direction of the raw RMSE comparison and should not be read as either a false positive or an ensemble advantage.

S10.1.4 Sensitivity analysis

The ensemble’s sensitivity to the three IDW parameters is tested to ensure that the reported results are not artifacts of a particular parameter choice. The default IDW power parameter is $\kappa = 1.0$. This is the standard choice for spatial interpolation of geophysical fields and reflects the expected decay of tidal coherence with distance at coastal scales of 100 to 1500 km. Sensitivity is tested across $\kappa = 1.0, 1.5, 2.0, 3.0$. Higher values give exponentially more weight to the nearest station, so the test asks whether a nearest-neighbor regime at $\kappa \geq 2.0$ differs materially from the smoother interpolation at $\kappa = 1.0$.

The basin-friction factor f_p is varied by multiplying the site default by 0.8, 1.0, 1.2, and 1.5, testing the sensitivity to the friction classification. At sites where the default is $f_p = 1.0$ the multiplier 0.8 represents a friction-free scenario and 1.5 represents strong frictional damping. Where the default is $f_p = 1.2$, as in the South China Sea and the Northwest Atlantic, the same multipliers probe propagation from open-ocean-like at $0.8 \times 1.2 = 0.96$ to strongly damped at $1.5 \times 1.2 = 1.80$.

The same-basin bonus multiplier β weights stations in the same tidal basin as the target more heavily and is tested across $\beta = 1, 1.5, 2, 3$. The default $\beta = 1$, disabled, was chosen after sensitivity analysis showed a negligible effect below 1.1% CV at all sites.

The pass criterion for each parameter group is a coefficient of variation below 15% across the tested values for the tidal range, MHHW, and anomaly std sub-scores. The anomaly mean sub-score is excluded from the pass gate because its value is near zero by construction as a detided residual and a CV is mathematically unstable for near-zero fields. Anomaly mean sensitivity is assessed as an absolute bias in millimeters rather than as a CV, and the bias remains below 0.001 m at all eight sites across every tested IDW power and basin-friction value, with the values available from the corresponding author on request. Station exclusion uses a separate criterion that tests the ensemble’s dependence on any single station, requiring a maximum percentage change below 20% when one station is removed.

S10.1.5 n-confound subsample analysis (n-CSA)

The apparent n -dependence could in principle arise from a sample-size artifact of LOO-CV. At two stations the RMSE rests on two fold comparisons and may be less reliable than at three or more, regardless of the true accuracy of the ensemble. To decouple the n -dependence from this statistical artifact, we designed a subsample analysis that artificially reduces the station count at sites with three or more stations and

re-evaluates LOO-CV on each reduced set. If the RMSE degrades gracefully as stations are removed and the upper end of the interquartile range (IQR) remains below 0.50 m for all reduced counts, then the n -dependence is a real accuracy signal rather than a statistical artifact. Conversely, if the RMSE jumps abruptly at three stations, the operating envelope is sharp and that count is empirically critical.

The analysis was applied at the six sites with at least three stations. The Philippines, Argentina, and the US West Coast have four stations and yield subsamples of two, three, and four, while the North Sea, the Mediterranean, and the US East Coast have three stations and yield subsamples of two and three. The two-station sites (Raja Ampat, Iceland/Norway) offer no reduced sets and contribute only their full-set value. The main text uses the Philippines and the North Sea as the illustrative cases because they span contrasting regimes, a well-constrained Philippine basin against a spatially heterogeneous North Sea. For each target station count n' we drew 100 random subsamples of n' stations without replacement within a draw. When n' is small the number of unique combinations $\binom{n}{n'}$ falls below 100, so the 100 draws over-sample the available combinations. From four stations at $n' = 2$ there are $\binom{4}{2} = 6$ unique combinations and each is sampled about 17 times on average. We ran LOO-CV on each subsample and report the median, 25th to 75th percentile (IQR), and 5th to 95th percentile of the resulting RMSE distribution (Table S14).

S10.1.6 Bootstrap confidence intervals for LOO-CV RMSE

The LOO-CV RMSE at each site is a single point estimate based on two to four fold comparisons. To quantify its sampling variability we apply a bootstrap to the per-fold LOO-CV error vectors. The per-fold errors are resampled with replacement in 1000 replicates, the RMSE is recomputed for each resampled set, and the 2.5th and 97.5th percentiles of the resulting RMSE distribution form the 95% confidence interval. This CI quantifies how much the RMSE estimate would vary if different stations were held out. Quantifying the uncertainty of the ensemble prediction at the target site would instead require a substantially more expensive hourly-data bootstrap that re-runs the harmonic analysis on each resample.

The CI width provides a diagnostic of site reliability. A narrow CI below 0.15 m half-width indicates that the per-site RMSE is stable across different held-out combinations and can be interpreted with confidence. A wide CI at or above 0.15 m half-width indicates that the result is sensitive to which stations are included and should be interpreted cautiously. At the two-station sites (Raja Ampat, Iceland/Norway) the CI rests on two per-fold error values that fundamentally limit its resolution regardless of the number of bootstrap replicates. As the Raja Ampat and Iceland/Norway intervals illustrate, the bootstrap CI width is driven by station heterogeneity within each site rather than by the number of bootstrap replicates.

S10.1.7 Adversarial robustness tests

Adversarial tests assess whether the protocol degrades gracefully under realistic data-quality failures rather than producing catastrophically wrong estimates. Five categories of adversarial tests are applied, covering six scenarios in the acceptance gate.

Noise injection adds Gaussian noise ($\sigma = 0.1$ m and 0.5 m) to all station hourly records before harmonic analysis. The ensemble is recomputed and the maximum ratio of noisy-to-clean sub-score is assessed. The pass criterion is a ratio < 2.0 for non-zero fields and absolute bias < 1 mm for near-zero fields (anomaly mean). The $\sigma = 0.5$ m scenario is physically unrealistic for modern tide gauge instruments and fails at all sites. This scenario is retained to document the failure mode rather than to discriminate protocol performance. Only the $\sigma = 0.1$ m scenario is included in the acceptance gate.

Data gaps randomly remove 10%, 25%, and 50% of hourly records from each station before harmonic analysis. The pass criterion is a maximum bias < 0.10 m (10% gaps) or < 0.15 m (25% gaps). A gap scenario also fails if more than half of a site's stations can no longer be harmonically analyzed. The 50% gap scenario is excluded from the acceptance gate because it represents a failure mode of more than half the year missing, a failure that would preclude any meaningful tidal analysis regardless of the protocol. Random removal is a conservative test of gap tolerance. Contiguous gaps (instrument outages) of the same total duration would impose greater statistical leverage on the harmonic fit and are considered a more severe scenario not tested here.

Station dropout removes each UHSLC station in turn and recomputes the ensemble from the remaining stations. The pass criterion is a maximum bias < 0.15 m across all sub-scores when any single station is

removed. This test evaluates the ensemble’s reliance on individual stations. At sites with limited redundancy (two or three stations), removing one station substantially reduces the IDW network and may degrade accuracy.

Wrong-basin exclusion introduces a synthetic station 5° from the target with a tidal range of 5.0 m and an MHHW of 2.0 m, labeled “wrong_basin”. The pass criterion is that this station receives an IDW weight below 0.3, with observed values of 0.045 to 0.268 across the eight sites. Because the same-basin bonus is disabled with $\beta = 1.0$, cross-basin stations are not excluded by design. The weight is set by the distance kernel, so the distant synthetic station is down-weighted.

Spike detection inserts a +2 m spike at the midpoint of the QC-surviving hours of one station’s hourly record. The spike therefore cannot land on a missing hour or on an hour that the quality control already rejects. Quality control runs in two stages. Stage 1 rejects a point whose deviation from the midpoint of its two hourly neighbors exceeds 1.0 m and is flagged only at the local deviation maximum so neighboring valid hours are preserved. Stage 2 applies the original rate check of greater than 2.0 m/h to the remaining series. The pass criterion is that stage 1 sets the spike hour itself to NaN. The two-stage detector removes the spike at all eight sites independent of the local tidal slope at the injection hour. A rate check scaled to the local tidal rate rather than the fixed threshold remains planned for future versions.

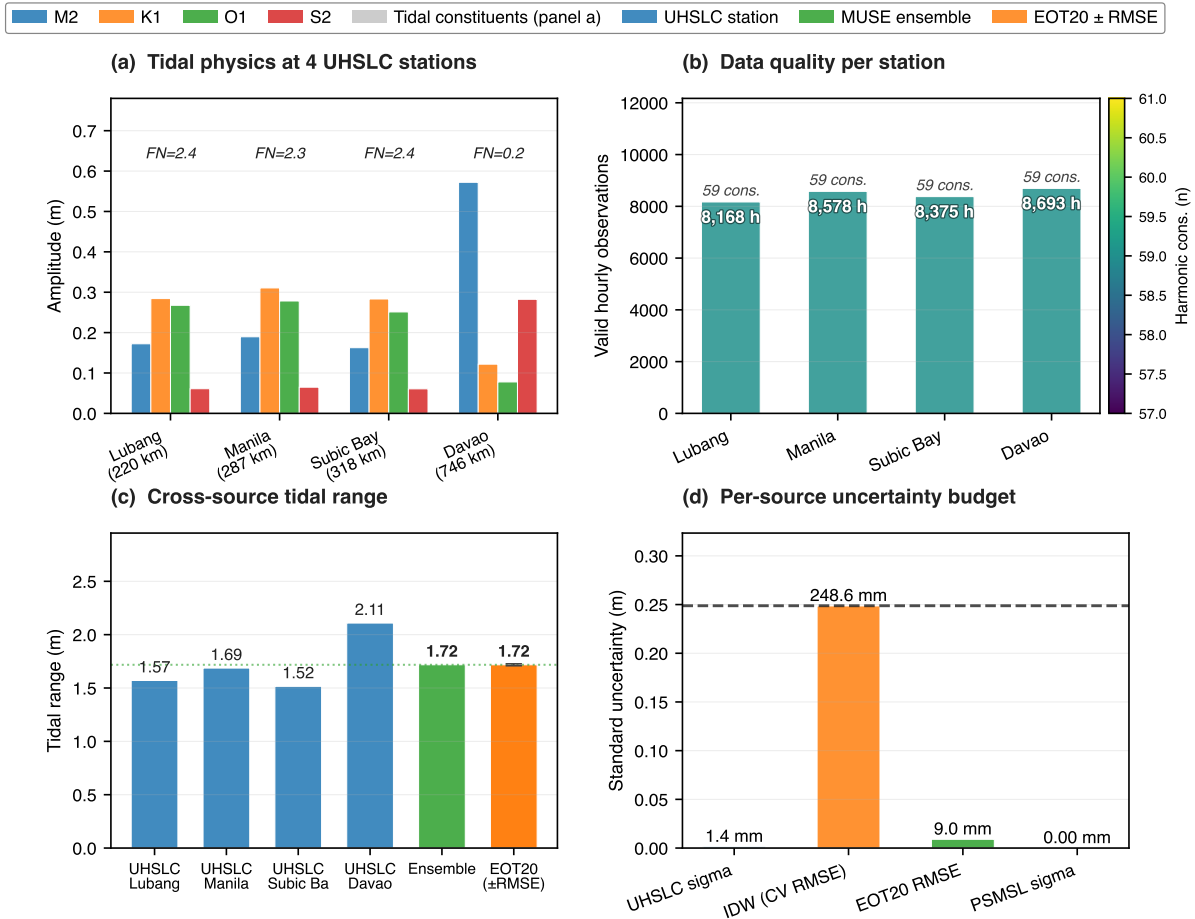


Figure S3: Philippines site output: (a) constituent amplitudes, (b) data quality, (c) cross-source range, (d) uncertainty budget.

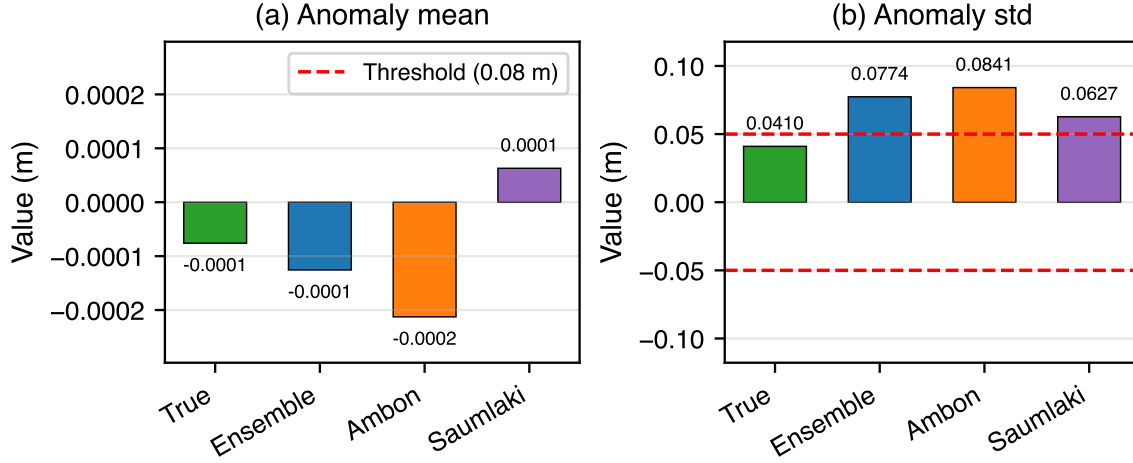


Figure S4: Raja Ampat anomaly validation: (a) mean and (b) std for Bitung reference, MUSE, and stations.

Table S7: EOT20 cluster-level accuracy: RMS of UHSLC–EOT20 amplitude biases across M2, S2, K1, O1.

Site	n	Total RMSE (m)	Mean bias (m)	M2 RMSE (m)	EOT20 weight
Philippines	4	0.015	−0.001	0.009	0.25
Raja Ampat	2	0.026	+0.006	0.041	0.25
North Sea	3	0.052	+0.026	0.059	0.25
Mediterranean	3	0.008	+0.000	0.005	0.25
Argentina	4	0.033	−0.014	0.040	0.25
Iceland/Norway	2	0.008	−0.005	0.011	0.25
US East Coast	3	0.018	−0.009	0.033	0.25
US West Coast	4	0.063	+0.011	0.118	0.25

Table S8: Site-level f_p classification of the eight sites (a priori categorical rather than depth-derived with depth shown for context and sensitivity in Table S2).

Site	Basin	h (m)	f_p	Source
Philippines	South China Sea	>200	1.2	This study
Raja Ampat	Halmahera/Seram/Arafura	>200	1.0	This study
North Sea	North Sea	30–50	1.0	This study
Mediterranean	Mediterranean	>200	1.0	This study
Argentina	South Atlantic	50–100	1.0	This study
Iceland/Norway	Norwegian Sea	>200	1.0	This study
US East Coast	Northwest Atlantic	50–100	1.2	This study
US West Coast	Northeast Pacific	>200	1.0	This study

Table S9: Per-source ensemble weights; normalized EOT20 share: 20% (range, MHHW) and 0% (anomaly).

Source i	Range R	MHHW	Anomaly $\bar{a}$	Anomaly σ_a
UHSLC (IDW)	1	1	1	1
EOT20	0.25	0.25	0	0

Table S10: Summary of testing methods and acceptance criteria.

Method	Purpose	Acceptance criterion
LOO-CV	Primary validation	$\text{RMSE} \leq 0.50 \text{ m}$ (tidal range)
Baselines	Compare vs. simple methods	Raw RMSE comparison
DM test	Statistical significance	$p < 0.05$ (exploratory)
Sensitivity	Parameter robustness	$\text{CV} < 15\%$ (parameters); $< 20\%$ (station exclusion)
n-CSA	n -dependence check	Graceful degradation
Bootstrap CI	RMSE uncertainty	CI width reported
Adversarial	Failure mode testing	6 sub-tests pass/fail

Table S11: Threshold derivation from per-source uncertainties.[†]

Sub-score	σ_{UHSLC}	σ_{IDW}	σ_{EOT20}	Quadrature σ	2σ threshold
Tidal range	0.02	0.20	0.15	0.251	0.50
MHHW	0.02	n/a	0.10	0.102	0.20
Anomaly mean	0.03	0.02	n/a	0.036	0.08
Anomaly std	0.025	n/a	n/a	0.025	0.05

[†] The anomaly-mean a priori threshold is rounded up to 0.08 m for conservatism ($2\sigma = 0.072 \text{ m}$). The anomaly rows keep their per-source values fixed a priori in the validation protocol ($\sigma_{\text{UHSLC}} = 0.03$ and 0.025 m ; $\sigma_{\text{IDW}} = 0.02 \text{ m}$ for the mean), which exceed the harmonic half-width.

Table S12: Pearson correlation between PSMSL monthly MSL anomaly and UHSLC non-tidal residual.

Site	n	r	p
Philippines	12	−0.55	0.062
Raja Ampat	5	−0.70	0.188
North Sea	7	−0.80	0.029
Mediterranean	12	+0.80	0.002
Argentina	0	n/a	n/a
US East Coast	12	+0.82	0.001
US West Coast	12	−0.24	0.450
Iceland/Norway	12	+0.67	0.018

Table S13: Anomaly standard deviation: LOO-CV RMSE of MUSE against the zero-anomaly (EOT20) reference error.[†]

Site	n	MUSE RMSE (m)	EOT20 [†] (m)	Reduction (%)
Philippines	4	0.020	0.068	70
Raja Ampat	2	0.021	0.074	71
North Sea	3	0.095	0.226	58
Mediterranean	3	0.046	0.109	58
Argentina	4	0.064	0.218	70
US East Coast	3	0.039	0.140	72
US West Coast	4	0.057	0.149	62
Iceland/Norway	2	0.023	0.155	85

[†] The EOT20 column is the RMS of the held-out stations' anomaly standard deviations. EOT20 predicts zero anomaly by construction, so this column is the reference error of a zero-anomaly prediction.

Table S14: Subsample analysis: median LOO-CV RMSE (IQR) for tidal range vs effective station count.

Site	Full- n RMSE		$n' = 2$	$n' = 3$		$n' = 4$	IQR at $n' = 2$
Philippines	0.249	0.278	[0.124, 0.486]	0.262	[0.135, 0.293]	0.249 [0.249, 0.249]	0.362
Raja Ampat	0.353	0.353	[0.353, 0.353]	n/a		n/a	0.000
Argentina	2.155	2.820	[0.560, 2.921]	2.208	[2.114, 2.325]	2.155 [2.155, 2.155]	2.360
North Sea	1.975	3.151	[0.211, 3.247]	1.975	[1.975, 1.975] [†]	n/a	3.036
Mediterranean	0.730	0.542	[0.406, 0.542]	0.730	[0.730, 0.730]	n/a	0.137
US East Coast	1.208	1.406	[0.358, 1.406]	1.208	[1.208, 1.208]	n/a	1.048
US West Coast	0.442	0.494	[0.371, 0.756]	0.485	[0.425, 0.620]	0.442 [0.442, 0.442]	0.386
Iceland/Norway	0.212	0.212	[0.212, 0.212]	n/a		n/a	0.000

[†] $n' = 3$ equals the full station set (single combination), so no IQR is available.

Table S15: Protocol validation at Raja Ampat against the Bitung proxy reference (ID 33; 630 km from the target centroid, opposite side of the Halmahera Sea; excluded from the ensemble).

Sub-score	True	Virtual	Bias	Threshold	Pass
Tidal range (m)	1.836	2.253	0.417	0.20	Fail
MHHW (m)	0.515	0.623	0.108	0.15	Pass
Anomaly mean (m)	0.000	0.000	0.000	0.05	Pass
Anomaly std (m)	0.041	0.077	0.036	0.03	Fail

Table S16: Comparison with prior virtual tide gauge and tidal-interpolation studies.

Study	Sources	Purpose	Uncertainty	Continents	Index valid.?
Hawkins 2019	3	RSL trend	Trend $\pm\sigma$	Global	No
Abd Rahman 2024a	2	Chart datum	None	SE Asia	No
Abd Rahman 2024b	2	Maritime-boundary datum	None	SE Asia	No
Turner 2010; Hess 2002	1	Tidal levels / datum	None	Regional	No
This study	2	Index proxy	Per-value	4	Yes

Abd Rahman 2024a is the engineering-journal chart-datum study and Abd Rahman 2024b is the International Journal of Remote Sensing datum study. The source count for this study covers the two estimation sources (UHSLC and EOT20) and PSMSL enters as a consistency reference only (Section 3.4 of the main text). Index valid.? marks whether the study reports a per-value uncertainty that a composite coastal index could propagate. The interpolation studies Turner 2010 and Hess 2002 provide tidal levels without anomaly metrics or per-source uncertainties.

Table S17: LOO-CV baseline comparison at the three additional sites (RMSE, m). Bold = MUSE ensemble.

Site	Method	Tidal range	MHHW	Anomaly mean	Anomaly std
Iceland/Norway	MUSE ensemble	0.212	0.092	0.000	0.023
	IDW only	0.234	0.102	0.000	0.023
	EOT20 only	0.054	0.009	0.000	0.155
	Single nearest	0.234	0.102	0.000	0.023
	Unweighted mean	0.234	0.102	0.000	0.023
US East Coast	MUSE ensemble	1.208	0.501	0.000	0.039
	IDW only	1.306	0.544	0.000	0.037
	EOT20 only	0.072	0.011	0.000	0.140
	Single nearest	1.569	0.652	0.000	0.045
	Unweighted mean	1.234	0.514	0.000	0.032
US West Coast	MUSE ensemble	0.442	0.152	0.000	0.057
	IDW only	0.455	0.160	0.000	0.059
	EOT20 only	0.222	0.107	0.000	0.149
	Single nearest	0.339	0.138	0.000	0.044
	Unweighted mean	0.627	0.198	0.000	0.077

Table S18: Station-exclusion sensitivity: maximum percentage change in each sub-score when any single station is removed from the ensemble. Pass requires all sub-score changes < 20%.

Site	Tidal range	MHHW	Anom. μ	Anom. σ	Pass (<20%)
Philippines	2.4%	2.1%	0.6%	10.4%	Yes
Raja Ampat	7.7%	13.8%	18.9%	18.9%	Yes
North Sea	72.5%	71.0%	20.7%	30.4%	No
Mediterranean	146.0%	143.2%	6.1%	13.6%	No
Argentina	21.4%	22.4%	2.3%	9.7%	No
US East Coast	25.5%	27.7%	15.8%	20.7%	No
US West Coast	5.8%	6.3%	1.7%	32.2%	No
Iceland/Norway	8.5%	10.6%	15.8%	11.4%	Yes

Table S19: Tidal-range definition sensitivity: the manuscript’s four-constituent sum ($R_4 = 2 \sum_{\mathcal{F}_R} A_f$, Eq. 2 of the main text) against the Eq.-1 subset sum ($R_{\mathcal{F}}$, the 11-member set of Eq. 1 of the main text) and the mean diurnal range of the full nodal-corrected reconstruction ($\text{GDR} = \text{MHHW} - \text{MLLW}$). The harmonic fits retain up to 59 auto-selected constituents while $R_{\mathcal{F}}$ is restricted to the Eq.-1 subset for comparability. UHSLC values are station means per site from the cached harmonic fits; EOT20 rows are the target-point extractions.

Site	Source	R_4 (m)	$R_{\mathcal{F}}$ (m)	GDR (m)	$R_{\mathcal{F}} - R_4$ (%)	GDR- R_4 (%)
Philippines	UHSLC	1.72	2.14	1.061	+24.6	-38.5
Raja Ampat	UHSLC	2.51	3.15	1.459	+25.8	-42.0
North Sea	UHSLC	1.45	1.92	1.146	+38.5	-17.8
Mediterranean	UHSLC	0.84	1.05	0.543	+28.0	-35.4
Argentina	UHSLC	3.41	4.59	2.531	+34.7	-27.9
US East Coast	UHSLC	2.51	3.20	1.941	+27.3	-23.6
US West Coast	UHSLC	3.29	4.12	2.081	+25.3	-36.9
Iceland/Norway	UHSLC	1.88	2.34	1.343	+24.4	-28.6
Philippines	EOT20	2.012	2.51	1.102	+24.8	-45.2
Raja Ampat	EOT20	1.470	1.84	0.864	+24.9	-41.2
North Sea	EOT20	1.207	1.49	0.878	+23.8	-27.3
Mediterranean	EOT20	0.268	0.343	0.159	+28.1	-40.6
Argentina	EOT20	2.270	2.81	1.770	+23.7	-22.0
US East Coast	EOT20	1.370	1.72	0.999	+25.8	-27.1
US West Coast	EOT20	3.124	3.87	1.941	+23.9	-37.9
Iceland/Norway	EOT20	1.791	2.22	1.286	+23.9	-28.2

Table S20: Target-level context: EOT20 extracted at the target centroid against the MUSE ensemble estimate at the same target. Both columns are estimates and no in situ truth exists at the ungauged target while the station-level LOO-CV RMSEs of Table 3 of the main text and Table S17 quantify each method’s validated accuracy at station locations.

Site	EOT20@tgt R	MUSE R	$ \Delta R $	EOT20@tgt MHHW	MUSE MHHW	$ \Delta $ MHHW
	(m)	(m)			(m)	
Philippines	2.012	1.718	0.294	0.593	0.543	0.050
Raja Ampat	1.470	2.253	0.782	0.451	0.623	0.172
North Sea	1.207	1.653	0.446	0.433	0.623	0.190
Mediterranean	0.268	0.374	0.106	0.079	0.118	0.039
Argentina	2.270	3.572	1.302	0.883	1.340	0.457
US East Coast	1.370	1.865	0.496	0.525	0.709	0.184
US West Coast	3.124	3.102	0.022	0.919	0.915	0.005
Iceland/Norway	1.791	1.803	0.012	0.644	0.635	0.009