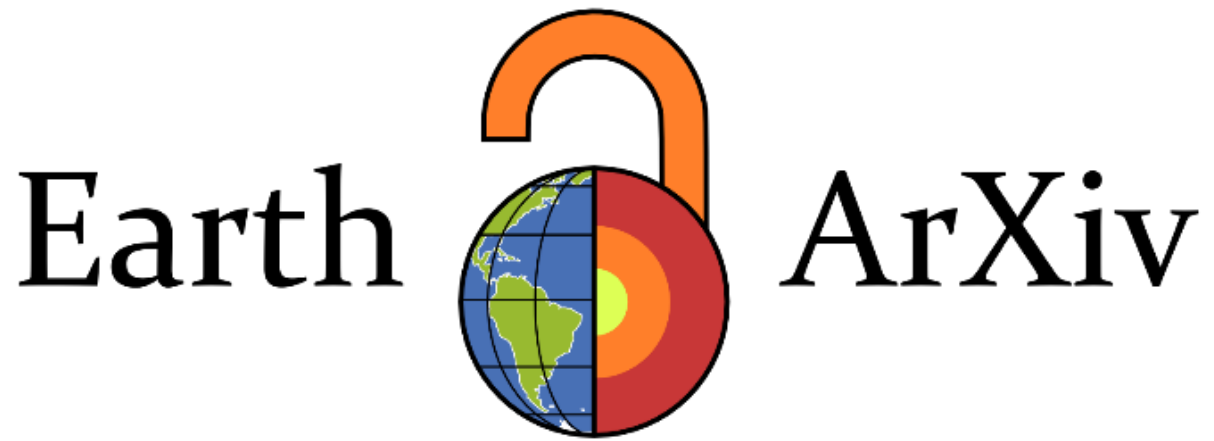




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A scaling law for stagnant-lid convection with composite visco-elasto-plastic rheology: application to Europa's ice shell

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Summary

Scaling laws relating the Nusselt number (Nu) and the Rayleigh number (Ra) are the standard tool for translating numerical models of convection into predictions of surface heat flux and the thicknesses of the stagnant lid and the rheological sublayer. Existing scaling frameworks for icy moons normalise this relationship using a temperature-dependence parameter, γ , calibrated against viscous and incompressible models with composite creep rheology (diffusion, dislocation, grain boundary sliding, and basal slip). These calibrations, however, omit physics included in recent models of Europa's ice shell: visco-elasto-plastic (VEP) deformation and a free surface, needed to capture realistic surface stress and topography, and compressible (anelastic liquid approximation, ALA) flow, required once shear and adiabatic heating become non-negligible. Whether this additional physics alters the Nu - Ra - γ scaling remains untested, even though heat flux and ice shell thickness estimates for Europa rely on that scaling and would need reassessment if this breaks down under more realistic rheology and boundary conditions. Therefore, we investigate whether the additional physics disturbs this scaling, using numerical simulations of convection in Europa's ice shell at 60, 40 and 20 km, chosen to bracket independent estimates of Europa's shell thickness. At each thickness, we ran every combination of four modelling choices: i) rheology (viscoplastic, VP, vs. visco-elasto-plastic, VEP), ii) surface boundary condition (free-slip vs. free-surface), iii) creep mechanism (with or without grain boundary sliding, GBS), and iv) compressibility (i.e., the Boussinesq approximation, BA, vs. the anelastic liquid approximation, ALA). We find that the 20-km cases do not convect within the 100 Myr model runtime. We also note

the standardised form of the rheological-sublayer parameter γ_{rh} matches published expectations only for our diffusion–dislocation cases, and even there departs from the value we measure directly by up to 8.5 (51 per cent). Additionally, we find that γ_{rh} tracks ice shell thickness rather than cleanly measuring temperature-dependent viscosity for our models. Instead, we find that boundary-layer depth normalises the Nu – Ra relationship far better than either form of the γ_{rh} , and we report the dimensional depth of the rheological sublayer d_{rh} , because d_{rh} keeps the interior Rayleigh number, Ra_i , as an independent, statistically significant predictor of the Nusselt number whereas its nondimensional form d_{rh}/d_{shell} does not. Of the four additional physical ingredients, only the presence of GBS creep significantly affects the boundary-layer geometry. Elasticity and the top boundary have no detectable effect, and compressibility only has a small and inconsistently significant one. What does matter is the choice of normalising variable itself: the standardised γ_{rh-std} formulation is only dependent on the creep-mechanism contributions and cannot distinguish our two shell thicknesses, whereas the dimensional depth of the rheological sublayer, d_{rh} , can. More broadly, these results argue for revisiting how composite-rheology convection models of icy-satellite ice shells are normalised.

keywords: Planetary interiors, Numerical modelling, Numerical approximations and analysis, Planetary tectonics, Heat generation and transport, Extraterrestrial

1 Introduction

Numerical models of thermal convection in stagnant-lid ice shells are usually distilled into a compact scaling relationship between the Nusselt number, Nu , i.e., the ratio of the total surface heat flux to the conductive heat flux, and the Rayleigh number, Ra , which measures the vigour of convection relative to diffusive heat loss. For a fluid with strongly temperature-dependent viscosity, such as ice, the simplest relationship $Nu = \alpha Ra^\beta$ (Schubert et al., 2001) is not sufficient on its own, because the same Ra can correspond to different viscosity contrasts across the convecting layer. This effect has been documented since Davaille and Jaupart (1993)’s laboratory experiments on transient high-Rayleigh-number convection with large viscosity contrasts, and addressed by a lineage of stagnant-lid scaling studies of increasing complexity (e.g., Deschamps, 2021; Deschamps and Lin, 2014; Deschamps and Vilella, 2021; Mitri and Showman, 2005; Reese et al., 1998; Solomatov and Moresi, 1997, 2000).

Recent work by Harel et al. (2020) addressed this same viscosity-contrast problem for planetary ice shells by introducing a Frank–Kamenetskii type variable, γ , to capture the temperature-dependence of viscosity, along with two refinements evaluated in the rheological sublayer beneath the stagnant lid: γ_{rh} , measured locally in each model, and γ_{rh-std} , its standardised form predicted from the fractional contribution of diffusion creep. Under the Frank–Kamenetskii approximation, Harel et al. (2020) fit $Nu = 1.90 Ra_i^{1/5} \gamma^{-1}$ for stagnant-lid convection in the stationary regime, close to previous studies (Dumoulin et al., 1999; Moresi and Solomatov, 1995). For a composite viscosity law spanning diffusion, dislocation, grain boundary sliding and basal slip, the authors instead fit $Nu = 0.65 Ra_i^{1/3} (c_g \gamma_{rh})^{-4/3}$,

restricted to time-dependent convection regimes. Similarly, Schulz et al. (2020) tested a broad range of activation enthalpies together with dislocation creep. Their equivalent parameter, θ , varies over only 11.4–12.8 across their suite, too narrow a range to constrain a scaling, and cannot be determined in advance because it depends on the interior temperature and pressure. They instead adopt a purely geometric normalisation based on the mean stagnant-lid thickness, fitting $Nu = 0.37\langle L \rangle^{-0.666} Ra_{har}^{0.071}$ where $\langle L \rangle$ is the mean stagnant-lid thickness non-dimensionalised by the layer depth and Ra_{har} is an effective Rayleigh number based on the harmonic average of the viscosity near the base of the lid. Most recently, Grigné (2023) compared viscosity laws for stagnant-lid convection and showed that the viscous temperature scale used to normalise most existing scalings holds for the Frank–Kamenetskii approximation but fails for a true Arrhenius law at high viscosity contrast. This highlights that the normalising variable can determine whether a scaling law fitted to one set of models can still predict the heat flux of another. We address that question for Europa’s ice shell, where heat flux and shell thickness are routinely estimated from such scalings.

Two threads of prior work bear on Europa’s ice shell specifically. One on which creep mechanisms control its thermal scaling, the other on which rheological and boundary-condition choices control its surface stress. Neither has yet been combined with the other. Harel et al. (2020) showed that including grain boundary sliding (GBS) alongside diffusion and dislocation creep in a composite viscosity law (Behn et al., 2021; Goldsby and Kohlstedt, 2001) changes the boundary-layer geometry that sets Nu , but did so within a viscous and incompressible model with a non-deforming surface. Capturing Europa’s realistic surface deformation instead requires visco-elasto-plastic (VEP) rheology with a free-surface boundary condition, which substantially changes the magnitude and pattern of surface stress relative to viscous, free-slip models (Cramer et al., 2012a; Grima and Becker, 2024; Patočka et al., 2019; Thielmann et al., 2015); this is the approach recently used to model brittle failure and surface downwelling in Europa’s own ice shell (Kihoulou et al., 2025). On the other hand, flow under the anelastic liquid approximation (ALA) is required once shear and adiabatic heating become non-negligible (Leng and Zhong, 2008), irrespective of the surface boundary condition.

Complementary to Harel et al. (2020)’s test suite, Thielmann et al. (2015) varied the surface boundary condition and elasticity while holding the creep mechanism fixed. Over Rayleigh numbers of 10^5 – 10^7 , they found that neither effects the Nusselt number, root-mean-square velocity, or stagnant-lid thickness while surface stress changed by two orders of magnitude. We test whether that conclusion holds when the creep mechanism is composite. To do so we use the finite element code ASPECT (Bangerth et al., 2024; Clevenger and Heister, 2021; Heister et al., 2017; Kronbichler et al., 2012; Rose et al., 2017) to run a suite of convection simulations of Europa’s ice shell varying shell thickness between two end-members drawn from the literature (Nimmo and Manga, 2009; Schenk, 2002; Soderlund et al., 2020), with the 20-km case as our thinnest end-member and the 60-km case as our thickest. We also include a 40-km case, representative of estimates in which the shell contains both a conductive lid and a convecting interior and close to the conductive thickness inferred from Juno microwave radiometry (Levin et al., 2025; Rhoden and Walker, 2027). Across all model

thicknesses, we explore a combination of creep mechanisms (diffusion, diffusion-dislocation, diffusion-GBS-dislocation), rheology (VP, VEP), top boundary condition (free-slip, free-surface), and compressibility (BA, ALA). However, we note that convection does not develop in the 20-km cases within the model run time of 100 Myr, and they are excluded from the analyses below. With 40- and 60-km cases, we test whether the γ_{rh-std} -based normalisation developed by Harel et al. (2020) extends to this broader combination of physical ingredients, compare six candidate normalising variables (γ_{rh} , γ_{rh-std} , d_{rh} , d_{sl} , and d_{rh} and d_{sl} each nondimensionalised by ice shell thickness) against a nested Ra_i -only baseline, and isolate the effect of each physical ingredient within that common framework. A companion study (in preparation) treats the surface-deformation consequences of elasticity, a free surface and compressibility.

2 Methods

2.1 Modelling approach

We model thermal convection in Europa’s ice shell in two dimensions using the finite-element code ASPECT (Bangerth et al., 2024; Clevenger and Heister, 2021; Heister et al., 2017; Kronbichler et al., 2012; Rose et al., 2017). Our reference case is a viscoplastic (VP) rheology under the incompressible Boussinesq approximation (BA), at infinite Prandtl number and with no internal heating, for which we solve conservation of mass (Eq. 1), momentum (Eq. 2) and energy (Eq. 3):

$$\nabla \cdot \mathbf{v} = 0, \quad (1)$$

$$-\nabla \cdot (2\eta\dot{\boldsymbol{\epsilon}}) + \nabla P = \rho\mathbf{g}, \quad (2)$$

$$\rho_0 C_p \frac{DT}{Dt} = \nabla \cdot k \nabla T, \quad (3)$$

where $\mathbf{v}$ is velocity, η is viscosity, $\dot{\boldsymbol{\epsilon}}$ is the strain-rate tensor, P is pressure, ρ is density, $\mathbf{g}$ is gravitational acceleration, T is temperature, t is time, and k is thermal conductivity. Parameter values are given in Table 1.

Under the Boussinesq approximation, density is constant everywhere except in the buoyancy term of the momentum equation (Eq. 2), where it varies linearly with temperature,

$$\rho = \rho_0(1 - \alpha(T - T_0)), \quad (4)$$

where ρ_0 is the reference density, α is the thermal expansion coefficient, and T_0 is the temperature at the top of the domain.

2.2 Rheology

We test three creep-mechanism configurations for viscous deformation: i) diffusion and dislocation creep, and iii) diffusion and dislocation creep combined with grain boundary sliding (GBS), a stress- and grain-size-dependent mechanism favoured for ice deformation alongside dislocation creep (Behn et al., 2021). We additionally ran four diffusion-creep-only simulations (Table S2) to extend the range of p_{diff} sampled in Figs. 1. We implement GBS using the flow law of Goldsby and Kohlstedt (2001). Each mechanism follows a power-law viscosity (Goldsby and Kohlstedt, 2001; Kalousová et al., 2016):

$$\eta_i = \frac{1}{2} A_i^{-\frac{1}{n_i}} d^{\frac{m_i}{n_i}} |\dot{\epsilon}|^{\frac{1-n_i}{n_i}} \exp\left(\frac{E_i + PV_i}{n_i RT}\right), \quad (5)$$

where i denotes diffusion, dislocation, or GBS creep, A is the prefactor, n is the stress exponent, m is the grain-size exponent, $|\dot{\epsilon}| = (\dot{\epsilon} : \dot{\epsilon}/2)^{1/2}$ is the square root of the second invariant of the strain-rate tensor, E is activation energy, V is activation volume, and R is the gas constant. We combine the active mechanisms by harmonic averaging; for our reference case of diffusion-dislocation creep,

$$\eta = \left(\frac{1}{\eta_{diff}} + \frac{1}{\eta_{disl}} \right)^{-1}, \quad (6)$$

and we give the three-mechanism form used when GBS is included below.

Plastic deformation occurs where the deviatoric stress, τ , exceeds the Drucker–Prager yield limit, τ_y (Glerum et al., 2018; Moresi et al., 2003),

$$\tau_y = P \sin(\phi) + C_o \cos(\phi), \quad (7)$$

where C_o is cohesion and ϕ is the angle of friction. In this condition, viscosity of plastic deformation is used instead of that of viscous deformation (Glerum et al., 2018),

$$\eta_p = \frac{\tau_y}{2|\dot{\epsilon}|}. \quad (8)$$

Viscosity is limited to the range $10^{12} \leq \eta \leq 10^{20}$ Pa s (Table 1). The lower limit is never reached: the weakest ice in our models lies at the bottom boundary, where the diffusion-creep viscosity at 270 K is of order 10^{14} Pa s. The upper limit is reached within the stagnant lid, so the imposed total viscosity contrast across the shell, $\ln(\eta_{top}/\eta_{bottom})$, is approximately 13.

2.2.1 Visco-elasto-plastic rheology

Our VEP simulations use the same creep laws under incompressible flow, but modify the momentum equation to (Sandiford et al., 2021):

$$-\nabla \cdot (2\eta_{eff} \dot{\epsilon}) + \nabla P = -\rho \mathbf{g} + \nabla \cdot \mathbf{F}_e, \quad (9)$$

where η_{eff} is the effective viscosity and $\mathbf{F}_e$ is the elastic deformation force. ASPECT's Maxwell viscoelastic implementation tracks stress history via compositional fields in an Eulerian reference frame (Sandiford et al., 2021). The viscoelastic strain rate relates to the stress, $\boldsymbol{\tau}$, and its rate, $D\boldsymbol{\tau}/Dt$, as:

$$\dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}^v + \dot{\boldsymbol{\epsilon}}^e = \frac{\boldsymbol{\tau}}{2\eta} + \frac{1}{2\mu} \frac{D\boldsymbol{\tau}}{Dt}, \quad (10)$$

where $\dot{\boldsymbol{\epsilon}}^v$ and $\dot{\boldsymbol{\epsilon}}^e$ are the viscous and elastic strain rates and μ is the elastic modulus. Following Sandiford et al. (2021)'s discretisation using the Zaremba–Jaumann stress rate, Eq. 10 is rewritten to give the stress entering the Stokes equation at time t ,

$$\boldsymbol{\tau} = \eta_{eff} \left(2\dot{\boldsymbol{\epsilon}} + \frac{1}{\mu\Delta t} \tilde{\boldsymbol{\tau}}^t \right), \quad \eta_{eff} = \frac{\eta}{1 + \frac{\eta}{\mu\Delta t}}, \quad (11)$$

$$\tilde{\boldsymbol{\tau}}^t = \boldsymbol{\tau}^{t-\Delta t} + \mathbf{v} \cdot \nabla \boldsymbol{\tau}^{t-\Delta t} - \mathbf{W} \boldsymbol{\tau}^{t-\Delta t} + \boldsymbol{\tau}^{t-\Delta t} \mathbf{W}, \quad (12)$$

where $\tilde{\boldsymbol{\tau}}^t$ is the advected, rotated stress history from the previous time step, Δt is the elastic time step (equal to the numerical time step in our models, from tens to a thousand years), and $\mathbf{W}$ is the spin tensor. The corresponding surface Deborah number, $De_s = \eta_s \kappa / (\mu d_{shell}^2)$ (Thielmann et al., 2015), is 1.9×10^{-5} at 40 km and 8.4×10^{-6} at 60 km using our viscosity cap. Thielmann et al. (2015) varied De_s between 10^{-4} and 10^6 at Rayleigh numbers of 10^5 – 10^7 , and found the Nusselt number, root-mean-square velocity and stagnant-lid thickness insensitive to it. Our Rayleigh numbers fall inside their range, and our Deborah numbers are a factor of 5–12 below the range's lower end, where elasticity is expected to matter least. Both follow from our imposed viscosity ceiling of 10^{20} Pa s, which is reached within the stagnant lid.

An effective strain rate is defined (Sandiford et al., 2021):

$$\dot{\boldsymbol{\epsilon}}_{eff} = 2\dot{\boldsymbol{\epsilon}} + \frac{1}{\mu\Delta t} \tilde{\boldsymbol{\tau}}^t, \quad (13)$$

which is then used to rescale the plastic viscosity locally:

$$\eta_p = \frac{\tau_y}{|\dot{\boldsymbol{\epsilon}}_{eff}|}. \quad (14)$$

2.2.2 Diffusion, dislocation, and grain boundary sliding creep

In cases with GBS included, viscosity, η_{gbs} , follows the same flow-law form as Eq. 5, with parameters given in Table 1. We combine the three mechanism viscosities by harmonic averaging;

$$\eta = \left(\frac{1}{\eta_{diff}} + \frac{1}{\eta_{disl}} + \frac{1}{\eta_{gbs}} \right)^{-1}. \quad (15)$$

Each mechanism's viscosity is evaluated at the same converged local strain rate, so the mechanisms share a strain rate rather than a stress; because their stress exponents differ

($n = 1, 4$ and 1.8 ; Table 1), this is an approximation to the iso-stress sum of Goldsby and Kohlstedt (2001) rather than an identity with it.

The flow law we use for grain boundary sliding is Goldsby and Kohlstedt (2001)’s GBS-accommodated basal slip, their superplastic $n = 1.8$ regime. We do not include their complementary basal-slip-limited branch ($n = 2.4$), which acts in series with GBS and becomes rate-limiting at low stress; because diffusion creep dominates the composite in all our models, omitting it has only a small effect on the total viscosity.

Following Harel et al. (2020)’s comparison method, we use this same per-mechanism decomposition to compute each creep mechanism’s fractional contribution to the total deformation (p_{diff} , p_{dist} , p_{gbs}), reported in Section 3.1. Note that our p_{gbs} therefore measures the $n = 1.8$ branch alone, whereas Harel et al. (2020)’s composite law includes basal slip.

2.2.3 Compressible flow

Compressible flow under the anelastic liquid approximation (ALA) accounts for shear heating, adiabatic heating, and density changes with pressure and adiabatic temperature. The energy equation (Eq. 3) therefore is modified to include these heating terms (Schubert et al., 2001):

$$\rho C_p \frac{DT}{Dt} - \alpha T \frac{Dp}{Dt} = \nabla \cdot k \nabla T + 2\eta \left(\varepsilon - \frac{1}{3} \text{tr} \varepsilon \mathbf{1} \right) : \left(\varepsilon - \frac{1}{3} \text{tr} \varepsilon \mathbf{1} \right). \quad (16)$$

The density equation (Eq. 4) becomes

$$\rho = \rho_0 e^{K(p-p_0)} (1 - \alpha(T - T_{adiabat})), \quad (17)$$

where K is the reference compressibility, p_0 is the surface pressure, and $T_{adiabat}$ is the adiabatic temperature.

2.3 Initial and boundary conditions

We test ice shell thickness at 60, 40 and 20 km, chosen to bracket independent estimates of Europa’s shell thickness, which range from a lower bound near 19 km from crater morphology (Schenk, 2002), through roughly 30 ± 10 km of conductive ice inferred from Juno microwave radiometry (Levin et al., 2025; Rhoden and Walker, 2027), to the thicker shells permitted by multi-ring basin formation (Wakita et al., 2024) and by Europa’s global shape (Nimmo et al., 2007): 60 km is our thickest end-member, 40 km is representative of estimates in which the shell contains both a conductive lid and a convecting interior, and 20 km is our thinnest end-member. The 20-km cases do not develop convection within the 100 Myr model duration and are excluded from all Nusselt-Rayleigh analysis below.

Top and bottom temperatures are fixed at 110 K and 270 K (e.g., Nimmo and Manga, 2009); side boundaries are insulating. The initial temperature linearly increases between the top and bottom temperatures, with a random perturbation of amplitude $10^{-3} \Delta T$.

For VEP cases, we track stress history using six compositional fields: ve_stress_xx , ve_stress_xy , ve_stress_yy , and their previous-time-step counterparts. We initialise these at zero throughout the domain and fix them to zero flux at the boundaries. All velocity boundaries are free-slip in the reference model; in a subset of cases the top boundary is instead a free surface, allowing surface mesh deformation and the development of topography. We use a value of 0.5 for ASPECT’s Free surface stabilisation parameter θ , and adopt a vertical surface velocity projection. We run each model for 100 Myr, close to the oldest surface ages estimated for Europa (Soderlund et al., 2020). The model’s initial time step size is 10 years, and the size can increase up to 20 % per each time step. The time step size is regulated by a CFL number of 0.5, and the maximum time step size is a thousand years.

The initial mesh resolution is 500 m, and it is refined to 250-m both at the lower 40 % of model thickness and 2 km from the top boundary. The initial bottom refinement is held for 200 kyr. Interpolation across the change in element size produces velocity perturbation before the onset of convection, and retaining the fine resolution until convection is established allows for a stably converged solution for the velocity. Test runs showed 200 kyr to be sufficient at all thicknesses tested. A high resolution at the top boundary is maintained throughout the model runtime to resolve the physical ingredients’ near-surface effect (Crameri et al., 2012b; Thielmann et al., 2015). The resolution in low-velocity convecting regions is adaptively coarsened up to 2 km.

2.4 Model physical ingredient variations

Starting from the VP, free-slip, diffusion-dislocation-creep, BA reference case, we vary four aspects of the model in turn to isolate their effect on the resulting convective scaling. We test: i) rheology, as VP or VEP, to determine whether elastic stress storage feeds back on the thermal solution; ii) the top boundary condition, as free-slip or free-surface, to determine whether allowing the surface to deform changes the convective scaling in a way that matters once elasticity is also present; iii) creep mechanism, as diffusion and dislocation creep with or without GBS, to isolate the effect of this additional, weakening mechanism on the boundary-layer geometry; and iv) the energy and density equations, as incompressible (BA) or compressible (ALA), to test whether shear and adiabatic heating become significant once the shell is deforming more realistically. Every combination of these four choices is run, giving $2 \times 2 \times 2 \times 2 = 16$ simulations at each of the 40- and 60-km thicknesses, for 32 simulations in total (Table 2). A further 10 non-convecting simulations at 20 km, spanning a subset of the same combinations, show that convection does not initiate at this thickness within the model duration and are not analysed further here (Table S1).

2.5 Thermal diagnostics

For each model we track with horizontally averaged profiles, following Harel et al. (2020): i) the depth of the stagnant lid, d_{sl} , taken as the depth at which the tangent to the $v \cdot T$ profile at its inflection point falls to zero, ii) the depth of the rheological sublayer, d_{rh} , the depth at which the convective and conductive components of the vertical heat flux are equal;

since the rheological sublayer lies beneath the stagnant lid, d_{rh} always lies deeper than d_{sl} ; iii) the Nusselt number, Nu for the temperature evolution across the ice shell thickness, d_{shell} ; and iv) the interior Rayleigh number, $Ra_i = \alpha \rho g \Delta T d_{shell}^3 / \kappa \eta_{mixed}$, using the viscosity of the well-mixed convecting interior, η_{mixed} .

The temperature dependence of viscosity, γ_{rh} , is measured as $-\Delta T \frac{d \ln(\eta)}{dT}$, where ΔT is the temperature contrast across the shell and the derivative is evaluated as a finite difference between the top and bottom of the rheological sublayer. γ_{rh-std} is the standardised variable of γ_{rh} expressed by p_{diff} , eq. 25 in Harel et al. (2020). All diagnostics are averaged over the 10 Myr after the time of thermal equilibrium, defined as the point at which the surface heat flux becomes time-independent. For the five cases that reach equilibrium after 90 Myr (Table 3), the averaging window is correspondingly shorter, and we average over the final 10 Myr of the run (90–100 Myr).

Following Harel et al. (2020), we classify each model into one of three convection regimes based on the evolution of surface heat flux and the temperature field. In the stationary regime, the convection cells do not change with time. In the intermediate regime, secondary instabilities develop near the upwelling and downwelling limbs of otherwise stable cells. In the time-dependent regime, the instabilities vary continuously, and the surface heat flux fluctuates. We classify the convection regime at the end time of the models, regardless of that in the previous time. Only stationary and intermediate regimes occur in our suite (Table 3).

2.6 Statistical methods

We compare candidate scaling laws using ordinary least-squares (OLS) regression on log-transformed variables, $\log Nu \sim a + b \log Ra_i + c \log X$, for each candidate normalising variable X (namely γ_{rh} , γ_{rh-std} , d_{rh} , d_{sl} , and d_{rh} and d_{sl} each nondimensionalised by ice shell thickness, d_{shell}), and the final regression form follows:

$$Nu = A Ra_i^B X^C \quad (18)$$

where A is 10^a and $B = b$ and $C = c$. Ra_i is a diagnostic variable of the converged solution rather than an independently prescribed input parameter, so we do not read a large exponent on $\log Ra_i$ as evidence of an independent physical control. Instead, with a candidate X in the model, we read the 95 % confidence interval (CI) on B : if it excludes zero, Ra_i carries information that X does not capture; if it includes zero, X has absorbed that information.

We added four binary dummy variables to the log-transformed regression to test which physical ingredients affect the left-hand-side variable, as $\log Nu \sim a + \log Ra_i + b \log Ra_i + c \log X + \sum_{j=1}^4 d_j (\text{physical ingredient } j)$. Each binary variable is zero for creep without GBS, VP rheology, a free-slip top boundary, and BA, respectively, and unity for the alternative in

each case. The final scaling law form follows:

$$Nu = A Ra_i^B X^C \prod_{j=1}^4 D_j, \quad D_j = 1 \text{ in use of } \begin{cases} \text{diffusion} - \text{dislocation} \\ VP \\ \text{free slip} \\ BA \end{cases} \quad (19)$$

where $D_j = 10^{d_j}$. We treat D_j as an independent physical control when its CI excludes unity.

We stress-test the resulting comparison with a leave-one-out procedure across all 32 cases, refitting each candidate model with each case held out in turn and recording both the stability of each fitted coefficient (via the standard leave-one-out cross validation variance estimator, Efron and Tibshirani 1993) and, more stringently, each model’s out-of-sample prediction error for the held-out case. We summarise each model’s out-of-sample performance by the predictive coefficient of determination, $R_{pred}^2 = 1 - \text{PRESS}/\text{SST}$, where $\text{PRESS} = \sum_i (y_i - \hat{y}_i^{(-i)})^2$ is the sum of squared leave-one-out prediction errors, $\hat{y}_i^{(-i)}$ is the prediction for case i from the model refitted without it, and $y = \log_{10} Nu$.

We note that our cases form a complete factorial design (Montgomery, 2017), meaning every combination of the four modelling choices described above (rheology, boundary condition, creep mechanism, compressibility) is represented exactly once, rather than a random sample with replication; the confidence intervals and significance language used throughout this study should accordingly be read as a measure of how much a term improves the fit relative to the model’s own lack-of-fit, not as a formal inferential claim about generalisation to a wider population.

3 Results

3.1 Limits of the γ_{rh} and γ_{rh-std} normalisation for composite-rheology convection

Diffusion creep dominates the total deformation ($p_{diff} > 50\%$) in every one of the 32 cases, including those with GBS enabled (54.3–64.4%). The whole suite therefore falls on the single branch of Harel et al. (2020)’s piecewise correction factor in which $c_g = 1$. Our $p_{diff} - p_{gbs}$ values sit 3.8–5.6 percentage points below their eq. 24 (Fig. 1 and Table 3). The offset is accounted for by dislocation creep, which contributes 5.2–7.5% here against the 1.4–1.9% implicit in their regression, plausibly reflecting the different diffusion-creep activation energy in our flow law.

Our results show that γ_{rh} increases systematically with the ice shell thickness independently of temperature dependence, so that 40- and 60-km cases separate into two offset populations rather than collapsing onto a single trend (Fig. 2). The diffusion–dislocation cases isolate this effect clearly. Across them, p_{diff} varies only between 87.2 and 92.5 %, so the creep-mechanism contributes in almost the same proportions, and the well-mixed interior

Table 1: Model parameters

Parameter	Symbol	Units	Value
Thermal diffusivity	κ	$m^2 s^{-1}$	$1 \cdot 10^{-6}$
Thermal conductivity	k	$W m^{-1} K^{-1}$	1.9257
Specific heat capacity	C_p	$J K^{-1} kg^{-1}$	2100
Gravitational acceleration	g	$m s^{-2}$	1.3
Reference density	ρ_0	$kg m^{-3}$	917
Thermal expansion coefficient	α	K^{-1}	$1.6 \cdot 10^{-4}$
Top temperature	T_0	K	110
Bottom temperature	T_1	K	270
Grain size	d	m	10^{-3}
Gas constant	R	$J K^{-1} mol^{-1}$	8.314
Diffusion creep			
Prefactor	A	$Pa^{-1} s^{-1}$	$3.3 \cdot 10^{-10}$
Stress exponent	n	.	1
Grain exponent	m	.	2
Activation energy	E	$J mol^{-1}$	$59 \cdot 10^3$
Activation volume	V	$m mol^{-1}$	$-13 \cdot 10^{-6}$
Dislocation creep			
Prefactor	A	$Pa^{-1} s^{-1}$	$4.0 \cdot 10^{-19}$
Stress exponent	n	.	4
Grain exponent	m	.	0
Activation energy	E	$J mol^{-1}$	$60 \cdot 10^3$
Activation volume	V	$m mol^{-1}$	$-13 \cdot 10^{-6}$
Grain boundary sliding creep			
Prefactor	A	$Pa^{-1} s^{-1}$	$6.2 \cdot 10^{-14}$
Stress exponent	n	.	1.8
Grain exponent	m	.	1.4
Activation energy	E	$J mol^{-1}$	$49 \cdot 10^3$
Activation volume	V	$m mol^{-1}$	$-13 \cdot 10^{-6}$
Elastic modulus	μ	Pa	$3.3 \cdot 10^9$
Cohesion	C_o	Pa	10^6
Angle of friction	ϕ	$^\circ$	30
Maximum viscosity	η_{max}	$Pa s$	10^{20}
Minimum viscosity	η_{min}	$Pa s$	10^{12}
Reference compressibility	K	Pa^{-1}	10^{-10}

Table 2: The 32 convecting simulations analysed in this study (40- and 60-km thicknesses; a further 10 non-convecting 20-km simulations, spanning a subset of the same combinations, are excluded from the Nusselt-Rayleigh analysis and are not listed here). VP and VEP denote viscoplastic and visco-elasto-plastic deformation; diff, disl, and gbs denote diffusion, dislocation, and grain boundary sliding creep; BA and ALA denote the Boussinesq and anelastic liquid approximations.

Number	Thickness	Rheology	Top boundary	Compressibility	Creep
1	40	VP	Free slip	BA	diff-disl
2	40	VP	Free slip	BA	diff-gbs-disl
3	40	VP	Free slip	ALA	diff-disl
4	40	VP	Free slip	ALA	diff-gbs-disl
5	40	VP	Free surface	BA	diff-disl
6	40	VP	Free surface	BA	diff-gbs-disl
7	40	VP	Free surface	ALA	diff-disl
8	40	VP	Free surface	ALA	diff-gbs-disl
9	40	VEP	Free slip	BA	diff-disl
10	40	VEP	Free slip	BA	diff-gbs-disl
11	40	VEP	Free slip	ALA	diff-disl
12	40	VEP	Free slip	ALA	diff-gbs-disl
13	40	VEP	Free surface	BA	diff-disl
14	40	VEP	Free surface	BA	diff-gbs-disl
15	40	VEP	Free surface	ALA	diff-disl
16	40	VEP	Free surface	ALA	diff-gbs-disl
17	60	VP	Free slip	BA	diff-disl
18	60	VP	Free slip	BA	diff-gbs-disl
19	60	VP	Free slip	ALA	diff-disl
20	60	VP	Free slip	ALA	diff-gbs-disl
21	60	VP	Free surface	BA	diff-disl
22	60	VP	Free surface	BA	diff-gbs-disl
23	60	VP	Free surface	ALA	diff-disl
24	60	VP	Free surface	ALA	diff-gbs-disl
25	60	VEP	Free slip	BA	diff-disl
26	60	VEP	Free slip	BA	diff-gbs-disl
27	60	VEP	Free slip	ALA	diff-disl
28	60	VEP	Free slip	ALA	diff-gbs-disl
29	60	VEP	Free surface	BA	diff-disl
30	60	VEP	Free surface	BA	diff-gbs-disl
31	60	VEP	Free surface	ALA	diff-disl
32	60	VEP	Free surface	ALA	diff-gbs-disl

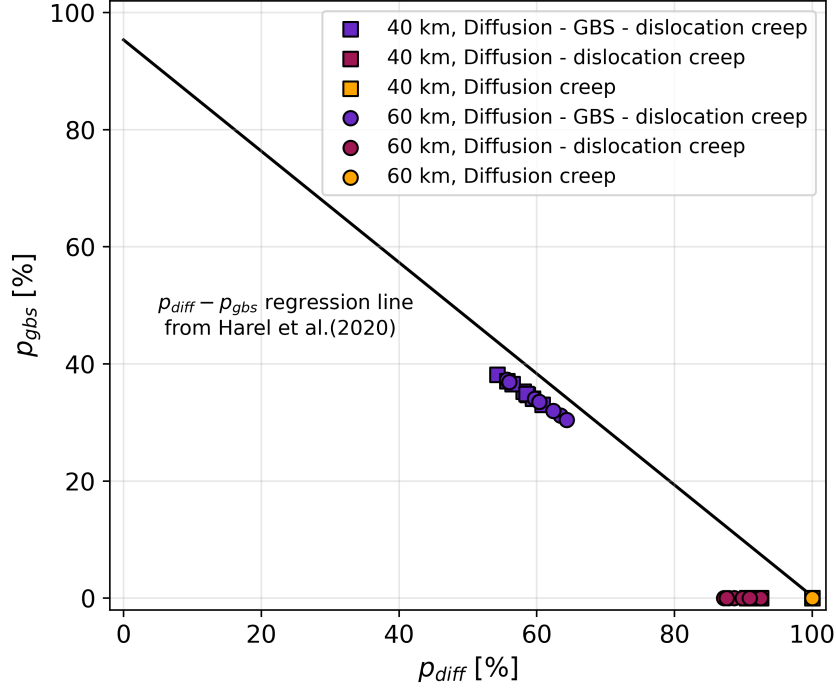


Figure 1: Plot of the relationship between the contribution of grain boundary sliding and diffusion creep to total creep mechanisms using 36 simulations noted in Table 2 and S2. The marker colour denotes the creep mechanism: indigo for diffusion-GBS-dislocation creep, maroon for diffusion-dislocation creep, and orange for diffusion creep. The marker shape denotes the ice shell thickness; square for 40 km and circle for 60 km. The black line indicates the regression line of $p_{diff}-p_{gbs}$ as Eq. 24 in Harel et al. (2020). Markers sit slightly below the regression line because dislocation creep contributes more here than in Harel et al. (2020)’s suite. The three clusters correspond to which creep mechanisms are active, not to shell thickness: 40- and 60-km cases overlap within each cluster, and their mean p_{diff} differs by only about 2 percentage points, against a 23-point separation between clusters. The contributions are therefore set by the choice of creep mechanisms rather than by shell thickness.

Table 3: Convective variables for 32 convecting simulations. t_{eq} [Myr] is the time of thermal equilibrium, Ra_i is the Rayleigh number of well-mixed interior, Nu is the Nusselt number, γ_{rh} is the temperature dependence of viscosity, γ_{rh-std} is the eq. 25 from Harel et al. (2020), d_{rh} [km] is the bottom depth of rheological sublayer, d_{sl} [km] is the bottom depth of stagnant lid, d_{rh}/d_{shell} and d_{sl}/d_{shell} are the nondimensional counterparts of d_{rh} and d_{sl} by d_{shell} , and p_{diff} , p_{gbs} , p_{disl} [%] are the contributions of diffusion, GBS and dislocation creep within the rheological sublayer (between d_{sl} - d_{rh}), respectively, and the convection regimes are classified following Harel et al. (2020).

No.	t_{eq}	Ra_i	Nu	γ_{rh}	γ_{rh-std}	d_{rh}	d_{sl}	d_{rh}/d_{shell}	d_{sl}/d_{shell}	p_{diff}	p_{gbs}	p_{disl}	Convection
1	22.2	$2.43 \cdot 10^6$	1.88	17.55	24.57	19.90	17.22	0.498	0.430	90.5	0.0	9.5	Stationary
2	20.1	$2.68 \cdot 10^6$	2.06	17.84	18.04	17.90	15.04	0.448	0.376	54.3	38.2	7.5	Stationary
3	18.6	$2.40 \cdot 10^6$	1.85	17.36	24.82	20.13	17.52	0.503	0.438	92.0	0.0	8.0	Stationary
4	19.1	$2.61 \cdot 10^6$	2.03	17.12	18.84	18.20	15.35	0.455	0.384	58.7	34.7	6.6	Stationary
5	51.7	$2.39 \cdot 10^6$	1.85	18.97	24.72	20.10	17.27	0.503	0.432	91.4	0.0	8.6	Stationary
6	32.9	$2.71 \cdot 10^6$	2.09	17.01	18.30	17.90	15.31	0.448	0.383	55.7	37.1	7.2	Stationary
7	25.6	$2.43 \cdot 10^6$	1.87	16.97	24.90	20.00	17.38	0.500	0.435	92.4	0.0	7.6	Stationary
8	20.9	$2.62 \cdot 10^6$	2.13	15.00	18.72	17.60	15.21	0.440	0.380	58.1	35.3	6.6	Stationary
9	27.2	$2.44 \cdot 10^6$	1.88	17.38	24.70	19.94	17.27	0.498	0.432	91.3	0.0	8.7	Stationary
10	21.6	$2.69 \cdot 10^6$	2.06	16.91	19.22	18.00	15.21	0.450	0.380	60.8	33.0	6.1	Stationary
11	20.7	$2.42 \cdot 10^6$	1.87	16.46	24.93	20.00	17.48	0.500	0.437	92.5	0.0	7.5	Stationary
12	19.0	$2.67 \cdot 10^6$	2.10	15.38	18.43	17.81	15.30	0.445	0.382	56.5	36.6	7.0	Stationary
13	25.4	$2.43 \cdot 10^6$	1.86	17.50	24.82	20.03	17.34	0.501	0.434	92.0	0.0	8.0	Stationary
14	26.1	$2.68 \cdot 10^6$	2.04	17.28	18.97	18.30	15.48	0.458	0.387	59.5	34.1	6.5	Stationary
15	21.3	$2.39 \cdot 10^6$	1.85	17.24	24.80	20.20	17.53	0.505	0.438	91.8	0.0	8.2	Stationary
16	24.0	$2.61 \cdot 10^6$	2.02	16.73	18.80	18.30	15.46	0.458	0.387	58.5	34.9	6.6	Stationary
17	51.2	$9.02 \cdot 10^6$	2.33	23.78	23.97	23.45	19.08	0.391	0.318	87.2	0.0	12.8	Stationary
18	53.7	$9.90 \cdot 10^6$	2.63	19.71	18.33	21.09	17.61	0.352	0.294	55.9	37.0	7.1	Intermediate
19	53.1	$8.71 \cdot 10^6$	2.35	23.20	24.09	23.33	19.49	0.389	0.325	87.9	0.0	12.1	Stationary
20	89.7	$9.87 \cdot 10^6$	2.50	17.74	19.69	22.44	19.36	0.374	0.323	63.5	31.2	5.3	Intermediate
21	91.4	$9.26 \cdot 10^6$	2.33	22.89	24.23	23.76	19.86	0.396	0.331	88.7	0.0	11.3	Stationary
22	99.6	$1.06 \cdot 10^7$	2.53	18.61	19.85	22.42	19.16	0.374	0.319	64.4	30.4	5.2	Intermediate
23	67.8	$9.52 \cdot 10^6$	2.20	23.99	24.46	25.27	21.25	0.421	0.354	90.0	0.0	10.0	Intermediate
24	99.6	$9.73 \cdot 10^6$	2.52	18.66	19.50	22.16	18.94	0.369	0.316	62.4	32.0	5.6	Intermediate
25	35.7	$9.15 \cdot 10^6$	2.30	22.07	24.45	24.05	19.84	0.401	0.331	89.9	0.0	10.1	Stationary
26	47.8	$1.01 \cdot 10^7$	2.59	19.21	19.02	21.57	18.19	0.360	0.303	59.7	34.1	6.2	Stationary
27	83.2	$9.56 \cdot 10^6$	2.15	22.17	24.92	26.14	22.13	0.436	0.369	92.5	0.0	7.5	Intermediate
28	81.9	$9.37 \cdot 10^6$	2.59	18.84	18.29	21.21	17.81	0.353	0.297	55.7	37.3	7.0	Intermediate
29	45.2	$9.13 \cdot 10^6$	2.30	22.71	24.04	23.85	19.70	0.397	0.328	87.6	0.0	12.4	Stationary
30	95.2	$9.87 \cdot 10^6$	2.65	19.24	18.35	20.68	17.44	0.345	0.291	56.0	36.9	7.1	Stationary
31	89.0	$9.09 \cdot 10^6$	2.23	22.30	24.64	24.90	21.30	0.415	0.355	90.9	0.0	9.1	Intermediate
32	92.9	$9.61 \cdot 10^6$	2.60	19.39	19.14	21.25	18.31	0.354	0.305	60.4	33.5	6.1	Intermediate

temperature stays within 259.9–261.3 K (Table S3), so that a Frank–Kamenetskii parameter, $E\Delta T/RT_i^2$, would vary by less than 2 % whatever value of E is assumed. Yet, the measured γ_{rh} separates cleanly by thickness, from 16.46–18.97 at 40 km to 22.07–23.99 at 60 km, with no overlap. The spread in γ_{rh} across our suite therefore does not track temperature dependence; it tracks the depth of the rheological sublayer, as Section 4.1 shows directly.

γ_{rh-std} fails for the complementary reason. This is because γ_{rh-std} is a function of p_{diff} alone, so it carries no information beyond the creep-mechanism contributions (i.e., the contributions of diffusion vis-a-vis dislocation creep) and cannot distinguish shell thicknesses. Over the diffusion–dislocation cases, it is confined to 23.97–24.93, while the measured γ_{rh} spans 16.46–23.99; the discrepancy reaches 8.47 (51.4 %) there, and 9.75 (59.0 %) in the diffusion-only cases of Table S2, against at most 3.72 (24.8 %) where GBS is active. Because γ_{rh-std} depends on p_{diff} alone, the same value can also arise for different reasons in different suites. Our grain size is fixed at 1 mm, so p_{diff} varies here only with which creep mechanisms are active, whereas in Harel et al. (2020) it varies with grain size. With GBS suppressed, diffusion creep accounts for 87.2–92.5 % of the deformation, which their eq. 23 would associate with grain sizes of 0.3–0.4 mm, and the corresponding γ_{rh-std} (23.97–24.93) sits just below their finest-grain band (25–30 for $d_g = 0.1$ mm) despite our imposed grain size of 1 mm. With GBS active, p_{diff} falls to 54.3–64.4 %, below the 71.7 % their eq. 23 predicts at 1 mm, and γ_{rh-std} (18.04–19.85) lies between their two bands (Table 3).

We conclude that neither γ_{rh} nor γ_{rh-std} , as defined by Harel et al. (2020), provides a normalising variable that transfers to this composite-rheology, multi-approximation dataset without modification.

3.2 A two-variable scaling law: interior Rayleigh number (Ra_i) and rheological sublayer depth (d_{rh})

We test six candidate normalising variables in the nested models of eq. 18: the two temperature-dependence variables of Harel et al. (2020), γ_{rh} and γ_{rh-std} ; the two dimensional boundary-layer depth in dimensional form (km), d_{rh} and d_{sl} ; and each depth nondimensionalised by the ice shell thickness, d_{rh}/d_{shell} and d_{sl}/d_{shell} (Table 3). Table 4 compares the coefficients and exponents of the Ra_i -only and Ra_i -plus-candidate models.

The dimensional rheological-sublayer depth, d_{rh} , gives the best-performing model ($R^2 = 0.996$), ahead of its own nondimensional form d_{rh}/d_{shell} (also $R^2 = 0.996$), the dimensional stagnant-lid depth d_{sl} ($R^2 = 0.982$), the nondimensional stagnant-lid depth d_{sl}/d_{shell} ($R^2 = 0.976$), γ_{rh-std} ($R^2 = 0.972$), and γ_{rh} ($R^2 = 0.910$). The two leading forms are separated only in the fourth decimal place (0.9963 against 0.9957), so goodness of fit alone does not choose between them. Critically, the two best-performing dimensional variables behave differently from their own normalised forms: with d_{rh} as the candidate variable, Ra_i ’s exponent is 0.27 (95% CI [0.267, 0.282], remaining within [0.269, 0.281] across all 32 leave-one-out folds), whereas with d_{rh}/d_{shell} instead it falls to 0.001 (CI [−0.009, 0.011]), with the confidence interval containing zero in all 32 folds. The same pattern holds for d_{sl} versus d_{sl}/d_{shell} .

This is the expected signature of the near-definitional relationship $Nu \approx D/\delta$ in the

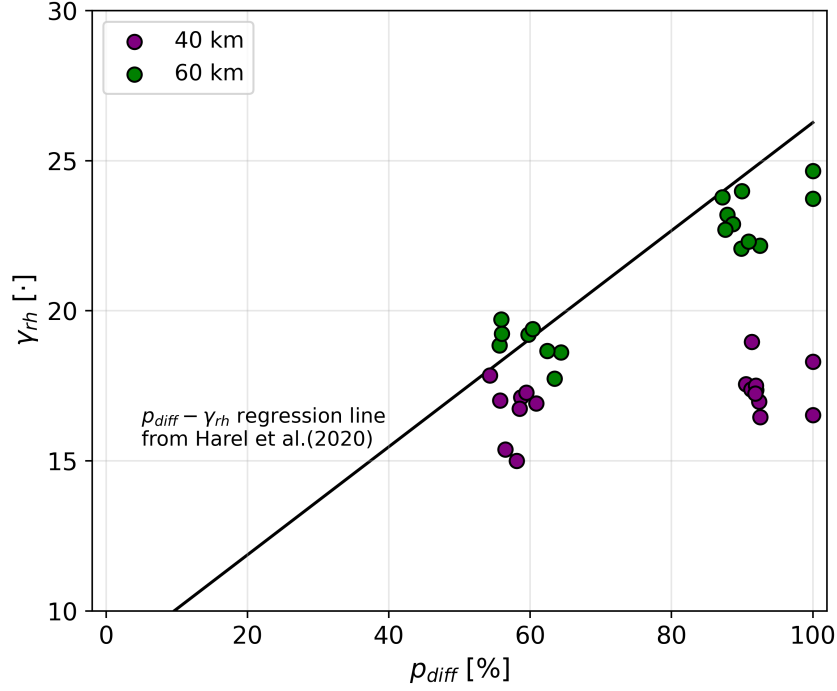


Figure 2: γ_{rh} against the contribution of diffusion creep to total creep, p_{diff} , for the 36 simulations of Tables 2 and S2. Purple markers denote 40-km cases and green markers 60-km cases. The black line is eq. 25 of Harel et al. (2020); γ_{rh-std} is by construction the value of this line at each case's p_{diff} . Because that line is a function of p_{diff} alone, it can match only one shell thickness: the 60-km cases straddle it (offsets -1.4 to $+2.8$), whereas the 40-km cases lie below it without exception (offsets $+0.2$ to $+9.8$, the largest of these in the diffusion-only cases). Measured γ_{rh} rises with shell thickness, from a mean of 17.1 at 40 km to 21.3 at 60 km. γ_{rh} measured in the rheological sublayer therefore carries thickness information that eq. 25 cannot reproduce.

well-mixed-interior limit (cf. Schulz et al., 2020, their fig. 12 discussion of the diminishing role of Ra as the lid thickens). Nondimensionalising a boundary-layer thickness by d_{shell} removes the same shell-thickness information that Ra_i carries (since $Ra_i \propto d_{shell}^3$), so Ra_i becomes redundant once that information is supplied directly. The effect is visible directly in the distribution of each candidate against Nu (Fig. 3). In the regression of each candidate as $Nu = A X^C D_{thickness}$, where $D_{thickness}$ is a binary dummy variable for ice shell thickness, the 60-km cases sit at a factor of 1.48 [1.46, 1.50] higher in Nu than the 40-km cases at the same d_{rh} , and 1.43 [1.40, 1.46] higher at the same d_{sl} . For the nondimensional forms, the two thicknesses are indistinguishable: 1.00 [0.99, 1.01] at the same d_{rh}/d_{shell} and 1.00 [0.97, 1.03] at the same d_{sl}/d_{shell} . Ra_i differs by a factor of 3.8 between the two thicknesses, so with d_{rh} as the candidate variable the Ra_i term supplies most of the required separation: at $B = 0.274$ it contributes a factor of 1.44, against the 1.48 observed. With d_{rh}/d_{shell} instead, the two thicknesses already lie on one line, leaving nothing for Ra_i to explain, and its fitted exponent drops to 0.001. Using d_{rh} in its dimensional form avoids this and gives a two-parameter scaling law in which convective vigour (Ra_i) and boundary-layer geometry (d_{rh}) each carry distinguishable explanatory power, stable across all 32 folds:

$$Nu \approx 0.38 Ra_i^{0.27} d_{rh}^{-0.82}. \quad (20)$$

A leave-one-out cross-validation confirms this is not an artefact of the full sample. d_{rh} gives the best out-of-sample accuracy of the six candidate variables (predictive $R^2 = 0.995$) and the best within-fold fit in 31 of the 32 refits (Table S4 to S10). The cross-validation reproduces the near-tie out of the sample, returning 0.995 for d_{rh}/d_{shell} as well. The choice between the two therefore rests on the behaviour of the Ra_i exponent described above, rather than on predictive accuracy.

The boundary-layer relationship underlying all of this is tighter than the fitted scaling law itself. Across all 32 cases $Nu d_{rh}/d_{shell} = 0.928$ with a standard deviation of 0.9% (range 0.911–0.946), while the same product formed with d_{sl} scatters nearly three times as widely ($0.789 \pm 2.5\%$). Fitting $\log Nu$ against $\log d_{rh}$ by OLS regression within a single shell thickness (16 cases each) returns an exponent of -0.970 ± 0.052 at 40 km and -0.977 ± 0.084 at 60 km, neither excluding the value of -1 implied by $Nu \approx D/\delta$. This one-parameter relationship, $Nu \approx 0.93 d_{shell}/d_{rh}$, accounts for $R^2 = 0.994$ of the variance in $\log Nu$, against 0.996 for Eq. 20 and 0.997 once the four dummy variables are added. We retain the fuller form because a near-definitional relationship cannot address whether the additional physical ingredients shift the thermal scaling; that requires those ingredients to enter as separable terms.

3.3 Physical ingredients as controllers of convection

To test whether the scaling law should be separated by the selection of additional physical ingredients in our models, we first regressed the nested model with binary dummy variables for creep mechanism (GBS included vs. not), elasticity (VEP vs. VP), top boundary condition (free surface vs. free slip), and compressibility (ALA vs. BA), as in Eq. 19 (Table 5). Fig. 4 shows this fit in the nested model $Nu \sim A Ra_i^B \prod_{j=1}^4 D_j$ and shows that the scaling

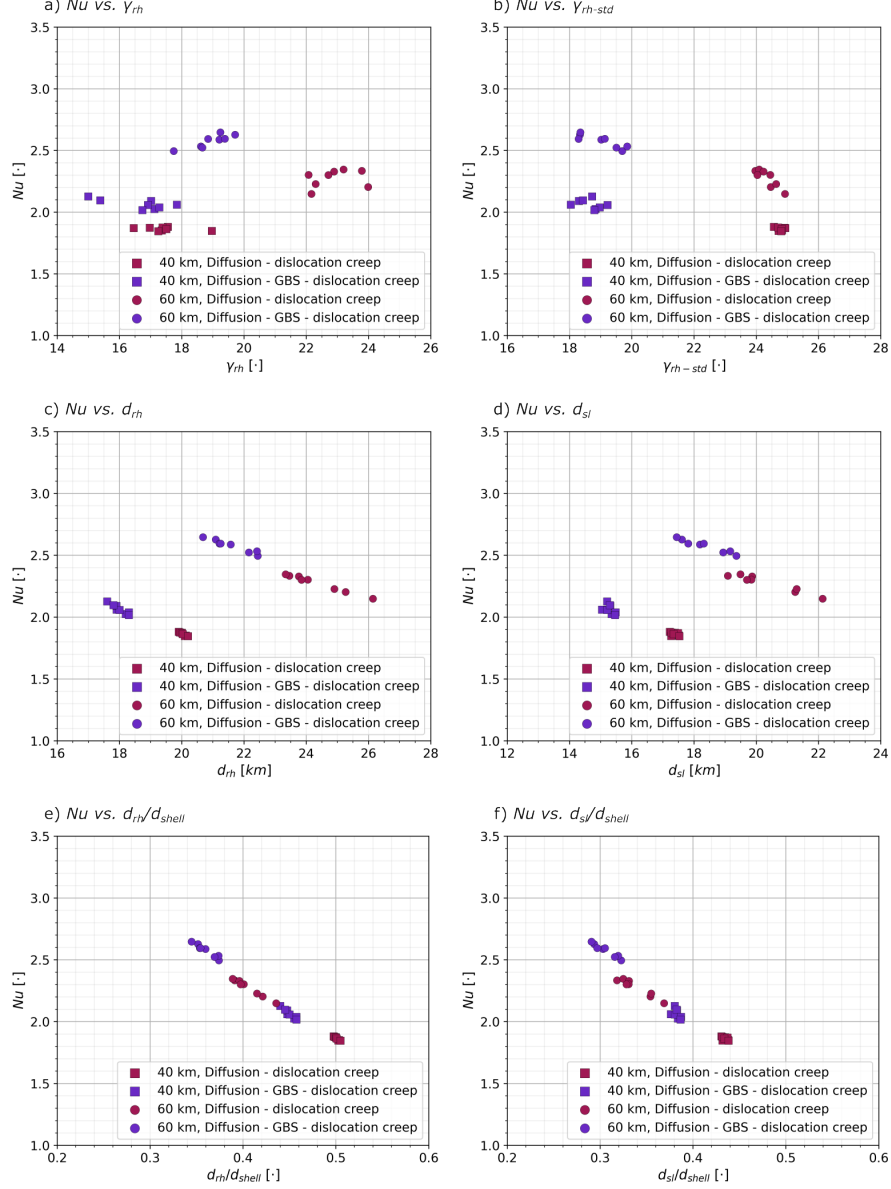


Figure 3: Plot of Nu of each candidate normalising variable; a) γ_{rh} , b) γ_{rh-std} , c) d_{rh} , d) d_{sl} , e) d_{rh}/d_{shell} and f) d_{sl}/d_{shell} from Table 3. Marker colours denote creep mechanisms (maroon for diffusion-dislocation creep and purple for diffusion-GBS-dislocation creep). Marker shape indicates ice shell thickness (square for 40 km and circle for 60 km). Nu itself separates by thickness, spanning 1.85–2.13 at 40 km against 2.15–2.65 at 60 km, and within each thickness it is higher when GBS is active. Panel a) shows no useful relationship. In b), the points fall close to a line, but γ_{rh-std} takes much the same values at both thicknesses, so that line carries no thickness information. In c) and d) the two thicknesses form parallel but vertically offset trends, with the creep mechanism separating the points within each. In e) and f) the same four groups appear, but the offset is gone, and all 32 cases fall on a single line. This is why d_{rh} serves as the most accurate normalising variable while leaving Ra_i an independent role, and why its nondimensional form removes that role.

Table 4: Nested comparison of Ra_i -only ($Nu \sim A Ra_i^B$) and Ra_i -plus-candidate nested model ($Nu \sim A Ra_i^B X^C$) for six candidate normalising variables X , ordered by R^2 . The values of coefficients and exponents are written with 95% confidence intervals.

X	R^2	A (95% CI)	B (95% CI)	C (95% CI)
d_{rh}	0.996	0.380 [0.352, 0.410]	0.274 [0.267, 0.281]	-0.816 [-0.858, -0.774]
d_{rh}/d_{shell}	0.996	0.940 [0.837, 1.056]	0.001 [-0.009, 0.011]	-0.960 [-1.013, -0.906]
d_{sl}	0.982	0.375 [0.317, 0.443]	0.255 [0.240, 0.269]	-0.752 [-0.840, -0.664]
d_{sl}/d_{shell}	0.976	0.829 [0.634, 1.084]	0.006 [-0.018, 0.030]	-0.857 [-0.974, -0.739]
γ_{rh-std}	0.972	0.610 [0.470, 0.791]	0.158 [0.146, 0.170]	-0.379 [-0.437, -0.322]
γ_{rh}	0.910	0.249 [0.177, 0.349]	0.234 [0.202, 0.265]	-0.487 [-0.651, -0.324]
Ra_i only	0.795	0.178 [0.110, 0.286]	0.163 [0.132, 0.194]	

Table 5: Nested comparison of Ra_i -only ($Nu \sim A Ra_i^B$) and Ra_i -plus-candidate nested model ($Nu \sim A Ra_i^B X^C$) for six candidate normalising variables X with coefficients of physical ingredients D_j , ordered by R^2 . The values of coefficients and exponents are written with 95% confidence intervals.

X	R^2	A (95% CI)		B (95% CI)		C (95% CI)		D_1 (95% CI)		D_2 (95% CI)		D_3 (95% CI)		D_4 (95% CI)	
d_{rh}	0.997	0.443	[0.392, 0.499]	0.299	[0.282, 0.316]	-0.989	[-1.107, -0.871]	0.977	[0.961, 0.992]	0.999	[0.994, 1.004]	1.000	[0.995, 1.005]	1.006	[1.001, 1.012]
d_{rh}/d_{shell}	0.997	0.714	[0.600, 0.850]	0.028	[0.012, 0.044]	-0.797	[-0.893, -0.701]	1.021	[1.010, 1.032]	0.998	[0.993, 1.003]	0.999	[0.994, 1.004]	0.999	[0.993, 1.004]
d_{sl}/d_{shell}	0.988	0.491	[0.351, 0.687]	0.058	[0.026, 0.091]	-0.559	[-0.735, -0.383]	1.045	[1.024, 1.067]	0.998	[0.987, 1.009]	0.997	[0.986, 1.008]	1.002	[0.991, 1.014]
d_{sl}	0.986	0.331	[0.257, 0.428]	0.238	[0.208, 0.268]	-0.626	[-0.851, -0.400]	1.022	[0.991, 1.054]	0.999	[0.987, 1.010]	0.997	[0.985, 1.009]	1.007	[0.994, 1.021]
γ_{rh-std}	0.973	0.700	[0.196, 2.503]	0.158	[0.146, 0.171]	-0.421	[-0.816, -0.026]	0.989	[0.889, 1.099]	0.997	[0.981, 1.014]	1.001	[0.984, 1.018]	0.992	[0.976, 1.009]
γ_{rh}	0.970	0.200	[0.158, 0.252]	0.177	[0.147, 0.206]	-0.125	[-0.297, 0.048]	1.087	[1.057, 1.119]	0.995	[0.978, 1.013]	1.003	[0.986, 1.020]	0.984	[0.966, 1.002]
Ra_i only	0.968	0.183	[0.149, 0.225]	0.158	[0.145, 0.171]			1.105	[1.086, 1.125]	0.997	[0.980, 1.015]	1.003	[0.986, 1.021]	0.988	[0.971, 1.006]

law should be separated by the creep mechanism. The coefficient D_1 is 1.105 (CI [1.086, 1.125]). The remaining coefficients have a CI spanning unity; $D_2 = 0.997$ (CI [0.980, 1.015]), $D_3 = 1.003$ (CI [0.986, 1.021]), and $D_4 = 0.988$ (CI [0.971, 1.006]). This is because GBS creep works as an intermediate creep mechanism between diffusion and dislocation creep (fig. 6 and 7 in Harel et al. (2020)).

The creep mechanism remains the primary controller for our two-variable scaling law with d_{rh} , $Nu \sim A Ra_i^B d_{rh}^C \prod_{j=1}^4 D_j$. As in the dummy-free comparison, d_{rh} and d_{rh}/d_{shell} are indistinguishable in goodness of fit ($R^2 = 0.997$ for both; predictive R^2 of 0.9959 and 0.9958), and both are well ahead of the remaining candidates (Table 5). What separates them is again the Ra_i exponent, which stays at 0.299 ([0.282, 0.316]) with d_{rh} but collapses to 0.028 ([0.012, 0.044]) with d_{rh}/d_{shell} . Neither model shows noticeable sensitivity to leave-one-out folds (Table S17 to S24). Meanwhile, D_4 is 1.006 and has a 95 % CI that does not span unity, [1.001, 1.012]. Its value is smaller compared to D_1 , but it is a detectable effect throughout the leave-one-out cross validation. Omitting the insignificant coefficients, D_2 and D_3 , the scaling law with d_{rh} that separates between diffusion-dislocation creep and diffusion-GBS-dislocation creep follows:

$$Nu \approx 0.44 Ra_i^{0.30} d_{rh}^{-0.99} D_1 D_4, \quad D_1 = \begin{cases} 1, & \text{without GBS} \\ 0.977, & \text{with GBS} \end{cases}, \quad D_4 = \begin{cases} 1, & BA \\ 1.006, & ALA \end{cases} \quad (21)$$

For the other nested models in Table 5, the equivalent scaling law with d_{sl} has a lower R^2 and shows the CI of all D_j spanning unity: [0.991, 1.054], [0.987, 1.010], [0.985, 1.009], and [0.994, 1.021], respectively (Table 5). The nested model with γ_{rh} , CI of its exponent C spans zero (CI [-0.297, 0.048]), meaning that its effect can be explained by other variables in the nested model. The nested model with γ_{rh-std} has CIs for the coefficients A and D_j that span unity. However, its R^2 , 0.973, is smaller than that of the nested model with d_{sl} . The nested models of d_{sl}/d_{shell} show the same tendency as the aforementioned d_{rh}/d_{shell} . The B exponent does not span zero, but its value of B exponent is still an order of magnitude smaller than the exponent C . We also find that adding a convective-regime dummy variable (stationary vs. intermediate 60-km cases) to the final $Nu \sim A Ra_i^B d_{rh}^C$ scaling law leaves its coefficient statistically indistinguishable from unity in every leave-one-out fold, confirming that the regime distinction is not useful for the headline result (Table S25).

3.4 Grain boundary sliding creep controls the boundary-layer geometry

d_{rh} was regressed against Ra_i and the binary dummy variables, changing the left-hand side of eq. 19, to test which of the additional physical ingredients in our model suite significantly affects the boundary-layer geometry.

Only the creep-mechanism term is robustly significant, both in the full-sample fit, $D_1 = 0.882$, 95% CI [0.867, 0.898], and across every one of the 32 leave-one-out folds (Table S26).

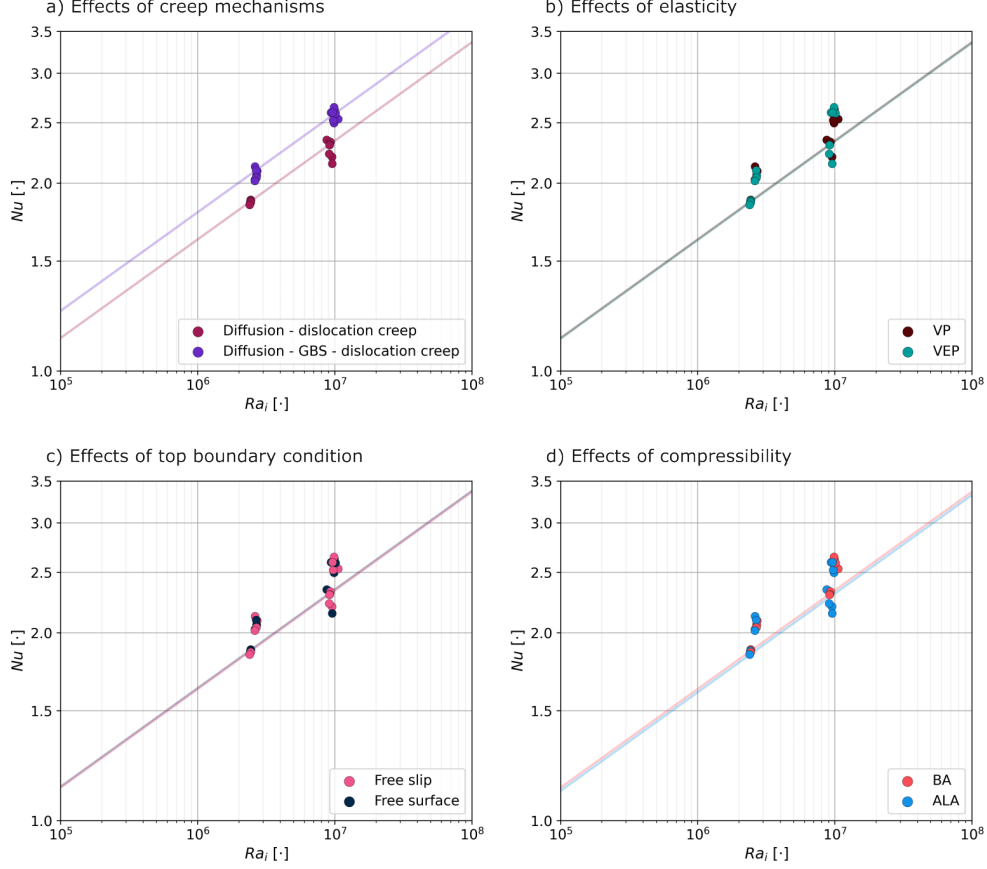


Figure 4: Nu against Ra_i for the 32 cases, with the regression lines of the nested model $Nu \sim A Ra_i^B \prod_{j=1}^4 D_j$ (eq. 19) overlaid. Each panel colours the same 32 cases by one physical ingredient: a) with and without GBS, b) VP or VEP deformation, c) free-slip or free-surface top boundary condition, d) BA or ALA. The two lines in a panel are the fitted model evaluated at the two values of that panel's dummy variable, so they share the exponent B and differ only by the multiplier D_j . In a) they are separated by $D_1 = 1.105$ [1.086, 1.125], which excludes unity; they overlap one another in b), c), and d), with $D_2 = 0.997$, $D_3 = 1.003$, and $D_4 = 0.988$, all spanning unity (Table 5). Only the creep mechanism has a detectable effect on the scaling.

Enabling GBS thins the rheological sublayer by 2.0 km in the 40-km shells and 2.7 km in the 60-km shells.

Elasticity and the top boundary condition have no detectable effect in any fold, $D_2 = 1.001$, 95% CI [0.985, 1.019] and $D_3 = 0.996$, 95% CI [0.979, 1.013]. Compressibility has a small, physically consistent effect, $D_4 = 1.018$, 95% CI [1.001, 1.036], always in the same direction above unity across every fold, where ALA thickens the sublayer, consistent with shear and adiabatic heating weakening convection. Its significance is, however, marginal and sample-sensitive: the leave-one-out cross validation finds its confidence interval excludes the null in 20 of 32 folds, short of the consistency shown by the creep-mechanism term. We report this as a small, directionally consistent but not confidently significant effect.

Since Nu is fixed by d_{rh} to within 0.9% across the suite (Section 3.2), these are also the controls on the thermal scaling itself. Regressing $Nu d_{rh}/d_{shell}$ on the same four dummy variables returns every coefficient within 1% of unity, so the ingredients act on heat transport only by moving the boundary layer, not by altering the relationship between Nu and its depth. These results indicate that of the four physical ingredients distinguishing composite-rheology, viscoelastic, free-surface and compressible ice shell models from simpler ones, only the creep-mechanism choice materially affects the thermal scaling examined here, with a small but not consistently significant effect from compressibility. However, elasticity and the top boundary condition have no detectable effect at the level of this scaling law, and their consequences are largely confined to the surface deformation examined in the companion study.

4 Discussion

4.1 Normalising variables and physical ingredients

Our comparison of normalising variables is a comparison of what each variable knows about shell thickness. Within a single thickness, every candidate but γ_{rh} describes Nu comparably well; they differ in whether they also carry the information that distinguishes a 40-km shell from a 60-km one, and in what form. γ_{rh} carries this information, but inadvertently: it varies with the depth of the rheological sublayer rather than with any change in temperature dependence (Section 3.1), so it acts as a depth variable in disguise, and a poor one. γ_{rh-std} carries no shell thickness information, since it is a function of p_{diff} alone. The dimensional depths carry this information but not sufficiently on their own. The 40- and 60-km cases follow parallel but vertically offset trends, so d_{rh} does not by itself fix Nu , and Ra_i is needed to say which trend applies. Dividing by d_{shell} collapses the two trends onto one, leaving Ra_i nothing to contribute and driving its exponent to zero. We stress that d_{rh} is itself a diagnostic of the converged solution rather than a prescribed parameter, so this is not a route to predicting Europa’s heat flux without a model. Its value is diagnostic rather than predictive: it says which modelling choices matter, and how. Because the ingredients act on Nu only by moving d_{rh} , and we find that only the creep mechanism moves d_{rh} appreciably, a model’s predicted heat flux is sensitive to which creep mechanisms its flow law admits

and barely to the other three choices. Simplifying the treatment of elasticity or the surface boundary condition costs nothing in the thermal result, and simplifying compressibility costs little.

The difference between published forms is not a disagreement about the physics of stagnant-lid convection but about where the shell-thickness information sits. Whether this is represented by the normalising variable, or alongside it in Ra . Comparing fitted exponents between studies that normalise differently is therefore comparing partial derivatives taken at fixed values of different quantities. Our own suite shows how large the effect can be with the same 32 models returning an Ra_i exponent of 0.27 with d_{rh} in the model and 0.001 with d_{rh}/d_{shell} .

Underneath the statistical comparison lies a simple physical relation. The product $Nu d_{rh}/d_{shell}$ is constant to within 0.9% across our whole suite, and none of the four physical ingredients shifts it in a detectable way ($CI > 0.6$ in every case). Heat transport in these models is set by the depth of the rheological sublayer and by nothing else resolvable at this precision; the ingredients act on Nu only insofar as they move d_{rh} .

That reframes our creep-mechanism result. Enabling GBS thins the rheological sublayer by 2.0 km in the 40-km shells and 2.7 km in the 60-km shells ($CI < 10^{-4}$ in both), whereas elasticity and the top boundary condition move it immeasurably and compressibility only marginally (Section 3.4). The creep mechanism is a first-order control on the thermal scaling not because it changes how heat is transported, but because it is the only ingredient that appreciably changes the geometry that sets it. We stress that d_{rh} is itself a diagnostic of the converged solution rather than a prescribed parameter, so this is not a route to predicting Europa’s heat flux without a model. Its value is the opposite: it localises where the modelling choices act. A composite flow law that admits GBS and one that does not will differ in predicted heat flux through their boundary-layer geometry alone, and the remaining ingredients will not differ at all.

4.2 Boundary-layer depth and not the local temperature dependence as the normalising variable: support from independent studies

Two independent studies point the same way: that a boundary-layer depth, rather than a temperature-dependence parameter, is the variable that carries a stagnant-lid heat-flux scaling.

Schulz et al. (2020) reach this by an independent route. Their equivalent temperature-dependence parameter varies over only 11.4–12.8 across their suite and cannot be determined in advance, so they normalise instead on the mean stagnant-lid thickness. From that fit, they conclude that the heat flux through a stagnant lid is set by the thickness of the lid and the viscosity beneath it, rather than by the deformation mechanism or the lid’s shape, and that the Nusselt number becomes progressively less sensitive to the Rayleigh number as the lid thickens, which they attribute to the lid’s insulating effect (their eq. 36).

Our result is a variant of the same finding. However, we use the deeper rheological-

sublayer boundary rather than the stagnant-lid boundary itself. The apparent discrepancy in the residual Rayleigh-number exponent — 0.071 in Schulz et al. (2020) against 0.27 in ours — is the effect described in section 4.1. The $\langle L \rangle$ in Schulz et al. (2020)’s work is a stagnant-lid thickness nondimensionalised by the layer depth, the same construction that drives the Ra_i exponent down to 0.001–0.058 in our own nondimensional fits. Their Ra_{har} is defined on the viscosity near the base of the lid rather than on the well-mixed interior, so the correspondence is qualitative; but once the normalisation is accounted for, the two results agree.

Grigné (2023) reaches a related conclusion from a different direction, showing that several widely used viscosity laws and normalisation choices for stagnant-lid convection give materially different temperature and heat-flux predictions for the same underlying physics. Taken with our result, this suggests that the choice of normalising variable for temperature-dependent-viscosity convection remains an open methodological question. That is so even before additional rheological ingredients such as GBS creep, viscoelasticity, or compressibility are introduced.

4.3 A physical basis for separating thermal scaling from surface deformation

The decoupling of effects of other physical ingredients has a physical basis, not just an empirical one. Elastic stress relaxes on the Maxwell timescale (η/μ , from minutes at our lower viscosity cap to of order 10^3 yr at our imposed higher viscosity cap (Section 4.5), and of order days at the viscosity of the well-mixed interior). This is far shorter than the Myr timescale over which the thermal boundary layer develops. Once relaxed, deformation reverts to the same creep law that sets d_{rh} , so elasticity leaves no lasting imprint on the time-averaged boundary-layer geometry. In our models, every case develops a stagnant lid, so the surface velocity is negligible compared with the convective velocity and surface velocity is not coupled to the upwellings and downwellings. The free-surface boundary condition therefore leaves no discernible imprint either. Compressibility is the only partial exception, since shear and adiabatic heating act as source terms throughout the convecting layer, including near d_{rh} and d_{sl} , rather than only at the surface, consistent with the small effect reported above. This is why elasticity, the top boundary condition and, mostly, compressibility can be layered onto the thermal scaling established here without revisiting that scaling itself. Therefore, elasticity and a free surface remain necessary where surface stress, topography, or lid mobility are the object of study. As discussed in Thielmann et al. (2015), a free surface raises stresses in the stagnant lid by up to two orders of magnitude, with the largest effect at the low Deborah numbers applicable to icy shells. Our result is that this does not propagate into the thermal solution, so a viscoplastic model with a free-slip surface reproduces the same heat flux, lid thickness, and boundary-layer geometry at lower cost, provided the surface is not itself the object of study.

4.4 Applicability to Europa’s ice shell

Framing the above discussion within the context of Europa and dimensionalising k to $1.9257 \text{ W m}^{-1} \text{ K}^{-1}$ and $\Delta T = 160 \text{ K}$ (Table 1), the Nusselt numbers of Table 3 correspond to surface heat fluxes of $14.3\text{--}16.4 \text{ mW m}^{-2}$ at 40 km and $11.0\text{--}13.6 \text{ mW m}^{-2}$ at 60 km, against conductive values of 7.70 and 5.14 mW m^{-2} . These follow from the imposed temperature contrast and from a thermal conductivity held constant at its melting-point value. Therefore, we do compare them against published estimates of Europa’s surface heat flux, which assume a temperature-dependent conductivity roughly twice as large across a cold lid (Rhoden and Walker, 2027) and include internal heating that our models omit.

Our 20-km cases do not convect within 100 Myr while the 40- and 60-km cases do, placing the convective threshold of our suite between these thicknesses. This is consistent with Hussmann et al. (2002), who find convection initiating at 27 km, and with Nimmo and Manga (2009), who note that non-Newtonian rheologies require thicker layers still. It also falls inside the approximately 30-km conductive shell inferred from Juno microwave radiometry (Levin et al., 2025). Rhoden and Walker (2027) map Europa’s shell thickness on the assumption that the shell is conductive, but do suggest regardless that convection becomes likely above 30 km. It is the upper end of the Juno range (44.9 km), which is the regime our models describe. Europa’s most updated inferred thickness therefore does not by itself settle whether its shell convects.

The fact that ingredients discussed in Section 3.2 and 3.3 act on Nu only by moving d_{rh} , and only the creep mechanism moves d_{rh} appreciably (Section 4.1), implies that the uncertainty in modelling Europa’s thermal structure is located in the flow law rather than in the numerical approach chosen. In other words, Europa’s thermal regime depends on whether grain boundary sliding operates in its ice shell, which in turn depends on grain size. This changes the predicted surface heat flux by roughly a tenth at fixed thickness, whereas the treatment of elasticity, the surface boundary condition and compressibility can be simplified at little cost to the thermal result.

Interestingly, in Green et al. (2021), modelling the crystallization of Europa’s shell in one dimension with stagnant-lid scaling laws, find that the thickness of the conductive lid is almost insensitive to the imposed tidal heating and is instead controlled by the creep mechanism and grain size. They further find that grain-size-sensitive creep yields a thinner lid than diffusion creep. Our results reproduce both the insensitivity and its sign through a different diagnostic: grain boundary sliding is the only ingredient that displaces d_{rh} , raising Nu by 11 % at 40 km and 13 % at 60 km.

This is important because Europa’s surface heat flux and shell thickness are routinely obtained from scalings of this kind, and the studies compared in Section 4.2 normalise them differently from one another. Differences between such estimates therefore need not reflect differences in the physics modelled. Instead, part of the spread is attributable to the choice of normalising variable.

4.5 Limitations

The Rayleigh number range tested spans only a factor of 4–5 (from two thickness clusters, 40 and 60 km), and any fitted exponent should be read as provisional to that range rather than as a validated general scaling law. The leave-one-out cross validation stability reported in Table 4 bounds how much the fitted exponents could plausibly move under this constraint, but does not substitute for a wider Ra range.

The convection regimes of our scaling laws are different from that of Harel et al. (2020). The scaling laws in our study are tested for stationary and intermediate convection regimes, while that in Harel et al. (2020) was tested for a time-dependent convection regime. This is because our settings focused on convection within the possible ice shell thickness of Europa; near 30-km conductive ice shell thicknesses as found from the Juno mission’s data (Levin et al., 2025) and allowing additional thickness up to 60 km for the convection layer. Therefore, Ra_i of our models is limited up to near $1 \cdot 10^7$, which is inevitably smaller than Ra_i of time-dependent convection. This may have caused different trends in temperature dependencies on viscosity. Ra_i increases in these thicknesses by realistic internal heating on Europa (i.e., tidal heating), but the effects of internal heating on the scaling law of the ice shell was studied in Deschamps and Vilella (2021).

We observed both stationary and intermediate convection regimes in the 60-km ice shell, whereas all 16 of the 40-km cases were stationary. However, the convection regime is not tied to the value of Ra_i or combinations of physical ingredients (Table 3). Intermediate regimes occur more often in the cases that include GBS creep, but stationary regimes also occur with GBS (Table 3), so the control on the convection regime may be more complex than any of these alone. A further possibility is that the convection regime changes after our 100-Myr runtime, which sits at the old end of estimates for Europa’s surface age. Every case did reach thermal equilibrium within that runtime, and both regimes appear among the cases that equilibrated before 60 Myr (Table 3), so regime is not simply a function of how long a model was run. Five cases nonetheless equilibrated after 90 Myr, leaving little of the run in which to observe the equilibrated state, and for those we cannot exclude a later change of regime.

Furthermore, the viscosity in our models is limited to the range 10^{12} – 10^{20} Pa s (Table 1). The upper bound/cap is reached in the cold near-surface ice, where the unmodified composite flow law would give substantially higher values, so the viscosity contrast across the layer is set in part by this numerical limit rather than by the rheology alone. Two of our arguments depend on it. The Maxwell relaxation time quoted in Section 4.3 is evaluated at the cap and would be longer without it, so the conclusion that elasticity leaves no imprint on the boundary-layer geometry holds for the capped rheology we solve rather than for ice in general. Separately, Grigné (2023) shows that the normalisations used for stagnant-lid convection can fail at large viscosity contrast, and we have not established that the contrast our models reach lies within the range over which those normalisations have been validated.

Finally, our basal boundary is a fixed, laterally uniform temperature. Ocean circulation beneath the shell is expected to deliver a heat flux that varies by several tens of per cent with latitude, with the pattern depending on the convective regime of the ocean (Soderlund

et al., 2020, 2014), and the resulting melting and crystallization at the base feed back on shell thickness. A laterally varying basal condition would modulate the local boundary-layer thickness and therefore the local Nu , but because our scaling collapses onto d_{rh} rather than onto the imposed temperature contrast, we expect it to shift the position of the shell along the scaling rather than to alter the scaling itself.

5 Conclusion

We tested whether the γ_{rh-std} -based normalisation developed for composite-creep stagnant-lid convection extends to the additional physical ingredients needed to model Europa’s surface deformation realistically: visco-elasto-plastic rheology, a free-surface boundary condition, and compressible flow under the anelastic liquid approximation. We find that it extends only partially. γ_{rh} tracks ice shell thickness rather than cleanly isolating the temperature dependence of viscosity, and γ_{rh-std} depends only on the diffusion-creep fraction, so it cannot distinguish between our two shell thicknesses. In their place, the dimensional depth of the rheological sublayer, d_{rh} , is the best-performing of the six normalising variables we tested. It also keeps the Ra_i and the boundary-layer geometry as separable terms. Dividing the depth by the shell thickness makes the two redundant with one another, and the fitted Ra_i exponent collapses to zero.

Of the physical ingredients we tested beyond composite creep rheology, only the presence of grain boundary sliding significantly affects Nu , and it does so by changing the boundary-layer geometry instead of the relationship between Nu and that geometry. We find that across all 32 cases $Nu d_{rh}/d_{shell}$ is constant to within 0.9 %, and none of the four ingredients shifts it in a detectable way. Elasticity and the free-surface top boundary condition leave d_{rh} itself unchanged, and compressibility changes it by a small and less consistently detected amount.

This gives a physical basis for treating the thermal scaling established here separately from elasticity-, free-surface-, and compressibility-driven surface stress and topography addressed in a companion study. The ingredients needed to model Europa’s surface deformation realistically can be added to composite-creep models without revisiting their thermal scaling. However, what does matter, independent of any of these additions, is the choice of the normalising variable itself, which remains an open methodological question for temperature-dependent-viscosity convection.

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7 Data availability

The parameter files of the numerical models and python codes for model analyses in the study are available via download at Zenodo (<https://doi.org/10.5281/zenodo.23196440>). ASPECT version 3.1.0-pre (Bangerth et al., 2024; Clevenger and Heister, 2021; Heister et al., 2017; Kronbichler et al., 2012; Rose et al., 2017) used in these computations is freely available under the GPL v2.0 or later license through its software landing page <https://geodynamics.org/resources/aspect> or <https://aspect.geodynamics.org> and is being actively developed on GitHub and can be accessed via (<https://github.com/geodynamics/aspect>).

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Supplementary materials for A scaling law for stagnant-lid convection with composite visco-elasto-plastic rheology: application to Europa’s ice shell

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S1 Models not included in scaling law tests

Tables S1 to S2 show the combinations of physical ingredients that were run but not included in the scaling law tests. Simulations with 20-km ice shell thickness in Table S1 were omitted from the scaling law as they do not have convection until the end of runtime. Simulations with diffusion creep in Table S2 were only used to compare γ_{rh} of other creep mechanisms (Fig. 1 and 2).

Table S1: The 10 non-convecting 20-km simulations in this study. VP and VEP denote viscoplastic and visco-elasto-plastic deformation; diff, disl, and gbs denote diffusion, dislocation, and grain boundary sliding creep; BA and ALA denote the Boussinesq and anelastic liquid approximations.

Number	Rheology	Top boundary	Creep	Compressibility
S1	VP	free-slip	diff-disl	BA
S2–S5	VP	free-surface	diff-disl / diff-gbs-disl	BA / ALA
S6	VEP	free-slip	diff-disl	BA
S7–S10	VEP	free-surface	diff-disl / diff-gbs-disl	BA / ALA

Table S2: The 4 simulations with diffusion-creep mechanism in this study, showing model settings and results of γ_{rh} . VP and VEP denote viscoplastic and visco-elasto-plastic deformation; BA denotes the Boussinesq approximations.

Number	Thickness [km]	Rheology	Top boundary	Compressibility	γ_{rh}
S11	40	VP	free-slip	BA	16.52
S12	40	VEP	free-surface	BA	18.30
S13	60	VP	free-slip	BA	24.66
S14	60	VEP	free-surface	BA	23.73

15 S2 Temperature of well mixed interior

16 Table S3 shows the temperature of the well-mixed interior of all the cases
 17 included in the scaling law tests. The measurement of it follows Harel et al.
 (2020).

Table S3: Convective variable, T_i [K], for 32 convecting simulations, where T_i is the temperature of well-mixed interior.

Number	T_i
1	261.2
2	260.2
3	261.0
4	260.0
5	261.1
6	260.3
7	261.1
8	260.4
9	261.3
10	260.2
11	261.0
12	260.2
13	261.3
14	260.2
15	261.0
16	259.9
17	260.8
18	260.5
19	261.0
20	260.5
21	261.1
22	261.0
23	261.3
24	260.5
25	261.0
26	260.7
27	261.3
28	260.2
29	260.9
30	260.1
31	261.1
32	260.4

S3 Leave-one-out cross validation results

Table S4 to S26 show the results of leave-one-out cross validation of each nested model. Left case denotes the case number that is omitted (Table 2), R^2 is the coefficient of determination. A is the coefficient of scaling law. B is the exponent of Ra_i , C is the exponent of X (second variable), D is the coefficient of binary dummy variables. Their 95 % confidence interval (CI) is also noted in square bracket. The detail explanation is in Section 2.6. Note that effects of coefficients and exponents are undetectable when their 95 % CI spans over unity and zero, respectively.

Table S4: Comparison of nested model $Nu = A Ra_i^B$ in 32 leave-one-out cross validation. The predictive R^2 is 0.7666.

Left case	R^2	A (95% CI)	B (95% CI)
1	0.787	0.183 [0.112, 0.298]	0.161 [0.129, 0.193]
2	0.798	0.172 [0.106, 0.280]	0.165 [0.133, 0.196]
3	0.787	0.185 [0.113, 0.301]	0.160 [0.129, 0.192]
4	0.794	0.173 [0.106, 0.283]	0.164 [0.132, 0.196]
5	0.787	0.185 [0.114, 0.302]	0.160 [0.129, 0.192]
6	0.801	0.171 [0.105, 0.276]	0.165 [0.134, 0.196]
7	0.787	0.183 [0.112, 0.299]	0.161 [0.129, 0.193]
8	0.809	0.167 [0.104, 0.269]	0.166 [0.136, 0.197]
9	0.787	0.183 [0.112, 0.299]	0.161 [0.129, 0.193]
10	0.797	0.172 [0.106, 0.280]	0.165 [0.133, 0.196]
11	0.787	0.183 [0.112, 0.299]	0.161 [0.129, 0.193]
12	0.803	0.170 [0.105, 0.275]	0.166 [0.134, 0.197]
13	0.787	0.184 [0.113, 0.300]	0.160 [0.129, 0.192]
14	0.795	0.173 [0.106, 0.283]	0.164 [0.133, 0.196]
15	0.787	0.185 [0.114, 0.302]	0.160 [0.128, 0.192]
16	0.794	0.174 [0.107, 0.284]	0.164 [0.132, 0.196]
17	0.795	0.174 [0.107, 0.284]	0.164 [0.132, 0.196]
18	0.791	0.188 [0.117, 0.303]	0.159 [0.128, 0.190]
19	0.793	0.175 [0.108, 0.286]	0.164 [0.132, 0.195]
20	0.787	0.181 [0.111, 0.295]	0.162 [0.130, 0.193]
21	0.796	0.173 [0.106, 0.282]	0.164 [0.133, 0.196]
22	0.786	0.182 [0.111, 0.297]	0.161 [0.129, 0.193]
23	0.816	0.165 [0.104, 0.264]	0.168 [0.137, 0.198]
24	0.788	0.182 [0.112, 0.297]	0.161 [0.129, 0.193]
25	0.798	0.172 [0.106, 0.279]	0.165 [0.133, 0.196]
26	0.788	0.186 [0.114, 0.301]	0.160 [0.128, 0.191]
27	0.829	0.162 [0.103, 0.254]	0.169 [0.140, 0.198]
28	0.792	0.187 [0.116, 0.301]	0.159 [0.128, 0.190]
29	0.798	0.172 [0.106, 0.279]	0.165 [0.133, 0.196]
30	0.793	0.189 [0.118, 0.304]	0.158 [0.128, 0.189]
31	0.808	0.168 [0.105, 0.271]	0.166 [0.136, 0.197]
32	0.791	0.187 [0.115, 0.301]	0.159 [0.128, 0.191]

Table S5: Comparison of nested model $Nu = A Ra_i^B \gamma_{rh}^C$ in 32 leave-one-out cross validation. The predictive R^2 is 0.8905.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.906	0.252	[0.177, 0.357]	0.232	[0.200, 0.265]	-0.485	[-0.652, -0.318]
2	0.918	0.239	[0.172, 0.334]	0.238	[0.207, 0.269]	-0.497	[-0.657, -0.338]
3	0.907	0.256	[0.181, 0.362]	0.231	[0.199, 0.263]	-0.484	[-0.648, -0.319]
4	0.911	0.243	[0.172, 0.344]	0.235	[0.203, 0.267]	-0.487	[-0.652, -0.322]
5	0.904	0.248	[0.175, 0.351]	0.235	[0.201, 0.269]	-0.492	[-0.664, -0.319]
6	0.915	0.239	[0.170, 0.337]	0.235	[0.204, 0.267]	-0.484	[-0.646, -0.322]
7	0.908	0.257	[0.181, 0.363]	0.232	[0.200, 0.264]	-0.488	[-0.652, -0.323]
8	0.910	0.245	[0.171, 0.350]	0.233	[0.201, 0.266]	-0.479	[-0.653, -0.304]
9	0.907	0.253	[0.179, 0.359]	0.232	[0.200, 0.264]	-0.485	[-0.651, -0.319]
10	0.912	0.242	[0.171, 0.343]	0.235	[0.203, 0.267]	-0.485	[-0.650, -0.320]
11	0.911	0.261	[0.186, 0.368]	0.232	[0.201, 0.263]	-0.495	[-0.657, -0.333]
12	0.910	0.247	[0.173, 0.353]	0.234	[0.201, 0.266]	-0.484	[-0.656, -0.312]
13	0.907	0.254	[0.179, 0.360]	0.232	[0.199, 0.264]	-0.484	[-0.649, -0.318]
14	0.912	0.243	[0.172, 0.343]	0.235	[0.203, 0.267]	-0.488	[-0.653, -0.323]
15	0.908	0.257	[0.182, 0.363]	0.231	[0.199, 0.263]	-0.484	[-0.648, -0.321]
16	0.909	0.246	[0.173, 0.350]	0.234	[0.202, 0.266]	-0.486	[-0.653, -0.319]
17	0.913	0.262	[0.185, 0.372]	0.236	[0.204, 0.268]	-0.519	[-0.690, -0.347]
18	0.908	0.256	[0.182, 0.360]	0.229	[0.197, 0.262]	-0.475	[-0.639, -0.312]
19	0.913	0.261	[0.184, 0.369]	0.236	[0.204, 0.267]	-0.514	[-0.683, -0.345]
20	0.915	0.245	[0.176, 0.341]	0.244	[0.211, 0.277]	-0.536	[-0.706, -0.366]
21	0.909	0.253	[0.177, 0.360]	0.234	[0.202, 0.266]	-0.495	[-0.666, -0.324]
22	0.908	0.244	[0.173, 0.345]	0.239	[0.205, 0.273]	-0.509	[-0.679, -0.338]
23	0.911	0.241	[0.168, 0.344]	0.232	[0.200, 0.265]	-0.469	[-0.643, -0.294]
24	0.906	0.247	[0.174, 0.349]	0.236	[0.202, 0.270]	-0.497	[-0.668, -0.326]
25	0.910	0.246	[0.173, 0.350]	0.234	[0.202, 0.266]	-0.483	[-0.652, -0.315]
26	0.904	0.251	[0.177, 0.355]	0.232	[0.199, 0.266]	-0.482	[-0.651, -0.312]
27	0.928	0.228	[0.166, 0.312]	0.234	[0.205, 0.263]	-0.459	[-0.609, -0.308]
28	0.905	0.251	[0.178, 0.355]	0.231	[0.198, 0.264]	-0.476	[-0.646, -0.307]
29	0.909	0.249	[0.175, 0.356]	0.234	[0.201, 0.266]	-0.488	[-0.659, -0.317]
30	0.906	0.254	[0.181, 0.358]	0.229	[0.196, 0.262]	-0.471	[-0.637, -0.306]
31	0.913	0.240	[0.169, 0.339]	0.233	[0.202, 0.265]	-0.473	[-0.639, -0.306]
32	0.906	0.253	[0.179, 0.357]	0.230	[0.198, 0.263]	-0.477	[-0.643, -0.310]

Table S6: Comparison of nested model $Nu = A Ra_i^B \gamma_{rh-std}^C$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9651.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.971	0.609	[0.468, 0.792]	0.159	[0.147, 0.171]	-0.383	[-0.442, -0.324]
2	0.973	0.642	[0.490, 0.841]	0.157	[0.145, 0.169]	-0.389	[-0.448, -0.330]
3	0.970	0.610	[0.468, 0.795]	0.158	[0.146, 0.171]	-0.380	[-0.440, -0.321]
4	0.973	0.634	[0.485, 0.829]	0.157	[0.145, 0.169]	-0.385	[-0.444, -0.327]
5	0.970	0.610	[0.468, 0.795]	0.158	[0.146, 0.170]	-0.380	[-0.439, -0.320]
6	0.972	0.619	[0.470, 0.815]	0.158	[0.146, 0.170]	-0.382	[-0.442, -0.322]
7	0.971	0.610	[0.469, 0.792]	0.159	[0.147, 0.171]	-0.384	[-0.443, -0.324]
8	0.973	0.586	[0.448, 0.767]	0.159	[0.148, 0.171]	-0.373	[-0.431, -0.314]
9	0.971	0.609	[0.468, 0.793]	0.159	[0.147, 0.171]	-0.383	[-0.442, -0.323]
10	0.971	0.614	[0.467, 0.806]	0.158	[0.146, 0.170]	-0.380	[-0.440, -0.321]
11	0.971	0.610	[0.469, 0.793]	0.159	[0.147, 0.171]	-0.384	[-0.443, -0.324]
12	0.972	0.611	[0.463, 0.805]	0.158	[0.146, 0.170]	-0.379	[-0.440, -0.319]
13	0.970	0.610	[0.468, 0.794]	0.159	[0.147, 0.171]	-0.382	[-0.441, -0.322]
14	0.972	0.627	[0.479, 0.822]	0.157	[0.145, 0.169]	-0.384	[-0.443, -0.325]
15	0.970	0.610	[0.468, 0.795]	0.158	[0.146, 0.170]	-0.380	[-0.440, -0.320]
16	0.973	0.640	[0.490, 0.834]	0.157	[0.145, 0.168]	-0.387	[-0.445, -0.329]
17	0.972	0.622	[0.475, 0.815]	0.157	[0.145, 0.169]	-0.382	[-0.441, -0.323]
18	0.970	0.609	[0.468, 0.791]	0.157	[0.145, 0.169]	-0.375	[-0.434, -0.315]
19	0.973	0.633	[0.486, 0.824]	0.157	[0.145, 0.169]	-0.385	[-0.443, -0.327]
20	0.971	0.608	[0.467, 0.793]	0.159	[0.146, 0.171]	-0.380	[-0.439, -0.321]
21	0.972	0.621	[0.473, 0.814]	0.158	[0.145, 0.170]	-0.382	[-0.441, -0.323]
22	0.970	0.610	[0.467, 0.795]	0.158	[0.146, 0.170]	-0.379	[-0.438, -0.320]
23	0.976	0.568	[0.442, 0.730]	0.161	[0.149, 0.172]	-0.368	[-0.423, -0.313]
24	0.970	0.610	[0.468, 0.796]	0.158	[0.146, 0.170]	-0.379	[-0.438, -0.320]
25	0.971	0.615	[0.468, 0.808]	0.158	[0.146, 0.170]	-0.381	[-0.440, -0.321]
26	0.970	0.611	[0.469, 0.795]	0.158	[0.145, 0.170]	-0.377	[-0.436, -0.317]
27	0.981	0.546	[0.436, 0.683]	0.162	[0.152, 0.172]	-0.360	[-0.409, -0.311]
28	0.970	0.608	[0.467, 0.792]	0.158	[0.146, 0.170]	-0.376	[-0.436, -0.316]
29	0.971	0.609	[0.464, 0.800]	0.158	[0.146, 0.170]	-0.379	[-0.439, -0.319]
30	0.971	0.608	[0.469, 0.788]	0.157	[0.145, 0.169]	-0.372	[-0.431, -0.314]
31	0.973	0.588	[0.450, 0.768]	0.159	[0.148, 0.171]	-0.373	[-0.431, -0.314]
32	0.971	0.611	[0.472, 0.792]	0.157	[0.145, 0.169]	-0.374	[-0.432, -0.316]

Table S7: Comparison of nested model $Nu = A Ra_i^B d_{rh}^C$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9954.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.996	0.379	[0.351, 0.409]	0.275	[0.268, 0.282]	-0.819	[-0.861, -0.777]
2	0.997	0.387	[0.359, 0.417]	0.275	[0.268, 0.281]	-0.824	[-0.865, -0.783]
3	0.996	0.379	[0.351, 0.410]	0.275	[0.267, 0.282]	-0.818	[-0.862, -0.775]
4	0.997	0.385	[0.356, 0.415]	0.274	[0.267, 0.281]	-0.820	[-0.862, -0.779]
5	0.996	0.380	[0.351, 0.410]	0.275	[0.267, 0.282]	-0.817	[-0.861, -0.774]
6	0.996	0.381	[0.352, 0.413]	0.274	[0.267, 0.282]	-0.818	[-0.861, -0.774]
7	0.996	0.378	[0.351, 0.408]	0.275	[0.268, 0.283]	-0.820	[-0.862, -0.778]
8	0.997	0.375	[0.347, 0.405]	0.274	[0.267, 0.281]	-0.810	[-0.852, -0.767]
9	0.996	0.379	[0.351, 0.409]	0.275	[0.268, 0.282]	-0.818	[-0.861, -0.776]
10	0.997	0.386	[0.358, 0.416]	0.275	[0.268, 0.281]	-0.822	[-0.864, -0.781]
11	0.996	0.379	[0.351, 0.408]	0.275	[0.268, 0.283]	-0.820	[-0.862, -0.777]
12	0.996	0.380	[0.351, 0.412]	0.274	[0.267, 0.282]	-0.817	[-0.860, -0.773]
13	0.996	0.380	[0.351, 0.410]	0.275	[0.267, 0.282]	-0.817	[-0.861, -0.774]
14	0.996	0.383	[0.354, 0.414]	0.274	[0.267, 0.281]	-0.819	[-0.861, -0.776]
15	0.996	0.379	[0.351, 0.410]	0.275	[0.267, 0.282]	-0.819	[-0.862, -0.776]
16	0.997	0.384	[0.356, 0.415]	0.274	[0.267, 0.281]	-0.820	[-0.861, -0.778]
17	0.996	0.379	[0.351, 0.410]	0.274	[0.267, 0.282]	-0.816	[-0.859, -0.773]
18	0.996	0.380	[0.352, 0.411]	0.274	[0.266, 0.281]	-0.813	[-0.857, -0.770]
19	0.996	0.383	[0.355, 0.413]	0.274	[0.267, 0.281]	-0.818	[-0.860, -0.776]
20	0.996	0.381	[0.353, 0.412]	0.274	[0.267, 0.281]	-0.816	[-0.858, -0.773]
21	0.996	0.380	[0.351, 0.411]	0.274	[0.267, 0.282]	-0.816	[-0.859, -0.773]
22	0.996	0.380	[0.351, 0.411]	0.274	[0.267, 0.282]	-0.817	[-0.859, -0.774]
23	0.997	0.372	[0.345, 0.401]	0.274	[0.267, 0.280]	-0.805	[-0.847, -0.763]
24	0.996	0.382	[0.354, 0.412]	0.274	[0.266, 0.281]	-0.814	[-0.856, -0.772]
25	0.996	0.380	[0.351, 0.412]	0.274	[0.267, 0.282]	-0.817	[-0.860, -0.773]
26	0.996	0.380	[0.352, 0.411]	0.274	[0.267, 0.282]	-0.815	[-0.859, -0.772]
27	0.997	0.371	[0.343, 0.401]	0.273	[0.266, 0.280]	-0.802	[-0.845, -0.758]
28	0.996	0.381	[0.354, 0.410]	0.273	[0.266, 0.280]	-0.810	[-0.852, -0.768]
29	0.996	0.378	[0.349, 0.408]	0.274	[0.267, 0.281]	-0.814	[-0.857, -0.771]
30	0.996	0.380	[0.352, 0.410]	0.275	[0.267, 0.283]	-0.820	[-0.864, -0.775]
31	0.996	0.379	[0.350, 0.411]	0.274	[0.267, 0.281]	-0.815	[-0.859, -0.771]
32	0.996	0.381	[0.353, 0.411]	0.273	[0.266, 0.281]	-0.813	[-0.856, -0.770]

Table S8: Comparison of nested model $Nu \sim A Ra_i^B d_{sl}^C$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9787.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.982	0.373	[0.315, 0.442]	0.256	[0.241, 0.270]	-0.756	[-0.846, -0.666]
2	0.984	0.389	[0.328, 0.460]	0.255	[0.241, 0.269]	-0.768	[-0.856, -0.680]
3	0.981	0.373	[0.315, 0.442]	0.256	[0.241, 0.271]	-0.757	[-0.848, -0.667]
4	0.983	0.385	[0.324, 0.456]	0.255	[0.241, 0.269]	-0.761	[-0.850, -0.673]
5	0.981	0.375	[0.316, 0.445]	0.254	[0.239, 0.269]	-0.751	[-0.842, -0.660]
6	0.982	0.375	[0.315, 0.447]	0.255	[0.240, 0.269]	-0.752	[-0.843, -0.661]
7	0.982	0.373	[0.315, 0.441]	0.256	[0.241, 0.271]	-0.758	[-0.848, -0.669]
8	0.983	0.365	[0.307, 0.432]	0.254	[0.240, 0.268]	-0.740	[-0.829, -0.652]
9	0.982	0.373	[0.315, 0.443]	0.255	[0.241, 0.270]	-0.755	[-0.845, -0.665]
10	0.983	0.384	[0.324, 0.456]	0.255	[0.241, 0.269]	-0.763	[-0.852, -0.673]
11	0.982	0.372	[0.315, 0.440]	0.257	[0.242, 0.271]	-0.761	[-0.850, -0.672]
12	0.982	0.372	[0.312, 0.444]	0.254	[0.240, 0.269]	-0.749	[-0.840, -0.658]
13	0.981	0.374	[0.315, 0.444]	0.255	[0.240, 0.270]	-0.754	[-0.845, -0.663]
14	0.983	0.381	[0.321, 0.453]	0.255	[0.240, 0.269]	-0.758	[-0.847, -0.668]
15	0.981	0.374	[0.315, 0.443]	0.256	[0.241, 0.270]	-0.756	[-0.847, -0.665]
16	0.983	0.384	[0.324, 0.455]	0.255	[0.241, 0.269]	-0.760	[-0.848, -0.672]
17	0.984	0.367	[0.313, 0.432]	0.256	[0.242, 0.270]	-0.752	[-0.836, -0.668]
18	0.981	0.375	[0.316, 0.445]	0.254	[0.239, 0.269]	-0.749	[-0.841, -0.657]
19	0.982	0.375	[0.315, 0.445]	0.255	[0.240, 0.269]	-0.752	[-0.841, -0.662]
20	0.983	0.382	[0.324, 0.451]	0.253	[0.239, 0.267]	-0.752	[-0.838, -0.666]
21	0.982	0.372	[0.313, 0.441]	0.255	[0.240, 0.269]	-0.749	[-0.839, -0.660]
22	0.982	0.379	[0.320, 0.449]	0.254	[0.239, 0.268]	-0.750	[-0.839, -0.662]
23	0.983	0.363	[0.306, 0.431]	0.254	[0.240, 0.268]	-0.736	[-0.826, -0.647]
24	0.983	0.380	[0.322, 0.449]	0.253	[0.239, 0.267]	-0.749	[-0.836, -0.662]
25	0.983	0.368	[0.311, 0.436]	0.255	[0.241, 0.269]	-0.747	[-0.835, -0.659]
26	0.981	0.376	[0.317, 0.446]	0.254	[0.239, 0.269]	-0.748	[-0.839, -0.658]
27	0.983	0.363	[0.304, 0.433]	0.253	[0.239, 0.268]	-0.733	[-0.827, -0.639]
28	0.981	0.376	[0.318, 0.445]	0.253	[0.238, 0.268]	-0.745	[-0.835, -0.654]
29	0.983	0.367	[0.311, 0.433]	0.255	[0.241, 0.269]	-0.747	[-0.833, -0.661]
30	0.981	0.375	[0.316, 0.445]	0.254	[0.239, 0.269]	-0.748	[-0.841, -0.656]
31	0.982	0.377	[0.316, 0.450]	0.255	[0.240, 0.269]	-0.755	[-0.848, -0.661]
32	0.983	0.379	[0.323, 0.446]	0.252	[0.238, 0.266]	-0.742	[-0.827, -0.656]

Table S9: Comparison of nested model $Nu \sim A Ra_i^B (d_{rh}/d_{shell})^C$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9949.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.996	0.940	[0.835, 1.057]	0.001	[-0.009, 0.012]	-0.961	[-1.016, -0.906]
2	0.996	0.951	[0.844, 1.073]	0.000	[-0.010, 0.011]	-0.964	[-1.018, -0.909]
3	0.995	0.940	[0.835, 1.058]	0.001	[-0.009, 0.012]	-0.959	[-1.014, -0.904]
4	0.996	0.952	[0.845, 1.072]	0.000	[-0.010, 0.011]	-0.963	[-1.017, -0.909]
5	0.995	0.941	[0.836, 1.058]	0.001	[-0.009, 0.012]	-0.957	[-1.012, -0.903]
6	0.996	0.928	[0.824, 1.047]	0.002	[-0.008, 0.013]	-0.956	[-1.010, -0.901]
7	0.996	0.940	[0.835, 1.057]	0.001	[-0.009, 0.011]	-0.962	[-1.016, -0.907]
8	0.996	0.923	[0.818, 1.042]	0.003	[-0.008, 0.013]	-0.953	[-1.008, -0.898]
9	0.996	0.940	[0.835, 1.058]	0.001	[-0.009, 0.012]	-0.961	[-1.016, -0.906]
10	0.996	0.945	[0.837, 1.066]	0.001	[-0.009, 0.011]	-0.961	[-1.016, -0.906]
11	0.996	0.940	[0.835, 1.058]	0.001	[-0.009, 0.012]	-0.961	[-1.016, -0.906]
12	0.996	0.932	[0.825, 1.052]	0.002	[-0.008, 0.013]	-0.957	[-1.012, -0.902]
13	0.995	0.940	[0.835, 1.058]	0.001	[-0.009, 0.012]	-0.960	[-1.014, -0.905]
14	0.996	0.936	[0.830, 1.055]	0.002	[-0.009, 0.012]	-0.959	[-1.013, -0.904]
15	0.995	0.940	[0.835, 1.058]	0.001	[-0.009, 0.012]	-0.959	[-1.014, -0.904]
16	0.996	0.950	[0.844, 1.070]	0.001	[-0.010, 0.011]	-0.962	[-1.016, -0.909]
17	0.996	0.928	[0.828, 1.039]	0.002	[-0.007, 0.012]	-0.957	[-1.009, -0.906]
18	0.995	0.939	[0.834, 1.057]	0.002	[-0.009, 0.012]	-0.958	[-1.013, -0.902]
19	0.996	0.931	[0.831, 1.042]	0.002	[-0.008, 0.012]	-0.959	[-1.010, -0.907]
20	0.996	0.946	[0.843, 1.061]	0.001	[-0.009, 0.011]	-0.958	[-1.011, -0.906]
21	0.996	0.936	[0.830, 1.055]	0.002	[-0.009, 0.012]	-0.958	[-1.013, -0.904]
22	0.997	0.960	[0.870, 1.059]	0.000	[-0.008, 0.009]	-0.959	[-1.004, -0.914]
23	0.996	0.940	[0.828, 1.067]	0.001	[-0.010, 0.012]	-0.960	[-1.017, -0.902]
24	0.996	0.942	[0.839, 1.058]	0.001	[-0.009, 0.011]	-0.957	[-1.010, -0.904]
25	0.996	0.935	[0.829, 1.055]	0.002	[-0.009, 0.012]	-0.958	[-1.013, -0.903]
26	0.996	0.940	[0.837, 1.055]	0.002	[-0.008, 0.012]	-0.955	[-1.009, -0.901]
27	0.996	0.967	[0.849, 1.101]	-0.001	[-0.013, 0.010]	-0.972	[-1.031, -0.913]
28	0.995	0.944	[0.839, 1.062]	0.001	[-0.009, 0.011]	-0.964	[-1.020, -0.908]
29	0.996	0.925	[0.824, 1.038]	0.003	[-0.007, 0.013]	-0.955	[-1.008, -0.903]
30	0.996	0.949	[0.845, 1.066]	0.000	[-0.010, 0.010]	-0.970	[-1.025, -0.914]
31	0.996	0.934	[0.825, 1.056]	0.002	[-0.009, 0.013]	-0.957	[-1.013, -0.901]
32	0.995	0.942	[0.837, 1.060]	0.001	[-0.009, 0.011]	-0.962	[-1.018, -0.906]

Table S10: Comparison of nested model $Nu \sim A Ra_i^B (d_{sl}/d_{shell})^C$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9724.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.975	0.828	[0.631, 1.088]	0.006	[-0.018, 0.031]	-0.859	[-0.980, -0.738]
2	0.977	0.862	[0.652, 1.139]	0.003	[-0.022, 0.028]	-0.870	[-0.991, -0.750]
3	0.975	0.830	[0.631, 1.090]	0.006	[-0.018, 0.030]	-0.859	[-0.981, -0.738]
4	0.977	0.857	[0.650, 1.130]	0.003	[-0.021, 0.028]	-0.867	[-0.986, -0.747]
5	0.975	0.830	[0.633, 1.090]	0.006	[-0.018, 0.031]	-0.853	[-0.973, -0.732]
6	0.976	0.815	[0.617, 1.078]	0.008	[-0.017, 0.033]	-0.852	[-0.972, -0.731]
7	0.975	0.829	[0.632, 1.088]	0.006	[-0.019, 0.030]	-0.862	[-0.983, -0.741]
8	0.977	0.795	[0.603, 1.047]	0.010	[-0.015, 0.034]	-0.843	[-0.962, -0.724]
9	0.975	0.829	[0.631, 1.089]	0.006	[-0.018, 0.030]	-0.859	[-0.980, -0.738]
10	0.976	0.845	[0.639, 1.119]	0.005	[-0.020, 0.030]	-0.863	[-0.984, -0.742]
11	0.976	0.830	[0.634, 1.088]	0.006	[-0.019, 0.030]	-0.865	[-0.985, -0.744]
12	0.977	0.812	[0.614, 1.073]	0.008	[-0.017, 0.033]	-0.850	[-0.971, -0.729]
13	0.975	0.829	[0.631, 1.089]	0.006	[-0.018, 0.031]	-0.858	[-0.979, -0.736]
14	0.976	0.835	[0.632, 1.104]	0.006	[-0.019, 0.030]	-0.859	[-0.980, -0.738]
15	0.975	0.829	[0.631, 1.090]	0.006	[-0.018, 0.031]	-0.858	[-0.980, -0.736]
16	0.977	0.852	[0.647, 1.124]	0.004	[-0.021, 0.029]	-0.865	[-0.984, -0.745]
17	0.981	0.815	[0.638, 1.041]	0.007	[-0.015, 0.029]	-0.865	[-0.972, -0.757]
18	0.974	0.827	[0.629, 1.088]	0.007	[-0.018, 0.031]	-0.854	[-0.978, -0.730]
19	0.977	0.819	[0.626, 1.070]	0.007	[-0.017, 0.031]	-0.857	[-0.974, -0.741]
20	0.978	0.857	[0.660, 1.112]	0.004	[-0.020, 0.027]	-0.861	[-0.974, -0.747]
21	0.977	0.816	[0.620, 1.074]	0.007	[-0.017, 0.032]	-0.853	[-0.972, -0.734]
22	0.979	0.866	[0.671, 1.118]	0.003	[-0.020, 0.026]	-0.862	[-0.973, -0.751]
23	0.977	0.805	[0.603, 1.074]	0.009	[-0.017, 0.035]	-0.845	[-0.970, -0.721]
24	0.977	0.841	[0.645, 1.097]	0.005	[-0.018, 0.029]	-0.854	[-0.970, -0.738]
25	0.978	0.806	[0.617, 1.053]	0.008	[-0.015, 0.032]	-0.851	[-0.967, -0.735]
26	0.975	0.830	[0.634, 1.087]	0.007	[-0.018, 0.031]	-0.850	[-0.969, -0.730]
27	0.976	0.823	[0.607, 1.115]	0.007	[-0.021, 0.034]	-0.854	[-0.986, -0.721]
28	0.975	0.829	[0.630, 1.090]	0.006	[-0.018, 0.031]	-0.856	[-0.980, -0.733]
29	0.979	0.804	[0.619, 1.044]	0.009	[-0.015, 0.032]	-0.852	[-0.966, -0.739]
30	0.974	0.828	[0.629, 1.089]	0.006	[-0.018, 0.031]	-0.855	[-0.979, -0.730]
31	0.976	0.833	[0.624, 1.110]	0.006	[-0.020, 0.032]	-0.858	[-0.983, -0.734]
32	0.976	0.828	[0.635, 1.079]	0.007	[-0.017, 0.031]	-0.846	[-0.963, -0.728]

Table S11: Comparison of nested model $Nu \sim A Ra_i^B \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9509.

Left case	R^2	A (95% CI)		B (95% CI)	
1	0.966	0.182	[0.147, 0.226]	0.158	[0.144, 0.172]
2	0.968	0.186	[0.151, 0.230]	0.157	[0.143, 0.170]
3	0.966	0.183	[0.147, 0.226]	0.158	[0.144, 0.172]
4	0.968	0.186	[0.151, 0.230]	0.157	[0.143, 0.170]
5	0.966	0.184	[0.149, 0.229]	0.157	[0.144, 0.171]
6	0.968	0.184	[0.148, 0.227]	0.158	[0.144, 0.171]
7	0.967	0.180	[0.146, 0.222]	0.159	[0.145, 0.172]
8	0.970	0.178	[0.145, 0.218]	0.160	[0.146, 0.173]
9	0.966	0.182	[0.147, 0.226]	0.158	[0.144, 0.172]
10	0.968	0.186	[0.151, 0.229]	0.157	[0.143, 0.170]
11	0.967	0.181	[0.146, 0.223]	0.159	[0.145, 0.172]
12	0.968	0.181	[0.147, 0.224]	0.159	[0.145, 0.172]
13	0.966	0.183	[0.148, 0.227]	0.158	[0.144, 0.172]
14	0.969	0.187	[0.152, 0.231]	0.157	[0.143, 0.170]
15	0.966	0.182	[0.147, 0.225]	0.158	[0.144, 0.172]
16	0.968	0.186	[0.151, 0.229]	0.157	[0.143, 0.170]
17	0.968	0.185	[0.150, 0.228]	0.157	[0.143, 0.171]
18	0.966	0.186	[0.151, 0.229]	0.157	[0.143, 0.170]
19	0.971	0.189	[0.154, 0.230]	0.156	[0.143, 0.169]
20	0.968	0.179	[0.146, 0.221]	0.159	[0.146, 0.173]
21	0.968	0.185	[0.150, 0.228]	0.157	[0.143, 0.171]
22	0.968	0.179	[0.146, 0.220]	0.159	[0.146, 0.173]
23	0.971	0.178	[0.145, 0.217]	0.160	[0.147, 0.173]
24	0.966	0.182	[0.147, 0.226]	0.158	[0.144, 0.172]
25	0.967	0.183	[0.148, 0.227]	0.158	[0.144, 0.172]
26	0.965	0.183	[0.148, 0.227]	0.158	[0.144, 0.172]
27	0.978	0.172	[0.144, 0.205]	0.162	[0.151, 0.173]
28	0.968	0.188	[0.153, 0.231]	0.156	[0.143, 0.170]
29	0.968	0.184	[0.149, 0.227]	0.158	[0.144, 0.171]
30	0.968	0.188	[0.153, 0.231]	0.156	[0.143, 0.169]
31	0.968	0.181	[0.146, 0.223]	0.159	[0.145, 0.172]
32	0.968	0.188	[0.153, 0.231]	0.156	[0.143, 0.170]

Table S12: D_j coefficient comparison of nested model $Nu \sim A Ra_i^B \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation.

Left case	D_1 (95% CI)		D_2 (95% CI)		D_3 (95% CI)		D_4 (95% CI)	
1	1.106	[0.036, 0.052]	0.998	[0.980, 1.016]	1.003	[0.985, 1.022]	0.989	[0.971, 1.007]
2	1.107	[0.036, 0.052]	0.996	[0.978, 1.014]	1.005	[0.987, 1.023]	0.987	[0.969, 1.005]
3	1.106	[0.036, 0.052]	0.997	[0.979, 1.016]	1.003	[0.985, 1.022]	0.988	[0.970, 1.006]
4	1.107	[0.036, 0.052]	0.996	[0.978, 1.014]	1.005	[0.987, 1.023]	0.990	[0.972, 1.008]
5	1.105	[0.035, 0.051]	0.997	[0.979, 1.015]	1.003	[0.985, 1.021]	0.988	[0.970, 1.006]
6	1.106	[0.036, 0.052]	0.997	[0.979, 1.015]	1.003	[0.985, 1.022]	0.988	[0.970, 1.006]
7	1.107	[0.036, 0.052]	0.999	[0.981, 1.017]	1.005	[0.987, 1.023]	0.987	[0.969, 1.005]
8	1.102	[0.035, 0.050]	1.000	[0.982, 1.017]	1.006	[0.989, 1.024]	0.986	[0.969, 1.003]
9	1.106	[0.036, 0.052]	0.997	[0.979, 1.015]	1.003	[0.985, 1.022]	0.989	[0.971, 1.007]
10	1.107	[0.036, 0.052]	0.999	[0.981, 1.017]	1.005	[0.987, 1.023]	0.987	[0.969, 1.005]
11	1.107	[0.036, 0.052]	0.996	[0.978, 1.014]	1.002	[0.984, 1.021]	0.987	[0.969, 1.005]
12	1.104	[0.035, 0.051]	0.996	[0.978, 1.014]	1.003	[0.984, 1.021]	0.987	[0.970, 1.005]
13	1.105	[0.036, 0.051]	0.997	[0.979, 1.015]	1.004	[0.985, 1.022]	0.988	[0.971, 1.007]
14	1.107	[0.036, 0.052]	0.999	[0.981, 1.017]	1.002	[0.984, 1.020]	0.987	[0.969, 1.004]
15	1.106	[0.036, 0.052]	0.997	[0.979, 1.015]	1.004	[0.986, 1.023]	0.988	[0.970, 1.006]
16	1.107	[0.036, 0.052]	0.999	[0.981, 1.017]	1.002	[0.984, 1.020]	0.990	[0.972, 1.008]
17	1.107	[0.036, 0.052]	0.998	[0.980, 1.017]	1.002	[0.984, 1.021]	0.989	[0.972, 1.007]
18	1.104	[0.035, 0.051]	0.999	[0.981, 1.017]	1.002	[0.984, 1.020]	0.990	[0.972, 1.008]
19	1.109	[0.037, 0.052]	1.000	[0.983, 1.017]	1.001	[0.984, 1.018]	0.986	[0.969, 1.003]
20	1.107	[0.036, 0.052]	0.996	[0.978, 1.014]	1.005	[0.987, 1.023]	0.990	[0.972, 1.008]
21	1.106	[0.036, 0.052]	0.998	[0.980, 1.016]	1.004	[0.986, 1.023]	0.989	[0.971, 1.007]
22	1.107	[0.037, 0.052]	0.995	[0.978, 1.013]	1.002	[0.984, 1.020]	0.986	[0.969, 1.004]
23	1.102	[0.035, 0.050]	0.994	[0.977, 1.012]	1.001	[0.984, 1.018]	0.991	[0.974, 1.008]
24	1.106	[0.036, 0.052]	0.997	[0.979, 1.015]	1.003	[0.985, 1.022]	0.989	[0.971, 1.007]
25	1.105	[0.036, 0.051]	0.997	[0.979, 1.016]	1.003	[0.985, 1.022]	0.988	[0.971, 1.007]
26	1.105	[0.035, 0.051]	0.997	[0.979, 1.015]	1.003	[0.985, 1.022]	0.988	[0.971, 1.007]
27	1.100	[0.035, 0.048]	1.002	[0.987, 1.017]	1.008	[0.993, 1.024]	0.993	[0.978, 1.008]
28	1.103	[0.035, 0.050]	0.995	[0.978, 1.013]	1.001	[0.984, 1.019]	0.986	[0.969, 1.004]
29	1.106	[0.036, 0.052]	0.997	[0.979, 1.015]	1.004	[0.986, 1.022]	0.989	[0.971, 1.007]
30	1.103	[0.035, 0.050]	0.995	[0.977, 1.012]	1.006	[0.989, 1.024]	0.991	[0.974, 1.008]
31	1.104	[0.035, 0.051]	0.998	[0.980, 1.017]	1.002	[0.984, 1.021]	0.990	[0.972, 1.008]
32	1.103	[0.035, 0.050]	0.995	[0.978, 1.013]	1.006	[0.988, 1.024]	0.986	[0.969, 1.004]

Table S13: Comparison of nested model $Nu \sim A Ra_i^B \gamma_{rh}^C \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9498.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.969	0.200	[0.156, 0.257]	0.177	[0.147, 0.207]	-0.126	[-0.306, 0.055]
2	0.970	0.200	[0.157, 0.253]	0.174	[0.142, 0.206]	-0.112	[-0.298, 0.074]
3	0.968	0.200	[0.156, 0.255]	0.177	[0.147, 0.207]	-0.125	[-0.302, 0.053]
4	0.970	0.200	[0.157, 0.253]	0.174	[0.142, 0.206]	-0.110	[-0.295, 0.075]
5	0.968	0.200	[0.157, 0.255]	0.176	[0.146, 0.207]	-0.123	[-0.301, 0.054]
6	0.970	0.199	[0.157, 0.254]	0.177	[0.146, 0.207]	-0.125	[-0.303, 0.053]
7	0.969	0.196	[0.153, 0.252]	0.176	[0.146, 0.206]	-0.115	[-0.296, 0.065]
8	0.972	0.192	[0.150, 0.244]	0.175	[0.145, 0.204]	-0.101	[-0.278, 0.076]
9	0.969	0.200	[0.156, 0.256]	0.177	[0.147, 0.207]	-0.125	[-0.305, 0.055]
10	0.971	0.201	[0.158, 0.254]	0.175	[0.144, 0.205]	-0.116	[-0.294, 0.062]
11	0.969	0.197	[0.154, 0.253]	0.176	[0.146, 0.206]	-0.116	[-0.299, 0.066]
12	0.971	0.198	[0.155, 0.251]	0.177	[0.147, 0.207]	-0.122	[-0.298, 0.055]
13	0.969	0.201	[0.157, 0.257]	0.177	[0.147, 0.207]	-0.127	[-0.306, 0.052]
14	0.971	0.201	[0.158, 0.254]	0.173	[0.142, 0.204]	-0.109	[-0.289, 0.071]
15	0.968	0.199	[0.156, 0.254]	0.177	[0.147, 0.207]	-0.123	[-0.300, 0.054]
16	0.970	0.200	[0.158, 0.254]	0.174	[0.143, 0.206]	-0.113	[-0.296, 0.069]
17	0.971	0.206	[0.161, 0.262]	0.178	[0.149, 0.208]	-0.143	[-0.320, 0.034]
18	0.969	0.202	[0.159, 0.256]	0.175	[0.145, 0.205]	-0.120	[-0.296, 0.055]
19	0.976	0.215	[0.172, 0.269]	0.182	[0.155, 0.209]	-0.176	[-0.341, -0.011]
20	0.972	0.200	[0.159, 0.250]	0.186	[0.156, 0.217]	-0.175	[-0.353, 0.003]
21	0.970	0.202	[0.159, 0.256]	0.176	[0.146, 0.206]	-0.126	[-0.302, 0.050]
22	0.973	0.201	[0.161, 0.250]	0.188	[0.158, 0.219]	-0.183	[-0.359, -0.008]
23	0.972	0.190	[0.149, 0.242]	0.173	[0.143, 0.202]	-0.086	[-0.267, 0.095]
24	0.969	0.199	[0.157, 0.252]	0.178	[0.148, 0.209]	-0.131	[-0.309, 0.047]
25	0.970	0.200	[0.157, 0.254]	0.177	[0.147, 0.207]	-0.125	[-0.301, 0.052]
26	0.968	0.199	[0.157, 0.254]	0.177	[0.146, 0.207]	-0.125	[-0.303, 0.053]
27	0.980	0.185	[0.151, 0.226]	0.177	[0.152, 0.202]	-0.101	[-0.247, 0.045]
28	0.970	0.205	[0.162, 0.259]	0.175	[0.146, 0.204]	-0.123	[-0.294, 0.048]
29	0.970	0.201	[0.158, 0.256]	0.177	[0.147, 0.206]	-0.126	[-0.303, 0.051]
30	0.970	0.203	[0.161, 0.255]	0.173	[0.144, 0.202]	-0.111	[-0.283, 0.060]
31	0.971	0.197	[0.154, 0.251]	0.176	[0.147, 0.206]	-0.118	[-0.296, 0.059]
32	0.971	0.206	[0.163, 0.261]	0.176	[0.147, 0.205]	-0.132	[-0.302, 0.038]

Table S14: D_j coefficient comparison of nested model $Nu \sim A Ra_i^B \gamma_{rh}^C \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation.

Left case	D_1 (95% CI)		D_2 (95% CI)		D_3 (95% CI)		D_4 (95% CI)	
1	1.087	[1.055, 1.120]	0.995	[0.977, 1.013]	1.003	[0.985, 1.021]	0.984	[0.965, 1.003]
2	1.090	[1.057, 1.125]	0.995	[0.977, 1.013]	1.004	[0.986, 1.022]	0.984	[0.966, 1.002]
3	1.087	[1.056, 1.120]	0.995	[0.977, 1.014]	1.003	[0.985, 1.021]	0.984	[0.966, 1.003]
4	1.090	[1.057, 1.125]	0.995	[0.977, 1.013]	1.004	[0.986, 1.022]	0.985	[0.967, 1.005]
5	1.087	[1.056, 1.119]	0.995	[0.977, 1.013]	1.003	[0.985, 1.021]	0.984	[0.966, 1.003]
6	1.087	[1.055, 1.120]	0.995	[0.977, 1.013]	1.003	[0.985, 1.021]	0.984	[0.966, 1.003]
7	1.090	[1.057, 1.123]	0.996	[0.978, 1.015]	1.004	[0.986, 1.022]	0.984	[0.965, 1.002]
8	1.089	[1.058, 1.120]	0.998	[0.980, 1.015]	1.005	[0.988, 1.023]	0.983	[0.965, 1.001]
9	1.087	[1.055, 1.120]	0.995	[0.977, 1.013]	1.003	[0.985, 1.021]	0.984	[0.965, 1.003]
10	1.090	[1.058, 1.123]	0.996	[0.978, 1.015]	1.004	[0.986, 1.022]	0.983	[0.965, 1.002]
11	1.089	[1.057, 1.123]	0.995	[0.977, 1.013]	1.002	[0.985, 1.020]	0.984	[0.966, 1.002]
12	1.087	[1.056, 1.119]	0.994	[0.977, 1.012]	1.002	[0.984, 1.020]	0.983	[0.965, 1.002]
13	1.087	[1.055, 1.120]	0.996	[0.978, 1.014]	1.003	[0.985, 1.021]	0.984	[0.965, 1.003]
14	1.091	[1.059, 1.125]	0.997	[0.979, 1.015]	1.002	[0.984, 1.020]	0.983	[0.965, 1.002]
15	1.088	[1.056, 1.121]	0.995	[0.977, 1.013]	1.003	[0.985, 1.021]	0.984	[0.965, 1.003]
16	1.090	[1.057, 1.124]	0.996	[0.978, 1.015]	1.002	[0.984, 1.020]	0.985	[0.967, 1.004]
17	1.087	[1.056, 1.118]	0.997	[0.979, 1.014]	1.001	[0.984, 1.019]	0.985	[0.967, 1.003]
18	1.087	[1.056, 1.119]	0.997	[0.979, 1.015]	1.002	[0.984, 1.020]	0.985	[0.967, 1.004]
19	1.084	[1.056, 1.113]	0.998	[0.982, 1.014]	0.999	[0.983, 1.015]	0.979	[0.962, 0.996]
20	1.083	[1.053, 1.114]	0.992	[0.975, 1.009]	1.006	[0.989, 1.023]	0.985	[0.968, 1.003]
21	1.088	[1.057, 1.120]	0.996	[0.978, 1.014]	1.004	[0.986, 1.022]	0.985	[0.967, 1.004]
22	1.082	[1.053, 1.112]	0.991	[0.974, 1.008]	0.999	[0.983, 1.016]	0.979	[0.961, 0.997]
23	1.090	[1.060, 1.122]	0.994	[0.976, 1.011]	1.001	[0.984, 1.018]	0.988	[0.969, 1.006]
24	1.087	[1.056, 1.119]	0.994	[0.976, 1.013]	1.002	[0.984, 1.020]	0.985	[0.966, 1.003]
25	1.087	[1.056, 1.120]	0.995	[0.977, 1.013]	1.003	[0.985, 1.021]	0.984	[0.966, 1.003]
26	1.087	[1.056, 1.120]	0.995	[0.978, 1.014]	1.003	[0.985, 1.021]	0.984	[0.965, 1.003]
27	1.085	[1.060, 1.112]	1.000	[0.985, 1.015]	1.008	[0.993, 1.023]	0.990	[0.974, 1.005]
28	1.085	[1.055, 1.117]	0.993	[0.976, 1.011]	1.001	[0.984, 1.018]	0.982	[0.964, 1.000]
29	1.088	[1.057, 1.120]	0.995	[0.977, 1.013]	1.003	[0.986, 1.022]	0.985	[0.966, 1.003]
30	1.087	[1.057, 1.118]	0.993	[0.976, 1.011]	1.005	[0.988, 1.023]	0.987	[0.969, 1.005]
31	1.087	[1.056, 1.119]	0.996	[0.978, 1.014]	1.002	[0.984, 1.020]	0.985	[0.967, 1.004]
32	1.084	[1.054, 1.115]	0.993	[0.976, 1.010]	1.005	[0.988, 1.023]	0.982	[0.964, 1.000]

Table S15: Comparison of nested model $Nu \sim A Ra_i^B \gamma_{rh-std}^C \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9565.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.972	0.710	[0.193, 2.603]	0.159	[0.146, 0.172]	-0.428	[-0.832, -0.024]
2	0.976	1.041	[0.283, 3.834]	0.156	[0.144, 0.168]	-0.534	[-0.935, -0.134]
3	0.971	0.709	[0.193, 2.608]	0.159	[0.146, 0.171]	-0.427	[-0.832, -0.022]
4	0.974	0.717	[0.200, 2.571]	0.157	[0.144, 0.170]	-0.423	[-0.818, -0.027]
5	0.971	0.697	[0.189, 2.569]	0.158	[0.145, 0.171]	-0.418	[-0.824, -0.013]
6	0.973	0.791	[0.207, 3.031]	0.157	[0.145, 0.170]	-0.455	[-0.868, -0.041]
7	0.973	0.711	[0.198, 2.555]	0.159	[0.147, 0.172]	-0.432	[-0.829, -0.035]
8	0.975	0.618	[0.173, 2.209]	0.160	[0.147, 0.172]	-0.389	[-0.783, 0.004]
9	0.972	0.723	[0.197, 2.658]	0.159	[0.146, 0.172]	-0.434	[-0.839, -0.029]
10	0.973	0.669	[0.175, 2.553]	0.158	[0.145, 0.171]	-0.405	[-0.823, 0.014]
11	0.973	0.745	[0.207, 2.680]	0.159	[0.147, 0.172]	-0.446	[-0.844, -0.048]
12	0.973	0.682	[0.178, 2.608]	0.158	[0.145, 0.171]	-0.413	[-0.827, 0.001]
13	0.971	0.712	[0.193, 2.626]	0.159	[0.146, 0.171]	-0.428	[-0.834, -0.022]
14	0.974	0.678	[0.188, 2.443]	0.157	[0.144, 0.170]	-0.405	[-0.803, -0.007]
15	0.971	0.698	[0.190, 2.560]	0.159	[0.146, 0.171]	-0.422	[-0.825, -0.019]
16	0.974	0.761	[0.212, 2.737]	0.157	[0.144, 0.169]	-0.441	[-0.837, -0.045]
17	0.973	0.679	[0.183, 2.521]	0.158	[0.145, 0.171]	-0.410	[-0.818, -0.001]
18	0.971	0.668	[0.180, 2.482]	0.158	[0.145, 0.170]	-0.403	[-0.812, 0.006]
19	0.975	0.603	[0.169, 2.151]	0.157	[0.144, 0.169]	-0.367	[-0.763, 0.030]
20	0.972	0.661	[0.157, 2.781]	0.158	[0.145, 0.171]	-0.404	[-0.845, 0.037]
21	0.973	0.688	[0.186, 2.540]	0.158	[0.145, 0.171]	-0.414	[-0.820, -0.008]
22	0.971	0.655	[0.141, 3.038]	0.158	[0.145, 0.171]	-0.401	[-0.871, 0.069]
23	0.978	0.784	[0.238, 2.580]	0.161	[0.149, 0.173]	-0.467	[-0.838, -0.097]
24	0.972	0.740	[0.191, 2.860]	0.158	[0.145, 0.171]	-0.436	[-0.853, -0.020]
25	0.973	0.702	[0.191, 2.590]	0.158	[0.145, 0.171]	-0.421	[-0.825, -0.017]
26	0.971	0.739	[0.197, 2.770]	0.158	[0.145, 0.170]	-0.435	[-0.843, -0.027]
27	0.982	0.539	[0.184, 1.577]	0.162	[0.151, 0.172]	-0.357	[-0.689, -0.026]
28	0.972	0.614	[0.161, 2.339]	0.157	[0.144, 0.170]	-0.375	[-0.794, 0.044]
29	0.973	0.716	[0.191, 2.683]	0.158	[0.145, 0.171]	-0.429	[-0.840, -0.017]
30	0.972	0.607	[0.165, 2.233]	0.157	[0.144, 0.169]	-0.370	[-0.777, 0.036]
31	0.974	0.711	[0.197, 2.564]	0.159	[0.146, 0.172]	-0.430	[-0.829, -0.032]
32	0.974	0.766	[0.220, 2.662]	0.156	[0.144, 0.169]	-0.440	[-0.826, -0.054]

Table S16: D_j coefficient comparison of nested model $Nu \sim A Ra_i^B \gamma_{rh-std}^C \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation.

Left case	D_1 (95% CI)		D_2 (95% CI)		D_3 (95% CI)		D_4 (95% CI)	
1	0.987	[0.886, 1.100]	0.998	[0.981, 1.015]	1.000	[0.983, 1.018]	0.993	[0.976, 1.011]
2	0.962	[0.865, 1.070]	0.995	[0.979, 1.011]	1.003	[0.987, 1.019]	0.991	[0.975, 1.007]
3	0.987	[0.886, 1.101]	0.998	[0.981, 1.015]	1.000	[0.983, 1.018]	0.992	[0.975, 1.009]
4	0.990	[0.890, 1.101]	0.996	[0.979, 1.013]	1.002	[0.986, 1.020]	0.994	[0.977, 1.011]
5	0.989	[0.887, 1.102]	0.997	[0.980, 1.014]	1.001	[0.984, 1.018]	0.992	[0.975, 1.010]
6	0.981	[0.878, 1.095]	0.996	[0.980, 1.013]	1.000	[0.982, 1.017]	0.992	[0.975, 1.009]
7	0.987	[0.887, 1.098]	0.999	[0.982, 1.016]	1.002	[0.985, 1.019]	0.991	[0.974, 1.008]
8	0.995	[0.895, 1.105]	0.999	[0.983, 1.016]	1.003	[0.986, 1.020]	0.990	[0.973, 1.007]
9	0.986	[0.884, 1.099]	0.997	[0.980, 1.014]	1.000	[0.983, 1.017]	0.993	[0.976, 1.011]
10	0.993	[0.887, 1.113]	0.998	[0.981, 1.015]	1.001	[0.984, 1.019]	0.992	[0.974, 1.010]
11	0.983	[0.884, 1.094]	0.996	[0.979, 1.013]	0.999	[0.982, 1.016]	0.991	[0.974, 1.008]
12	0.990	[0.887, 1.106]	0.997	[0.980, 1.014]	1.001	[0.984, 1.018]	0.992	[0.975, 1.010]
13	0.987	[0.885, 1.100]	0.997	[0.980, 1.014]	1.001	[0.984, 1.019]	0.993	[0.976, 1.011]
14	0.994	[0.893, 1.107]	0.999	[0.982, 1.016]	0.999	[0.983, 1.017]	0.991	[0.974, 1.008]
15	0.989	[0.887, 1.102]	0.997	[0.980, 1.014]	1.001	[0.984, 1.019]	0.992	[0.975, 1.009]
16	0.985	[0.886, 1.095]	0.999	[0.983, 1.016]	0.999	[0.982, 1.016]	0.994	[0.977, 1.012]
17	0.992	[0.888, 1.108]	0.998	[0.981, 1.015]	1.000	[0.983, 1.018]	0.993	[0.976, 1.010]
18	0.992	[0.889, 1.107]	0.998	[0.981, 1.015]	1.000	[0.983, 1.018]	0.993	[0.976, 1.010]
19	1.005	[0.903, 1.119]	1.000	[0.983, 1.016]	0.999	[0.983, 1.016]	0.990	[0.973, 1.007]
20	0.993	[0.881, 1.119]	0.997	[0.980, 1.014]	1.001	[0.984, 1.019]	0.993	[0.976, 1.010]
21	0.991	[0.888, 1.105]	0.998	[0.981, 1.015]	1.001	[0.984, 1.019]	0.993	[0.976, 1.010]
22	0.994	[0.875, 1.129]	0.997	[0.980, 1.014]	1.001	[0.984, 1.018]	0.992	[0.974, 1.010]
23	0.973	[0.880, 1.075]	0.994	[0.979, 1.010]	0.997	[0.982, 1.013]	0.996	[0.980, 1.012]
24	0.984	[0.879, 1.101]	0.998	[0.981, 1.015]	1.001	[0.984, 1.019]	0.992	[0.975, 1.009]
25	0.989	[0.887, 1.102]	0.997	[0.980, 1.015]	1.001	[0.984, 1.018]	0.993	[0.975, 1.010]
26	0.984	[0.882, 1.099]	0.997	[0.980, 1.014]	1.000	[0.983, 1.018]	0.993	[0.976, 1.011]
27	1.000	[0.915, 1.093]	1.002	[0.988, 1.016]	1.006	[0.992, 1.020]	0.996	[0.982, 1.011]
28	0.999	[0.894, 1.117]	0.996	[0.979, 1.013]	1.000	[0.983, 1.017]	0.991	[0.973, 1.008]
29	0.986	[0.882, 1.102]	0.998	[0.981, 1.015]	1.000	[0.983, 1.018]	0.992	[0.975, 1.010]
30	1.000	[0.897, 1.115]	0.996	[0.979, 1.013]	1.003	[0.986, 1.020]	0.994	[0.977, 1.011]
31	0.985	[0.884, 1.096]	0.999	[0.982, 1.016]	0.999	[0.983, 1.017]	0.994	[0.977, 1.011]
32	0.981	[0.884, 1.089]	0.995	[0.979, 1.011]	1.003	[0.987, 1.020]	0.990	[0.974, 1.007]

Table S17: Comparison of nested model $Nu \sim A Ra_i^B d_{rh}^C \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9959.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.998	0.440	[0.390, 0.496]	0.300	[0.283, 0.317]	-0.990	[-1.108, -0.873]
2	0.998	0.444	[0.396, 0.498]	0.297	[0.281, 0.314]	-0.981	[-1.094, -0.869]
3	0.997	0.442	[0.391, 0.501]	0.299	[0.281, 0.317]	-0.989	[-1.110, -0.868]
4	0.998	0.443	[0.395, 0.497]	0.297	[0.280, 0.314]	-0.980	[-1.094, -0.866]
5	0.997	0.442	[0.391, 0.499]	0.300	[0.282, 0.318]	-0.993	[-1.113, -0.873]
6	0.997	0.443	[0.391, 0.501]	0.299	[0.281, 0.317]	-0.989	[-1.110, -0.868]
7	0.997	0.440	[0.388, 0.498]	0.299	[0.281, 0.316]	-0.985	[-1.106, -0.864]
8	0.998	0.436	[0.384, 0.496]	0.298	[0.280, 0.316]	-0.978	[-1.101, -0.854]
9	0.998	0.440	[0.390, 0.496]	0.300	[0.282, 0.317]	-0.990	[-1.108, -0.873]
10	0.998	0.442	[0.392, 0.499]	0.298	[0.280, 0.315]	-0.982	[-1.101, -0.864]
11	0.997	0.439	[0.388, 0.497]	0.299	[0.281, 0.316]	-0.985	[-1.105, -0.865]
12	0.998	0.446	[0.393, 0.505]	0.300	[0.282, 0.317]	-0.994	[-1.115, -0.873]
13	0.997	0.441	[0.390, 0.498]	0.300	[0.282, 0.317]	-0.990	[-1.110, -0.871]
14	0.997	0.443	[0.392, 0.501]	0.300	[0.281, 0.318]	-0.992	[-1.116, -0.868]
15	0.997	0.441	[0.390, 0.499]	0.299	[0.282, 0.317]	-0.988	[-1.108, -0.868]
16	0.998	0.443	[0.393, 0.499]	0.298	[0.280, 0.315]	-0.982	[-1.099, -0.865]
17	0.998	0.445	[0.394, 0.502]	0.301	[0.283, 0.319]	-1.000	[-1.121, -0.880]
18	0.997	0.442	[0.391, 0.500]	0.298	[0.280, 0.316]	-0.985	[-1.106, -0.864]
19	0.997	0.445	[0.392, 0.505]	0.300	[0.281, 0.319]	-0.997	[-1.126, -0.868]
20	0.998	0.454	[0.400, 0.515]	0.302	[0.284, 0.319]	-1.009	[-1.131, -0.888]
21	0.998	0.443	[0.392, 0.501]	0.300	[0.282, 0.318]	-0.994	[-1.115, -0.873]
22	0.998	0.454	[0.399, 0.515]	0.302	[0.284, 0.319]	-1.010	[-1.133, -0.886]
23	0.998	0.427	[0.380, 0.480]	0.296	[0.280, 0.312]	-0.961	[-1.074, -0.849]
24	0.998	0.451	[0.401, 0.508]	0.300	[0.284, 0.317]	-1.002	[-1.117, -0.886]
25	0.997	0.443	[0.392, 0.501]	0.299	[0.281, 0.317]	-0.989	[-1.110, -0.869]
26	0.997	0.445	[0.394, 0.503]	0.299	[0.282, 0.316]	-0.990	[-1.109, -0.871]
27	0.998	0.428	[0.369, 0.497]	0.295	[0.275, 0.315]	-0.958	[-1.102, -0.815]
28	0.997	0.440	[0.391, 0.496]	0.297	[0.279, 0.314]	-0.975	[-1.095, -0.856]
29	0.998	0.443	[0.393, 0.499]	0.300	[0.283, 0.318]	-0.995	[-1.114, -0.876]
30	0.997	0.451	[0.399, 0.509]	0.304	[0.285, 0.322]	-1.017	[-1.142, -0.891]
31	0.997	0.442	[0.390, 0.501]	0.299	[0.281, 0.317]	-0.988	[-1.110, -0.866]
32	0.997	0.441	[0.390, 0.500]	0.298	[0.280, 0.317]	-0.984	[-1.108, -0.860]

Table S18: D_j coefficient comparison of nested model $Nu \sim A Ra_i^B d_{rh}^C \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation.

Left case	D_1 (95% CI)		D_2 (95% CI)		D_3 (95% CI)		D_4 (95% CI)	
1	0.977	[0.962, 0.992]	0.999	[0.994, 1.004]	0.999	[0.994, 1.004]	1.007	[1.001, 1.012]
2	0.978	[0.964, 0.993]	0.998	[0.993, 1.003]	1.001	[0.996, 1.006]	1.005	[1.000, 1.011]
3	0.977	[0.961, 0.992]	0.999	[0.993, 1.004]	1.000	[0.995, 1.005]	1.006	[1.000, 1.012]
4	0.979	[0.964, 0.993]	0.998	[0.993, 1.003]	1.001	[0.996, 1.006]	1.007	[1.002, 1.012]
5	0.976	[0.961, 0.992]	0.999	[0.994, 1.004]	1.000	[0.995, 1.005]	1.007	[1.001, 1.012]
6	0.977	[0.961, 0.992]	0.999	[0.993, 1.004]	1.000	[0.995, 1.005]	1.006	[1.001, 1.012]
7	0.977	[0.962, 0.993]	0.999	[0.994, 1.004]	1.000	[0.995, 1.005]	1.006	[1.000, 1.012]
8	0.978	[0.962, 0.993]	0.999	[0.994, 1.004]	1.000	[0.995, 1.005]	1.006	[1.000, 1.011]
9	0.977	[0.962, 0.992]	0.998	[0.993, 1.003]	0.999	[0.994, 1.004]	1.007	[1.001, 1.012]
10	0.978	[0.963, 0.993]	0.999	[0.994, 1.004]	1.000	[0.995, 1.005]	1.006	[1.000, 1.011]
11	0.977	[0.962, 0.993]	0.998	[0.993, 1.003]	0.999	[0.994, 1.005]	1.006	[1.000, 1.011]
12	0.976	[0.961, 0.992]	0.999	[0.994, 1.004]	1.000	[0.995, 1.005]	1.007	[1.001, 1.012]
13	0.977	[0.961, 0.992]	0.998	[0.993, 1.003]	1.000	[0.995, 1.005]	1.007	[1.001, 1.012]
14	0.976	[0.960, 0.992]	0.999	[0.993, 1.004]	1.000	[0.995, 1.005]	1.006	[1.001, 1.012]
15	0.977	[0.961, 0.993]	0.998	[0.993, 1.004]	1.000	[0.995, 1.005]	1.006	[1.000, 1.012]
16	0.978	[0.963, 0.993]	0.999	[0.994, 1.004]	0.999	[0.994, 1.004]	1.007	[1.001, 1.012]
17	0.975	[0.959, 0.991]	0.998	[0.993, 1.003]	1.000	[0.995, 1.005]	1.006	[1.000, 1.011]
18	0.977	[0.961, 0.993]	0.999	[0.994, 1.004]	1.000	[0.994, 1.005]	1.006	[1.001, 1.012]
19	0.975	[0.959, 0.993]	0.998	[0.993, 1.004]	1.000	[0.995, 1.005]	1.007	[1.001, 1.012]
20	0.974	[0.958, 0.989]	0.999	[0.994, 1.004]	0.999	[0.994, 1.004]	1.006	[1.001, 1.011]
21	0.976	[0.960, 0.992]	0.998	[0.993, 1.004]	1.000	[0.994, 1.005]	1.006	[1.000, 1.012]
22	0.974	[0.958, 0.990]	0.999	[0.994, 1.004]	1.000	[0.995, 1.005]	1.007	[1.001, 1.013]
23	0.979	[0.965, 0.993]	0.998	[0.993, 1.002]	0.999	[0.994, 1.004]	1.007	[1.002, 1.012]
24	0.974	[0.959, 0.989]	0.999	[0.994, 1.004]	1.001	[0.996, 1.005]	1.006	[1.000, 1.011]
25	0.977	[0.961, 0.992]	0.998	[0.993, 1.004]	1.000	[0.994, 1.005]	1.006	[1.001, 1.012]
26	0.976	[0.961, 0.992]	0.998	[0.993, 1.003]	0.999	[0.994, 1.005]	1.007	[1.001, 1.012]
27	0.980	[0.962, 0.998]	0.999	[0.994, 1.004]	1.000	[0.995, 1.006]	1.006	[1.001, 1.012]
28	0.978	[0.963, 0.993]	0.998	[0.993, 1.003]	0.999	[0.994, 1.004]	1.005	[1.000, 1.011]
29	0.975	[0.960, 0.991]	0.999	[0.994, 1.004]	0.999	[0.994, 1.004]	1.006	[1.000, 1.011]
30	0.974	[0.958, 0.990]	0.999	[0.994, 1.004]	0.999	[0.994, 1.004]	1.006	[1.001, 1.012]
31	0.977	[0.961, 0.992]	0.999	[0.994, 1.004]	1.000	[0.995, 1.005]	1.006	[1.001, 1.012]
32	0.977	[0.961, 0.993]	0.998	[0.993, 1.004]	1.000	[0.995, 1.005]	1.006	[1.000, 1.012]

Table S19: Comparison of nested model $Nu \sim A Ra_i^B d_{sl} \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9765.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.986	0.332	[0.257, 0.427]	0.242	[0.211, 0.272]	-0.644	[-0.871, -0.418]
2	0.987	0.339	[0.264, 0.436]	0.237	[0.208, 0.267]	-0.629	[-0.850, -0.408]
3	0.985	0.330	[0.255, 0.428]	0.239	[0.208, 0.270]	-0.629	[-0.858, -0.399]
4	0.988	0.344	[0.269, 0.440]	0.238	[0.210, 0.267]	-0.640	[-0.856, -0.425]
5	0.985	0.331	[0.255, 0.430]	0.239	[0.207, 0.270]	-0.627	[-0.859, -0.396]
6	0.986	0.332	[0.255, 0.431]	0.238	[0.207, 0.269]	-0.625	[-0.856, -0.395]
7	0.986	0.326	[0.251, 0.424]	0.238	[0.207, 0.269]	-0.619	[-0.848, -0.390]
8	0.986	0.320	[0.245, 0.418]	0.236	[0.205, 0.267]	-0.602	[-0.834, -0.370]
9	0.986	0.332	[0.258, 0.428]	0.242	[0.211, 0.272]	-0.644	[-0.871, -0.418]
10	0.986	0.334	[0.259, 0.431]	0.237	[0.206, 0.267]	-0.621	[-0.847, -0.395]
11	0.986	0.327	[0.254, 0.422]	0.239	[0.209, 0.270]	-0.628	[-0.852, -0.404]
12	0.986	0.331	[0.253, 0.431]	0.238	[0.207, 0.269]	-0.624	[-0.856, -0.391]
13	0.985	0.331	[0.255, 0.428]	0.240	[0.209, 0.271]	-0.634	[-0.863, -0.404]
14	0.986	0.332	[0.258, 0.429]	0.236	[0.205, 0.267]	-0.614	[-0.842, -0.387]
15	0.985	0.330	[0.254, 0.429]	0.238	[0.207, 0.269]	-0.625	[-0.855, -0.394]
16	0.988	0.347	[0.271, 0.443]	0.239	[0.211, 0.268]	-0.648	[-0.863, -0.433]
17	0.986	0.341	[0.263, 0.442]	0.245	[0.212, 0.277]	-0.669	[-0.909, -0.429]
18	0.985	0.332	[0.256, 0.430]	0.237	[0.206, 0.268]	-0.618	[-0.848, -0.389]
19	0.986	0.328	[0.251, 0.429]	0.236	[0.202, 0.270]	-0.611	[-0.860, -0.362]
20	0.986	0.347	[0.261, 0.461]	0.242	[0.210, 0.274]	-0.662	[-0.909, -0.415]
21	0.986	0.332	[0.256, 0.430]	0.238	[0.207, 0.270]	-0.626	[-0.859, -0.393]
22	0.985	0.342	[0.257, 0.456]	0.241	[0.208, 0.274]	-0.652	[-0.901, -0.402]
23	0.988	0.317	[0.248, 0.405]	0.237	[0.208, 0.266]	-0.604	[-0.818, -0.390]
24	0.985	0.337	[0.259, 0.440]	0.239	[0.208, 0.270]	-0.636	[-0.868, -0.404]
25	0.986	0.331	[0.255, 0.429]	0.239	[0.208, 0.270]	-0.630	[-0.860, -0.399]
26	0.985	0.338	[0.260, 0.439]	0.239	[0.208, 0.269]	-0.635	[-0.864, -0.405]
27	0.988	0.289	[0.219, 0.382]	0.226	[0.195, 0.257]	-0.516	[-0.756, -0.276]
28	0.985	0.330	[0.255, 0.426]	0.235	[0.204, 0.267]	-0.608	[-0.841, -0.375]
29	0.986	0.335	[0.259, 0.433]	0.242	[0.210, 0.273]	-0.647	[-0.879, -0.415]
30	0.985	0.328	[0.253, 0.425]	0.235	[0.203, 0.267]	-0.604	[-0.840, -0.368]
31	0.986	0.330	[0.252, 0.430]	0.238	[0.207, 0.269]	-0.622	[-0.855, -0.389]
32	0.986	0.333	[0.259, 0.428]	0.235	[0.205, 0.265]	-0.611	[-0.834, -0.389]

Table S20: D_j coefficient comparison of nested model $Nu \sim A Ra_i^B d_{sl}^C \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation.

Left case	D_1 (95% CI)		D_2 (95% CI)		D_3 (95% CI)		D_4 (95% CI)	
1	1.021	[0.990, 1.052]	1.000	[0.988, 1.012]	0.996	[0.984, 1.008]	1.009	[0.995, 1.024]
2	1.023	[0.993, 1.054]	0.997	[0.985, 1.009]	0.999	[0.987, 1.011]	1.006	[0.992, 1.020]
3	1.022	[0.991, 1.054]	0.999	[0.987, 1.011]	0.997	[0.984, 1.009]	1.007	[0.993, 1.021]
4	1.022	[0.993, 1.052]	0.997	[0.985, 1.008]	0.999	[0.988, 1.011]	1.010	[0.997, 1.023]
5	1.022	[0.990, 1.054]	0.999	[0.987, 1.011]	0.998	[0.985, 1.010]	1.008	[0.993, 1.022]
6	1.022	[0.990, 1.055]	0.999	[0.986, 1.011]	0.997	[0.985, 1.010]	1.007	[0.993, 1.022]
7	1.023	[0.992, 1.056]	0.999	[0.987, 1.012]	0.998	[0.986, 1.011]	1.006	[0.992, 1.021]
8	1.024	[0.992, 1.056]	1.000	[0.988, 1.012]	0.999	[0.986, 1.011]	1.006	[0.991, 1.020]
9	1.021	[0.990, 1.052]	0.997	[0.985, 1.009]	0.996	[0.984, 1.008]	1.009	[0.995, 1.023]
10	1.024	[0.992, 1.056]	1.000	[0.988, 1.012]	0.999	[0.986, 1.011]	1.006	[0.992, 1.020]
11	1.023	[0.992, 1.055]	0.997	[0.985, 1.009]	0.996	[0.984, 1.008]	1.006	[0.992, 1.020]
12	1.022	[0.990, 1.055]	0.998	[0.986, 1.011]	0.997	[0.985, 1.010]	1.007	[0.993, 1.022]
13	1.022	[0.990, 1.054]	0.998	[0.986, 1.010]	0.998	[0.986, 1.011]	1.008	[0.994, 1.023]
14	1.024	[0.993, 1.057]	1.000	[0.988, 1.012]	0.996	[0.984, 1.009]	1.006	[0.992, 1.020]
15	1.022	[0.991, 1.055]	0.998	[0.986, 1.011]	0.998	[0.985, 1.010]	1.007	[0.993, 1.021]
16	1.021	[0.992, 1.051]	1.001	[0.989, 1.012]	0.995	[0.984, 1.007]	1.010	[0.997, 1.024]
17	1.015	[0.982, 1.049]	0.997	[0.985, 1.009]	0.998	[0.986, 1.010]	1.007	[0.994, 1.021]
18	1.022	[0.991, 1.054]	0.999	[0.987, 1.012]	0.997	[0.984, 1.009]	1.008	[0.994, 1.022]
19	1.024	[0.989, 1.060]	0.999	[0.987, 1.011]	0.997	[0.985, 1.010]	1.006	[0.992, 1.022]
20	1.016	[0.982, 1.052]	1.000	[0.987, 1.012]	0.996	[0.984, 1.009]	1.007	[0.994, 1.021]
21	1.022	[0.989, 1.055]	0.999	[0.986, 1.011]	0.997	[0.985, 1.010]	1.007	[0.993, 1.022]
22	1.018	[0.983, 1.054]	0.999	[0.987, 1.012]	0.998	[0.986, 1.010]	1.009	[0.994, 1.024]
23	1.022	[0.993, 1.052]	0.996	[0.985, 1.008]	0.995	[0.984, 1.007]	1.009	[0.996, 1.022]
24	1.020	[0.988, 1.053]	0.999	[0.987, 1.012]	0.998	[0.986, 1.010]	1.007	[0.993, 1.021]
25	1.021	[0.989, 1.054]	0.999	[0.987, 1.011]	0.998	[0.986, 1.010]	1.007	[0.993, 1.021]
26	1.020	[0.988, 1.052]	0.998	[0.986, 1.010]	0.996	[0.984, 1.009]	1.008	[0.994, 1.023]
27	1.033	[1.002, 1.066]	1.001	[0.989, 1.012]	1.001	[0.989, 1.013]	1.006	[0.994, 1.020]
28	1.023	[0.992, 1.056]	0.998	[0.986, 1.010]	0.997	[0.985, 1.009]	1.006	[0.992, 1.020]
29	1.018	[0.986, 1.051]	1.000	[0.988, 1.012]	0.996	[0.984, 1.009]	1.007	[0.993, 1.021]
30	1.024	[0.992, 1.057]	0.998	[0.986, 1.010]	0.998	[0.986, 1.011]	1.008	[0.994, 1.022]
31	1.022	[0.990, 1.055]	0.999	[0.987, 1.011]	0.997	[0.985, 1.010]	1.007	[0.993, 1.022]
32	1.022	[0.992, 1.053]	0.997	[0.985, 1.009]	0.999	[0.987, 1.011]	1.005	[0.992, 1.019]

Table S21: Comparison of nested model $Nu \sim A Ra_i^B (d_{rh}/d_{shell})^C \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9958.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.997	0.710	[0.595, 0.847]	0.028	[0.012, 0.044]	-0.797	[-0.894, -0.700]
2	0.998	0.714	[0.604, 0.843]	0.028	[0.013, 0.044]	-0.791	[-0.883, -0.698]
3	0.997	0.713	[0.596, 0.853]	0.028	[0.011, 0.044]	-0.797	[-0.895, -0.698]
4	0.998	0.714	[0.607, 0.840]	0.028	[0.013, 0.043]	-0.790	[-0.880, -0.700]
5	0.997	0.717	[0.601, 0.855]	0.028	[0.011, 0.044]	-0.796	[-0.893, -0.699]
6	0.997	0.713	[0.597, 0.852]	0.028	[0.011, 0.044]	-0.798	[-0.896, -0.700]
7	0.997	0.703	[0.589, 0.839]	0.029	[0.013, 0.045]	-0.792	[-0.889, -0.695]
8	0.998	0.691	[0.578, 0.825]	0.031	[0.014, 0.047]	-0.782	[-0.879, -0.686]
9	0.997	0.710	[0.597, 0.846]	0.028	[0.012, 0.044]	-0.798	[-0.894, -0.702]
10	0.998	0.711	[0.597, 0.848]	0.028	[0.012, 0.044]	-0.792	[-0.889, -0.695]
11	0.998	0.704	[0.592, 0.837]	0.029	[0.013, 0.045]	-0.793	[-0.888, -0.698]
12	0.998	0.707	[0.593, 0.844]	0.029	[0.012, 0.045]	-0.794	[-0.891, -0.697]
13	0.997	0.712	[0.596, 0.851]	0.028	[0.011, 0.044]	-0.797	[-0.895, -0.699]
14	0.997	0.711	[0.595, 0.850]	0.028	[0.012, 0.045]	-0.793	[-0.893, -0.693]
15	0.997	0.712	[0.595, 0.851]	0.028	[0.011, 0.045]	-0.796	[-0.894, -0.698]
16	0.998	0.714	[0.605, 0.843]	0.028	[0.013, 0.043]	-0.791	[-0.883, -0.700]
17	0.998	0.723	[0.607, 0.862]	0.026	[0.010, 0.043]	-0.808	[-0.905, -0.710]
18	0.997	0.712	[0.596, 0.850]	0.028	[0.012, 0.045]	-0.793	[-0.892, -0.695]
19	0.997	0.726	[0.603, 0.874]	0.026	[0.009, 0.043]	-0.809	[-0.914, -0.703]
20	0.997	0.722	[0.600, 0.868]	0.027	[0.010, 0.044]	-0.802	[-0.902, -0.701]
21	0.997	0.714	[0.597, 0.854]	0.028	[0.011, 0.044]	-0.797	[-0.896, -0.698]
22	0.998	0.774	[0.650, 0.922]	0.020	[0.004, 0.036]	-0.837	[-0.932, -0.742]
23	0.998	0.693	[0.576, 0.834]	0.030	[0.013, 0.048]	-0.783	[-0.884, -0.682]
24	0.997	0.718	[0.600, 0.859]	0.027	[0.011, 0.044]	-0.799	[-0.897, -0.700]
25	0.997	0.715	[0.598, 0.854]	0.028	[0.011, 0.044]	-0.797	[-0.895, -0.699]
26	0.997	0.723	[0.607, 0.860]	0.027	[0.011, 0.043]	-0.800	[-0.895, -0.704]
27	0.997	0.701	[0.560, 0.877]	0.029	[0.009, 0.050]	-0.787	[-0.908, -0.667]
28	0.997	0.714	[0.595, 0.856]	0.028	[0.011, 0.045]	-0.797	[-0.898, -0.695]
29	0.998	0.717	[0.602, 0.853]	0.027	[0.011, 0.043]	-0.802	[-0.899, -0.706]
30	0.997	0.724	[0.603, 0.869]	0.026	[0.009, 0.043]	-0.807	[-0.910, -0.704]
31	0.997	0.709	[0.592, 0.848]	0.028	[0.012, 0.045]	-0.794	[-0.893, -0.696]
32	0.997	0.713	[0.595, 0.854]	0.028	[0.011, 0.045]	-0.796	[-0.897, -0.694]

Table S22: D_j coefficient comparison of nested model $Nu \sim A Ra_i^B (d_{rh}/d_{shell})^C \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation.

Left case	D_1 (95% CI)		D_2 (95% CI)		D_3 (95% CI)		D_4 (95% CI)	
1	1.021	[1.010, 1.033]	0.998	[0.993, 1.003]	0.999	[0.994, 1.004]	0.999	[0.994, 1.004]
2	1.023	[1.012, 1.033]	0.997	[0.992, 1.002]	1.000	[0.995, 1.005]	0.998	[0.993, 1.003]
3	1.021	[1.010, 1.033]	0.998	[0.993, 1.003]	0.999	[0.994, 1.005]	0.998	[0.993, 1.004]
4	1.023	[1.012, 1.033]	0.997	[0.992, 1.002]	1.000	[0.996, 1.005]	0.999	[0.995, 1.004]
5	1.021	[1.010, 1.032]	0.997	[0.992, 1.003]	0.999	[0.994, 1.004]	0.998	[0.993, 1.003]
6	1.021	[1.009, 1.032]	0.998	[0.993, 1.003]	0.999	[0.994, 1.005]	0.999	[0.993, 1.004]
7	1.022	[1.011, 1.033]	0.998	[0.993, 1.003]	1.000	[0.995, 1.005]	0.998	[0.993, 1.003]
8	1.022	[1.011, 1.033]	0.998	[0.993, 1.003]	1.000	[0.995, 1.005]	0.998	[0.992, 1.003]
9	1.021	[1.010, 1.033]	0.997	[0.992, 1.002]	0.999	[0.994, 1.004]	0.999	[0.994, 1.004]
10	1.022	[1.011, 1.033]	0.998	[0.993, 1.003]	1.000	[0.995, 1.005]	0.998	[0.993, 1.003]
11	1.022	[1.011, 1.033]	0.997	[0.992, 1.002]	0.999	[0.994, 1.004]	0.998	[0.993, 1.003]
12	1.021	[1.010, 1.032]	0.997	[0.992, 1.002]	0.999	[0.994, 1.004]	0.998	[0.993, 1.003]
13	1.021	[1.010, 1.033]	0.998	[0.992, 1.003]	0.999	[0.994, 1.005]	0.999	[0.993, 1.004]
14	1.022	[1.010, 1.033]	0.998	[0.993, 1.003]	0.999	[0.994, 1.004]	0.998	[0.993, 1.004]
15	1.021	[1.010, 1.033]	0.998	[0.992, 1.003]	0.999	[0.994, 1.005]	0.998	[0.993, 1.004]
16	1.023	[1.012, 1.033]	0.999	[0.994, 1.004]	0.998	[0.994, 1.003]	0.999	[0.994, 1.004]
17	1.019	[1.008, 1.031]	0.997	[0.992, 1.002]	1.000	[0.995, 1.005]	0.998	[0.993, 1.003]
18	1.021	[1.010, 1.032]	0.998	[0.993, 1.003]	0.999	[0.994, 1.004]	0.999	[0.993, 1.004]
19	1.020	[1.007, 1.032]	0.997	[0.992, 1.003]	1.000	[0.994, 1.005]	0.999	[0.994, 1.004]
20	1.020	[1.009, 1.032]	0.998	[0.993, 1.003]	0.999	[0.994, 1.004]	0.998	[0.993, 1.004]
21	1.021	[1.010, 1.033]	0.998	[0.992, 1.003]	0.999	[0.994, 1.005]	0.998	[0.993, 1.004]
22	1.016	[1.005, 1.027]	0.999	[0.994, 1.004]	1.000	[0.996, 1.005]	1.000	[0.995, 1.005]
23	1.022	[1.011, 1.033]	0.997	[0.992, 1.002]	0.999	[0.994, 1.004]	0.999	[0.994, 1.004]
24	1.021	[1.009, 1.032]	0.998	[0.993, 1.003]	1.000	[0.994, 1.005]	0.998	[0.993, 1.004]
25	1.021	[1.010, 1.033]	0.998	[0.992, 1.003]	0.999	[0.994, 1.005]	0.999	[0.993, 1.004]
26	1.020	[1.009, 1.031]	0.997	[0.992, 1.002]	0.999	[0.994, 1.004]	0.999	[0.994, 1.004]
27	1.022	[1.009, 1.035]	0.998	[0.993, 1.003]	1.000	[0.994, 1.005]	0.999	[0.993, 1.004]
28	1.021	[1.010, 1.033]	0.998	[0.992, 1.003]	0.999	[0.994, 1.005]	0.998	[0.993, 1.004]
29	1.020	[1.009, 1.031]	0.998	[0.993, 1.003]	0.999	[0.994, 1.004]	0.998	[0.993, 1.003]
30	1.020	[1.009, 1.032]	0.998	[0.993, 1.003]	0.999	[0.994, 1.004]	0.998	[0.993, 1.004]
31	1.021	[1.010, 1.032]	0.998	[0.993, 1.003]	0.999	[0.994, 1.004]	0.999	[0.993, 1.004]
32	1.021	[1.010, 1.033]	0.998	[0.992, 1.003]	0.999	[0.994, 1.005]	0.998	[0.993, 1.004]

Table S23: Comparison of nested model $Nu \sim A Ra_i^B (d_{sl}/d_{shell})^C \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation. The predictive R^2 is 0.9800.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)	
1	0.988	0.493	[0.352, 0.689]	0.058	[0.025, 0.090]	-0.570	[-0.746, -0.393]
2	0.989	0.500	[0.360, 0.696]	0.057	[0.025, 0.089]	-0.560	[-0.732, -0.387]
3	0.987	0.490	[0.348, 0.689]	0.059	[0.026, 0.091]	-0.561	[-0.741, -0.382]
4	0.990	0.512	[0.372, 0.705]	0.055	[0.024, 0.086]	-0.570	[-0.737, -0.404]
5	0.987	0.492	[0.349, 0.694]	0.058	[0.025, 0.091]	-0.558	[-0.738, -0.378]
6	0.988	0.491	[0.348, 0.692]	0.058	[0.025, 0.092]	-0.560	[-0.740, -0.379]
7	0.988	0.481	[0.342, 0.678]	0.060	[0.027, 0.093]	-0.554	[-0.732, -0.377]
8	0.989	0.468	[0.331, 0.662]	0.063	[0.029, 0.096]	-0.541	[-0.720, -0.361]
9	0.988	0.495	[0.354, 0.691]	0.057	[0.025, 0.089]	-0.572	[-0.748, -0.395]
10	0.988	0.492	[0.351, 0.690]	0.059	[0.026, 0.091]	-0.554	[-0.732, -0.377]
11	0.988	0.487	[0.350, 0.677]	0.059	[0.027, 0.091]	-0.563	[-0.736, -0.389]
12	0.988	0.486	[0.344, 0.686]	0.059	[0.026, 0.093]	-0.556	[-0.736, -0.376]
13	0.988	0.490	[0.349, 0.690]	0.058	[0.025, 0.091]	-0.563	[-0.742, -0.384]
14	0.989	0.490	[0.351, 0.685]	0.059	[0.027, 0.092]	-0.550	[-0.727, -0.374]
15	0.987	0.489	[0.346, 0.690]	0.059	[0.026, 0.092]	-0.558	[-0.738, -0.378]
16	0.990	0.519	[0.378, 0.713]	0.054	[0.023, 0.084]	-0.577	[-0.742, -0.412]
17	0.989	0.516	[0.366, 0.729]	0.053	[0.020, 0.087]	-0.595	[-0.780, -0.409]
18	0.987	0.490	[0.348, 0.688]	0.059	[0.026, 0.092]	-0.553	[-0.733, -0.374]
19	0.988	0.488	[0.340, 0.700]	0.059	[0.024, 0.094]	-0.555	[-0.751, -0.359]
20	0.988	0.512	[0.354, 0.739]	0.055	[0.019, 0.090]	-0.578	[-0.767, -0.388]
21	0.988	0.491	[0.348, 0.692]	0.059	[0.025, 0.092]	-0.559	[-0.740, -0.377]
22	0.988	0.537	[0.367, 0.785]	0.050	[0.013, 0.086]	-0.602	[-0.798, -0.407]
23	0.989	0.462	[0.330, 0.646]	0.064	[0.032, 0.096]	-0.535	[-0.709, -0.361]
24	0.988	0.495	[0.350, 0.701]	0.058	[0.024, 0.091]	-0.562	[-0.743, -0.381]
25	0.988	0.491	[0.349, 0.691]	0.058	[0.025, 0.091]	-0.562	[-0.742, -0.382]
26	0.988	0.505	[0.358, 0.712]	0.056	[0.023, 0.089]	-0.568	[-0.747, -0.390]
27	0.989	0.414	[0.282, 0.609]	0.075	[0.038, 0.112]	-0.478	[-0.675, -0.280]
28	0.987	0.486	[0.343, 0.688]	0.060	[0.026, 0.094]	-0.550	[-0.736, -0.364]
29	0.988	0.501	[0.357, 0.703]	0.056	[0.023, 0.089]	-0.577	[-0.757, -0.396]
30	0.987	0.481	[0.341, 0.679]	0.061	[0.027, 0.094]	-0.543	[-0.727, -0.359]
31	0.988	0.487	[0.343, 0.690]	0.059	[0.026, 0.093]	-0.556	[-0.737, -0.375]
32	0.988	0.486	[0.349, 0.679]	0.060	[0.028, 0.092]	-0.545	[-0.721, -0.369]

Table S24: D_j coefficient comparison of nested model $Nu \sim A Ra_i^B \prod_{j=1}^4 D_j$ in 32 leave-one-out cross validation.

Left case	D_1 (95% CI)		D_2 (95% CI)		D_3 (95% CI)		D_4 (95% CI)	
1	1.045	[1.024, 1.067]	0.999	[0.988, 1.010]	0.996	[0.985, 1.007]	1.004	[0.992, 1.016]
2	1.047	[1.026, 1.068]	0.997	[0.986, 1.007]	0.999	[0.988, 1.010]	1.001	[0.989, 1.013]
3	1.046	[1.024, 1.068]	0.999	[0.987, 1.010]	0.997	[0.985, 1.008]	1.002	[0.990, 1.014]
4	1.046	[1.026, 1.067]	0.996	[0.986, 1.006]	0.999	[0.989, 1.010]	1.005	[0.993, 1.016]
5	1.045	[1.023, 1.068]	0.998	[0.986, 1.009]	0.997	[0.986, 1.008]	1.002	[0.990, 1.014]
6	1.045	[1.023, 1.068]	0.998	[0.987, 1.010]	0.997	[0.986, 1.009]	1.002	[0.990, 1.015]
7	1.047	[1.025, 1.069]	0.999	[0.988, 1.010]	0.998	[0.987, 1.010]	1.001	[0.989, 1.014]
8	1.046	[1.025, 1.068]	0.999	[0.988, 1.010]	0.999	[0.987, 1.010]	1.001	[0.989, 1.013]
9	1.045	[1.024, 1.067]	0.997	[0.986, 1.008]	0.996	[0.985, 1.007]	1.004	[0.992, 1.016]
10	1.047	[1.025, 1.069]	0.999	[0.988, 1.010]	0.998	[0.987, 1.010]	1.001	[0.989, 1.013]
11	1.046	[1.025, 1.068]	0.997	[0.986, 1.008]	0.996	[0.985, 1.007]	1.001	[0.989, 1.013]
12	1.045	[1.023, 1.067]	0.997	[0.986, 1.009]	0.997	[0.985, 1.008]	1.002	[0.990, 1.014]
13	1.046	[1.024, 1.068]	0.997	[0.986, 1.009]	0.998	[0.986, 1.009]	1.003	[0.991, 1.015]
14	1.047	[1.026, 1.070]	0.999	[0.988, 1.010]	0.996	[0.985, 1.007]	1.001	[0.989, 1.013]
15	1.046	[1.024, 1.068]	0.998	[0.986, 1.009]	0.997	[0.986, 1.009]	1.002	[0.990, 1.014]
16	1.046	[1.026, 1.066]	1.000	[0.990, 1.011]	0.995	[0.984, 1.005]	1.005	[0.994, 1.016]
17	1.040	[1.017, 1.064]	0.997	[0.986, 1.008]	0.998	[0.987, 1.009]	1.002	[0.990, 1.014]
18	1.045	[1.024, 1.067]	0.999	[0.987, 1.010]	0.997	[0.985, 1.008]	1.003	[0.991, 1.015]
19	1.046	[1.022, 1.071]	0.998	[0.987, 1.010]	0.997	[0.986, 1.009]	1.002	[0.989, 1.015]
20	1.043	[1.019, 1.067]	0.999	[0.987, 1.010]	0.996	[0.985, 1.008]	1.002	[0.990, 1.014]
21	1.045	[1.023, 1.068]	0.998	[0.987, 1.009]	0.997	[0.986, 1.009]	1.002	[0.990, 1.015]
22	1.040	[1.016, 1.064]	0.999	[0.988, 1.011]	0.998	[0.987, 1.009]	1.005	[0.992, 1.017]
23	1.046	[1.025, 1.067]	0.996	[0.986, 1.007]	0.996	[0.985, 1.007]	1.003	[0.992, 1.015]
24	1.045	[1.022, 1.067]	0.998	[0.987, 1.010]	0.997	[0.986, 1.009]	1.002	[0.990, 1.014]
25	1.044	[1.022, 1.067]	0.998	[0.987, 1.010]	0.998	[0.986, 1.009]	1.002	[0.990, 1.014]
26	1.043	[1.022, 1.066]	0.997	[0.986, 1.008]	0.996	[0.985, 1.008]	1.004	[0.991, 1.016]
27	1.052	[1.029, 1.074]	1.000	[0.989, 1.011]	1.000	[0.989, 1.011]	1.002	[0.991, 1.014]
28	1.046	[1.024, 1.068]	0.998	[0.986, 1.009]	0.997	[0.986, 1.008]	1.002	[0.989, 1.014]
29	1.042	[1.020, 1.065]	0.999	[0.988, 1.010]	0.996	[0.985, 1.007]	1.002	[0.990, 1.014]
30	1.046	[1.024, 1.069]	0.997	[0.986, 1.008]	0.998	[0.987, 1.010]	1.003	[0.991, 1.015]
31	1.045	[1.023, 1.068]	0.998	[0.987, 1.010]	0.997	[0.986, 1.008]	1.003	[0.991, 1.015]
32	1.045	[1.024, 1.067]	0.997	[0.986, 1.008]	0.999	[0.987, 1.010]	1.001	[0.989, 1.013]

Table S25: Comparison of nested model $Nu \sim A Ra_i^B d_{rh}^C D_{convection}$ and its 32 leave-one-out cross validation. The predictive R^2 is 0.9952.

Left case	R^2	A (95% CI)		B (95% CI)		C (95% CI)		D_1 (95% CI)	
None	0.996	0.383	[0.349, 0.421]	0.274	[0.266, 0.282]	-0.816	[-0.859, -0.774]	1.001	[0.993, 1.010]
1	0.996	0.382	[0.347, 0.420]	0.275	[0.266, 0.283]	-0.819	[-0.862, -0.776]	1.001	[0.993, 1.010]
2	0.997	0.390	[0.356, 0.427]	0.274	[0.266, 0.282]	-0.824	[-0.865, -0.783]	1.001	[0.993, 1.009]
3	0.996	0.383	[0.348, 0.421]	0.274	[0.266, 0.283]	-0.818	[-0.862, -0.775]	1.001	[0.993, 1.010]
4	0.997	0.388	[0.353, 0.426]	0.274	[0.266, 0.282]	-0.820	[-0.863, -0.778]	1.001	[0.993, 1.009]
5	0.996	0.383	[0.348, 0.421]	0.274	[0.266, 0.282]	-0.817	[-0.861, -0.773]	1.001	[0.993, 1.010]
6	0.996	0.385	[0.349, 0.424]	0.274	[0.266, 0.282]	-0.818	[-0.862, -0.774]	1.001	[0.993, 1.010]
7	0.996	0.382	[0.348, 0.419]	0.275	[0.266, 0.283]	-0.820	[-0.863, -0.777]	1.001	[0.993, 1.010]
8	0.997	0.378	[0.344, 0.415]	0.273	[0.265, 0.281]	-0.810	[-0.853, -0.766]	1.001	[0.993, 1.010]
9	0.996	0.382	[0.348, 0.420]	0.274	[0.266, 0.283]	-0.818	[-0.862, -0.775]	1.001	[0.993, 1.010]
10	0.997	0.389	[0.354, 0.427]	0.274	[0.266, 0.282]	-0.822	[-0.865, -0.780]	1.001	[0.993, 1.009]
11	0.996	0.382	[0.348, 0.419]	0.275	[0.266, 0.283]	-0.820	[-0.863, -0.777]	1.001	[0.993, 1.010]
12	0.996	0.384	[0.348, 0.423]	0.274	[0.266, 0.282]	-0.817	[-0.861, -0.772]	1.001	[0.993, 1.010]
13	0.996	0.383	[0.348, 0.421]	0.274	[0.266, 0.282]	-0.817	[-0.861, -0.774]	1.001	[0.993, 1.010]
14	0.996	0.386	[0.351, 0.425]	0.274	[0.266, 0.282]	-0.819	[-0.862, -0.775]	1.001	[0.993, 1.010]
15	0.996	0.382	[0.348, 0.421]	0.274	[0.266, 0.283]	-0.819	[-0.863, -0.775]	1.001	[0.993, 1.010]
16	0.997	0.388	[0.353, 0.426]	0.274	[0.266, 0.282]	-0.820	[-0.862, -0.778]	1.001	[0.993, 1.009]
17	0.996	0.383	[0.346, 0.423]	0.274	[0.266, 0.282]	-0.816	[-0.860, -0.772]	1.001	[0.992, 1.010]
18	0.996	0.382	[0.348, 0.421]	0.273	[0.265, 0.282]	-0.814	[-0.858, -0.769]	1.001	[0.992, 1.009]
19	0.996	0.390	[0.354, 0.430]	0.273	[0.265, 0.281]	-0.819	[-0.861, -0.776]	1.003	[0.994, 1.011]
20	0.996	0.383	[0.348, 0.421]	0.274	[0.265, 0.282]	-0.816	[-0.859, -0.772]	1.001	[0.992, 1.010]
21	0.996	0.383	[0.346, 0.424]	0.274	[0.265, 0.282]	-0.816	[-0.860, -0.772]	1.001	[0.993, 1.010]
22	0.996	0.383	[0.349, 0.422]	0.274	[0.266, 0.282]	-0.817	[-0.860, -0.773]	1.002	[0.993, 1.010]
23	0.997	0.379	[0.347, 0.415]	0.272	[0.264, 0.280]	-0.804	[-0.846, -0.762]	1.003	[0.995, 1.011]
24	0.996	0.383	[0.348, 0.420]	0.273	[0.265, 0.281]	-0.814	[-0.857, -0.771]	1.000	[0.992, 1.009]
25	0.996	0.385	[0.347, 0.426]	0.274	[0.265, 0.282]	-0.817	[-0.861, -0.773]	1.002	[0.993, 1.010]
26	0.996	0.385	[0.349, 0.425]	0.273	[0.264, 0.282]	-0.815	[-0.859, -0.771]	1.002	[0.993, 1.011]
27	0.997	0.378	[0.345, 0.414]	0.272	[0.264, 0.280]	-0.800	[-0.844, -0.757]	1.003	[0.995, 1.011]
28	0.996	0.381	[0.347, 0.418]	0.273	[0.265, 0.281]	-0.810	[-0.853, -0.767]	1.000	[0.992, 1.008]
29	0.996	0.379	[0.343, 0.419]	0.274	[0.266, 0.282]	-0.814	[-0.858, -0.771]	1.000	[0.992, 1.009]
30	0.996	0.382	[0.346, 0.421]	0.275	[0.265, 0.284]	-0.819	[-0.865, -0.774]	1.001	[0.992, 1.010]
31	0.996	0.383	[0.348, 0.421]	0.273	[0.265, 0.282]	-0.815	[-0.860, -0.770]	1.002	[0.993, 1.010]
32	0.996	0.382	[0.347, 0.420]	0.273	[0.265, 0.281]	-0.813	[-0.857, -0.769]	1.001	[0.992, 1.009]

Table S26: Comparison of nested model $d_{rh} \sim A Ra_i^B \prod_{j=1}^4 D_j$ and its 32 leave-one-out models. The predictive R^2 is 0.9463.

Left number	R^2	A (95% CI)		B (95% CI)		D_1 (95% CI)		D_2 (95% CI)		D_3 (95% CI)		D_4 (95% CI)	
None	0.965	2.441	[2.000, 2.978]	0.143	[0.130, 0.156]	0.882	[0.867, 0.898]	1.001	[0.985, 1.019]	0.996	[0.979, 1.013]	1.018	[1.001, 1.036]
1	0.964	2.435	[1.977, 2.998]	0.143	[0.130, 0.156]	0.882	[0.867, 0.898]	1.002	[0.984, 1.020]	0.996	[0.979, 1.014]	1.018	[1.001, 1.037]
2	0.963	2.423	[1.970, 2.980]	0.143	[0.130, 0.157]	0.882	[0.866, 0.898]	1.002	[0.984, 1.020]	0.996	[0.978, 1.013]	1.019	[1.001, 1.037]
3	0.965	2.447	[1.988, 3.010]	0.143	[0.129, 0.156]	0.882	[0.867, 0.898]	1.001	[0.984, 1.019]	0.996	[0.979, 1.014]	1.018	[1.001, 1.037]
4	0.963	2.420	[1.970, 2.973]	0.143	[0.130, 0.157]	0.882	[0.866, 0.897]	1.002	[0.985, 1.020]	0.995	[0.978, 1.013]	1.018	[1.000, 1.036]
5	0.965	2.411	[1.958, 2.969]	0.143	[0.130, 0.157]	0.883	[0.868, 0.899]	1.002	[0.985, 1.020]	0.997	[0.980, 1.015]	1.019	[1.001, 1.037]
6	0.962	2.435	[1.978, 2.998]	0.143	[0.130, 0.156]	0.882	[0.867, 0.898]	1.002	[0.984, 1.020]	0.996	[0.979, 1.014]	1.018	[1.001, 1.037]
7	0.965	2.476	[2.014, 3.043]	0.142	[0.129, 0.155]	0.881	[0.866, 0.897]	1.000	[0.983, 1.018]	0.995	[0.978, 1.013]	1.019	[1.002, 1.037]
8	0.964	2.505	[2.049, 3.061]	0.141	[0.128, 0.154]	0.884	[0.869, 0.900]	0.999	[0.982, 1.017]	0.994	[0.977, 1.011]	1.020	[1.003, 1.038]
9	0.964	2.436	[1.980, 2.997]	0.143	[0.130, 0.156]	0.882	[0.867, 0.898]	1.001	[0.984, 1.019]	0.996	[0.979, 1.014]	1.018	[1.001, 1.037]
10	0.963	2.418	[1.969, 2.969]	0.143	[0.130, 0.157]	0.882	[0.866, 0.897]	1.001	[0.983, 1.018]	0.995	[0.978, 1.013]	1.019	[1.001, 1.037]
11	0.965	2.464	[2.007, 3.027]	0.142	[0.129, 0.155]	0.882	[0.866, 0.897]	1.002	[0.985, 1.020]	0.997	[0.980, 1.015]	1.019	[1.001, 1.037]
12	0.963	2.471	[2.016, 3.029]	0.142	[0.129, 0.155]	0.884	[0.868, 0.899]	1.003	[0.985, 1.020]	0.998	[0.980, 1.015]	1.020	[1.002, 1.038]
13	0.965	2.432	[1.975, 2.995]	0.143	[0.130, 0.156]	0.883	[0.867, 0.898]	1.001	[0.984, 1.019]	0.997	[0.979, 1.014]	1.019	[1.001, 1.037]
14	0.965	2.385	[1.949, 2.919]	0.144	[0.131, 0.157]	0.881	[0.865, 0.896]	1.000	[0.982, 1.017]	0.998	[0.981, 1.016]	1.020	[1.003, 1.038]
15	0.965	2.450	[1.991, 3.014]	0.143	[0.129, 0.156]	0.882	[0.867, 0.898]	1.002	[0.984, 1.020]	0.996	[0.978, 1.014]	1.019	[1.001, 1.037]
16	0.963	2.419	[1.969, 2.972]	0.143	[0.130, 0.157]	0.882	[0.866, 0.897]	1.001	[0.983, 1.018]	0.997	[0.980, 1.015]	1.017	[1.000, 1.036]
17	0.965	2.403	[1.963, 2.941]	0.144	[0.131, 0.157]	0.881	[0.866, 0.896]	1.000	[0.983, 1.017]	0.998	[0.981, 1.015]	1.017	[0.999, 1.035]
18	0.965	2.410	[1.964, 2.958]	0.144	[0.130, 0.157]	0.883	[0.868, 0.899]	1.000	[0.983, 1.018]	0.997	[0.980, 1.015]	1.017	[0.999, 1.035]
19	0.968	2.365	[1.951, 2.867]	0.145	[0.132, 0.157]	0.880	[0.865, 0.894]	0.998	[0.982, 1.015]	0.999	[0.983, 1.016]	1.021	[1.005, 1.038]
20	0.967	2.509	[2.055, 3.063]	0.141	[0.128, 0.154]	0.880	[0.866, 0.896]	1.004	[0.987, 1.021]	0.994	[0.977, 1.011]	1.016	[0.999, 1.033]
21	0.964	2.415	[1.970, 2.960]	0.144	[0.130, 0.157]	0.881	[0.866, 0.897]	1.000	[0.983, 1.018]	0.995	[0.978, 1.013]	1.017	[0.999, 1.035]
22	0.968	2.510	[2.061, 3.056]	0.141	[0.128, 0.154]	0.880	[0.866, 0.895]	1.004	[0.987, 1.021]	0.999	[0.982, 1.016]	1.021	[1.004, 1.038]
23	0.963	2.491	[2.037, 3.047]	0.141	[0.128, 0.154]	0.884	[0.869, 0.900]	1.003	[0.986, 1.021]	0.998	[0.981, 1.016]	1.016	[0.999, 1.034]
24	0.965	2.471	[2.014, 3.032]	0.142	[0.129, 0.155]	0.881	[0.866, 0.897]	1.003	[0.985, 1.020]	0.997	[0.980, 1.015]	1.017	[0.999, 1.035]
25	0.963	2.442	[1.986, 3.003]	0.143	[0.129, 0.156]	0.882	[0.867, 0.898]	1.001	[0.984, 1.019]	0.996	[0.979, 1.014]	1.018	[1.000, 1.037]
26	0.965	2.448	[1.989, 3.012]	0.143	[0.129, 0.156]	0.882	[0.867, 0.898]	1.001	[0.984, 1.019]	0.996	[0.978, 1.014]	1.019	[1.001, 1.037]
27	0.972	2.594	[2.182, 3.083]	0.139	[0.128, 0.150]	0.887	[0.874, 0.900]	0.997	[0.982, 1.012]	0.992	[0.977, 1.006]	1.013	[0.999, 1.029]
28	0.966	2.392	[1.952, 2.933]	0.144	[0.131, 0.157]	0.884	[0.868, 0.899]	1.003	[0.986, 1.021]	0.998	[0.981, 1.015]	1.020	[1.002, 1.038]
29	0.963	2.420	[1.972, 2.971]	0.143	[0.130, 0.157]	0.882	[0.866, 0.897]	1.002	[0.985, 1.020]	0.995	[0.978, 1.013]	1.017	[1.000, 1.036]
30	0.969	2.362	[1.949, 2.863]	0.145	[0.133, 0.158]	0.885	[0.870, 0.900]	1.004	[0.988, 1.021]	0.993	[0.977, 1.010]	1.015	[0.999, 1.032]
31	0.962	2.472	[2.014, 3.034]	0.142	[0.129, 0.155]	0.883	[0.868, 0.899]	1.000	[0.983, 1.018]	0.997	[0.980, 1.015]	1.017	[0.999, 1.035]
32	0.967	2.383	[1.950, 2.914]	0.144	[0.131, 0.157]	0.884	[0.869, 0.899]	1.003	[0.986, 1.021]	0.994	[0.977, 1.011]	1.020	[1.003, 1.038]