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Wave-Driven Sliding, Ratcheting, and Overturning of Coral Transplant Units: An Idealized Nonsmooth Contact Model

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Abstract

Coral transplantation attaches fragments to a structure placed on the seabed, and that structure must stay put long enough for the fragments to fuse to it. We examine that requirement with an idealized model in which a transplant unit is a planar rigid body on a rigid bed under two unilateral frictional contacts, loaded through a Morison description of near-bed wave kinematics and integrated by a nonsmooth contact time-stepping scheme. No measurement enters the model and no coefficient is calibrated. Three closed-form results follow from the balance of forces at rest. Whether a unit slides or tips is decided by comparing the base half-width divided by the height at which the flow pushes hardest against the friction coefficient of the bed; lift cancels exactly, and no material or wave property enters. Material affects the thresholds only through the submerged weight, so changing it rescales every threshold by one factor and reorders nothing. A wave riding on a current drives a net drift per cycle rising as the square of the overshoot above threshold. Five reference structures carrying growing colonies are compared at four idealized Indonesian site classes. Every unit loses margin as its corals grow, so success in the biological sense erodes the margin that permitted it, and an internal solitary wave (ISW) at transplant depth triggers a sliding step rather than an overturning. Across samples varying every uncalibrated coefficient together, the ranking of the structures proves far firmer than the thresholds. The solver is verified against closed-form impact, sliding, and rocking results.

Keywords: Coral restoration; Transplant unit stability; Nonsmooth contact dynamics; Rocking block; Internal solitary waves

1 Introduction

Coral reefs supply coastal protection of a magnitude that is now quantified rather than asserted: the reef crest dissipates on the order of nine tenths of the incident wave energy, and the loss of that structure transfers the residual directly to the shoreline behind it [9]. Where the framework has already been removed, by blast fishing, bleaching, or anchor damage, natural recovery on mobile rubble is slow, and active restoration is increasingly undertaken to reestablish both the living cover and the structure that carries it [24, 3]. The record of that effort is uneven. Multiregional follow-up of restored assemblages finds survival spanning the full range from near-total loss to near-total persistence across sites that used comparable biological material, which points to site conditions and to deployment practice rather than to the corals themselves [12].

Active restoration has become a standard response to reef degradation, and the majority of projects attach coral fragments to some form of artificial substrate before returning them to the seabed [3]. The substrate ranges from cast concrete blocks and perforated domes to welded rebar tables, sand-coated steel frames, and tetrahedral modules, and the choice is usually made on grounds of cost, availability, and handling rather than on any stated mechanical criterion [4]. Indonesian projects span this whole range. Large deployments on rubble fields in the Spermonde Archipelago have used hexagonal sand-coated steel frames laid in interlocking webs [30, 28], while elsewhere fragments have been fixed to substrates whose durability proved insufficient over the first years [7, 8].

The reasons that restoration fails are varied, and most of them are biological or chemical. Bleaching, sedimentation, disease, predation, and the simple absence of maintenance all end projects [3, 2]. A separate and narrower failure mode is mechanical. If a unit slides, migrates, buries itself in sand, or overturns, the fragments it carries are lost whatever their physiological state, and the loss is attributable to the structure rather than to the site. Reviews of substrate stabilization identify the stability of small structures as a recurring practical problem and note that quantitative guidance is scarce [4]. Long-term follow-up of low-cost rehabilitation has likewise found that the fate of the structure, rather than of the coral attached to it, often decides the outcome [10].

The flow environment in which that mechanical failure occurs is itself well characterized. Wave-driven and wave-breaking-driven motions dominate the circulation over shallow reefs, and the near-bed orbital velocities they

produce are the principal hydrodynamic forcing on anything resting there [21]. The discipline that connects such flows to the survival of benthic objects, biological or manufactured, is ecomechanics, which proceeds by comparing an imposed load against a material or geometric capacity and has been applied successfully to organisms across the intertidal and subtidal range [5].

The mechanics needed to address this are old and well established in other settings. The hydrodynamic load on a body small compared with the wavelength follows the Morison parameterization [23], whose relative weighting of drag and inertia is organized by the Keulegan-Carpenter number [17]. The rocking of a rigid block about its base corners, including the loss of angular momentum at each impact, was solved by Housner [14] and remains the reference problem for bodies that may overturn. Numerical integration of rigid bodies under unilateral frictional contact is carried out by nonsmooth time-stepping schemes that treat the contact impulse as a measure [22, 16, 1], and such schemes have been validated against laboratory measurements of planar blocks that rock, slide, and slide while rocking on a frictional support [31]. Separately, the mechanics of coral colonies themselves under hydrodynamic disturbance has been developed in detail, with dislodgement expressed through a colony shape factor and compared against the strength of the attachment [20, 19]. A further element of the Indonesian flow environment is the internal solitary wave (ISW), generated at sills and propagating through the straits of the archipelago, which delivers a transient near-bed current at depths where the surface wave field alone would be quiescent [26, 27]. What has not been assembled is the corresponding treatment of the manufactured object that carries the coral, in the configuration and at the scales that restoration practice actually uses.

This study assembles it, deliberately at the level of an idealized reduced model. A single planar formulation, verified against closed-form solutions, is applied to five reference structures in six materials across four idealized site classes, with one solver, one set of load coefficients, and one contact law, so that differences among the outcomes can be attributed to geometry and material alone. The aim is to rank the mechanical controls, to identify which failure mode each geometry reaches first, and to establish how much of that ranking survives the uncertainty in the force coefficients. It is not to predict the fate of any unit at any site. No field, laboratory, or reanalysis data are used. Each structure is defined by a small set of round dimensions that lie within the ranges reported for deployed designs, and every quantity reported here follows from the model equations. Throughout, the symbol $\equiv$ introduces a definition and $=$ denotes an equality that is derived or imposed as a model relation.

2 Methods

2.1 Model description

The configuration is shown in Fig. 1. A transplant unit rests on a flat, rigid bed at depth h beneath the still water line. Motion is confined to the vertical plane that contains the direction of wave propagation, which is the worst case for a given wave height in this formulation because the entire load then acts in the plane of the two base contacts. The horizontal coordinate is x , positive in the direction of propagation, and z is measured upward from the still water line, so that the bed lies at $z = -h$.

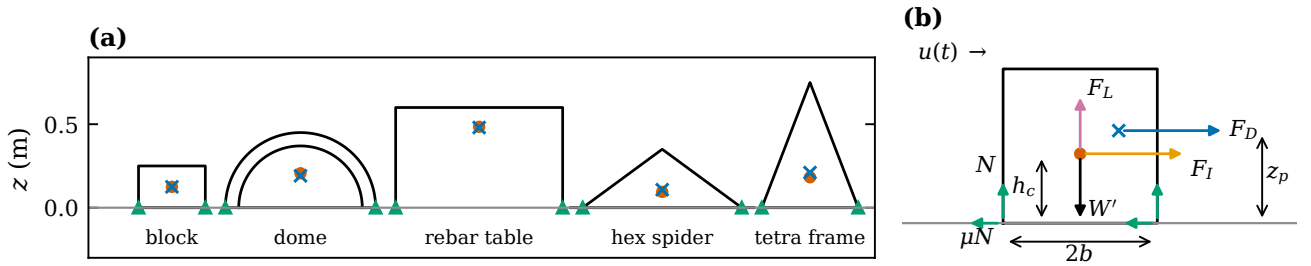


Figure 1: Model geometry. (a) Edge-on outlines of the five reference units drawn to scale, with the center of mass (filled circle), the center of pressure (cross), and the two base contacts that act as rocking pivots (triangles). (b) Free-body diagram of a unit at rest: Morison drag F_D at the center of pressure z_p , the Froude-Krylov and added-mass force F_I and the lift F_L at the center of mass h_c , the submerged weight W' , and the normal and friction reactions at contacts separated by $2b$.

The unit is a rigid body $\mathcal{B}$ of mass m , solid volume V , and density ρ_s , whose configuration is

$$\mathbf{q} \equiv (X, Z, \theta)^T \in \mathbb{R}^2 \times \mathbb{S}^1, \quad \mathbf{v} \equiv \dot{\mathbf{q}}, \quad (1)$$

with (X, Z) the center of mass and θ the rotation, taken positive counterclockwise. At rest the center of mass lies at height h_c above the bed, the two base contacts lie at body-fixed offsets $(\pm b, -h_c)$ from it, and the overall height is H_u .

The load that acts on this body is set by the water motion at the bed, which is obtained from linear wave theory. The water, of density ρ under gravitational acceleration g , is treated as inviscid, incompressible, and irrotational outside the immediate neighborhood of the unit, so that the velocity admits a potential φ satisfying

$$\nabla^2 \varphi = 0 \quad \text{in } -h < z < \eta, \quad \frac{\partial \varphi}{\partial z} = 0 \quad \text{at } z = -h, \quad (2)$$

with $\eta(x, t)$ the surface elevation. Linearizing the kinematic and dynamic free-surface conditions about $z = 0$ and separating variables for a progressive wave of amplitude $a \equiv H_w/2$, where H_w is the crest-to-trough wave height, with wavenumber $k \equiv 2\pi/L$ for wavelength L and angular frequency $\omega \equiv 2\pi/T$ for period T , gives

$$\varphi = \frac{ag}{\omega} \frac{\cosh[k(z+h)]}{\cosh(kh)} \sin(kx - \omega t), \quad \eta = a \cos(kx - \omega t), \quad \omega^2 = gk \tanh(kh). \quad (3)$$

The dispersion relation is the solvability condition of Eq. (2) under the linearized free-surface conditions. Differentiating the potential and eliminating gk with the dispersion relation, using $\cosh(kh) \tanh(kh) = \sinh(kh)$, gives the orbital velocity components

$$u = \frac{\partial \varphi}{\partial x} = a\omega \frac{\cosh[k(z+h)]}{\sinh(kh)} \cos(kx - \omega t), \quad w = \frac{\partial \varphi}{\partial z} = a\omega \frac{\sinh[k(z+h)]}{\sinh(kh)} \sin(kx - \omega t). \quad (4)$$

At the bed the vertical component vanishes identically and the horizontal amplitude is

$$u_b \equiv \max_t |u| \Big|_{z=-h} = \frac{a\omega}{\sinh(kh)}. \quad (5)$$

Equation (5) is the single channel through which the surface wave reaches the unit, and the factor $\sinh^{-1}(kh)$ is why long swell loads the bed at depths where wind sea does not. The transcendental dispersion relation is solved by Newton iteration from the explicit approximation of Fenton and McKee [6].

Carrying the free-surface expansion to second order adds a bound harmonic whose near-bed amplitude is

$$u_2 = \frac{3}{4} \frac{a^2 \omega k}{\sinh^4(kh)}, \quad (6)$$

which makes the onshore half-cycle stronger and shorter than the offshore half-cycle. The second-order series develops a spurious secondary crest once the Ursell number $Ur \equiv H_w L^2 / h^3$, which measures the ratio of nonlinearity to dispersion, exceeds about 26. Beyond that limit the harmonic is set to zero and the wave is represented by its linear component alone, which we report rather than correct.

A steady current U_c is superposed, and an ISW is represented in the two-layer weakly nonlinear limit. With an upper layer of thickness h_1 and a lower layer of thickness h_2 separated by a density step $\Delta\rho$, the reduced gravity is $g' \equiv g\Delta\rho/\rho$ and the interface displacement ζ obeys the Korteweg-de Vries (KdV) equation

$$\frac{\partial \zeta}{\partial t} + c_0 \frac{\partial \zeta}{\partial x} + \alpha_1 \zeta \frac{\partial \zeta}{\partial x} + \beta \frac{\partial^3 \zeta}{\partial x^3} = 0, \quad c_0 \equiv \sqrt{\frac{g' h_1 h_2}{h_1 + h_2}}, \quad \alpha_1 \equiv \frac{3c_0}{2} \frac{h_1 - h_2}{h_1 h_2}, \quad \beta \equiv \frac{c_0 h_1 h_2}{6}, \quad (7)$$

whose solitary solution is

$$\zeta = \zeta_0 \operatorname{sech}^2 \left(\frac{x - \mathcal{V}t}{\lambda} \right), \quad \mathcal{V} = c_0 + \frac{\alpha_1 \zeta_0}{3}, \quad \lambda^2 = \frac{12\beta}{\alpha_1 \zeta_0}. \quad (8)$$

To the same order the layer velocities are $u_2^{\text{lo}} = c_0 \zeta / h_2$ and $u_1^{\text{up}} = -c_0 \zeta / h_1$, so that $u_1^{\text{up}} h_1 + u_2^{\text{lo}} h_2 = 0$ and the wave transports no net volume. With the pycnocline closer to the bed than to the surface, $\alpha_1 > 0$ and the soliton is one of elevation, as expected for a wave that has shoaled onto a shelf [13]. Such waves are routinely observed in Indonesian straits [27]. The unit therefore experiences

$$u(t) = U_c + u_b \cos(\omega t) + u_2 \cos(2\omega t) + \frac{c_0 \zeta_0}{h_2} \operatorname{sech}^2 \left(\frac{\mathcal{V}(t - t_0)}{\lambda} \right), \quad (9)$$

evaluated at the bed and taken uniform over the unit, which is admissible because the unit spans about one metre against wavelengths of tens of metres.

This ambient velocity must now be converted into a force on the unit. Because the unit is small compared with the wavelength, so that it does not appreciably scatter the incident wave, the load is represented by the Morison decomposition [23]. Writing A for the frontal area presented to the flow, A_L for the plan area seen from above, C_D , C_M , and C_L for the dimensionless drag, inertia, and lift coefficients, and $C_a \equiv C_M - 1$ for the added-mass coefficient, the horizontal force on a body whose center of mass translates at velocity $v \equiv \dot{X}$ and whose center of pressure moves at the horizontal velocity v_P , which differs from v by the rotational contribution $-\dot{\theta}$ times the lever arm of that point above the center of mass, is

$$F_x = \rho V \dot{u} + C_a \rho V (\dot{u} - \dot{v}) + \frac{1}{2} \rho C_D A (u - v_P) |u - v_P|, \quad (10)$$

in which $\dot{u}$ is the local fluid acceleration and $\dot{v}$ the body acceleration. The three terms have distinct physical origins. The first is the Froude-Krylov force, the resultant of the pressure field that the undisturbed wave would impose over the volume the body occupies, and it is present even if the body were replaced by an equal volume of water. The second is the added-mass force, which arises because a body accelerating relative to the surrounding fluid must also accelerate a quantity of that fluid, and it therefore depends on the relative acceleration rather than on either acceleration alone. The first two are collectively the inertial load, and they are in quadrature with the velocity, so they peak when the orbital flow reverses. The third is the quadratic drag, written in the signed form that preserves its direction through flow reversal and denoted F_D below; it peaks at maximum velocity, a quarter period away from the inertial maximum. The lift, directed upward, is $F_L = \frac{1}{2} \rho C_L A_L (u - v)^2$. Collecting the added-mass term that is proportional to the body acceleration onto the left side of the equation of motion

defines the effective masses

$$m_\star \equiv m + C_a \rho V, \quad I_\star \equiv m_\star \kappa^2, \quad \mathcal{F} \equiv \rho V (1 + C_a), \quad (11)$$

with $m_\star$ the effective translational mass, $I_\star$ the effective rotational inertia, κ the radius of gyration about the center of mass, and $\mathcal{F}$ the coefficient that multiplies the fluid acceleration. The remaining inertial load is $F_I = \mathcal{F}\dot{u}$. The relative importance of drag and inertia is set by the Keulegan-Carpenter number $KC \equiv u_b T / D$ [17], with D a characteristic cross-stream width, which measures the excursion of a fluid particle in one wave period against the size of the body; the configurations examined here are drag-dominated at the design conditions. Applying C_a to the rotation as well as to the translations is a modelling choice, and the sensitivity of the results to it is measured rather than assumed.

The submerged weight is

$$W' \equiv (\rho_s - \rho) g V, \quad (12)$$

which is the only quantity in the quasi-static balance that depends on the material. For a frame assembled from slender members, V , the frontal area A , the center of pressure z_p , and the inertia are all obtained by integrating over the members, so that one construction serves the solid and the open geometries alike.

With the load so defined, the remaining ingredient is the law that governs the two points at which the unit touches the bed. Contact there is unilateral, since the bed can push but not pull, and frictional, so the equations of motion are not smooth: the velocity may jump at an impact and the contact force is not a function of the configuration alone. The appropriate setting is therefore the measure differential inclusion of nonsmooth contact dynamics. With $\mathbb{M} \equiv \text{diag}(m_\star, m_\star, I_\star)$ the generalized mass matrix and $\mathbf{f}(t, \mathbf{q}, \mathbf{v})$ the applied generalized force, which collects the hydrodynamic load and the submerged weight, the motion is governed by

$$\mathbb{M} d\mathbf{v} = \mathbf{f}(t, \mathbf{q}, \mathbf{v}) dt + \sum_{i \in \{-, +\}} \mathbb{H}_i(\mathbf{q})^\top d\mathbf{P}_i, \quad d\mathbf{q} = \mathbf{v} dt, \quad (13)$$

where $d\mathbf{v}$ is a differential measure that admits atoms at impacts, dt is the Lebesgue measure, the index $i \in \{-, +\}$ labels the upstream and downstream base contacts, $\mathbb{H}_i$ is the contact Jacobian that maps generalized velocities to the local velocity at contact i , and $d\mathbf{P}_i$ is the impulse measure there [22, 16]. Let $\mathbf{r}_i(\theta) \equiv \mathbb{R}(\theta) (\pm b, -h_c)^\top$ be the rotated contact offset, with $\mathbb{R}(\theta)$ the planar rotation matrix and $r_{i,x}, r_{i,z}$ its horizontal and vertical components. The gap g_i , which is the signed height of contact i above the bed, together with the normal and tangential velocities there, are

$$g_i \equiv Z + r_{i,z}, \quad U_{n,i} \equiv \dot{Z} + \dot{\theta} r_{i,x}, \quad U_{t,i} \equiv \dot{X} - \dot{\theta} r_{i,z}, \quad (14)$$

so that the contact Jacobian rows are $\mathbb{H}_{n,i} = (0, 1, r_{i,x})$ and $\mathbb{H}_{t,i} = (1, 0, -r_{i,z})$. Writing $P_{n,i}$ and $P_{t,i}$ for the normal and tangential components of the contact impulse, the contact obeys the Signorini complementarity condition, which states that the gap and the normal impulse are both non-negative and never positive together, and Coulomb friction with coefficient μ ,

$$0 \leq g_i \perp P_{n,i} \geq 0, \quad |P_{t,i}| \leq \mu P_{n,i}, \quad U_{t,i} \neq 0 \Rightarrow P_{t,i} = -\mu P_{n,i} \frac{U_{t,i}}{|U_{t,i}|}, \quad (15)$$

the last relation being the principle of maximal dissipation. Impacts are closed by the Newton restitution law $U_{n,i}^+ = -e U_{n,i}^-$ on contacts that are active, with e the normal restitution coefficient. The bed is taken rigid and flat, which excludes scour, burial, and bearing failure.

Before integrating this system it is worth asking what it predicts at rest, because the answer is available in closed form and governs the interpretation of everything computed afterwards. Setting $\mathbf{v} = \mathbf{0}$ and requiring that the contact forces of Eq. (15) exist gives three admissibility conditions that can be written in closed form and that organize the whole of what follows. With the unit at rest and the loads evaluated from Eq. (9), sliding is impending when the tangential demand reaches the frictional capacity,

$$|F_D + F_I| = \mu (W' - F_L), \quad (16)$$

where the lift reduces the normal reaction. Tipping is impending when the resultant passes through the downstream contact, so that taking moments about that point gives

$$|F_D z_p + F_I h_c| + F_L b = W' b, \quad (17)$$

and lift-off is impending when $F_L = W'$, where W' is the submerged weight of Eq. (12) and z_p, h_c are the heights above the bed at which the drag and the inertial load respectively act. Dividing each relation by its capacity defines the dimensionless demands

$$\mathcal{S}_s \equiv \frac{|F_D + F_I|}{\mu (W' - F_L)}, \quad \mathcal{S}_t \equiv \frac{|F_D z_p + F_I h_c| + F_L b}{W' b}, \quad \mathcal{S}_\ell \equiv \frac{F_L}{W'}, \quad (18)$$

each maximized over the wave phase. The unit is admissible at rest while $\max(\mathcal{S}_s, \mathcal{S}_t, \mathcal{S}_\ell) < 1$.

The structure of the problem is clearest in the drag-dominated limit $F_I \rightarrow 0$. Define

$$\Theta \equiv \frac{F_D}{W'}, \quad \Lambda \equiv \frac{C_L A_L}{C_D A}, \quad (19)$$

in which Θ is the drag load normalized by the submerged weight and Λ is the ratio of lift to drag capacity of the geometry, so that $F_L = \Lambda F_D$ identically, because both forces are quadratic in the same velocity. Substituting

into Eq. (16) gives $\Theta(1 + \mu\Lambda) = \mu$, and into Eq. (17) gives $\Theta(z_p + \Lambda b) = b$. The three onset values are therefore

$$\Theta_s = \frac{\mu}{1 + \mu\Lambda}, \quad \Theta_t = \frac{1}{z_p/b + \Lambda}, \quad \Theta_\ell = \frac{1}{\Lambda}. \quad (20)$$

Sliding precedes tipping when $\Theta_s < \Theta_t$, that is when $\mu(z_p/b + \Lambda) < 1 + \mu\Lambda$. The terms in Λ cancel, and the condition reduces to

$$\mu \frac{z_p}{b} < 1 \iff \frac{b}{z_p} > \mu. \quad (21)$$

Three consequences follow. First, lift shifts both onsets downward but cannot change which one is reached first, so an uncertain C_L cannot corrupt the selection of the failure mode. Second, the selection is purely geometric and frictional: it involves no material property, no wave property, and no drag coefficient. Third, because coral growth adds frontal area high on the unit and so raises z_p while leaving b unchanged, a unit may begin its life on the sliding side of Eq. (21) and cross to the tipping side as its colonies develop.

Material enters only through W' . At fixed geometry in the drag-dominated limit, Eq. (20) gives $\frac{1}{2}\rho C_D A u_c^2 = \Theta_{\min} W'$, so the critical near-bed velocity obeys

$$u_c \propto \sqrt{\frac{W'}{C_D A}} \propto \sqrt{\rho_s - \rho}, \quad (22)$$

and the critical wave height inherits the same square-root dependence through Eq. (5). Changing the material therefore rescales every threshold by a single factor and reorders nothing.

The hold-down force F_a required to meet a stated safety factor $\mathcal{N}$ follows by adding F_a to W' in Eq. (18) and requiring each demand to fall to $\mathcal{N}^{-1}$,

$$F_a = \max \left\{ \frac{\mathcal{N} |F_D + F_I|}{\mu} + F_L - W', \mathcal{N} \left(\frac{|F_D z_p + F_I h_c|}{b} + F_L \right) - W', \mathcal{N} F_L - W', 0 \right\}, \quad (23)$$

maximized over the wave phase. This is the quantity a restoration team can act on, since F_a/g is the effective submerged ballast mass to be added.

Exceeding the sliding threshold once does not by itself move a unit far, and the question that matters in practice is what accumulates over many waves. A symmetric wave drives equal and opposite slip episodes and produces no net displacement. A current breaks that symmetry, and the resulting drift admits a closed form near threshold. Let $r(\phi) \equiv |F_x| / [\mu N]$ be the ratio of the horizontal load to the frictional capacity with the unit at rest, where $N \equiv W' - F_L$ is the normal reaction, over the wave phase $\phi \equiv \omega t$. Let r attain its maximum $1 + \varepsilon$ at phase ϕ_p , so that ε is the fractional overshoot of the threshold, and expand about that maximum,

$$r(\phi) = (1 + \varepsilon) \left[1 - \frac{1}{2} c (\phi - \phi_p)^2 \right] + O((\phi - \phi_p)^3), \quad c \equiv - \frac{1}{r} \frac{d^2 r}{d\phi^2} \Big|_{\phi_p}. \quad (24)$$

in which c is the dimensionless curvature of the load ratio at its peak. Writing $\mathcal{X}$ for the horizontal displacement accumulated in one slip episode, $\tau \equiv (\phi - \phi_p)/\omega$ for the time measured from the peak, and $a_\star \equiv \mu N / m_\star$ for the acceleration scale set by the frictional capacity, the acceleration during slip is

$$\ddot{\mathcal{X}} = a_\star [r - 1] = a_\star \left(\varepsilon - \frac{1}{2} \kappa \tau^2 \right), \quad \kappa \equiv (1 + \varepsilon) c \omega^2, \quad (25)$$

with κ the curvature of the acceleration in time. Slip begins where the bracket changes sign, at $\tau = -\tau_0$ with $\tau_0 \equiv \sqrt{2\varepsilon/\kappa}$. Integrating once from rest,

$$\dot{\mathcal{X}}(\tau) = \frac{a_\star \kappa}{6} (3\tau_0^2 \tau + 2\tau_0^3 - \tau^3), \quad (26)$$

which vanishes again when $\tau^3 - 3\tau_0^2 \tau - 2\tau_0^3 = 0$. Factoring gives $(\tau - 2\tau_0)(\tau + \tau_0)^2 = 0$, so the episode ends at $\tau = 2\tau_0$ and lasts exactly $3\tau_0$. Integrating Eq. (26) over the episode,

$$\Delta_0 = \int_{-\tau_0}^{2\tau_0} \dot{\mathcal{X}} d\tau = \frac{a_\star \kappa}{6} \left[\frac{3}{2} \tau_0^2 \tau^2 + 2\tau_0^3 \tau - \frac{1}{4} \tau^4 \right]_{-\tau_0}^{2\tau_0} = \frac{9}{8} a_\star \kappa \tau_0^4 = \frac{9}{2} \frac{\mu N}{m_\star} \frac{\varepsilon^2}{\kappa}, \quad (27)$$

which is exact for the parabolic excess and is the leading term for any smooth load as $\varepsilon \rightarrow 0$. For pure drag under $u = U_c + u_b \cos \omega t$ one finds $c = 2u_b/(U_c + u_b)$, so the drift vanishes with the current, as it must.

Equation (27) neglects that drag acts on the relative velocity. A slipping unit therefore feels a restoring load $\partial F_D / \partial v = -\rho C_D A |u|$, which acts as a weak linear damping of rate $\gamma \equiv \rho C_D A |u_p| / m_\star$, with u_p the ambient velocity at the load peak. Treating it as a regular perturbation, $\dot{\mathcal{X}} = \dot{\mathcal{X}}_0 - \gamma \int_{-\tau_0}^\tau \dot{\mathcal{X}}_0 ds + O(\gamma^2)$, and integrating the correction over the episode gives $\int_{-\tau_0}^{2\tau_0} \int_{-\tau_0}^\tau \dot{\mathcal{X}}_0 ds d\tau = \frac{27}{20} a_\star \kappa \tau_0^5$. Dividing by Eq. (27) and defining the damping number $\Gamma \equiv \gamma \tau_0$,

$$\Delta = \Delta_0 \left[1 - \frac{6}{5} \Gamma + O(\Gamma^2) \right], \quad \frac{27/20}{9/8} = \frac{6}{5}. \quad (28)$$

Since $\Gamma \propto \varepsilon^{1/2}$, the correction vanishes at onset but only slowly. When the current itself varies slowly compared with the wave period, as during the passage of a tide or of an internal solitary wave, the displacement is obtained by integrating the steady per-cycle drift $\mathcal{D}(U_c)$, meaning the value of Δ attained at a fixed current U_c , along the

slowly varying current,

$$\mathcal{X} = \frac{1}{T} \int \mathcal{D}(U_c(t)) dt. \quad (29)$$

All of the above describes a bare structure, but a transplant unit is deployed in order that it should not stay bare, and the colonies it carries alter its own mechanics as they grow. Colonies are represented as hemispheres of radius r_c mounted at a fixed height z_m above the bed, with bulk density ρ_c , solid fraction ς , that is, the fraction of the hemispherical envelope occupied by skeleton, and drag coefficient $C_{D,c}$. For n colonies the frontal area increment is $\frac{1}{2}n\pi r_c^2$ with centroid $4r_c/(3\pi)$ above the mounting point, and the solid volume increment is $\frac{2}{3}n\varsigma\pi r_c^3$. The drag area and center of pressure of the assembly follow from the area-weighted combination

$$(C_{DA})' = C_{DA} + \frac{1}{2}C_{D,c}n\pi r_c^2, \quad z_p' = \frac{C_{DA}z_p + \frac{1}{2}C_{D,c}n\pi r_c^2(z_m + \frac{4r_c}{3\pi})}{(C_{DA})'}, \quad (30)$$

in which a prime denotes the property of the colonized assembly, and with the mass, center of mass, and inertia updated by the parallel-axis theorem. Because the added area grows as r_c^2 while the added submerged weight grows as r_c^3 , the two effects partly cancel, and which one dominates depends on how heavy the bare unit is. Colonies are not followed beyond $r_c = 0.10$ m, since neither colony merging nor the canopy flow that would then develop is represented. The mechanics of the colonies themselves, including their own dislodgement, is outside the present scope and has been treated elsewhere [20, 19].

It remains to specify the objects to which this formulation is applied. Five reference structures span the range in common use: a solid cast block, a perforated hemispherical dome, a welded rebar table on legs, a hexagonal sand-coated frame of the kind deployed on Indonesian rubble fields [30], and a tetrahedral rod frame. Their derived mechanical data are listed in Table 1. The dimensions are a parameter set rather than measurements of any particular design. Because the quasi-static thresholds depend on the geometry only through $W'/(C_{DA})$ and z_p/b , a reader with a different unit can rescale the results through Eq. (22) rather than recompute them.

Table 1: Reference transplant units in their native materials. Here m is mass, W' submerged weight, b the pivot half-distance, z_p the center-of-pressure height, h_c the center-of-mass height, and C_{DA} the drag area. The last two columns give b/z_p as deployed and with reference colonies of 0.10 m radius mounted on top, to be compared with μ through Eq. (21). All values are illustrative.

Unit	Material	m (kg)	W' (N)	b (m)	z_p (m)	C_{DA} (m ²)	b/z_p	b/z_p grown
Solid block	concrete	92.0	500.3	0.200	0.125	0.120	1.60	1.10
Perforated dome	concrete	136.5	742.1	0.450	0.191	0.200	2.36	1.59
Rebar table	steel	9.8	83.8	0.500	0.480	0.115	1.04	0.83
Hexagonal frame	coated steel	6.2	46.4	0.476	0.108	0.095	4.41	1.90
Tetrahedral frame	concrete	51.7	281.1	0.289	0.210	0.382	1.37	0.88

Four idealized site classes stand for settings that practitioners recognize: a sheltered bay, a coast exposed to monsoon wind sea, a south-facing coast exposed to Indian Ocean swell, and a strait carrying a persistent current. Each is defined by a design wave height, a period, a current, and a reference depth, listed in Table 2, and the corresponding near-bed forcing is shown in Fig. 2. None is a measurement at a site. The four uncalibrated coefficients are assigned the ranges C_D factor [0.7, 1.5], C_L factor [0, 2], $C_a \in [0.5, 1.5]$, and $\mu \in [0.4, 0.8]$, which are carried through every result that follows.

Table 2: Idealized site classes. H_w is the design wave height, T the period, U_c the superposed current, and h the reference depth. All values are illustrative and none is a measurement.

Site class	H_w (m)	T (s)	U_c (m s ⁻¹)	h (m)
Sheltered bay	0.8	5.0	0.0	5.0
Monsoon-exposed	2.0	8.0	0.0	8.0
Swell-exposed	3.0	14.0	0.0	10.0
Strait with current	1.2	6.0	0.8	8.0

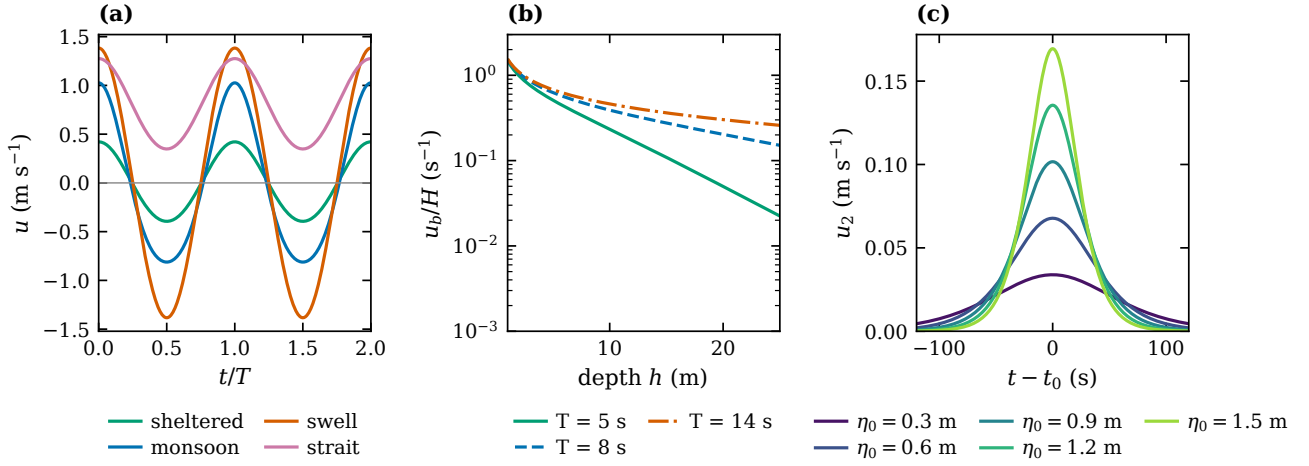


Figure 2: Near-bed forcing. (a) Velocity over two periods of the design wave of each site class at its reference depth, on the class current. The second-order Stokes harmonic of Eq. (6) is included where $Ur < 26$; the swell class exceeds that limit and is shown by its linear component. (b) Near-bed amplitude per metre of wave height, u_b/H_w , against depth for three periods, showing the $\sinh^{-1}(kh)$ attenuation of Eq. (5). (c) Lower-layer velocity of the two-layer soliton of Eq. (8) at a fixed point, for crest displacements of 0.3 to 1.5 m.

2.2 Numerical implementation

Equation (13) is integrated by the Moreau-Jean time-stepping scheme [22, 16, 1], which discretizes the measure rather than the acceleration and therefore requires no event detection. With step $\hbar$ and the midpoint rule $\vartheta = \frac{1}{2}$, one step from $(\mathbf{q}_k, \mathbf{v}_k)$ is

$$\mathbf{q}_m = \mathbf{q}_k + \frac{1}{2}\hbar \mathbf{v}_k, \quad \mathbf{v}^{\text{free}} = \mathbf{v}_k + \hbar \mathbb{M}^{-1} \mathbf{f}(t_{k+1/2}, \mathbf{q}_m, \mathbf{v}_k), \quad (31)$$

$$\mathbf{v}_{k+1} = \mathbf{v}^{\text{free}} + \mathbb{M}^{-1} \mathbb{H}(\mathbf{q}_m)^\top \mathbf{P}, \quad \mathbf{q}_{k+1} = \mathbf{q}_k + \frac{1}{2}\hbar (\mathbf{v}_k + \mathbf{v}_{k+1}), \quad (32)$$

in which the impulse $\mathbf{P}$ is determined on the contacts whose gap at $\mathbf{q}_m$ is closed by the velocity-level complementarity problem

$$0 \leq U_{n,i}^+ + e \min(U_{n,i}^-, 0) \perp P_{n,i} \geq 0, \quad P_{t,i} \in -\mu P_{n,i} \text{Sign}(U_{t,i}^+), \quad (33)$$

with Sign the set-valued sign. Writing $\mathbb{W} \equiv \mathbb{H}\mathbb{M}^{-1}\mathbb{H}^\top$ for the Delassus operator and $\mathbf{U}^{\text{free}} = \mathbb{H}\mathbf{v}^{\text{free}}$, the problem reads $\mathbf{U}^+ = \mathbb{W}\mathbf{P} + \mathbf{U}^{\text{free}}$ subject to Eq. (33). For a planar body with two contacts $\mathbb{W}$ is 4×4 , and the problem is solved by projected Gauss-Seidel sweeps in which the normal impulse of each contact is projected onto $[0, \infty)$ and the tangential impulse onto $[-\mu P_n, \mu P_n]$. Twenty to thirty sweeps reduce the complementarity residual to the level reported below. A contact counts as closed when its gap falls below 10^{-8} m; without that tolerance, round-off decides whether a resting contact is active and the two implementations diverge.

The impact map implied by Eq. (33) can be obtained in closed form and provides the first verification target. A body rocking about the contact at $(-b, 0)$ with angular velocity $\dot{\theta}^-$ strikes the contact at $(b, 0)$. Angular momentum about the new pivot is conserved through the impulsive phase, so with $\mathbf{r}_{G/B} = (-b, h_c)$ and $\mathbf{v}_G = \dot{\theta}^- \hat{\mathbf{z}} \times (b, h_c)$,

$$I_G \dot{\theta}^- + m_\star (\mathbf{r}_{G/B} \times \mathbf{v}_G)_z = (I_G + m_\star R^2) \dot{\theta}^+, \quad R \equiv \sqrt{b^2 + h_c^2}, \quad (34)$$

and evaluating the cross product gives the restitution ratio

$$\frac{\dot{\theta}^+}{\dot{\theta}^-} = \frac{I_G + m_\star (h_c^2 - b^2)}{I_G + m_\star R^2}. \quad (35)$$

For a solid rectangle in vacuum, $I_G = m(b^2 + h_c^2)/3$, and substituting with the slenderness angle $\alpha \equiv \arctan(b/h_c)$, so that $\sin \alpha = b/R$, reduces Eq. (35) to

$$\frac{\dot{\theta}^+}{\dot{\theta}^-} = \frac{2h_c^2 - b^2}{2R^2} = 1 - \frac{3}{2} \sin^2 \alpha, \quad (36)$$

which is Housner's classical result [14].

Two reduced models, sharing no code path with the time-stepper, serve as independent references. The first integrates pure sliding without rotation, $m_\star \dot{\mathcal{X}} = F_x(t, \mathcal{X}) - \varsigma \mu N$ with $\varsigma = \text{sgn} \dot{\mathcal{X}}$, sticking whenever $\dot{\mathcal{X}} = 0$ and $|F_x| \leq \mu N$. The second integrates rocking about a stuck corner, $I_O \ddot{\theta} = \mathcal{T}(t, \theta, \dot{\theta})$ with $I_O = I_G + m_\star R^2$, applying Eq. (35) at each passage through $\theta = 0$. Both are advanced with an eighth-order explicit Runge-Kutta pair and exact event location.

The production runs use $\hbar = T/400$. Each unit is advanced independently, so the kernel is written over a batch and compiled, which makes the Monte Carlo sampling of the coefficient ranges tractable on one core. The library is written in Python with NumPy [11], SciPy [29], Numba [18], and Matplotlib [15]. Five verification

tests are used: the restitution ratio against Eq. (36); the time to first impact in free rocking against the exact nonlinear pivot equation; the drift per cycle against the reduced sliding model; the peak rotation against the reduced rocking model; and the onset under a slowly ramped current against Eq. (20). The dispersion relation and the soliton of Eq. (8) are checked against their own governing equations, and the compiled kernel is checked against an independent reference implementation written in array form.

3 Results

The solver passes each verification test (Fig. 3). The restitution ratio agrees with Eq. (36) to 4.4×10^{-3} in absolute value over slenderness angles from 11° to 46° . The dispersion relation is satisfied to 2.2×10^{-16} , and the soliton of Eq. (8) satisfies Eq. (7) to a relative residual of 5.5×10^{-9} . The compiled kernel and the array reference agree to 1.7×10^{-16} in displacement and 8.4×10^{-17} in rotation, which is round-off. The worst complementarity residual over all steps is 7.4×10^{-6} . Convergence against the two reduced models is first order, as expected for a nonsmooth trajectory: the error in the drift per cycle falls from 1.3×10^{-4} to 9.5×10^{-6} m and the error in the peak rotation from 2.5×10^{-2} to 2.2×10^{-4} rad as the step is refined by a factor of sixteen. The error in the time to first impact in free rocking falls from 2.4×10^{-5} to 6.9×10^{-6} s and then flattens, a floor set by the closed-gap tolerance rather than by the step. Onsets under a slowly ramped current reproduce Eq. (20) to a maximum relative error of 8.4×10^{-4} , with the predicted first mode recovered in 100% of cases, and the drift per cycle agrees with the event-driven sliding model to 2.3×10^{-3} in relative terms.

These checks establish that the implemented equations are solved correctly, and they do so along three independent axes. The agreement with the impact map and with Housner’s restitution tests the treatment of the velocity jump, which is the part of a nonsmooth formulation most easily got wrong. The first-order convergence against two event-driven solvers that share no code with the time-stepper tests the integration of the smooth phases between events and the detection of the events themselves; first order is the expected rate for a trajectory with velocity discontinuities, and obtaining a higher apparent rate would indicate that the test problem was not exercising the nonsmooth behaviour. The agreement of the ramped onsets with the closed forms tests the quasi-static limit of the full solver against the algebra of Eqs. (20) and (21), and the recovery of the predicted first mode in every case is the stronger of the two statements there, since the mode is a discrete outcome that a small numerical error could nonetheless flip near the criterion. None of this is validation. The closed-gap tolerance that floors the free-rocking error at about 7×10^{-6} s is a reminder that the contact is regularized at a scale far below any physical one, which is adequate for the thresholds reported here but would matter for a question posed at that scale.

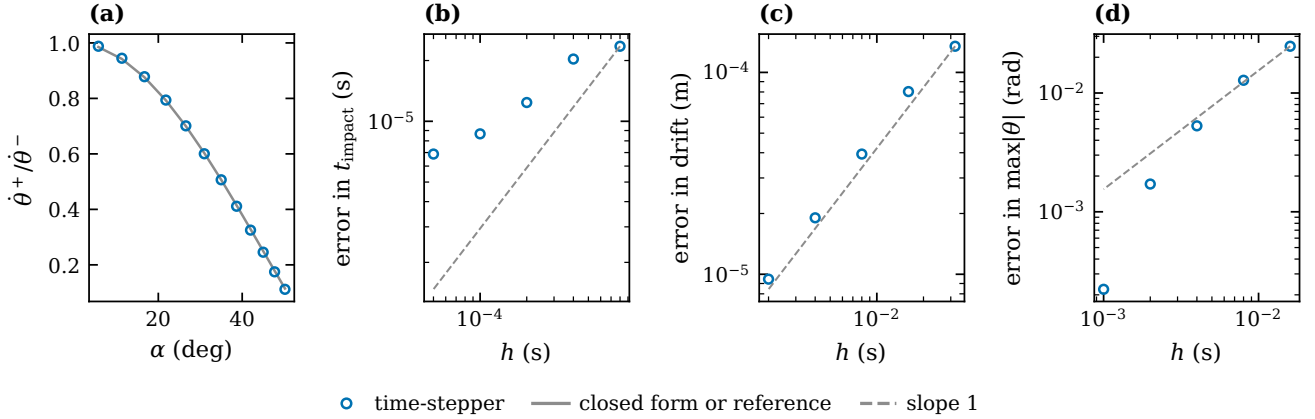


Figure 3: Verification. (a) Angular-velocity ratio across an impact in vacuum, from the time-stepper (circles) against Eq. (36) (line). (b) Error in the time to the first impact in free rocking against step size, with the exact nonlinear pivot equation as reference. (c) Error of the drift per cycle against the event-driven sliding model. (d) Error of the peak rotation against the event-driven single-pivot rocking model. The grey lines have slope one.

The failure-mode criterion of Eq. (21) separates the five units cleanly (Fig. 4a). At $\mu = 0.6$ every unit lies on the sliding side, with b/z_p from 1.04 for the rebar table to 4.41 for the hexagonal frame (Table 1). Growth moves all five toward the criterion, and the grown rebar table and tetrahedral frame fall to $b/z_p = 0.83$ and 0.88 , which places them on the tipping side for any μ above those values. The onsets computed under a slowly ramped current lie on the closed forms of Eq. (20) across two decades of z_p/b and for both $\Lambda = 0$ and $\Lambda = 0.5$ (Fig. 4b), and the mode that the simulation reaches first is in every case the one that Eq. (21) predicts.

The margin to the criterion is worth reading against the friction range carried through this study, $\mu \in [0.4, 0.8]$. As deployed, the smallest ratio among the five units is 1.04 for the rebar table, which exceeds the upper end of that range, so every unit slides first at every friction value considered and the mode selection is unambiguous. Once colonies have grown, the rebar table and the tetrahedral frame fall to 0.83 and 0.88, which sit just above the upper end; the mode selection for those two geometries becomes sensitive to the substrate in a way it was not on

the day of deployment. The practical consequence is specific and slightly counterintuitive. Staking a unit, which raises the effective friction, is the standard remedy for sliding, and it works: raising μ to 0.9 removes sliding as the governing mode for these geometries. But 0.9 is above both 0.83 and 0.88, so the same intervention that cures the sliding of a grown rebar table or tetrahedral frame moves it across the criterion and makes overturning the mode that governs. An anchoring scheme that resists sliding without also resisting rotation about the downstream contact therefore solves one problem by creating another, and the two frames with the slenderest grown geometry are precisely the ones for which this matters. The solid block, the dome, and the hexagonal frame retain grown ratios of 1.10, 1.59, and 1.90 and remain on the sliding side whatever is done to the substrate.

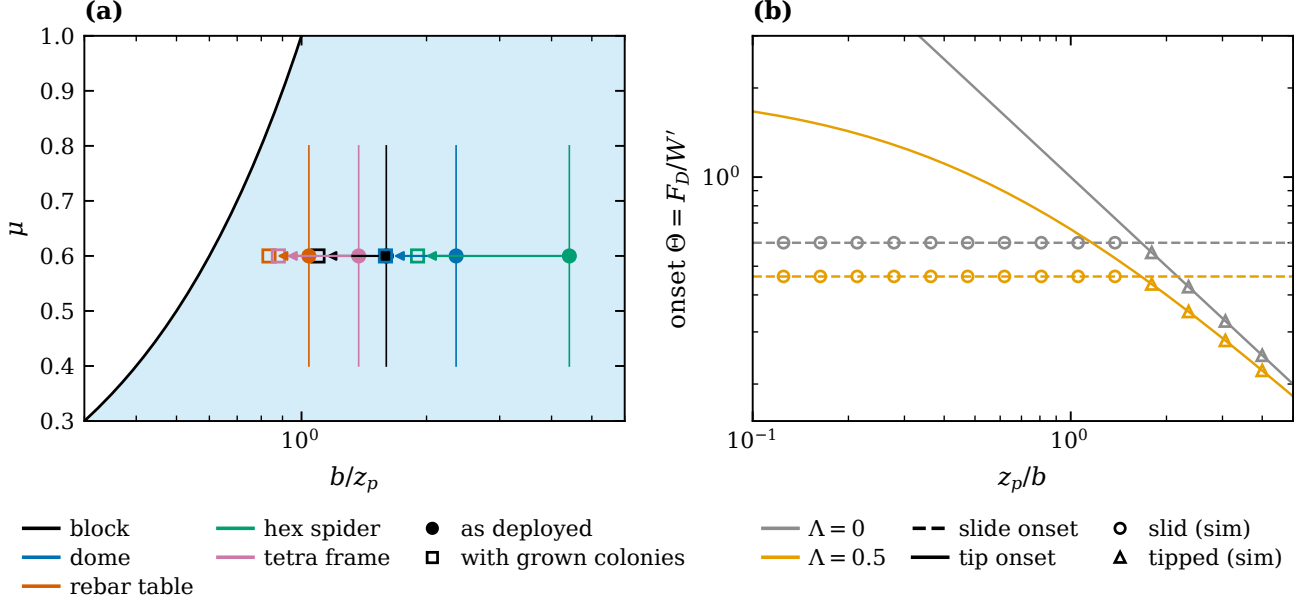


Figure 4: Failure-mode selection. (a) The plane of b/z_p against μ . Units above the diagonal slide first and those below tip first, independently of lift. Circles mark the five units as deployed, squares the same units carrying grown colonies, and the arrows the displacement caused by growth. (b) Onset value of $\Theta = F_D/W'$ under a slowly ramped current from the time-stepper (markers) against the closed forms of Eq. (20) (lines), for blocks of varying height at two values of Λ .

Critical wave heights over depth and period are shown in Fig. 5. At the monsoon reference condition of 8 m depth and 8 s period, the solid block in concrete requires 3.38 m to move and the dome 3.18 m, while the tetrahedral frame requires 1.81 m, the rebar table in steel 1.82 m, and the hexagonal frame in coated steel 1.51 m. Changing the material rescales every entry by $\sqrt{\rho_s - \rho}$ as Eq. (22) requires, so the same block in polymer requires 1.80 m and in solid steel 6.86 m, with the ordering of the structures preserved throughout.

The structure of Fig. 5a to 5e repays attention, because its shape is set by the $\sinh^{-1}(kh)$ factor of Eq. (5) and not by anything particular to transplant units. The critical height rises steeply with depth at fixed period, so a unit that is marginal at 5 m is comfortable at 12 m, and it falls with period at fixed depth, so a long swell of modest height is more dangerous than a shorter wind sea of the same height. Reading the four site classes of Table 2 against these surfaces separates them cleanly. The sheltered bay class is benign for every structure examined. The monsoon class, at 2 m and 8 s on 8 m of water, is where the two solid units stand well clear and the three frames do not. The swell class, at 3 m and 14 s, is the most demanding of the four despite being shallower-water than one might expect, because the long period carries the orbital motion to the bed with little attenuation; it is also the class whose Ursell number exceeds the second-order limit, so it is reported on its linear component alone and the true onshore excursion there is somewhat larger than shown. The strait class, at only 1.2 m and 6 s, is the mildest of the three exposed classes by wave height and yet is the one in which units are most likely to be lost, because its superposed current of 0.8 m s^{-1} converts a symmetric wave into the ratchet of Eq. (27). A classification of restoration sites by wave height alone would rank these four in a different order than a classification by what actually moves a unit.

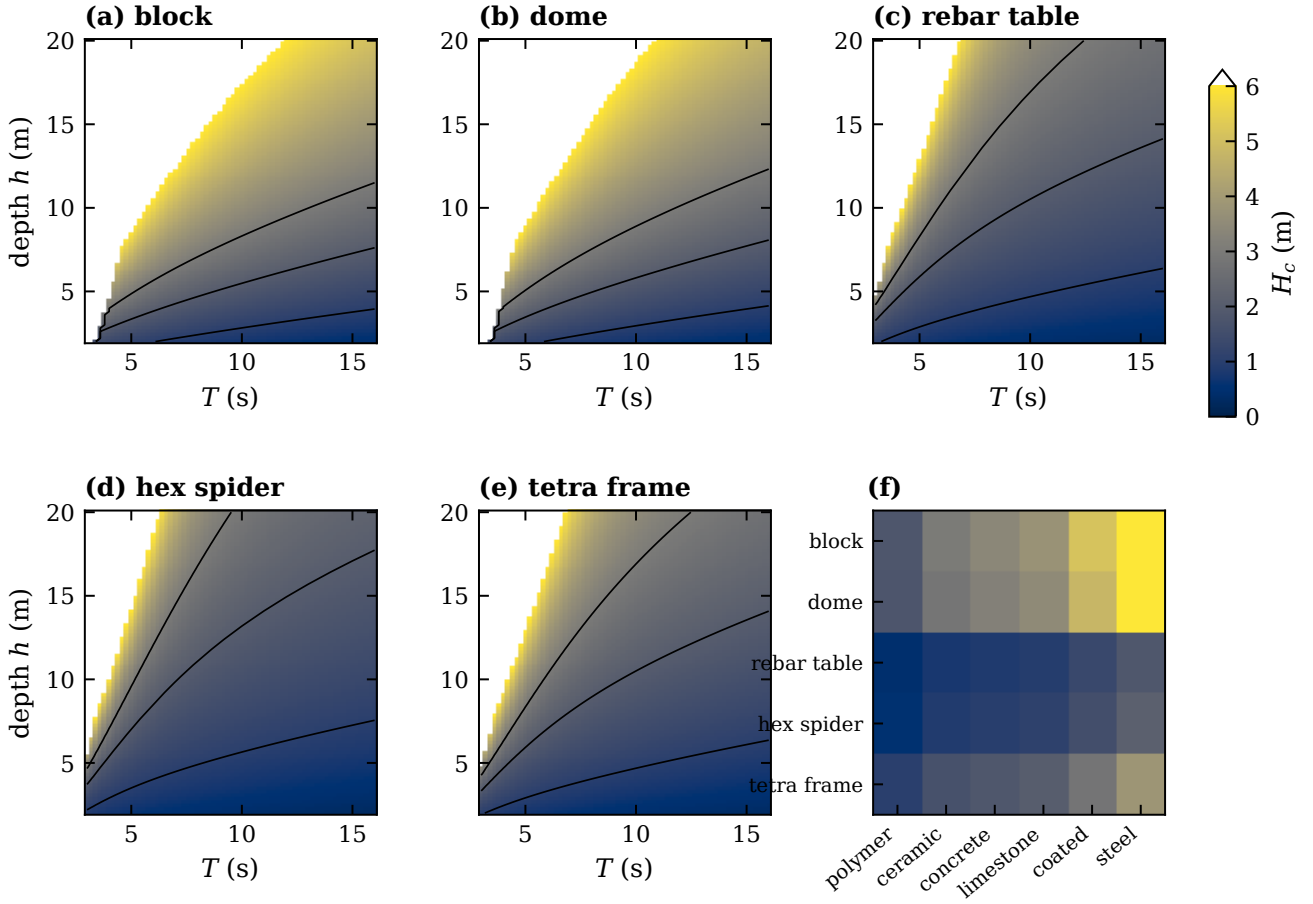


Figure 5: Critical wave height. (a) to (e) The smallest regular wave that moves each unit, over water depth and wave period, with the second-order harmonic included where admissible. White marks combinations at which no wave below the depth-limited breaking height moves the unit. Contours at 1, 2, and 3 m. (f) Critical wave height at the monsoon reference condition for every structure in every material, geometry held fixed.

Drift under waves riding on a current is confirmed by three independent calculations (Fig. 6). For the concrete block at $T = 8$ s and $u_b = 1.8$ m s⁻¹, the time-stepper, the event-driven sliding model, and Eq. (27) agree near threshold, and the drift per cycle rises from 10^{-4} m at $U_c/u_b = 0.2$ to 0.33 m at $U_c/u_b = 0.4$. Suppressing the relative-velocity dependence of the drag recovers Eq. (27) to within 0.5% as $\varepsilon \rightarrow 0$, and restoring it reduces the drift by the factor of Eq. (28), verified for Γ up to 0.62. Expressed as a rate at the monsoon design wave, the solid block and the dome do not move at all, while the three frames migrate at rates that rise steeply once the current exceeds a few centimetres per second.

The practical weight of this result lies in how it accumulates. A drift of a tenth of a millimetre per wave is invisible on any single dive, but at a period of 8 s it is some eleven thousand cycles a day, and a monsoon season of a hundred days is of order a million cycles. The ε^2 scaling of Eq. (27) is what makes this difficult to anticipate from observation. Because the drift vanishes quadratically at onset, a unit held at 90% of its threshold produces essentially nothing and a unit held at a hundred and ten percent produces a migration that is visible within a season; there is no intermediate regime in which the accumulation is slow but detectable, and so no early warning. The transition between the two states can be driven by a change in any of the quantities in the threshold, and the easiest of these to change inadvertently is the exposure, through the loss of a neighbouring reef structure, a shift in the seasonal wave direction, or the growth of the colonies themselves. The factor $1 - \frac{6}{5}\Gamma$ of Eq. (28) works in the practitioner's favour throughout, since it reduces the drift and is largest where the overshoot is largest, but it is a correction of order tens of percent at the damping numbers reached here, not a mechanism that arrests the migration.

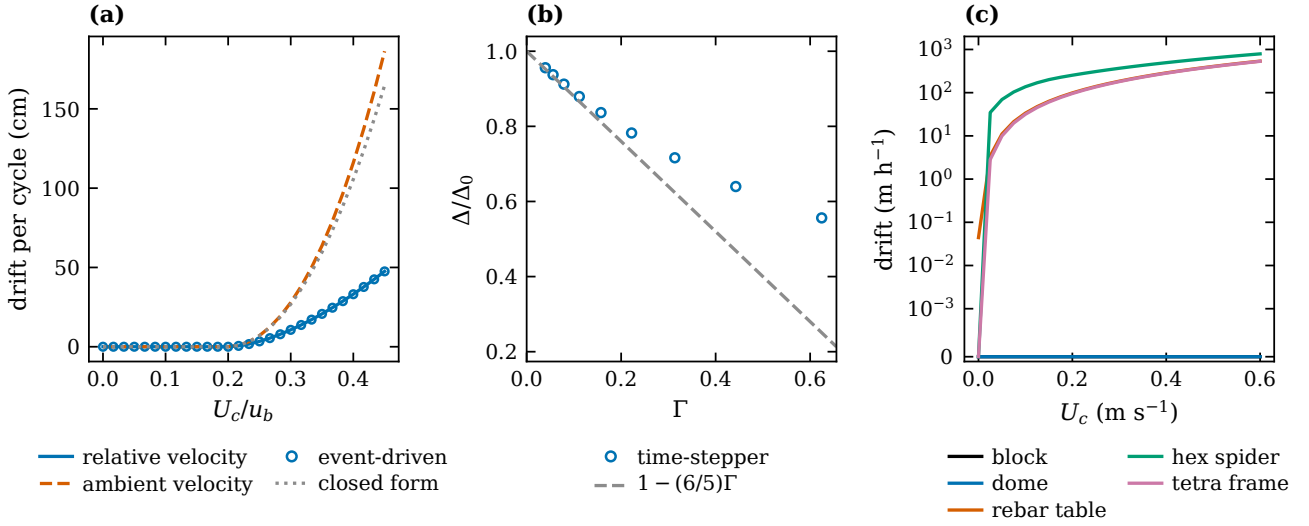


Figure 6: Drift under waves on a current. (a) Net drift per wave cycle of the concrete block against the current ratio, from the time-stepper with drag on the relative velocity, with drag on the ambient velocity alone, from the event-driven reference, and from Eq. (27). (b) Drift with relative-velocity drag over the estimate Δ_0 , against the damping number, with the first-order law of Eq. (28). (c) Displacement per hour of each unit under the monsoon design wave against a superposed current.

An ISW at transplant depth is a transient current rather than an impulsive load (Fig. 7). For a pycnocline 3 m above a 15 m bed with a density step of 5 kg m^{-3} , a crest displacement of 1.5 m gives a near-bed peak of 0.17 m s^{-1} lasting about 60 s, with a soliton half-width of 11 m travelling at 0.40 m s^{-1} . That current consumes a substantial share of the wave margin: the critical wave height of the hexagonal frame at 15 m depth and 14 s period falls from 1.87 m to about 1.0 m as the superposed current rises to 0.35 m s^{-1} . Units held at 95% of their critical height therefore step downstream while the soliton is overhead and stop when it has passed, by 0.11 m for the hexagonal frame and 0.01 m for the rebar table at $\zeta_0 = 1.5 \text{ m}$, with the concrete block not moving. The quasi-steady estimate of Eq. (29) reproduces the simulated step over the range in which the step is resolved.

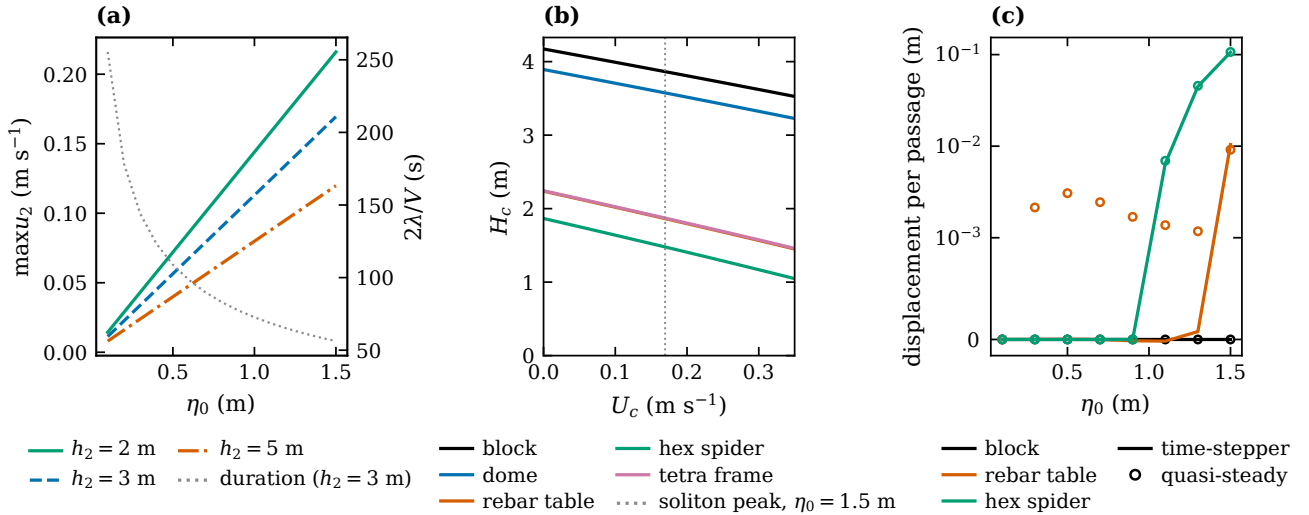


Figure 7: Internal solitary waves. (a) Near-bed peak velocity against crest displacement for lower layers of 2, 3, and 5 m under a 15 m bed, with the pulse duration for the 3 m case. (b) Critical wave height of each unit at 15 m depth and 14 s period against a superposed current, with the soliton peak of the reference state marked. (c) Net displacement per passage for units held at 95% of their critical wave height, from the time-stepper and from Eq. (29).

Growth erodes the margin that the units were built for (Fig. 8). Measured against the monsoon design wave, the ratio H_c/H_{design} falls over five years of colony growth at 0.03 m yr^{-1} from 1.67 to 1.50 for the block, from 1.55 to 1.32 for the dome, from 0.89 to 0.82 for the tetrahedral frame, from 0.83 to 0.70 for the rebar table, and from 0.71 to 0.66 for the hexagonal frame. Two features of this are worth separating. The solid units begin well above unity and remain there, so growth costs them margin without costing them adequacy. The three frames begin below unity, so at this site class they are inadequate on the day they are deployed and growth only deepens

the deficit. Staking the units against sliding, modelled by raising μ to 0.9, raises the five-year ratios to 1.73, 1.52, 0.98, 0.82, and 0.80. Mounting height has no effect at all while sliding governs, because sliding depends on the total load and not on where it acts, and begins to matter only once μ exceeds b/z_p and tipping takes over (Fig. 8b).

The insensitivity of sliding to mounting height is the clearest instance in these results of a modelling prediction that contradicts an intuition widely acted upon. Restoration practice frequently places fragments high on a structure, for light, for flow, and to keep them clear of sediment, and the mechanical cost of doing so is usually assumed to be a loss of stability. Within this formulation that cost is exactly zero while sliding governs, because the frictional capacity responds to the resultant force and is indifferent to its line of action. Mounting height enters only through the tipping moment, and therefore only once the substrate is rough enough, or the anchoring effective enough, that tipping is the first mode reached. The practical reading is that the decision about where to mount fragments is not a stability decision for an unanchored unit on a smooth bed, and becomes one at exactly the point where the anchoring succeeds. The two decisions are coupled, and Fig. 8b locates the coupling: the ratio curves for colonies mounted on top and at half height are identical to the left of $\mu = b/z_p$ and separate to the right of it.

The asymmetry between the solid and open units in Fig. 8a also deserves a quantitative statement, since it is what the hold-down calculation of Table 3 later acts on. The absolute loss of margin over five years is similar in relative terms across the five, between 10 and 15%, but its consequence is not, because the units do not start from comparable places. The block and the dome begin at 1.67 and 1.55 and end at 1.50 and 1.32, so growth consumes margin they can afford. The frames begin at 0.89, 0.83, and 0.71 and end at 0.82, 0.70, and 0.66; they are already inadequate at this site class before any colony has grown, and the growth term is a second-order concern beside that. Staking lifts the tetrahedral frame from 0.82 to 0.98 after five years and the other two frames to 0.82 and 0.80, so that even a staked frame sits just short of the design wave at this site class. Staking does most but not all of the work required at the monsoon class, and the remainder has to come from added mass, from siting at greater depth, or from accepting a lower safety factor than the 1.5 used in Table 3.

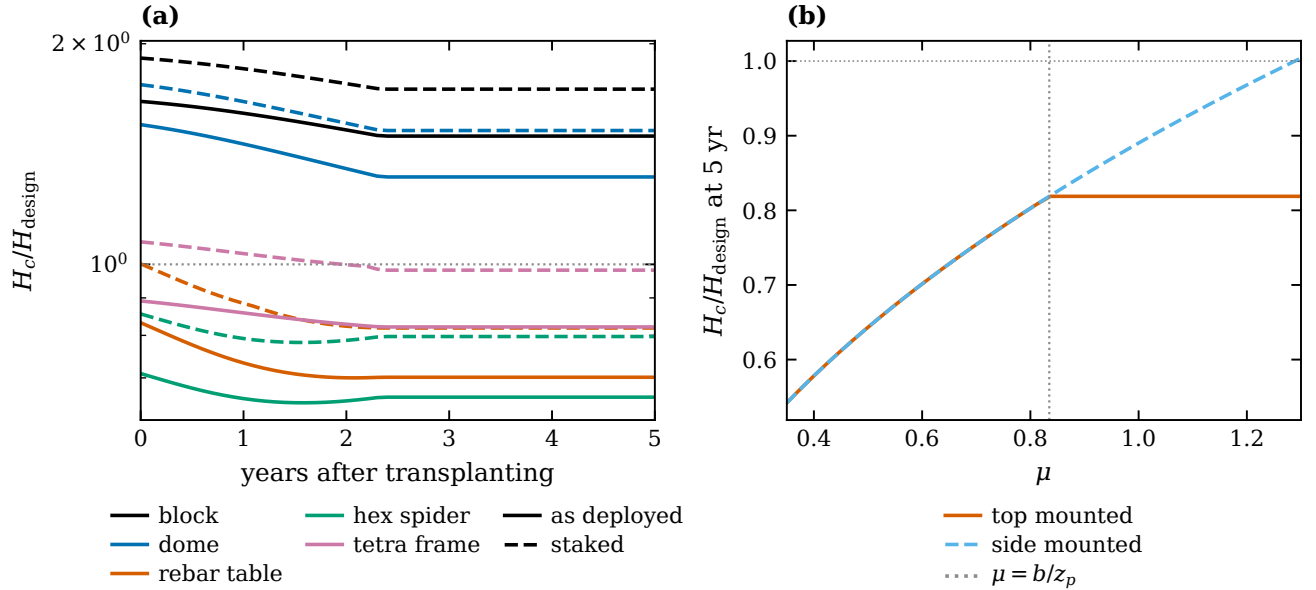


Figure 8: Coral growth against the design wave. (a) Critical wave height over the monsoon design height as colonies grow at 0.03 m yr^{-1} of radius, capped at 0.10 m. Solid lines are the units as deployed and dashed lines the same units staked against sliding. (b) The same ratio after five years for the rebar table against the friction coefficient, with colonies mounted on top and at half height. The switch at $\mu = b/z_p$ is marked.

The sensitivity of these findings to the three weakest modelling choices is reported in Fig. 9. Applying a sheltering factor to the frontal area of the rod frames, which the rod-sum construction neglects, raises their critical wave heights by up to 26% at a factor of 0.6 and leaves the solid units untouched. Over 4000 samples in which that factor, the colony drag coefficient, the colony bulk density, and the four uncalibrated coefficients are drawn together from their ranges, the interquartile spread of the critical wave height is 0.17 to 0.29 of its median, and the ranking of the five structures is reproduced exactly in 97.5% of samples. Applying the added mass to the rotation as well as to the translations moves the overturning threshold of the staked grown rebar table by 0.4%, so that choice is immaterial here.

The three tests are deliberately of different kinds, and the contrast between them is informative. The sheltering factor is a structural omission: the rod-sum construction assumes that every member of a frame sees the undisturbed flow, which is false for a frame of any density, and the test measures how wrong the answer becomes as that assumption is relaxed. Its 26% effect is the largest single uncertainty identified, and it is also the one most

directly addressable, since the drag of a specific frame geometry is a well-posed computational fluid dynamics (CFD) problem. The coefficient ensemble is a parametric uncertainty: every quantity that was assigned a range rather than a value is varied simultaneously, and the result separates two things that are often conflated. The absolute thresholds are soft, with an interquartile spread of 0.17 to 0.29 of the median, which is to say that a critical wave height reported here should be read as indicative to within a factor of roughly one and a half. The ordering is firm, reproduced exactly in 97.5% of samples, because the quantities that are uncertain act on all five structures in the same direction and largely cancel in the comparison. The third test, on the rotational added mass, is a modelling choice with no structural or parametric content, and its 0.4% effect confirms that it does not need to be resolved. The pattern across the three is that the conclusions of this study which concern relative merit are robust, and those which concern absolute survival are not.

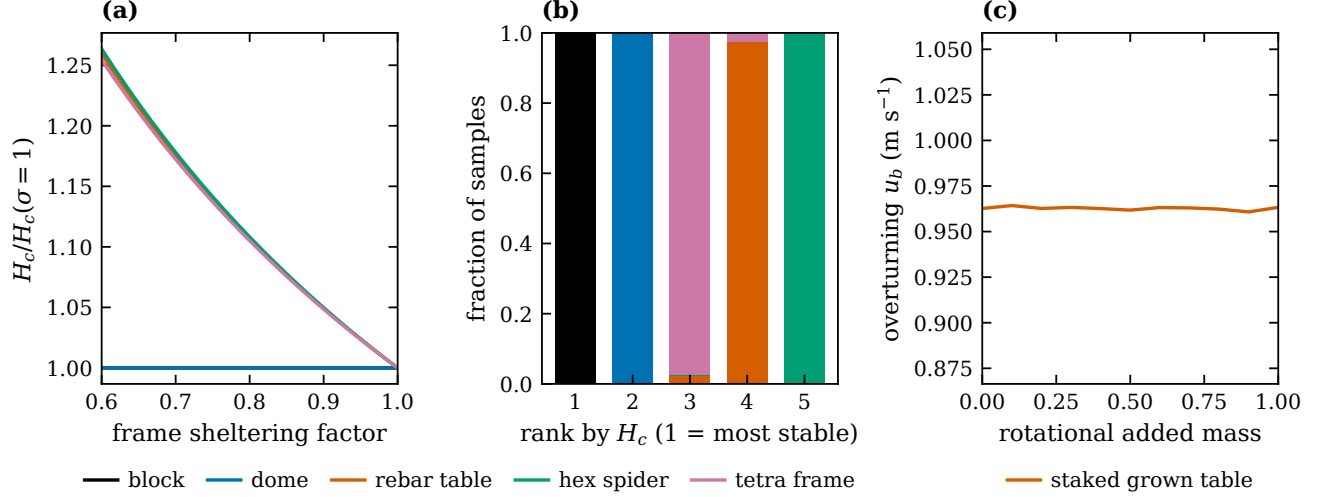


Figure 9: Sensitivity to the three weakest assumptions. (a) Critical wave height of each unit against a sheltering factor applied to the frontal area of the rod frames, relative to no sheltering. (b) Frequency with which each unit occupies each rank by critical wave height over 4000 samples that vary every uncalibrated coefficient together. (c) Overturning threshold of the staked grown rebar table against the fraction of the added mass applied to the rotation, on axes spanning plus or minus ten percent.

The hold-down requirement of Eq. (23) at a safety factor of 1.5, evaluated at the ninetieth percentile over the coefficient ranges, is given in Table 3 for the monsoon class. The solid block and the dome need no addition below 5 m depth, while the frames need between 2 and 61 kg of effective submerged ballast depending on depth, and the tetrahedral frame needs more than either because of its large frontal area. In every entry of the table the governing mode is sliding.

Table 3: Hold-down requirement at the monsoon site class ($H_w = 2$ m, $T = 8$ s), from Eq. (23) at a safety factor of 1.5, reported at the ninetieth percentile over the coefficient ranges. F_a is the required downward force through the base and F_a/g the equivalent effective submerged ballast mass. The governing mode is sliding throughout.

Unit	$h = 3$ m		$h = 5$ m		$h = 8$ m		$h = 12$ m	
	F_a (N)	(kg)	F_a (N)	(kg)	F_a (N)	(kg)	F_a (N)	(kg)
Solid block	363	37	0	0	0	0	0	0
Perforated dome	743	76	87	9	0	0	0	0
Rebar table	600	61	290	30	118	12	24	2
Hexagonal frame	520	53	263	27	120	12	43	4
Tetrahedral frame	1990	203	963	98	391	40	82	8

4 Discussion

The result with the clearest practical content is Eq. (21). Whether a transplant unit slides or tips is decided by one geometric ratio against one frictional property, and lift drops out of the comparison exactly. This matters because lift is the least well constrained of the four coefficients in a configuration of this kind, and because it means that a practitioner who knows only the base width, the height at which the flow pushes hardest, and the roughness of the bed can determine the failure mode without knowing anything about the wave climate. This is the ecomechanical posture in its simplest form, a demand compared against a capacity [5], with the useful feature that the comparison collapses to a single inequality between two quantities a diver can measure with a tape and a slope board. The same criterion explains why the hexagonal frames used on Indonesian rubble fields

[30, 28] behave as they do: with $b/z_p = 4.41$ they are among the most tip-resistant geometries that can be built, and their vulnerability is entirely to sliding, which is a different problem with a different remedy.

The distinction between the two modes is not merely taxonomic, because the two fail differently and leave different evidence. A unit that tips does so once, decisively, and is found on its side or inverted with its fragments crushed or shaded; the event is unmistakable on the next dive and is naturally attributed to the storm that caused it. A unit that slides moves incrementally and may be found upright, intact, and apparently healthy some metres from where it was placed, or not found at all. Monitoring protocols that record the condition of surviving units will capture the first mode and may systematically miss the second, and a unit that has migrated out of a surveyed plot is indistinguishable in the record from one that was never deployed or that was recorded in error. Since the present results place all five geometries on the sliding side of the criterion as deployed, and since sliding is the governing mode in every entry of Table 3, this asymmetry in detectability may be a reason why the mechanical component of restoration loss is harder to find in the literature than the biological one.

That distinction organizes the practical reading of Table 3. The frames fail by sliding at every depth and site class examined here, and sliding is not a defect of their shape but a consequence of their mass. A 6 kg hexagonal frame carrying twelve fragments needs about 12 kg of effective ballast at 8 m depth in the monsoon class, against a 92 kg solid block that needs none. Measured per fragment carried, the block costs 23 kg of material and the frame 0.52 kg, a ratio of about forty-five, and the block also holds its fragments lower in the boundary layer where sediment accumulates. A light open frame that is properly pinned is therefore the more efficient structure by a wide margin, and the substantive design decision is the anchoring rather than the shape. This is consistent with the emphasis that reviews of substrate stabilization place on fixing method rather than on module type [4], with the demonstrated economy of attachment hardware designed for rapid fixing at scale [25], and with field experience that attributes failure to the durability and the attachment of the substrate [7, 10].

That conclusion has an economic form worth making explicit, because restoration budgets are finite and the trade is between mass and anchoring. Mass is bought once, at the cost of material, of the vessel and lifting capacity needed to deploy it, and of the labour of placing it; it is also irreversible, in that a unit heavy enough to resist the design wave is heavy enough to be difficult to reposition if the siting proves wrong. Anchoring is bought per unit, at a cost that scales with the hold-down force of Eq. (23) rather than with the mass of the structure, and it is reversible. Table 3 puts numbers on the comparison at the monsoon class: the hexagonal frame needs 12 kg of effective ballast at 8 m depth, against the 92 kg of concrete that the solid block carries permanently in order to need none. The frame achieves the same mechanical outcome for roughly an eighth of the mass, carries twelve fragments rather than four, and holds them higher in the water column. Against this, the anchoring has to work, and the failure of an anchoring scheme is a single point of failure in a way that the mass of a block is not, which is the substantive argument for solid units where maintenance access is poor. The choice is therefore between a design whose stability is guaranteed by its material and one whose stability is guaranteed by an installation step, and the second is preferable wherever the installation can be verified and revisited.

The material result has a complementary reading. Because the critical velocity scales as the square root of the density excess, the available range of construction materials spans a narrower band of stability than their cost and durability would suggest. Moving from concrete to solid steel raises the critical wave height of the reference block from 3.38 to 6.86 m, slightly more than a doubling, while moving to a polymer of near-neutral buoyancy drops it to 1.80 m. Since the ordering of the structures is preserved throughout, the material decision and the geometric decision are separable: a project can choose its structure on mechanical grounds and its material on grounds of cost, local availability, durability in seawater, and the settlement behaviour of coral larvae on the surface, without the two choices interfering. This separability is a consequence of the square-root scaling being a single multiplicative factor, and it would not survive a formulation in which material entered the drag or the contact as well as the weight.

The growth result inverts an intuition. Restoration structures are assessed at deployment, when they are bare, and a structure that survives its first season is taken to have been adequately specified. The present model indicates that the margin falls monotonically as the colonies develop, because the added frontal area grows as r_c^2 while the added submerged weight grows as r_c^3 and the balance between them favors the area at the sizes that matter. Success in the biological sense therefore erodes the mechanical margin that permitted it, by 10 to 15% over five years in these configurations. The operational consequence is simple and inexpensive: a unit should be specified for its grown state, and the monitoring dives that restoration projects already conduct should include a check on anchoring once colony height reaches an appreciable fraction of the base half-width. We emphasize that the quantitative rate depends on the assumed colony density and solid fraction, which are uncalibrated here, whereas the sign of the effect does not. It is worth noting that the same geometric growth that erodes the unit's margin also raises the dislodgement risk of the colonies themselves, since colony shape factor and attachment strength evolve together as a fragment develops [20, 19]; the two failure paths are distinct but they sharpen on the same timescale, and a project that monitors only one of them is monitoring half the problem. The wide spread in restoration outcomes across sites with comparable biological material [12] is consistent with a mechanical component that is not currently specified at deployment.

Drift under waves riding on a current is the failure mode most likely to go unrecognized in the field. A unit that migrates a few millimetres per wave leaves no dramatic evidence, and over a monsoon season it may travel far enough to leave the restored patch or to reach sand. Because Eq. (27) scales as ε^2 , the drift is negligible until the load approaches the threshold and then rises steeply, which means that a site may appear stable for

years and then lose units quickly following a modest change in exposure. ISWs, which are generated at the sills of Indonesian straits and propagate in packets visible from orbit [26, 27, 13], enter this mechanism rather than the overturning one. The near-bed velocity they deliver at transplant depths, about 0.17 m s^{-1} for the reference state examined here, is far too small to overturn any of the five units, but it is a current, and a current is exactly what converts a symmetric wave into a ratchet. The 0.11 m step computed for the hexagonal frame per soliton passage is small, and its significance lies in the frequency with which such waves recur. Internal waves in these straits arrive in packets on the semidiurnal and diurnal tidal cycles rather than as isolated events, so a step of this size is better thought of as a quantum delivered a few times a day than as a rare perturbation. A unit held near its threshold in such a setting migrates on a schedule set by the tide rather than by the weather, which is a different monitoring problem from the storm-driven one that restoration practice is organized around, and one that a seasonal survey timed to the calm period will not detect. We emphasize that the reference stratification used here is illustrative, and that the near-bed velocity a real packet delivers depends on the layer thicknesses and the density step at the site, which are measurable and which would be the first thing to establish before applying this part of the model anywhere specific.

A further implication concerns how the margin should be set. Design practice in coastal engineering sets a safety factor against a return period, and the value of 1.5 used in Table 3 follows that habit. The results here suggest that for transplant units a factor of this kind should be interpreted differently, because two of the mechanisms examined erode the margin systematically rather than stochastically. Colony growth is monotone and predictable; the loss of 10 to 15% over five years is not a tail risk but a trend, and a factor intended to absorb it is being spent on something that will certainly occur. The drift mechanism is worse in this respect, since its quadratic onset means that a unit sitting at any overshoot at all accumulates displacement continuously. A margin specified at deployment against the design wave therefore covers a configuration that will not persist, and the more defensible procedure is to specify against the grown state at the intended service life and to treat the deployment condition as the comfortable case rather than the design case.

Several limitations bound these conclusions, and the most important are structural rather than parametric. The motion is planar, so oblique and directionally spread seas, rocking about a corner, and yaw are absent; head-on loading is the worst case within this formulation but not in three dimensions. The bed is rigid and flat, which removes scour, burial, rubble mobility, and bearing failure, and these are common on precisely the degraded reefs where restoration is attempted. Waves are non-breaking, which places reef flats and the surf zone outside the scope; since breaking is the dominant driver of flow over shallow reef crests [21], the formulation should be read as applying to the fore-reef and back-reef settings in which transplant units are usually placed rather than to the crest itself. The frontal area is held fixed as the unit rotates, which is adequate at small rotation and wrong near overturning. Frame drag is obtained by summing the members with no allowance for sheltering, which the sensitivity analysis shows to be the single widest uncertainty, moving the frame thresholds by up to 26%. The force coefficients are ranges, not measurements. Nothing here has been compared against field or laboratory observation, and the quantitative thresholds should be read accordingly. What does survive the uncertainty is the ordering: the ranking of the five structures was reproduced in 97.5% of samples that varied every uncertain quantity at once, and the closed-form results of Eqs. (21), (22), and (27) are derivations and hold independently of any coefficient. Resolving the frame drag coefficients by CFD, extending the contact formulation to three dimensions, coupling several units through tethers, and admitting a deformable bed would each narrow a specific gap identified here. The most direct of these is experimental: planar rocking and sliding blocks on frictional supports have been measured against nonsmooth time-stepping predictions in air [31], and the corresponding flume experiment on a submerged transplant unit under oscillatory flow would test the present thresholds against the one thing this study does not have, which is data.

5 Conclusions

We examined the mechanical failure of coral transplant units with an idealized model in which the unit is a planar rigid body on a rigid bed under two unilateral frictional contacts, loaded through a Morison description of near-bed wave kinematics and integrated by a nonsmooth contact time-stepping scheme, with no calibration and no external data, and verified against closed-form impact, sliding, and rocking results. Whether a unit slides or tips is decided by comparing the base half-width divided by the height at which the flow pushes hardest against the friction coefficient of the bed, a criterion from which lift cancels identically and which involves no material property and no wave property, so that a practitioner can determine the failure mode of a design from its dimensions and the roughness of the substrate alone. Material enters the thresholds only through the submerged weight, so denser construction rescales every threshold by one factor and reorders nothing, and the choice between concrete, steel, and coated steel is therefore a question of durability, cost, and handling rather than of stability. A wave riding on a current drives a net drift per cycle that rises as the square of the overshoot above threshold, so that a site may appear stable for years and then begin losing units quickly after a modest change in exposure, and ISWs at transplant depth enter this drift mechanism rather than the overturning one. Coral growth lowers the critical wave height of every structure examined, because the frontal area the colonies add grows faster than the submerged weight they add at the sizes that matter, so success in the biological sense erodes the mechanical margin that permitted it and a transplant unit should be specified for its grown state rather than for the day it is deployed. Among the structures compared, the solid units are the most stable and by a wide margin the least efficient per fragment carried, while the light open frames fail by sliding at every depth examined and are

corrected by modest ballast or pinning, which makes the anchoring rather than the shape the substantive design decision. The ranking of the structures survived the great majority of samples that varied every uncalibrated coefficient together, so the ordering is far firmer than the thresholds themselves, and that asymmetry is the proper way to read these results: as a guide to relative choices among designs, not as a predictor of absolute survival at any particular site. Resolving the drag of porous frames carrying grown colonies, extending the contact formulation to three dimensions, coupling adjacent units through tethers, and representing a deformable bed would show how far these ratios carry over to absolute loads, and a flume experiment on a submerged unit under oscillatory flow would test the ranking found here against the measurements that a site-specific assessment requires.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

Declaration of generative AI use

During the preparation of this work, the authors used Claude Sonnet 5 solely for the purposes of English grammar, vocabulary refinement, and improving the overall readability of the manuscript. The authors maintain full responsibility for the conceptualization, model development, mathematical derivations, computational execution, and the analysis and interpretation presented in this study.

Code and data availability

This study uses no external data. The analysis code and all of its outputs, including CSV tables of the data behind each figure panel, the nine figures in vector and raster form, plain-text computation reports, animations, and the scripts that regenerate them, are openly available at <https://github.com/sandyherho/reefunit> under the MIT License.

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