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Universality and Information Loss in the Stratigraphic Record

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Abstract

Sedimentary rocks are the main archive of Earth history, yet most of the time they span is not represented by rock. Material deposited at any moment survives only if the depositional surface never later erodes below it, so the stratigraphic column keeps only the lowest levels that a fluctuating surface will reach in the future. We treat this rule as a filter on a randomly evolving surface and connect it to the statistical physics of records and first passage. The fraction of time that is preserved depends on a single ratio between the resolution of observation and a crossover time set by net accumulation and surface roughness. At fine resolution this dependence is universal, the same for erosion by many small events or by rare catastrophic ones, provided successive events are independent. The Sadler effect, the decline of apparent accumulation rate with measurement interval, is governed by the persistence of the surface and changes form only when erosion has memory. With net accumulation, the age of a column as a function of height advances in independent jumps, which yields the complete statistics of hiatuses and shows that the longest gap produced by ordinary fluctuations grows only slowly with the length of the record. The same description predicts that an astronomical cycle appears in depth broadened, shifted toward longer apparent periods, and coherent over a limited number of repetitions. These results provide null models for unconformities and quantitative limits on the timing information that sedimentary sequences retain.

Keywords: cyclostratigraphy; persistence; record statistics; Sadler effect; stratigraphic completeness; subordinators; unconformities

1 Introduction

Sedimentary rocks are the principal archive of Earth history. Ages of events, durations of processes, rates of environmental change, and the astronomical calibration of the geological timescale are all read from stacks of layers whose thickness is converted into time. The conversion rests on an assumption that is known to fail: that the column records time continuously. Deposition at any site alternates with nondeposition and erosion, and every episode of erosion removes part of what was deposited before it. The consequence is quantitative and systematic. Accumulation rates measured over short intervals exceed those measured over long intervals by orders of magnitude, a decline known as the Sadler effect [21]. Over the timescales on which most geological questions are posed, the time represented by rock is a small fraction of the time that elapsed [21, 20]. Unconformities, from diastems spanning hours to surfaces that represent hundreds of millions of years [10], are the visible part of this missing time, and most of it leaves no visible trace at all.

The incompleteness of the record has been studied for more than four decades. Stochastic models of a depositional surface that rises and falls reproduce the Sadler effect and have been used to estimate the completeness of sections [28, 27, 9, 22]. Experiments and field studies have identified a compensation timescale beyond which deposition fills topography evenly and the record becomes steady [26, 20], and analyses of unsteady sedimentation have shown how missing time distorts the apparent timing and duration of abrupt events [11]. These studies established that incompleteness is the expected outcome of ordinary depositional dynamics. They have not, however, produced a framework that states which properties of the record are fixed by that dynamics in general and which carry information about a particular setting. Most results are obtained for a specific model, by simulation, or by fitting compilations of rates that combine many environments, and the answers they give are therefore difficult to transfer from one setting to another or to turn into tests.

Three questions illustrate the gap, and each bears on a practical problem in stratigraphy. The first concerns the origin of incompleteness. It is natural to expect that a setting dominated by rare, catastrophic erosion, such as a shelf swept by storms or a slope cut by turbidity currents, preserves less of its history than one reworked by small and frequent events. Whether this expectation is correct determines whether the shape of a Sadler curve can be used to infer the style of erosion. The second question concerns the interpretation of gaps. A hiatus of a given duration can be produced by the ordinary fluctuations of a depositional system or by an external change in forcing, such as uplift, a fall in base level, or glaciation. Distinguishing the two requires knowing how long the longest gap in a section should be when nothing unusual happened, and no such null distribution is available. The question is sharpest for major unconformities, whose interpretation as records of single events or of protracted, diachronous erosion remains contested [10]. The third question concerns chronology. Cyclostratigraphy converts

depth into time by identifying astronomical cycles in sedimentary sequences [7], and testing whether a sequence records astronomical forcing is itself a subject of active debate [17]. Hiatuses that are not marked in the rock distort this conversion, but the amount of chronological information that survives them, and the form in which the distortion appears, have not been determined.

These questions share a common structure that the existing models do not exploit. Material deposited at a given moment survives only if the depositional surface never later descends below it. The preserved column is therefore not a sample of the surface history but its future minimum, and the preserved layers are the lower records of that history read backward in time. This identification places stratigraphic completeness within the statistical physics of records and first passage, where two classes of results are directly relevant. The statistics of records of random walks are universal, in the sense that some of them do not depend on the distribution of the steps at all [24, 25, 15, 14, 5]. Persistence exponents characterize how long a fluctuating surface avoids returning below a previous level and how that behavior changes when the fluctuations carry memory [12, 19, 1]. Survival of a layer is a question about the order in which the surface visits different levels, and it is this ordering, not the magnitude of individual events, that the theory of records addresses. The record also has a natural interpretation in information theory. Because the deposition of a layer leaves no mark when that layer is later removed, the stratigraphic filter deletes information without flagging the deletion, which destroys synchronization as well as content [18].

In this study, we develop the consequences of treating the stratigraphic column as the future minimum of a stochastic surface. We derive the completeness of the record exactly for a random walk with net accumulation and show that its behavior at fine resolution is universal across all symmetric fluctuation laws of finite variance. We show that the Sadler exponent equals the persistence exponent of the surface, so that it is insensitive to the size distribution of erosional events and sensitive only to memory in the surface dynamics. In the limit of fine resolution, we show that the age of the column as a function of height is a subordinator, a process with independent jumps, which gives the number of hiatuses of each duration and the distribution of the longest hiatus in closed form. The same description determines how a periodic forcing appears in the depth domain: its spectral line is broadened, displaced toward longer apparent periods, and coherent over a finite number of cycles, and the information that two horizons retain about elapsed time can be computed exactly. Departures from these forms provide signatures of heavy-tailed erosion and of memory that can be sought in real sections.

The aim is to supply null models and testable predictions, not to describe any particular basin. The model is one-dimensional and idealized and uses no observational data. Every analytical result is verified against direct Monte Carlo simulation. To our knowledge, the identification of the age-depth map as a subordinator, the closed-form law of the longest hiatus, and the depth-domain line shape of an astronomical cycle have not been reported before, although the stochastic view of completeness that they extend goes back several decades [28, 27].

2 Theory

2.1 The stratigraphic filter

Let $h(t)$ be the elevation of the depositional surface at a fixed location. Sediment deposited at time t lies at height $h(t)$ and is removed if the surface later falls below that height. It survives to the present if and only if

$$h(t) = m(t), \quad m(t) \equiv \inf_{s \geq t} h(s), \quad (1)$$

where $m(t)$ is the future minimum. The preserved column is therefore the graph of m , and the map $h \mapsto m$ is many-to-one: every history that shares the same future minimum produces the same rock. Two observables of the column follow directly. The height in the column at which material of age t lies is $m(t)$, and the age of the material at height z is the last passage time

$$L(z) \equiv \sup\{t : h(t) \leq z\}. \quad (2)$$

A hiatus is an interval of time that is absent from the column, that is, a maximal interval on which $h(t) > m(t)$. In the column it appears as a jump of $L(z)$ at a single height (Fig. 1).

We consider the surface at discrete times $t_n = n\Delta t$, where Δt is the resolution at which layers are distinguished, with

$$h_n = h_{n-1} + \mu + \xi_n, \quad \mu \equiv v\Delta t > 0, \quad (3)$$

where v is the long-term accumulation rate and the ξ_n are fluctuations with zero median. For fluctuations with finite variance we write $\mathbb{E}\xi^2 \equiv \sigma^2 = 2D\Delta t$, which defines a diffusivity D that does not depend on Δt when the surface is a diffusion observed at resolution Δt . Drift and fluctuations balance at the crossover time

$$t_c \equiv \frac{D}{v^2}, \quad (4)$$

since vt and $\sqrt{2Dt}$ are equal at $t = 2t_c$. Equivalently, $t_c = \ell_*/(2v)$ with $\ell_* \equiv 2D/v$ the vertical scale at which the surface roughness and the net accumulation are comparable. The reduced resolution is $\epsilon \equiv \Delta t/t_c = 2(\mu/\sigma)^2$.

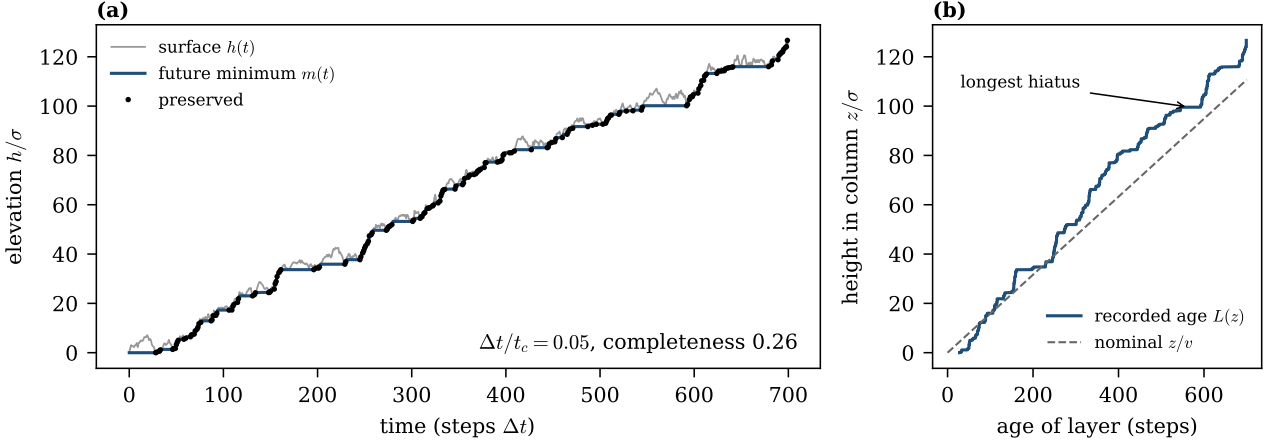


Figure 1: The stratigraphic filter. (a) A realization of Eq. (3) with Gaussian fluctuations at $\Delta t/t_c = 0.05$ (gray), its future minimum $m(t)$ (blue), and the preserved layers (black). (b) The resulting column: the age $L(z)$ of the material at each height, with the nominal age z/v (dashed). Horizontal segments of $L(z)$ are hiatuses.

2.2 Completeness

Layer n is preserved if and only if $h_{n+k} > h_n$ for every $k \geq 1$, that is, if the walk $S_k \equiv h_{n+k} - h_n = k\mu + \sum_{j=1}^k \xi_{n+j}$ never returns to or below zero. Since the increments are stationary, the probability of this event is the same for every n , and by ergodicity it equals the completeness c , the long-run fraction of layers that survive. Let $\tau_- \equiv \inf\{k \geq 1 : S_k \leq 0\}$. For independent increments the Spitzer–Baxter identity [25] gives

$$c = \mathbb{P}(\tau_- = \infty) = \exp \left[- \sum_{n=1}^{\infty} \frac{1}{n} \mathbb{P}(S_n \leq 0) \right]. \quad (5)$$

The series converges whenever $\mu > 0$ and the mean step is finite. For Gaussian fluctuations $\mathbb{P}(S_n \leq 0) = \Phi(-a\sqrt{n})$ with $a \equiv \mu/\sigma = (\epsilon/2)^{1/2}$ and Φ the standard normal distribution function, so that c is a function of ϵ alone.

The behavior at fine resolution follows from a second identity. Let $\tau_+ \equiv \inf\{k \geq 1 : S_k > 0\}$ be the first strict ascending ladder epoch and $H_+ \equiv S_{\tau_+}$ the ladder height. Duality gives $\mathbb{E}\tau_+ = 1/\mathbb{P}(\tau_- = \infty)$ [13], and Wald's identity gives $\mathbb{E}H_+ = \mu \mathbb{E}\tau_+$. Hence

$$c = \frac{\mu}{\mathbb{E}H_+}. \quad (6)$$

For symmetric continuous fluctuations with variance σ^2 , the ladder height of the driftless walk has mean $\sigma/\sqrt{2}$, because the general expression $\mathbb{E}H_+ = (\sigma/\sqrt{2}) \exp\{\sum_n n^{-1}[\mathbb{P}(S_n \leq 0) - \frac{1}{2}]\}$ reduces to its prefactor when $\mathbb{P}(S_n \leq 0) = \frac{1}{2}$ [25, 13]. Since $\mathbb{E}H_+$ is continuous in μ ,

$$c \simeq \frac{\sqrt{2}\mu}{\sigma} = \left(\frac{\Delta t}{t_c} \right)^{1/2}, \quad \Delta t \ll t_c, \quad (7)$$

with a prefactor of exactly one for every symmetric law of finite variance.

For Gaussian steps the first correction follows from Eq. (5). The Mellin transform of $\Phi(-y)$ has simple poles at $s = 0, -1, -3, \dots$, with residues given by the coefficients of the expansion $\Phi(-y) = \frac{1}{2} - y/\sqrt{2\pi} + y^3/(6\sqrt{2\pi}) - \dots$. Representing the sum in Eq. (5) as a Mellin integral against $\zeta(1+s/2)$, the double pole at $s = 0$ produces the logarithmic term fixed by Eq. (7), and the pole at $s = -1$ produces a term $-a\zeta(\frac{1}{2})/\sqrt{2\pi}$ in the sum, as in the analogous expansion of the maximum of a random walk [3]. The expansion contains only odd powers of a , and therefore

$$c = \epsilon^{1/2} \exp \left[\frac{\zeta(\frac{1}{2})}{2\sqrt{\pi}} \epsilon^{1/2} + O(\epsilon^{3/2}) \right], \quad \zeta(\frac{1}{2}) = -1.46035 \dots \quad (8)$$

The Sadler effect follows. The accumulation rate measured over an interval Δt between preserved horizons exceeds the long-term rate by the factor $1/c$ [21], so it scales as $\Delta t^{-\gamma}$ for $\Delta t \ll t_c$ with Sadler exponent $\gamma = 1/2$, and it approaches v for $\Delta t \gg t_c$.

The exponent $1/2$ is a persistence exponent [1], and the argument extends beyond finite variance. Suppose the fluctuations accumulated over a time t have scale $s_1 t^\kappa$, with $\kappa = 1/\alpha$ for symmetric α -stable increments ($1 < \alpha \leq 2$) and $\kappa = H$ for fractional Brownian motion [16]. Drift dominates after the crossover time $t_c = (s_1/v)^{1/(1-\kappa)}$, which reduces to D/v^2 up to a constant factor in the Brownian case. A layer survives if the surface, which behaves as the driftless process for $t \lesssim t_c$, avoids returning below the layer over the $t_c/\Delta t$ steps before drift takes

over. If the survival probability of the driftless process over N steps decays as $N^{-\theta}$, then

$$c \sim \left(\frac{\Delta t}{t_c}\right)^\theta, \quad \gamma = \theta. \quad (9)$$

By the Sparre Andersen theorem, $\theta = 1/2$ for every process with independent, symmetric, and continuous increments, whatever their distribution [24, 14]. For fractional Brownian motion $\theta = 1 - H$ [19, 12]. The Sadler exponent is therefore insensitive to the tails of the erosion distribution and sensitive to memory. The scaling argument fixes the exponent, not the prefactor, which for stable laws is not universal.

2.3 The driftless limit

When net accumulation vanishes, $t_c \rightarrow \infty$ and the completeness of a column of N layers decreases without bound. Reading the column from the top downward, layer n is preserved if it is a strict lower record of the reversed sequence $h_N, h_{N-1}, \dots, h_1$. For independent, symmetric, and continuous increments, the mean number of records of an n -step walk, counting the starting point, is

$$\mathbb{E}R_n = (2n+1) \binom{2n}{n} 2^{-2n} \simeq 2\sqrt{\frac{n}{\pi}}, \quad (10)$$

independent of the step distribution [15]. A column of N layers therefore preserves $\mathbb{E}R_{N-1}$ of them on average. The same universality holds for the longest gap between successive records, whose mean is a fixed fraction of N [15, 5]. In this regime the longest hiatus in a section is comparable to the duration of the section itself, so a gap that spans most of a record is typical, not exceptional. For fractional Brownian motion the survival argument gives a mean of order $N^{1-\theta} = N^H$ preserved layers, so that a renewal description of the hiatuses requires their density to decay as $\tau^{-(2-\theta)} = \tau^{-(1+H)}$.

2.4 The age-depth map as a subordinator

In the continuum limit $\Delta t \ll t_c$ with finite variance, the surface converges to a Brownian motion with drift, $h(t) = vt + \sqrt{2D}W(t)$. Fix a time T_0 far in the future and set $Y(s) \equiv h(T_0 - s) - h(T_0)$, a Brownian motion with drift $-v$. The future minimum of h at time t is the running minimum of Y over $[0, T_0 - t]$, shifted by $h(T_0)$. Consequently the last passage time of Eq. (2) satisfies $T_0 - L(z) = \sigma_y$, where σ_y is the first time Y reaches the level $y = z - h(T_0) < 0$. The first-passage times of a Brownian motion with drift toward the level form a subordinator in the level [13]. Hence $z \mapsto L(z)$ has stationary, independent, and nonnegative increments, with

$$\mathbb{E}e^{-\lambda[L(z+\Delta z) - L(z)]} = e^{-\Delta z \Phi(\lambda)}, \quad \Phi(\lambda) = \frac{\sqrt{v^2 + 4D\lambda} - v}{2D} = \int_0^\infty (1 - e^{-\lambda\tau}) \nu(d\tau), \quad (11)$$

where the Lévy measure is

$$\nu(d\tau) = \frac{1}{\sqrt{4\pi D\tau^3}} \exp\left(-\frac{\tau}{4t_c}\right) d\tau. \quad (12)$$

The second equality in Eq. (11) follows from $\int_0^\infty (1 - e^{-\lambda\tau}) \tau^{-3/2} e^{-b\tau} d\tau = 2\sqrt{\pi}(\sqrt{b+\lambda} - \sqrt{b})$ with $b = v^2/4D$. The subordinator has no drift term, so the preserved ages form a set of zero Lebesgue measure, consistent with $c \rightarrow 0$ as $\Delta t \rightarrow 0$ in Eq. (7). Its mean, $\mathbb{E}[L(z + \Delta z) - L(z)] = \Phi'(0)\Delta z = \Delta z/v$, recovers the long-term rate, and its variance,

$$\text{Var}[L(z + \Delta z) - L(z)] = -\Phi''(0) \Delta z = \frac{2D}{v^3} \Delta z = 2t_c \frac{\Delta z}{v}, \quad (13)$$

is the variance of the age misregistration accumulated over a thickness Δz when ages are interpolated with the long-term rate.

Each jump of L is a hiatus, and the jumps of a subordinator form a Poisson point process with intensity $dz \nu(d\tau)$. The number of hiatuses longer than ℓ in a column of thickness $Z = vT$ is therefore Poisson with mean $Z\bar{\nu}(\ell)$, where $\bar{\nu}(\ell) \equiv \int_\ell^\infty \nu(d\tau)$. Integrating Eq. (12) and using $v^2 t_c = D$,

$$Z\bar{\nu}(\ell) = \frac{\hat{T}}{4\sqrt{\pi}} \Gamma\left(-\frac{1}{2}, \frac{\ell}{4t_c}\right), \quad \hat{T} \equiv \frac{T}{t_c}, \quad (14)$$

with $\Gamma(\cdot, \cdot)$ the upper incomplete gamma function. Hiatus durations are distributed as $\tau^{-3/2}$ below t_c , the first-passage law, and are cut off exponentially on the scale $4t_c$. The longest hiatus $\ell_{\max}$ exceeds ℓ unless the Poisson count is zero, so

$$\mathbb{P}(\ell_{\max} < \ell) = \exp\left[-\frac{\hat{T}}{4\sqrt{\pi}} \Gamma\left(-\frac{1}{2}, \frac{\ell}{4t_c}\right)\right]. \quad (15)$$

For large $y = \ell/4t_c$, $\Gamma(-\frac{1}{2}, y) \simeq y^{-3/2} e^{-y}$, and setting the exponent to a constant gives

$$\ell_{\max} \simeq 4t_c \left[\ln \hat{T} - \frac{3}{2} \ln \ln \hat{T} + O(1) \right]. \quad (16)$$

In a section with net accumulation, the longest hiatus produced by stationary fluctuations grows only logarithmically with the duration of the section.

2.5 Periodic forcing in the depth domain

Suppose a proxy records a periodic forcing, $s(t) = \cos(\omega t + \varphi)$ with period $P = 2\pi/\omega$, and that every preserved layer carries the value of s at the time of its deposition. The proxy at height z is $s(L(z))$, and its depth-domain structure follows from the complex coherence $C(\Delta z) \equiv \mathbb{E} \exp\{i\omega[L(z + \Delta z) - L(z)]\}$. Setting $\lambda = -i\omega$ in Eq. (11) gives the exact result

$$C(\Delta z) = e^{-\Delta z \Psi(\omega)}, \quad \Psi(\omega) = \frac{v}{2D} \left(\sqrt{1 - 4i\omega t_c} - 1 \right). \quad (17)$$

The depth-domain power spectrum of $e^{i\omega L(z)}$ is therefore a Lorentzian centered at the apparent wavenumber $k_* = -\text{Im } \Psi$ with half-width $\text{Re } \Psi$. Writing $k_0 \equiv \omega/v$ for the nominal wavenumber and $u \equiv \omega t_c$, the expansion $\sqrt{1 - 4iu} = 1 - 2iu + 2u^2 + 4iu^3 - 10u^4 + \dots$ gives

$$\frac{\text{Re } \Psi}{k_0} = u - 5u^3 + O(u^5), \quad \frac{k_*}{k_0} = 1 - 2u^2 + O(u^4), \quad (18)$$

while for $u \gg 1$ both ratios approach $(2u)^{-1/2}$. The cycle is broadened, and it is also displaced toward smaller wavenumber. An age model that converts depth to time with the long-term rate v therefore assigns the cycle an apparent period $P/(1 - 2u^2)$, longer than the true period. The quality factor of the line is

$$Q \equiv \frac{k_*}{2 \text{Re } \Psi} \simeq \frac{1}{2\omega t_c} = \frac{P}{4\pi t_c}, \quad (19)$$

which approaches $1/2$ for $u \gg 1$, where the cycle is indistinguishable from a broadband signal. The coherence decays by a factor e over the thickness $1/\text{Re } \Psi$, which corresponds to

$$N_{\text{coh}} = \frac{v}{P \text{Re } \Psi} \simeq \frac{P}{4\pi^2 t_c} \quad (20)$$

nominal cycles of the forcing.

These forms are universal within the Brownian class. For any symmetric law of finite variance at $\Delta t \ll t_c$, the surface converges to the Brownian limit, so Eqs. (13)–(20) hold for $\omega \Delta t \ll 1$. Outside this class they change in recognizable ways. For symmetric α -stable fluctuations the misregistration over a thickness Δz is dominated by the fluctuation accumulated over the time $\Delta z/v$, so its scale grows as $\Delta z^{1/\alpha}$, and the half-width of the line grows as $|\omega|^\alpha$ at small ω instead of ω^2 . For fractional Brownian surfaces the misregistration increments are no longer independent; their scale grows as Δz^H , so that $|C|$ decays as $\exp(-b \Delta z^{2H})$ and the line is not Lorentzian.

2.6 Information loss

Two measures of the information that survives the filter follow from these results. The first concerns the record as a communication channel. Each deposited layer is deleted with probability $1 - c$, and the deletion leaves no mark in the column. The stratigraphic record is therefore a deletion channel, not an erasure channel. Deletion channels are harder to analyze than erasure channels, and their capacity is not known in closed form, but it cannot exceed that of the erasure channel with the same deletion probability [18, 23]. The information about any signal written in the sediment is thus at most c times the information written per layer, and the number of preserved layers per unit thickness is $c/(v\Delta t) \simeq 1/(v\sqrt{\Delta t} t_c)$ by Eq. (7). Correlated deletions, which the hiatus statistics of Eq. (14) describe, add a loss of synchronization that the bound does not capture.

The second measure quantifies that loss directly. Given two horizons a thickness Δz apart and the long-term rate v , the record constrains the elapsed time modulo the forcing period through the distribution of the phase $\phi = \omega[L(z + \Delta z) - L(z)] \bmod 2\pi$. Its Fourier coefficients are $e^{-\Delta z \Psi(k\omega)}$ for $k \geq 1$, so its density is

$$f(\phi) = \frac{1}{2\pi} \left[1 + 2 \text{Re} \sum_{k=1}^{\infty} e^{-\Delta z \Psi(k\omega)} e^{-ik\phi} \right]. \quad (21)$$

We define the chronometric information as the Kullback–Leibler divergence of f from the uniform density,

$$\mathcal{I}(\Delta z) \equiv \int_{-\pi}^{\pi} f(\phi) \log_2[2\pi f(\phi)] d\phi. \quad (22)$$

It is the number of bits by which the record narrows the elapsed phase relative to complete ignorance, and it vanishes when the record carries no information about the timing of the forcing. In units of nominal cycles, $n \equiv \Delta z k_0/2\pi$, the exponent in Eq. (21) is $\Delta z \Psi(k\omega) = n(\pi/u)(\sqrt{1 - 4iku} - 1)$, so $\mathcal{I}$ depends only on n and $u = \omega t_c$. For $u \ll 1$ and $n \lesssim N_{\text{coh}}$ the phase is a wrapped Gaussian with variance $4\pi u n$ and $\mathcal{I} \simeq \log_2[2\pi(8\pi^2 e u n)^{-1/2}]$, which declines logarithmically with depth. For $n \gtrsim N_{\text{coh}}$ it is dominated by the first harmonic, $\mathcal{I} \simeq |C|^2/\ln 2$, and decays exponentially.

3 Numerical methods

Surfaces are generated from Eq. (3) with the per-step drift μ set by the reduced resolution ϵ . Finite-variance fluctuations have unit variance and are drawn from Gaussian, Laplace, and rescaled Student t distributions with three degrees of freedom, so that $\epsilon = 2\mu^2$. Symmetric α -stable fluctuations with characteristic function $\exp(-|k|^\alpha)$ are generated by the Chambers–Mallows–Stuck method [2], with $\epsilon \equiv \mu^{\alpha/(\alpha-1)}$, which reduces to the Gaussian definition at $\alpha = 2$. Fractional Gaussian noise of unit variance is generated exactly by circulant

embedding [4], with $\epsilon \equiv (\sqrt{2}\mu)^{1/(1-H)}$, which reduces to the Gaussian definition at $H = 1/2$. For stable and fractional surfaces these definitions fix t_c up to a constant factor, which affects prefactors but not exponents.

The filter of Eq. (1) is evaluated in $O(N)$ operations as a reversed cumulative minimum. Layer n is marked as preserved when h_n is strictly smaller than the minimum of $h_{n+1}, \dots, h_N$. Since the future of the last layers is truncated, completeness is measured after discarding the final $200t_c/\Delta t$ steps or one quarter of the series, whichever is smaller. Completeness is computed at 13 values of ϵ between 10^{-3} and 10 from three realizations of 2^{21} steps for each finite-variance and fractional law and from twelve realizations for stable laws. Equation (5) is summed to $n = \max(200, 60/a^2)$ terms. The driftless statistics use 4000 walks of 10^4 steps for each law. Hiatus densities in the driftless case are pooled from 2000 walks of 2^{15} steps.

The predictions of the subordinator are tested on Gaussian surfaces. Hiatus counts per unit thickness use ten realizations of 2^{21} steps at $\epsilon = 0.01$, with the preserved thickness taken from the simulated column. The longest hiatus is computed at $\epsilon = 0.02$ for $30 \leq T/t_c \leq 10^4$, with 2000 realizations at the shortest duration and 80 at the longest, and with $40t_c$ of additional surface history beyond T so that the future minimum is resolved. The depth-domain coherence is computed at $\epsilon = 0.005$ from columns of 2^{22} steps. The age $L(z)$ is sampled on a uniform height grid with spacing 5μ , by assigning to each height the age of the preserved layer immediately below it. For each of twelve frequencies with $0.05 \leq \omega t_c \leq 3$, the coherence is averaged over all pairs of grid points at each separation. The half-width is the slope of $-\ln |C|$ over separations with $0.15 < |C| < 0.9$, and k_* is k_0 plus the slope of the unwrapped phase of $Ce^{-ik_0\Delta z}$. The misregistration $L(z + \Delta z) - L(z) - \Delta z/v$ is computed from eight realizations of 2^{21} steps per law at $\epsilon = 0.005$, and its scaling exponents are fitted for $\Delta z/(vt_c) \geq 10$. The interquartile range is used as the scale because the variance does not exist for stable laws. The chronometric information is evaluated from Eqs. (21) and (22) with 600 harmonics on 8001 phase points. The code is written in Python with NumPy [6], SciPy [29], and Matplotlib [8]. The generators are `noise` and `fgn`, the filter is `preserved`, and the exact expressions are `spitzer_c`, `mean_records`, and `psi`.

4 Results

The simulated completeness of Gaussian surfaces agrees with Eq. (5) to within 0.3 percent for $\epsilon \geq 10^{-2}$ and to within 3 percent at $\epsilon = 10^{-3}$, where the truncation of the record is most severe (Fig. 2a). The two-term asymptote of Eq. (8) reproduces the exact sum to a relative error of 5×10^{-6} at $\epsilon = 10^{-2}$ and 1.5×10^{-4} at $\epsilon = 0.1$, consistent with an error of order $\epsilon^{3/2}$. At $\epsilon = 10^{-3}$, the ratio $c/\epsilon^{1/2}$ is 1.015, 0.982, and 1.026 for Gaussian, Laplace, and Student t fluctuations, which differ in their kurtosis but share the prefactor of Eq. (7). The laws separate only near $\epsilon \sim 1$, where the record is nearly complete. Stable fluctuations have the same exponent with a different prefactor. The local exponent fitted for $\epsilon \leq 10^{-2}$ is 0.46 for $\alpha = 1.5$ and 0.41 for $\alpha = 1.2$, approaching $1/2$ more slowly as α approaches one, where the crossover to drift-dominated behavior is increasingly gradual. Fractional surfaces depart from $1/2$ as predicted by Eq. (9) (Fig. 2b). The fitted exponents are 0.68, 0.50, and 0.27 for $H = 0.3, 0.5$, and 0.7 , compared with $1 - H = 0.7, 0.5$, and 0.3 .

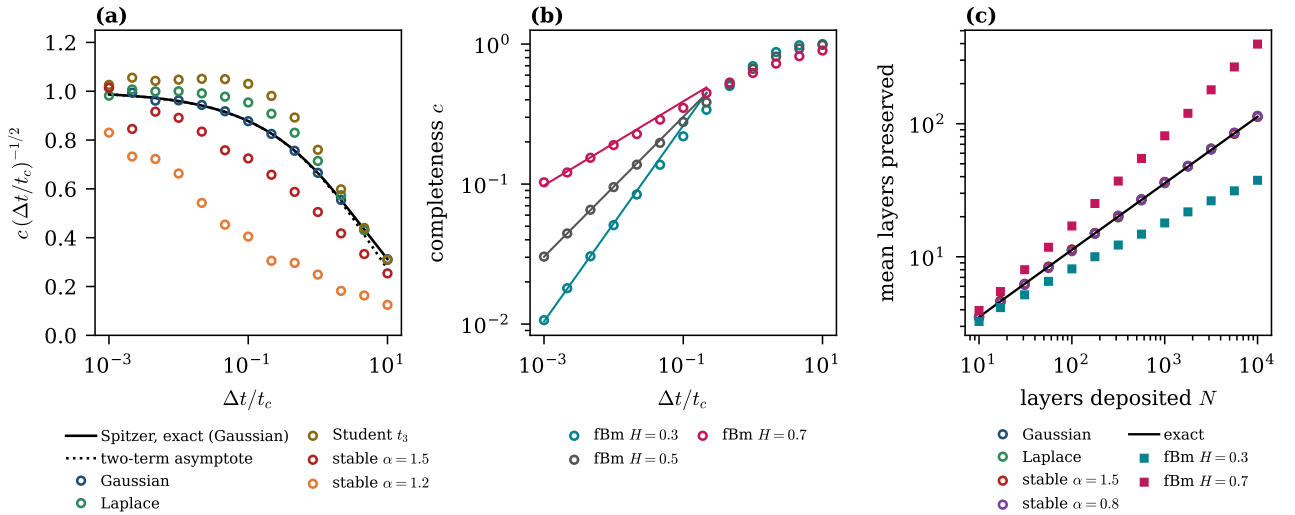


Figure 2: Completeness. (a) Completeness divided by $(\Delta t/t_c)^{1/2}$ for five fluctuation laws, with the exact Gaussian result of Eq. (5) and the asymptote of Eq. (8). (b) Completeness of fractional Brownian surfaces, with lines of slope $1 - H$. (c) Mean number of preserved layers in a driftless column of N layers, with the exact result of Eq. (10).

In the driftless limit, the mean number of preserved layers agrees with Eq. (10) to within 2.7 percent for $10 \leq N \leq 10^4$ and for all four independent-increment laws, including stable fluctuations with $\alpha = 0.8$, which

have no finite mean (Fig. 2c). Fractional surfaces depart from this universal curve, with fitted exponents of 0.33 and 0.68 for $H = 0.3$ and 0.7 , close to the predicted N^H . The mean longest hiatus, measured as a fraction of the column duration, is 0.623, 0.630, 0.629, and 0.624 for the Gaussian, Laplace, $\alpha = 1.5$, and $\alpha = 0.8$ laws, each within one standard error (0.004) of the constant 0.6265 of the longest lasting record [15, 5].

The distribution of hiatus durations in driftless columns is shown in Fig. 3a. Its density decays with fitted slopes between -1.53 and -1.54 for the four independent-increment laws, close to the first-passage value of $-3/2$, and with slopes of -1.34 and -1.72 for fractional surfaces with $H = 0.3$ and 0.7 . These are close to $-(2 - \theta) = -(1 + H)$, the exponent required for consistency with the N^H growth of the preserved count if successive hiatuses are treated as a renewal sequence. With net accumulation, the number of hiatuses longer than ℓ per unit thickness agrees with Eq. (14) to within 2.5 percent for $0.09 \leq \ell/t_c \leq 6$ (Fig. 3b). Below this range the discreteness of the walk matters, and above it the counts are small. The median longest hiatus agrees with Eq. (15) to within 3 percent for $T/t_c \geq 100$, rising from $6.7 t_c$ at $T/t_c = 100$ to $20.4 t_c$ at $T/t_c = 10^4$ (Fig. 3c, Table 1). A hundredfold increase in the duration of the record thus only triples the longest gap.

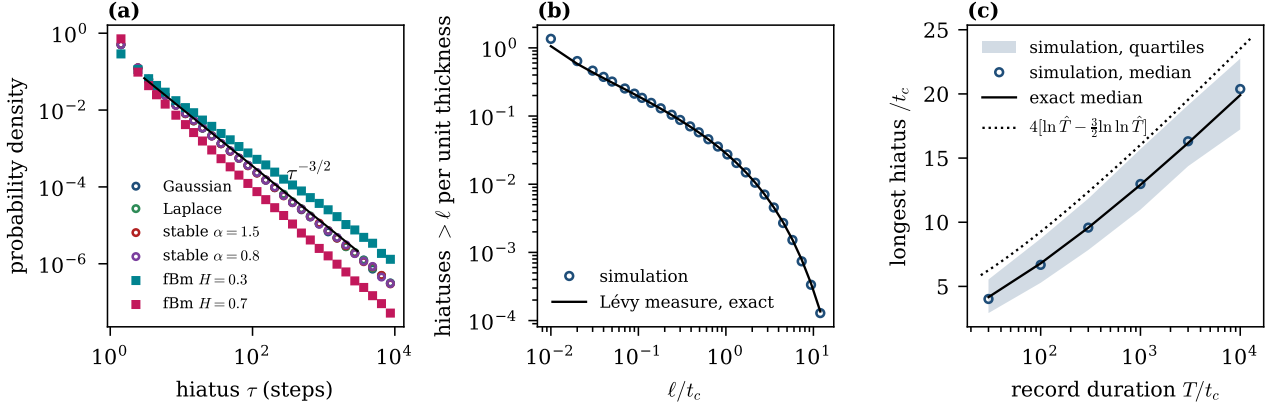


Figure 3: Hiatus statistics. (a) Probability density of hiatus durations in driftless columns, with a $\tau^{-3/2}$ reference. (b) Number of hiatuses longer than ℓ per unit thickness for Gaussian surfaces at $\Delta t/t_c = 0.01$, with Eq. (14). (c) Longest hiatus against record duration, with the exact median from Eq. (15) and the asymptote of Eq. (16) without its constant term.

Table 1: Median longest hiatus, in units of t_c , from simulation and from Eq. (15), for Gaussian surfaces at $\Delta t/t_c = 0.02$.

T/t_c	30	100	300	10^3	3×10^3	10^4
Simulation	4.02	6.68	9.58	12.98	16.31	20.38
Eq. (15)	4.16	6.80	9.58	12.92	16.17	19.91

The depth-domain coherence confirms Eq. (17) (Fig. 4a, b). For $0.15 \leq \omega t_c \leq 3$, the fitted half-width lies within -1 to $+13$ percent of $\text{Re } \Psi$ for Gaussian surfaces, within -4 to $+3$ percent for Laplace surfaces, and within -2 to $+9$ percent for Student t surfaces, and the apparent wavenumber lies within 2 percent of k_* for all three laws. Below $\omega t_c = 0.1$ the coherence does not decay enough over the simulated column for a reliable fit, and those points are omitted. The half-width relative to k_0 rises as ωt_c at small frequency, reaches a maximum of 0.30 near $\omega t_c = 1$, and then declines as $(2\omega t_c)^{-1/2}$. The apparent wavenumber falls to 0.98, 0.88, 0.63, and 0.39 of its nominal value at $\omega t_c = 0.1, 0.3, 1$, and 3 . The misregistration of Eq. (13) has a variance within 3 percent of $2t_c \Delta z/v$ for Gaussian surfaces at $\Delta z/(vt_c) \geq 150$. Its interquartile range grows with fitted exponents of 0.54 and 0.52 for Gaussian and Laplace surfaces, 0.75 for stable surfaces with $\alpha = 1.5$, and 0.31 and 0.80 for fractional surfaces with $H = 0.3$ and 0.7 (Fig. 4c). The predicted exponents are $1/2$, $1/\alpha = 0.67$, and H . Agreement is close for the Brownian and antipersistent cases and within about 0.1 for the heavy-tailed and persistent cases, in which convergence to the asymptotic regime is slow over the accessible range of Δz .

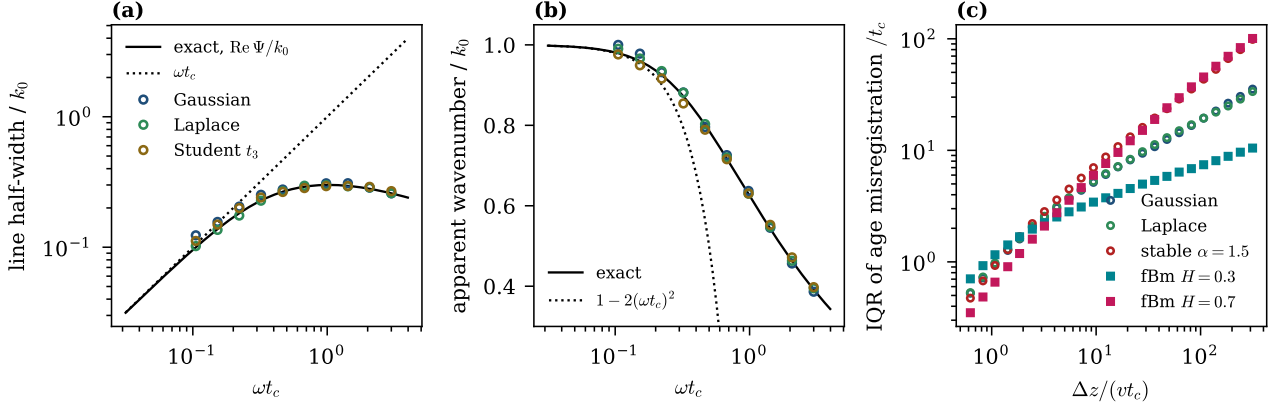


Figure 4: A periodic forcing seen in depth. (a) Half-width of the depth-domain spectral line divided by the nominal wavenumber k_0 , with $\text{Re } \Psi/k_0$ from Eq. (17) and its small-frequency limit. (b) Apparent wavenumber divided by k_0 , with $-\text{Im } \Psi/k_0$ and the expansion of Eq. (18). (c) Interquartile range of the age misregistration against thickness, for five surface laws.

The chronometric information of Eq. (22) is shown in Fig. 5a. Between horizons one nominal cycle apart it is 2.97, 2.10, 1.32, and 0.50 bits for $\omega t_c = 0.003$, 0.01, 0.03, and 0.1, within 0.06 bits of the wrapped-Gaussian approximation in each case. It falls below 0.1 bits after 72, 21.5, 7.2, and 2.3 cycles, between 1.35 and 1.42 times N_{coh} of Eq. (20). The coherence horizon is mapped against t_c and the forcing period in Fig. 5b. Retaining ten coherent cycles requires $t_c \leq P/(40\pi^2)$, which is about 1.0 kyr for the 405 kyr eccentricity cycle and about 0.05 kyr for a 20 kyr precession cycle.

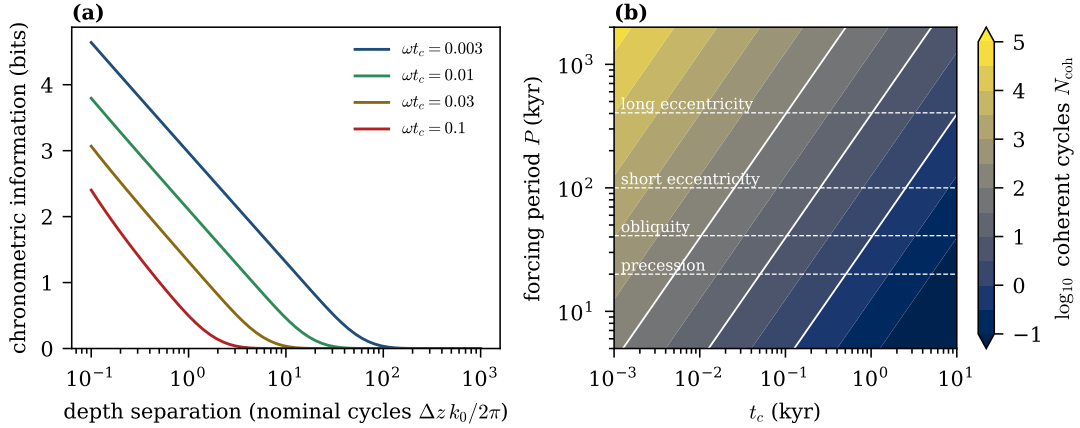


Figure 5: Information loss. (a) Chronometric information carried by two horizons about the elapsed phase of the forcing, against their separation in nominal cycles, from Eqs. (21) and (22). (b) Number of coherent cycles N_{coh} of Eq. (20) against t_c and forcing period, with white contours at 1, 10, and 100 cycles and nominal Milankovitch periods marked.

Table 2: Universality classes of the stratigraphic filter, with predicted and measured exponents. The Sadler exponent γ is fitted for $\Delta t/t_c \leq 10^{-2}$, the preserved count of a driftless column of N layers grows as $N^{1-\theta}$, the hiatus density decays as $\tau^{-(2-\theta)}$, and the misregistration scale grows as Δz^χ . Measured values follow the predicted ones in parentheses.

Surface	$\gamma = \theta$	$1 - \theta$	$2 - \theta$	χ
Gaussian	0.475 (0.5) ^a	exact ^b	1.54 (1.5)	0.54 (0.5)
Laplace	0.506 (0.5)	exact ^b	1.53 (1.5)	0.52 (0.5)
Stable, $\alpha = 1.5$	0.46 (0.5)	exact ^b	1.53 (1.5)	0.75 (0.67)
fBm, $H = 0.3$	0.68 (0.7)	0.33 (0.3)	1.34 (1.3)	0.31 (0.3)
fBm, $H = 0.7$	0.27 (0.3)	0.68 (0.7)	1.72 (1.7)	0.80 (0.7)

^aLowered by the correction of Eq. (8). ^bWithin 2.7 percent of Eq. (10).

5 Discussion

The central physical result is that whether a layer survives depends on the order in which the surface visits different levels, not on the size of the steps by which it gets there. Survival requires that every later elevation lie above the layer, which is a statement about the signs of partial sums of the elevation changes. For independent and symmetric changes, the combinatorics of these signs is the same for every step distribution, which is the content of the Sparre Andersen theorem [24, 14]. A single large storm or turbidity current removes many layers at once, but under symmetric fluctuations it is balanced, in the ordering sense, by equally rare intervals of rapid aggradation, and the probability that a given layer is the last one reached from above is unchanged. A shelf swept by rare catastrophic storms and a lagoon reworked by small, frequent events therefore preserve the same fraction of their history at fine resolution, and they share the same Sadler exponent and the same distribution of short hiatuses (Table 2). The magnitude of erosional events matters only through the variance of the surface fluctuations, which enters the crossover time t_c , and through the scale of the misregistration of ages, which grows faster than the square root of thickness when the events are heavy tailed.

The universality has two boundaries that are geologically meaningful. The first is asymmetry. Many depositional systems build up slowly and erode abruptly, or the reverse, as in event beds deposited in hours and reworked over years. Asymmetry leaves the exponent $1/2$ unchanged for fluctuations of finite variance but alters the prefactor of the completeness, because the mean ladder height then differs from $\sigma/\sqrt{2}$. The second boundary is memory, which changes the exponent itself. At a single location, compensation is a negative feedback: topographic lows attract sediment and highs are bypassed, so that elevation changes over successive intervals are anticorrelated [26]. In the language of the model, compensation makes the surface antipersistent, with $H < 1/2$, at timescales shorter than the compensation time. The model then predicts a Sadler exponent larger than $1/2$, slower growth of the number of preserved beds with the number deposited, and a heavier tail of long hiatuses. Persistent behavior, with $H > 1/2$, would arise where erosion and deposition cluster in time, for example during prolonged occupation of a site by a channel or under accommodation that fluctuates slowly, and would produce the opposite signatures. If Sadler exponents measured within a single depositional setting depart from $1/2$, the model attributes the departure to such feedbacks, not to the size distribution of erosional events.

The three classes can be distinguished in principle by two measurements made in the same setting: the Sadler exponent γ and the scaling exponent χ of age misregistration between independently dated horizons. Brownian surfaces give $(1/2, 1/2)$, heavy-tailed independent erosion gives $(1/2, 1/\alpha)$, and surfaces with memory give $(1 - H, H)$. Compilations of accumulation rates that combine many environments [21, 9, 22] mix settings with different crossover times, and a mixture of curves with different crossovers can steepen or flatten an apparent Sadler slope even when every component belongs to the Brownian class. The diagnosis is therefore meaningful only within a setting whose crossover time can be treated as constant.

The crossover time $t_c = D/v^2$ is the single scale of the model, and it has a direct stratigraphic meaning. Below t_c , the surface roughness dominates and the record is shredded by its own fluctuations. Above t_c , net accumulation dominates and the record becomes nearly complete. The same scale appears in the compensation timescale of experimental and field studies, beyond which deposition fills topography evenly [26, 20]. Written as $\ell_*/(2v)$, it is the time needed to accumulate half the vertical scale ℓ_* at which roughness and net accumulation balance. A surface with $\ell_* = 1$ m and a net rate of 0.1 mm yr⁻¹ has $t_c = 5$ kyr. In a given section, t_c could be estimated from the resolution at which the Sadler curve flattens, from the exponential cutoff of the hiatus distribution at $4t_c$, or from the growth of age misregistration between dated horizons. The values quoted here illustrate the scaling and are not estimates for any setting.

The hiatus statistics of Eq. (14) suggest an objective way to separate discontinuities produced by the depositional system itself from those imposed on it. Below t_c the durations of hiatuses follow a power law, which has no characteristic scale. Diastems, bed boundaries, and bedset boundaries then form a single continuum, and a hierarchy of bounding surfaces of increasing order can emerge from one stationary process without separate mechanisms for each order. Above about $4t_c$ the distribution is cut off exponentially, so stationary fluctuations rarely produce gaps much longer than this scale. A population of gaps that stands apart from this cutoff points to forcing external to the depositional system, such as a fall in base level, tectonic uplift, or a change in climate. In the vocabulary of sequence stratigraphy, the model provides a null distribution against which a candidate sequence boundary can be distinguished from an autogenic surface of erosion, provided t_c is known for the setting.

Equation (15) turns this distinction into a statistical test for the longest gap in a section. With net accumulation, the longest hiatus produced by stationary fluctuations grows only logarithmically with the duration of the record and reaches about $20t_c$ in a record $10^4 t_c$ long. Lengthening the record does little to produce larger gaps, because the exponential cutoff of the hiatus distribution dominates its power law. An unconformity many times longer than this bound, in a setting whose t_c is known, is unlikely under the null model and calls for a change in forcing.

The driftless limit reverses this conclusion. Where net accumulation vanishes over long intervals, as it may on stable cratons, the longest gap occupies on average about 0.63 of the record, whatever the distribution of erosional events. In such a setting a hiatus spanning most of the available time is the expected outcome, not an anomaly. This bears on the Great Unconformity, which has been attributed to Neoproterozoic glacial erosion [10]. The model implies that the duration of the gap at a single site cannot by itself discriminate between a

discrete global event and the ordinary statistics of a surface with little net accumulation over a very long interval. What can discriminate is synchronicity. Independent basins governed by stationary dynamics would show gaps whose bounding ages are uncorrelated beyond the distance over which their surfaces interact, whereas a global event would impose correlated boundaries. A spatially coupled version of the model would convert this argument into a quantitative test.

Age models are commonly built by interpolating between dated horizons with a constant or smoothly varying accumulation rate. Equation (13) quantifies the error this introduces when the hiatuses between the horizons are unmarked. Misregistration accumulates like a random walk in thickness, so its standard deviation grows as the square root of the interval. Between two horizons of known age separated by a duration T , the increments of the age are constrained at both ends, and to leading order the variance of the interpolated age at the midpoint is $t_c T/2$, one quarter of the unconstrained variance over the whole interval. Holding the interpolation error below a target δ therefore requires dated horizons separated by less than about $2\delta^2/t_c$. This criterion depends on the crossover time and the desired precision, not on the accumulation rate alone, and it complements earlier assessments of how unsteady sedimentation blurs the timing of abrupt events [11]. For heavy-tailed or persistent surfaces the misregistration grows faster than the square root of thickness, and the required spacing is correspondingly smaller.

The model also exposes a bias in which parts of the history are preserved. Material survives only if it is followed by net aggradation, so the record samples preferentially those moments that precede a rise of the surface. In the model the forcing is recorded passively and this selection affects only timing. If the forcing also modulates accumulation, as when sea level controls both the composition of sediment and the space available for it, the preserved record will over-represent the phases of the forcing that favor aggradation. Coupling the proxy to the drift is a natural extension of the present framework.

The results for periodic forcing state when an astronomical cycle survives in depth and how it is distorted. A cycle of period P remains a resolved spectral line only if $P \gg 4\pi t_c$, and it remains phase coherent over about $P/(4\pi^2 t_c)$ repetitions. Long cycles are therefore far more robust than short ones, which is consistent with the use of the 405 kyr eccentricity cycle as a stable metronome in astrochronology [7, 17]. Two consequences can be tested. First, the broadening of the line is not a loss of amplitude and cannot be removed by filtering, because it reflects a random wandering of phase in depth that grows with distance from any fixed point. Evolutionary spectral methods that track changes in accumulation rate can follow slow variations of rate, but they cannot recover the phase lost within hiatuses shorter than their window. Second, the shift of the line toward longer apparent periods grows with $(\omega t_c)^2$, so short cycles are stretched more than long ones. In a section where precession and eccentricity are both recorded, the apparent number of precession cycles per eccentricity cycle should be smaller than the astronomical ratio, by an amount that increases with t_c . A systematic deficit of this kind in bundling ratios would be a signature of unmarked hiatuses, and its magnitude would provide an independent estimate of t_c .

The information-theoretic view clarifies why this loss is irreducible. An erasure channel marks the symbols it destroys, so the receiver knows where information is missing. The stratigraphic record deletes layers without marking the deletions, which makes it a deletion channel, and deletion channels destroy synchronization as well as content [18]. No improvement in the resolution or precision of a proxy can restore synchronization within a sequence, because the missing time leaves no trace. Only independent clocks, such as dated volcanic ash beds or magnetic reversals, can resynchronize the record. They play the role of the synchronization markers inserted into communication signals, and the chronometric information of Eq. (22) indicates that they must be spaced at less than the coherence horizon to keep the record locked to the forcing. In this sense, the coherence horizon is the natural unit for planning the density of geochronological sampling in a cyclostratigraphic study.

From the perspective of complex systems, the stratigraphic filter is an extremal dynamics: the output retains only the running extremes of the input, and almost all of the history is discarded. The universality found here has the same origin as universality near critical points. The observables that survive coarse graining, in this case the ordering of elevations, depend only on symmetry and on the existence of a single crossover scale, not on the microscopic details of transport. The map from history to rock is many to one, so its information loss is an intrinsic property of the operator, not a consequence of imperfect observation. Memory changes the universality class, as long-range correlations do in other interface problems [12, 1], and the age-depth map provides a physical realization of a subordinator, a process whose jumps record the excursions of the surface above its future minimum.

The model describes the surface at a single location as a stochastic process in one dimension. Lateral transport, the migration of channels and lobes, and the three-dimensional geometry of erosion surfaces enter only through the statistics of elevation at that location, and spatial correlations between neighboring sections are not represented. Compaction, diagenesis, and bioturbation, which further distort the depth scale and mix proxy signals across layers, are omitted. The fluctuations are stationary and symmetric, whereas real accommodation changes with tectonics and sea level, and many depositional systems aggrade and erode asymmetrically. The forcing is assumed to be recorded without noise in every preserved layer and to have no effect on accumulation. The exponents for heavy-tailed and persistent surfaces rest on scaling arguments, and their numerical confirmation is less precise than that of the exact Brownian results. Finally, the accumulation rate and the fluctuation diffusivity are treated as known, whereas in practice both must be inferred from a record that the filter has already degraded, which is itself an inverse problem.

6 Conclusions

Treating the stratigraphic column as the record of the lowest levels that a fluctuating depositional surface will later reach turns long-standing questions about the completeness of the geological record into problems of record statistics and first passage. The fraction of time preserved depends on a single ratio between the resolution of observation and a crossover time set by net accumulation and surface roughness. At fine resolution this dependence is universal: it is the same for erosion by many small events and by rare catastrophic ones, provided successive events are independent. The Sadler effect is governed by the persistence of the depositional surface and changes form only when erosion has memory, as compensational filling of topography may produce. With net accumulation, the age of a column advances with height in independent jumps. This description yields the complete statistics of hiatuses, shows that ordinary fluctuations produce gaps with no characteristic scale up to the crossover time but rarely beyond it, and implies that the longest gap grows only slowly with the length of a record. Where net accumulation vanishes, by contrast, a gap spanning most of the record is expected, so the duration of an unconformity at a single site cannot by itself establish an extraordinary cause. The same description predicts that an astronomical cycle appears in depth broadened, stretched toward longer apparent periods, and coherent over a limited number of repetitions, because unmarked hiatuses destroy synchronization in a way that no improvement in proxy precision can repair. These results provide null models for unconformities and sequence boundaries, criteria for the spacing of dated horizons, and testable signatures of the dynamics of depositional surfaces. Extending the framework to spatially coupled basins would allow the synchronicity of major unconformities to be tested against the same null.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this article.

Declaration of generative AI use

During the preparation of this work, the authors used Claude Sonnet 5 solely for the purposes of English grammar, vocabulary refinement, and improving the overall readability of the manuscript. The authors maintain full responsibility for the conceptualization, model development, mathematical derivations, computational execution, and the analysis and interpretation presented in this study.

Code and data availability

This study uses no external data. The simulation code, the outputs behind every figure, and the scripts that regenerate them are openly available at https://github.com/sandyherho/stratigraphic_universality under the MIT License.

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