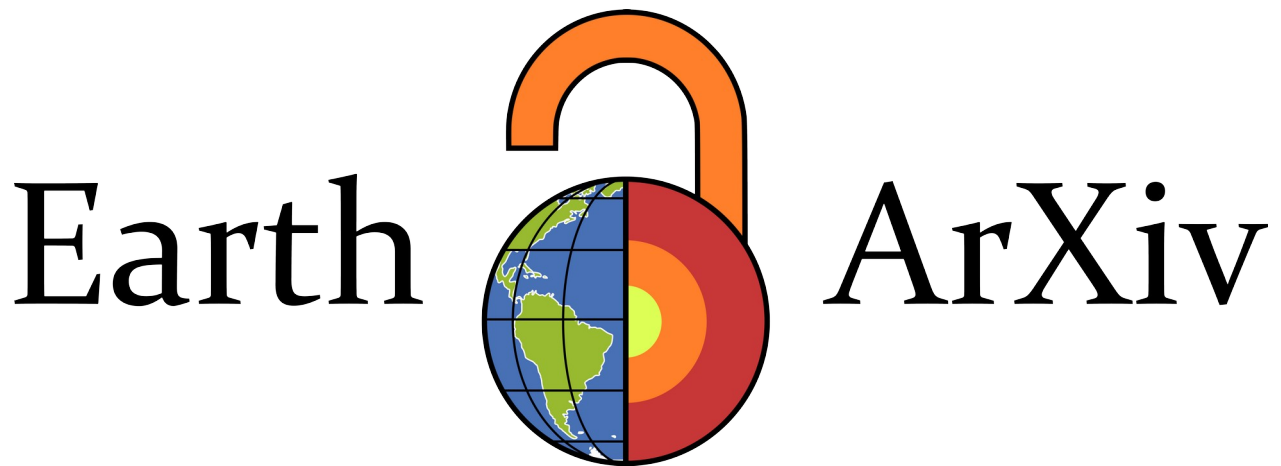




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The Ill-Posed Question of Global Mean Temperature

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Abstract

The question "Does a global temperature exist?" was answered in the negative by C. Essex, R. McKittrick, and B. Andresen (EMA), who showed that different choices of mean yield different signs of the global temperature trend.¹ We take EMA's result as our point of departure and argue that its root cause is more fundamental than the particular data set or averaging formula employed: the very notion of a mean presupposes a specified *purpose* — the physical condition that the single representative value is required to reproduce. Once this is made explicit, it becomes clear that the question "what is the mean of these temperatures?" is ill-posed until the purpose is named. In particular, the conventional arithmetic mean of station temperatures fails this requirement. We first illustrate this with elementary examples, including cases in which a legitimate mean lies outside the range of the data, contrary to a claim made by EMA. We then extend EMA's analysis to daily minimum and maximum temperatures from 1371 NOAA stations over 1939–2025, computing 33 different means for each. Depending on the mean adopted, the "global" mean temperature ranges over tens of degrees, while the inferred warming trend varies by a factor of 36 — from 0.015 to 0.539 °C/decade. We argue that the difficulty is not resolved by choosing a better averaging formula, since any scalar statistic discards the spatial structure that carries the physical information. We conclude that inferences drawn from global mean temperature about weather and climate are at present premature: climate science seems to be still at its infancy.

1 What is a mean?

What is the mean of 20 and 80? Is it 22, or 32? Or, maybe, it is 40, 50, 60, 78? Or, even 123? As we shall see, all those numbers are a legitimate mean between 20 and 80. If we ask the first passer-by in the street, they will probably answer: 50! The answer, before being wrong, is naive. But the question is even more naive, and whoever ventures to ask it does not seem to know what a mean is. The point is not that the person asking has

¹See reference [1]

failed to specify which mean; the point is that, without specifying which mean one refers to, the question is ill-posed — whereas once the type of mean is specified, the question becomes merely notional, fit for a middle-school test.

Let us then go straight to the heart of the matter and clarify what the mean of a sequence of numbers representing a clearly specified *situation* actually is. Let us first clarify why one would ever want to construct a mean at all. The reason is this: it stems from the wish to replace the multiplicity of the sequence with a single number. Yes, but which one? The fact is that one must specify not only the *situation* that generates those numbers, but also the *purpose* that single number is required to fulfill. Once the *situation* is defined and the *purpose* specified, the mean of a sequence of numbers generated by that situation, with respect to that particular purpose, is that single number which, substituted for every number in the sequence, fulfill that purpose. Clearly, under this definition, there are many possible means of a sequence of numbers.

One further point must be clarified here: it is not true that the mean value of a sequence of numbers must lie within the range of the sequence. This is the only correction we feel we should offer to the otherwise impeccable EMA article, where the Authors write: “One is faced with having an infinity of averages, having all possible values within the range of the data set.” There are indeed infinitely as many possible mean values as many purposes may be required; however, as we shall see, they need not lie between the smallest and the largest value of the sequence of numbers one wishes to average.

Let us return to the numbers 20 and 80, and consider the following *situation*: a car running at 20 km/h over the first half of a journey and 80 km/h over the second half. *Purpose*: let the mean speed that single value that, maintained over the whole route, would take the same amount of time. It is not hard to see that the mean speed is, in this case, the harmonic mean of the two numbers: 32 km/h. However, if we change the *purpose* and ask for that single speed that would consume the same amount of fuel, we would obtain a different value, depending on how fuel consumption changes with speed, which in turn depends on the car’s engine: it could turn out to be 60 km/h, but for our present purpose it is irrelevant to know.

Since we will be discussing temperatures, let us suppose now that 20 and 80 are indeed two temperatures. Until we specify the situation and the purpose, asking for the mean of these two temperatures is a question devoid of meaning. Here is one possible situation: 20 is the temperature (in Celsius) of a 1 L jug of water, and 80 is the temperature of a 1 kg lead ball. That is not enough: we must specify the purpose. Let it be that of bringing the two systems into thermal contact, that is, immersing the lead sphere in the jug of water and waiting for thermal equilibrium. The mean we seek is then that temperature value, common to the water and the lead, that would fulfill the same purpose as when the two systems are, respectively, at 20 and 80 Celsius. As is well known, the equilibrium temperature is a sort of average weighted by the heat capacities of water and lead, and

the result turns out to be 22°C . Now, same purpose, same numbers, but a different situation: if the water is at 80°C and the lead at 20°C , the mean turns out to be 78°C .

An interesting question now arises — one which we shall come back on later: upon going from the (20,80) state to the (22,22) state the global water-lead system has warmed up or has cooled down? And what about from the (80,20)-state to the (78,78)-state? According to the arithmetic means, there would be a cooling in the former case and a warming in the latter. However, there is no reason to call for the arithmetic mean, so we set aside the question for the time being.

Let us consider one more example. A fictitious cold planet small enough to need only two weather stations. The stations recorded yesterday the temperatures 20 K and 80 K, and today the temperatures 40 K and 40 K. Depending on whether the arithmetic, geometric, or harmonic mean is used as the global mean temperature, our small planet would have had, yesterday, a temperature of 50, 32, or 40 K; and today a temperature of 40 K in all three cases. Thus, depending on the mean used, the planet has either cooled down, kept a stable climate, or warmed up.

In their work, EMA carried out several kinds of means on real data from 12 weather stations and found exactly this eccentric result: for those 12 stations, the “global” climate has warmed or cooled depending on the mean used. In commenting on this eccentric result, the Authors conclude the article by answering the question in its title, and the answer is: No, there is nothing that can be qualified as a global mean temperature. To drive home the sense of their conclusion, they write: “Individual telephone numbers are both meaningful and useful, while the sum or average over telephone numbers in a directory has no meaning.”

Should someone object that computing the mean of telephone numbers is an arithmetic operation which, while valid, carries no informative value because there is no common factor linking the telephone numbers of different people (so that the EMA metaphor would not hold), consider this other situation. Consider the foot lengths of a population (say, the inhabitants of a city). What meaning does their arithmetic mean have (or, for that matter, any other mean)? Once again, none, unless the purpose is specified. Here is a plausible one. It involves a young shoe manufacturer who is just starting out and, at present, can only produce shoes of a single size; he would want to know that single number representative of the population so as to maximize his sales. It is clear that none of the means cited so far will do: he might not selling a single pair of shoes. Just as clearly, the single number fitting the required purpose is that statistical index called the “mode.” A geneticist studying the foot size of a population, might require another purpose whereby some other position index might be more profitable.

The message should by now be clear: We have grounds enough to view with suspicion the number that is passed off as the global mean temperature and — as we shall see — with even greater suspicion the number passed off as the global warming trend. Before analyzing the eccentric EMA result in more detail, as promised, let us look at an example

of a situation in which the mean value can lie outside the range of the data. Consider the following *situation*. Let us take an angle of α width and vertex at O ; let x and y be the distances from O of two random points A and B , each lying on one of the two sides of the angle. And let d be their distance.

The *purpose* of the required mean value of the two random variables with values x and y is the following: it should be that single value that these random variables should share so as to leave the value of the distance d unchanged. It is readily seen that the value $m = x = y$ is:

$$m = \sqrt{\frac{x^2 + y^2 - 2xy\cos\alpha}{2(1 - \cos\alpha)}}$$

This is a quantity that — if $\alpha < \pi/3$ — can well be greater than the larger of the values x or y . For instance, with $\alpha = \pi/6$, the mean between $x = 20$ and $y = 80$ would be $m = 123$.

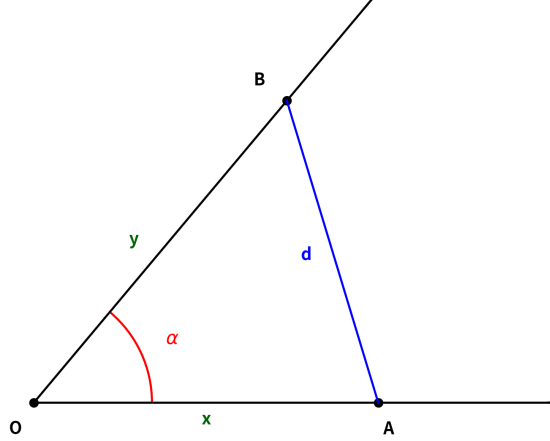


Figure 1: Example of a mean value lying outside the range of the data.

All of the above teaches us that it makes no sense to declare a mean between two or more numbers unless one specifies the *situation* from which those numbers arise and the precise *purpose* one wishes fulfilled by the mean.

2 What a global mean temperature would like to be

In the case of climatology, the *situation* appears fairly clear: we have the sequence of daily minimum and maximum temperature readings from weather stations located at well-defined geographic coordinates (longitude, latitude, elevation above sea level, which we may collectively denote by $\mathbf{x}$), each recorded at time t . The set of these numbers is a (scalar) field $T(\mathbf{x}, t)$ which, for lack of a better term, we call “climate”. But what is the *purpose* that determines that single number that we may call a mean? As we have seen, there could be as many purposes as one wished.

There can be many purposes for a mean between temperatures, but one in particular attracts, more than all others, the attention of scientists and of humanity as a whole: the weather. The widespread fear is, in fact, that changes in the global climate — which, it would seem, are changes in the sense of global warming — might induce extreme and unwelcome weather phenomena. Let us set aside the fact that no one asks whether this presumed global warming might, instead, induce more pleasant weather, and focus on the approach that has inspired most of the research, namely: to record temperatures; compute, somehow, their global mean value over time; observe the rate at which it grows; and, assuming a resulting damage from the warming, quantify the actions needed to contain both, warming and damage. With the backing of bodies regarded as scientific and so accredited, hundreds of countries around the world have sent their representatives to Paris, where they have pledged to keep the warming down. And they have gone as far as to take their commitment so seriously as to provide a precise figure: below 1.5°C .

One way or another, the *situation* is clear. The *purpose*, too, would seem clear: to determine that single number representative of all the temperature readings which, once substituted for them, would produce the same weather. It is evident that this number does not exist, neither globally nor locally: there is no way to represent the scalar field $T(\mathbf{x}, t)$ by a single number indicative of the weather. And this holds not only for global weather — whatever that may mean — but also for the weather localized in space and confined in time.

Another, perhaps more reasonable, purpose might be this: If we replaced all the temperatures recorded today with today’s mean value and all those recorded tomorrow with tomorrow’s mean value, we would obtain the same climate variation as that given by the original values.

We have already seen that the two pairs of temperatures (20, 80) before, and (40, 40) after, indicate — between before and after — a warming, a climate stability, or a cooling, depending on whether one considers the harmonic, geometric, or arithmetic mean. It is not difficult to construct a larger data set providing the same oddity. A set of this kind is shown in Table 1, comprising 30 values for each of two locations (A and B) over three years, which we call Year 1, 2, and 3.

The “time” evolution of the arithmetic, quartic,² geometric, and harmonic means is shown in Fig. 2:³ the system made up of the two locations is, in Year 3, colder or warmer than in Year 1 depending on which mean is used.

The values in Table 1 are an example showing how relative the concept of global warming from a collection of temperature data can be (Fig. 2). Clearly, those given are constructed values, built on two locations, one “cold” and the other “warm”, with the thermal range between the two locations high in Year 1, intermediate in Year 2, and low in Year 3. One may ask whether this could correspond to a real situation. EMA

²The quartic mean is given by eq. (1) below for $n = 4$.

³Temperature values have of course been transformed into kelvin before evaluating the means.

Table 1: Fictitious TA / TB sequences in Celsius for three fictitious years.

Year 1								
n	TA (°C)	TB (°C)	n	TA (°C)	TB (°C)	n	TA (°C)	TB (°C)
1	-15.26	14.54	11	-16.12	14.05	21	-15.97	13.29
2	-15.01	14.83	12	-15.62	13.84	22	-16.31	13.79
3	-14.13	14.49	13	-16.13	13.98	23	-15.08	15.47
4	-15.23	14.88	14	-14.41	13.46	24	-16.21	14.42
5	-15.13	14.39	15	-14.69	15.13	25	-15.89	15.64
6	-14.94	13.96	16	-16.25	14.25	26	-15.77	14.24
7	-15.91	15.68	17	-13.99	15.32	27	-16.28	13.47
8	-15.12	15.68	18	-14.03	14.22	28	-15.24	14.80
9	-14.84	15.31	19	-14.78	15.59	29	-15.29	15.16
10	-14.45	14.99	20	-14.87	15.32	30	-14.33	13.94

Year 2								
n	TA (°C)	TB (°C)	n	TA (°C)	TB (°C)	n	TA (°C)	TB (°C)
1	-10.99	8.17	11	-9.86	8.55	21	-8.87	8.26
2	-10.40	8.28	12	-10.13	8.75	22	-9.27	8.12
3	-8.89	9.51	13	-10.74	9.96	23	-10.47	9.43
4	-9.38	8.45	14	-9.44	8.82	24	-9.08	9.92
5	-10.92	8.37	15	-10.89	9.04	25	-10.69	8.13
6	-10.61	7.84	16	-9.66	9.74	26	-10.25	9.94
7	-10.96	7.88	17	-10.92	8.10	27	-9.15	9.78
8	-11.06	9.06	18	-10.19	8.03	28	-9.66	9.11
9	-9.29	8.24	19	-10.69	9.84	29	-10.96	8.67
10	-10.77	9.10	20	-10.55	9.62	30	-8.83	7.91

Year 3								
n	TA (°C)	TB (°C)	n	TA (°C)	TB (°C)	n	TA (°C)	TB (°C)
1	-4.26	2.17	11	-4.18	1.67	21	-3.28	2.41
2	-2.04	1.91	12	-3.80	2.26	22	-4.20	2.71
3	-3.78	0.85	13	-3.01	1.28	23	-3.93	1.35
4	-2.66	0.59	14	-2.30	2.91	24	-3.46	2.15
5	-3.73	0.55	15	-2.88	0.69	25	-2.98	2.67
6	-2.37	2.69	16	-3.68	1.82	26	-4.03	2.60
7	-2.92	2.19	17	-2.15	2.28	27	-3.48	1.51
8	-3.65	2.82	18	-3.86	2.67	28	-2.21	2.41
9	-3.93	0.56	19	-4.31	2.28	29	-2.00	2.58
10	-2.62	2.04	20	-3.70	2.20	30	-2.77	1.88

considered data from 12 real weather stations and concluded that, depending on the mean used, that data yielded either a warming or a cooling of the “world” made up of those 12 stations.

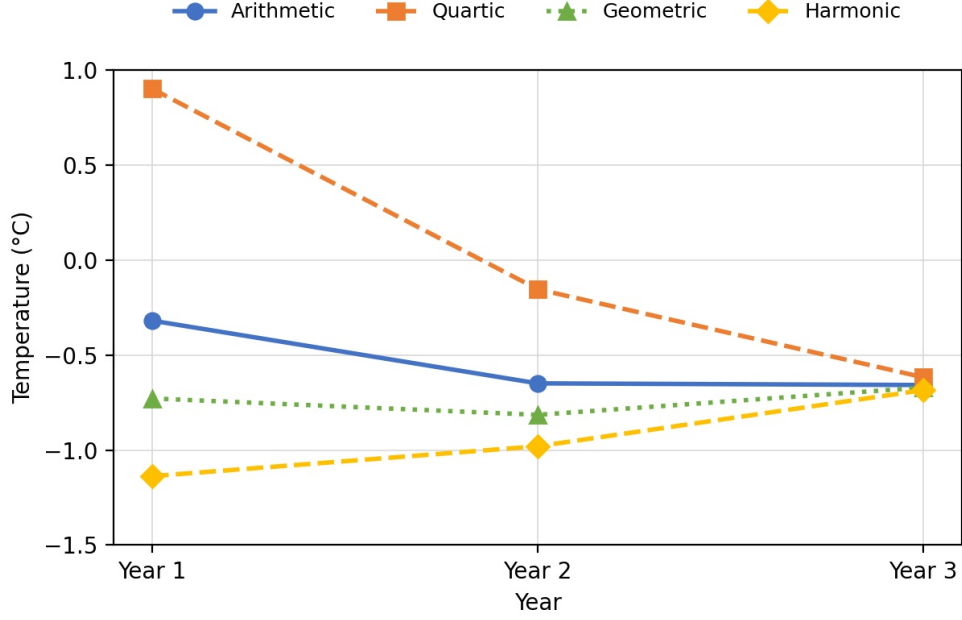


Figure 2: The “fictitious” world made up of locations A and B, where the temperature values in Table 1 were recorded, shows global warming or global cooling depending on the mean used.

Why does this happen? Consider, for example, the n -th power mean in the EMA article:

$$M_n = \left(\frac{1}{N} \sum_{j=1}^N T_j^n \right)^{1/n}. \quad (1)$$

For small values of the ratio σ/μ , where σ is the standard deviation and μ the arithmetic mean, the following approximation holds:

$$M_n \simeq \mu + \frac{n-1}{2} \frac{\sigma^2}{\mu}. \quad (2)$$

The trend of the n -th power mean, then, does not necessarily follow that of the arithmetic mean but also depends on the variance, whose term can dominate and reverse the sign if the variance’s contribution has the opposite sign to that of the mean. Indeed, the 12 stations chosen by EMA are located in places with highly variable temperatures and very different local trends (such as Antarctica, the Tropics, Greenland, the Arizona desert).

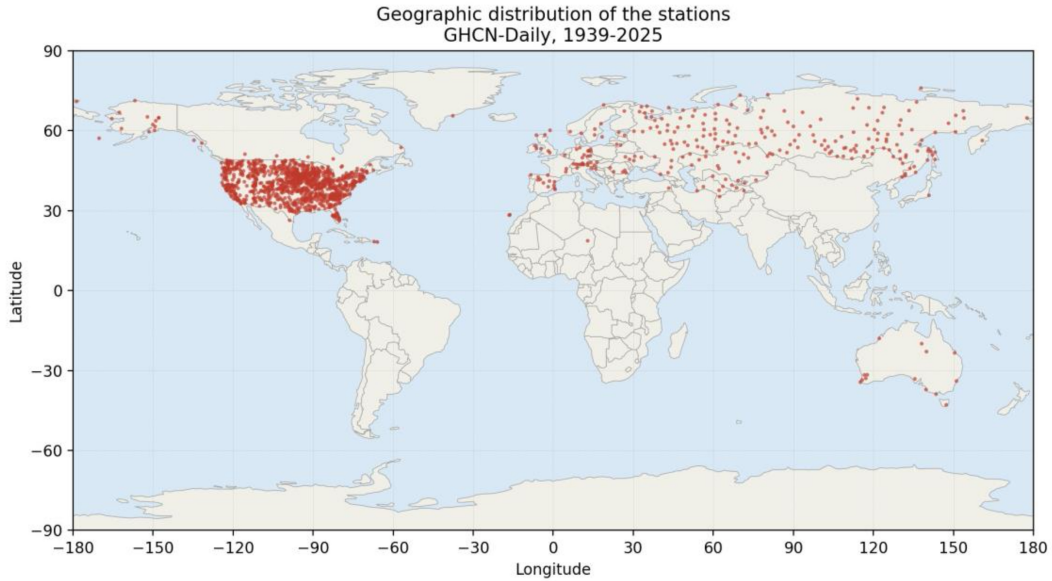


Figure 3: Geographic location of the stations considered in this study.

3 Calculation, results and comments

While the crucial importance of the EMA result — whose conclusion needs no revision as regards the non-existence of a global mean temperature⁴ — stands firm, it is of interest to examine the matter more deeply, on a larger data sample and over a longer time span. The initial idea was to consider the daily maximum and minimum temperature values recorded by weather stations in the years 1939–2025. For this purpose, we used the NOAA database.⁵ For those 87 years, there were 127,240 operating weather stations, but, for obvious reasons of homogeneity, we chose to take only those stations that were operational for all the years considered, which left us with only 1371 stations.⁶ The spatial distribution of the stations (Fig. 3) is anything but uniform over the surface of our planet, but for the purposes of this article this is irrelevant.

We then computed the n -th power means M_n and the n -th exponential means A_n , defined, respectively, in eq. (1) and in the following eq. (3), of the minimum and maximum temperatures, TMIN and TMAX, recorded daily by the 1371 stations, thereby producing annual means for all 87 years 1939–2025:⁷

$$A_n = \frac{1}{n} \ln \left(\frac{1}{N} \sum_{j=1}^N \exp n T_j \right) \quad (3)$$

⁴Indeed, the mere example of the temperature pairs (20, 80) and (40, 40) considered in the text would suffice to invalidate the numerology underlying the calculation of global mean temperatures.

⁵See references [2, 3]

⁶If, over time, stations are added in warmer locations (or removed from colder ones), a warming would be observed that has nothing to do with the planet; the opposite effect would occur if, over time, stations are added in colder locations (or removed from warmer ones).

⁷Equations (1) and (3) correspond to equations (23) of the EMA article.

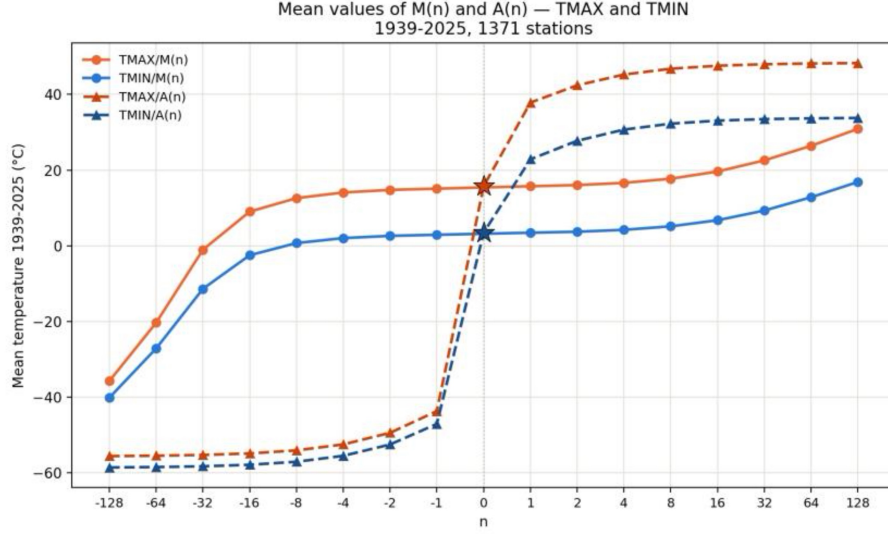


Figure 4: Mean value of the minimum temperatures (blue curves) and maximum temperatures (red curves) recorded at 1371 stations over the years 1939–2025, computed using eqs. (1) and (3), each for 16 values of n , as shown on the horizontal axis. The case $n = 0$ is obtained by computing, for the mean M_0 , the geometric mean, and, for the mean A_0 , by setting it equal to the arithmetic mean, as it should be. Depending on which of the 33 different means is used, the mean value of the global minimum temperature turns out to lie between -58.6°C and 33.8°C ; that of the global maximum temperature turns out to lie between -55.6°C and 48.3°C .

We then determined the temperature anomalies, each with respect to its own mean. Finally, we performed a linear regression on the anomalies for each of the computed means, specifically for n equal to the positive and negative values of the following powers of 2: $-128, -64, -32, -16, -8, -4, -2, -1, 0, 1, 2, 4, 8, 16, 32, 64, 128$.⁸ The results are shown in Figs. 4 and 5.⁹

Figure 4 shows, for each type of mean, the “global” means of the minimum (blue curves) and maximum (red curves) temperatures for the years 1939–2025. The curves with circular markers follow the various n -th power means [eq. (1)]; those with triangular markers follow the various n -th exponential means [eq. (3)]. The two points marked with a star at $n = 0$ on the A_n curve have a value equal to that computed for M_1 (i.e., the arithmetic mean).¹⁰ Note that the values $\text{TMIN}(M_1) \simeq 3.4^\circ\text{C}$ and $\text{TMAX}(M_1) \simeq 15.7^\circ\text{C}$ — which, with every caveat given that the station coverage in this study is far from being

⁸Note that M_1 and the limit as $n \rightarrow 0$ of A_n give the arithmetic mean, M_{-1} is nothing other than the harmonic mean, and the limit as $n \rightarrow 0$ of M_n is the geometric mean.

⁹All p -values associated with the computed values are significant at the $p < 0.05$ level (mostly at $p < 10^{-10}$), except for the trends in TMAX from M_{32} , M_{64} and M_{128} , with $p = 0.11$, $p = 0.46$ and $p = 0.19$, respectively.

¹⁰Likewise, the points at $n = 0$ on the M_n curves have a mean temperature value equal to that computed, independently, from the geometric mean.

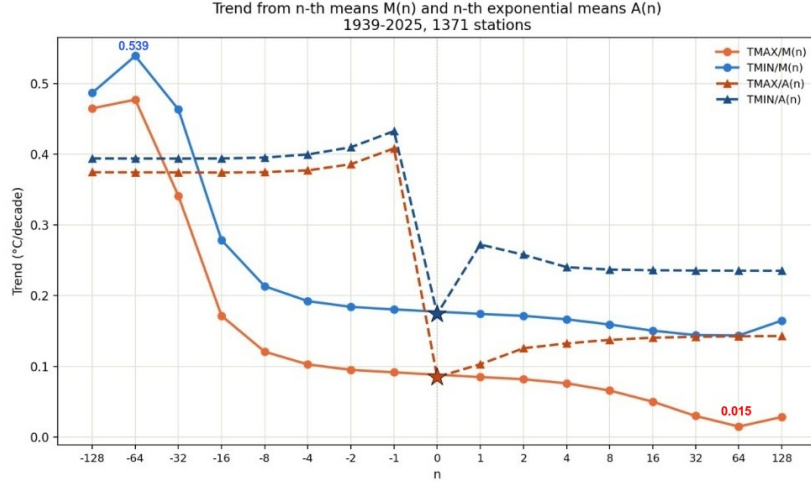


Figure 5: The notation is the same as in Fig. 4: the values of the “global” warming trend are shown as a function of the mean used.

uniformly distributed in space — are well compatible with those usually quoted as the global mean temperature. However, depending on which of the 33 different means is used, the mean value of the global minimum temperature turns out to lie between -58.6°C and 33.8°C ; that of the global maximum temperature turns out to lie between -55.6°C and 48.3°C .

Should the large variability just found come as a surprise, it is worth noting that the very emission spectrum of the Earth (which is not a blackbody spectrum!) yields, scene by scene and channel by channel, brightness temperatures — i.e., the temperature of the blackbody that locally best reproduces the observed radiance — ranging all the way from about -98°C , recorded by satellite thermal-infrared sensors over the East Antarctic Plateau,¹¹ to about $+81^{\circ}\text{C}$, recorded by the MODIS instrument over Iran’s Lut Desert.¹²

At this point, two remarks are worth making. First, one must not be misled by the fact that the values just mentioned appear “extreme”, while those given by the arithmetic mean seem more reasonable because “intermediate”: the numbers 22 or 78 are — between the values 20 and 80 — legitimate and well-defined means relative to well-defined situations and purposes, even though they are not “somewhere in the middle” of the original values; moreover, as already seen, it is not even excluded that, depending on the purpose, an appropriate mean value may lie outside the range of the data.¹³ Once again, until the *purpose* is well identified, anything is possible regarding the most appropriate mean value.

The second remark is that the matter is less important than one might think, and perhaps it is an academic one: what value to assign to the Earth’s mean temperature

¹¹See reference [4]

¹²See references [5, 6]

¹³This is not our case, since the means chosen are all internal (i.e., bounded within the data range).

from a given set of registered temperatures is perhaps more a curiosity than anything else. The point at which the matter becomes serious is when that mean temperature is used to draw inferences about the global climate, and, from it, about local weather phenomena. This brings us to the second and far more important result, namely the one shown in Fig. 5, which represents a measure of the mean global *warming* that occurred over the years 1939–2025.

It is interesting, first of all, to note that all the means considered, computed on both the minimum and the maximum temperature values, indicate a temperature increase; which, we wish to stress, is a warming of the data rather than of the planet, given that its mean temperature — from temperature records, at least — has not even been defined yet. In any case, in the data from the 1371 stations, and over the 1939–2025 time span, no trend reversal appears such as the one found by EMA for the 12 stations and the 21-year interval those authors considered.

It should nonetheless be stressed that, once a global mean temperature has a meaning, the trend would be an objective physical property and so it would be for computed trend. However, the trend computed from the 33 means here considered takes on values ranging all the way from $0.015^{\circ}\text{C}/\text{decade}$ to $0.539^{\circ}\text{C}/\text{decade}$: this is a discrepancy of a factor 36, that holds until the question on how to compute the global mean temperature has an answer. And no answer has been given so far.

What should we conclude? We would perhaps be close enough to the truth if we admitted that any inference drawn from any statistic on the scalar temperature field — particularly regarding the occurrence of weather phenomena — is, at best, premature and, given the current state of the art, reckless. Perhaps studying the temperature gradient vector field rather than the temperature scalar field could bear greater fruit. In any case, far from being able to say that “the science is settled”, it would be more correct to say that climatology is still in its infancy.

References

1. Essex C, McKittrick R, and Andresen B. Does a Global Temperature Exist? *Journal of Non-Equilibrium Thermodynamics* 2007;32:1–27.
2. Menne MJ, Durre I, Korzeniewski B, et al. Global Historical Climatology Network - Daily (GHCN-Daily), Version 3. Dataset. [Accessed August 2026]. 2012. DOI: 10.7289/V5D21VHZ.
3. Menne MJ, Durre I, Vose RS, Gleason BE, and Houston TG. An Overview of the Global Historical Climatology Network-Daily Database. *Journal of Atmospheric and Oceanic Technology* 2012;29:897–910.
4. Scambos TA, Campbell GG, Pope A, et al. Ultralow Surface Temperatures in East Antarctica from Satellite Thermal Infrared Mapping: The Coldest Places on Earth. *Geophysical Research Letters* 2018;45:6124–33.
5. Mildrexler DJ, Zhao M, and Running SW. Where Are the Hottest Spots on Earth? *Eos, Transactions American Geophysical Union* 2006;87:461.
6. Azarderakhsh M, Prakash S, Zhao Y, and AghaKouchak A. Satellite-Based Analysis of Extreme Land Surface Temperatures and Diurnal Variability Across the Hottest Place on Earth. *IEEE Geoscience and Remote Sensing Letters* 2020;17:2025–9.