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Abstract: Erosion can fundamentally alter landslide acceleration through the produced momentum. However, existing analytical landslide velocity models do not account for the energy dissipation associated with erosion-induced system acceleration. Here, I present a novel, complete dynamical velocity model for erosive landslides that incorporates two intrinsically coupled mechanically essential processes: erosion-induced acceleration and mobility booster through the net momentum production. This coupling is established analytically by formulating a simple equation for landslide velocity with erosion and deriving its elegant exact solution. I demonstrate that these two processes jointly govern the mobility of erosive landslides. My model applies to both non-deformable and deformable landslides, a key advantage, as erosive landslides are inherently deformable. The newly introduced booster velocity, defined as the ratio of erosion-induced acceleration to the mobility booster rate, provides the first precise mechanical criterion for determining landslide mobility: the lower the booster velocity, the higher the landslide mobility, a phenomenal understanding. This criterion incorporates all governing mechanisms: erosion rate, energy generator, and resulting acceleration. Remarkably, this result becomes possible only when erosion-induced acceleration is considered. However, contrary to prevailing perceptions, erosion-rate alone is insufficient to determine landslide mobility, irrespective of its magnitude. The model ascertains that accurate simulation of landslide dynamics requires accounting for erosion-induced reductions in frictional energy dissipation and inertia, alongside system acceleration, as these mechanisms collectively control landslide mobility. Neglecting erosion-induced reduced-inertia leads to physically inconsistent predictions and can substantially underestimate landslide mobility, potentially resulting in catastrophic consequences for people and infrastructure in landslide-prone mountain slopes and valleys.

1 Introduction

Erosion is one of the most challenging and influential processes in the simulation of hazardous geophysical mass flows, including landslides, avalanches, and debris flows, as it substantially affects flow dynamics, sediment transport, and the evolution of mountain landscapes (Huggel et al., 2005; Santi et al., 2008; Evans et al., 2009; de Haas et al., 2020). As these flows propagate downslope, they interact with the underlying bed material and entrain sediment when the shear stress exerted by the moving mass exceeds the bed's resisting shear strength. This process, known as sediment entrainment, increases the total flow volume, sometimes by several orders of magnitude, thereby amplifying the potential for destruction (Theule et al., 2015; Cuomo et al., 2016; Somos-Valenzuela et al., 2016; Mergili et al., 2018; Liu et al., 2019). This erosion-induced bulking can significantly alter key flow characteristics, including velocity, flow depth, runout distance, inundation extent, impact forces, and deposition patterns (Dowling and Santi, 2014; Mergili et al., 2020; Pudasaini, 2025).

The landslide mobility is measured with respect to the travel distance or the run-out which in turn is mechanically controlled by the landslide velocity. This means the mobility of the erosive landslide is enhanced if its velocity is higher than the reference velocity without erosion. So, I formally define the mobility enhancement of an erosive landslide as follows: The mobility of the erosive landslide is enhanced if its velocity is higher than

the reference velocity of the landslide without erosion.

However, although the incorporation of additional bed material increases the mass of the moving landslide and may enhance its mobility, the resulting change in velocity depends on the coupled mechanical effects of entrainment, particularly erosion-induced net momentum production and changes in flow inertia. Consequently, erosion can lead to increased or decreased flow velocity. Among the various characteristics of erosive landslides, mobility is particularly important. High mobility is commonly manifested in long runout distances, extensive inundation areas, and large impact forces, which can substantially increase risks to human populations and infrastructure. Therefore, erosion and associated flow bulking in landslide-prone areas and debris-flow channels are major concerns for geoscientists, civil and environmental engineers, and land-use planners (Huggel et al., 2005; Evans et al., 2009; Li et al., 2019; Mergili et al., 2020; Liu et al., 2021; Zhu et al., 2026).

In recent decades, research on erosion and sediment entrainment in catastrophic mountain mass flows has expanded considerably, leading to the development of numerous models to describe these processes (Egashira et al., 2001; Fraccarollo and Capart, 2002; McDougall and Hungr, 2005; Le and Pitman, 2009; Christen et al., 2010; Iverson and Ouyang, 2015; Frank et al., 2016). However, many existing models are not physics-based and do not explicitly account for the net momentum production associated with erosion, thereby limiting their mechanical consistency in describing the dynamics of erosive mass flows. To address this limitation, Pudasaini and Fischer (2020) developed a process-based mechanical erosion model that explicitly incorporates erosion-induced momentum production into the momentum balance equation. Their formulation demonstrated that the effective reduction in frictional resistance associated with erosion is mechanically equivalent to the momentum production resulting from sediment entrainment. This finding provides a fundamental explanation for how erosion may enhance mass-flow mobility. By incorporating the mechanical effects of erosion directly into the governing momentum balance, the model established a physically consistent foundation for the mathematical description and dynamic analysis of erosive mass flows.

Building on the framework developed by Pudasaini and Fischer (2020), Pudasaini and Krautblatter (2021) investigated the fundamental role of erosion in controlling landslide mobility by deriving explicit mechanical criteria for mobility changes as a function of the erosion velocity of the mobilized bed material. Their model provides a mechanistic explanation of the conditions under which erosion enhances or reduces landslide mobility by consistently accounting for the effects of reduced inertia and erosion-induced net momentum production. This formulation enables the quantitative assessment of erosion-driven changes in landslide mobility within a physically consistent mechanical framework. Subsequently, building on these developments, Pudasaini (2025) established a unified, mechanically consistent, and comprehensive erosion model for multiphase mass flows, integrating the relevant physical mechanisms governing erosion and momentum exchange among the constituent phases. Collectively, these physics-based models (Pudasaini and Fischer, 2020; Pudasaini and Krautblatter, 2021; Pudasaini, 2025) provide a rigorous theoretical foundation for physically meaningful simulations of erosive landslide dynamics. In particular, they highlight the necessity of explicitly incorporating mechanically derived erosion rates and the associated net momentum production into the governing equations to accurately capture the evolution and mobility of erosive mass flows. The significance of this mechanical framework for multiphase mass flows is further demonstrated by its application to the simulation of catastrophic real-world events (Pudasaini and Mergili, 2019; Qiao et al., 2019; Shen et al., 2019; Liu and He, 2020; Mergili et al., 2020; Liu et al., 2021; Zhu et al., 2026). These applications underscore the importance of mechanically consistent erosion formulations for improving the physical representation and predictive capability of numerical models of complex geophysical mass flows.

However, existing analytical models of landslide velocity neglect the energy dissipation associated with erosion-induced system acceleration. Here, I present a novel dynamical velocity model for erosive landslides that explicitly accounts for this acceleration and couples it with erosion-induced net momentum production. The model demonstrates how these two intrinsically coupled processes jointly govern landslide mobility and cannot be treated independently. By incorporating both mechanisms within a unified analytical framework, the proposed model provides a more comprehensive and mechanically consistent description of erosive landslide dynamics, which is essential for ensuring the physical validity of analytical models for erosive landslide mobility.

2 The model construction

For ease of reference, assume a flow in steady-state. Then, suppose that instantaneously erosion takes place. This introduces the erosion induced net momentum production resulting in immediate and simultaneous emergence of temporal acceleration. These dynamics cannot be isolated. Considering one, while ignoring the other is mechanically inconsistent. All these aspects are realized here for the first time controlling the mobility of landslide with erosion. This is the main essence that will be clearer in the sequel.

Let t [s] is time, x [m] the distance along the slope, h [m] is the flow depth, v [ms^{-1}] and u [ms^{-1}] are the mean flow velocity without erosion and with erosion, α the net driving force [ms^{-2}], β [m^{-1}] the viscous drag coefficient, E [s^{-1}] is the erosion rate per unit flow depth, u^b is the erosion velocity and $u^b = \lambda^b u$, where $\lambda^b \in (0, 1)$ is the erosion drift, and $\Lambda^b = (2\lambda^b - 1)$ is the energy generator (Pudasaini and Fischer, 2020; Pudasaini and Krautblatter, 2021).

First, consider the reference landslide motion down a channel in steady-state (Pudasaini, 2026):

$$v \frac{\partial v}{\partial x} = \alpha - \beta v^2. \quad (1)$$

Next, assume that instantaneously erosion takes place. Then, because of erosion, the system begins to accelerate in time at each location, and (1) expands into (Pudasaini and Krautblatter, 2021)

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \alpha - \beta u^2 + \Lambda^b E u. \quad (2)$$

So, the erosion velocity u^b (via λ^b) is a measure of gain in temporal acceleration in relation to the erosion rate E that together introduces the acceleration $\partial u / \partial t$, which, however, was ignored in Pudasaini and Krautblatter (2021).

2.1 The complete velocity of erosive landslide

Now, write (2) as:

$$u \frac{\partial u}{\partial x} = \alpha - \beta u^2 + \left(\Lambda^b E u - \frac{\partial u}{\partial t} \right). \quad (3)$$

Assume that the erosion mechanism is such that the system accelerates in a constant rate, i.e., $\partial u / \partial t = \alpha_e$, where the subscript $_e$ indicates the quantity associated with erosion. More on this is elaborated later. Then, (3) becomes:

$$u \frac{\partial u}{\partial x} = \alpha - \beta u^2 + \left(\Lambda^b E u - \alpha_e \right). \quad (4)$$

This can be written equivalently in terms of u as:

$$u \frac{\partial u}{\partial x} = (\alpha - \alpha_e) + \left(\Lambda^b E \right) u - \beta u^2. \quad (5)$$

For convenience, introduce the notations: $A = (\alpha - \alpha_e)$, $B = (\Lambda^b E)$, $C = \beta$, and write (5) in compact form:

$$u \frac{\partial u}{\partial x} = A + B u - C u^2. \quad (6)$$

In (6), dynamically, A, B, C play the role of the reduced (because $A < \alpha$) net driving force (induced by the particle acceleration consuming energy supplied additionally by erosion induced net momentum production $\Lambda^b E u$), erosion-induced mobility booster rate (for $B > 0$) and the viscous drag, respectively. So, $\Lambda^b E u$ is the mobility booster (energy booster). This is a novel dynamical model for the erosive landslide. The model (6) is a direct extension of the erosive velocity presented in Pudasaini and Krautblatter (2021). I call (6) the dynamical model for the complete velocity of erosive landslide. It is called complete because it formally includes the mechanically essential component, the erosion-induced system acceleration α_e through A , that was ignored in Pudasaini and Krautblatter (2021) in the spatial description of landslide velocity. Structurally, equation of type (6), which is a non-linear ordinary differential equation of first order, is wide spread in science and technology, in modelling physical, engineering and environmental phenomena.

2.2 Exact-analytical solution to the velocity of erosive landslide

Analytical solution exists for (1), and takes the form (Pudasaini, 2026):

$$v = \sqrt{\frac{\alpha}{\beta} \left[1 - \left(1 - \frac{\beta}{\alpha} v_0^2 \right) \frac{1}{\exp(2\beta(x - x_0))} \right]}, \quad (7)$$

where $v = v_0$ at $x = x_0$. Following some standard mathematical procedures in solving non-linear ordinary differential equations, the model (6) can be solved analytically to yield an elegant exact solution:

$$\left(\frac{u_0 - p_1}{u - p_1} \right)^{p_1} \left(\frac{u - p_2}{u_0 - p_2} \right)^{p_2} = \exp[s(x - x_0)], \quad (8)$$

equivalently:

$$x = x_0 + \frac{1}{s} \ln \left| \left(\frac{u_0 - p_1}{u - p_1} \right)^{p_1} \left(\frac{u - p_2}{u_0 - p_2} \right)^{p_2} \right|, \quad (9)$$

where,

$$s = \sqrt{B^2 + 4AC}, \quad p_1 = \frac{B + s}{2C}, \quad p_2 = \frac{B - s}{2C}. \quad (10)$$

The solution (8) or (9) contains important mechanics of both the non-erosive and erosive landslides via the composite mechanical structures s, p_1, p_2 , and $u = u_0$ at $x = x_0$. When α_e is not a constant and varies with position, the situation becomes complex and a close form solution is less likely. For this reason, the analysis is restricted to (9) with constant α_e .

Whether the velocity with erosion given by (9) will increase or decrease relative to that without erosion given by (7) depends on the competition between the erosion related energy dissipation in A (because $A < \alpha$) and the mobility booster rate B (as B takes positive values). I analyze this below.

2.3 Mechanical conditions for mobility enhancement

To establish simple, quantitative, mechanical conditions when the erosive landslide will have enhanced mobility, we need to compare the velocity of this system with the system without erosion, i.e., compare u in (8) with v in (7). As defined earlier, this contrast in velocity in the measure of enhanced mobility. This means, the mobility is enhanced if $u > v$ and reduced if $u < v$.

2.3.1 Incipient dynamics

Since $u_0 = v_0$ for incipient motion around x_0 , we, respectively, have:

$$\left. \frac{dv}{dx} \right|_{x_0} = \frac{\alpha - \beta v_0^2}{u_0}, \quad (11)$$

and

$$\left. \frac{du}{dx} \right|_{x_0} = \frac{A + Bu_0 - Cu_0^2}{u_0}, \quad (12)$$

So, their difference is

$$\left(\frac{du}{dx} - \frac{dv}{dx} \right) \Big|_{x_0} = \frac{Bu_0 - \alpha_e}{u_0}. \quad (13)$$

If u rises faster than v in the close vicinity of x_0 , for the erosion-induced enhanced velocity, (13) must be a positive quantity. With the definition of $B = \Lambda^b E$, this results in the incipient condition for $u > v$:

$$\frac{\alpha_e}{\Lambda^b E} < u_0. \quad (14)$$

This effectively means, erosion must start at substantially higher velocity u_0 . I call u_0 the erosive transition velocity such that the inequality (14) holds. This is a novel perspective for erosive landslide. Interestingly, if erosion starts at low transition velocity with relatively high erosion rate and erosion velocity (through Λ^b), then, erosion induced acceleration can be relatively high. In other words, if erosion starts at high transition velocity with relatively low erosion rate and erosion velocity, then erosion induced acceleration can be relatively low. However, there are several possibilities.

2.3.2 Long-time dynamics as system approaches equilibrium

Now, consider the long time equilibrium states for both the systems, without erosion and with erosion. Then, as in the limit the velocity gradients vanish, we respectively have:

$$\frac{\alpha - \beta v_m^2}{u_m} \approx 0, \quad (15)$$

and

$$\frac{A + Bu - Cu_m^2}{u_m} \approx 0, \quad (16)$$

where the subscript $_m$ indicates the maximum velocity of the system. Equivalently,

$$v_m = \sqrt{\frac{\alpha}{\beta}}, \quad (17)$$

and

$$u_m = \frac{B + \sqrt{B^2 + 4AC}}{2C}. \quad (18)$$

So, the long-time condition requires that, if globally u surpasses v all the way to the asymptotic limit, $u_m > v_m$ is a must, leading to the condition:

$$\sqrt{B^2 + 4AC} > 2\sqrt{\alpha C} - B, \quad (19)$$

resulting from the combination of (17) and (18).

Simplifications:

The general condition (19) can be simplified in terms of α_e and $\Lambda^b E$ containing all the information associated with erosion mechanics in relation to the system limit without erosion, i.e., $\sqrt{\alpha/\beta}$.

I. If $2\sqrt{\alpha C} - B > 0$

For this condition, after some simple calculations with (19), results in

$$\frac{\alpha_e}{\Lambda^b E} < \sqrt{\frac{\alpha}{\beta}}. \quad (20)$$

So, the precise mechanical conditions are given by $\alpha_e/(\Lambda^b E)$, all involving erosion, that is constrained by $\sqrt{\alpha/\beta}$ and u_0 , both not involving erosion. This is appreciable.

The condition (20), together with (14), has the great physical implication providing the first clear and complete quantitative mechanical criterion for erosion-induced enhanced landslide mobility involving all the mechanisms governing the system: the erosion rate, energy generator and the produced acceleration, and the transition velocity. This is astonishing as it was made possible only by considering the erosion induced produced system acceleration. To further elaborate its importance, introduce the notation for the quantity on the left side of (20) as:

$$S_p = \frac{\alpha_e}{\Lambda^b E}. \quad (21)$$

It has the dimension of velocity. I call S_p the erosion-induced booster velocity as the ratio between the erosion-induced system acceleration and the mobility booster rate. This single mechanical construct (entity)

completely governs the erosive landslide dynamics: the lower the booster velocity the higher the landslide mobility. The booster velocity provides a relative measure of the energy dissipated in accelerating the system following erosion-induced net momentum production.

II. If $2\sqrt{\alpha C} - B < 0$

Since $2\sqrt{\alpha C} - B < 0$, (19) always holds provided $B^2 + 4AC > 0$. This leads to

$$\frac{\alpha_e}{\Lambda^b E} < \frac{\alpha}{\Lambda^b E} + \frac{\Lambda^b E}{4\beta}. \quad (22)$$

Therefore, the natural global conditions are: minimum of (14) and (20), or (14) and (22). However, (22) may be less likely situation in reality.

2.3.3 The time and spatial domains

Earlier we assumed a controlled erosion process such that the temporal acceleration caused by erosion induced net momentum production can be approximated by a constant rate, i.e., the variation in the erosion induced acceleration is negligible. This means $\partial u / \partial t \approx \alpha_e$, and thus, $u \approx \alpha_e T_p$, for some time, say T_p , until when the system can cope with such induced acceleration. However, there must be a limit in T_p , because the acceleration cannot increase unbounded. This can be constrained by demanding that the momentum balance (5) is identically satisfied by $u = \alpha_e T_p$. After some calculations, this yields:

$$T_p = \frac{1}{2\beta\alpha_e} \left[\Lambda^b E \pm \sqrt{(\Lambda^b E)^2 + 4\beta(\alpha - 2\alpha_e)} \right]. \quad (23)$$

For sufficiently high α , $(\Lambda^b E)^2 + 4\beta(\alpha - 2\alpha_e) > 0$ and larger than $\Lambda^b E$. So, generally, the positive branch of (23) defines the physically valid time domain:

$$T_p = \frac{1}{2\beta\alpha_e} \left[\Lambda^b E + \sqrt{(\Lambda^b E)^2 + 4\beta(\alpha - 2\alpha_e)} \right]. \quad (24)$$

However, if $\alpha < 2\alpha_e$ then the negative branch of (23) could also describe T_p .

As β is a very small positive number and α_e is also a small positive number, T_p can be significantly large. This means, T_p covers a large time domain to capture the erosion process in realistic flow situations. This is practically important.

Since $\partial x / \partial t = u = \alpha_e T_p$, with (24), the corresponding travel distance X_p can be obtained:

$$X_p = x_0 + \frac{1}{2}\alpha_e T_p^2. \quad (25)$$

The time and spatial domains provided in (24) and (25) contain all the information on the mechanisms without and with erosion.

3 Results

3.1 Enhanced mobility

Erosion often enhances the landslide mobility, a natural phenomenon. The competition between the energy added into the system through the produced net momentum ($\Lambda^b E u$) and the energy consumed in accelerating the system (α_e), both with erosion, in contrast to non-erosive situation determines whether the landslide mobility with erosion will be enhanced or reduced. So, how the system determines the enhanced mobility depends on the relative actions of λ^b through B , and α_e through A in (6). This is what is proven here. For this, following Pudasaini and Krautblatter (2021), the parameters are chosen reasonably as: $\alpha = 7.0$,

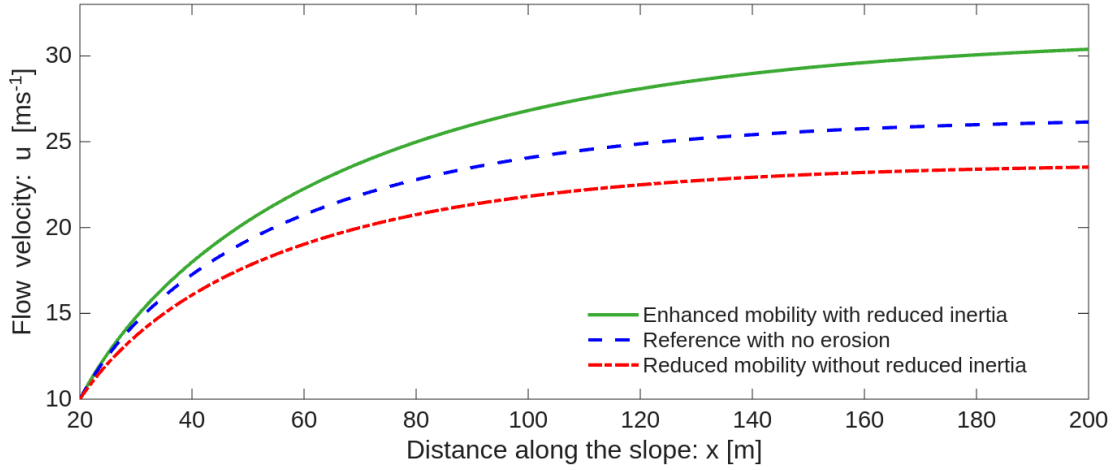


Figure 1: Mechanically enhanced or unphysically reduced mobility of an erosive landslide as given by the solution (9) of the dynamical velocity model (6) with respect to the reference state without erosion. Erosion begins at $x_0 = 20$ m with the transition velocity of $u_0 = 10 \text{ ms}^{-1}$.

$\beta = 0.01, \lambda^b = 0.9, E = 0.15, \alpha_e = 1.0$, and $u_0 = 10 \text{ ms}^{-1}$. These parameters are considered consistently, because, the high erosion rate $E = 0.15$ is associated with the very high erosion velocity (through λ^b : as $\lambda^b \rightarrow 1$). This suggests that $B = \Lambda^b E$ is a positive (and relatively) high quantity. This means, the erosion supplies significant additional energy to the landslide. This also indicates that the erosion induced acceleration $\alpha_e > 0$ is a significant quantity. Since $B = 0.12$ and the transition velocity is 10, α_e around 1.0 (constrained by the formal mechanics in (14) and (20)) is a legitimate selection, because the erosive system dissipates energy while accelerating the bed materials proportional to the energy produced by the net momentum gain, but this must be below the gained energy. Otherwise, the situation becomes unphysical, because, no energy is available to produce further access acceleration. Thus, for the considered situation of a highly erosive landslide, it is natural to perceive that, most likely, the mobility will be significantly enhanced. I essentially ascertain that both the erosion-induced reduced frictional energy dissipation (Pudasaini and Fischer, 2020) and the reduced inertia (Pudasaini and Krautblatter, 2021) in Λ^b , and the induced acceleration (α_e : considered here), all must be regarded in mechanically meaningfully simulating erosive landslides. This further strengthens the physical fact discovered in Pudasaini and Krautblatter (2021) on the mobility of erosive landslides.

Figure 1 presents the results as given by the solution (9) of the complete dynamical velocity model for erosive landslide (6) for three different situations: erosion-induced enhanced mobility with reduced inertia, reference without erosion, and unphysically reduced mobility due to illegitimate neglection of erosion-induced reduced inertia. As compared with no erosion reference, erosion with proper consideration of the reduced inertia largely enhances the landslide mobility, as an expected natural phenomenon.

Although it is unphysical to do so (Pudasaini and Krautblatter, 2021), momentarily assume that one wishes to neglect the reduced inertia in erosion. Denote this by $*$. Then, this forces Λ^b to be $\Lambda_*^b = (\lambda^b - 1)$, which is always a negative quantity, because $\lambda^b \in (0, 1)$. But then, as $\lambda^b \rightarrow 1$, $\Lambda_*^b \rightarrow 0$, and thus, $B_* \rightarrow 0$, both from negative side. Seemingly the flow decelerates negligibly. This means, one may think to take $\alpha_{e*} < 0$ as a negligibly small quantity. Consequently, even for a highly and effortlessly mobilizable bed material (because $\lambda^b \rightarrow 1$ and $E > 0$ are large quantities), virtually this does not effect the dynamics of the heavily erosive landslide. This is not anticipated, and contradicts the natural phenomena of erosive landslides. This meaningless situation is not displayed in Fig. 1.

There is another even worse contradiction associated with ignoring the erosion-induced reduced inertia. Because of the highly erosive situation ($\lambda^b \rightarrow 1$ and $E > 0$), in reality, the physical system demands that the erosion-induced acceleration α_{e*} must be a significantly high positive quantity, say 1.0 as considered first, not that as α_{e*} , a negligibly small negative quantity. The solution as presented in Fig. 1 with $\alpha_{e*} = 1.0$ and $\Lambda_*^b = (\lambda^b - 1)$

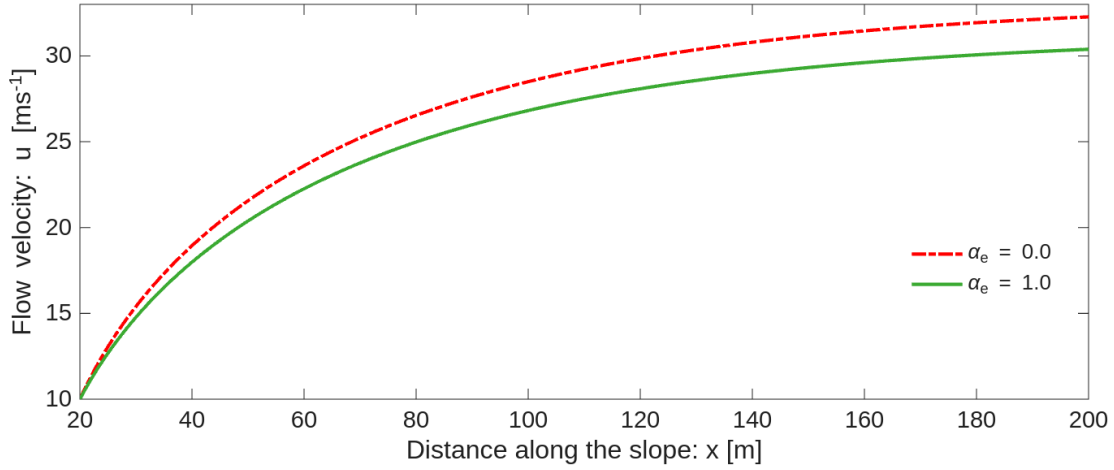


Figure 2: Acceleration control on the enhanced mobility for different values of α_e where $\alpha_e = 1.0$ corresponds to the enhanced mobility curve in Fig. 1. While $\alpha_e = 0.0$ does not dissipate any energy in accelerating particles in erosion this situation is unphysical.

without considering reduced inertia in erosion clearly manifests a mechanically highly contradictory situation the one that is not observable in nature. This contradiction stems from the fact that the illegitimately forced Λ_*^b unphysically ignored the erosion-induced reduced inertia. Such a neglect can result in catastrophic consequences while developing hazard mitigation and zoning, and early warning in landslide prone mountain slopes and valleys, as such, the mobility is dramatically reduced artificially. This is so, because, the landslide velocity is the measure of its mobility, and the destructive impact pressure is proportional to the square of velocity. This clearly demonstrates the catastrophic consequences if one attempts to unphysically ignore the essence of reduced inertia in simulating erosive landslide dynamics.

3.2 Acceleration control on enhanced mobility

The landslide velocity model with erosion (6) includes the induced acceleration, namely α_e through A . This is a must, because, as explained earlier, the erosion phenomenon alters the dynamical state of landslide. Assuming no change in system acceleration while entraining mechanically significantly weaker bed material is physically inconsistent. So, it is desirably to analyze how the induced system acceleration α_e controls the dynamics of erosive mass transport. For this, consider $E = 0.15$, $\lambda^b = 0.9$ (enhanced mobility), and take different values for α_e , $\alpha_e = \{0.0, 1.0\}$. This means, no energy consumed in acceleration and energy consumed as in Fig. 1 (enhanced mobility curve). Results are displayed in Fig. 2. When no energy is dissipated in accelerating the particles as such acceleration is switched off ($\alpha_e = 0.0$) the flow mobility increases significantly. However, this is not physically meaningful because the system must consume some energy in erosion while accelerating the particle. This indicates that (the spatial description of) the landslide velocity with erosion in Pudasaini and Krautblatter (2021) is incomplete as they did not include the energy dissipation due to erosion-induced system acceleration.

One can similarly show that neglect of erosion-induced reduced inertia also results in unphysical results for $\lambda^b < 1/2$. The results presented in Fig. 1 - Fig. 2 highlight the mechanical strength and essence of the proposed principle on how erosion-induced momentum production and system acceleration determine the landslide mobility. Yet, which value of $\alpha_e > 0$ corresponds the reality should be further studied experimentally, which, however, is out of scope here.

4 Discussion

The advancements I made in this study is twofold. First, it formally establishes erosion-induced acceleration as an intrinsic component of erosive landslide dynamics that cannot be ignored. Second, it demonstrates that

accounting for the erosion-induced reduction in inertia is essential to a mechanically consistent description of enhanced landslide mobility.

Building on the erosion-induced mobility enhancement mechanisms associated with reduced friction and inertia identified by Pudasaini and Krautblatter (2021), this study developed a novel exact analytical model that elucidates how erosion-induced net momentum production and system acceleration jointly govern landslide mobility. For the first time, their inherent coupling is demonstrated analytically through a simple dynamical equation for landslide velocity and its exact solution, explicitly incorporating the essential mechanical effects of erosion. The analysis formally establishes that erosion-induced net momentum production triggers and governs the temporal acceleration of the landslide system. As erosion occurs, the associated momentum production gives rise to acceleration, linking these two processes through an intrinsic mechanical relationship. Consequently, momentum production and system acceleration cannot be treated as independent mechanisms: accounting for one while neglecting the other (as in existing models) is mechanically inconsistent.

Unlike existing analytical velocity models, the proposed framework applies to both non-deformable and deformable landslide masses, making it particularly relevant to erosive landslide dynamics. The results demonstrate that the erosion rate alone does not determine the resulting change in landslide mobility. Rather, mobility emerges from the combined effects of erosion-induced net momentum production (that accounts for the erosion rate and the energy generator) and the system acceleration, with the accompanying reduction in inertia playing an essential role in consistently describing the system’s dynamical response. Furthermore, the model establishes an analytical criterion for determining whether erosion enhances or reduces landslide mobility under explicit mechanical conditions through the booster velocity. By integrating these coupled effects within a unified analytical framework, this study provides a more comprehensive and mechanically consistent description of erosive landslide dynamics and clarifies the fundamental role of erosion in governing landslide acceleration and mobility.

5 Summary

As the dominant erosion process significantly alters the dynamical state of landslide, inclusion of the induced acceleration is a must. Because of the highly erosive situation in reality, the physical system suggests that the erosion-induced acceleration can be significant. However, the existing analytical landslide velocity models with erosion are incomplete as they do not include the energy dissipation due to erosion-induced system acceleration. Here, I presented the first complete dynamical velocity model for the erosive landslide as it formally combines the two mechanically essential components, the erosion-induced system acceleration together with the mobility booster. The model and its solution contain important mechanics of erosive landslide via some composite mechanical structures. The newly constructed erosion-induced booster velocity as the ratio between the erosion-induced system acceleration and the mobility booster rate, both involving erosion, offers the precise mechanical condition for the enhanced mobility. The booster velocity, constrained by the formal mechanics, completely governs the erosive landslide dynamics: the lower the booster velocity the higher the landslide mobility. This provides the first clear and complete quantitative mechanical criterion for erosion-induced enhanced landslide mobility involving all the mechanisms governing the system: the erosion rate, energy generator, produced acceleration, and the transition velocity. This is astonishing as it was made possible only by considering the erosion induced produced system acceleration. The new model essentially ascertains the physical fact that the erosion-induced reduced frictional energy dissipation, reduced inertia, and the induced acceleration, all must be accounted for in mechanically meaningfully simulating erosive landslides. Results show that erosion with proper consideration of the reduced inertia largely enhances the landslide mobility, as expected, highlighting the mechanical essence of the proposed principle on how erosion-induced momentum production and system acceleration determine the landslide mobility. However, neglecting the reduction in inertia due to erosion leads to a clear contradiction. Such a neglect can result in catastrophic consequences while developing hazard mitigation and zoning, and early warning in landslide prone mountain slopes and valleys.

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