

# **Geodetic and gravimetric constraints on regional glacier mass changes in High Mountain Asia**

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## Abstract

Regional glacier assessments often require mass-change estimates over periods that differ from the intervals measured geodetically. We quantify how native cumulative-mass adjustments and within-interval variation contribute to the regional estimates obtained by adding GRACE. A linear joint inversion represents glacier, lake, non-glacier and exterior mass across High Mountain Asia. Geodetic-only, gravity-only and joint estimates share source geometry, auxiliary observations and temporal regularization. Evaluation uses excluded CryoSat contributor records for complete RGI regions 13 to 15 over January 2011 to January 2015 and January 2015 to January 2019. Joint regional-period mass-rate RMSE is 4.37 Gt yr<sup>-1</sup>, compared with 8.01 for geodesy and 14.80 for gravity. Adding gravity improves interperiod estimates relative to geodesy, while geodetic constraints mainly reduce the gravity-only mean offset. Under a fixed linear-bridge decomposition, improvement is associated with adjustments to cumulative mass at the native geodetic endpoints. Adding only this component to the geodetic history gives a diagnostic RMSE of 3.43 Gt yr<sup>-1</sup>; adding only within-interval variation gives 8.51. The joint solution also has substantial geodetic residuals and poorer results in missing-period and local glacier evaluations. The regional gain reflects reconciliation of cumulative mass constraints under the specified decomposition. Evaluating external-period rates alongside native cumulative budgets distinguishes this improvement from recovery of temporal variation within the original geodetic intervals.

Keywords: glacier mass balance; GRACE; geodetic observations; High Mountain Asia; native observation support; joint inversion

## 1 Introduction

Glacier mass change in High Mountain Asia (HMA) varies across mountain ranges and observation periods. Early satellite assessments identified contrasting behaviour across the Himalaya, Karakoram, Pamir and neighbouring ranges (Gardelle et al., 2013; Kääb et al., 2012; Yao et al., 2012). Subsequent elevation records documented widespread loss alongside persistent regional differences (Brun et al., 2017; Shean et al., 2020). Long-term decline in the Tien Shan and accelerating loss across the Himalaya further demonstrate that a regional mass budget depends on its temporal extent as well as its geographic boundary (Farinotti et al., 2015; Maurer et al., 2019). Both the accumulated mass change and its evolution are therefore relevant to assessing glacier imbalance and its consequences for mountain water resources (Miles et al., 2021).

The growing satellite record allows glacier mass estimates to be compared across large regions and reconciled over common assessment periods (Gardner et al., 2013; The GlacMBIE Team, 2025; Zemp et al., 2019). The contributing observations span different intervals. Geodetic observations constrain cumulative change between acquisition dates. Rates over periods with intermediate endpoints require

the intervening mass history. Successive five-year measurements still leave that history incompletely determined. Annual estimates, or multiyear estimates with different endpoints, consequently require additional observations or assumptions about temporal evolution. This distinction between cumulative mass and its temporal distribution also underlies the calibration of glaciological records against geodetic measurements (Zemp et al., 2013).

Optical elevation differencing provides extensive spatial coverage and is central to regional and global glacier mass assessments (Hugonnet et al., 2021a; Shean et al., 2020). Temporal variation between acquisition dates remains unresolved by spatial detail alone. Reconstructions with different within-interval trajectories can reproduce the same cumulative mass change. Regional uncertainty also depends on spatially correlated elevation errors, data gaps and the conversion from volume to mass (Hugonnet et al., 2022; Huss, 2013; McNabb et al., 2019; Rolstad et al., 2009). Reconstruction therefore requires observation operators that retain the original intervals and distinguish measured cumulative change from the estimated intervening trajectory.

Satellite gravimetry provides complementary temporal information. GRACE and GRACE Follow-On repeatedly observe changes in the gravity field associated with surface mass redistribution (Landerer et al., 2020; Tapley et al., 2004; Wahr et al., 1998). Their observations have supported glacier mass estimates globally and across HMA (Jacob et al., 2012; Wouters et al., 2019; Yi and Sun, 2014). Repeated gravity measurements provide temporal constraints beyond interval-integrated geodetic measurements. The spatial gravity response extends beyond glacier outlines and depends on spectral truncation, source geometry and regularization (Landerer and Swenson, 2012; Swenson and Wahr, 2002; Wahr et al., 2006). Estimating glacier mass from these observations therefore requires separating its contribution from other varying mass sources.

This separation is particularly relevant to HMA and the Tibetan Plateau, where lake gains coexist with glacier loss and their gravity responses overlap (Zhang et al., 2017, 2013). Observed lake growth cannot generally be attributed to glacier mass loss alone (Brun et al., 2020; Ke et al., 2022); precipitation and the broader basin water balance also affect regional storage (Treichler et al., 2019; Zhang et al., 2021). A gravimetric glacier estimate depends on concentrated lake loads, distributed land-water changes and mass outside the immediate study region. Geodetic and lake observations help distinguish these contributions, while the representation of the surrounding mass field determines how the remaining gravity signal is allocated.

Several approaches combine these sources of information. Laser altimetry and gravity have been used to extend glacier records across the interval between ICESat missions (Wang et al., 2021). Annual glaciological anomalies have been transferred spatially and calibrated against long-term geodetic changes (Dussaillant et al., 2025). Component-based gravity inversions jointly represent glacier and hydrological sources over China and globally (Xiong et al., 2026, 2025), while observation-based lake corrections account for concentrated storage signals in GRACE products (Deggim et al., 2021). Recent HMA reconstruction has also combined machine learning with a mass-conservation adjustment (Wu et al., 2026). These studies demonstrate complementary uses of geodetic, gravimetric and hydrological observations. When several geodetic periods are already available, the source of improved agreement over different assessment periods remains unresolved: redistribution within the measured intervals or revision of their cumulative mass budgets.

Isolating these contributions requires comparable estimates with explicit observation constraints. Simultaneous changes to source geometry, regularization and fitted data obscure the contribution of each observing system. Regional mean rates and differences between periods measure distinct aspects of reconstruction accuracy. An estimate can reproduce the interperiod difference while retaining a substantial mean offset. Both errors can occur in regularized gravity inversions because neighbouring

sources have overlapping responses (Loomis et al., 2019; Luthcke et al., 2013; Schrama et al., 2014). A common mass representation isolates the effects of geodetic and gravimetric information on cumulative change and temporal variation.

External evaluation must match the spatial and temporal extent of the observations. CryoSat swath altimetry provides regional histories that resolve changes beyond a single multiyear mean (Gourmelen et al., 2018; Jakob et al., 2021; Jakob and Gourmelen, 2023). Optical stereo measurements and ICESat-2 provide complementary information over more restricted glacier footprints (Berthier et al., 2014; Falaschi et al., 2023a; Smith et al., 2019), and recent ICESat and ICESat-2 estimates extend the regional evidence (Hassan et al., 2026). The information these observations provide about reconstructed mass histories depends on acquisition dates, sampling and source-specific errors. Evaluation against regionalized altimetry and prediction of a withheld geodetic interval use different fitting observations. Comparing these tasks clarifies the observational conditions under which joint reconstruction is useful.

We reconstruct glacier mass changes across complete RGI regions 13 to 15 using a common linear representation of glacier, lake, non-glacier and exterior mass. Geodetic-only, gravity-only and joint estimates share geometry, auxiliary observations and temporal regularization. The principal comparison retains all four native five-year intervals in estimates that include geodetic observations. CryoSat contributor records excluded from fitting provide evaluation over two fixed four-year periods. We examine regional mass histories, separate mean-rate and interperiod errors, and decompose the joint adjustment into changes at the native geodetic endpoints and variation between them. Further comparisons assess reduced geodetic availability, local glacier footprints and prescribed uncertainty alternatives. Together, these analyses establish which regional estimates benefit from joint observations and distinguish changes to the measured cumulative budgets from changes to their temporal distribution.

## **2 Study region and observations**

### **2.1 Study region and spatial representation**

The study covers complete Randolph Glacier Inventory version 6 (RGI6) regions 13, 14 and 15: Central Asia, South Asia west and South Asia east (Pfeffer et al., 2014; RGI Consortium, 2017). Their glacier areas are 49,303.415, 33,568.298 and 14,734.204 km<sup>2</sup>, respectively. Together they contain 95,536 inventory objects and 97,605.917 km<sup>2</sup> of glacier area. This HMA domain extends beyond the Tibetan Plateau and matches the complete inventory regions represented by the external observations.

Fifteen second-order RGI regions define the glacier sources in the inversion. All model comparisons use identical source geometry. Within each source, mass change is proportional to glacier area, equivalent to a uniform specific mass change within that second-order region. Summing these sources gives the three first-order regional estimates. The estimates retain common regional boundaries; individual-glacier differences within each source remain unresolved. Figure 1 shows the glacier distribution, geodetic coverage and observation periods.

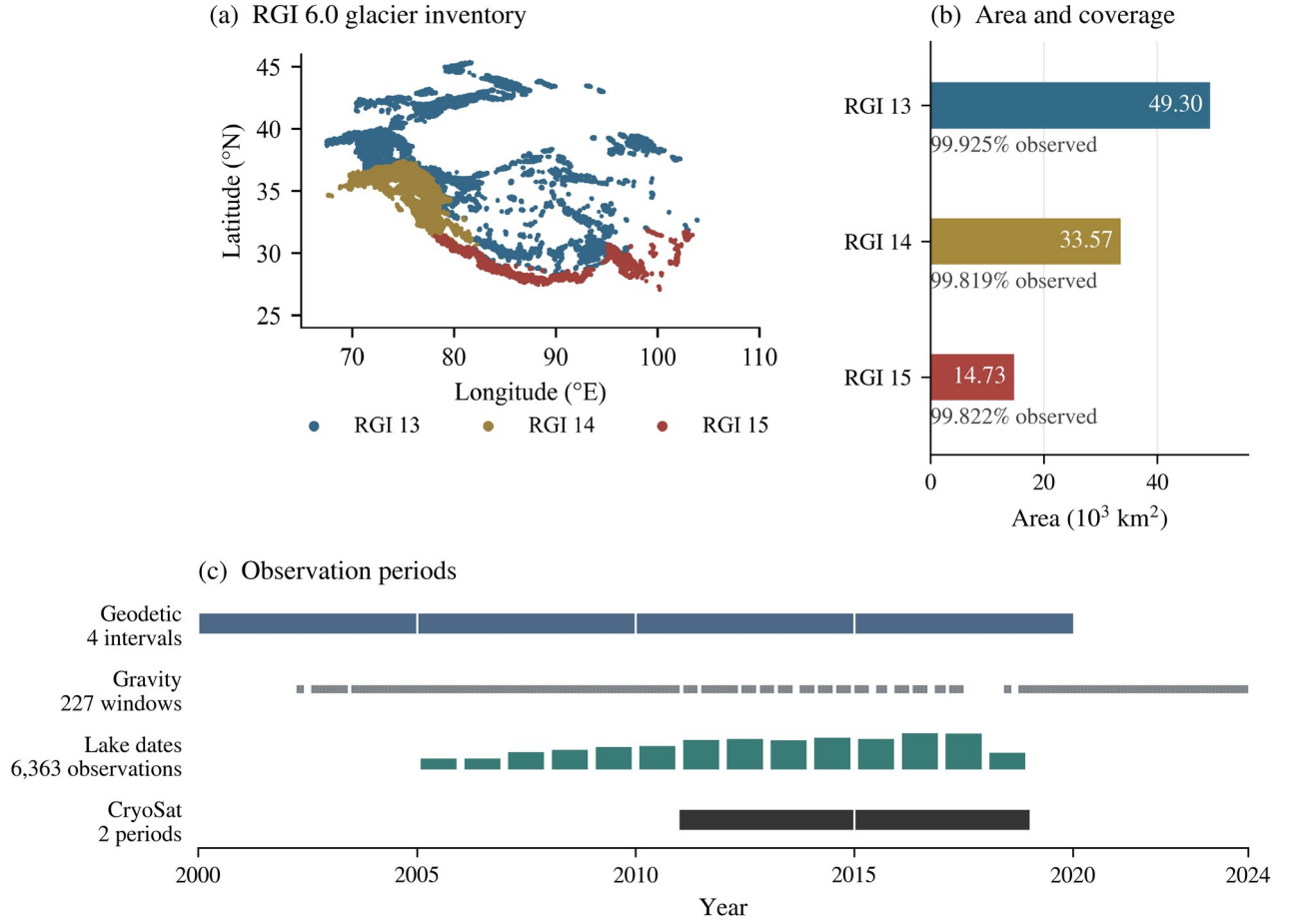


Figure 1. Spatial and temporal support of the observations. (a) The RGI 6.0 inventory map shows the three complete reporting regions: Central Asia (RGI 13), South Asia west (RGI 14) and South Asia east (RGI 15). (b) Full inventory area and the fraction observed in all four geodetic intervals. (c) Four native geodetic intervals, 227 gravity windows, annual lake-observation counts relative to their maximum, and the two CryoSat comparison periods. Storage observations cover 44 of the 46 lake geometries.

## 2.2 Multiperiod geodetic observations

The geodetic dataset provides four native five-year intervals: January 2000 to January 2005, January 2005 to January 2010, January 2010 to January 2015 and January 2015 to January 2020 (Hugonnet et al., 2021a, 2021b). We use the same cohort of glacier objects with valid observations in all four intervals. It covers 99.925%, 99.819% and 99.822% of the inventory areas in regions 13, 14 and 15, respectively. The common cohort holds observed area constant across periods, eliminating changes in spatial coverage from successive regional comparisons.

Published mass rates are multiplied by their actual interval durations to obtain cumulative changes, retaining the source dataset’s mass conversion. For each second-order region, the observed-area fraction relates the measured change to the full-inventory mass state. The small unobserved fraction is assigned the same specific mass change as the observed portion of that region. The observed cohort and full-inventory state retain their respective spatial supports. Section 3 describes the geodetic uncertainty and correlations between consecutive intervals. When all four intervals are fitted, their twenty-year sum is omitted as a redundant observation.

## 2.3 Gravimetric observations and temporal coverage

The gravity record comprises 227 official ITSG degree-60 solutions from April 2002 to December 2023, combining ITSG-Grace2018 with subsequent operational solutions from the same processing family (Behzadpour et al., 2021; Kvas et al., 2019; Mayer-Gürr et al., 2018). Start and end epochs are

taken from the SINEX metadata, and model predictions are averaged over these actual windows. Nominal months serve as solution identifiers; SINEX epochs determine the measurement durations. Missing windows, including the interval between the GRACE and GRACE Follow-On missions, remain unobserved.

The forward model and observations use the same spherical-harmonic coefficients. Degrees 0 and 1 and coefficients  $C_{20}$  and  $C_{30}$  are excluded; geocentre and satellite-laser-ranging replacements are therefore not used (Cheng et al., 2013; Loomis et al., 2020; Swenson et al., 2008). ICE-6G\_C trends are removed from the remaining coefficients (Peltier et al., 2015). Observations and predictions are referenced to the same available gravity windows in 2004 to 2009. The resulting spectral range determines the observable combinations of mass. Fine spatial integration represents source geometry within this fixed spectral range.

## **2.4 Lake observations and the surrounding mass field**

The model includes 46 lake geometries, of which 44 have 6,363 storage observations derived from altimetry, Landsat shorelines and hypsometric information for Tibetan lakes (Li et al., 2019a, 2019b, 2019c). We retain their acquisition dates and reported uncertainties. The two lakes without storage series are constrained by the remaining observations and priors. Fixed footprints are taken from the 2009 member of Lake\_TP (Zhou et al., 2024). For each observed lake, an unknown constant is estimated and removed to express storage changes independently of the arbitrary vertical or volume reference.

The prescribed land-water background is the sum of GLDAS-Noah 2.1 soil water in four layers, snow water equivalent and canopy interception over 2000 to 2023 (Beaudoing et al., 2020; Rodell et al., 2004). Monthly states are integrated over the actual gravity windows. This background represents part of terrestrial water storage; additional non-glacier states account for departures from it. Forty-five geographic tiles within 65 to 110° E and 24 to 46° N represent local non-glacier adjustments, while 36 exterior regions represent mass outside that rectangle. These lake, local and exterior sources are included in every comparison. The joint solution distributes gravity variations among glacier and surrounding mass according to the observational constraints and source priors.

## **2.5 External regional and glacier-footprint observations**

The principal external evaluation uses three Jakob/Gourmelen contributor series from GlaMBIE Dataset 1.0.0, released on 16 July 2024 (The GlaMBIE Team, 2024). The evaluation uses the individual submissions separately from the combined GlaMBIE estimate. The entire CryoSat source family, including previously derived radar targets, is excluded from fitting, parameter selection and covariance estimation. Evaluation covers January 2011 to January 2015 and January 2015 to January 2019. Their eight-year union summarizes the same observations over the combined interval.

The submission variable changes contains successive monthly elevation increments in metres. We sum 48 increments for each four-year period and convert the accumulated elevation change to mass using a density of 850 kg m<sup>-3</sup> (Huss, 2013). The original CryoSat processing uses 90-day moving medians at 30-day steps (Jakob et al., 2021); centred 90-day model averages at the period endpoints approximate this temporal averaging. The comparison measures accumulated change between the specified dates; the source publication reports a regression trend for July 2010 to July 2019. These regionalized records have incomplete direct radar coverage. Their shared error structure is considered in the uncertainty analysis.

Local evaluation uses 59 Pléiades records from 53 glaciers and 84 ATL06 version 007 patch records from eight glaciers (Falaschi et al., 2023b, 2023a; Neumann et al., 2019; Smith et al., 2025; Smith et al.,

2019). The western and eastern Pléiades elevation-difference fields come from one study; their seven 5 km blocks describe spatial grouping within that source family. The released fields contain gap-filled areas, and the original observation masks are unavailable. ATL06 comparisons use actual acquisition times and the original slope operators. All intervals cross January 2020, with none lying wholly beyond the geodetic record. The regional reconstruction is evaluated over these local footprints against continuation of each glacier’s latest geodetic rate. This evaluation is retrospective: related observations had been examined previously, and the evaluation records are excluded from the present fit.

*Table 1. Observations and their roles in the regional reconstruction. Dataset versions and persistent identifiers are given in Table S1. Counts refer to retained records or represented sources; their statistical dependence follows the observation design.*

| Observation                 | Spatial support                                          | Temporal support                              | Role                                     |
|-----------------------------|----------------------------------------------------------|-----------------------------------------------|------------------------------------------|
| Multiperiod geodesy         | 15 glacier groups; 60 source-period values               | Four native five-year intervals, 2000 to 2020 | Cumulative glacier mass constraint       |
| GRACE and GRACE-FO          | 3,715 retained harmonic modes; 227 solutions             | Actual observation windows, 2002 to 2023      | Joint mass constraint                    |
| Lake storage                | 44 observed lakes among 46 geometries; 6,363 records     | Actual acquisition dates                      | Concentrated non-glacier mass constraint |
| GLDAS-Noah 2.1              | 45 local non-glacier tiles                               | Monthly, 2000 to 2023                         | Prescribed partial-storage background    |
| CryoSat contributor records | Three complete RGI regions                               | 2011 to 2015 and 2015 to 2019                 | Excluded regional evaluation             |
| Pléiades / ATL06            | 59 records from 53 glaciers / 84 patches from 8 glaciers | Actual footprint intervals crossing 2020      | Excluded local evaluation                |

### 3 Methods

#### 3.1 Common representation of cumulative mass

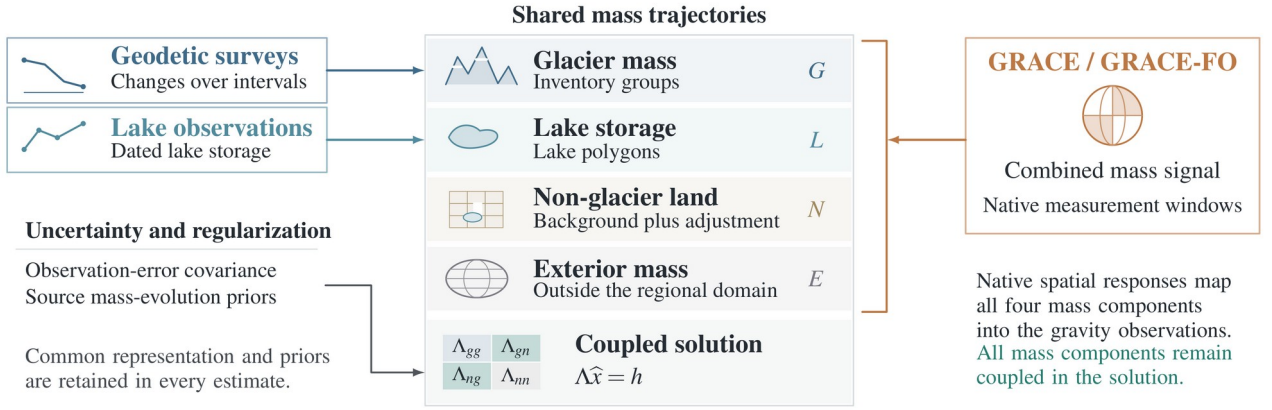
The model comprises 142 mass sources: 15 glacier regions, 46 lakes, 45 local non-glacier regions and 36 exterior regions. For source  $s$ , cumulative mass  $m_s(t)$  is expressed in Gt relative to 1 January 2000. Twenty-four piecewise-linear annual basis functions describe the subsequent knots through January 2024, and two harmonics represent the annual cycle. With  $\tau$  denoting elapsed time in 365.25-day years,

$$\begin{aligned}
 m_s(t) &= b(t)^T x_s, \\
 b(t) &= [\phi_1(t), \dots, \phi_{24}(t), \sin(2\pi\tau), \cos(2\pi\tau) - 1]^T.
 \end{aligned} \tag{1}$$

Omitting the reference-knot coefficient and shifting the cosine term constrain each trajectory to zero at the reference date. The annual hat functions vary linearly in elapsed time between their calendar dates. The coefficient matrix  $X$  has 142 rows and 26 columns, with row  $s$  equal to  $x_s^T$ ;  $x$  stacks these rows into a 3,692-element vector. Within each second-order inventory region, glacier mass is distributed in proportion to area. Summation to the three first-order regions is therefore exact for the specified geometry. The same temporal basis represents all mass components continuously (Figures 2 and 3).

The observation operators evaluate this continuous representation over their respective dates and intervals. Monthly estimates before the first gravity window, during missing windows and after the geodetic record are determined by the available observations and temporal prior. Monthly values between observations are model estimates of the mass trajectory.

### (a) Observational constraints on shared mass trajectories



### (b) Common model, different information

|                      | Geodesy<br>$D_0$ | Gravity<br>$G_0$ | Joint<br>$DG_0$ |
|----------------------|------------------|------------------|-----------------|
| Geodetic information | ●                | ○                | ●               |
| Gravity information  | ○                | ●                | ●               |

● included ○ omitted

Same lake observations, source geometry and priors

### (c) Matched external evaluation

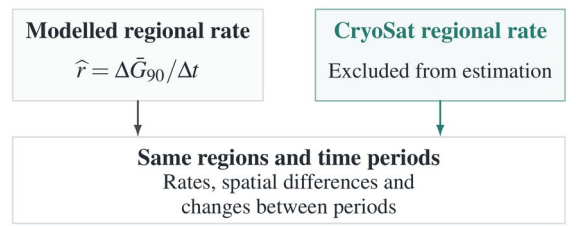


Figure 2. Observation constraints and comparative design of the joint mass reconstruction. (a) Geodetic interval changes constrain glacier mass, and dated lake observations constrain lake storage after removal of an unknown offset. GRACE and GRACE-FO respond to combined glacier, lake, non-glacier land and exterior loads through native spatial and temporal operators. Observation-error covariance and source priors enter the full normal system, whose off-diagonal terms couple glacier and surrounding mass. Symbols denote source classes schematically. (b) Geodetic, gravity and joint estimates use identical source geometry, lake information and priors. The auxiliary-only P estimate excludes both geodetic and gravity information. (c) Model and CryoSat rates use matched regions and periods. Model rates are differences of centred 90-day endpoint means divided by elapsed time. CryoSat records are withheld from fitting, parameter selection and error-model estimation.

## 3.2 Native observation operators

For glacier source  $s$  and native geodetic interval  $p$ , the mass change measured over the observed cohort is related to the full-inventory state by

$$y_{D,s,p} = f_s [b(t_{p,1}) - b(t_{p,0})]^T x_s + \epsilon_{D,s,p}. \quad (2)$$

Here  $f_s$  is the observed-area fraction, fixed across the four native intervals. Multiplying each rate by its interval duration in 365.25-day years gives the measured mass change. The complete geodetic operator  $H_D$  has 60 source-period rows. Its four intervals are fitted without their redundant twenty-year sum. In the missing-period experiments, one linear transformation selects the available intervals and total and is applied consistently to the observations, design matrix and covariance.

Lake observations constrain cumulative mass at their acquisition dates, subject to an unknown lake-specific reference offset. Eliminating this constant with covariance  $\Sigma_L$  gives the precision matrix

$$P_L = \Sigma_L^{-1} - \Sigma_L^{-1} \mathbf{1} (\mathbf{1}^T \Sigma_L^{-1} \mathbf{1})^{-1} \mathbf{1}^T \Sigma_L^{-1}. \quad (3)$$

The lake design matrix  $H_L$  contains rows  $b(t_j)^T$ . Weighting by  $P_L$  removes the unknown constant while retaining the observed storage variations. Forty-four lakes contribute observation rows; the other two lake geometries remain in the estimated mass field.

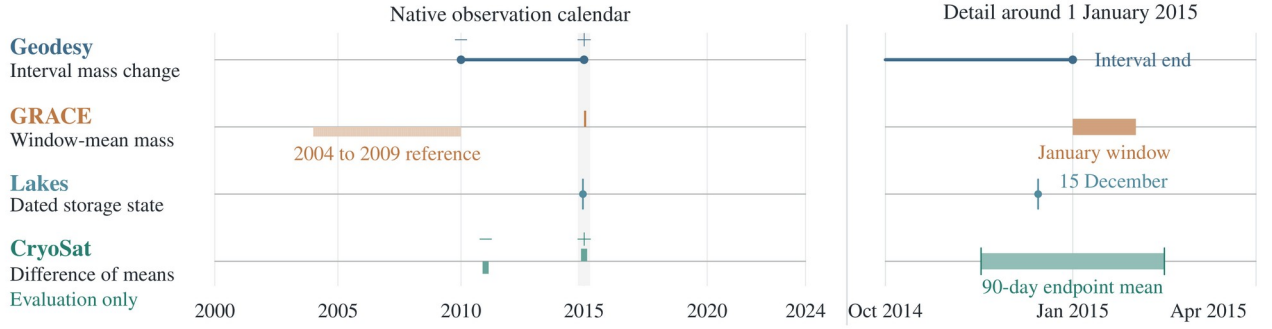
The gravity operator combines the temporal basis with the spatial loading matrix  $A$ , whose columns contain 3,715 spherical-harmonic responses per unit source mass. These responses incorporate elastic loading and degree-dependent Love numbers (Farrell, 1972; Gégout et al., 2010; Wahr et al., 1998). Across all observation windows, the predicted gravity coefficients are

$$\hat{Y}_G = T X^T A^T + F A_N^T. \quad (4)$$

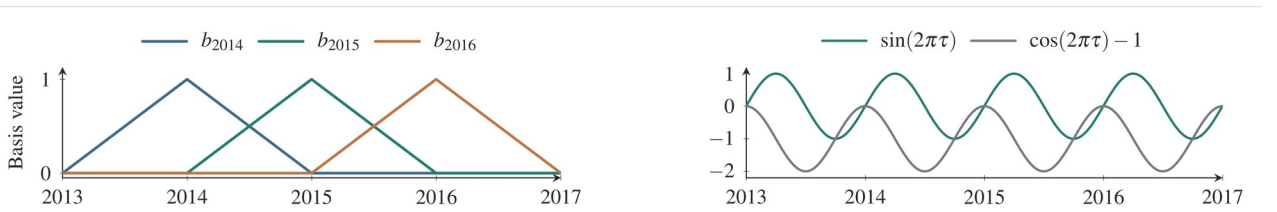
$T$  contains window-averaged basis functions relative to the available 2004 to 2009 reference-window mean.  $F$  contains the GLDAS background averaged and referenced in the same way, and  $A_N$  selects the local non-glacier responses. Basis integrals use 12-point Gauss-Legendre quadrature, with windows divided at annual knots. Observed and predicted GRACE coefficients have identical scaling and spectral limits. Degrees 0 and 1 and  $C_{20}$  and  $C_{30}$  are excluded from both; ICE-6G\_C trends are subtracted from the observations. Spatial integration on a fine grid represents the source geometry within the degree-60 response.

The prescribed GLDAS contribution is subtracted before estimation and restored when calculating total gravity predictions.  $N$  represents the estimated correction to this background. Lake and exterior mass retain distinct geometries and gravity contributions throughout the inversion.

**(a) Observation type and temporal support**



**(b) Temporal basis for cumulative mass**



**(c) Observation-specific temporal weights**

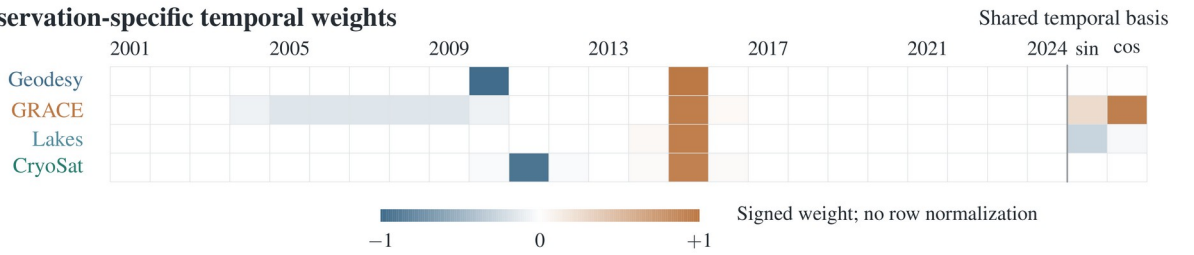


Figure 3. Temporal support of observations and regional rate calculation. (a) The 2010 to 2015 geodetic interval, January 2015 gravity window referenced to 2004 to 2009, lake acquisition on 15 December 2014, and 2011 to 2015 CryoSat comparison. The January 2015 enlargement distinguishes the interval endpoint, window average, acquisition date and centred 90-day mean. Signs indicate addition and subtraction. The CryoSat mean approximates the provider's temporal support for evaluation. (b) Three annual hat functions and both annual harmonics illustrate the continuous basis. (c) Exact temporal weights use a common colour scale without row normalization. The geodetic row includes coverage, gravity includes reference subtraction, and lake weights precede offset elimination. The CryoSat row gives a mass difference, divided by elapsed time for a rate. Spatial loading and aggregation are applied separately.

### 3.3 Observation error covariance

Geodetic errors are correlated across space and time. Let  $\sigma_{sbp}$  denote the mass-change uncertainty for source  $s$ , spatial block  $b$  and period  $p$ . Assuming coherent errors within  $2^\circ$  blocks defined by object centres gives

$$(\Sigma_D)_{sp,s'q} = \sum_b \sigma_{sbp} \sigma_{s'bq} R_{pq}, R_{pq} = \begin{cases} 1, & p=q, \\ -0.5, & |p-q|=1, \\ 0, & \text{otherwise.} \end{cases} \quad (5)$$

A relative diagonal term of  $10^{-8}$  stabilizes the covariance. Block uncertainties are converted from rate to mass units using the interval duration. This covariance approximation represents spatial error correlation and dependence between adjacent intervals sharing an endpoint, both relevant to elevation differencing (Hugonnet et al., 2022; Rolstad et al., 2009). Alternatives with independent periods or positive period correlations are evaluated separately.

Gravity covariance combines formal coefficient uncertainty with an empirical model of representation error.  $D_f$  contains the median formal standard deviations over fitted windows. Spatial least-squares residuals, standardized by  $D_f$ , are estimated from odd-year windows before 2020 without glacier or radar response data.  $U$  contains the twelve leading right singular vectors of these residuals, scaled by their singular values,  $\sqrt{0.8}$  and the inverse square root of the number of covariance-development windows. The coefficient covariance is

$$\Sigma_c = D_f \left[ \text{diag}(1 + 0.2 \overline{r^2}) + U U^T \right] D_f. \quad (6)$$

The overbar denotes the mean squared residual for each coefficient. This coefficient covariance,  $\Sigma_c$ , is repeated across conditionally independent gravity windows to form the full block covariance  $\Sigma_G$ . Correlated GRACE errors motivate this covariance structure, although it differs from decorrelation filtering (Kusche, 2007; Swenson and Wahr, 2006). For lakes, reported volume errors have a minimum standard deviation of  $0.005 / \sqrt{3}$  Gt. A mission-shared level uncertainty of 0.11 m, converted using lake area, represents an additional correlated error. These models specify the observation weights. Cross-system covariance remains unspecified because a complete estimate is unavailable.

For each lake and altimetry mission, the shared error is represented by a vector whose entries equal the level uncertainty multiplied by the reported water-surface area. The sum of these outer products represents correlations among observations from the same mission. Lake-specific offsets are removed after including this covariance, retaining the relative storage information among irregular acquisition dates.

### 3.4 Regularization and joint estimation

The glacier prior penalizes changes in annual mass increments and allows a broad initial rate. Define  $u_s = \max(A_s / 10^9, 0.005)$ , where  $A_s$  is source area in  $\text{m}^2$ , and let  $z_s$  contain the 24 annual coefficients.  $D_2$  is the second-difference matrix with the fixed reference column removed, and  $d_1$  selects the first increment. The penalty is

$$J_{Q,s} = \frac{\|D_2 z_s\|^2}{(a u_s)^2} + \frac{(d_1^T z_s)^2}{(2 u_s)^2} + \frac{10^{-9} \|z_s\|^2}{u_s^2} + \frac{x_{s,25}^2 + x_{s,26}^2}{(0.5 u_s)^2}. \quad (7)$$

The common annual-rate-change scale is  $a = 0.3$  m w.e.  $\text{yr}^{-1}$  per annual step. Lake, non-glacier and exterior annual coefficients have an amplitude penalty proportional to  $I + 0.25 D_2^T D_2$ , with respective

scales of  $10u_s$ ,  $0.4u_s$  and  $0.3u_s$  Gt. Their harmonic coefficients use the same amplitude scales. Together, the penalties define the prior precision  $Q$ .

For residuals  $r_k = y_k - H_k x - b_k$ , the indicators  $I_D$  and  $I_G$  specify whether geodetic and gravity observations are included. The objective function is

$$J(x) = I_D r_D^T \Sigma_D^{-1} r_D + I_G r_G^T \Sigma_G^{-1} r_G + r_L^T P_L r_L + x^T Q x. \quad (8)$$

The gravity quadratic form sums over native observation windows; lake and prior terms are common to every comparison. The covariance-weighted objective follows established inverse-problem formulations and continuous-time gravity mass estimation (Luthcke et al., 2013; Sabaka et al., 2010; Tarantola and Valette, 1982). The inversion estimates glacier and surrounding mass simultaneously and retains their coupled contributions to gravity observations.

Partition the normal precision  $\Lambda$  and right-hand side  $h$  into glacier coefficients  $g$  and the remaining coefficients  $n$ . Schur elimination of  $n$  gives

$$\begin{aligned} S &= \Lambda_{gg} - \Lambda_{gn} \Lambda_{nn}^{-1} \Lambda_{ng}, \\ \hat{x}_g &= S^{-1} (h_g - \Lambda_{gn} \Lambda_{nn}^{-1} h_n), \\ \hat{x}_n &= \Lambda_{nn}^{-1} (h_n - \Lambda_{ng} \hat{x}_g). \end{aligned} \quad (9)$$

The reduced system contains 390 glacier coefficients and is solved by Cholesky factorization. The eliminated coefficients are subsequently recovered from the same normal equations, retaining their covariance with glacier mass. External evaluation observations are excluded from the objective.

Area scaling gives the glacier prior the same specific-mass interpretation across regions of different sizes. Finite background and exterior penalties allow gravity variations to be distributed among sources. Schur elimination propagates uncertainty in the estimated surrounding mass and its covariance with glacier mass into the glacier solution.

*Table 2. State dimensions and fixed parameters of the common-parameter comparison. All observation combinations share these values; the separately selected estimates are specified in Table S2.*

| Quantity                       | Value                                  | Interpretation                                            |
|--------------------------------|----------------------------------------|-----------------------------------------------------------|
| Spatial sources                | 15 G; 46 L; 45 N; 36 E                 | 142 simultaneous mass sources                             |
| Time coefficients              | 24 annual knots + 2 harmonics          | 26 coefficients per source; 3,692 total                   |
| Glacier Schur system           | 390 coefficients                       | Coupling with other sources retained                      |
| Gravity response               | 3,715 modes; degree $\leq 60$          | Degree 0, 1, $C_{20}$ and $C_{30}$ omitted                |
| Annual increment-change scale  | 0.3 m w.e. yr <sup>-1</sup> per step   | Common glacier curvature prior                            |
| Initial-rate / harmonic scales | 2 m w.e. yr <sup>-1</sup> / 0.5 m w.e. | Broad initial change and annual cycle                     |
| Geodetic adjacent correlation  | -0.5                                   | Assumed correlation between intervals sharing an endpoint |
| Gravity residual covariance    | Rank 12 plus diagonal                  | Estimated before 2020 in odd years                        |
| External endpoint average      | 90 days                                | Identical rate calculation for all estimates              |

### 3.5 Observation combinations and parameter selection

The principal comparison fixes source geometry, auxiliary information, covariance assumptions and  $a=0.3$ .  $D_0$  includes all four geodetic periods,  $G_0$  includes gravity, and  $DG_0$  includes both. P uses only

auxiliary observations and priors.  $D_0$ -to- $DG_0$  measures the effect of adding gravity, while  $G_0$ -to- $DG_0$  measures the effect of adding geodesy within the same mass representation (Figure 2b; Table S2).

Separately selected estimates are denoted  $D^*$ ,  $G^*$  and  $DG^*$ . Candidate smoothers use  $a \in \{0.1, 0.3, 1.0\}$ ; direct geodetic candidates comprise piecewise interpolation, a constant rate and ridge trends with coefficients  $\{0.01, 1, 100\}$ . Geodetic interval-pair omissions select  $D^*$  and  $DG^*$  without external evaluation data. Full-history  $D^*$  selects ridge 0.01 and reproduces the observed interval rates;  $DG^*$  selects  $a=0.1$ .  $G^*$  selects  $a=1.0$  from gravity residuals in the withheld years 2012 and 2016. These estimates quantify the effect of parameter selection alongside the common-parameter comparison.

The missing-period experiment omits each of the six pairs of five-year intervals across all glacier sources, retaining two intervals and the known twenty-year total. The remaining observation operator has rank three; adding the omitted-period contrast raises its rank to four. Inner selection uses only the available observations. Errors across folds are dependent because their observation periods overlap. Sensitivity experiments around  $DG^*$  vary geodetic temporal correlation, non-glacier and exterior prior standard deviations by factors 0.5 and 2, and GIA correction. None is selected using the external evaluation.

### 3.6 External observation operators and error measures

CryoSat evaluation uses the excluded contributor series over the two fixed four-year periods. Predicted cumulative mass is averaged over 90-day windows centred at each endpoint. For region  $R$ ,

$$\hat{r}_{Rp} = \frac{\sum_{s \in R} [\bar{b}_{90}(t_{p,1}) - \bar{b}_{90}(t_{p,0})]^T \hat{x}_s}{\Delta t_p}, \hat{r}_{Rp}^{spec} = \frac{10^9 \hat{r}_{Rp}}{A_R}. \quad (10)$$

The first estimate is a mass rate in  $\text{Gt yr}^{-1}$ ; the second is a specific mass rate in  $\text{m w.e. yr}^{-1}$ . Observed elevation increments are summed over the same period and converted using  $850 \text{ kg m}^{-3}$  and the full inventory area (Huss, 2013; Jakob et al., 2021). Endpoint averaging approximates the original temporal smoothing; the radar-return weights are unavailable. RMSE is calculated from equally weighted regional mean squared errors. The aggregate HMA estimate is derived from the same three regions and is omitted from the metrics to avoid counting those observations twice.

For the two period rates, define the mean  $m = (r_1 + r_2)/2$  and interperiod difference  $d = r_2 - r_1$ . Their errors satisfy

$$\frac{e_1^2 + e_2^2}{2} = e_m^2 + \frac{e_d^2}{4}. \quad (11)$$

Spatial contrasts are defined as Central Asia minus west and west minus east. Mean rates, interperiod differences and spatial contrasts are transformations of the same observations. Radar-error propagation considers independent, rolling-overlap and coherent scenarios, shared density uncertainty and separate area sensitivity. These are conditional scenario ranges, without confidence-level calibration. Pléiades estimates use changes between acquisition dates; ATL06 estimates use the original acquisition-time slope operators. Aggregation retains the glacier and source-field hierarchy. All modern intervals cross 2020, and local continuation uses each glacier's latest geodetic rate.

Mass-rate metrics retain the influence of regional area, whereas specific-rate metrics give equal weight to regional area-normalized rates. The radar scenarios use a shared density standard deviation of  $60 \text{ kg m}^{-3}$ , density factors of 790 and  $910 \text{ kg m}^{-3}$ , and a separate  $\pm 10\%$  area conversion. Rolling-overlap

covariance decreases linearly to zero across three monthly steps. Coherent-error cancellation in period differences means that the fully coherent scenario does not necessarily give the largest uncertainty.

### 3.7 Endpoint decomposition and conditional uncertainty

We decompose the  $D_0$ -to- $DG_0$  glacier adjustment over 2000 to 2020 to distinguish changes at native geodetic endpoints from variation between them. Let  $I_5$  denote linear interpolation through the adjustment values at January 2000, 2005, 2010, 2015 and 2020. For each source,

$$\begin{aligned}\delta m_s(t) &= m_{s,DG_0}(t) - m_{s,D_0}(t), \\ e_s(t) &= I_5[\delta m_s](t), \\ w_s(t) &= \delta m_s(t) - e_s(t), H_D w = 0.\end{aligned}\tag{12}$$

The endpoint component changes cumulative mass at the native dates. The remainder vanishes at each endpoint, and  $w$  contains its coefficients in the common basis. Both components are evaluated with the same CryoSat operator. Adding either to  $D_0$  yields a diagnostic trajectory without refitting gravity or surrounding mass. The chosen linear bridge defines a diagnostic partition that is non-orthogonal under the external operator and has no unique physical interpretation.

Let  $\delta r_e$  and  $\delta r_w$  denote the endpoint and within-interval rate adjustments, respectively, and  $e_D$  the original  $D_0$  error. The reduction in MSE is

$$\Delta MSE = -2\overline{e_D \delta r_e} - \overline{(\delta r_e)^2} - 2\overline{e_D \delta r_w} - \overline{(\delta r_w)^2} - 2\overline{\delta r_e \delta r_w}.\tag{13}$$

The final term represents interaction between the two adjustments. This identity separates the individual and combined effects of the two adjustments. Seasonal and non-seasonal contributions to the within-interval adjustment are evaluated through the same period operator.

Conditional uncertainty is obtained from the normal-precision blocks. With  $B = -\Lambda_{nn}^{-1}\Lambda_{ng}$  and external observation operator  $K_{ext}$ ,

$$\begin{aligned}V_g &= S^{-1}, & V_{ng} &= B V_g, \\ V_n &= \Lambda_{nn}^{-1} + B V_g B^T, & V_{ext} &= K_{ext} V_g K_{ext}^T.\end{aligned}\tag{14}$$

These matrices quantify uncertainty and coupling among sources conditional on geometry, observation covariance and priors. Structural uncertainty and external sensor errors are excluded. Geodetic fit is assessed using covariance-weighted residual RMS and symmetric whitening. For an eigendecomposition  $\Sigma_D = V D V^T$ , the whitened residual is  $z = V D^{-1/2} V^T e$ . Its squared norm equals  $e^T \Sigma_D^{-1} e$  and is invariant to consistent row reordering. Sums of  $z_i^2$  by region or period describe contributions to the adopted quadratic form, although correlation mixes information across observations. Predictions are evaluated at the original observation dates and verified against the stored solutions before analysis.

## 4 Results

### 4.1 Regional mass histories and fit to geodetic observations

The three observation combinations yield different regional mass histories (Figure 4). With a common January 2000 reference, divergence between the trajectories represents differences in cumulative mass change. By January 2020, Central Asian changes are  $-192.66$  Gt for  $D_0$ ,  $-658.33$  Gt for  $G_0$  and  $-137.20$  Gt for  $DG_0$ . Geodetic constraints therefore substantially reduce the gravity-only loss. South Asia west changes by  $-90.97$ ,  $+35.86$  and  $-28.59$  Gt, respectively; South Asia east changes by  $-137.18$ ,

–113.63 and –145.03 Gt. Relative to geodesy alone, joint estimation reduces the estimated loss in Central Asia and the west while increasing it slightly in the east. These twenty-year cumulative changes characterize the fitted histories; the external altimetry comparisons cover shorter intervals.

The adjustments also vary through time. Joint and geodetic-only Central Asian histories differ by 1.64 Gt in January 2010 and 55.46 Gt in January 2020. In the west, the difference is already 30.78 Gt in January 2010 and reaches 62.38 Gt by January 2020. Eastern differences are –4.88 and –7.85 Gt at those endpoints. Adding gravity changes both the timing and regional distribution of cumulative loss, with adjustments that vary among regions and dates. Figure 4 includes the seasonal variation represented by the common basis. External evaluation covers multiyear changes, leaving individual monthly fluctuations unvalidated.

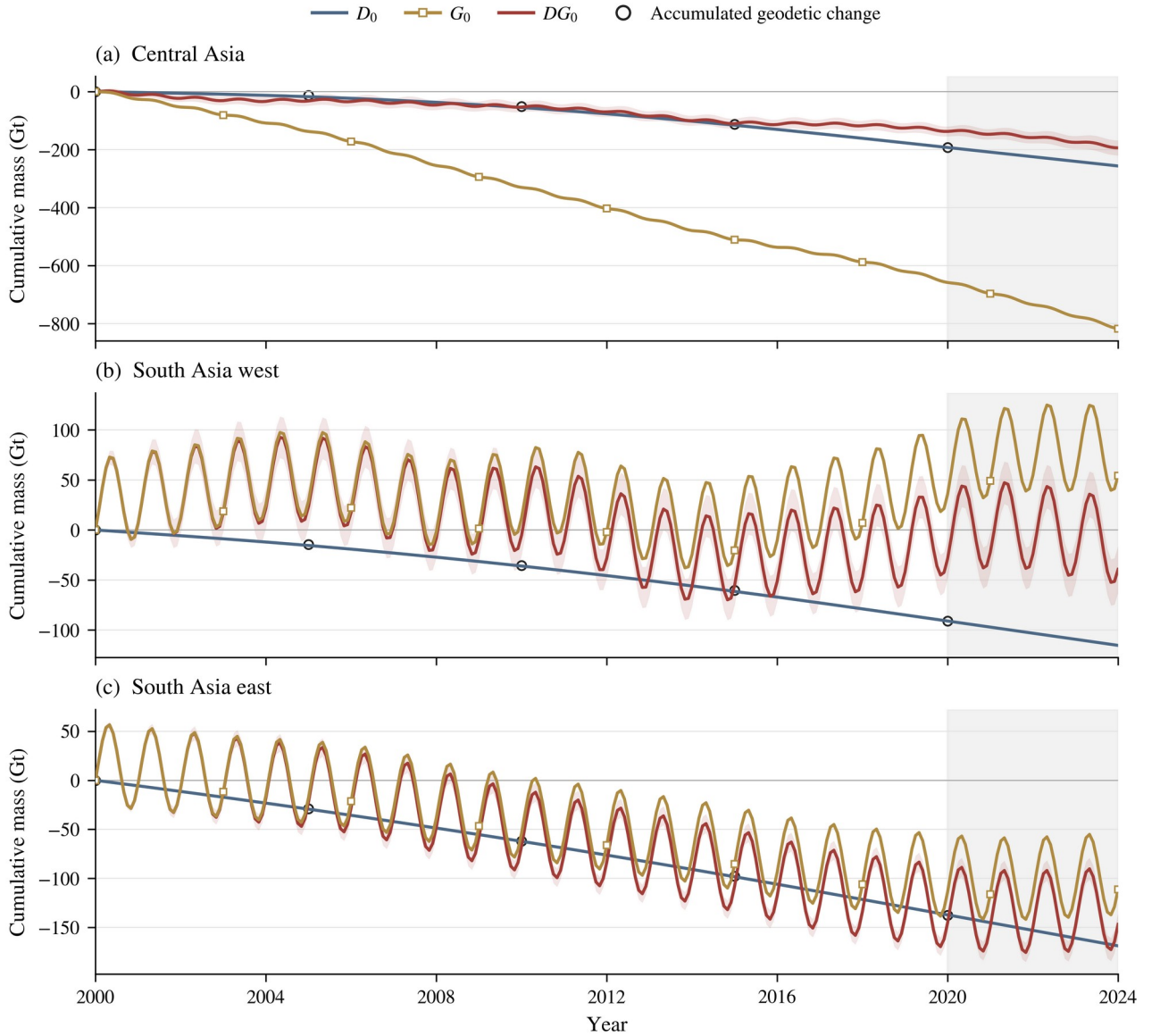


Figure 4. Regional cumulative glacier mass histories referenced to January 2000. Open circles accumulate native geodetic changes after applying the fixed coverage factors. Open squares identify  $G_0$  where curves overlap. Vertical scales differ among regions. The fitted annual harmonics describe monthly variation; external evaluation addresses multiyear changes. Shading around  $DG_0$  represents one conditional model standard deviation and excludes unresolved source and structural errors. Grey shading marks years after the final geodetic endpoint. Gravity observations extend through December 2023; estimates beyond the geodetic record are retrospective.

The joint adjustment changes the fit to the geodetic observations (Figure 5a,b; Table 3). With the full observation covariance, standardized residual RMS is 0.033 for  $D_0$ , 25.978 for  $G_0$  and 3.764 for  $DG_0$ .

Although all four native intervals enter  $D_0$  and  $DG_0$ , the joint estimate permits deviations from their measured changes. On the original observed-area support,  $DG_0$  rate errors for January 2015 to January 2020 are +10.557 Gt yr<sup>-1</sup> in Central Asia, +11.489 in the west and +0.501 in the east. Positive errors indicate less loss than observed geodetically. The equivalent full-inventory errors are slightly larger because of the coverage correction; fitted-residual comparisons below use the original observed areas.

Most of the correlated geodetic discrepancy occurs in the later intervals. Symmetric whitening assigns 42.5% of  $DG_0$ 's squared residual to 2010 to 2015 and 33.7% to 2015 to 2020, a combined 76.2%. These contributions depend on the specified covariance. Correlated source and period errors can give substantial quadratic contributions even where regional residuals partly cancel. Signed residuals and whitened contributions in the supplementary tables identify the intervals responsible for the geodetic discrepancy.

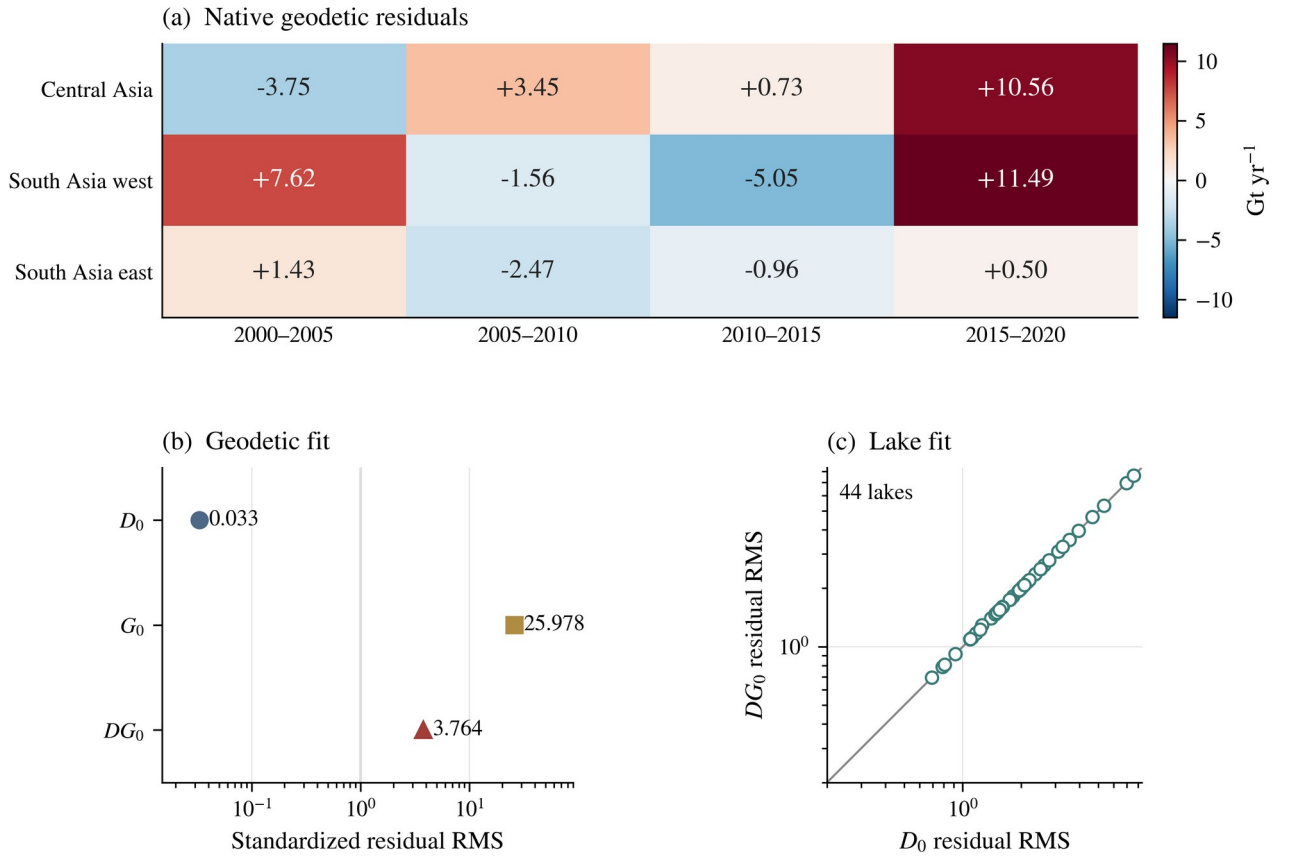


Figure 5. Fit to native geodetic and lake observations. (a)  $DG_0$  minus observed geodetic mass rates on the original observed-area support. (b) Standardized geodetic residual RMS under the full prescribed covariance; the vertical reference marks unity.  $G_0$  does not receive geodetic observations. (c) Standardized residual RMS for each of the 44 observed lakes, using native dates, mission covariance and fitted-offset removal. The solid diagonal denotes equal discrepancy. These diagnostics quantify agreement on native observation supports; external accuracy is evaluated separately.

Table 3. Native-observation residuals under common error metrics. Geodetic observations enter  $D_0$  and  $DG_0$ ; gravity enters  $G_0$  and  $DG_0$ ; lake data enter every estimate. The statistics quantify fit on the stated observation supports.

| Estimate | Geodetic RMS | Gravity RMS | Median lake RMS |
|----------|--------------|-------------|-----------------|
| $D_0$    | 0.033        | 1.222       | 1.750           |
| $G_0$    | 25.978       | 0.998       | 1.750           |
| $DG_0$   | 3.764        | 0.999       | 1.750           |

## 4.2 Gravity residuals and coupled mass adjustments

Including gravity observations reduces their prediction error (Figure 6). Under the same structured covariance, pooled standardized residual RMS is 1.222 for  $D_0$ , 0.998 for  $G_0$  and 0.999 for  $DG_0$ .  $G_0$  and  $DG_0$  therefore fit the gravity observations almost equally well despite their markedly different glacier histories. Geodetic information restricts the allocation of mass to glaciers while preserving this gravity agreement. The similar spectral residuals demonstrate the ambiguity of glacier mass allocation from gravity fit alone.

Normalization by formal coefficient errors changes the residual scale. Across degrees 2 to 20, RMS decreases from 26.86 for  $D_0$  to 24.20 for  $DG_0$ ; above degree 20 it remains close to 14.07 in both estimates. The approximately unit residuals in Figure 6a instead use the structured covariance, which includes empirical representation-error modes and a diagonal residual component. Consequently, its smaller standardized residuals reflect the adopted error model, not a corresponding reduction in measurement error.

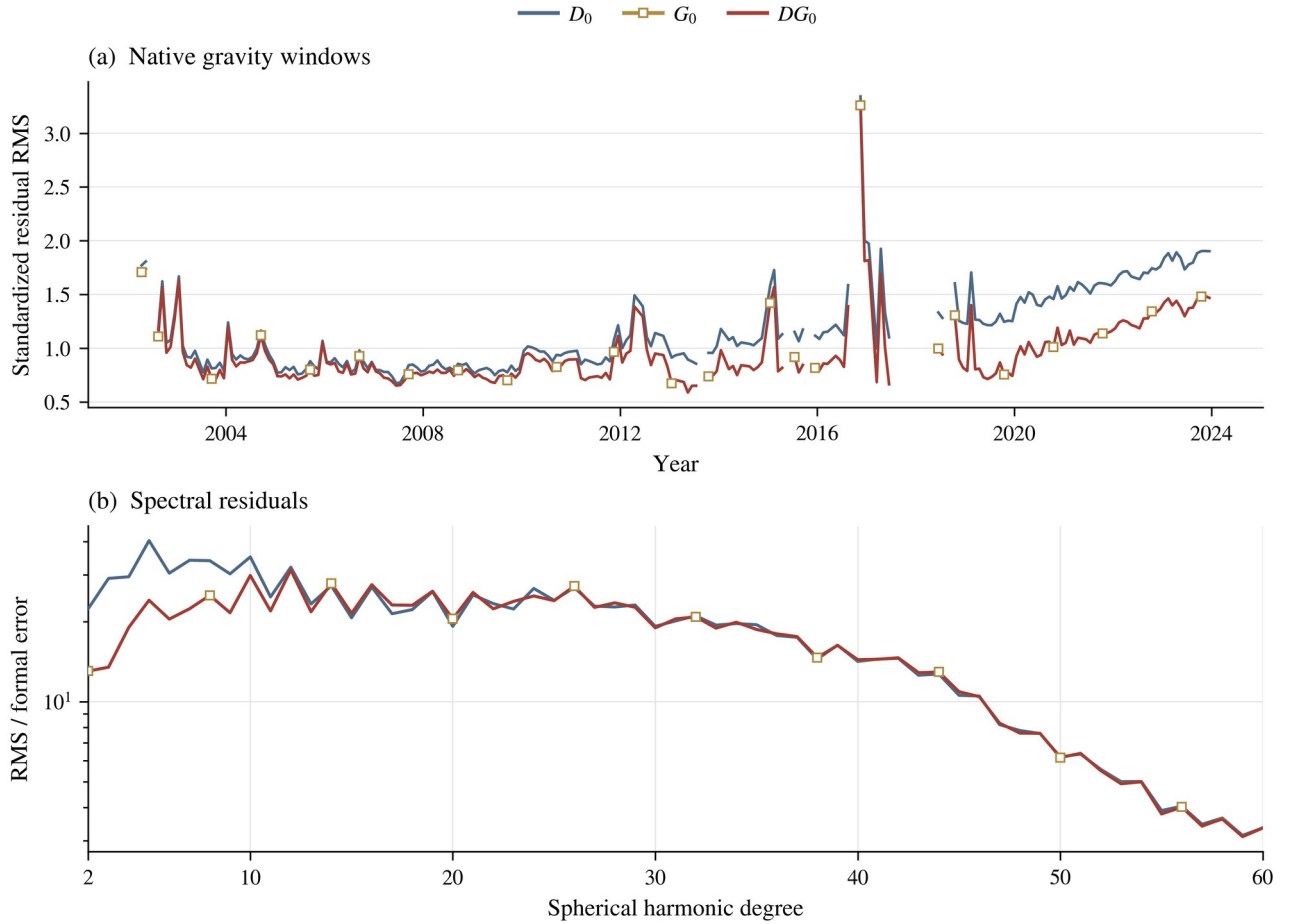


Figure 6. Gravity residuals on native time windows and spectral supports. (a) Window-specific residual RMS under the common structured error model; curves are separated across long gaps. (b) Residual RMS by retained spherical harmonic degree, scaled only by formal coefficient errors. Sparse squares distinguish  $G_0$  where it nearly overlaps  $DG_0$ . The distinct normalization in panel b reveals residual structure represented by the empirical covariance in panel a. Both residual calculations retain native gravity data and exclude external glacier observations.

Lake fits change little among observation combinations (Figure 5c). Across the 44 observed lakes, median mass RMSE after removal of the fitted constant is 0.118 Gt for  $D_0$  and 0.117 Gt for  $DG_0$ . Median standardized residual RMS is approximately 1.75 for all three estimates. The common lake observations continue to constrain storage variation at their acquisition dates while the joint solution

adjusts the wider mass field. The two lakes without storage observations are estimated from the remaining model terms and are excluded from these fit statistics.

Gravity alters all mass components. For January 2010 to January 2015,  $DG_0$  minus  $D_0$  changes the summed glacier, lake, local non-glacier and exterior rates by  $-5.09$ ,  $-7.00$ ,  $+64.77$  and  $+533.11$   $\text{Gt yr}^{-1}$ , respectively. For January 2015 to January 2020, the corresponding adjustments are  $+21.58$ ,  $+5.94$ ,  $-89.42$  and  $+322.95$   $\text{Gt yr}^{-1}$ . The components cover different spatial extents: non-glacier mass corrects the prescribed GLDAS background, and exterior mass spans regions outside the local rectangle. The sums quantify coupled adjustments over these different domains, rather than a closed HMA water budget. Omission of the gravity monopole also leaves global total mass unconstrained, so the exterior sum is a model adjustment.

#### 4.3 Regional estimates evaluated against CryoSat

Joint estimation gives the lowest pooled error against CryoSat observations withheld from fitting (Figure 7; Table 4). Across six regional-period values, mass-rate RMSE decreases from  $8.010$   $\text{Gt yr}^{-1}$  for  $D_0$  and  $14.798$  for  $G_0$  to  $4.373$  for  $DG_0$ . Equal-region specific-rate errors are  $0.2266$ ,  $0.3047$  and  $0.1241$  m w.e.  $\text{yr}^{-1}$ , respectively. The improvement therefore occurs in both mass and specific-mass units. The HMA aggregate provides regional context but is excluded from the metrics because it sums the same three constituent regions.

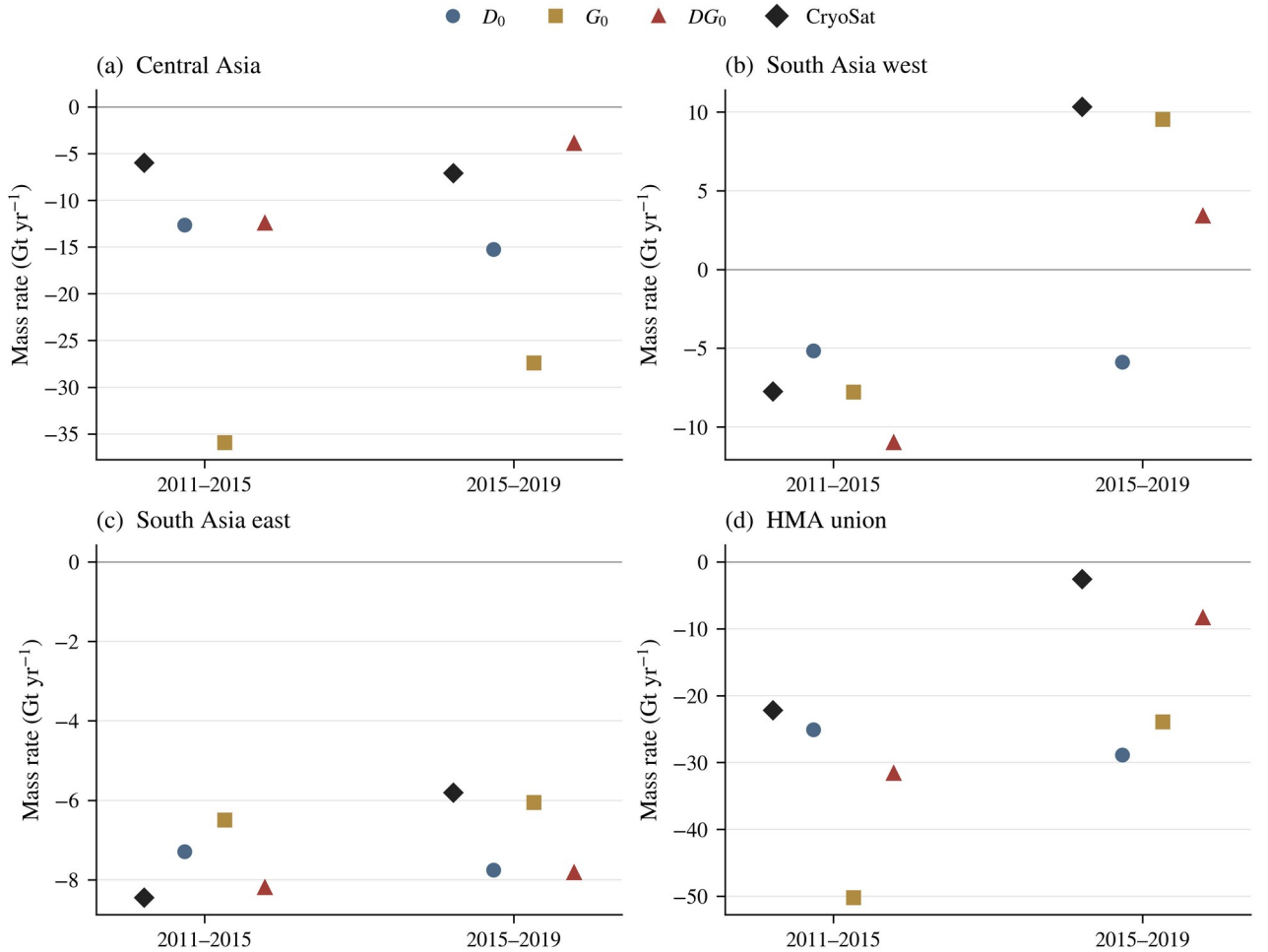


Figure 7. Regional glacier mass rates over the two fixed CryoSat periods. All model estimates use the same 90-day endpoint averaging operator and full inventory areas. Horizontal offsets separate estimates for the same period; vertical scales differ among panels. The HMA union is the dependent sum of the three regions and is excluded from equal-region error metrics. CryoSat symbols show point estimates. Shared source uncertainty is evaluated separately under the prescribed covariance scenarios.

Central Asia improves mainly in mean rate. CryoSat gives a two-period mean of  $-0.1328$  m w.e.  $\text{yr}^{-1}$ , compared with  $-0.2828$  for  $D_0$ ,  $-0.6425$  for  $G_0$  and  $-0.1659$  for  $DG_0$ . The joint mean is closest, with a large reduction in the negative gravity-only offset. For the interperiod difference, the CryoSat value is  $-0.0229$  m w.e.  $\text{yr}^{-1}$ , whereas both  $G_0$  and  $DG_0$  give approximately  $+0.173$  and  $D_0$  gives  $-0.0526$ . Joint estimation improves the average loss rate but retains the wrong direction of the smaller interperiod change.

Western South Asia shows the largest adjustment in interperiod change. CryoSat rates increase from  $-0.2310$  to  $+0.3077$  m w.e.  $\text{yr}^{-1}$ , an interperiod difference of  $+0.5387$ .  $D_0$  remains negative in both periods, with a difference of  $-0.0213$ .  $DG_0$  estimates  $-0.3277$  and  $+0.1010$ , recovering the direction and 79.6% of the observed amplitude.  $G_0$  is closer, with a difference of  $+0.5163$ , or 95.8% of the CryoSat amplitude. Its mean of  $+0.0261$  is also closer to the observed  $+0.0384$  than the  $DG_0$  mean of  $-0.1134$ . In this region, gravity alone best represents both the mean and the period difference.

Eastern South Asia shows weaker recovery of the interperiod change. CryoSat gives a mean of  $-0.4840$  and a difference of  $+0.1791$  m w.e.  $\text{yr}^{-1}$ . The respective  $D_0$  values are  $-0.5103$  and  $-0.0316$ ;  $DG_0$  gives  $-0.5435$  and  $+0.0249$ .  $DG_0$  recovers 13.9% of the observed amplitude while increasing the mean-rate error, yet its two-period MSE remains slightly below  $D_0$ . Across all regions,  $DG_0$  improves four of six individual rates and the two-period MSE in each region; the early western and late eastern rates deteriorate. The area-weighted HMA interperiod change reaches 118.8% of the CryoSat aggregate, showing that regional errors can yield an apparently strong aggregate response despite unresolved regional discrepancies.

Table 4. Common-parameter errors against excluded CryoSat records. The mass column is in  $\text{Gt yr}^{-1}$ ; other columns are in m w.e.  $\text{yr}^{-1}$ . Period, mean, change and contrast metrics use six, three, three and four dependent quantities, respectively. HMA is excluded.

| Estimate | Period mass | Period specific | Mean   | Interperiod | Spatial contrast |
|----------|-------------|-----------------|--------|-------------|------------------|
| $D_0$    | 8.010       | 0.2266          | 0.1464 | 0.3459      | 0.2596           |
| $G_0$    | 14.798      | 0.3047          | 0.2962 | 0.1426      | 0.3662           |
| $DG_0$   | 4.373       | 0.1241          | 0.0960 | 0.1573      | 0.1524           |

#### 4.4 Mean rate and interperiod error contributions

Mean and interperiod errors reveal different contributions from geodesy and gravity (Figure 8). Mean-rate RMSE is  $0.1464$  m w.e.  $\text{yr}^{-1}$  for  $D_0$ ,  $0.2962$  for  $G_0$  and  $0.0960$  for  $DG_0$ . Interperiod RMSE is  $0.3459$ ,  $0.1426$  and  $0.1573$ , respectively. Adding geodesy to gravity substantially improves regional means but slightly increases the error in period differences. Adding gravity to geodesy improves both pooled quantities. Their combined effect gives  $DG_0$  the lowest regional-period error, while  $G_0$  retains the lowest interperiod error.

Interperiod terms account for 66.0% of the  $D_0$ -to- $DG_0$  reduction in equal-region specific-rate MSE. For  $G_0$ -to- $DG_0$ , mean-rate terms account for 101.4% of the improvement, partly offset by a 1.4% interperiod penalty. These proportions follow the exact error identity for the six rates and quantify the contributions of their mean and interperiod errors. Section 4.5 examines whether those changes arise from revised mass at native geodetic endpoints or from variation within the native intervals.

Spatial-contrast RMSE decreases from  $0.2596$  m w.e.  $\text{yr}^{-1}$  for  $D_0$  and  $0.3662$  for  $G_0$  to  $0.1524$  for  $DG_0$ . Three of the four period-specific contrasts improve relative to each single-observation estimate. The exceptions differ: the early west-minus-east contrast deteriorates relative to  $D_0$ , and the late west-

minus-east contrast deteriorates relative to  $G_0$ . The eight-year Central-Asia-minus-west contrast also worsens relative to  $D_0$  and is reported separately from the period-specific metric. The lower pooled spatial error therefore coexists with deterioration in these individual regional contrasts.

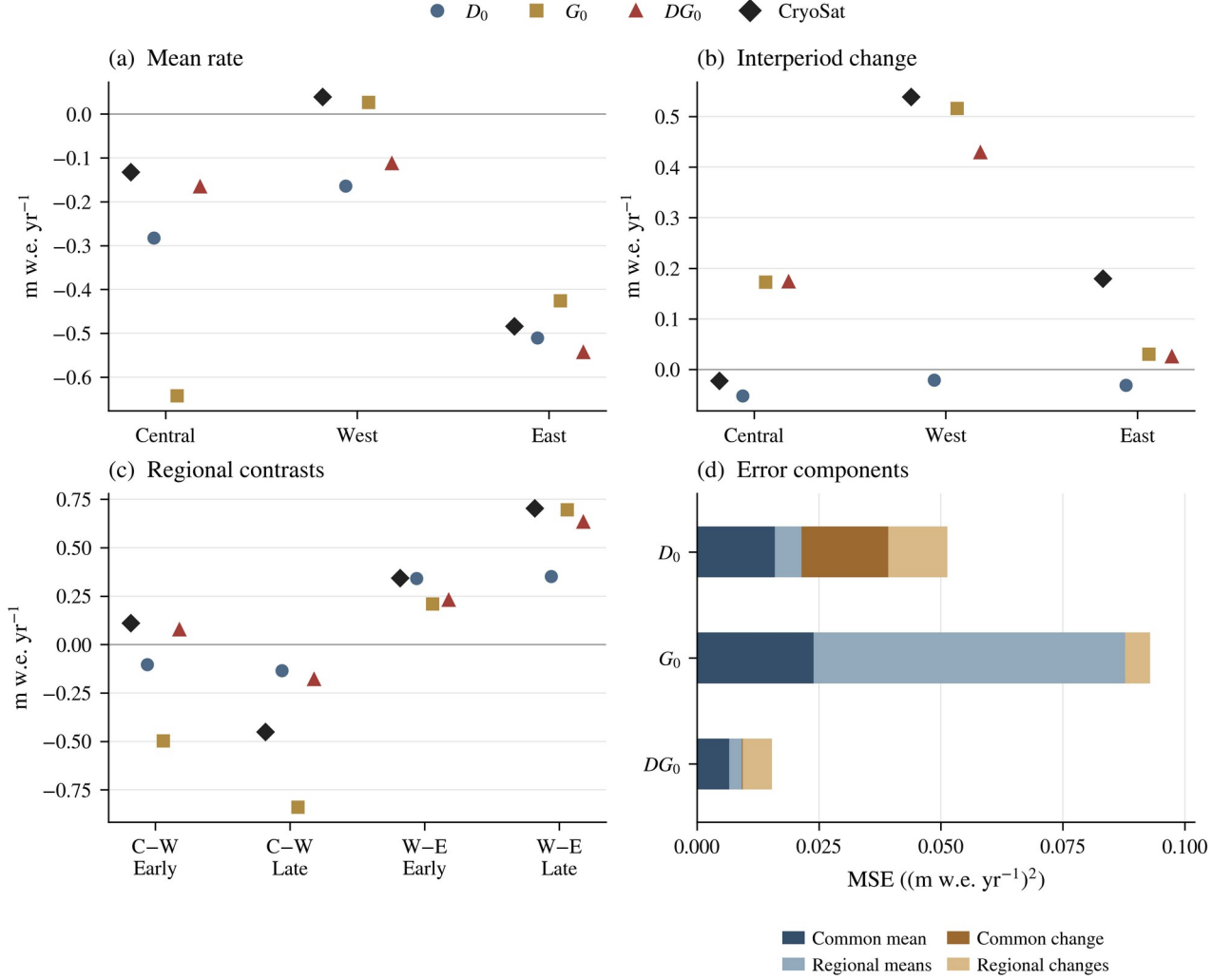


Figure 8. Mean, interperiod and spatial structure of the same six regional-period comparisons. (a) Mean of the two period rates. (b) Later minus earlier period rate. (c) The two predetermined contrasts in each period, where C, W and E denote Central Asia, South Asia west and South Asia east. (d) The exact four-term decomposition of equal-region specific-rate MSE into common and between-region mean errors and one quarter of the corresponding interperiod errors. All panels summarize the same observation family and retain its shared dependence.

#### 4.5 Endpoint and within-interval contributions

Under the fixed linear bridge through native geodetic endpoints, cumulative-mass adjustments account for the CryoSat improvement (Figure 9; Table 5). Adding the endpoint component alone to  $D_0$  lowers regional-period RMSE from 8.010 to 3.428 Gt yr<sup>-1</sup>, or from 0.2266 to 0.0957 m w.e. yr<sup>-1</sup>. Adding the within-interval component gives the complete  $DG_0$  solution, with errors of 4.373 Gt yr<sup>-1</sup> and 0.1241 m w.e. yr<sup>-1</sup>. Within-interval variation offsets part of the gain from endpoint adjustment.

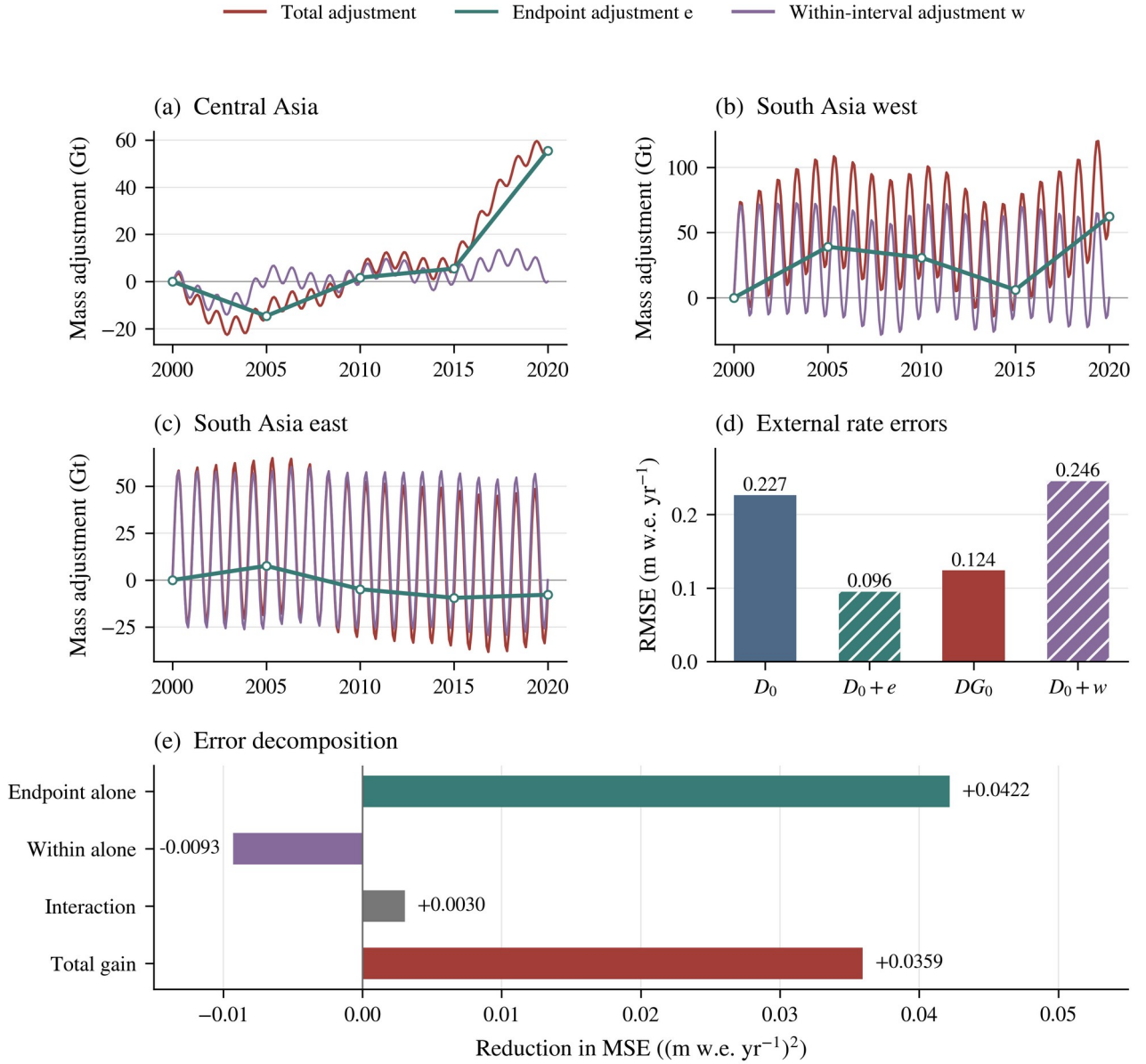


Figure 9. Endpoint and within-interval contributions to the joint adjustment. (a-c) Under the fixed decomposition,  $DG_0$  minus  $D_0$  is separated into the linear bridge through the January 2000, 2005, 2010, 2015 and 2020 updates and a remainder with zero native geodetic response. Open circles mark the native bridge endpoints. The visible seasonal oscillations contribute at most  $0.0023 \text{ m w.e. yr}^{-1}$  to the six evaluated rate updates. (d) External errors after adding endpoint ( $e$ ) or within-interval ( $w$ ) adjustments to  $D_0$ ; hatched bars identify these diagnostic additions, which were not separately refitted. (e) The two standalone MSE gains and their interaction reproduce the total gain. Negative gain denotes increased error.

The within-interval component also increases error when added to  $D_0$  alone: mass-rate RMSE reaches  $8.509 \text{ Gt yr}^{-1}$  and specific-rate RMSE reaches  $0.2463 \text{ m w.e. yr}^{-1}$ . Interperiod RMSE is  $0.3459$  for  $D_0$ ,  $0.1255$  after endpoint adjustment alone,  $0.1573$  for  $DG_0$  and  $0.3646$  after within-interval adjustment alone. Revising mass at the native five-year endpoints therefore improves the evaluated four-year period differences. The lower interperiod error of  $DG_0$  does not, by itself, establish useful recovery of the variation between those endpoints.

Table 5. External errors of the fitted estimates and fixed diagnostic component additions. Endpoint and within-interval additions use the unchanged external operator. They are derived from  $D_0$  and  $DG_0$  without refitting other observations.

| Trajectory                         | Period RMSE (Gt yr <sup>-1</sup> ) | Period RMSE (m w.e. yr <sup>-1</sup> ) | Interperiod RMSE (m w.e. yr <sup>-1</sup> ) |
|------------------------------------|------------------------------------|----------------------------------------|---------------------------------------------|
| $D_0$                              | 8.010                              | 0.2266                                 | 0.3459                                      |
| $D_0$ + endpoint adjustment        | 3.428                              | 0.0957                                 | 0.1255                                      |
| $DG_0$                             | 4.373                              | 0.1241                                 | 0.1573                                      |
| $D_0$ + within-interval adjustment | 8.509                              | 0.2463                                 | 0.3646                                      |

The specific-rate MSE reduction of 0.035943 (m w.e. yr<sup>-1</sup>)<sup>2</sup> comprises an endpoint-only contribution of +0.042194, a within-interval-only contribution of -0.009300 and an interaction of +0.003049 in the same squared units. The interaction arises because the adjustments are not orthogonal under the external period operator. Adding within-interval variation after endpoint adjustment increases two-period MSE in each region. In Central Asia, the within-interval component slightly improves  $D_0$  when added alone, but that benefit disappears in combination with the endpoint component. The regional response depends on each adjustment and their interaction.

The within-interval component vanishes at the native dates and has zero response, to numerical precision, under the coverage-weighted geodetic operator. Annual harmonics contribute at most 0.0023 m w.e. yr<sup>-1</sup> to the six evaluated within-interval rate adjustments, so the result is not primarily seasonal. The prescribed linear bridge defines this attribution. The component trajectories decompose the fitted joint adjustment; they are not separately fitted models.

#### 4.6 Conditional uncertainty and structural sensitivity

Joint observations reduce conditional glacier uncertainty in the adopted linear model (Figure 10). Standard deviations for the six regional-period rates decrease by 44.7% to 59.0% relative to  $D_0$  and by 6.0% to 34.7% relative to  $G_0$ . The early and late  $DG_0$  specific-rate standard deviations are 0.0394 and 0.0427 m w.e. yr<sup>-1</sup> in Central Asia, 0.0570 and 0.0599 in the west, and 0.0640 and 0.0709 in the east. Correlation matrices preserve dependencies arising from the common mass representation, observation covariance and temporal prior. This variance reduction follows from adding observations to the fixed linear Gaussian model. It quantifies uncertainty within that model; structural error and uncharacterized source correlations remain outside the calculation.

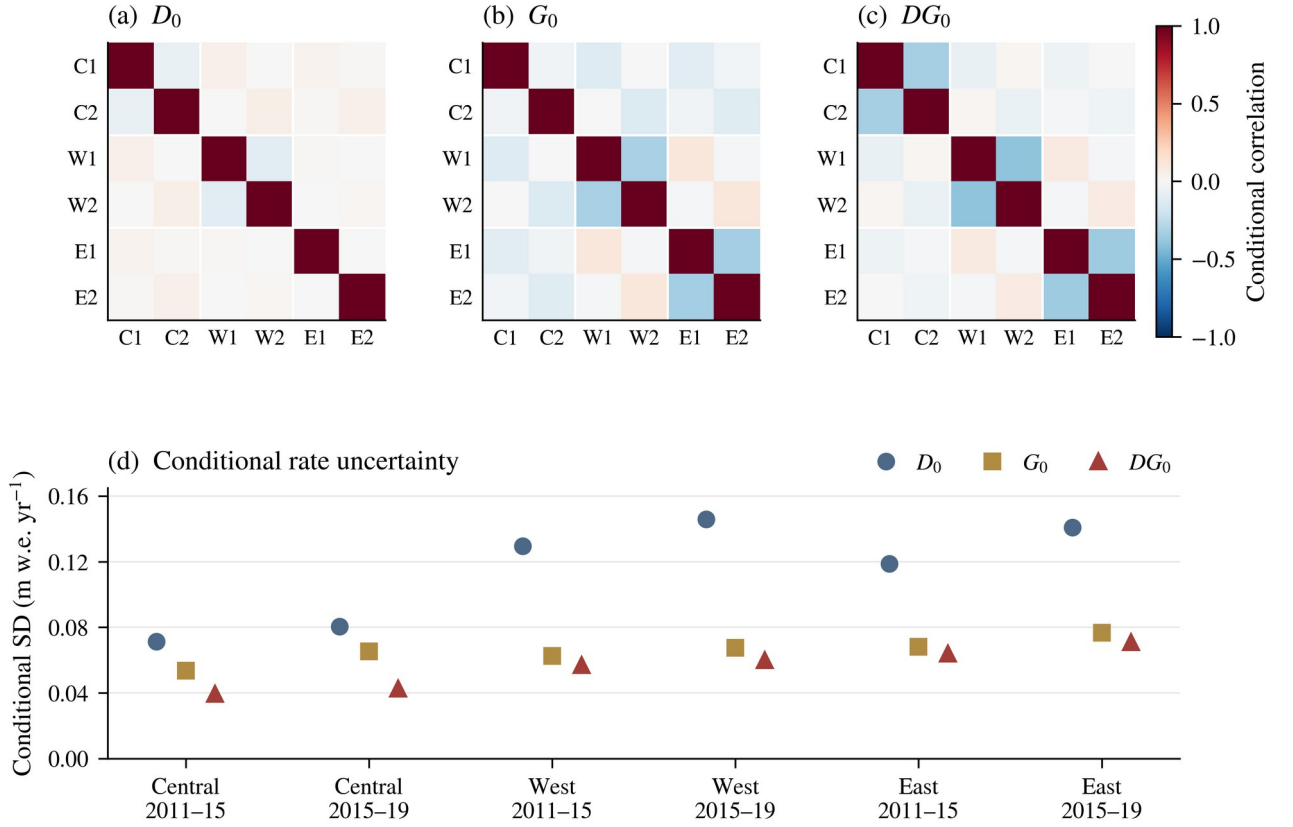


Figure 10. Conditional uncertainty of the six regional-period glacier estimates. (a-c) Correlations propagated from each fitted glacier covariance through the same external evaluation operator. C, W and E denote the three regions; 1 and 2 denote 2011-2015 and 2015-2019. White dividers separate the regional two-period blocks. (d) Marginal standard deviations, conditional on source geometry, prescribed observation covariance and regularization. These within-model standard deviations exclude unresolved cross-source errors and density, area and structural uncertainty.

Sensitivity experiments around the separately selected  $DG^*$  estimate retain lower pooled CryoSat error while changing its magnitude and regional distribution (Figure 11). Across the background-prior, geodetic-covariance and GIA alternatives, specific-rate RMSE ranges from 0.1190 to 0.1384 m w.e. yr<sup>-1</sup> and interperiod RMSE from 0.1768 to 0.1989. Regional-period errors remain below the corresponding D references, while interperiod errors exceed the common  $G_0$  value. This result applies to the tested alternatives around  $DG^*$ ; sensitivity across other common-parameter configurations has not been established.

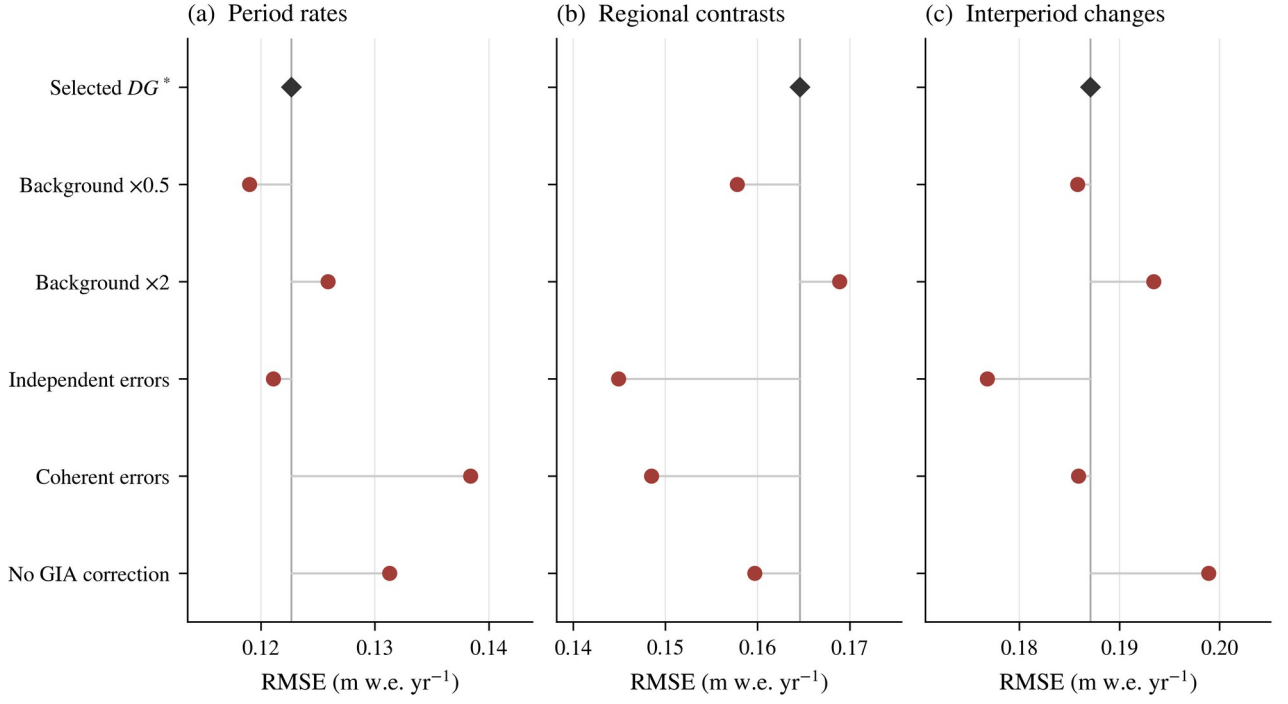


Figure 11. Structural sensitivity of the separately selected  $DG^*$  estimate, marked by the black diamond and vertical reference in each panel. Grey horizontal segments connect each alternative to the same reference. Background multipliers change non-glacier and exterior prior standard deviations; independent and coherent errors denote alternative prescribed geodetic error relations; the final alternative omits GIA correction. All panels use the same CryoSat observations to evaluate sensitivity around  $DG^*$ . The alternatives retain this common evaluation dependence and do not cover the full sensitivity of  $DG_0$ .

External source uncertainty differs from conditional model uncertainty. Shared density and area conversions affect several estimates together, while temporal correlation among radar increments changes the uncertainty of accumulated differences. Independent-increment, 90-day-overlap and fully coherent scenarios therefore produce different mean-rate and interperiod uncertainty scales (Table S11). The unavailable full radar covariance limits these calculations to conditional uncertainty scenarios. Amplitude ratios and RMSE reductions quantify agreement with the withheld CryoSat point estimates; the covariance scenarios describe their shared errors.

#### 4.7 Observation availability and footprint dependence

Separate parameter selection retains the pooled CryoSat improvement but changes its regional distribution.  $D^*$  and  $DG^*$  have mass-rate RMSEs of 8.187 and 3.989 Gt yr<sup>-1</sup> and interperiod RMSEs of 0.3589 and 0.1871 m w.e. yr<sup>-1</sup>.  $DG^*$  recovers 45.3% of the western and 35.6% of the eastern amplitudes. Central Asia retains the wrong interperiod direction, and the eastern mean remains less accurate than  $D^*$ . Interperiod terms account for 60.1% of this equal-region MSE reduction, and western South Asia contributes 74.7%. These proportions describe the separately selected estimates. The common-parameter results and  $DG_0$  endpoint decomposition use the fixed common parameters.

The joint estimate performs worse when native periods are withheld (Figure 12a,b). With two native intervals and the known twenty-year total available, common-parameter rate RMSE increases from 1.238 for  $D_0$  to 6.603 Gt yr<sup>-1</sup> for  $DG_0$ , and interperiod RMSE increases from 2.438 to 10.647 Gt yr<sup>-1</sup>. The corresponding selected  $D^*$  and  $DG^*$  errors increase from 1.698 to 4.419 and from 3.392 to 6.321 Gt yr<sup>-1</sup>. Results include all six omission pairs. These results describe reconstruction with reduced geodetic information. The changed fitting data account for the different estimation task, so the opposite rankings alone do not identify a sensor disagreement.

Local results also vary with regularization and observation footprint (Figure 12c,d). Pléiades equal-field elevation-rate RMSE is 0.6763 m yr<sup>-1</sup> for continuation of each glacier's latest rate, 0.7853 for  $D_0$  and 2.9333 for  $DG_0$ .  $D^*$  and  $DG^*$  give 0.7926 and 0.4780, respectively, while coherent geodetic errors increase the  $DG^*$  error to 1.3937. ATL06 specific-rate RMSE is 0.6108 m w.e. yr<sup>-1</sup> for local continuation, 0.7748 for  $D_0$ , 1.8259 for  $DG_0$  and 0.6194 for  $DG^*$ . No interval lies wholly beyond the geodetic record. Local reconstruction shows no consistent joint advantage, and monitoring wholly beyond the geodetic record remains untested.

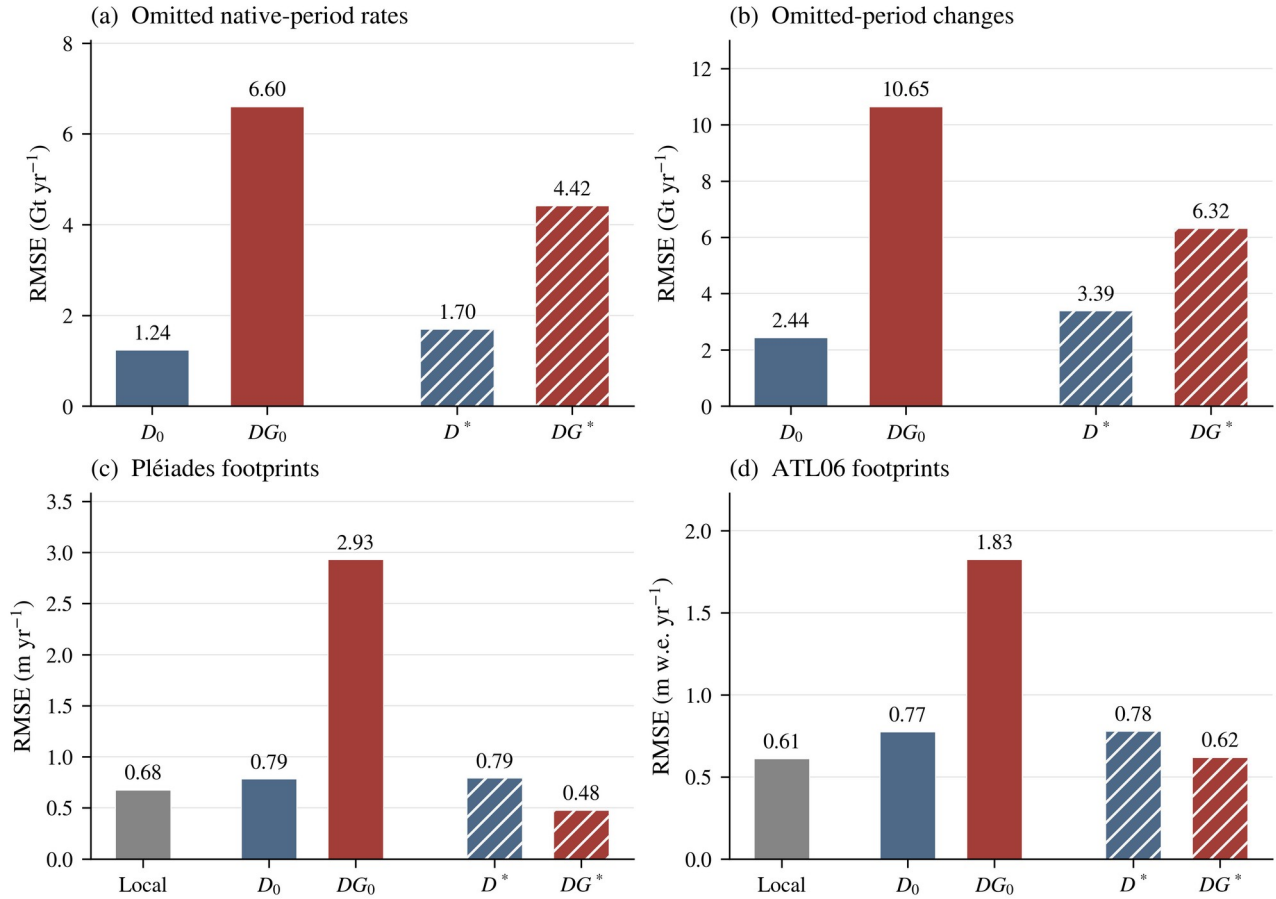


Figure 12. Dependence on fitting information and evaluation support. Solid method bars use common parameters; hatched, starred bars use separate parameter selection. (a,b) Six pair-omission fits use two native geodetic intervals and the known twenty-year total. The CryoSat comparison uses the complete geodetic history. (c) Pléiades elevation-rate errors give equal weight to two released fields from one study and retain the documented 762 kg m<sup>-3</sup> conversion. (d) ATL06 specific-rate errors retain one source group. Local denotes continuation of each glacier's last geodetic rate. All modern comparison intervals straddle January 2020 and include part of the geodetic history.

## 5 Discussion

### 5.1 The origin of the improvement in regional estimates

Adding GRACE changes both cumulative glacier mass at the geodetic endpoints and the trajectory connecting them. Geodetic observations enter the joint inversion with finite uncertainty, so their interval-integrated changes can be revised. Such revisions alter rates over assessment periods that intersect the native intervals, even if the trajectory between endpoints remains linear. Under the fixed linear bridge, cumulative-mass adjustments account for the CryoSat improvement, while additional within-interval variation offsets part of that gain. The regional benefit therefore reflects reconciliation of cumulative mass budgets, with no corresponding gain from the additional shorter-period variation.

The distinction is consequential for interpreting the interperiod error reduction. Interperiod terms account for 66.0% of the  $D_0$ -to- $DG_0$  reduction in specific-rate MSE, yet a revised five-year cumulative change can itself alter the difference between two offset four-year rates. The mean and interperiod decomposition identifies the external quantities that improve; the endpoint decomposition identifies the associated changes in the reconstructed history. The two decompositions describe complementary aspects of the same estimates. Evaluating both components through the same period operator, including their cross term, separates improvements associated with native cumulative mass from those associated with variation between its endpoints.

This interpretation depends on the specified linear bridge. The endpoint-only trajectory is derived from the two fitted solutions, with no separate optimization against gravity or the surrounding mass field. Its lower CryoSat error identifies the contribution of cumulative-mass revision to the existing joint adjustment; it does not establish an alternative joint solution. Annual-history calibration against geodesy and gravimetric bridging of altimetry records address related temporal limitations (Dussaillant et al., 2025; Wang et al., 2021). For these applications, joint evaluation of fitted cumulative changes and external rates distinguishes mass-budget reconciliation from improved temporal allocation.

## 5.2 Regional differences in observational complementarity

Observational complementarity varies across the three regions. In Central Asia, geodetic constraints reduce the strongly negative gravity-only mean toward the CryoSat estimate, although the joint interperiod direction remains opposite to the small negative radar difference. In western South Asia, adding gravity to geodesy produces the clearest recovery of the interperiod change; gravity alone is nevertheless closer to CryoSat in both mean rate and interperiod amplitude. Eastern South Asia shows a weaker change in period structure and a deterioration of the eight-year mean. These regional differences explain the lowest pooled error of the joint estimate and the stronger performance of individual observation combinations for particular quantities.

Regional glacier histories are known to differ across HMA mountain ranges (Brun et al., 2017; Gardelle et al., 2013; Kääb et al., 2012; Yao et al., 2012). The reconstructed adjustments combine that geographic setting with the sensitivity of the observations and the freedom allowed by the source representation. The regional trajectories reveal distinct observational contributions: reduced mean bias in one region and recovery of interperiod change in another. Published thinning and acceleration estimates provide physical context for these contrasts (Farinotti et al., 2015; Jakob et al., 2021; Maurer et al., 2019). Attribution to precipitation, circulation or glacier dynamics requires process observations beyond those used in this mass inversion.

Aggregation can obscure these regional distinctions. The joint HMA interperiod amplitude exceeds the CryoSat estimate even though both southern regions recover only part of their individual amplitudes. Central Asia's incorrect change direction contributes to the aggregate excess. Regional mean rates, interperiod changes and spatial contrasts reveal complementary features of the same observations. They also provide a basis for comparing studies whose regional boundaries, sampling and assessment periods differ, factors that have affected previous interpretations of HMA mass change (Kääb et al., 2015; Piermattei et al., 2024).

## 5.3 Mass separation and the cost of reconciling observations

The external improvement is accompanied by substantial adjustment of the fitted geodetic budgets.  $DG_0$  has a correlated standardized geodetic residual RMS of 3.764, compared with 0.033 for  $D_0$ . For 2015 to 2020, its native-support rates exceed the observed geodetic values by 10.557 Gt yr<sup>-1</sup> in Central Asia and 11.489 Gt yr<sup>-1</sup> in western South Asia. These are also regions with large improvements against

CryoSat. The concurrent external gain and geodetic discrepancy reveal a changed balance among observation constraints. Distinguishing geodetic error from gravity-representation error and other model assumptions requires evidence beyond the external RMSE.

Gravity agreement improves concurrently: standardized residual RMS decreases from 1.222 for  $D_0$  to 0.999 for  $DG_0$ , close to the gravity-only value of 0.998. Geodetic information thus changes the glacier allocation while preserving nearly the same gravity fit. This occurs because gravity constrains combinations of glacier and surrounding mass, leaving freedom in their separation. Residuals near unity measure agreement under the adopted covariance and source representation, with glacier accuracy assessed separately against external observations. The native geodetic residuals and endpoint decomposition expose the mass adjustments that a spectral fit alone would conceal.

The covariance model determines how those adjustments are penalized. Spatially correlated elevation errors and shared acquisition epochs can make combinations of moderate marginal residuals consequential in a joint inversion (Hugonnet et al., 2022; Rolstad et al., 2009). The different rankings obtained from marginal and correlated residual summaries demonstrate this dependence. Blockwise spatial coherence and the prescribed endpoint correlation represent some shared-error structure, but cross-source covariance remains incompletely characterized. The concentration of standardized residuals identifies the regions and intervals that remain difficult to reconcile under the adopted assumptions; it does not identify their physical causes.

Uncertainty in the surrounding mass field further affects glacier separation. Regional storage comprises several varying components (Li et al., 2022; Zhang et al., 2023), and lake growth can partly oppose glacier loss in overlapping gravity footprints (Brun et al., 2020; Zhang et al., 2013). Non-glacier storage also contains processes only partly represented by GLDAS. Groundwater exchange is relevant to some Tibetan lake budgets (Lei et al., 2022), while the estimated background correction represents unresolved non-glacier storage collectively. The source adjustments and joint covariance quantify how the inversion distributes mass among glacier, lake, non-glacier and exterior regions. Their physical interpretation remains conditional on geometry, regularization and the treatment of leakage, as in other regional gravimetric estimates (Loomis et al., 2019; Save et al., 2016; Schrama et al., 2014; Watkins et al., 2015; Wiese et al., 2016).

#### 5.4 Conditions for using the joint reconstruction

The regional CryoSat improvement applies when the complete multiperiod geodetic record is available. Removing native intervals changes the estimation problem: GRACE must then help recover information that was directly constrained in the full-history fit. The poorer joint results establish that the regional benefit does not extend to the tested missing-period reconstruction. The rankings refer to different fitting information, rather than a direct comparison of sensor consistency. Within each of fifteen second-order regions, the imposed uniform specific mass change also limits the reconstruction to regional glacier mass.

The local comparisons show additional dependence on regularization and observation footprint. Common-parameter  $DG_0$  performs poorly against both Pléiades and ATL06. Separate parameter selection improves the Pléiades comparison, but ATL06 remains less accurate than local rate continuation, and coherent geodetic errors weaken the Pléiades gain. The regional benefit therefore does not extend uniformly to local glacier estimates. All modern intervals cross January 2020, and gravity continues through 2023. The comparisons therefore evaluate retrospective reconstruction with incomplete later geodetic coverage; prospective monitoring remains untested.

External observation errors also affect interpretation. Coregistration, missing elevation data, radar surface sensitivity and regionalization affect elevation-based mass estimates (Gourmelen et al., 2018; McNabb et al., 2019; Nuth and Kääb, 2011; Piermattei et al., 2024). Density conversion and overlapping temporal sampling introduce shared errors in CryoSat period estimates (Huss, 2013; Jakob et al., 2021). Their marginal errors permit conditional propagation scenarios, but the full covariance needed for calibrated confidence limits on the six regional-period estimates is unavailable. The result establishes closer agreement with the withheld CryoSat source over the specified regions and dates. Distinguishing the remaining glacier adjustment from compensation by surrounding mass would require fuller source covariance and regional observations sensitive to those separate contributions.

## 6 Conclusions

Joint geodetic and gravimetric estimation improves the specified regional-period glacier mass estimates in High Mountain Asia. With a common representation of glacier and surrounding mass, regional-period RMSE against excluded CryoSat records decreases from 8.01 Gt yr<sup>-1</sup> for geodesy and 14.80 Gt yr<sup>-1</sup> for gravity to 4.37 Gt yr<sup>-1</sup>. Gravity improves interperiod estimates relative to geodesy, whereas geodetic information mainly corrects the mean offset of the gravity-only solution. The contributions vary by region, and individual observation combinations remain more accurate for some regional means and period changes.

Under the fixed linear bridge through native geodetic endpoints, adjustments to cumulative mass lower the diagnostic RMSE to 3.43 Gt yr<sup>-1</sup>, while added within-interval variation offsets part of that gain. Substantial fitted geodetic residuals, sensitivity to source weighting and poorer missing-period and local results restrict the benefit to the evaluated regional application. Evaluating both native cumulative budgets and external-period rates distinguishes mass-budget reconciliation from improved temporal allocation in joint glacier assessments.

## Data and code availability

Public data sources, versions and persistent identifiers are listed in Table S1. The CryoSat contributor series are distributed with GLaMBIE Dataset 1.0.0 (The GLaMBIE Team, 2024). The accompanying analysis archive contains the common-parameter saved states, native observation operators and date indices, the fixed endpoint-decomposition code, the numerical data underlying the figures, and editable figure sources. Selected-parameter, omission and modern-footprint results are retained as the archived comparison tables with their original information conditions. The figure and manuscript sources preserve the distinction between the common-parameter estimates and those additional comparisons.

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# Supplementary Information

Geodetic and gravimetric constraints on regional glacier mass changes in High Mountain Asia

## S1 Data sources and observation support

Table S1 lists the input datasets and processing references. Lake storage and geometry come from distinct datasets. Li et al. (2019b) provide water-level and derived volume observations, and the 2009 member of Lake\_TP v2 defines the fixed footprints (Zhou et al., 2024). Lake volume changes in km<sup>3</sup> are converted to mass in Gt using a water density of 1000 kg m<sup>-3</sup>.

The GLDAS background spans January 2000 to December 2023 and includes all 36 monthly fields for 2000-2002. Four soil-moisture layers, snow water equivalent and canopy interception, in kg m<sup>-2</sup>, are summed and divided by 1000 to obtain metres of water. For each non-glacier tile, the area-weighted mean over valid cells is referenced to 2004-2009 and multiplied by the tile area excluding glacier and lake footprints. The minimum valid-data fraction is 0.8831 of tile area. Interpolation supplies missing auxiliary tile months where required; gravity observations begin in 2002. Monthly means are integrated over the native gravity windows using their actual overlap durations.

Table S1. Data sources and processing references. The gravity observations retain the AOD1B RL06 conventions of the original ITSG solutions. Dataset versions and persistent identifiers specify the input records.

| Input               | Role and observation support                                               | Source                                                                       |
|---------------------|----------------------------------------------------------------------------|------------------------------------------------------------------------------|
| RGI6                | Full glacier polygons; O1/O2 regions                                       | RGI Consortium (2017); 10.7265/4m1f-gd79                                     |
| Hugonnet mass rates | Original four five-year intervals, 2000-2020                               | Hugonnet et al. (2021a), article; dataset 10.6096/13                         |
| ITSG gravity        | Unconstrained n60; native SINEX windows, 2002-2023                         | Mayer-Gürr et al. (2018); Kvas et al. (2019); Behzadpour et al. (2021)       |
| GLDAS Noah 2.1      | Monthly 0.25° soil, snow, canopy, 2000-2023                                | Rodell et al. (2004); Beaudoin et al. (2020); 10.5067/SXAVCZFAQLNO           |
| Load Love numbers   | Load_Love2_CE.dat; column-four k <sub>n</sub>                              | Gégout et al. (2010); 10.13140/RG.2.1.1866.7045                              |
| GIA                 | ICE-6G_C (VM5a); retained Stokes terms                                     | Peltier et al. (2015); 10.1002/2014JB011176                                  |
| Lake observations   | 44 observed date/error series among 46 modeled geometries; derived storage | Li et al. (2019b); 10.1594/PANGAEA.898411; hypsometry 10.1594/PANGAEA.898409 |
| Lake footprints     | Lake_TP v2; fixed 2009 member                                              | Zhou et al. (2024); 10.5281/zenodo.10686952                                  |
| CryoSat submissions | GlaMBIE Dataset 1.0.0, 2024-07-16; Jakob/Gourmelen                         | The GlaMBIE Team (2024); Jakob et al. (2021); 10.5904/wgms-glambie-2024-07   |
| Pléiades fields     | Two west/east 2019-2022 fields; released footprints                        | Falaschi et al. (2023a), article; data 10.5281/zenodo.10392969               |
| ATL06               | Version 007; acquisition dates and observation footprints                  | Smith et al. (2025; 2019); 10.5067/ATLAS/ATL06.007                           |

The contributor archive of GlaMBIE Dataset 1.0.0, released on 16 July 2024 (The GlaMBIE Team, 2024), contains the following three regional files. Each has 100 contiguous monthly intervals from decimal year 2011.0 to 2019.33333333333333. The changes field contains regionalized elevation increments in metres.

glambie\_input\_20240716/13\_central\_asia/  
central\_asia\_altimetry\_Jakob\_Gourmelen\_Jakob\_et\_al.csv  
glambie\_input\_20240716/14\_south\_asia\_west/  
south\_asia\_west\_altimetry\_Jakob\_Gourmelen\_Jakob\_et\_al.csv  
glambie\_input\_20240716/15\_south\_asia\_east/  
south\_asia\_east\_altimetry\_Jakob\_Gourmelen\_Jakob\_et\_al.csv

For increments  $c_j = \text{changes}_j$ , cumulative height is  $h_k = h_0 + \sum_{j \leq k} c_j$ . Over  $[a, b]$ , the sum includes only rows wholly within the specified period:  $\Delta h(a, b) = \sum c_j$ , with a tolerance of  $10^{-8}$  in decimal-year membership. There are 48 rows for each four-year interval and 96 for their union. The arbitrary starting height cancels. Each changes value is an elevation increment; summation therefore requires no multiplication by interval duration. Mass change and mean rate are calculated as  $\Delta M = 850 A \Delta h / 10^{12} \text{ Gt}$  and  $r = \Delta M / \Delta t$ , where  $A$  is full RGI-region area in  $\text{m}^2$  and  $\Delta t$  is actual elapsed days/365.25. Specific rate is  $0.85 \Delta h / \Delta t \text{ m w.e. yr}^{-1}$ . The adopted density and  $\pm 60 \text{ kg m}^{-3}$  scenario follow the geodetic conversion framework of Huss (2013);  $1000 \text{ kg m}^{-3}$  defines water equivalent here, without a second GLaMBIE water-density correction.

Jakob et al. (2021) describe a weighted long-term linear elevation trend for July 2010–July 2019 and a time series beginning January 2011 with 90-day moving medians at 30-day steps. The January-to-January sum of submitted increments is a different statistic from that fitted trend. For model source mass  $M(t)$ , the evaluation rate is  $[\bar{M}_{90}(b) - \bar{M}_{90}(a)] / \Delta t$ , where  $\bar{M}_{90}(t)$  averages  $t-45$  to  $t+45$  days. Twelve-point Gauss-Legendre quadrature is split at annual knots. The model mean approximates the median-derived observation. Exact radar-return weights and detailed resampling information are unavailable.

The principal comparison uses the complete RGI regions. Geodetic observations apply to the common valid cohort within each second-order region, whose coverage fraction multiplies the represented full-region mass change. The same group-specific mass estimate applies to the small unobserved area. Independent inventory intersections define the gravity geometry. Pléiades and ATL06 comparisons retain their own observation footprints and acquisition dates.

## S2 Observation operators and regularization

Let  $b(t)$  contain the 24 annual hat functions for January 2001 to January 2024, followed by  $\sin(2\pi\tau)$  and  $\cos(2\pi\tau) - 1$ , where  $\tau$  is elapsed days since 1 January 2000 divided by 365.25. Omitting the January 2000 hat fixes the reference mass. Source  $s$  has  $m_s(t) = b(t)^T x_s$  in Gt. The matrix  $X$  contains the source coefficient rows, and  $x = \text{vec}_s(X)$  stacks them in source-major order. The  $G$ ,  $L$ ,  $N$  and  $E$  classes contain 15, 46, 45 and 36 sources, respectively; 44 lake sources have retained observations. Glacier mass within each group is distributed in proportion to inventory-intersection area.

For glacier source  $s$  and interval  $p$ , the geodetic prediction is  $f_s [b(t_{p,1}) - b(t_{p,0})]^T x_s$ , where  $f_s$  is the fixed observed-area fraction. Observed mass change equals the summed published object mass rates for the common valid cohort multiplied by the exact interval duration. Let  $\sigma_{sbp}$  be the sum of published mass-rate standard errors for source  $s$ , 2-degree object-centre block  $b$  and period  $p$ , multiplied by that duration. The 60-row covariance is

$$(\Sigma_D)_{[s,p],[s',q]} = R_{pq} \sum_b \sigma_{sbp} \sigma_{s'bp}. \quad (S1)$$

$R$  has unit diagonal,  $-0.5$  for adjacent periods and zero elsewhere. A diagonal stabilizer of  $10^{-8} \text{diag}(\Sigma_D)$  is then added. Shared blocks induce covariance across glacier groups. The independent-period alternative uses  $R=I$ , whereas the coherent alternative uses  $+0.5$  for all off-diagonal entries. These approximate shared elevation and endpoint errors. When  $M$  transforms native observations into the known twenty-year total and retained intervals, the design, observations and covariance become  $M H_D$ ,  $M y_D$  and  $M \Sigma_D M^T$ . The redundant total is omitted when all four intervals are supplied.

For each observed lake,  $\Sigma_L = \text{diag}(\sigma_i^2) + V V^T$ . Each  $\sigma_i$  is the larger of the reported volume error and  $0.005/\sqrt{3}$  Gt. A mission column of  $V$  contains  $0.11 A_i/1000$  Gt for that mission's observations and zero otherwise, where  $A_i$  is the reported lake area in  $\text{km}^2$ . The unknown lake offset is eliminated using

$$P_L = \Sigma_L^{-1} - \Sigma_L^{-1} \mathbf{1} (\mathbf{1}^T \Sigma_L^{-1} \mathbf{1})^{-1} \mathbf{1}^T \Sigma_L^{-1}. \quad (S2)$$

The lake normal-equation terms are  $H_L^T P_L H_L$  and  $H_L^T P_L y_L$ , where  $H_L$  evaluates  $b$  at the actual acquisition dates. This formulation uses relative storage observations directly, without estimating absolute lake volume. Glacier and lake areas are removed from the non-glacier spatial response to avoid representing their loads twice. The spatial representation uses static outlines and the specified geometric intersection approximation.

The gravity observations comprise 3,715 fully normalized real cosine and sine coefficients at degrees 2 to 60, excluding  $C_{20}$  and  $C_{30}$  (Tapley et al., 2004). For a unit-Gt source, the spatial response is the normalized area-weighted mean of the appropriate associated Legendre function and longitude sine or cosine, multiplied by  $10^{12}(1+k_n)/[(GM/G_N)(2n+1)]$ . The constants are  $GM = 3.9860044150 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$  and  $G_N = 6.67430 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ ; spherical integration uses radius 6,378,136.3 m. A common factor of  $10^{10}$  scales observations, responses and formal errors for numerical computation. The  $k_n$  values are column four of Load\_Love2\_CE.dat, identified by the distributor with the Gégout PREM calculation (Gégout et al., 2010).

Glacier and lake responses use polygon-area integration on the native 0.05-degree grid. Non-glacier sources are approximately 5-degree rectangles, truncated at the local boundary and excluding glacier and lake loads. Exterior sources are 30 degrees in latitude by 60 degrees in longitude, excluding overlap with the local rectangle. Rectangular integration uses 96-point latitude quadrature and analytic longitude averages. The GRACE-appropriate ICE-6G\_C Stokes trends are subtracted, excluding its absolute-sea-level low-degree block. Observations and model window means share the available 2004 to 2009 reference windows. Missing gravity months supply no observations.

Let  $D_f$  contain the median formal gravity standard deviations over fitted windows, floored at  $10^{-8}$  in internal scaled units. A spatial least-squares fit to standardized gravity observations in odd years before 2020 supplies residual matrix  $R_g$ , without glacier or radar response data. For its singular values  $d_j$  and right singular vectors  $v_j$ , the columns of  $U$  are up to the first 12 vectors  $v_j d_j \sqrt{0.8/n_c}$ , where  $n_c$  is the number of covariance-development windows. The per-window coefficient covariance is

$$\Sigma_c = D_f \left[ \text{diag}(1 + 0.2 \overline{R_g^2}) + U U^T \right] D_f. \quad (S3)$$

The overbar is the mode-wise mean of squared residuals. The unit diagonal preserves formal variance; the empirical diagonal and low-rank terms represent additional discrepancy. The Woodbury identity evaluates the inverse. The same  $\Sigma_c$  applies to each conditionally independent window, forming  $\Sigma_G$ ;

covariance between windows is not estimated. Atmospheric and ocean de-aliasing is inherited from the official unconstrained ITSG solutions. The calculation uses neither a filtered Level-2B product nor a CSR training target or replacement  $C_{20}/C_{30}$  series (Behzadpour et al., 2021; Kvas et al., 2019).

For source area  $A_s$  in  $\text{m}^2$ , define  $u_s = \max(A_s/10^9, 0.005)$ , the Gt scale corresponding to one metre of water equivalent with the numerical floor. Let  $D_1$  and  $D_2$  be first- and second-difference operators on the 25 annual knots after removal of the fixed January 2000 column. For the 24 annual coefficients  $z_s$  and two harmonic coefficients  $v_s$ , the glacier penalty is

$$J_{Q,s} = \frac{\|D_2 z_s\|^2}{(a u_s)^2} + \frac{(d_1^T z_s)^2}{(2 u_s)^2} + \frac{\|v_s\|^2}{(0.5 u_s)^2} + \frac{10^{-9} \|z_s\|^2}{u_s^2}. \quad (\text{S4})$$

$d_1^T$  is the first row of  $D_1$ . The common value is  $a = 0.3 \text{ m w.e. yr}^{-1}$  per annual step. The prior penalizes knot increments without an additional day-count normalization; actual durations enter the observation and reporting operators. The final term is a numerical ridge. For  $L$ ,  $N$  and  $E$ , the amplitude  $a_s$  is  $10 u_s$ ,  $0.4 u_s$  and  $0.3 u_s$  Gt, respectively. Their penalty is  $[z_s^T (I + 0.25 D_2^T D_2) z_s + v_s^T v_s] / a_s^2$ . The background multiplier scales only the  $N$  and  $E$  amplitudes and equals one in the principal comparison. All source penalties form the prior precision  $Q$ .

With source-major vectorization, the gravity contribution to the normal precision is  $(A^T \Sigma_c^{-1} A) \otimes (T^T T)$ . Its right-hand side is  $\text{vec}_s(A^T \Sigma_c^{-1} \tilde{Y}_G^T T)$ , where  $\tilde{Y}_G = Y_G - F A_N^T$  removes the prescribed GLDAS background. The matrix  $T$  contains the referenced native-window means. Adding the geodetic, lake and prior terms gives the full normal precision  $\Lambda$  and right-hand side  $h$ . Partitioning glacier coefficients  $g$  and the remaining coefficients  $n$  gives

$$\begin{aligned} S &= \Lambda_{gg} - \Lambda_{gn} \Lambda_{nn}^{-1} \Lambda_{ng}, \\ \hat{x}_g &= S^{-1} (h_g - \Lambda_{gn} \Lambda_{nn}^{-1} h_n), \\ \hat{x}_n &= \Lambda_{nn}^{-1} (h_n - \Lambda_{ng} \hat{x}_g). \end{aligned} \quad (\text{S5})$$

The solution retains the conditional precision of the surrounding mass, its regression on glacier coefficients, the glacier covariance and the complete coefficient matrix. Source priors and lake observations are identical for  $P$ ,  $D_0$ ,  $G_0$  and  $DG_0$ ; observation indicators specify the geodetic and gravity terms included in each estimate.

Table S2. Full-history estimates. Source geometry, lake observations and non-glacier and exterior priors are common to all estimates. Candidate smoothing scales are 0.1, 0.3 and 1.0. Missing-period estimates use the available intervals and known total in place of the complete geodetic record.

| Arm             | Geodetic | Gravity     | Scale / estimator | Choice                         |
|-----------------|----------|-------------|-------------------|--------------------------------|
| P               | No       | No          | 0.3               | Auxiliary-only                 |
| D <sub>0</sub>  | All four | No          | 0.3               | Common-state geodetic          |
| G <sub>0</sub>  | None     | 227 windows | 0.3               | Common-state gravity           |
| DG <sub>0</sub> | All four | 227 windows | 0.3               | Common-state joint             |
| D*              | All four | No          | Direct ridge 0.01 | Native pair-omission selection |
| G*              | None     | 227 windows | 1.0               | Held gravity 2012, 2016        |
| DG*             | All four | 227 windows | 0.1               | Native pair-omission selection |

Parameter selection evaluates all six two-period omissions and minimizes specific-rate MSE with equal weights for glacier groups and withheld periods. Direct candidates interpolate available interval rates, use their duration-weighted mean, or fit a ridge trend with intercept and normalized midpoint  $(t - 10)/10$ . The intercept stabilizer is  $10^{-12}$  and the slope penalty is 0.01, 1 or 100. Available rates are reproduced exactly, and a common adjustment to the missing rates enforces the known twenty-year total. Each direct candidate consequently reproduces all four rates when the complete record is available. Loss ties are resolved lexicographically. Joint candidates vary only the smoother within the common mass representation.  $G^*$  is selected from gravity residuals in 2012 and 2016 without glacier response data.

Within each outer two-period omission, inner selection removes one remaining interval at a time and retains the known total. Each of the six outer pairs is omitted across all glacier groups simultaneously. The remaining operator has rank three; the withheld-period contrast raises the rank to four. Overlapping periods make the fold errors dependent. Repeated dates and spatial resampling preserve this dependence.

### S3 Regional mass changes and contrasts

Tables S3 and S4 present the regional estimates for both assessment periods, their means and the specified spatial contrasts. D<sub>0</sub>, G<sub>0</sub> and DG<sub>0</sub> share  $a = 0.3$ . CryoSat values are the submitted point estimates converted on the full inventory areas. HMA is the sum of regional masses and is excluded from the equal-three-region RMSE. Mass-rate and specific-rate metrics use different spatial weightings and are reported separately.

Table S5 reports the separately selected results. The 60.1% interperiod share and 74.7% western share are based on D\* and DG\* in m w.e. yr<sup>-1</sup> with equal-region MSE; the corresponding regional shares in Gt units are different. Amplitude ratios express predicted interperiod change relative to the CryoSat point estimate. The observed interperiod difference in Central Asia is small and negative. A large negative ratio therefore indicates that the estimated change has the opposite sign.

Table S3(a). Common-parameter regional rates (m w.e. yr<sup>-1</sup>). Period 1 is 2011-2015; period 2 is 2015-2019; mean is their average; difference is later minus earlier. Ratio (%) divides predicted interperiod difference by the CryoSat point estimate. Negative ratios indicate opposite signs of estimated and observed interperiod change. HMA is a dependent area-weighted aggregate. Unrounded Gt and specific-rate values accompany this table electronically.

| Region | Estimate        | Period 1 | Period 2 | Mean    | Difference | Ratio (%) |
|--------|-----------------|----------|----------|---------|------------|-----------|
| RG113  | CryoSat         | -0.1213  | -0.1443  | -0.1328 | -0.0229    | n/a       |
| RG113  | D <sub>0</sub>  | -0.2565  | -0.3091  | -0.2828 | -0.0526    | +229.3    |
| RG113  | G <sub>0</sub>  | -0.7288  | -0.5561  | -0.6425 | 0.1727     | -752.8    |
| RG113  | DG <sub>0</sub> | -0.2523  | -0.0795  | -0.1659 | 0.1729     | -753.5    |
| RG114  | CryoSat         | -0.2310  | 0.3077   | 0.0384  | 0.5387     | n/a       |
| RG114  | D <sub>0</sub>  | -0.1538  | -0.1751  | -0.1645 | -0.0213    | -3.9      |
| RG114  | G <sub>0</sub>  | -0.2320  | 0.2842   | 0.0261  | 0.5163     | +95.8     |
| RG114  | DG <sub>0</sub> | -0.3277  | 0.1010   | -0.1134 | 0.4287     | +79.6     |
| RG115  | CryoSat         | -0.5735  | -0.3944  | -0.4840 | 0.1791     | n/a       |
| RG115  | D <sub>0</sub>  | -0.4945  | -0.5261  | -0.5103 | -0.0316    | -17.6     |
| RG115  | G <sub>0</sub>  | -0.4410  | -0.4110  | -0.4260 | 0.0301     | +16.8     |
| RG115  | DG <sub>0</sub> | -0.5560  | -0.5311  | -0.5435 | 0.0249     | +13.9     |
| HMA    | CryoSat         | -0.2273  | -0.0266  | -0.1270 | 0.2007     | n/a       |
| HMA    | D <sub>0</sub>  | -0.2571  | -0.2958  | -0.2764 | -0.0386    | -19.3     |
| HMA    | G <sub>0</sub>  | -0.5145  | -0.2452  | -0.3799 | 0.2693     | +134.2    |
| HMA    | DG <sub>0</sub> | -0.3241  | -0.0856  | -0.2048 | 0.2385     | +118.8    |

Table S3(b). Common-parameter regional mass rates (Gt yr<sup>-1</sup>), corresponding to Table S3(a). Periods are January 2011-January 2015 and January 2015-January 2019. HMA is the sum of the three regions; differences are later minus earlier rates.

| Region | Estimate        | Period 1 | Period 2 | Mean    | Difference |
|--------|-----------------|----------|----------|---------|------------|
| RG113  | CryoSat         | -5.983   | -7.114   | -6.548  | -1.131     |
| RG113  | D <sub>0</sub>  | -12.647  | -15.240  | -13.943 | -2.593     |
| RG113  | G <sub>0</sub>  | -35.934  | -27.419  | -31.677 | 8.516      |
| RG113  | DG <sub>0</sub> | -12.441  | -3.917   | -8.179  | 8.524      |
| RG114  | CryoSat         | -7.755   | 10.330   | 1.287   | 18.085     |
| RG114  | D <sub>0</sub>  | -5.164   | -5.878   | -5.521  | -0.714     |
| RG114  | G <sub>0</sub>  | -7.789   | 9.541    | 0.876   | 17.331     |
| RG114  | DG <sub>0</sub> | -11.001  | 3.390    | -3.805  | 14.391     |
| RG115  | CryoSat         | -8.450   | -5.811   | -7.131  | 2.639      |
| RG115  | D <sub>0</sub>  | -7.287   | -7.751   | -7.519  | -0.465     |
| RG115  | G <sub>0</sub>  | -6.498   | -6.055   | -6.277  | 0.443      |
| RG115  | DG <sub>0</sub> | -8.192   | -7.825   | -8.008  | 0.367      |
| HMA    | CryoSat         | -22.188  | -2.595   | -12.392 | 19.593     |
| HMA    | D <sub>0</sub>  | -25.097  | -28.869  | -26.983 | -3.772     |
| HMA    | G <sub>0</sub>  | -50.222  | -23.933  | -37.077 | 26.289     |
| HMA    | DG <sub>0</sub> | -31.633  | -8.352   | -19.993 | 23.282     |

Table S4. Regional specific-rate differences used in the spatial evaluation (m w.e. yr<sup>-1</sup>). The eight-year rows are retained separately from the four period-specific quantities entering spatial RMSE.

| Period    | Contrast | CryoSat | D <sub>0</sub> | G <sub>0</sub> | DG <sub>0</sub> |
|-----------|----------|---------|----------------|----------------|-----------------|
| 2011-2015 | 13-14    | 0.1097  | -0.1027        | -0.4968        | 0.0754          |
| 2011-2015 | 14-15    | 0.3425  | 0.3407         | 0.2090         | 0.2283          |
| 2015-2019 | 13-14    | -0.4520 | -0.1340        | -0.8404        | -0.1805         |
| 2015-2019 | 14-15    | 0.7021  | 0.3510         | 0.6952         | 0.6321          |
| 2011-2019 | 13-14    | -0.1712 | -0.1183        | -0.6686        | -0.0525         |
| 2011-2019 | 14-15    | 0.5223  | 0.3459         | 0.4521         | 0.4302          |

Table S5. Separately selected regional rates (m w.e. yr<sup>-1</sup>). Period 1 is 2011-2015; period 2 is 2015-2019; mean is their average; difference is later minus earlier. Ratio (%) divides predicted interperiod difference by the CryoSat point estimate. Negative ratios indicate opposite signs of estimated and observed interperiod change. HMA is a dependent area-weighted aggregate. Unrounded Gt and specific-rate values accompany this table electronically.

| Region | Estimate | Period 1 | Period 2 | Mean    | Difference | Ratio (%) |
|--------|----------|----------|----------|---------|------------|-----------|
| RG113  | CryoSat  | -0.1213  | -0.1443  | -0.1328 | -0.0229    | n/a       |
| RG113  | D*       | -0.2474  | -0.3231  | -0.2852 | -0.0757    | +330.1    |
| RG113  | G*       | -0.5712  | -0.3678  | -0.4695 | 0.2033     | -886.2    |
| RG113  | DG*      | -0.1851  | -0.1393  | -0.1622 | 0.0457     | -199.3    |
| RG114  | CryoSat  | -0.2310  | 0.3077   | 0.0384  | 0.5387     | n/a       |
| RG114  | D*       | -0.1475  | -0.1834  | -0.1655 | -0.0360    | -6.7      |
| RG114  | G*       | -0.4041  | 0.3722   | -0.0160 | 0.7763     | +144.1    |
| RG114  | DG*      | -0.2059  | 0.0379   | -0.0840 | 0.2439     | +45.3     |
| RG115  | CryoSat  | -0.5735  | -0.3944  | -0.4840 | 0.1791     | n/a       |
| RG115  | D*       | -0.4875  | -0.5395  | -0.5135 | -0.0520    | -29.0     |
| RG115  | G*       | -0.4436  | -0.3781  | -0.4108 | 0.0655     | +36.6     |
| RG115  | DG*      | -0.5716  | -0.5078  | -0.5397 | 0.0638     | +35.6     |
| HMA    | CryoSat  | -0.2273  | -0.0266  | -0.1270 | 0.2007     | n/a       |
| HMA    | D*       | -0.2493  | -0.3077  | -0.2785 | -0.0585    | -29.1     |
| HMA    | G*       | -0.4945  | -0.1149  | -0.3047 | 0.3796     | +189.1    |
| HMA    | DG*      | -0.2506  | -0.1340  | -0.1923 | 0.1166     | +58.1     |

## S4 Geodetic residuals and component adjustments

Residuals are calculated from the fitted mass trajectories using the original 60 geodetic observations and their covariance, with all estimation parameters unchanged. Symmetric whitening uses the eigendecomposition  $C_D = U \Lambda U^T$  and  $z = U \Lambda^{-1/2} U^T e$ . Its squared norm equals  $e^T C_D^{-1} e$  and the original Cholesky-whitened norm. Grouping  $z_i^2$  is invariant to a consistent row permutation, unlike attributing sequential Cholesky innovations. Symmetric whitening combines correlated observations, so the grouped fractions quantify the distribution of covariance-weighted misfit, with no independent attribution to local error sources.

DG\* has correlated RMS 3.882576 and marginal RMS 1.951758; DG<sub>0</sub> has correlated RMS 3.764086 and marginal RMS 2.282841. The reversed ranking shows that discrepancy depends on both inter-observation covariance and marginal uncertainty. The native-support rates in Table S6 are observed covered-cohort rates and coverage-weighted model rates. Their regional sums refer to the observed cohort, whereas the CryoSat comparison uses the full inventory. Signed errors are accompanied by the marginal error scale of the regional sum, calculated from the relevant covariance submatrix. This scale excludes cross-period combinations; the full correlated statistic includes them.

Table S6. Native-support full-fit residuals. Observed rates, errors (predicted minus observed) and the marginal standard deviation of the regional summed rate are in Gt yr<sup>-1</sup>. Last two columns are percentages of each fit's full correlated squared residual under symmetric whitening. Sum  $\sigma$  is not the standardization used for those shares. The supplementary numerical data include all sixty source-period residuals and their covariance.

| RGI | Period    | Observed | DG <sub>0</sub> error | DG* error | Sum $\sigma$ | DG <sub>0</sub> share | DG* share |
|-----|-----------|----------|-----------------------|-----------|--------------|-----------------------|-----------|
| 13  | 2000-2005 | -2.643   | -3.754                | -5.985    | 5.014        | 2.53                  | 0.38      |
| 13  | 2005-2010 | -7.652   | +3.450                | +2.097    | 4.614        | 6.57                  | 6.68      |
| 13  | 2010-2015 | -12.152  | +0.733                | +3.672    | 4.704        | 20.39                 | 19.09     |
| 13  | 2015-2020 | -15.942  | +10.557               | +9.622    | 4.953        | 15.65                 | 16.51     |
| 14  | 2000-2005 | -2.903   | +7.622                | +3.816    | 6.266        | 0.31                  | 2.52      |
| 14  | 2005-2010 | -4.200   | -1.562                | -0.406    | 5.616        | 13.16                 | 13.72     |
| 14  | 2010-2015 | -4.933   | -5.053                | -1.933    | 5.478        | 19.04                 | 21.02     |
| 14  | 2015-2020 | -6.156   | +11.489               | +7.701    | 5.914        | 15.88                 | 13.15     |
| 15  | 2000-2005 | -5.785   | +1.428                | -1.036    | 2.827        | 0.22                  | 0.10      |
| 15  | 2005-2010 | -6.561   | -2.467                | -1.879    | 2.655        | 1.02                  | 1.16      |
| 15  | 2010-2015 | -7.164   | -0.962                | -1.307    | 2.654        | 3.11                  | 3.17      |
| 15  | 2015-2020 | -7.940   | +0.501                | +0.713    | 2.806        | 2.13                  | 2.49      |

Component rates in Table S7 are calculated from the fitted source trajectories using January endpoints and actual elapsed durations. DG<sub>0</sub>–D<sub>0</sub> isolates the addition of gravity, DG<sub>0</sub>–G<sub>0</sub> the addition of geodesy, and DG\*–DG<sub>0</sub> the change in regularization at fixed observation input. The accompanying numerical data contain the common-parameter trajectories and component sums, together with the original separately selected estimates. Non-glacier and exterior adjustments are integrated over their respective source supports. The common GLDAS mean cancels from state differences.

The gravity observations exclude the global degree-zero mode, leaving broad exterior loads dependent on their prior. The resulting exterior sums and compensating local adjustments quantify coupling among represented mass sources; they lack independent validation as storage budgets. Component-sum consistency and regional glacier accuracy are separate properties. The non-glacier adjustment represents an undifferentiated storage correction, with no attribution to groundwater or a specific hydrological process.

Table S7. Component mass-rate adjustments ( $\text{Gt yr}^{-1}$ ). Native endpoints and exact durations are used. G sums full inventory glacier groups; L and N sum their local source supports; E sums exterior tiles. The common GLDAS mean cancels. These coupled model adjustments retain their estimation dependence; E lacks an observed global monopole, and the component budgets lack independent validation.

| Comparison    | Native period | Glacier G | Lake L | Non-glacier N | Exterior E |
|---------------|---------------|-----------|--------|---------------|------------|
| $DG_0 - D_0$  | 2000-2005     | +6.41     | -5.69  | +3.23         | -2251.37   |
| $DG_0 - D_0$  | 2005-2010     | -0.90     | +7.52  | -30.84        | +1562.09   |
| $DG_0 - D_0$  | 2010-2015     | -5.09     | -7.00  | +64.77        | +533.11    |
| $DG_0 - D_0$  | 2015-2020     | +21.58    | +5.94  | -89.42        | +322.95    |
| $DG_0 - G_0$  | 2000-2005     | +19.02    | +3.59  | +32.24        | -41.01     |
| $DG_0 - G_0$  | 2005-2010     | +30.27    | -5.07  | -26.43        | -112.37    |
| $DG_0 - G_0$  | 2010-2015     | +19.41    | -1.67  | -11.83        | +259.57    |
| $DG_0 - G_0$  | 2015-2020     | +16.35    | +3.10  | -21.37        | -129.60    |
| $DG^* - DG_0$ | 2000-2005     | -8.52     | +0.03  | -6.80         | -476.62    |
| $DG^* - DG_0$ | 2005-2010     | +0.40     | +0.40  | -11.37        | -572.31    |
| $DG^* - DG_0$ | 2010-2015     | +5.72     | -0.57  | +31.95        | +1911.32   |
| $DG^* - DG_0$ | 2015-2020     | -4.52     | +0.99  | -13.74        | -897.44    |

## S5 Additional evaluations and uncertainty scenarios

Table S8 summarizes the native-period omission evaluation. Its 36 regional-period values and 18 regional interperiod differences are dependent because they reuse six folds. The original archive contains the complete predictions, summarized regionally in Table S8. In each fold, the models use two native intervals and their known total; the full-history CryoSat evaluation uses all four intervals. The known total does not determine the contrast between the omitted periods. The two evaluations are summarized separately, with the complete set of regions retained in each.

Table S9 gives the modern-footprint comparisons. Pléiades west uses 2019-11-11 to 2022-11-07, and east uses 2019-10-29 to 2022-11-01. There are 51 western records and eight eastern records, with six glaciers appearing in both fields. Record errors are area-weighted within each glacier and field. Glaciers receive equal weight within each field, and both fields receive equal weight in the final MSE. Both fields belong to the Falaschi et al. (2023a) study. The seven-block summaries describe spatial variation within this single source family. The released fields include filled areas and object-specific raw-measurement coverage is unknown.

ATL06 version 007 provides the heights (Smith et al., 2025), with the retrieval described by Smith et al. (2019). The local-rate records use repeated-footprint pairs with the specified slope correction. The corresponding weights  $w_i$  are applied to model  $G(t_i)/\text{area}$  at the acquisition times, giving a rate  $\sum w_i G(t_i)/\text{area}$ . The supplied rates retain their adopted water-equivalent conversion. Records receive equal weight within each glacier, and glaciers receive equal weight within the single source group. The patches share this source dependence. Acquisition times and slope-correction weights for these records are retained in the original experiment archive.

Table S8. Native-period omission RMSEs. Thirty-six regional-period values and eighteen regional interperiod differences reuse six folds; they are not independent samples. The HMA aggregate is excluded. Each refitted D/DG estimate uses the two available periods and the known total.

| Method          | Rate (Gt yr <sup>-1</sup> ) | Rate (m w.e. yr <sup>-1</sup> ) | Interperiod (Gt yr <sup>-1</sup> ) | Interperiod (m w.e. yr <sup>-1</sup> ) |
|-----------------|-----------------------------|---------------------------------|------------------------------------|----------------------------------------|
| P               | 7.8984                      | 0.3101                          | 4.8808                             | 0.1128                                 |
| D <sub>0</sub>  | 1.2376                      | 0.0277                          | 2.4380                             | 0.0544                                 |
| G <sub>0</sub>  | 15.0687                     | 0.3359                          | 9.2474                             | 0.2540                                 |
| DG <sub>0</sub> | 6.6029                      | 0.1842                          | 10.6475                            | 0.2958                                 |
| D*              | 1.6979                      | 0.0441                          | 3.3920                             | 0.0880                                 |
| DG*             | 4.4187                      | 0.1217                          | 6.3213                             | 0.1724                                 |

Table S9. Modern-footprint RMSE. Pléiades elevation errors are in m yr<sup>-1</sup> for 762 and 850 kg m<sup>-3</sup> model conversions and equal weighting of two related source fields. ATL06 errors are in m w.e. yr<sup>-1</sup> on one source group; its supplied rates are not converted again. All records cross 2020.

| Estimate           | Pléiades: 762 | Pléiades: 850 | ATL06  |
|--------------------|---------------|---------------|--------|
| Local continuation | 0.6763        | 0.7127        | 0.6108 |
| P                  | 1.2890        | 1.2890        | 1.0911 |
| D <sub>0</sub>     | 0.7853        | 0.8339        | 0.7748 |
| G <sub>0</sub>     | 6.2285        | 5.4625        | 4.6387 |
| DG <sub>0</sub>    | 2.9333        | 2.5139        | 1.8259 |
| D*                 | 0.7926        | 0.8406        | 0.7790 |
| G*                 | 11.1364       | 9.8604        | 8.1306 |
| DG*                | 0.4780        | 0.4523        | 0.6194 |
| Background ×0.5    | 0.4676        | 0.4526        | 0.6167 |
| Background ×2      | 0.4815        | 0.4527        | 0.6202 |
| Independent errors | 0.7955        | 0.6459        | 0.7672 |
| Coherent errors    | 1.3937        | 1.1492        | 1.1482 |
| No GIA correction  | 0.4781        | 0.4520        | 0.6203 |

Table S10 presents sensitivity to the prescribed covariance and background assumptions around DG\*. These alternatives retain the original selection and characterize DG\* sensitivity; DG<sub>0</sub> has no corresponding complete sensitivity analysis. Table S9 includes the 850 kg m<sup>-3</sup> Pléiades conversion for comparison with the principal density assumption.

For submitted radar increments with standard errors  $\sigma_i$ , the independent scenario uses  $\text{diag}(\sigma_i^2)$ , the overlap proxy uses  $\sigma_i \sigma_j \max(0, 1 - |i - j|/3)$ , and the coherent scenario uses  $\sigma \sigma^T$ . Linear period sums and the mean/interperiod transformation propagate these covariances. A shared 60 kg m<sup>-3</sup> density term is added jointly to regional mean and interperiod uncertainty. Common area cancels from specific rates when the same area converts and normalizes mass; the  $\pm 10\%$  area scenario is separate in Gt. Cancellation of coherent errors in interperiod differences prevents this scenario from defining a monotonic upper confidence bound.

Table S10. Structural sensitivity of DG\* (m w.e. yr<sup>-1</sup>). Selection is independent of the external comparison. The glacier regularization remains a = 0.1, including in background and covariance alternatives.

| Estimate           | Rate RMSE | Spatial RMSE | Interperiod RMSE |
|--------------------|-----------|--------------|------------------|
| DG*                | 0.1227    | 0.1646       | 0.1871           |
| Background ×0.5    | 0.1190    | 0.1578       | 0.1858           |
| Background ×2      | 0.1259    | 0.1689       | 0.1934           |
| Independent errors | 0.1211    | 0.1449       | 0.1768           |
| Coherent errors    | 0.1384    | 0.1485       | 0.1859           |
| No GIA correction  | 0.1313    | 0.1597       | 0.1989           |

Table S11. Conditional propagated source-error scales (m w.e. yr<sup>-1</sup>), including shared 60 kg m<sup>-3</sup> density uncertainty. The scales follow the stated covariance scenarios and have no confidence-level calibration. Covariance between the mean and interperiod change is provided in the supplementary numerical data. The coherent scenario can reduce interperiod uncertainty through cancellation.

| Region | Increment-error scenario  | Mean scale | Interperiod scale |
|--------|---------------------------|------------|-------------------|
| RGI13  | independent increments    | 0.1713     | 0.3422            |
| RGI13  | 90day overlap proxy       | 0.2947     | 0.5845            |
| RGI13  | fully coherent increments | 1.6494     | 0.2168            |
| RGI14  | independent increments    | 0.3809     | 0.7627            |
| RGI14  | 90day overlap proxy       | 0.6563     | 1.3014            |
| RGI14  | fully coherent increments | 3.7084     | 0.2250            |
| RGI15  | independent increments    | 0.5886     | 1.1754            |
| RGI15  | 90day overlap proxy       | 1.0098     | 1.9985            |
| RGI15  | fully coherent increments | 5.6412     | 1.0198            |

The independent-increment interperiod scales are 0.342, 0.763 and 1.175 m w.e. yr<sup>-1</sup> for RGI13, RGI14 and RGI15; the overlap proxy raises them to 0.584, 1.301 and 1.999. These scales include the specified density scenario and depend on the assumed increment covariance. Calibration of confidence intervals and significance tests for the three regional interperiod signs requires covariance estimated from the underlying observations. A constant reference-height offset cancels; time-varying penetration, sampling and regionalization errors remain unresolved.

Replacing the 90-day endpoint means with instantaneous endpoints changes DG\* regional rates by at most 0.041 Gt yr<sup>-1</sup>. This comparison quantifies sensitivity to model temporal averaging; radar-return weights are unavailable. The density and area scenarios retain the fitted models and physical footprints. They quantify conversion uncertainty at fixed geometry and geodetic fit. Glacier-boundary changes are outside this comparison.

## S6 Model coefficients and spatial fields

The common-parameter estimates are stored in full\_D\_common.npz, full\_G.npz and full\_DG\_common.npz. SHARED\_STATE\_DG.nc and EVALUATED\_PERIOD\_FIELDS.nc contain the

separately selected DG\* estimate at  $a = 0.1$ . The latter represents glacier, lake, non-glacier and local total mass on a  $0.25^\circ$  reporting grid, with source-region identifiers for glacier aggregation. Aggregation of these fields reproduces the corresponding DG\* regional rates within  $7.2 \times 10^{-15} \text{ Gt yr}^{-1}$ . File identities distinguish the common-parameter estimates from DG\*.

The accompanying analysis materials identify the observation arrays, estimation code, temporal operators and coefficients used for each comparison. Public input sources are listed in Table S1. A public repository identifier for the derived study materials has not yet been assigned.

## S7 Fixed native endpoint decomposition

The decomposition is calculated from the common-parameter  $D_0$  and  $DG_0$  mass trajectories. For each glacier source, the difference trajectory is evaluated at 1 January 2000, 2005, 2010, 2015 and 2020. A piecewise-linear bridge through these five values defines the endpoint adjustment. The remainder is expressed in the same 26-coefficient basis and has zero response under the original coverage-weighted geodetic operator. Attribution is evaluated over 2000 to 2020. The constant bridge extension after 2020 lies outside these evaluated periods.

Both adjustment components are evaluated using the identical 90-day centred endpoint operator applied to the fitted estimates. Each component is added separately to  $D_0$  without refitting the remaining mass states. These diagnostic trajectories isolate contributions within the existing fit. The external MSE gain retains the interaction term in Eq. (13). Seasonal and non-seasonal contributions are evaluated with the same endpoint bridge. The maximum seasonal contribution to the six within-interval rate updates is  $0.002265 \text{ m w.e. yr}^{-1}$ .

Table S12. External specific-rate RMSEs of the fitted estimates and diagnostic component additions ( $\text{m w.e. yr}^{-1}$ ).

| Trajectory              | Period   | Mean     | Interperiod | Spatial  |
|-------------------------|----------|----------|-------------|----------|
| $D_0$                   | 0.226602 | 0.146430 | 0.345871    | 0.259573 |
| $G_0$                   | 0.304699 | 0.296239 | 0.142606    | 0.366233 |
| $DG_0$                  | 0.124119 | 0.096015 | 0.157308    | 0.152377 |
| $D_0 + \text{endpoint}$ | 0.095680 | 0.072209 | 0.125546    | 0.107826 |
| $D_0 + \text{within}$   | 0.246270 | 0.165555 | 0.364639    | 0.293708 |

Table S13. Specific-rate MSE gain and its interaction term ( $(\text{m w.e. yr}^{-1})^2$ ). Negative values increase error. Regional rows refer to their two period values; the all-region result gives equal weighting to the three regions.

| Region      | Endpoint alone | Within alone | Interaction | Total gain |
|-------------|----------------|--------------|-------------|------------|
| All regions | +0.042194      | -0.009300    | +0.003049   | +0.035943  |
| RG113       | +0.015082      | +0.002449    | -0.005495   | +0.012037  |
| RG114       | +0.105920      | -0.026466    | +0.014042   | +0.093496  |
| RG115       | +0.005579      | -0.003883    | +0.000601   | +0.002297  |

## S8 Native observation fits and conditional uncertainty

The reconstructed gravity fields are obtained from  $T X^T A^T$  plus the referenced GLDAS contribution on the non-glacier source columns. Gravity residuals of all three estimates are evaluated under the structured

error covariance used for  $DG_0$ . This gives standardized RMS 1.221546, 0.998035 and 0.998860 for  $D_0$ ,  $G_0$  and  $DG_0$ . Degree-dependent residuals are standardized by formal uncertainties alone and therefore differ from the covariance-weighted metric.

The original 60 geodetic source-period rows are evaluated on the observed-area support. Regional full-inventory rates are also supplied in the numerical tables after dividing each source contribution by its fixed coverage fraction. Labels distinguish the observed-area and full-inventory quantities. Symmetric whitening uses the positive eigendecomposition of the complete covariance; its squared coordinates sum to the original correlated quadratic form. The relative numerical difference is at most  $6.3 \times 10^{-15}$ .

Forty-six lake geometries enter the state, but retained observations occur at 44 sources and 6,363 dates. LI52\_0013 and LI52\_0026 have no retained native series. For each observed lake, the fitted constant is recovered under the same mission-correlated covariance and projected out when calculating residuals. Median standardized RMS is approximately 1.75 across lakes for each common-parameter estimate. This statistic quantifies fit consistency under the prescribed lake temporal representation.

Table S14. Conditional model standard deviations of the six regional-period specific mass rates (m w.e.  $\text{yr}^{-1}$ ). These values exclude uncertainty in the source geometry, covariance assumptions, external sensor errors and density or area conversion.

| Region | Period    | $D_0$    | $G_0$    | $DG_0$   |
|--------|-----------|----------|----------|----------|
| RGI13  | 2011-2015 | 0.071296 | 0.053645 | 0.039414 |
| RGI13  | 2015-2019 | 0.080516 | 0.065307 | 0.042654 |
| RGI14  | 2011-2015 | 0.129599 | 0.062514 | 0.057025 |
| RGI14  | 2015-2019 | 0.145988 | 0.067530 | 0.059853 |
| RGI15  | 2011-2015 | 0.118740 | 0.068054 | 0.063967 |
| RGI15  | 2015-2019 | 0.140966 | 0.076626 | 0.070930 |

## S9 Numerical evaluation of observation operators

The common-parameter model contains  $142 \times 26$  coefficients and predicts 227 gravity solutions, each containing 3,715 spherical-harmonic coefficients. The spatial loading matrix  $A$  and observation-window matrix  $T$  define the gravity operator in factorized form. Geodetic observations constrain interval mass differences on their observed-area support, while lake observations constrain states at individual acquisition dates.

Figure 3 illustrates the 2010 to 2015 geodetic interval, the January 2015 gravity observation window, the lake observation acquired on 15 December 2014, and the January 2011 to January 2015 external comparison. Coefficient weights are displayed without row normalization. The CryoSat weights represent an endpoint-averaged mass difference; the corresponding rate is obtained by dividing by the interval duration.

Independent evaluation of the common-parameter coefficients reproduces the external period rates within  $3.2 \times 10^{-14} \text{ Gt yr}^{-1}$  and the predicted gravity coefficients within  $1.7 \times 10^{-16}$  in the scaled coefficient units. The maximum response of the within-interval diagnostic under the geodetic operator is  $2.04 \times 10^{-14} \text{ Gt}$ . Aggregation of the original CryoSat increments reproduces the external specific rates within  $5.6 \times 10^{-17} \text{ m w.e. yr}^{-1}$ . These differences quantify numerical precision and verify that the rate and observation operators retain their intended temporal support.

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