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Clément Ubelmann¹, Cécile Anadon², Antoine Delepouille¹,
Gérald Dibarboure³, and Matthias Raynal³

¹ Datlas, Grenoble, France

² CLS, Toulouse, France

³ CNES, Toulouse, France

Clément Ubelmann — clement.ubelmann@datlas.fr

Cécile Anadon — cecile.anadon@groupcls.com

Antoine Delepouille — antoine.delepouille@datlas.fr

Gérald Dibarboure — gerald.dibarboure@cnes.fr

Matthias Raynal — matthias.raynal@cnes.fr

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Clément Ubelmann,^a Cécile Anadon,^b Antoine Delepoulle,^a Gérald Dibarboure,^c and Matthias Raynal^c

^a *Datlas, Grenoble, France*

^b *CLS, Toulouse, France*

^c *CNES, Toulouse, France*

Corresponding author: Clément Ubelmann, clement.ubelmann@datlas.fr

ABSTRACT

23 The SWOT KaRIN instrument provides wide-swath Sea Surface Height measurements
24 affected by cross-swath systematic errors largely driven by thermo-elastic distortions and
25 attitude knowledge errors. Using more than two years of science-orbit data, we evaluate a
26 calibration procedure structured in two principal steps to estimate the systematic errors. The
27 first step applies an empirical modeling approach—combining a solar–orbital model and a
28 geographically fixed model. This empirical phase reduces the dominant linear swath-tilt
29 variability from 17cm to 7cm. The second step implements a localized Optimal Interpolation
30 framework that exploits the k^{-2} power-law behavior of the remaining attitude errors to isolate
31 them from actual SLA signal to preserve. This combined approach reduces the final residual
32 formal error of the linear term to ~ 1 cm over open oceans and ~ 3 cm over continental regions,
33 providing a calibration scheme for global oceanographic and hydrological applications.

34 1. Introduction

35 The SWOT KaRIN instrument delivers Sea Level Anomalies (SLA) over a 120-km-wide
36 swath. These measurements contain both random errors, with weak or short spatial
37 correlation, and systematic errors that evolve slowly in the along-track direction. First post-
38 launch characterizations of systematic errors (Peral et al., 2024; Ubelmann et al., 2024;
39 Jacobs et al., 2024) have confirmed that KaRIN systematic errors are contained within
40 requirements. As anticipated, they are characterized by low-order polynomial functions of the
41 across-track coordinate, limited to degree two and largely dominated by the linear term
42 (swath tilt). A key feature of these errors is their strong energy at the orbital frequency and its
43 secondary harmonics, which poses significant challenges for calibration, particularly over
44 land.

45 With more than two years of data now available, a more detailed investigation of KaRIN
46 systematic errors has become possible. Over this period, the satellite has experienced
47 approximately 10 cycles of orbital plane rotation relative to the Sun, which largely controls
48 the thermal environment of the platform. This allows for the construction and evaluation of
49 empirical models of thermo-elastic distortion effects, aimed at explaining most of the
50 systematic errors using a limited number of parameters.

51 The first objective of this study is to provide this empirical description based on the
52 orientation of the orbital plane relative to the Sun and on the satellite position along the orbit.
53 We also characterize additional dependencies related to the yaw-flip state of the platform,
54 which are not yet fully understood.

55 The second adjective is to address an optimal calibration strategy to separate the
 56 systematic errors from actual SLA variability. The use of the empirical models will be the
 57 first step of the calibration that already explains the largest part of systematic errors. After its
 58 application, the spectral characterization of the signal will reveal clear power laws associated
 59 with Gyroscope systematic error residuals (-2 slope decay) and oceanic variability (spectral
 60 hump). We will propose an optimal least-square filter to isolate the systematic errors based
 61 on these spectra, that constitutes the second step of the calibration. Finally, the formal
 62 residual errors after calibration will be addressed.

63 2. The Solar-Orbital Frame

64 The SWOT orbit is non-sun-synchronous, meaning that the orbital plane (illustrated in the
 65 left panel of Figure 1) rotates around the Earth's poles relative to the Sun. The angle between
 66 the orbital plane and the sun is commonly noted β , varying between 0° and 180° as shown on
 67 the figure. This β angle primarily drives the thermal condition on the platform. However, for
 68 a given β that is non zero, the sun position in the platform reference evolves over the orbit
 69 cycle, with generally an earth eclipse during which the platform is in the earth eclips. To
 70 characterize this lightning cycle at a given β , we define the θ angle, varying between 0° and
 71 360° as shown on the figure, with respect to the vector e_θ which lies in the β plane and is
 72 orthogonal to the Sun direction. These two (β, θ) angles define the solar-orbital frame.

73 For SWOT orbit, the orbital plane rotation has an approximate period of 120 days. As
 74 illustrated on the right panel of Figure 1, β is not strictly repetitive over the successive cycles,
 75 as the Sun's position with respect to the equatorial plane also varies seasonally in the $\pm 23^\circ$
 76 earth inclination range. In particular, for certain combinations of season and orbital-plane
 77 position, β approaches zero. This situation occurred in August and October 2024. During
 78 these singular configurations, the Sun's position relative to the satellite remains nearly fixed
 79 over the orbit, potentially leading to strong and specific thermo-elastic responses.

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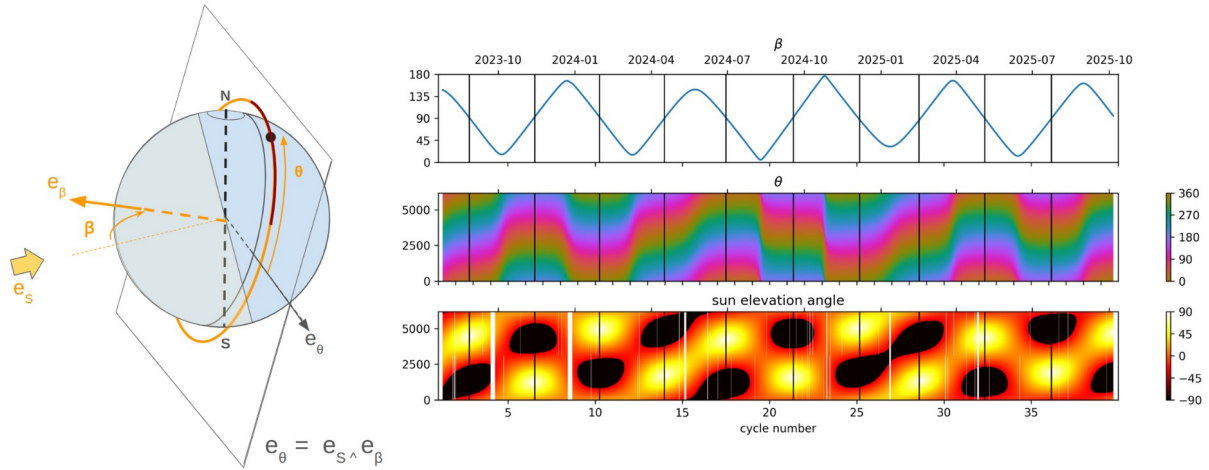


Fig. 1. Left panel: schematic representation of the orbital plane forming an angle β with respect to the Sun direction (yellow arrow). The satellite, represented by the black dot, rotates within this plane. The angle θ describes the satellite position along the orbit with respect to the vector e_θ , which lies in the β plane and is orthogonal to the Sun direction. Right panels: time evolution of the β angle between August 2023 and November 2025, with vertical lines indicating yaw-flip transitions occurring when β is close to 90° . The θ panel shows the evolution of θ as a function of time and orbital coordinate (in seconds). The Sun elevation angle, defined as $\arccos[\sin(\beta) \times \sin(\theta)]$, represents the angle between the satellite horizontal plane and the Sun direction. When this angle is below -28° , the satellite is in Earth eclipse, indicated by black shading.

3. Overview of the B, L, Q Terms

The systematic errors can be represented as polynomial functions of the across-track coordinate (noted x) up to degree two (bias, linear, and quadratic components; see Peral et al., 2024, Ubelmann et al., 2024). These polynomial coefficients are denoted as (B, L, Q, aB, aL, aQ). The terms (B, L, Q) correspond to the bias, linear, and quadratic components common to both swaths, while (aB, aL, aQ) describe the relative bias, linear, and quadratic differences between the right and left swaths. Under this formulation, the polynomial function writes:

$$\delta h(x) = B + \text{sgn}(x) * aB + x * L + |x| * aL + x^2 * Q + |x|x * aQ$$

These polynomial terms are estimated by a least-square fit across the swath for each along-track segment. When applied to the full SLA signal, this fitting procedure inevitably absorbs part of the oceanic variability projecting onto the polynomial basis. To mitigate this effect, we subtract a prior estimate of SLA variability derived from the nadir altimeter

constellation [DOI : 10.48670/moi-00145] independent from KaRIN data. The first part of this study focuses on very-low-frequency variability (longer than the orbital period), for which oceanic leakage into the polynomial fits is assumed negligible after applying the prior. For higher frequencies, the Ocean leakage becomes important even with the prior, and the separation between the two contributions will be tackled in section 6.

a. Variability with Beta Only (Low Frequency)

To investigate dependencies within the (β, θ) solar–orbital frame introduced above, we first focus on the β angle alone, as it evolves slowly in time compared to θ which varies continuously along the orbit. This slow evolution characterizes the very-low-frequency component of the systematic errors. We therefore analyze the temporal evolution of the polynomial coefficients averaged over a sliding one-orbit window and examine their dependence on β , as illustrated in Figure 2.

The KaRIN bias term can be analyzed jointly with the nadir altimeter bias (upper panel of Figure 2). The black and green curves represent the nadir and KaRIN biases relative to the prior, respectively. A significant correlation between the two highlights the presence of shared contributions, such as common orbital errors or geophysical signals affecting both instruments. Notable differences are also observed: in particular, the KaRIN bias exhibits enhanced variability at monthly timescales and an additional drift of approximately 0.5 cm yr^{-1} , as confirmed by the KaRIN–nadir difference shown in blue. However, regarding the β cycle, no clear dependence of the bias term is detected.

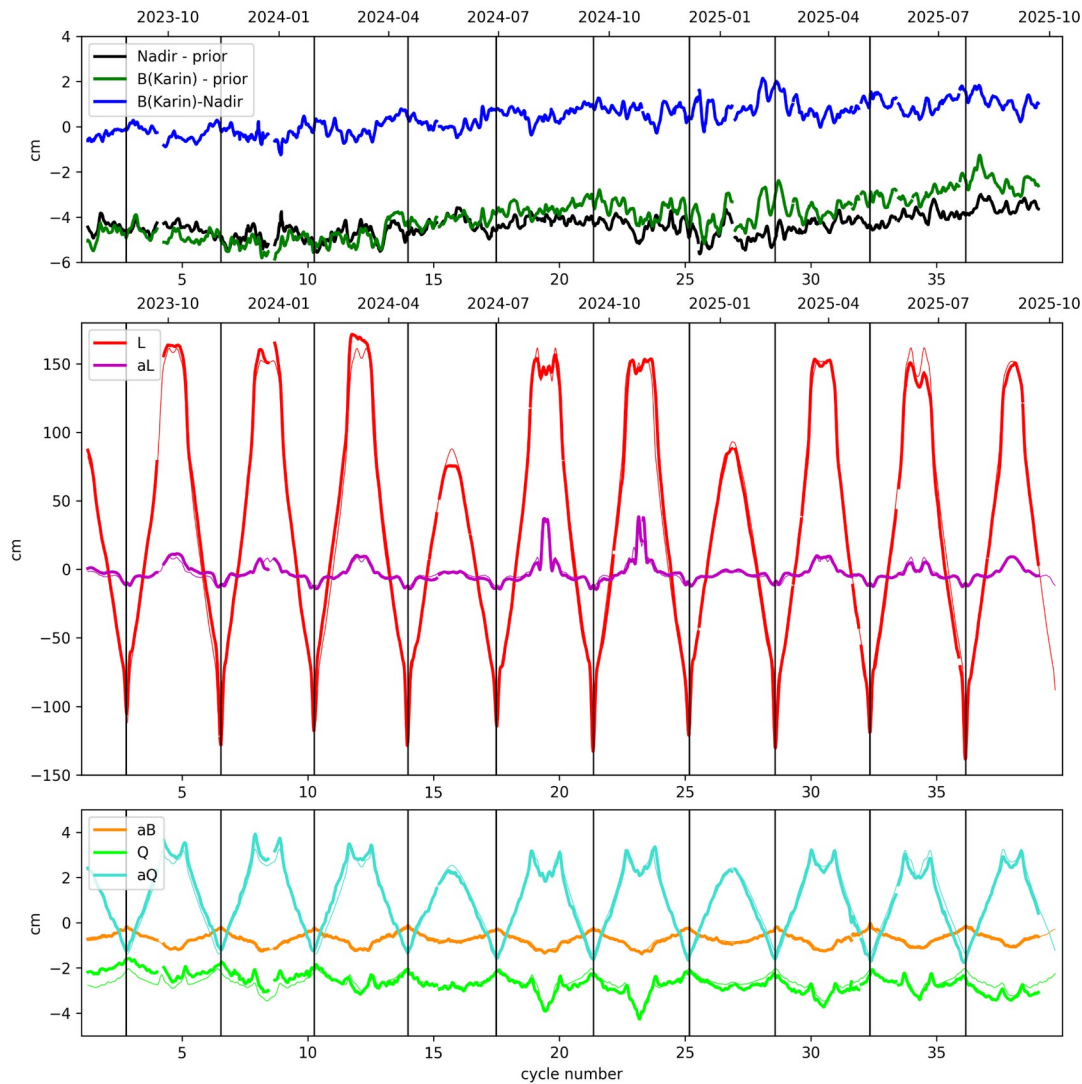
In contrast, the other KaRIN systematic error terms (aB , L , aL , Q , and aQ) show a clear dependence on the β cycle, as illustrated in the middle and lower panels of Figure 2. To quantify this dependence, a regression against β was fitted using the 11 available β cycles. The application of the fitted regressions are shown as thin curves of matching color. The difference between the fitted and observed signals represents the component not explained by β .

For the dominant linear term L , which exhibits variations exceeding 2 m peak-to-peak, the residual relative to the β fit is only about 20 centimeters. Similarly, for the aB , aL , Q , and aQ terms, the departures from the β fit remain small compared to their respective total variabilities. During the two episodes when β approaches zero, the aL term (purple curve) features particularly strong anomalies. A slow drift of approximately 0.3 cm yr^{-1} is also

138 observed in the Q term, which is not captured by the β fit and may reflect a gradual evolution
 139 of the interferometric baseline over the mission duration.

140 From a calibration perspective, these results indicate that a β -based empirical model
 141 provides a relevant description for predicting the very-low frequency behavior of all
 142 systematic error terms, with the exception of the bias that will be discussed in section 7.

143



144

145 Fig. 2. Upper panel: one-orbit sliding mean and low-pass filtered (1-day) time series of
 146 the bias terms indicated in the legend. The prior corresponds to the gridded SLA derived from
 147 the nadir satellite constellation excluding SWOT. Middle panel: same representation for the
 148 linear (L) and anti-linear (aL) terms, converted to centimeters at 40 km cross-track distance
 149 (1 cm at 40 km corresponds to 0.25 μ rad).

151 *b. Orbital Variability in the (β , θ) Frame*

Beyond the very low-frequency explained by β , systematic errors exhibit pronounced variability at the scale of individual orbits due to cyclic thermal forcing. From a calibration standpoint, this orbital-scale variability is critical, as it affects spatial scales that may be contaminated by oceanic leakage if not properly modeled.

To examine this behavior, we construct two-dimensional synoptic representations of the polynomial coefficients as functions of orbital coordinate (divided into 128 along-orbit segments) and orbit start time (low-frequency temporal axis). Figure 3 shows the results for the bias term, with nadir data included to isolate the KaRIN-specific contribution.

Consistent with the analysis of mean biases, no clear dependence on the β cycle is observed for the bias term. Instead, prominent patterns appear that are largely symmetric between ascending (0–3000 s) and descending (3000–6000 s) passes, suggesting a dominant dependence on geographical latitude rather than on orbital position. These structures are likely associated with geophysical errors, such as wave-related biases, particularly evident in the Southern Ocean latitudes where red bands appear in the (B–nadir) panel. Their amplification during austral winter further supports a geophysical origin likely related to sea state. Consequently, no empirical (β , θ) model is proposed for the bias term. A more in-depth analysis of geophysical error terms could later explain some patterns shown in this figure. A spectral characterization of this bias is proposed in section 7.

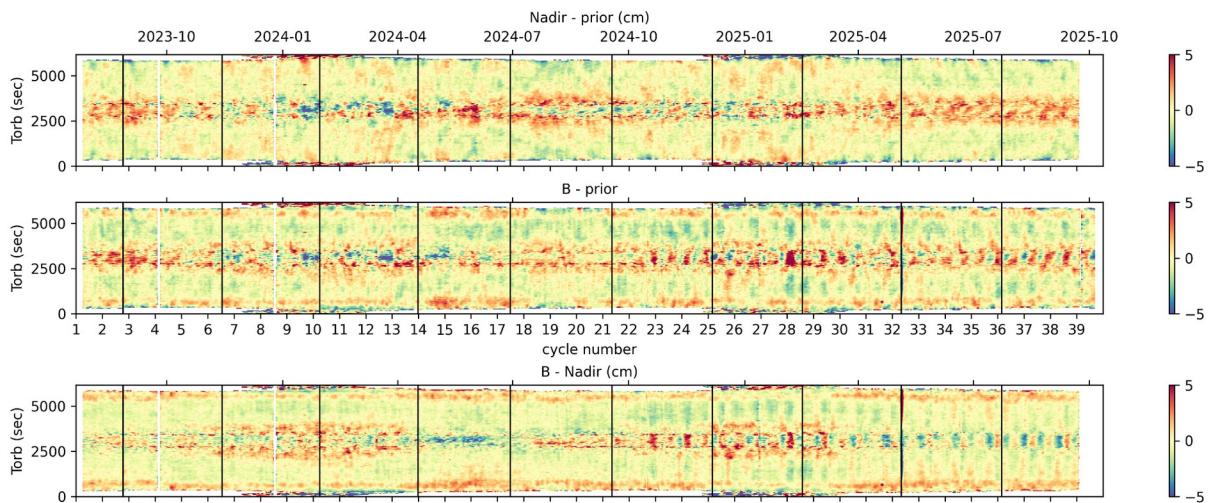


Fig. 3. Upper panel: difference between Level-2 SWOT nadir SLA and the prior (gridded SLA from the nadir constellation excluding SWOT) shown as a function of time and orbital coordinate. Middle panel: same representation for the KaRIN B term relative to the prior. Lower panel: KaRIN bias relative to nadir, defined as the difference between the two upper panels.

In contrast, the remaining polynomial terms (aB, L, aL, Q, and aQ), shown in Figure 4, feature markedly different behavior. Their variability is modulated by the β cycle and exhibits clear patterns consistent with the θ distribution shown in Figure 1. Notably, the relative bias term aB shows clear signatures associated with the Sun elevation angle: contours corresponding to eclipse transitions coincide with positive anomalies of approximately 4 mm, suggesting perturbations related to eclipse entry and exit.

The dominant linear term L exhibits orbital-scale variations reaching ± 40 cm, largely organized by θ and modulated by β . Similar structures are observed for the aL, Q, and aQ terms, although with much smaller amplitudes on the order of 1–2 cm.

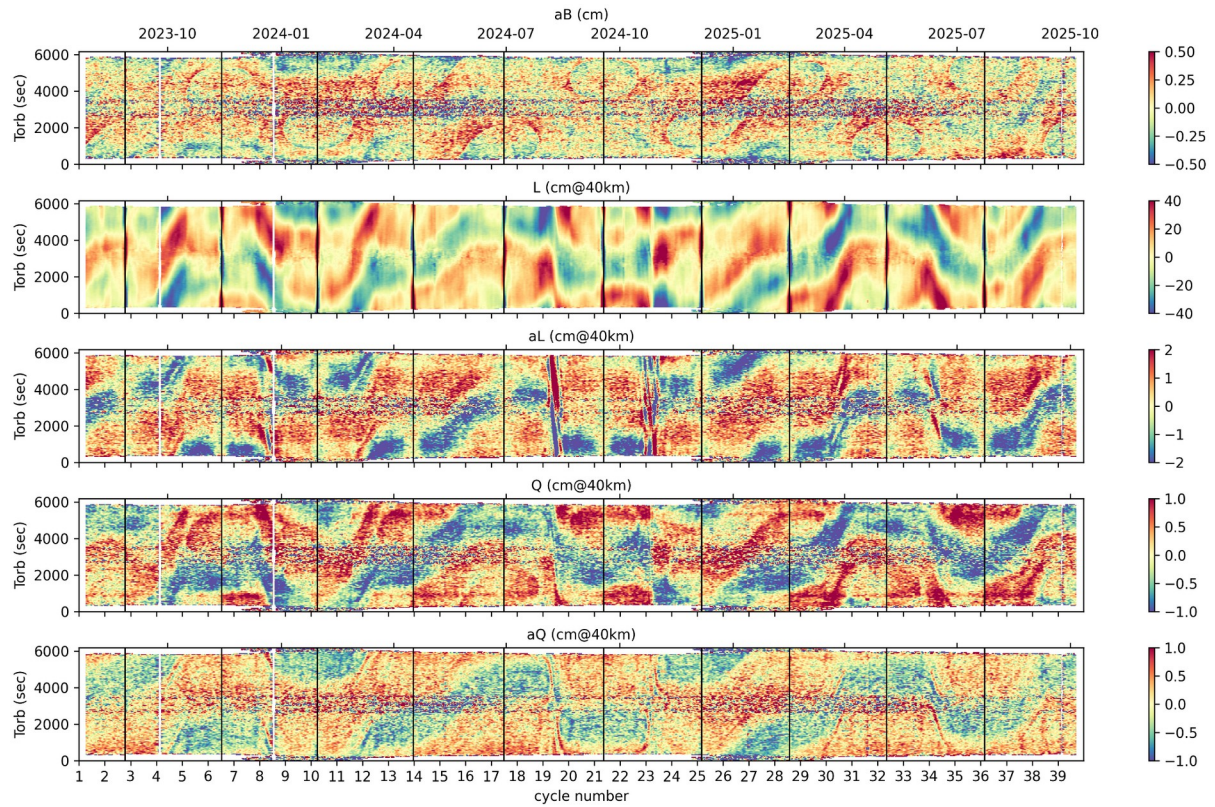


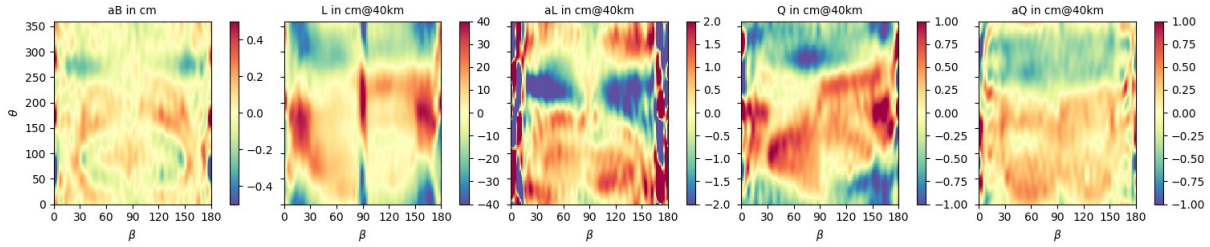
Fig. 4. Upper panel: orbital anomalies of the aB term (cm) as functions of time and orbital coordinate. Other panels: same representation for the L, aL, Q, and aQ terms, expressed in centimeters at 40 km cross-track distance.

4. The (β , θ) Model Fit

The variability of systematic error terms along the orbital coordinate and its clear dependencies with the β and θ angles shown in the last section justify the construction of empirical tables with these two angle parameters. To do so, we stacked all values of the

195 systematic error terms along 128 bins of β and θ values. Figure 5 represents the tables for the
 196 aB, L, aL, Q and aQ terms.

197



198

199 Fig. 5. Fit of the systematic error terms along β and θ .

200

201 We note the imprint of the earth eclipse zone on the aB term left panel. We also note the
 202 specific L perturbation for $\beta=90^\circ$, and the aL perturbation for β approaching 0° . Beyond this,
 203 we do not yet propose any interpretation of the observed features. However, their consistent
 204 character justifies the empirical table model as a prediction for the calibration.

205 The projection obtained from the (β, θ) model fit is shown in the left panels of Figure 6.
 206 Of particular interest at this stage are the residuals not captured by the model, displayed in the
 207 right panels of Figure 6.

208 While a large fraction of the variability is successfully removed (see the variance
 209 reduction between left and right panels), significant residual structures remain, especially for
 210 the dominant linear term L, with amplitudes reaching approximately 10-15 cm. These
 211 residuals therefore represent a non-negligible component of the systematic error that must be
 212 characterized. A zoom into the residuals over a single 21-day cycle reveals their higher-
 213 frequency content, as shown in Figure 7. For all terms except L, the residual variability
 214 appears largely white, consistent with geophysical leakage previously identified in
 215 polynomial fitting procedures (Ubelmann et al., 2024). However, the L term exhibits a
 216 pronounced oscillatory signal with a characteristic period of approximately one day and an
 217 amplitude of about 10 cm. The nature of this signal is investigated in the following section.

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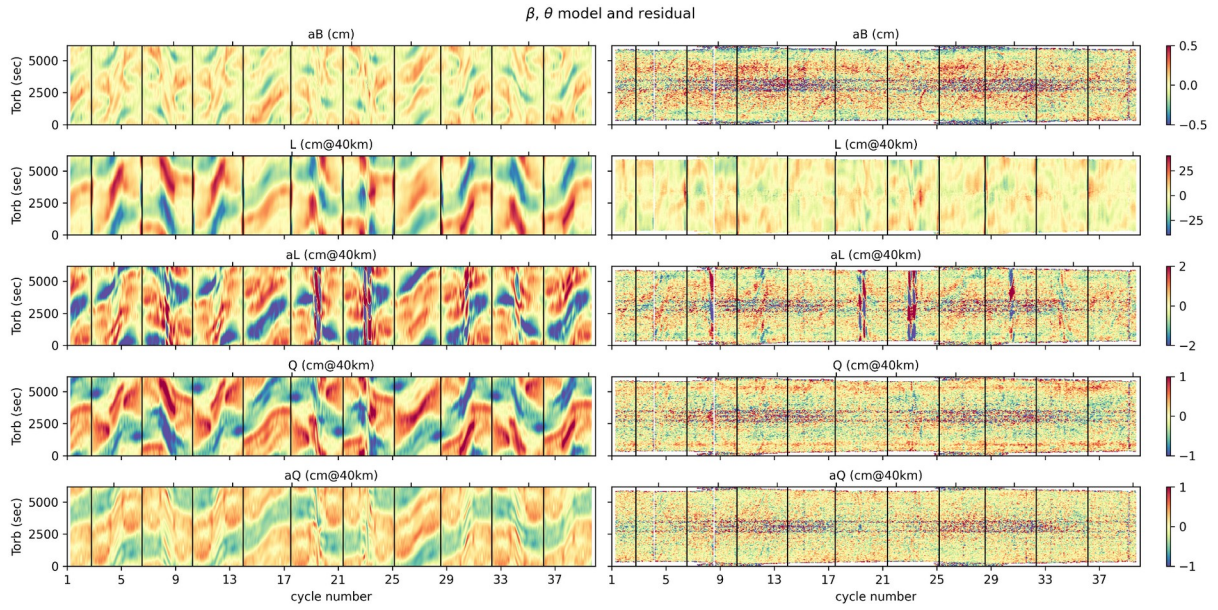


Fig. 6. Left panels: projection of the (β, θ) empirical fit for the aB , L , aL , Q , and aQ terms onto the time–orbital coordinate space. Right panels: residual anomalies after removing the (β, θ) fit.



Fig. 7. Same as the right panels of Fig. 6, zoomed over cycle #5.

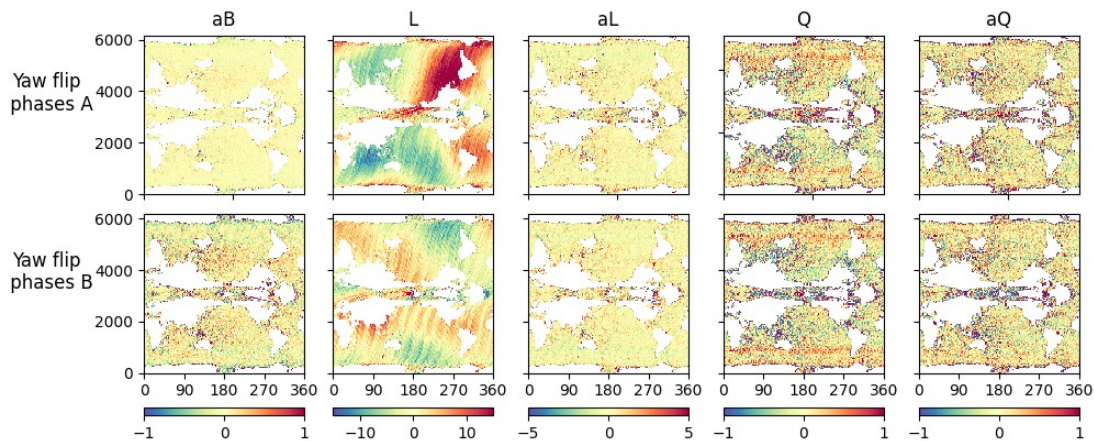
227 5. The Geographical Model Fit

228 Beyond the (β , θ) fit, the ~ 1 -day oscillatory patterns in the residuals on the L term is
 229 analysed here, in the objective of constructing a second empirical fit to capture additional
 230 systematic errors.

231 If the time series first suggests the presence of a dominant cycle near 1-day, the
 232 examination of the signal over extended time windows (>50 days) reveals that the phase is
 233 locked to the geographical position of the ground track (but different for ascending and
 234 descending swaths). The 1-day being therefore the aliasing manifestation of the geographical
 235 patterns with the 1-day sub-cycle of the SWOT science orbit. A striking and unexpected
 236 result is that both the amplitude and phase of this signal alternate between successive yaw-
 237 flip phases of the satellite. To better understand this behavior, we computed the temporal
 238 mean of each residual term as a function of longitude (x-axis) and orbital time (y-axis),
 239 separately for the two yaw-flip configurations. The resulting maps are shown in Figure 8.

240 This analysis confirms the presence of a persistent spatial pattern on the L term only. The
 241 amplitude indeed reaches up to 10 cm during yaw-flip phase A, consistent with the time-
 242 series analysis shown in Figure 7. For a given yaw-flip phase, the pattern differs markedly
 243 between ascending and descending passes. In particular, the eastern Pacific exhibits a strong
 244 anomaly during descending passes in yaw-flip phase A. None of the other polynomial terms
 245 display comparable spatial organization, with the exception of very weak signatures in Q near
 246 45°S (orbital times around 1000 s and 5200 s), which are also visible in Figure 6.

247



248

249 Fig. 8. Geographical distributions of the residual polynomial terms, separated by yaw-flip
 250 phase ($\beta > 90^\circ$ in the upper panels and $\beta < 90^\circ$ in the lower panels).

251

At present, no physical explanation has been identified for the origin of this systematic geographical pattern in the L term, nor for its alternation between yaw-flip phases. Nevertheless, given its large amplitude and clear predictability, this signal represents an important component to include in the empirical calibration strategy.

Because the raw geographical maps shown in Figure 8 remain slightly noisy, we apply a bi-harmonic low-pass filter retaining the first six harmonics along both the longitudinal and orbital-time dimensions. The resulting smoothed patterns are shown in Figure 9 and constitute the geographical component of the empirical model fit.

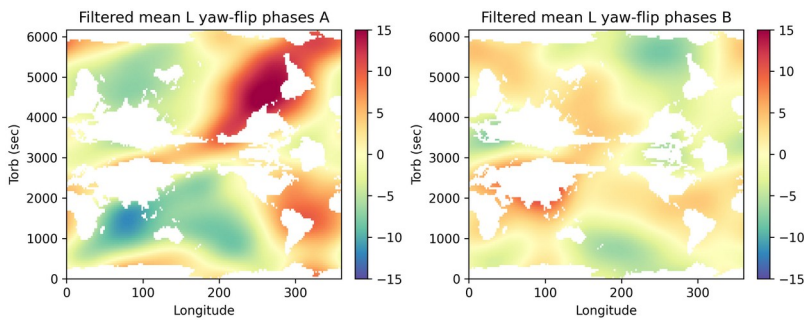
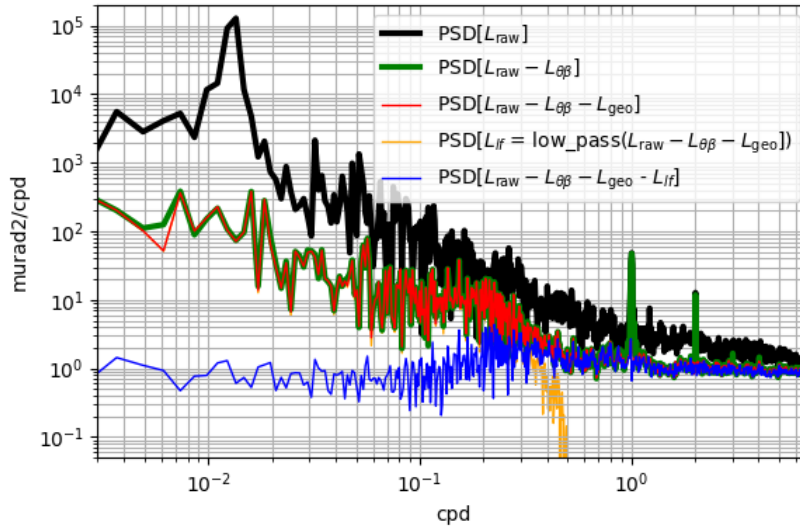


Fig. 9. Same as Fig. 8 for the L term only, after bi-harmonic low-pass filtering (first 6 harmonics along the orbital axis and first 6 harmonics along the zonal axis).

6. Spectral Assessment

The empirical model so far consists of two successive steps: (i) the (β, θ) model fit, and (ii) the geographical ascending/descending model fit applied separately for each yaw-flip phase. A spectral analysis is now proposed to quantify the explained and residual signal as a function of temporal scales.

We first consider spectra computed along the low-frequency (inter-orbit) time dimension at fixed orbital position. The power spectral densities of the dominant L term are averaged over all orbital positions and shown in Figure 10. The black curve corresponds to the raw L signal and exhibits a pronounced energy maximum near 90 days. After removal of the (β, θ) fit, the energy is strongly reduced at all time scales (green curve). A prominent spectral peak remains at the 1-day frequency, along with its secondary harmonics. These peaks reflect the aliased signature of the fixed geographical pattern: indeed, when the geographical fit is removed, the peaks disappear entirely (red curve). This result is encouraging as it confirms that the simple geographical model fit was sufficient to explain the 1-day oscillatory patterns.



280

281 Fig. 10. Power spectral density of the L term along the low-frequency time dimension.
 282 Black: raw signal. Green: after (β, θ) fit removal. Red: after removal of both (β, θ) and
 283 geographical components. Orange: 3-day low-pass filter of the Red. Blue: raw signal after
 284 removing the (β, θ) , geographical and low-frequency components.

285

286 After the (β, θ) and geographical fits, the residual signal features a more continuous
 287 spectral law, with a plateau at intermediate frequencies (> 3 days, right side of the graph) and
 288 a spectrum of slope near -1 above (left side of the graph). This latter part should not contain
 289 actual ocean signals that would have no reason to feature growing energy beyond the orbital
 290 period. Therefore we chose to isolate it with a 3-day low-pass filter, resulting in the orange
 291 spectrum on the figure. This constitutes a third additional part of the systematic error
 292 estimation, that can still be considered without Oceanic contamination. Then, the residual
 293 signal (blue curve) is likely constituted of systematic errors, but also actual Oceanic SLA
 294 projecting in the cross-swath slope L. A final separation based on Optimal Interpolation will
 295 be described in section 7.

296

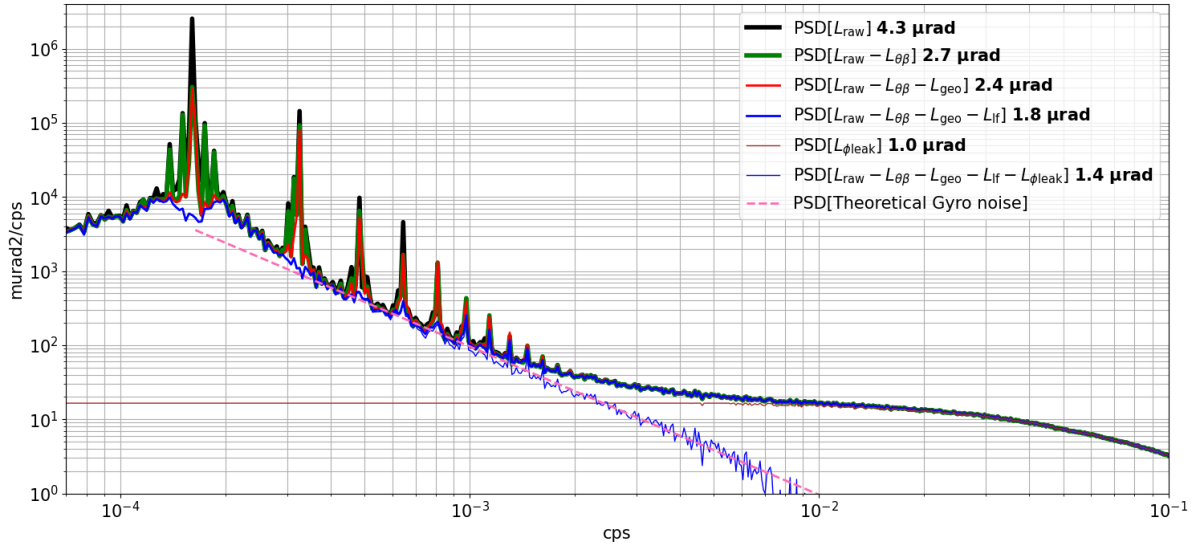


Fig. 11. Power spectral densities of the L term computed along the along-track time coordinate. Color coding identical to Figure 10.

Before tackling the separation, we also examine the spectral content of the L term when applying the successive estimates (two empirical fits plus the low frequency >3-day part), at medium frequencies using spectra computed along the SWOT grid time coordinate, following the methodology of Ubelmann et al. (2024). The resulting spectra, spanning over periods from 10,000 s to 10 s, are shown in Figure 11. The color coding is identical to Figure 10.

The raw signal (black curve) exhibits strong spectral peaks at the orbital frequency and its harmonics. Removal of the (β, θ) model fit leads to a significant attenuation of these peaks (green curve). Then, removal of the geographical component results in additional attenuation (red curve), particularly near the orbital harmonics. Finally, after removal of the low-frequency (>3 days) component, the residual signal spectrum (blue curve) no longer displays orbital peaks, indicating that most of the orbital signatures were actually the result of orbital aliasing of the (i) (β, θ) signal, (ii) the geographical signal and (iii) the >3days signal. Then, the residual energy follows a smooth spectral law over all frequencies as indicated by the blue curve. Below approximately 500 s, the spectrum reaches a plateau corresponding to actual SLA signal leaking in the polynomial fit of the L term (Ubelmann et al., 2024), before decreasing again below ~ 25 s.

By fitting and removing this leakage plateau in spectral space (pink horizontal line), the residual signal spectrum (thin blue curve) follows a clear k^{-2} power-law. Indeed, below the orbital frequency, roll residual errors were expected to be dominated by gyroscope noise,

whose theoretical spectrum was given by Esteban-Fernandez et al. (2016). The theoretical spectrum, converted to microradians, matches remarkably well the observed residual spectrum after leakage removal. Near the orbital frequency, a slight excess of energy remains, consistent with the suspected presence of residual star-tracker errors. This spectral characterization will be the basis for the optimal interpolation proposed in the next section, aiming at separating the residual systematic errors (not captured by the empirical models) from the actual SLA signal leakage to preserve in the KaRIN data.

Before optimal interpolation, we summarize here the successive contribution of the empirical model fits in microradians (μrad) of slope, from the integration of the power spectral densities up to twice the orbital period. The raw L signal exhibits a standard deviation of $4.3 \mu\text{rad}$ (equivalent to 17 cm at 40 km off-nadir). This is reduced to $2.7 \mu\text{rad}$ after removal of the (β, θ) fit, to $2.4 \mu\text{rad}$ after removal of the geographical component, and to $1.8 \mu\text{rad}$ after removal of the low-frequency (>3 day) fits. Actual SLA leakage accounts for approximately $1 \mu\text{rad}$ of variance. The remaining residual error (thin blue spectrum) is approximately $1.4 \mu\text{rad}$, corresponding to 5.6 cm at 40 km off-nadir.

7. Optimal Interpolation of the L Term

a. The Optimal Interpolation Framework

After removing the model fits from the raw signal, the residual variability—mostly composed of gyroscope and star-tracker errors together with geophysical leakage—is well characterized in spectral space (Figure 11). The calibration challenge is then to separate these contributions in physical space, so as to separate optimally actual systematic errors from the Sea Level Anomaly observations (and preserve the contribution from the leakage).

We therefore define the observation vector $\mathbf{y}$ as the fitted L term, which contains both the systematic error contribution and the geophysical leakage. The quantity to be estimated is the systematic error, denoted $\mathbf{x}$, while the geophysical leakage is treated as an observation error $\boldsymbol{\epsilon}$. If $\mathbf{H}$ denotes the observation operator mapping the continuous along-track grid (on which $\mathbf{x}$ is defined) to the observation space (where the L fit is available over ocean-only, unedited data), the observation equation writes: $\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\epsilon}$.

Let $\mathbf{B}$ be the covariance matrix of the signal $\mathbf{x}$, and $\mathbf{R}$ the covariance matrix of the observation error $\boldsymbol{\epsilon}$. The optimal interpolation (OI) estimate of $\mathbf{x}$, denoted $\mathbf{x}_a$, is given by the standard OI formula. This classical OI framework can be efficiently applied provided that the

number of observations remains manageable (typically $O(10^4)$). In practice, we consider segments of three consecutive orbits, with L fits sampled every 10 km, leading to approximately 15,000 observations per window. The accuracy of the estimate $\mathbf{x}_a$ and its associated uncertainty $\mathbf{P}_a$ critically depends on the correct specification of the signal covariance $\mathbf{B}$ and the observation error covariance $\mathbf{R}$.

b. Covariances of the Signal and Observation Errors

The spectra shown in Figure 11 can be used to derive the covariances for the matrices $\mathbf{B}$ (signal of systematic errors) and $\mathbf{R}$ (observation error from SLA leakage). From the power spectra, the inverse Fourier transform operator gives the associated autocovariance function.

However, the spectra in Figure 11 are global averages, whereas the SLA leakage is expected to vary strongly with geographical location. Since the level of the plateau can be easily estimated near the 100 s wavelength (corresponding to approximately 700 km along-track), we computed the spectral energy at this wavelength using geographically localized segments of 100 s duration. The resulting map is shown in Figure 12.

As expected, the geographical distribution closely resembles known patterns of oceanic mesoscale variability, with maxima located in energetic regions such as western boundary currents (Gulf Stream, Agulhas, Kuroshio, etc.). In these areas, the prior from the Level-4 gridded SLA is known to be less accurate, allowing more SLA variability to leak into the L fit. This strong similarity with mesoscale energy maps provides additional confidence that the identified spectral plateau indeed originates from SLA leakage, primarily due to SLA slope variability not resolved in the CMEMS Level-4 prior.

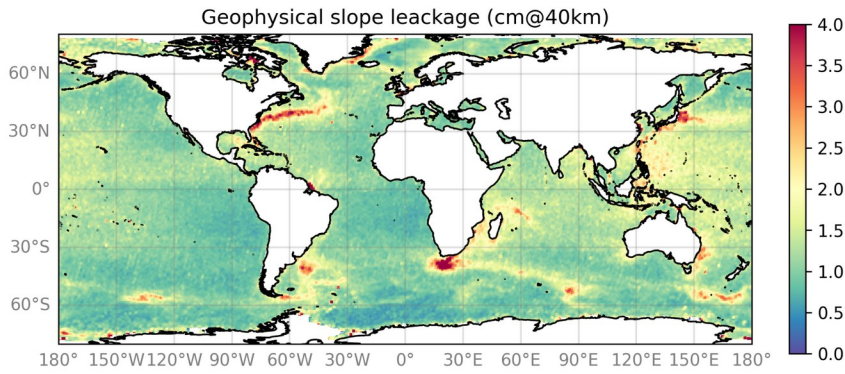


Fig. 12. Amplitude (cm at 40 km off-nadir) of the estimated geophysical leakage contribution in the L term.

Seasonal variations of this leakage are also expected, but we assumed them to be of secondary importance compared to geographical contrasts, and the available two-year dataset is likely insufficient to robustly resolve both effects. As a first-order approximation, the geographical distribution shown in Figure 12 is therefore used to define the observation error covariance matrix $\mathbf{R}$.

In contrast, the covariance of the signal to be estimated in matrix $\mathbf{B}$ (systematic error residual) is not expected to vary geographically. We assume that the averaged spectrum standing for the Gyroscope shown in Figure 11 is valid globally and at all times. This assumption is supported by the close agreement with the pre-launch expectations for gyroscope residual errors (Esteban-Fernandez, 2013) supposedly universal. The inverse Fourier transform of this spectrum directly provides the covariance function used to build the matrix $\mathbf{B}$.

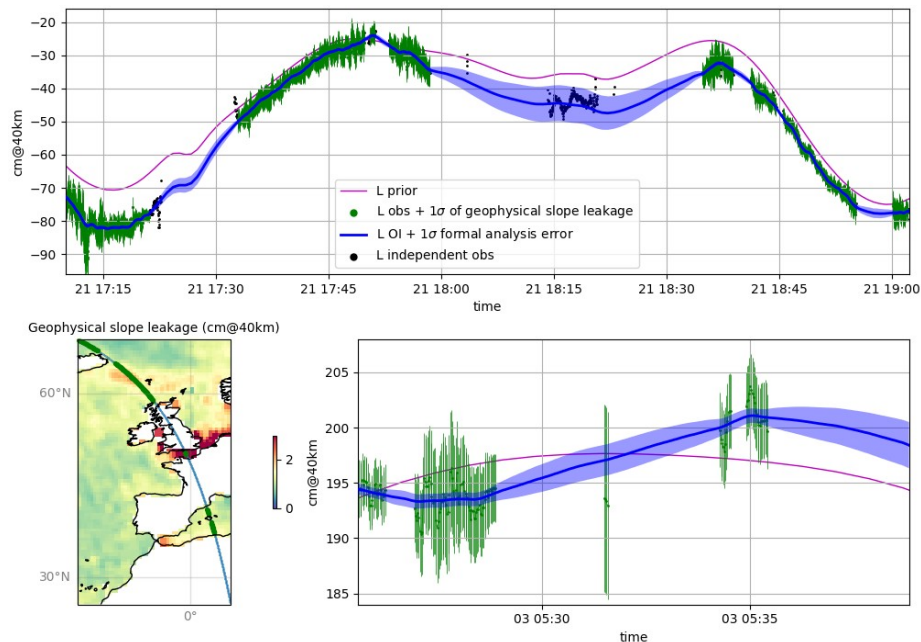
c. Results of the Optimal Interpolation

Using the implementation described above, the optimal interpolation is applied to the L fit observations over three-orbit windows, sliding by one orbit at a time. To avoid edge effects, only the central orbit of each window is retained. An illustrative example is shown in Figure 13. In the upper panel, the OI estimate is shown in blue (solid curve), with the shaded blue envelope representing the formal $1-\sigma$ uncertainty derived from the diagonal of $\mathbf{P}_a$. The prior estimate—obtained from the (β, θ) and geographical empirical models together with low-pass filtering—is shown in purple. The L fit observations are represented by green dots, with their

397 associated uncertainties (diagonal of $\mathbf{R}$) shown as green bars. Black dots indicate independent
398 L fit observations over the Arctic that are flagged in the Level-2 KaRIN product. In this
399 example, we can verify that independent observations of L are reasonably contained in the
400 estimated envelope of uncertainty.

401 The lower panels of Figure 13 illustrate the robustness of the OI scheme in regions
402 affected by potentially strong SLA leakage, such as the Channel Sea where tidal residuals are
403 large. The corresponding diagonal elements of $\mathbf{R}$ (zoomed from Figure 12) indeed indicate
404 large L observation errors in this region. As a result, the OI solution is only weakly
405 influenced by these observations, effectively preserving the underlying geophysical signal at
406 the cost of a slightly higher formal error.

407



408

409 Fig. 13. Illustration of the one-dimensional optimal interpolation of the L term from the
410 observations (green dots) and their estimated uncertainties (green bars). The OI estimate is
411 shown in blue, with its formal 1- σ analysis error. The black dots represent independent L fits
412 in the Arctic.

413

414 The optimal interpolation was applied to the full science orbit dataset. Figure 14 shows
415 seasonal mean geographical maps of the formal analysis error (1- σ), with boreal winter in the
416 upper panel and boreal summer in the lower panel. As expected, uncertainties increase away
417 from ocean-covered regions where reliable L fits are available. During boreal winter, sea-ice
418 coverage in the Arctic prevents accurate L estimation, leading to increased uncertainties over
419 continental regions from North America to Asia.

Over open oceans, uncertainties are significantly reduced thanks to the dense availability of L observations. The modulation by oceanic variability is also evident: residual errors are below 0.5 cm in low-energy regions and exceed 1 cm in western boundary currents and continental shelves.

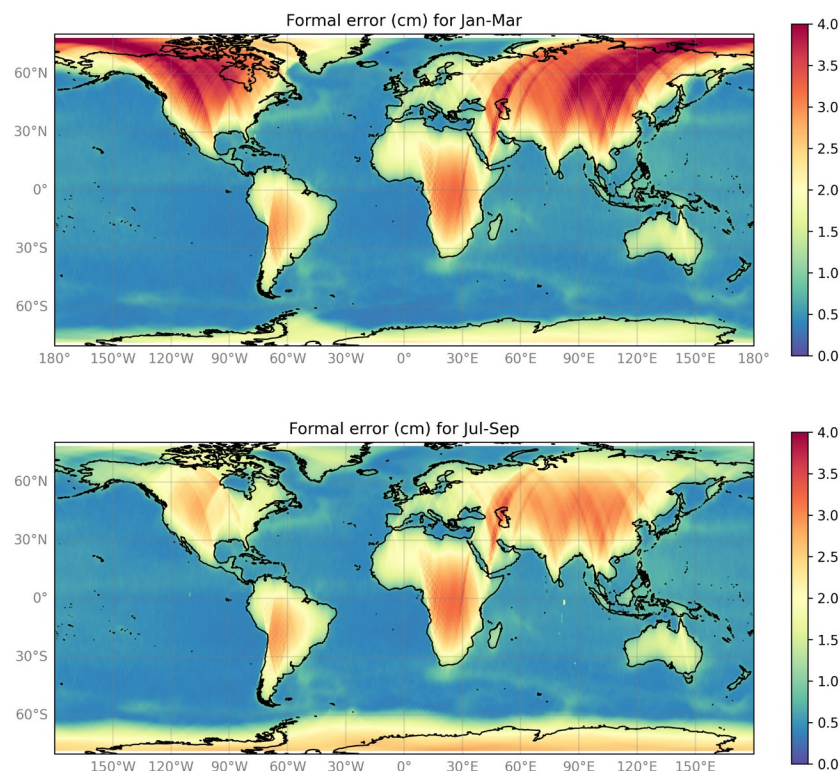


Fig. 14. Mean geographical distribution of the formal 1-σ analysis error of the optimal interpolation of the L term.

The uncertainty shown in Figure 14 corresponds to the diagonal elements of $\mathbf{P}_a$ and is represented by a single variance value at each location. However, this information alone is insufficient for applications such as data assimilation, where the spatial covariance structure of the errors is critical. In particular, treating these large-scale errors as uncorrelated noise would be inappropriate. We therefore propose a simple approach to reconstruct the autocovariance function of the residual roll errors. In spectral space, the residual error spectrum follows the gyroscope spectrum at medium and high frequencies and is truncated at

low frequencies thanks to the constraint provided by the OI. A practical approximation can be obtained by integrating the gyroscope spectrum from the highest frequencies toward lower frequencies until the target variance (as given by Figure 14) is reached. This defines a cutoff frequency below which the spectrum can be approximated by a plateau. The resulting spectrum can then be inverse-Fourier-transformed to obtain the corresponding autocovariance function of the residual systematic errors.

8. The Bias Term

The bias term was excluded from the empirical models as no clear dependencies were found. However, Figures 2 and 3 indicated the presence of systematic (KaRIN – Nadir) residuals on the order of several centimeters. To characterize these biases, power spectral densities (PSD) were computed along the SWOT grid time dimension (Figure 15). The spectra confirm that the KaRIN bias relative to the prior (black curve) contains significantly more energy than the Nadir bias relative to the prior (blue curve). Assuming that neither instrument suffers from significant omission errors, this excess energy must be attributed to an additional commission error. Two bounds for the KaRIN bias are thus defined: a lower bound (green), representing the difference between the KaRIN bias spectrum and the Nadir bias spectrum, which is positive across all wavenumbers above 50 km and integrates to an error of 1.8 cm; and an upper bound (black), corresponding to the KaRIN bias relative to the prior, which represents a conservative maximum integrating to 3.5 cm. While the exact magnitude of the KaRIN-specific bias remains difficult to isolate between these bounds, the excess of energy at wavelengths above 50 km is noteworthy. Consequently, we recommend calibrating the KaRIN bias using the Nadir data beyond 50 km wavelength. After calibration, the upper bound of residual error is 3.2 cm (integration of the blue spectrum) with a lower bound not yet characterized.

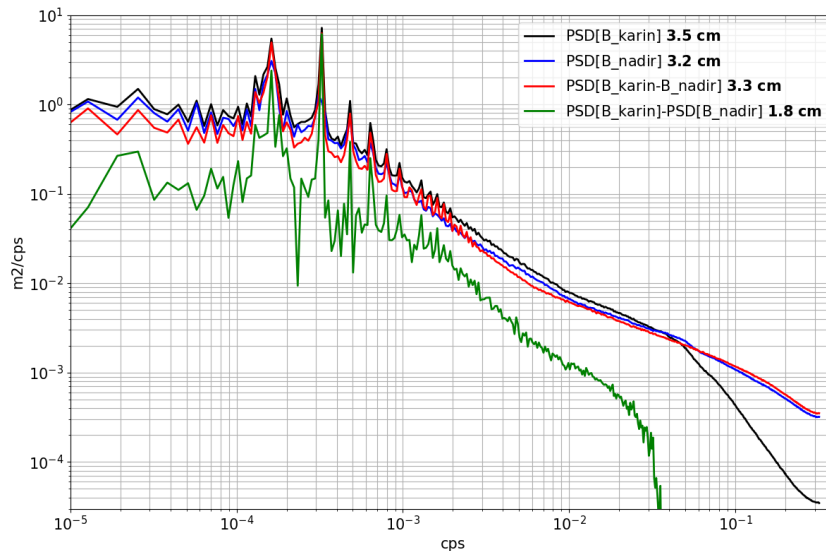


Fig. 15. Power spectral densities of the B term computed along the along-track time coordinate. The black and blue spectra correspond to the biases of the KaRIN and Nadir Altimeter respectively, with respect to the prior. The red spectrum corresponds to the relative KaRIN-Nadir biases. The green spectrum is the difference between the black and blue spectra, representing the excess of the KaRIN bias over the Nadir bias.

9. Conclusions

This study provides a comprehensive assessment of SWOT KaRIN systematic errors based on more than two years of science-orbit observations and introduces a calibration framework designed to separate systematic-error contributions from actual sea level variability.

The analysis demonstrates that a large fraction of the variability of the estimated systematic-error terms can be explained through a sequence of empirical corrections. A solar-orbital model captures the dominant thermo-elastic distortions associated with the spacecraft thermal environment, while a geographically fixed model accounts for persistent patterns linked to the yaw-flip configuration. An additional low-frequency filtering step removes slowly varying residual components that are not described by the empirical models. Together, these three stages reduce the standard deviation of the dominant swath-tilt error from 4.3 μrad (17 cm rms in the 10–60 km swath) to 1.8 μrad (7 cm) and eliminate most orbit-related signatures and their harmonics.

After removal of the empirical corrections, the residual systematic-error estimates exhibit a spectral behavior consistent with a combination of attitude-error residuals and geophysical leakage into the polynomial fits. Exploiting this spectral separation, a localized Optimal

Interpolation framework was developed to estimate the remaining systematic-error contribution while preserving the geophysical signal. The resulting formal uncertainty is generally below 1 cm rms over open-ocean regions and remains close to 3 cm rms over continental areas.

The analysis also reveals that the KaRIN bias behaves differently from the other polynomial error terms and shows no clear dependence on the solar–orbital configuration. Spectral comparisons with nadir altimetry nevertheless indicate an excess energy at wavelengths larger than 50 km, motivating an additional calibration strategy based on the SWOT nadir observations.

Overall, the methodology proposed here combines empirical modeling, spectral characterization, and statistical estimation to provide a consistent Level-3 calibration framework for SWOT KaRIN systematic errors. The results demonstrate that the dominant large-scale systematic-error contributions can be effectively characterized and mitigated using long-term in-flight observations, leading to residual error levels fairly below mission requirements over both oceanic and continental surfaces. More broadly, the framework developed in this study provides a practical basis for the calibration and uncertainty characterization of future wide-swath altimetric observing systems.

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Data Availability Statement.

All datasets discussed in this manuscript are preserved and distributed by the Agencies’ data repositories in compliance with FAIR requirements (e.g. Core Trust Seal certified, or ongoing certification from the Research Data Alliance). The Level-3 SWOT products generated in the frame of this study are distributed by the AVISO repository from CNES from the following DOI: <https://doi.org/10.24400/527896/a01-2023.017>. The SWOT Level 2 KaRIn Low Rate Sea Surface Height Data Product, Version 1.1. available on the two centers with two DOIs: <https://doi.org/10.24400/527896/a01-2023.015> (AVISO, beta prevalidated collection) or <https://doi.org/10.5067/SWOT-SSH-1.1> (PODAAC, collection ID:

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