

The remote reference as an instrumental variable in model space: time-series magnetotelluric inversion with noisy magnetic fields

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Abstract

Noise in the local magnetic field biases magnetotelluric (MT) responses, and a relevant, valid remote magnetic record removes the bias. For an earth model inverted from time series, remote referencing becomes estimating equations: the residual formed with the local field is orthogonal to the sensitivities driven by its projection on the remote field. Their solution, relevance and validity must be checked separately, and their scores over blocks of time supply the covariance of the root and a test of the remote site. Minimizing the misfit with the projected field as driver does not solve them. On observatory records rebuilt without noise it misses the model by up to 0.9 percent of the impedance, and the root, found by fixed point and Newton iterations, by 0.02 percent at most. The standard error shows neither failure of the instrument. A weak instrument draws the root toward the single-site response, as the share of the local field explained by the projection predicts. A remote site that the noise reaches moves the root toward the relation of the noise, and the difference of the roots with two remote sites, judged with the covariance of their common blocks, detects it. Near 25-kV railways, survey sites 1.5 to 12.5 km away fail this test in 49 of 61 testable pairs while trains run and in none of 63 at night, and a remote field 1090 km away passes it against four observatories in all 71 testable cases at the one percent level. With railway noise over whole days, the root and a standard robust code followed by a model recover the response on 61 and 58 of 80 records, so the method is not a better remote reference in itself. Its free-transfer-function form gives the full tensor and tipper of a railway record from service hours.

Keywords: magnetotellurics; remote reference; instrumental variable; time-series inversion; cultural noise; electrified railway; uncertainty

1 Introduction

A magnetotelluric (MT) response is a regression of the electric field on the magnetic field, and noise in the regressor biases it. Where the local magnetic record carries cultural noise, the part of that noise that is independent of the electric field attenuates the single-site impedance, and the part that is coherent with it pulls the impedance toward the relation of the noise. Neither effect decreases with the length of the record. The remote reference (Gamble et al., 1979) removes this bias with a second magnetic record whose noise

is independent of the local noise. It has long been understood as an instrumental variable estimator (Chave and Thomson, 1989, 2004). The remote field is useful because it is uncorrelated with the local noise, and not because it equals the local field.

Near electrified railways the method meets its limits. Lines fed with direct current contaminate periods from seconds to thousands of seconds at distances of kilometers to hundreds of kilometers (Larsen et al., 1996; Egbert et al., 2000; Pádua et al., 2002). Close to such a line, two quiet remote sites 500 km away did not recover periods beyond 5 s (Ichihara et al., 2018). At a 25-kV high-speed line fed with alternating current, Wang et al. (2023) found a near-field response at 0.01–10 Hz, little improvement from data recorded at night, and failure of single remote references 200–280 km away near 0.1 Hz. Processing of several stations at once (Egbert, 1997) detects and characterizes such noise from the coherence structure of an array when one is available, and robust remote-reference estimators are still being developed (Usui et al., 2024).

The companion paper (Oh, 2026) inverts MT time series directly. The residual between the electric field and its prediction from the magnetic field is weighted sample by sample before it is band-passed, in a Gaussian likelihood whose noise has a scale that varies in time, and the earth model is fitted without forming an impedance file. That paper treats contamination that is localized in time, which a single site can reject at its own temporal support. It leaves open the case in which the contamination is coherent in both channels and present at all hours. A single record then contains nothing that distinguishes the relation imposed by the source from that of the Earth, and a second site is required. The question of the present paper is how a remote magnetic record is used, and how it is tested, when the unknown is an earth model and not a transfer function.

Substituting the remote field for the local one is not the answer, because its incoherence with the true local field acts as noise in the regressor. The remote reference of a response parameterized by an earth model is a set of estimating equations. The residual, formed with the local field, is required to be orthogonal to the sensitivities of the model driven by the projection of the local field on the remote one. These equations are nonlinear two-stage least squares (Amemiya, 1974), a case of the generalized method of moments (Hansen, 1982), and the counterpart in the time domain of the two-stage remote reference of Chave and Thomson (2004). Yee et al. (1988) used a remote field as instrument in a recursive estimator of the impulse response in the time domain. Figure 1 sets out the likelihood, the equations, the way they are solved and the diagnostics that go with them.

Two properties of these equations are not obvious, and they are the technical contributions of this paper. The first concerns the solution. The natural implementation drives the prediction with the projected field and minimizes the misfit. With a projection estimated frequency by frequency and a misfit that weights octave bands and the sensitivities of a model, this minimum does not solve the equations. It carries an error that no noise causes, an error of construction, and the root of the equations has to be found instead. The second concerns the instrument. It can be weak or invalid, and the standard error warns of neither. A projection that fits part of the local noise biases the root by an amount that follows from the share of the local field that the remote one explains (Bound et al., 1995; Staiger and Stock, 1997). A remote site that the noise reaches moves the root toward the relation of the noise by a fraction that has a closed form in a scalar model. A second remote site supplies an over-identifying restriction (Hansen, 1982), and the difference of the two roots tests it in the manner of Hausman (1978). The scores of the equations over blocks of time give both the covariance of one root and that of the difference of two roots that share the local record, and the same scores therefore state the error of the root and test the remote site.

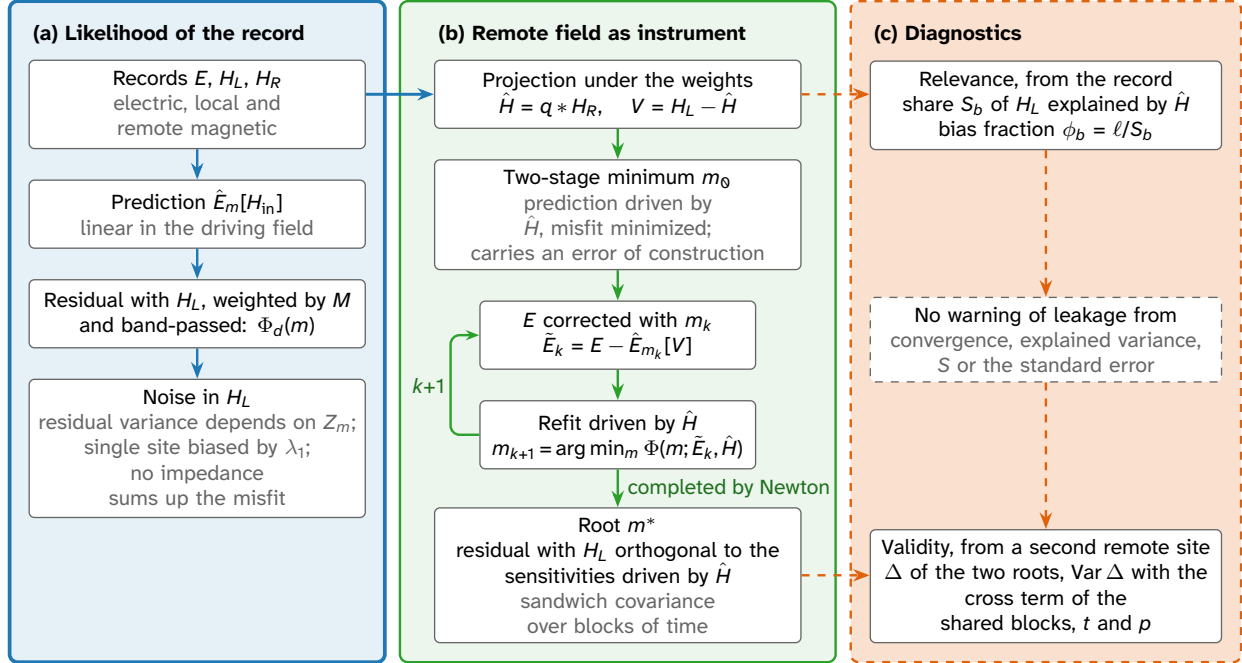


Figure 1. The remote field as instrument for an earth model, from the likelihood of the record to the estimating equations, their solution and the diagnostics of the instrument. (a) Likelihood of the record (Section 2). (b) The remote field as instrument (Section 3): the projection of equation 9, the two-stage minimum, the two steps of a pass of the fixed point of equation 12, with the next pass $k+1$ returning to the correction, and the root of equation 10, reached by Newton’s method from the last pass. (c) Diagnostics: relevance from the record (equations 16 and 17) and validity from a second remote site (equations 19 and 20). The dashed box lists what does not warn of leakage. Arrows of the solution are solid and those of the diagnostics dashed.

The paper makes three contributions. (1) Solving the equations. A fixed point iteration completed by Newton’s method finds their root, and the sandwich covariance formed with their Jacobian states its error. On field records without noise the two-stage minimum misses the model by up to 0.9 percent of the impedance at two observatories and 2.8 percent on Korean records, and the root does not. The equations also state when a transfer function followed by an inversion reproduces the root. (2) Testing the instrument. The share of the local field that the projection explains, reported band by band, measures relevance. A consistency test of two remote sites, with the covariance of their common blocks, detects leakage of the noise into one of them, and after a rejection the sign of the move of the root toward the single-site estimate suggests which. It is applied to the simultaneous records of a survey near 25-kV railways, taken in pairs, and to the 1-s records of four observatories as second remote sites for the distant field. (3) Tests against known and unknown responses. These use synthetic records; the daily records of two observatories, against a standard robust remote-reference code; recorded railway noise added to ten days of one of those records; and two Korean records, one with a near-field source and one beside a 25-kV railway.

Section 2 restates the likelihood of the companion paper as far as the estimating equations need it. Section 3 develops the instrument, its relation to a transfer function followed by an inversion, the two diagnostics and an errors-in-variables alternative, and defines the measures. Sections 4 to 7 report the tests against known and unknown responses, Section 8 tests the remote sites of a railway survey, and Section 9 sets out what the method adds and how it fails. Two appendices give cross-fitting and the single-site routes

in detail.

2 The time-series likelihood

This section restates the definitions of Oh (2026) that the estimating equations use. The results of that paper are not repeated.

2.1 Forward relation

For one off-diagonal element of the impedance tensor the plane-wave relation in the frequency domain is $E(f) = Z(f)H(f)$ at frequency f . For a conductivity model m with impedance $Z_m(f)$ from any forward engine, the predicted electric field is

$$\hat{E}_m[H_{\text{in}}](t) = \mathcal{F}^{-1}\{W(f)Z_m(f)\mathcal{F}[H_{\text{in}}](f)\}(t), \quad (1)$$

with H_{in} the driving magnetic field and $W(f)$ a smooth window equal to one over the analysis band. The bracket marks that the prediction is linear in the driving field, which the estimating equations below use. A multidimensional solver returns Z_m at a few forward frequencies, and $Z_m/\sqrt{f}$ is interpolated linearly in $\log f$ to the Fourier frequencies of the record. Where the impedance row has a diagonal element and the horizontal components of the source are correlated, the two-input form $\hat{E} = z_x * H_x + z_y * H_y$ is needed. In this paper the component that is not of interest is regressed out of the electric field and of the driving field before the inversion, the regression form of Oh (2026). This treats the diagonal element as a free, smooth function of frequency.

2.2 Noise model and profile likelihood

Write the record as $E_{\text{obs}}(t) = \hat{E}_m(t) + n(t)$ with the uniformly modulated noise of Priestley (1965),

$$n(t) = \sigma(t)u(t), \quad (2)$$

where u is stationary Gaussian noise and $\sigma(t) \geq 1$ is a scale that is large where the record is contaminated. Let $M(t) = 1/\sigma^2(t)$ be the weight, let $\mathcal{B}_b$ be band-pass operators on octave bands whose squared responses sum to one, $\sum_b W_b^2(f) = 1$, and let the inverse spectrum of u be $\sum_b W_b^2(f)/\sigma_b^2$, which is $1/\sigma_b^2$ where band b alone has weight. The Gaussian misfit of the residual $r = E_{\text{obs}} - \hat{E}_m$ over the analysis band is then

$$r^T \mathbf{C}^{-1} r = \sum_b \frac{1}{\sigma_b^2} \|\mathcal{B}_b(M^{1/2}r)\|^2. \quad (3)$$

The residual is weighted in time before it is filtered in frequency, as the inverse covariance of equation 2 prescribes. Maximizing over the band variances, with the log-determinant of the stationary covariance counted band by band as $\sum_b N_b \log \sigma_b^2$, gives the profile negative log-likelihood

$$\Phi_d(m) = \frac{1}{2} \sum_b N_b \log \text{RSS}_b(m), \quad \text{RSS}_b(m) = \|\mathcal{B}_b(M^{1/2}(E_{\text{obs}} - \hat{E}_m))\|^2, \quad (4)$$

with $N_b = 2 \sum_f W_b^2(f)$ over the positive Fourier frequencies the real degrees of freedom of band b . When samples are excised, N_b is multiplied by the mean weight, which is the retained fraction when the mask has

no tapers. This is an approximation, and it holds when the residual spectrum varies little over the frequencies that the mask mixes into the band (Oh, 2026). The model minimizes $\Phi(m) = \Phi_d(m) + \beta\Psi(m)$, with Ψ a smoothness prior (Constable et al., 1987). The one-dimensional inversions below use a fixed β , quoted as the standard deviation $(2\beta)^{-1/2}$ of the equivalent prior on the differences of $\log_{10}$ resistivity between adjacent layers.

2.3 Covariance

The Laplace approximation at the minimum gives

$$\Sigma_{\text{post}} = (\mathbf{H}_d + \beta \Psi'')^{-1}, \quad (5)$$

with $\mathbf{H}_d$ the Hessian of Φ_d and Ψ'' that of the prior. Equation 5 is correct only if the noise model is. It does not apply to an estimate that solves estimating equations instead of minimizing a likelihood. For both cases we use the sandwich covariance (White, 1982)

$$\Sigma_{\text{sw}} = \mathbf{H}^{-1} \mathbf{K} \mathbf{H}^{-T}, \quad \mathbf{K} = \sum_k \mathbf{s}_k \mathbf{s}_k^T, \quad (6)$$

with $\mathbf{H}$ the Jacobian of the estimating equations (the Hessian for a minimum) and $\mathbf{s}_k$ the contribution of block k to them. On field records, whose noise is not stationary, the blocks are blocks of time. Both covariances are conditional on one record, its weights and the forward relation. The variation of the response from one record to the next is not in them.

2.4 Why noise in the magnetic field needs more than an impedance

For fixed weights and band variances, equation 3 is quadratic in the transfer function. An impedance estimated under the same likelihood and carried with its full precision matrix is then a sufficient statistic for the earth model (Oh, 2026). Noise in the magnetic field breaks this. With independent noises of variance σ_E^2 and σ_H^2 in the two channels, the residual variance at frequency f is $\sigma_E^2 + |Z_m(f)|^2 \sigma_H^2$ (Clutton-Brock, 1967). It depends on the model, the misfit is no longer quadratic in the transfer function, and no impedance with error bars summarizes it. The remote instrument of the next section keeps the formulation in the time domain in this case without modeling σ_H . The errors-in-variables likelihood of Section 3.7 models it instead.

3 Noise in the magnetic field

3.1 A scalar model of the bias

Consider one Fourier coefficient of the records of a single input,

$$E = ZH + RC + \varepsilon_E, \quad H_L = H + C + \varepsilon_H, \quad H_R = \gamma H + \alpha C + U, \quad (7)$$

with H the natural field at the site, C the field that a cultural source produces in the local magnetic channel, and R the ratio of its electric to its magnetic field, which is the transfer function of a near-field source. The coefficient α is the amplitude of the same noise at the remote site relative to the local one, and γ is the regression of the remote on the local natural field. The terms ε_E , ε_H and U are noises uncorrelated with one another and with H and C , and C is uncorrelated with H . Let ρ be the power of C relative to that of

H over the weighted record and band, and v the power of ε_H relative to that of H . The coefficients may be complex, and α and γ then enter the expressions below as their conjugates. In the limit of many samples the single-site least-squares estimate is

$$Z_{SS} = Z + (R - Z) \lambda_1 - Z \frac{v}{1 + \rho + v}, \quad \lambda_1 = \frac{\rho}{1 + \rho + v}. \quad (8)$$

It mixes the response of the Earth and that of the noise in the proportion of the noise power in the local magnetic record, and independent noise in that record attenuates it.

3.2 The remote field as instrument

Substituting H_R for H_{in} in equation 1 reproduces the bias of equation 8, with the incoherence between H_R and the true local field as the noise of the regressor. The remote reference uses the projection of the local field on the remote one,

$$\hat{H} = q * H_R, \quad q(f) = \frac{\langle H_L \overline{H_R} \rangle_f}{\langle H_R \overline{H_R} \rangle_f}, \quad (9)$$

where $\langle \cdot \rangle_f$ averages the cross-spectra over a band around f . We use a bandwidth proportional to frequency, a third of an octave or less. In the two-input form both local components are projected on both remote components. The estimating equations of the remote reference in model space are

$$g_j(m) = \sum_b \frac{1}{\sigma_b^2} \left\langle \mathcal{B}_b \left(M^{1/2} \frac{\partial \hat{E}_m[\hat{H}]}{\partial m_j} \right), \mathcal{B}_b \left(M^{1/2} (E_{\text{obs}} - \hat{E}_m[H_L]) \right) \right\rangle = \beta \frac{\partial \Psi}{\partial m_j}. \quad (10)$$

The residual is formed with the local field and is required to be orthogonal to the sensitivities driven by the projected field, and σ_b^2 is the band variance of that residual. The right side is the gradient of the prior. With a free transfer function in place of $\hat{E}_m$, one frequency band and $\beta = 0$, equation 10 reduces to the two-stage remote reference of Chave and Thomson (2004).

3.3 The root and the two-stage minimum

The prediction is linear in the driving field. With $V = H_L - \hat{H}$, the part of the local field that the projection leaves, $E_{\text{obs}} - \hat{E}_m[\hat{H}] = E_{\text{obs}} - \hat{E}_m[H_L] + \hat{E}_m[V]$. Driving the prediction with $\hat{H}$ and minimizing the misfit 4, which we call the two-stage minimum, solves

$$\begin{aligned} & \sum_b \frac{1}{\tilde{\sigma}_b^2} \left\langle \mathcal{B}_b \left(M^{1/2} \frac{\partial \hat{E}_m[\hat{H}]}{\partial m_j} \right), \mathcal{B}_b \left(M^{1/2} (E_{\text{obs}} - \hat{E}_m[H_L]) \right) \right\rangle \\ & + \sum_b \frac{1}{\tilde{\sigma}_b^2} \left\langle \mathcal{B}_b \left(M^{1/2} \frac{\partial \hat{E}_m[\hat{H}]}{\partial m_j} \right), \mathcal{B}_b (M^{1/2} \hat{E}_m[V]) \right\rangle = \beta \frac{\partial \Psi}{\partial m_j}, \end{aligned} \quad (11)$$

with $\tilde{\sigma}_b^2$ the band variances of the residual of the second stage. Two conditions together make its solution the root of equation 10. First, the second sum vanishes, which means that the prediction driven by V is orthogonal to the sensitivities driven by $\hat{H}$ in the inner product of the misfit. Second, the ratio $\sigma_b^2 / \tilde{\sigma}_b^2$ is the same in every band, and equal to one if $\beta > 0$. The two conditions are sufficient and not necessary. Without them the solutions coincide only where the two departures cancel in every component of equation 11, and nothing in the construction makes them cancel. Neither condition holds in general. The ratio of the band variances differs between bands wherever the local magnetic record is noisy in some bands more than in

others. The projection makes V orthogonal to H_R on average over each smoothing band of the cross-spectra. The misfit weights octave bands, the weight $M(t)$ couples frequencies, and the sensitivities of a layered model weight the frequencies inside a band unequally. A sample correlation between V and $\hat{H}$ therefore remains in the inner product of the misfit. The two-stage minimum carries an error that no noise causes. Its size is measured in Section 5.2 on records without noise.

The error is not that of the forbidden regression of econometrics (Angrist and Pischke, 2009, Section 4.6.1). There the fitted value of an endogenous regressor is substituted into a model that is nonlinear in the regressor. The residual of the first stage then enters the second stage nonlinearly, it does not average out, and the estimate is inconsistent. The prediction $\hat{E}_m$ is nonlinear in the model but linear in the driving field, so this mechanism does not arise. In linear two-stage least squares the substitution of fitted values gives the instrumental variable estimate, because the residual of the first stage is orthogonal in the sample to every regressor of the second stage. The two-stage minimum departs from the root because the projection and the misfit use different inner products and the ratio of the band variances changes between bands, and not because the regressor enters nonlinearly.

We solve equation 10 by the fixed point

$$\tilde{E}_k = E_{\text{obs}} - \hat{E}_{m_k}[V], \quad m_{k+1} = \arg \min_m [\Phi_d(m; \tilde{E}_k, \hat{H}) + \beta \Psi(m)], \quad (12)$$

where $\Phi_d(m; \tilde{E}, \hat{H})$ is the misfit of equation 4 with the electric record replaced by $\tilde{E}$ and the driving field by $\hat{H}$. The corrected record $\tilde{E}_k$ is held fixed in the minimization of a pass, and the band variances are profiled within it. The weights and the projection stay fixed over the passes. The iteration is started from the two-stage minimum. At a limit m^* , $\tilde{E} - \hat{E}_{m^*}[\hat{H}] = E_{\text{obs}} - \hat{E}_{m^*}[H_L]$, and the stationarity of the inner minimization is equation 10. The band variances at the root are those of the residual formed with the local field, which contains the response of the model to the noise of the local magnetic record. The prior keeps its width. A given regularization weight therefore counts for more at the root than in the two-stage minimum where the local magnetic record is noisy, and for less where the noise of the two local channels is coherent. The passes stop when no layer changes by more than 10^{-5} in $\log_{10}$ resistivity, or after 40 passes. A few passes suffice where the remote field explains most of the local one. Where it explains little the step contracts slowly, and where it explains almost nothing of a clean local field the iteration need not converge. Newton's method applied to equation 10 itself, with its Jacobian formed by central differences, therefore completes the solution. A small last step does not by itself show that the equations are solved, so the result is judged on equation 10 directly, by the relative residual

$$\varepsilon = \max_j \frac{|g_j(m) - \beta \partial \Psi / \partial m_j|}{\sum_b \sigma_b^{-2} \|\mathcal{B}_b(M^{1/2} \partial \hat{E}_m[\hat{H}] / \partial m_j)\| \|\mathcal{B}_b(M^{1/2}(E_{\text{obs}} - \hat{E}_m[H_L]))\|}, \quad (13)$$

with g_j the left side of equation 10. The denominator bounds the data term of the equation by the Cauchy–Schwarz inequality. For a layer held on a bound of the fit, only the sign of the residual that the constrained minimum forbids counts. The Newton solution is kept where it lowers ε , and a result counts as a root when $\varepsilon < 10^{-4}$. The equations of a nonlinear model may have more than one root. The root in what follows is the one reached from the two-stage minimum. Its sandwich covariance takes for $\mathbf{H}$ the Jacobian of equation 10 at the root, the one that Newton's method uses.

The weights add a condition. A projection fitted to the whole record and used on part of it, the quiet hours for example, scales the two-stage minimum by the ratio of the two projections. The two differ when

the signal-to-noise ratio of the remote record differs between the whole record and the part. The root is nearly free of this, but the fixed point may then fail to reach it. The projection is therefore fitted under the weights of the misfit.

3.4 A transfer function followed by inversion

The usual practice estimates a remote-reference transfer function and then inverts it. The estimating equations state when this reproduces the root. With noise in the magnetic field no impedance with error bars summarizes the likelihood (Section 2), but the equations have a reduced form. Let a free transfer function be expanded in basis functions φ_k with real coefficients c_k , the real and imaginary parts at a node counted separately. Let $c(m)$ be the coefficients of the response of model m in that basis, and $\hat{E}_\varphi[H]$ the electric field predicted from H by the basis function φ . With the projection, the weights and the band variances held fixed, equation 10 for the free transfer function is linear, and its root $\hat{c}$ solves $\Gamma\hat{c} = d$, where

$$\Gamma_{jk} = \sum_b \frac{1}{\sigma_b^2} \left\langle \mathcal{B}_b(M^{1/2}\hat{E}_{\varphi_j}[\hat{H}]), \mathcal{B}_b(M^{1/2}\hat{E}_{\varphi_k}[H_L]) \right\rangle \quad (14)$$

and d holds the same inner products taken with E_{obs} in place of the second argument. If the responses of the model lie in the basis, equation 10 for the model becomes

$$J^T(d - \Gamma c(m)) = \beta \nabla \Psi(m), \quad J = \partial c / \partial m, \quad (15)$$

and the root in model space depends on the record only through d and Γ , or through $\hat{c} = \Gamma^{-1}d$ and Γ where Γ is invertible.

A transfer function followed by an inversion therefore reproduces the root under three conditions. First, the transfer function is the root of the same equations under the same weights and band variances, and not a two-stage minimum, which carries the error of Section 3.3. Second, it is carried with the kernel Γ , which couples the elements and the periods through the weights and the bands, and not with error bars of single elements. Third, the responses of the model lie in its basis. The usual practice, a robust estimate band by band followed by a fit with error bars and a floor, meets neither the first nor the second condition and is a different estimator. A transfer function solved as a root and carried with Γ would let an inversion use the instrument without the time series. We have not used the reduced form numerically here.

3.5 Relevance of the instrument

The share of the local field that the projection explains,

$$S_b = 1 - \frac{\|\mathcal{B}_b(M^{1/2}V)\|^2}{\|\mathcal{B}_b(M^{1/2}H_L)\|^2}, \quad (16)$$

measures the relevance of the instrument in band b , and S denotes its median over the bands. It is a share of the weighted energy of the band. The projection is fitted on smoothed cross-spectra and not in the inner product of the misfit, so S_b is not the coefficient of determination of the projection. A projection with L coefficients estimated from n_q Fourier coefficients carries on average the fraction $\ell = L/n_q$ of the power of the local noise into $\hat{H}$. It raises the share from S_b° , the share explained apart from this loading, to

$S_b = S_b^\circ + \ell(1 - S_b^\circ)$. In the scalar model of equation 7 with this loading, the estimate in band b is drawn from the earth response toward the single-site estimate of equation 8,

$$Z_{IV} - Z \approx \phi_b (Z_{SS} - Z), \quad \phi_b = \frac{\ell}{S_b^\circ + \ell(1 - S_b^\circ)} = \frac{\ell}{S_b}, \quad \kappa_b = (1 - S_b^\circ) \phi_b. \quad (17)$$

The fraction ϕ_b is the bias of two-stage least squares relative to that of least squares. For small S_b° it is the inverse of the expected F statistic of the first stage (see also Bound et al., 1995; Staiger and Stock, 1997), and it follows from the measured share. Where the local noise is independent of the electric field, the single-site estimate is attenuated by the fraction $1 - S_b^\circ$, and κ_b is the relative bias of the impedance. Where the remote field explains nothing, $\phi_b \rightarrow 1$ and the estimate is the single-site one. In this form the bias is a property of the instrument and not of the way the equations are solved. It is the same for the root and the two-stage minimum. Outside the scalar model, equation 17 is a guide to the size of the pull and not a correction. It is used below in that way, to select the components of the test with a second remote site (Section 3.6) and to read the measured pull. With a bandwidth of a third of an octave, $n_q \approx 0.23 T_{\text{rec}}/T$ for a record of length T_{rec} at period T . It is about 1400 at 90 s and 11 500 at 11 s for the 156 h of the Gyeongju record, and about 220 at 90 s for a record of one day. At 724–1024 s, the longest periods of the daily inversions, it is 20 to 30, and ℓ approaches 0.1.

Cross-fitting removes the loading of the local noise. The coefficients q used in one block of the record are estimated from the other blocks, the instrument of Angrist and Krueger (1995) estimated on a split sample, so that the instrument in a block carries none of the noise of that block. In the two-input form the chain contains further regressions: the orthogonalization of the projected driving field against the projected other component, and the regression of that component out of the electric field. If the projection alone is cross-fitted and the orthogonalization is done on the whole record, the orthogonalization returns the local noise with the opposite sign, and in the scalar test every regression of the chain had to be cross-fitted. The effect of cross-fitting in the full procedure is mixed (Appendix A), and we use it as a check and not as a default.

Convergence of the fixed point is a weaker warning than S . It slows where a clean local field is poorly explained, but where a cultural source fills the local magnetic record, V is mostly the field of the source and nearly uncorrelated with $\hat{H}$, and the fixed point converges quickly to a biased root (Section 6.4).

3.6 Validity of the instrument

The instrument is valid if the remote field is uncorrelated with the noise of the local channels. In the scalar model of equation 7, the solution of the estimating equations in the limit of many samples is

$$Z_{IV} = \frac{\langle E \overline{H_R} \rangle}{\langle H_L \overline{H_R} \rangle} = (1 - \lambda) Z + \lambda R, \quad \lambda = \frac{\alpha \rho}{\gamma + \alpha \rho}. \quad (18)$$

The noises ε_H and U do not enter λ . They weaken the instrument but do not bias its limit. The remote reference lowers the weight of the noise relation from λ_1 of equation 8 to λ when $\alpha(1 + \nu) < \gamma$, and for $\alpha \rho \ll \gamma$ the ratio is $\lambda/\lambda_1 \approx \alpha(1 + \rho + \nu)/\gamma$. For small λ the bias in $\log_{10} \rho_a$ is about $0.87 \text{Re}(R/Z - 1) \lambda$ and that of the phase $57.3^\circ \text{Im}(R/Z - 1) \lambda$. The bias is thus set by the product of $\alpha \rho$ and the contrast $|R/Z - 1|$ between the noise and the Earth, and not by α alone. In the two-input form with a correlation r_s between the two source components and noise in the driving component only, the same form holds with γ replaced by $\gamma' = \gamma(1 - r_s^2)$.

No diagnostic of a single record shows the violation. As α grows, the remote field explains more of the local record, because it now also explains its noise, and S rises. The fixed point converges as before. The share of the electric field that the model explains rises, and the root moves toward the single-site estimate, so that the difference between the two shrinks. The sandwich standard error grows, but more slowly than the bias (Section 4.4).

A second remote site tests the condition. Let $\hat{m}_A$ and $\hat{m}_B$ be the roots obtained from the same local record with remote sites A and B , and Δ_k the difference of their $\log_{10} \rho_a$ in component k , a period of an element of the response. Then

$$\text{Var } \Delta = \mathbf{J}_A \mathbf{V}_{AA} \mathbf{J}_A^T + \mathbf{J}_B \mathbf{V}_{BB} \mathbf{J}_B^T - \mathbf{J}_A \mathbf{V}_{AB} \mathbf{J}_B^T - \mathbf{J}_B \mathbf{V}_{AB} \mathbf{J}_A^T, \quad \mathbf{V}_{AB} = \mathbf{H}_A^{-1} \left(\sum_k \mathbf{s}_{A,k} \mathbf{s}_{B,k}^T \right) \mathbf{H}_B^{-T}, \quad (19)$$

with $\mathbf{J}$ the Jacobian of the response with respect to the model and $\mathbf{s}_{A,k}$, $\mathbf{s}_{B,k}$ the scores of the two roots in the same block k . The statistic is

$$t = \max_k \frac{|\Delta_k|}{\sqrt{(\text{Var } \Delta)_{kk}}}. \quad (20)$$

The two roots share the local record and its noise, so the variance of their difference is much smaller than the sum of their variances, and the cross term of equation 19 is what makes the test sensitive. The test compares the two estimators in the manner of Hausman (1978) and does not form the J statistic of the stacked moments (Hansen, 1982). It detects that two instruments disagree, and passing it bounds the leakage within its power without proving validity. It needs one of the two remote sites to be clean, and it does not see noise that reaches both sites in the same proportion. We therefore call it a consistency test of two remote sites.

Which instrument is invalid is read, where it can be, from the move

$$\mu = \text{Re} \frac{Z_B - Z_A}{Z_{SS} - Z_A}, \quad (21)$$

with Z_A and Z_B the responses of the two roots and Z_{SS} the single-site estimate on the same blocks. It is zero when the two roots agree and one when the root with B equals the single-site estimate. In the scalar model of equation 7, with a clean site A and no independent noise in the local magnetic record, μ is the ratio of λ of equation 18 for the field at B to λ_1 of equation 8. It exceeds one when $\alpha > \gamma$, that is, when the field at B carries the noise in a larger proportion to the natural field than the local record does. With a clean field at B instead,

$$\mu = -\frac{\lambda_A}{\lambda_1 - \lambda_A}, \quad (22)$$

where λ_A is the fraction of equation 18 for the field at A , so that λ_A/λ_1 is the move of the root with A toward the relation of the noise as a fraction of the move of the single-site estimate. After a rejection a positive μ therefore points to B as the invalid instrument, unless A carries the noise in a larger proportion than the local record.

Several remote sites have been used before, but not tested against one another in this way. Chave and Thomson (2004) combine two or more remote references in one estimate, which is good provided at least one of them is uncontaminated at a given frequency, and compare the estimates obtained with each graphically, without a formal test. Shalivahan and Bhattacharya (2002) compared responses obtained with remote sites at different distances to find how far a remote site can be. Processing of an array detects coherent noise from the principal components of the spectral matrix of many stations (Egbert, 1997), and interstation transfer functions combine the information of several sites (Campanyà et al., 2014). The test of equation 20 needs

two remote sites and no array. It judges the difference against the covariance of the common blocks, without which its standard deviation was 2.8 to 3.0 times too large in the synthetic test (Section 4.4). It holds for a free transfer function as for an earth model, because both are roots of equation 10 with scores over the same blocks.

3.7 Errors in variables

Without a remote record the noise can be modeled instead. With independent noises of variance σ_E^2 and σ_H^2 in the two channels and the true magnetic field treated as unknown, the profile likelihood of the model is

$$\Phi_d(m) = \frac{N}{2} \log \sum_f \frac{w(f) |D_m(f)|^2}{\eta + |Z_m(f)|^2}, \quad \eta = \sigma_E^2 / \sigma_H^2, \quad (23)$$

with D_m the Fourier transform of the weighted residual, $w(f) = \sum_b W_b^2(f)$ and $N = \sum_b N_b$. This is the Deming form of errors-in-variables regression (Fuller, 1987). It differs from an enlarged residual variance, which reweights the data and leaves the bias in place. It needs the noise ratio η , which a single station does not determine.

3.8 Measures and statistical tests

The error of a response is the difference of $\log_{10} \rho_a$ from the true or reference response. It is given in percent of the impedance magnitude, which is 115 times the difference in $\log_{10} \rho_a$, or as the root-mean-square (r.m.s.) difference of $\log_{10} \rho_a$ over the periods of a band. The error of a layered model is the r.m.s. difference of $\log_{10}$ resistivity from the true model over the layers. For a route applied to many records we quote the median over the records. The bias of a route is the median of its error over the records. The excess bias in a test with leakage is the bias minus that of the same records without leakage, as an r.m.s. over the periods.

Stated uncertainties are tested by coverage, the share of records whose interval of one or two standard errors (1.96 in the tests of the instrument) contains the true value. The scatter of a daily response from day to day is 1.48 times its median absolute deviation about a 31-day running median. Divided by the median stated standard error it gives the factor by which the stated errors are too small. Two routes run on the same days are compared by the ratio of their scatters, period by period. Rank correlations are Spearman's.

The consistency test of Section 3.6 rejects when the statistic t of equation 20 exceeds its 95 percent point on synthetic records without leakage, an empirical five percent level. In the field, where no record free of leakage is known, the null distribution of t is simulated from the estimated covariance of the components used, with 20 000 draws from a multivariate t distribution with $n_{\text{blk}} - 1$ degrees of freedom for n_{blk} blocks. The t distribution allows approximately for the covariance being estimated. The p of a test is the share of draws whose t exceeds the observed one. The share S of equation 16 is computed on the record as used, without and with noise, in the octave bands of the likelihood.

4 Synthetic tests

The synthetic records answer four questions for which the answer must be known. How should a remote record enter the inversion? When is the root a better estimate than the two-stage minimum? Are the stated

intervals right? What happens when the instrument is weak or invalid? The records are generated with the forward operator that is inverted, so the absolute errors are optimistic. Every route receives the same records, and the comparisons between routes are not affected.

4.1 Substitute or instrument

Design. Records of 2^{18} samples at 50 Hz are generated over a three-layer earth (100, 10 and 1000 Ωm , thicknesses 500 and 1500 m) with a source spectrum falling as $1/f$ and analyzed at 0.02–5 Hz. The local magnetic field carries impulsive noise with 44 percent of its variance. The remote field is the source field plus white noise at seven levels, from negligible to equal to the signal. Three routes are compared on 12 records per level: the remote field substituted for the local one; the local field projected on the remote field and driving the prediction, here the two-stage minimum; and a Fourier remote-reference impedance estimated and then inverted.

Results. Driven by the local field as recorded, the inversion has a model error of about one decade (Figure 2a). Substitution is exact when the remote field carries almost no noise, and it degrades quickly as the coherence of the remote field with the true local field falls. Its error passes that of the instrument at a squared coherence of about 0.997 and reaches 0.69 at 0.81. This is the errors-in-variables bias of Section 3.1 and not a limit that coherence places on remote referencing. The projected field keeps the error between 0.021 and 0.030 over the whole range. Its floor is the cost of estimating the projection from the noisy local record. The Fourier remote reference inverted afterwards gives 0.043 to 0.053. The instrument form is used in the rest of the paper.

4.2 The root and the two-stage minimum

Design. Records of 32768 samples at 1 Hz are generated over a five-layer earth ($\log_{10}$ resistivities 1.6, 1.3, 1.8, 2.2 and 2.0, interfaces at 1, 4, 14 and 44 km) and analyzed at 4–512 s. They have two horizontal source components with a correlation of 0.3 and an activity that varies along the record, a response of the other component, and noises with the spectral shape of the source, so that one power ratio holds in every band. The preprocessing is that of the daily inversions of Section 5, with detrending, tapering of the magnetic channels and a first difference. A baseline with small noise in every channel is varied one condition at a time (Table 1), with 160 records per condition, 80 for the records of one day and 60 for those without noise. The table counts the records on which every route returned an estimate, a standard robust code not reported here among them. The two-stage minimum and the root are compared by their bias and by the coverage of their sandwich intervals of 1.96 standard errors over 24 blocks.

Results. Where the instrument is weak because of noise at the remote site, the two-stage minimum is biased by 2 to 11 percent of the impedance and its intervals fail. The root stays below 2 percent in the median over the periods, and its intervals hold (Table 1). The rows without noise in the electric and local magnetic channels isolate the error of construction. The root is exact there, and the minimum is biased by an amount that grows as S falls. With the amplitude of the remote noise six times larger in the second half of the record, the minimum is biased by 3.7 percent and the root by 0.3 in the median over the periods. If the projection is fitted to the whole of that record and the misfit to its first half, the minimum misses by 82 percent. The root of the equations, found by Newton’s method from the root obtained with the projection of the first half, misses by 0.4 percent on 60 records, but the fixed point does not reach it, because the projection

of the whole record has a third of the amplitude of the local field on the first half. This is the condition on the weights of Section 3.3.

Where the noise is in the local magnetic field, the root is the more biased of the two, and at a noise power of 200 percent its intervals contain the truth in 80 percent of the records. The projection carries part of the local noise into the instrument (equation 17), and the root does not undo it. In its simplest form this bias is the same for both estimates. In the full procedure the root carries more of it, for reasons we have not separated. Noise common to the electric and the local magnetic channel, and noise at both sites, bias both estimates. A remote site reached by the local noise biases both by the same amount, with intervals that fail. Short records and records of one day keep the order of the two estimates. Every root of the table satisfies equation 10 to $\varepsilon < 10^{-11}$. The fixed point alone reached $\varepsilon < 10^{-4}$ within eight passes where $S \geq 0.22$ and within 23 at $S = 0.06$, and Newton's method only refined it. Four passes alone would have given the same responses within 0.008 percent of the impedance where $S \geq 0.22$ and within 1.7 percent at $S = 0.06$. The two-stage minimum has medians of ε of 3×10^{-3} to 0.5 over the records of a condition and does not solve the equations. Started from three other models on 24 records of every condition, the fixed point reached the same root within 10^{-5} in $\log_{10} \rho_a$. The Jacobian of equation 10 gives standard errors up to 7 percent larger than the Hessian of the misfit of the last pass, in the median over the records of a condition. On single records at $S = 0.06$ the factor reaches 2.5, and the coverage there rises from 0.87 to 0.92.

4.3 Coverage with noise in the local magnetic field

Design. The coverage simulation of Oh (2026) is extended to noise in the magnetic field. Each experiment is a record of 65 536 samples at 50 Hz over the three-layer earth of Section 4.1, analyzed at 0.02–5 Hz, with white noise in the electric field at 30 percent of the signal variance in the band. White noise is added to the local magnetic record at a level that equals the magnetic signal at 3 Hz and exceeds it above. The remote record carries independent noise of the same level. The three log-resistivities are estimated from three starting models. The coverage of the Hessian and of the sandwich intervals is counted over 500 records per row, with standard errors of 0.02 and 0.01 at the nominal levels of 0.68 and 0.95.

Results. Treating the magnetic record as exact biases the top layer by more than a decade, and its intervals never contain the truth (Table 2). The remote field as instrument removes the bias in every layer. The covariance must then be the sandwich form evaluated with the residual formed with the local field. The Hessian of the second stage understates the uncertainty, because the residual of the second stage does not contain the local magnetic noise to which the estimate is subject. The root and the two-stage minimum give the same estimates and coverage here, as expected where the remote field explains most of the local one. The errors-in-variables likelihood of equation 23 with the true noise ratio also removes the bias with right intervals. With the ratio wrong by a factor of two, the top layer is biased by half a decade in one direction and by more than a decade in the other, and the intervals of the middle layer never contain the truth. The errors-in-variables likelihood is therefore a tool for records whose magnetic noise is characterized independently. A remote record is to be preferred wherever one exists.

4.4 A remote site that the noise reaches

Design. The records of Section 4.2 receive a cultural noise C in the electric field and in the local driving field. Its ratio R of electric to magnetic field is real, independent of frequency and three times $|Z|$ at 30 s, so

Table 1. The two-stage minimum and the root on synthetic records of a five-layer earth (Section 4.2). Noise power in percent of the source power for the channel named; the baseline has 5, 2 and 10 percent in the electric, local magnetic and remote magnetic fields. S , median over the records. Bias, magnitude of the median error of the impedance in percent, as median and maximum over the 13 periods. Coverage, share of records within 1.96 sandwich standard errors, median over the periods (nominal 0.95).

condition	records	S	bias (percent)		coverage	
			minimum	root	minimum	root
baseline: E 5, local H 2, remote H 10	150	0.90	0.17 / 0.41	0.07 / 0.15	0.90	0.92
<i>noise in the local magnetic field</i>						
local H 10 (remote 2)	154	0.89	0.23 / 0.50	0.35 / 0.67	0.94	0.92
local H 50	153	0.66	0.22 / 0.76	0.67 / 1.60	0.90	0.88
local H 200	146	0.34	1.55 / 3.49	3.71 / 6.15	0.91	0.80
<i>noise common to E and the local H</i>						
30	154	0.70	1.85 / 4.16	0.75 / 2.52	0.84	0.88
300	148	0.24	9.22 / 18.75	6.17 / 11.08	0.48	0.78
300, in blocks over 30 percent	147	0.47	5.65 / 9.53	3.94 / 5.71	0.65	0.84
<i>noise at the remote site (weak instrument)</i>						
remote H 100	154	0.52	1.95 / 3.35	0.14 / 0.39	0.60	0.94
remote H 400	155	0.22	5.67 / 10.16	0.29 / 0.96	0.04	0.94
remote H 1900	156	0.06	11.41 / 26.15	1.83 / 2.61	0.01	0.92
local and remote H 100 each	153	0.27	4.07 / 6.50	4.13 / 6.72	0.82	0.80
<i>remote site reached by local noise (invalid instrument)</i>						
10 percent of local H noise 50	153	0.67	4.79 / 5.16	4.91 / 5.86	0.25	0.13
30 percent of local H noise 50	152	0.78	11.98 / 12.55	12.20 / 13.16	0.01	0.01
<i>remote noise that changes along the record length</i>						
remote H 10, amplitude $6\times$ in second half	155	0.37	3.72 / 5.53	0.33 / 0.75	0.28	0.90
<i>length</i>						
4096 samples	153	0.90	0.47 / 2.76	0.31 / 1.37	0.89	0.93
4096 samples, remote H 100	152	0.53	1.27 / 10.43	0.30 / 2.67	0.77	0.95
86400 samples, baseline	76	0.90	0.13 / 0.98	0.10 / 0.54	0.89	0.89
86400 samples, remote H 400	78	0.22	2.84 / 7.51	0.24 / 2.00	0.37	0.94
<i>no noise in E and local H</i>						
remote H 10	60	0.91	0.18 / 0.42	0.00 / 0.01	0.28	0.95
remote H 100	58	0.53	1.73 / 3.11	0.00 / 0.01	0.02	0.95
remote H 400	58	0.22	5.21 / 9.21	0.00 / 0.01	0.00	0.97

Table 2. Estimates and coverage with white noise in the local magnetic field, 500 records per row (Section 4.3). The three entries of a cell are the top (100 Ωm), middle (10 Ωm) and bottom (1000 Ωm) layers. Bias and scatter, mean and standard deviation of the error of $\log_{10}$ resistivity. Coverage at one and two standard deviations, for the inverse Hessian and for the sandwich covariance; the root has no Hessian of a misfit. η , the noise ratio of equation 23.

estimate	bias	scatter	coverage, inverse Hessian		coverage, sandwich	
	(decades)	(decades)	nominal 0.68	nominal 0.95	nominal 0.68	nominal 0.95
H treated as exact	-1.20/0.00/+0.45	0.021/0.024/0.163	0.00/0.58/0.03	0.00/0.89/0.56	0.00/0.68/0.02	0.00/0.96/0.17
two-stage minimum	0.00/0.00/+0.01	0.105/0.010/0.086	0.57/0.61/0.65	0.89/0.94/0.94	0.67/0.65/0.65	0.93/0.95/0.95
root of the estimating equations	0.00/0.00/+0.01	0.105/0.010/0.085	—	—	0.67/0.65/0.65	0.93/0.95/0.95
errors in variables, η true	+0.01/0.00/+0.01	0.082/0.009/0.081	0.62/0.65/0.69	0.95/0.96/0.96	0.66/0.69/0.68	0.97/0.97/0.95
η two times too large	-0.48/-0.06/+0.43	0.034/0.010/0.129	0.00/0.00/0.01	0.00/0.00/0.10	0.00/0.00/0.00	0.00/0.00/0.04
η two times too small	+1.72/+0.05/-0.23	0.467/0.009/0.064	0.31/0.00/0.00	0.93/0.00/0.04	0.12/0.00/0.01	0.40/0.00/0.07

that $|R/Z - 1|$ ranges from 1.1 at 5.7 s to 6.3 at 362 s. A share α of its amplitude reaches the remote record, for α from 0 to 0.5 and a noise power ρ of 0.3, 1 and 3 relative to the signal over the whole record. The records of a condition differ only in α , so that the excess bias over the record without leakage can be taken record by record. For the test, the same local record is inverted with a second remote site that the noise does not reach and that carries independent noise of 10 percent, on 200 records per condition. The statistic of equation 20 is taken over 13 half-octave periods of 5.7–362 s with the covariance of equation 19 from 24 common blocks. Its 95 percent points without leakage, 2.34, 2.38 and 2.36 for the three values of ρ , set the empirical five percent level. A further set of records has the noise in blocks of 1800 s that cover 30 percent of the record. It is inverted also with weights estimated from the residuals, as in Section 5.1.

Results. The excess bias of the root follows equation 18 (Figure 2b, Table 3). The measured excess is 1.02 to 1.09 times the closed form in $\log_{10} \rho_a$. The phase of the layered root follows the closed form less closely at the longest periods. The mixture $(1 - \lambda)Z + \lambda R$ is not the response of a layered earth, and the root follows its magnitude. The diagnostics of one record do not respond (Figure 2c). The share S rises with α , the fixed point converges no more slowly than without leakage, and every one of the 3360 fits reached a root. The coverage of the sandwich interval meanwhile falls from 0.88 without leakage to 0.36 at $\alpha = 0.05$ and 0.01 at $\alpha = 0.1$ for $\rho = 1$.

The second remote site detects the leakage (Figure 2d). For noise as strong as the natural field, the difference of the two roots rejects at the empirical five percent level in 76 percent of the records at $\alpha = 0.02$ and in 98 percent at $\alpha = 0.05$. For noise three times stronger, $\alpha = 0.01$ is detected in 82 percent of the records. The standard deviation of the difference from the common blocks is right, with a ratio to the actual scatter of 0.96 to 0.98 without leakage. Treated as independent, the two roots would have a standard deviation of the difference 2.8 to 3.0 times too large. A period beyond 1.96 such standard deviations would then flag 7 percent of the records at $\alpha = 0.02$ and $\rho = 1$. When both remote sites receive the noise in the same proportion, the rejection rate stays at 5 to 7 percent while the bias of both roots grows. With the noise confined to 30 percent of the record, weights estimated from the residuals reduce the bias at $\rho = 1$ and $\alpha = 0.2$ from 0.135 to 0.013 in $\log_{10} \rho_a$. With the noise over the whole record, the weights stay near one and change nothing.

4.5 Cross-fitting the projection

In a scalar test the cross-fitted projection removes the loading of equation 17 for one input, and with two inputs every regression of the chain has to be cross-fitted (Appendix A). In the full procedure, cross-fitting every regression lowers the bias of the root from 3.95 to 1.83 percent and raises the coverage from 0.80 to 0.86 at a local magnetic noise power of 200 percent, on all 160 records of that condition (146 in Table 1). It changes nothing where the local noise is moderate, raises the bias floor of the records of Section 4.4 without leakage, and leaves the bias of leakage unchanged.

5 The Parkfield and Hollister observatories

5.1 Records and inversion

The Parkfield and Hollister observatories of the Berkeley network in central California (BDSN, 2014; NCEDC, 2014), 120 km apart, recorded the electric and magnetic fields continuously for many years (Kap-

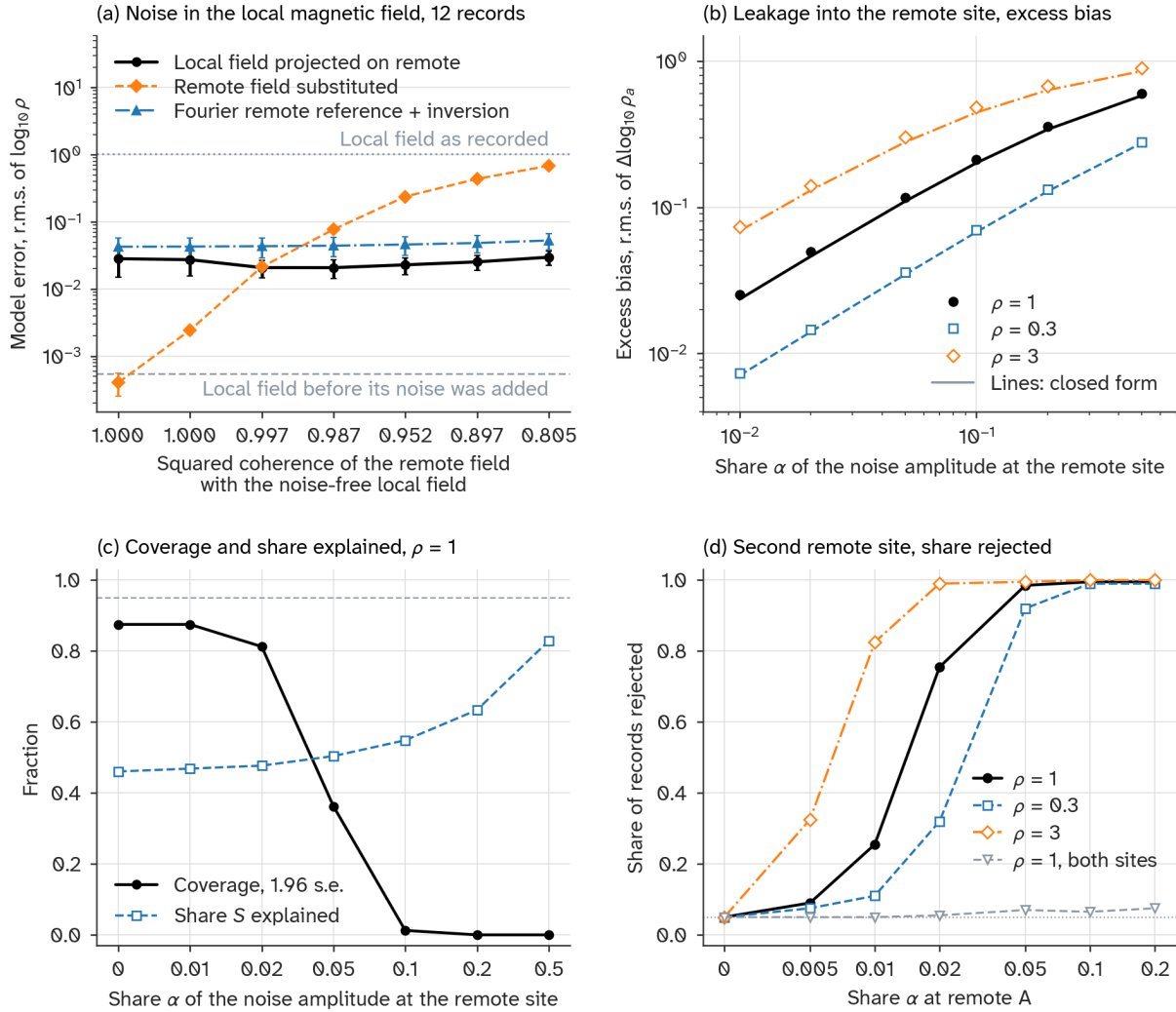


Figure 2. Synthetic tests of the instrument: the local field projected on the remote one against the remote field substituted for it, and a remote site that the noise reaches, with the bias of the root, the coverage of its standard errors and the test with a second remote site. (a) Section 4.1, impulsive noise in the local magnetic field: model error, r.m.s. of $\log_{10}$ resistivity averaged over the octaves of the band, against the squared coherence of the remote field with the noise-free local field, mean of 12 records with two standard errors; dotted and dashed lines, the local field as recorded and before its noise was added. (b) Section 4.4: excess bias of the root, r.m.s. of $\log_{10} \rho_a$ over 5.7–362 s, against the share α of the noise amplitude at the remote site, for a noise power ρ of 0.3, 1 and 3; markers, 80 records per condition; lines, equation 18. (c) At $\rho = 1$, coverage of 1.96 sandwich standard errors (median over the periods) and the share S , against α ; dashed line, nominal coverage. (d) Share of 200 records per condition in which the test with a second remote site rejects at the empirical five percent level (dotted line), against α at site A, with B clean and, for $\rho = 1$, with both sites reached alike.

Table 3. A remote site that the noise reaches, noise over the whole record (Section 4.4). λ , equation 18. Excess bias, r.m.s. over 5.7–362 s of the bias of the root in $\log_{10} \rho_a$ minus that without leakage, beside the closed form. S and coverage (1.96 sandwich standard errors, median over the periods), 80 records. Rejected, share of 200 records rejected by the test with a clean second remote site at the empirical five percent level.

ρ	α	λ	excess bias		S	coverage	rejected
			root	closed form			
0.3	0	0.000	0.000	0.000	0.70	0.93	0.05
	0.01	0.003	0.007	0.007	0.70	0.93	0.11
	0.02	0.007	0.015	0.014	0.71	0.91	0.32
	0.05	0.016	0.036	0.035	0.72	0.71	0.92
	0.1	0.032	0.070	0.067	0.74	0.23	0.99
	0.2	0.062	0.132	0.128	0.77	0.03	0.99
1	0	0.000	0.000	0.000	0.46	0.88	0.05
	0.01	0.011	0.025	0.023	0.47	0.88	0.26
	0.02	0.022	0.050	0.046	0.48	0.81	0.76
	0.05	0.053	0.117	0.110	0.50	0.36	0.98
	0.1	0.100	0.212	0.201	0.55	0.01	0.99
	0.2	0.182	0.356	0.340	0.63	0.00	0.99
3	0	0.000	0.000	0.000	0.24	0.81	0.05
	0.01	0.033	0.074	0.069	0.25	0.91	0.82
	0.02	0.063	0.140	0.130	0.27	0.60	0.99
	0.05	0.144	0.300	0.279	0.31	0.03	0.99
	0.1	0.252	0.483	0.444	0.38	0.00	1.00
	0.2	0.403	0.676	0.633	0.53	0.00	1.00

pler et al., 2010), with the responses of all channels in the station metadata. Each day of the 1-Hz records of Parkfield (October 2003 to December 2004) and of Hollister (November 2003 to December 2004) is inverted with the other observatory as remote. The inversion covers 4–1024 s in eight octaves for seven layers with fixed interfaces and the gain fixed, in the two-input form. Every channel is pre-whitened by a first difference, and a weak roughness prior of 0.2 decade acts between adjacent layers. Each day is a record of its own, and the sandwich covariance is summed over 24 one-hour blocks. Of each observatory two elements are inverted. At Parkfield the first relates the electric dipole at N78°E to the magnetic component at N177°E.

Three instrument routes are run on every day. These are the two-stage minimum, the root, and the root with weights. The weights are estimated band by band, in three passes, from what the projection leaves of the two local magnetic components and from the residual of equation 10. The same weights enter the projection, the two-input reduction and the misfit. The fixed point runs with the weights of a pass held fixed, and the projection is fitted again under the weights of the next. The same days are processed with the open robust code of Smaï and Wawrzyniak (2020) in remote-reference mode, by least squares, the M-estimate and bounded influence, with three window settings: eight periods and 71 percent overlap (its default), the same on pre-whitened series, and four periods and 50 percent overlap on pre-whitened series. The seven-layer model is then fitted to its estimates at 15 periods with their error bars and an error floor of one or three percent under the same prior. This gives eighteen settings. The model fitted to the estimates of the code is our addition to it, so that both routes deliver the same quantity. The comparisons use the days on which the projection explains at least half of both local components in the median of the six octaves of 16–1024 s, and on which every route returned an estimate. These are 265 to 267 days per element.

On the days on which the projection explains at least half of both local components, 1118 solutions for the two elements at the two observatories, four passes alone would have stopped short of a root ($\epsilon \geq 10^{-4}$) on 109, all at Parkfield. On 101 of them the fixed point had stopped moving, with ϵ of 3×10^{-4} in the median and 0.03 at most. The largest term of ϵ then belonged to the deepest layer, which the prior and not the data determines. Its denominator in equation 13 is four orders of magnitude below the largest in the median, so that the stopping rule of the inner minimization shows there, and Newton's method moved the response by less than 0.003 standard errors. On 16 January 2004, the day before the remote field loses its coherence with the local one, the share S_b of equation 16 is negative in the longest octave (-9 and -5 for the two local components), and the fixed point contracts by 0.98 per pass. Four passes then leave the second element 20 standard errors, or 11 percent of the impedance, from the root, the first element 1.8 standard errors, and the root with weights of the second element 0.5. On every other day four passes lie within 0.02 standard errors of the root. The responses below are the roots. They satisfy $\epsilon < 10^{-11}$, except on three days with a layer on a bound of the fit, where the constrained equations hold to 7×10^{-6} . The Jacobian of equation 10 changes the standard errors by less than one percent on 90 percent of these days.

5.2 Error of construction on records without noise

Design. On every day of November 2003 and July 2004 at either observatory on which the remote field explains at least half of the local one, 118 days in all, the electric channel is replaced by the prediction of the model of that day driven by the local magnetic field, plus a smooth response of the other component. The record then contains no noise, and its true model is known. It is passed through the whole procedure, with the projection estimated from the records as they are.

Results. A two-input least-squares fit with the local field returns the model to within 0.01 percent of the impedance. The two-stage minimum misses it by -0.5 to $+0.9$ percent, as the median over the days at the eight periods of Table 4 for each station, month and element, with a scatter of 0.1 to 0.8 percent from day to day (Figure 3c). The root misses it by 0.02 percent at most. The steps of the fixed point fall by two to three orders of magnitude from pass to pass on these days. On the Korean records of Section 7, treated in the same way, the minimum misses the model by up to 2.8 percent at Haenam and 2.0 percent at Gyeongju, and the root by 0.08 percent at most. The error of the two-stage minimum is therefore of the order of the repeatability of the response itself from day to day, which is a few tenths of a percent at Parkfield at periods above 20 s (Oh, 2026).

5.3 Repeatability against a standard robust code

Results. The scatter of the root with weights from day to day is lower than that of the two-stage minimum and of the twelve least-squares and M-estimate settings of the code at most periods of both observatories. It is as low as that of the setting that is best at each period, chosen after the fact (Table 4, Figure 3a, b). Over the 32 combinations of observatory, element and period, its scatter is 0.77 times that of the two-stage minimum in the median (quartiles 0.68 and 0.87) and 0.94 times that of the root without weights. Against each of the twelve least-squares and M-estimate settings it is 0.73 to 0.80 times, against each of the six settings with bounded influence 0.54 to 0.56 times, and against the best setting at each period 0.92 times, with a 95 percent interval of 0.89–1.01 (Figure 3f). The ratios against single settings change little with the rule that selects the days, with another robust measure of scatter, with the floor, the convention of the error

Table 4. Scatter from day to day of $|Z|$ in percent (Sections 3.8 and 5.3) at Parkfield and Hollister, each with the other as remote, on the days on which the remote field explains at least half of both local components. Robust code (Smaï and Wawrzyniak, 2020) followed by the seven-layer model: its M-estimate with the default windows and a floor of three percent, and the smallest scatter of its eighteen settings at each period.

period (s)	5.66	11.3	22.6	45.3	90.5	181	362	724
<i>Parkfield, first element (267 days)</i>								
two-stage minimum	1.16	1.14	0.99	0.62	0.38	0.82	1.21	1.31
root	0.86	0.62	0.45	0.26	0.36	0.54	0.84	0.87
root, weights from the residuals	0.70	0.59	0.46	0.25	0.33	0.55	0.73	0.73
robust code + model, default M	1.01	1.10	0.98	0.62	0.47	0.87	0.90	0.89
robust code + model, best of 18	0.69	0.76	0.49	0.32	0.37	0.78	0.83	0.76
<i>Parkfield, second element (267 days)</i>								
two-stage minimum	1.42	0.91	0.66	0.54	0.80	0.87	0.70	1.00
root	1.86	0.96	0.53	0.53	0.78	0.88	0.65	1.07
root, weights from the residuals	1.65	0.86	0.54	0.51	0.62	0.67	0.59	0.87
robust code + model, default M	1.15	1.05	0.86	0.65	0.63	0.62	0.64	0.90
robust code + model, best of 18	1.09	0.82	0.60	0.50	0.56	0.59	0.62	0.78
<i>Hollister, first element (265 days)</i>								
two-stage minimum	0.93	0.94	0.55	0.76	0.89	0.84	1.04	1.42
root	0.84	0.77	0.53	0.67	0.71	0.63	0.74	1.38
root, weights from the residuals	0.66	0.62	0.50	0.62	0.70	0.60	0.74	1.30
robust code + model, default M	1.26	0.84	0.62	0.76	0.86	0.88	1.08	1.64
robust code + model, best of 18	1.01	0.71	0.44	0.65	0.78	0.80	1.01	1.27
<i>Hollister, second element (266 days)</i>								
two-stage minimum	0.94	0.66	0.55	0.83	0.91	0.87	1.30	2.07
root	0.79	0.62	0.51	0.64	0.81	0.86	1.02	1.44
root, weights from the residuals	0.64	0.45	0.56	0.69	0.78	0.77	0.99	1.39
robust code + model, default M	1.40	0.98	0.60	0.90	1.12	1.05	1.66	2.59
robust code + model, best of 18	1.09	0.73	0.56	0.70	0.77	0.88	1.37	2.10

bars, the prior of the model fit and the number of periods given to the code, and between the two halves of the months.

The result does not hold everywhere. For the second element at Parkfield the ratio against single settings is 0.76 to 1.02, against the best one 1.07, and at the shortest periods most settings of the code are steadier. Band estimates of the two methods without a model show no difference in favor of the time series. Counted over all complete days in the root-mean-square, which gives full weight to the disturbed days, the ratios are 0.96 to 1.14 and the advantage disappears. On the 263 days that the selection leaves out, the iteration with weights stops short of a root ($\varepsilon \geq 10^{-4}$) on 54 of the 526 solutions, 41 of them at Hollister. The comparison is one of repeatability, in which a bias that is the same every day does not show. One code stands for the standard practice.

5.4 Stated errors

Design. The residual of another day is added to the prediction of the model fitted to a day. The noise then keeps its spectrum and its bursts and is independent of the driving field. The refitted responses have a known truth, and the scatter of their errors is compared with the median sandwich standard error.

Results. The sandwich standard errors of the root are close to the scatter of its errors, with ratios of 0.89

to 1.35 at Parkfield and 0.92 to 1.19 at Hollister for the first element (Figure 3e). For the two-stage minimum the ratios are 1.5 to 3.0 and 1.1 to 1.9. Its standard errors do not contain the error of its construction. On the recorded days the root, like the response with the local field, scatters by more than its standard errors state, by factors of 1.9 to 3.3 at Parkfield and of 1.2 to 2.2 at Hollister.

5.5 What the daily deviations share

Design. On the 282 days from 30 October 2003 to 31 December 2004 on which the projection explains at least half of both local components at Parkfield, the instrument response, the root, is compared with the response of the same day inverted in the same way with the local field (Oh, 2026). The comparison uses the level difference and the rank correlation of the deviations of the two responses from their running medians.

Results. The instrument response is higher than the response with the local field by 1.4 and 0.2 percent at 5.7 and 11 s, as the removal of attenuation by noise in the local magnetic channel would make it. It is the same within 0.1 percent at 16–45 s and higher by 0.3 to 0.9 percent at 91–724 s. The deviations of a day from the running median are shared by the two responses, with rank correlations of 0.69 to 0.86 at 5.7–45 s and 0.33 to 0.60 beyond for the first element and 0.63 to 0.85 at all periods for the second (Figure 3d). Two estimates whose errors have different origins agree on the deviation of a day. What varies is therefore the response contained in the record of that day, through source effects (Egbert et al., 2000), changes at the site or noise common to the two observatories. Its cause is not identified here.

6 Recorded railway noise added to an observatory record

6.1 Design

The instrument is tested where the answer is known by adding recorded railway noise to a quiet record. Ten days of June 2004 of the 40-Hz Parkfield record are decimated to 2.5 Hz and inverted, each as a record of its own, at 1–128 s for ten layers, with the gain fixed and the two-input form. The model explains 0.93 to 1.00 of the variance of the electric field in each octave. The noise is the daytime electric field of a recording made at Gyeongju near the 25-kV, 60-Hz railway of Section 7.3. It was taken from the files of one instrument without identifying its position and period, and only its waveform is used, at its own time scale. It is added to the electric channel and, divided by a real constant of three times the local impedance magnitude at 16 s, to the driving magnetic channel, which is how a grounded source nearby would add it. The other magnetic component and the remote record receive nothing. The noise is added in blocks of 10–30 min that cover 30 or 60 percent of the day, or over the whole day, with the nine hours of recorded noise repeated to fill it. Its energy in the analysis band is 0.5, 2, 7 or 100 times that of the record while it is on, at all periods or confined to 4–32 s. Railway noise is flatter in frequency than the natural field. At an energy ratio of 0.5 it is therefore still several times the record at 1–16 s. Two further cases add the noise over 30 percent of the day to the electric channel alone, at energy ratios of 2 and 7.

The measure is the r.m.s. difference of $\log_{10} \rho_a$ at 2–90 s, as a median over the ten days, and a day counts as recovered below 0.02, about 2.3 percent of the impedance. Each route is measured against its own response on the day without added noise, so that only the effect of the noise is counted and not the differences between routes that exist without it. For the single-site routes this reference is the single-site response of the day as recorded. The remote field is that of Hollister, 120 km away, and the instrument routes use no

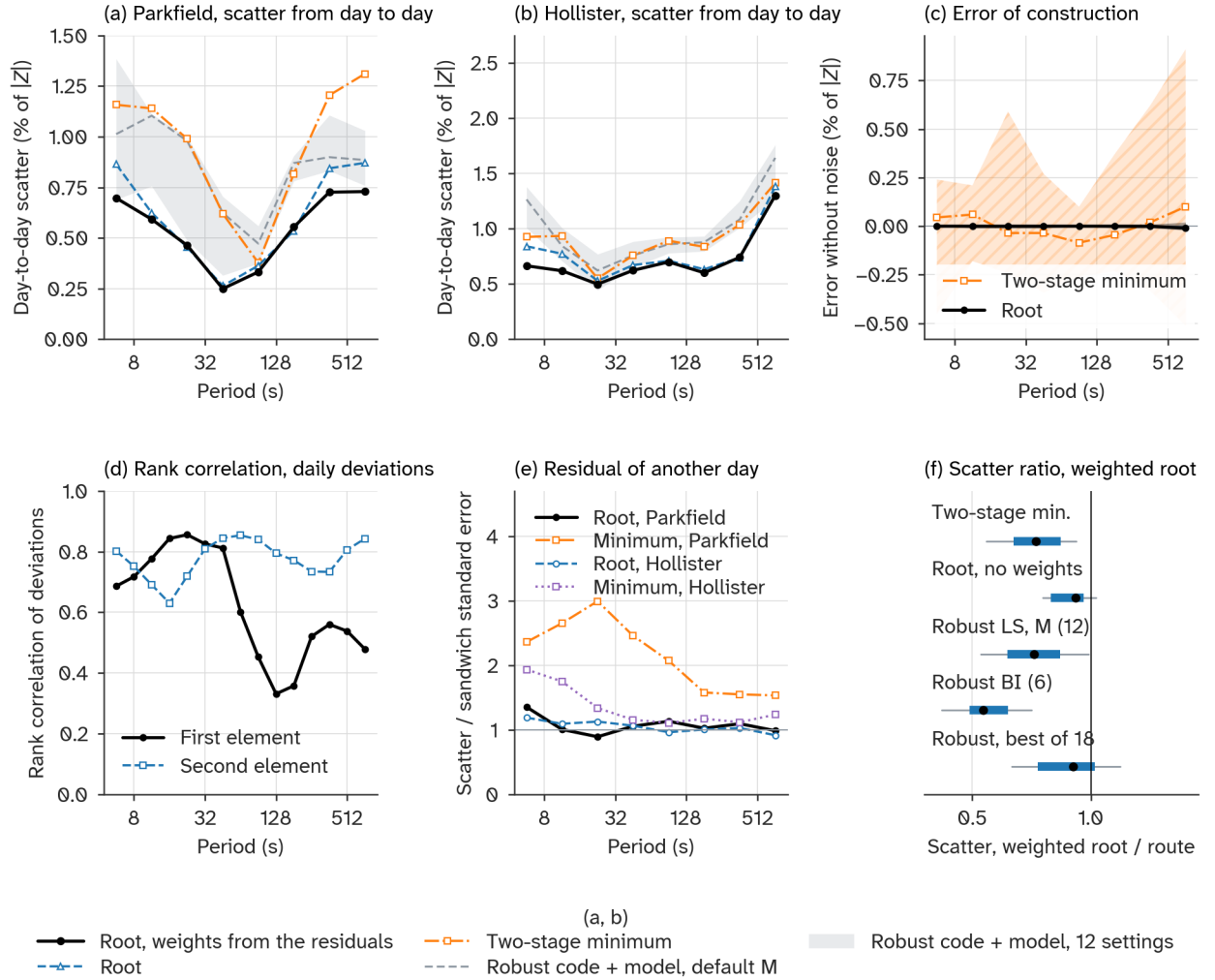


Figure 3. The instrument on the daily records of the Parkfield and Hollister observatories, each the remote site of the other: repeatability against a standard robust code followed by the same model, the error of construction of the two-stage minimum and checks of the stated errors. (a) Scatter from day to day of the first element at Parkfield (Section 5.3), percent of $|Z|$, with Hollister as remote; band, the twelve least-squares and M-estimate settings of the robust code followed by the model. (b) The same at Hollister. (c) Error of construction on records without noise (Section 5.2); lines, median of the eight medians of station, month and element; bands, their range. (d) Parkfield, rank correlation (Section 5.5) of the daily deviations of the response with the local field and of the root from their running medians. (e) Scatter of the error over the median sandwich standard error with the residual of another day (Section 5.4). (f) Scatter of the root with weights over that of each other route in 32 cells of observatory, element and period; dots, medians; thick bars, quartiles; thin bars, 10 and 90 percent points; numbers, settings of the code in a group (LS, least squares; M, M-estimate; BI, bounded influence).

mask. The single-site routes of Oh (2026) serve as baselines, seven in all. These are the inversion without a mask, the envelope detector, the amplitude detector applied band by band, and the weights inferred from the residual of a free transfer function and of the earth model, each started from the mask of the amplitude detector and from uniform weights. Where the noise covers part of the day the true mask is added. Figure 4 draws three of them, and Appendix B gives all of them over part of the day. The standard robust code of Section 5.1 is run on the same records in remote-reference mode, with least-squares estimates and a ten-layer model fitted to them with their errors and a floor of one percent.

6.2 Noise over part of the day

Results. With the noise over 30 or 60 percent of the day, the root of equation 10 without any mask stays close to its response without the added noise. In the median of the days it is within 0.002 to 0.016 in the twelve cases with noise up to seven times the energy of the record, and within 0.005 to 0.044 in the four cases with a hundred times (Figure 4a, b, d, e). This includes the cases with noise at all periods over 60 percent of the day, in which no weights derived from the record recover the response. Among the single-site routes (Appendix B), the true mask recovers 158 of the 160 records with coherent noise, and weights inferred from the residual complete the mask over 30 percent of the day. Over 30 percent of the day at a hundred times the energy of the record, the weights inferred from the residual of the earth model come closer than the instrument. Day by day, the root meets the criterion on 135 of the 160 records with coherent noise, the two-stage minimum on 122, and the standard code followed by the model on 126. The medians of the code range from 0.003 to 0.055. With the 20 records whose noise is in the electric channel alone, the root meets it on 151 of 180 records and the two-stage minimum on 138. On one of the ten days the projection explains almost nothing of the local field at 1–16 s. The fixed point there needs 24 passes in the median, against 2 to 5 on the other days, and the root misses the criterion on six of the 18 records of that day.

On one day with noise at all periods over 60 percent of it at twice the energy of the record, the single-site fit without a mask and the weights from the residual of the earth model both take the form of the added relation. That form is an apparent resistivity that rises nearly in proportion to the period and a phase of a few degrees (Figure 4h, i). The root returns the response of the day as recorded.

6.3 Noise over the whole day

Results. With the noise over the whole day, the time of the noise carries no information. The seven single-site routes recover the response on none of the 80 records, and leaving the three contaminated octaves out of the likelihood recovers one of the 40 with noise at 4–32 s (Table 5, Figure 4c, f). The root of equation 10, without a mask, meets the criterion on all 40 records with noise at 4–32 s. With noise at all periods it meets it on 18 of the 20 records up to twice the energy of the record, on three of ten at seven times and on none at a hundred times. That is 61 of 80 in all. The two-stage minimum meets it on 55, the model fitted to the least-squares remote-reference estimates of the standard code on 58, and the estimates of the code without a model on 15. In this test the root and standard remote-reference processing followed by a model perform alike. Measured instead against the single-site response of the day as recorded, the reference of Figure 4h, i, the counts are 61 for the root, 56 for the code followed by the model and 45 for the two-stage minimum. That reference contains the differences between routes that exist without the noise, and the error of construction of the two-stage minimum then adds to the effect of the noise.

Table 5. Recorded railway noise over the whole day (Section 6.3). Energy ratio, energy of the added noise in the analysis band relative to the record (for noise at 4–32 s, in those octaves). S , median over the octaves and the days. Entries, median over ten days of the measure of Section 6.1, with the days recovered in brackets. Single site, the best of the seven routes of Oh (2026). Code, least-squares remote-reference estimates of Smaï and Wawrzyniak (2020), and the ten-layer model fitted to them. Minimum and root without a mask.

noise	energy		single site	robust code		instrument	
	ratio	S		code	code + model	minimum	root
all periods	100	0.00	0.92 (0)	0.36 (0)	0.071 (0)	0.074 (0)	0.10 (0)
all periods	7	0.08	0.91 (0)	0.082 (0)	0.030 (2)	0.027 (3)	0.022 (3)
all periods	2	0.23	0.87 (0)	0.058 (0)	0.017 (6)	0.015 (8)	0.013 (8)
all periods	0.5	0.51	0.57 (0)	0.023 (5)	0.009 (10)	0.009 (10)	0.009 (10)
4–32 s	100	0.16	0.83 (0)	0.19 (0)	0.012 (10)	0.012 (7)	0.006 (10)
4–32 s	7	0.69	0.45 (0)	0.049 (0)	0.010 (10)	0.009 (9)	0.005 (10)
4–32 s	2	0.80	0.21 (0)	0.038 (3)	0.008 (10)	0.007 (9)	0.003 (10)
4–32 s	0.5	0.84	0.080 (0)	0.014 (7)	0.005 (10)	0.005 (9)	0.003 (10)
records within 0.02			0	15	58	55	61

6.4 The share explained and the convergence as warnings

Results. Where the root fails, it fails by bias and not by scatter. With noise at all periods over the whole day at a hundred times the energy of the record, the root lies above its response without noise on all ten days at 22.6 s, by 0.13 in $\log_{10} \rho_a$ in the median, toward the single-site response, which lies 1.04 above. The projection then explains less than a fifth of the local field in every octave and less than 0.005 of it at 1–16 s. The field of the source fills the local magnetic record, and the remote field cannot explain it. With these shares, equation 17 gives fractions ϕ_b of the way to the single-site response of 0.12 to 0.29 in the octaves of 1–16 s and 0.02 to 0.05 in those of 16–128 s. The root at 22.6 s, 0.13 of the way, lies between the two, and the layered model connects the octaves. On the 80 records with noise over the whole day, the root met the criterion on 36 of the 37 records whose share S exceeded 0.3, and on 25 of the 43 below (Figure 4g). S is a median over seven octaves, and the octave values are the better guide. With noise at 4–32 s at energy ratios of 2 and more, the share falls to 0.00–0.26 in the octaves of 4–16 s in the median of the days, and the root still recovers the response on all ten days of each case. The instrument is intact in the octaves outside the noise, and the layered model connects them.

The fixed point gives no warning. In the case with noise at all periods at a hundred times the energy of the record, it converged on all ten days, in 6 to 12 passes, to roots that miss the criterion. It converged on every other record as well, those on which the root missed the criterion included. The share explained, reported band by band, is the warning.

7 Korean records

7.1 Records, calibration and usable bands

Design. The Korean records were made with Phoenix broadband instruments at Haenam and Gyeongju, and the Japanese record at the Sawauchi remote-reference station, 1090–1100 km from the Gyeongju records (Table 6). They are decimated from 150 to 25 Hz with a single anti-alias filter and used from their first sample, without removal of the power-line harmonics at the original rate. The magnetic channels are calibrated

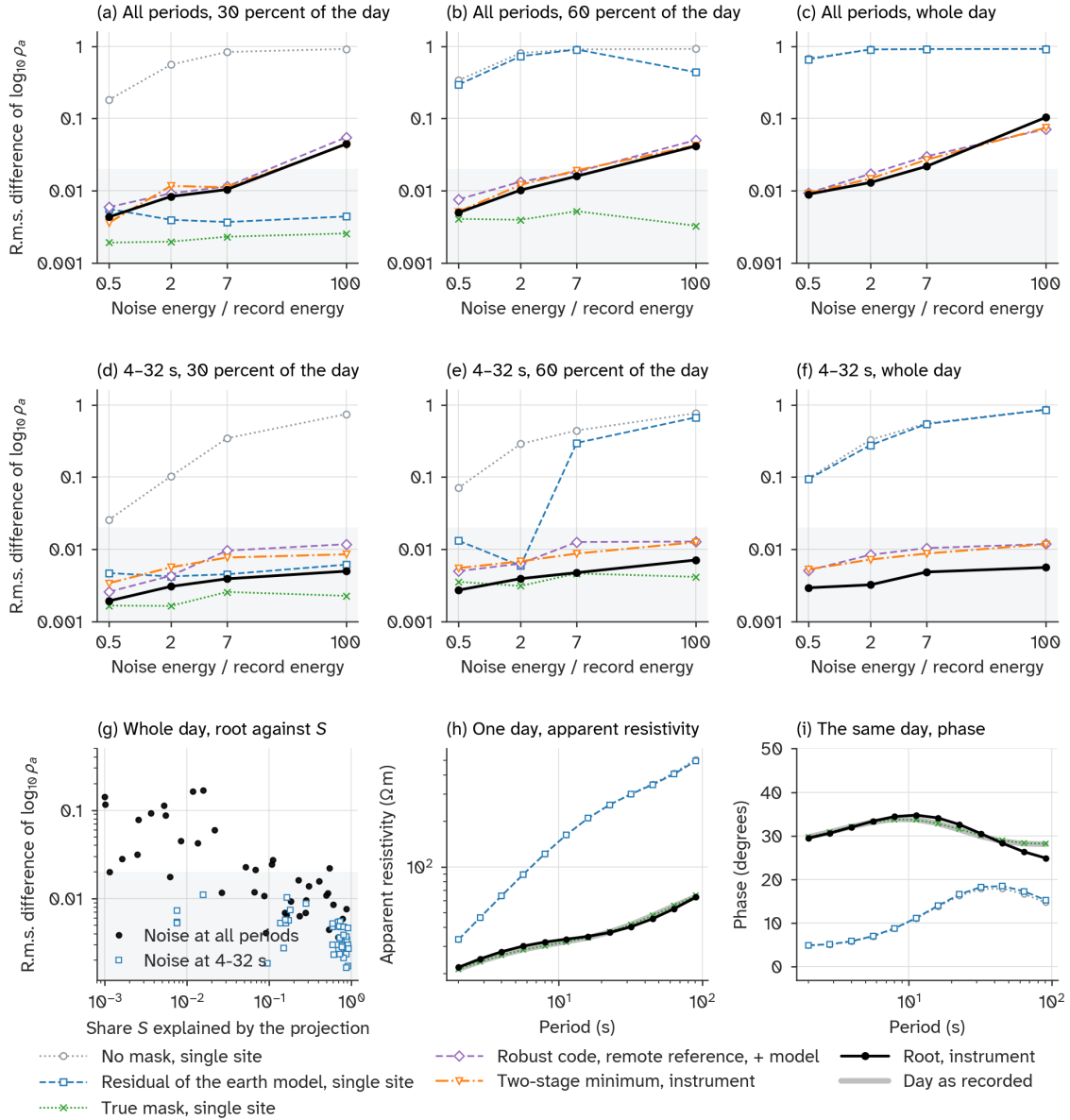


Figure 4. Recorded railway noise added to ten days of the Parkfield record: the difference of each route from its own response without the added noise, against the energy of the noise, for noise at all periods and at 4–32 s over three shares of the day, with one day as an example. (a) Noise at all periods over 30 percent of the day: median over ten days of the r.m.s. difference of $\log_{10} \rho_a$ at 2–90 s from the response of the same route without the added noise (Section 6.1), against the energy ratio; shading, below the criterion of 0.02. (b) Over 60 percent of the day. (c) Over the whole day, where the true mask is not defined. (d)–(f) The same for noise at 4–32 s, with the energy ratio taken in those octaves. (g) Whole day, difference of the root from its response without the noise against S , for the 80 records; $S < 0.001$ drawn at 0.001. (h) Apparent resistivity on one day with noise at all periods over 60 percent of it at twice the energy of the record; gray band, the day as recorded. (i) Phase of the same day.

Table 6. The Korean records and their remote records (Section 7.1). Position from the GPS of the record header, start and end in UTC (header times, for the Haenam records the start in the record name), and the distance and simultaneous duration of the remote record. The Gyeongju sites are numbered from west to east along the survey profile (Section 8.1). The remote stations are Sawauchi (39.5763°N, 140.8277°E) and Daecheong Lake (36.4335°N, 127.5297°E). The Sawauchi record simultaneous with site 23 consists of two runs.

record	position	period (UTC)	length	remote	distance, overlap
Gyeongju, site 26	35.7216°N, 129.4181°E	2023-08-19 18:01 to 08-26 17:59	168.0 h	Sawauchi	1091 km, 159 h
Gyeongju, site 23	35.7034°N, 129.3399°E	2023-08-12 03:24 to 08-19 00:11	164.8 h	Sawauchi	1098 km, 165 h
Haenam, station A	34.6706°N, 126.4995°E	2024-04-11 08:06 to 04-13 05:06	45.0 h	Daecheong	217 km, 23.6 h
Haenam, station B	34.6572°N, 126.4641°E	2024-04-13 07:48 to 04-15 03:36	43.8 h	Daecheong	220 km, 43.8 h

with the response files of the coils named in the record headers, and the electric channels with the electrode separations of the headers of each record, which differ between deployments of the same instrument. The stored samples are volts at the input of the instrument, so no gain is applied. No receiver response is applied either, which changes the phase by less than 0.02° at these periods. Remote records are aligned by their header times. An instrument number names a recording and not a site. One instrument, for example, recorded at Gyeongju in August 2023 and as the remote station of Haenam in April 2024. The records are therefore identified by the position in their headers and by their period.

None of these records has a known response, and cultural noise is the rule. Before inverting them we asked what, in the time series themselves, separates a usable band from a contaminated one. Eleven survey records were examined, with thirty days of the Parkfield and Hollister records as controls. The diagnostics are the share of an electric component that one linear relation explains in a typical segment, and how well the relation estimated at night predicts the electric field of the day (Banks, 1998).

Results. Coherence does not separate the classes. One linear relation explains 0.57 to 0.81 of an electric component in a typical segment at 4–23 s at the Gyeongju record nearest the railway, as much as at the quiet Sawauchi station. What separates them is the stability of the relation in time, the second diagnostic. The test is necessary and not sufficient, because it responds to a change of the relation and not to the presence of noise. At Haenam the relation at 0.25–2.8 s is stable and explains most of the electric field. Yet its phase is near zero and its apparent resistivity grows in proportion to the period, which is the response of a grounded near-field source (Qian and Pedersen, 1991). A band is called clean here when one relation explains at least 0.8 of the electric field in a typical segment, the relation fitted at night predicts the daytime field at least 0.9 times as well as the relation fitted by day does, and the response is not that of a near-field source. By that definition one electric component at Parkfield is clean in all 18 half-octave bands from 0.25 to 91 s. Of the Korean records one band of one component is, and that band is doubtful.

7.2 A near-field source at Haenam

Design. The single-site response at Haenam is that of a near-field source at 0.25–2.8 s, and the contractor's remote station, 217–220 km away, was recording. This is the situation for which equation 10 was written. Two stations of the survey, A and B, lie 3.6 km apart, and station B was recorded for 43.8 h together with the remote station. The electric time series are inverted at 0.25–2.8 s and at 32–90 s for five-layer models, with thicknesses eight times larger for the longer band, and the physical gain, in the two-input form. The prediction is driven in three ways: with the local field, with the remote field as instrument, and with the

remote field substituted for the local one. The roughness prior is weak, 0.7 decade between adjacent layers, and was adopted after a first fit under the prior fixed beforehand, 0.007 decade. At station A the transients of the remote record were bridged, 3.6 percent of its samples. One seven-layer model is also fitted with the instrument to the whole band of 0.25–90 s. The references are the contractor's remote-reference response and a remote reference in the frequency domain that we computed from the same records with jackknife errors.

Results. Driven by the local field, the model returns the single-site response at 0.25–2.8 s, with an apparent resistivity 1.2 to 1.5 (Z_{xy}) and 1.5 to 3.5 (Z_{yx}) times that of the contractor and a phase lower by 3° to 17° and 12° to 28° (Figure 5). With the remote field as instrument the ratios are 0.93 to 1.03 and 0.87 to 1.05 at station B. The r.m.s. differences from the contractor's response, 0.015 and 0.025 in $\log_{10} \rho_a$ and 6° and 4° in phase, are those of the remote reference in the frequency domain from the same records, 0.03 and 4° . Substituting the remote field for the local one gives 1.2 to 1.6 and 0.26 to 0.50 times the contractor's apparent resistivity. Under the prior fixed beforehand, which flattens the phases of the instrument responses, they differ from the contractor's by 0.01 to 0.12 in $\log_{10} \rho_a$ at the two stations, against 0.12 to 0.53 for the single-site inversions. The contractor's response is itself an edited estimate, with 73 and 82 percent of its crosspowers rejected at stations B and A according to its files. Agreement with it means that two independent treatments of the records give the same response. It is not a verification against a known earth.

At station A the remote field is a weaker instrument. The bridged samples of the remote record carry 80 to 97 percent of its energy at these periods. With them bridged, the projection has 5 to 18 percent of the energy of the local field, against 19 to 43 percent at station B. The instrument response is then 0.79 to 1.12 (Z_{yx}) and 1.3 to 1.9 (Z_{xy}) times the contractor's, against 2.3 to 5.1 for the single-site inversion. Without the bridging the projection has 0.1 to 6 percent of the energy. The Z_{xy} response then falls back toward the single-site one, while its standard error stays at 0.02 in $\log_{10} \rho_a$. With a weak instrument the bias grows before the standard error does, and the share of the local field that the projection explains gives the warning. Halves of these records inverted separately differ by two to three times their sandwich standard errors, as the days at Parkfield do.

The seven-layer model fitted with the instrument to the whole band includes the interval of 4–23 s where the projected field explains none of the electric field and the estimate in the frequency domain fails. It differs from the contractor's response at its 27 periods by 0.06 and 0.04 in $\log_{10} \rho_a$ and 4° . For Z_{xy} that is twice the difference of the two-stage minimum. At the root the octaves below 3 s, where the residual formed with the local field is small, weigh four to eight times more than those beyond 30 s, and one layered model does not follow both ends of that response. With one input and one remote channel the response is 8 to 16 percent low in $\rho_{a,xy}$, so the two-input form is needed.

Qualifications. Station A is the Haenam recording with the highest coherence between the electric and the magnetic field. The weak prior and the bridging of the remote transients were adopted after the first results had been seen. Both records carry line spectra of unidentified periodic sources, a harmonic family at 0.26 Hz at both stations and one at 1.22 Hz at station B. There the lines hold 29 and 10 percent of the energy of E_y and H_x at 0.25–2.8 s and make much of the single-site bias of Z_{yx} , which is 1.2 to 2.1 times the contractor's response with the lines removed, against 1.5 to 3.5. The instrument response does not change when the lines are removed. Some lines are common to the local and the remote recorder, however, and there the condition on the instrument fails. The factor of 1.3 to 1.9 that remains at station A is not that of a weak instrument alone. With the lines removed it is 1.1 to 1.5, and the rest is unexplained. The projection

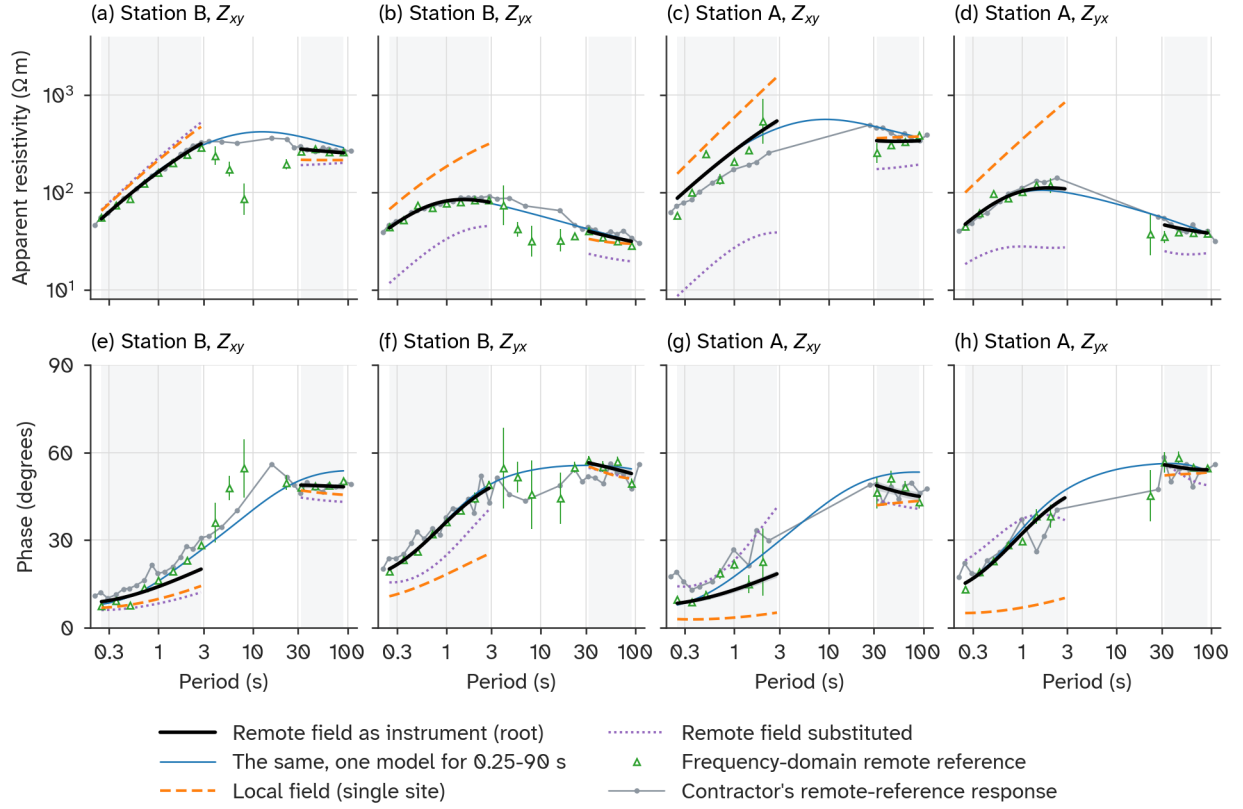


Figure 5. The off-diagonal elements at the two Haenam stations with the Daecheong field as instrument, beside the single-site fit, the remote field substituted for the local one and frequency-domain estimates. (a) Apparent resistivity of Z_{xy} at Haenam station B, 220 km from the remote station: five-layer models at 0.25–2.8 s and 32–90 s (shaded) and one seven-layer model over the whole band (Section 7.2); gray band on the root, one sandwich standard error; frequency-domain estimates with errors above 0.3 in $\log_{10} \rho_a$ or 15° not drawn. (b) Z_{yx} at station B. (c, d) Z_{xy} and Z_{yx} at station A, 217 km from the remote station. (e)–(h) Phase, as in (a)–(d).

at Haenam was fitted without the weights of the misfit, which excise 5 to 6 percent of the record. Refitting it under them changed the response at 32–90 s by less than 0.001 in $\log_{10} \rho_a$.

7.3 A railway record at Gyeongju

Design. No band of the two Gyeongju records (sites 26 and 23, seven days each) is clean. The four train-free hours (01–05 h) of a night do not determine a relation band by band at 2.8–45 s, where the railway noise lies. We call those hours train-free because the noise is at its night level in them. We hold no timetable of the two lines, the working figure for their service is 05:30–24:00, and trains at night on the conventional line are not excluded. Sites 23 and 26 lie 0.4 and 7 km from an electrified conventional railway and 14 and 21 km from a high-speed line, both 25 kV and 60 Hz, and galvanic distortion is strong at these sites. An earth model is constrained by all bands at once, and a remote record was running. The Sawauchi station, 1090 km away, recorded simultaneously with site 26, the record 7 km from the conventional line, for 159 h. Its field is tested as an instrument for the records of this survey in Section 8.

We invert that record at 0.5–90 s in eight octave bands, for seven layers to 50 km with the gain fixed,

in the two-input form, with a prior of 0.2 decade between adjacent layers. The record is cut into complete blocks of four hours with zero weight within 150 s of their ends, and each fit is started from the best half-space and from two perturbations of it. The record is inverted in three ways, and the responses are compared with two references (Figure 6). The first is the contractor's remote-reference response of the same recording, which has no estimates between 1.4 and 16 s. The second is a classical two-input remote-reference estimate that we computed from the same calibrated series at 11–90 s, without robust weighting or selection, with jackknife errors from leaving one day out.

Results for the references. The levels at this site are uncertain. The contractor's apparent resistivity is 1.15 to 1.57 times ours at 16–90 s for E_x-H_y and 2.3 to 3.6 times for E_y-H_x , with r.m.s. phase differences of 2° and 6° . The factors fall with period, and we have not been able to trace them. The contractor's response and ours use the same electrode separations and the same coil files, which therefore cannot make the difference. Two explanations remain open: attenuation of an estimate whose remote field is weakly coupled to the local one, and a contractor's response that mixes day and night, two thirds of its crosspowers having been rejected. The record also shows a static effect. In our estimates the two elements differ by a factor of two to three in apparent resistivity and by 12° at most in phase. We therefore measure against the estimate from the same series, which shares its remote record with the instrument, and regard the phases as the firmer comparison.

Results without a mask. Inverted whole with the local magnetic field (41 blocks, 164 h), the record gives apparent resistivities of 1 to 4 Ωm , 2.1 and 1.8 decades below the reference for the two elements. Over the whole record the Sawauchi field explains 3 to 16 percent of the energy of the local H_y at 11–90 s and 0.6 to 3 percent of that of H_x , against 48 to 91 percent in the train-free hours. In the service hours most of the regressor is local noise. For E_x-H_y the bias has the size that attenuation by noise in the regressor would give for that share. For E_y-H_x it is smaller, and the fitted response there is presumably in large part the relation between the electric and the magnetic part of the noise itself.

Results with the remote field as instrument. The record is inverted whole, service hours included, on the 39 blocks (156 h) that both stations recorded, with the Sawauchi field as instrument (root of equation 10). The inversion uses the three octave bands of 11–22, 22–45 and 45–90 s, in which the two fields are coherent. The response of the model, compared with the reference at the nodes of 16–90 s, differs from it by 0.04 in $\log_{10}\rho_a$ and 2° for E_x-H_y and by 0.05 and 4° for E_y-H_x . For E_x-H_y the band variances at the root are 9 to 18 times those of the second stage, because the local magnetic record is mostly railway noise, and the prior counts for that much more. A regularization weight ten times smaller gives 0.04 and 1° , and weights ten and a hundred times larger give 0.05 to 0.07. A projection estimated for each day from the other days gives 0.05. Leaving out one day at a time, each fit started from the solution of the whole record, moves the apparent resistivity at 32 s within 223–233 and 75–83 Ωm . The two-stage minimum moved within 189–258 and 74–86. With the projection of the whole record used on a part of it, the root is thus the steadier of the two. The model explains 27, 29 and 9 percent of the variance of E_x in the three octaves, and 3, 2 and 1 percent of that of E_y . The median energy of E_y at 10–22 h is ten to twenty-six times that of the train-free hours. With 10^4 degrees of freedom or more in each octave, a few percent suffice.

Results with weights from the hours of the night. With $M(t) = 1$ in the train-free hours (01–05 h on six nights, 14 percent of the record) and the local field, the response for E_x-H_y differs from the reference by 0.08 and 8° at 16–90 s. Between 1.4 and 16 s, where the natural signal is weakest and the contractor's file is empty, it is the interpolation of a layered earth between the bands that carry information. For E_y-H_x the

same inversion fails. At 2–11 s the relation fitted at night is that of a near-field source, with an apparent resistivity rising with the period to 2000 Ωm and a phase near zero. The layered model is drawn toward it, and its response at 16–90 s lies 0.3 to 0.4 above the reference. There it coincides with the contractor's response (0.07 in $\log_{10}\rho_a$), for which we have no explanation other than chance. Hours without trains are not hours without sources, and the instrument does not depend on them.

Scope. The noise is present throughout the service hours, eighteen to nineteen hours of the day, without a return to the night level between trains. The instrument is used at 11–90 s, and below 11 s the remote field is incoherent with the local one. For the second record, site 23, the simultaneous Sawauchi record is coherent with the local magnetic field at night, with a squared coherence of 0.3 to 0.8 at 32–90 s night by night. One hour of transients in the local H_y lowers it below 0.05 over the whole record. The train-free hours alone do not determine a model there. On the first day, with the Sawauchi field as instrument, the root gives 2.82 to 2.84 in $\log_{10}\rho_a$ for E_x – H_y at 8–128 s, within 0.10 of the robust remote-reference estimate.

7.4 The full tensor and the tipper at Gyeongju

Design. In a three-dimensional inversion of this survey, not shown here, the diagonal elements and the tipper of the contractor's files could hardly be used. A free transfer function with two inputs and three outputs (E_x , E_y , H_z) on half-octave nodes is solved as the root of equation 10 with the Sawauchi field as instrument, on 4-h blocks. The unknown is here the transfer function and not an earth model, the setting in which equation 10 corresponds to the two-stage remote reference of [Chave and Thomson \(2004\)](#). The projection is fitted under the weights of the misfit, and the sandwich covariance is summed over the blocks, 39 for the whole record, 33 for the service hours and 6 for the train-free hours. The root is solved on the whole record and on the service hours alone, and compared with the single-site estimate of the train-free hours and with the classical remote-reference estimate of the same series. The calibration chain is that of Section 7.1, with the vertical coil, whose response differs from that of the horizontal coils, calibrated with its own file. Five criteria were fixed before the computation. The median skew of the phase tensor of the train-free estimate is below 5° . The roots on the whole record and on the service hours have a median skew below 10° and diagonal elements within two standard errors of the train-free estimate in two of three octaves. The diagonal standard errors are below half of the off-diagonal element in two of three octaves. The magnitude of the tipper agrees within two standard errors in two of three octaves. After the criteria, the information that the service hours and the whole record provide is compared with that of the train-free hours by the ratio of the variances of the estimates with one day left out, or one night for the train-free hours.

Results. All five criteria are met (Figure 7). The whole record and the service hours agree in all 18 cells of six elements by three octaves at 11–90 s, each cell judged at every node of its band. Each agrees with the classical remote-reference estimate of the same series, with its jackknife errors, in all 18 cells. Each agrees with the single-site estimate of the train-free hours, within two standard errors of both estimates combined, in 15. The exceptions are Z_{xx} and Z_{yx} at 11–22 s, where the projection explains 0.006 to 0.03 of the local field, and Z_{yy} at 90 s, the edge of the band, where the root, the classical remote reference and the contractor's response have a skew of -13° to -16° against $0 \pm 6^\circ$ for the train-free hours.

Z_{xx} is not zero. The four estimates give $|Z_{xx}|/|Z_{xy}|$ of 0.3 to 0.7 at 16–90 s, with standard errors of 1 to 5 percent of $|Z_{xy}|$ for the root at 22–90 s. The skew of the phase tensor meanwhile stays below 5° in the median (2.3° , 3.7° and 4.6° for the train-free hours, the whole record and the service hours). A diagonal ten to thirty times its standard error, reproduced by the three other estimates, with a symmetric phase tensor is

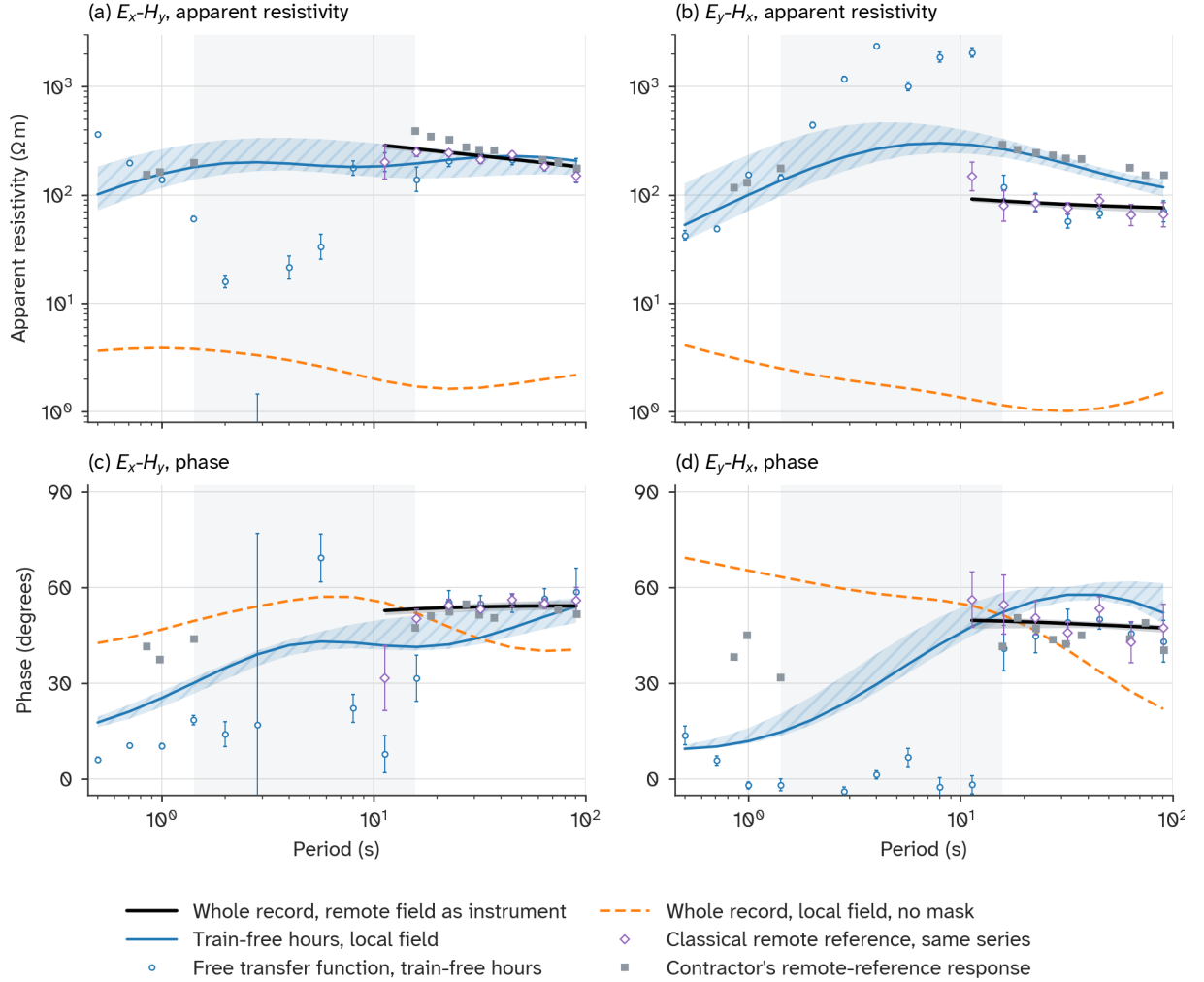


Figure 6. Two elements at Gyeongju site 26, 7 km from a 25-kV, 60-Hz railway: the whole record with the Sawauchi field as instrument, the train-free hours with the local field and the whole record with the local field and no mask, beside a free transfer function of the train-free hours, a classical remote-reference estimate and the contractor's response. (a) Apparent resistivity of E_x-H_y (Section 7.3). Bands: for the train-free hours (01–05 h), the range with one night left out; for the instrument (Sawauchi field, 11–90 s), the range with one day left out. A free transfer function fitted to the train-free hours under the same likelihood (17 half-octave nodes) has errors from its inverse Hessian, the classical remote reference jackknife errors leaving one day out, and the contractor's file no estimates in the shaded interval. (b) E_y-H_x . (c, d) Phase of the two elements.

consistent with galvanic distortion of a nearly two-dimensional response and not with noise. Z_{yy} is small, 0.1 to 0.3 of $|Z_{yx}|$ at 22–64 s, with standard errors of 8 to 26 percent of $|Z_{yx}|$ at 22–90 s. The tipper has $|T|$ of 0.62 to 0.93 with standard errors of 0.01 to 0.06 for the root on the whole record at 16–90 s, and its real induction arrows are not reversed and point between west-southwest and west-northwest. It is determined by the train-free hours alone (explained variance of H_z of 0.72 to 0.82 at 22–90 s) and reproduced by the root.

The service hours add information to the train-free hours and do not repeat them. In the median over the seven periods, the 132 service hours give 4.8 times the information of the six train-free nights for Z_{xx} and

2.2 times for Z_{xy} , about as much for Z_{yx} (1.1) and half as much for Z_{yy} and the tipper. The whole record gives 5.2, 2.6, 1.4 and 0.8 for the four elements and 0.7 and 0.5 for T_x and T_y . It gives less than the train-free hours for Z_{yy} and the tipper, because its estimate weights the service hours and the train-free hours alike within each band. At 11 s, where the projection explains least, the service hours give less than the train-free hours for Z_{xy} , Z_{yy} and the tipper. Per hour, a service hour carries 0.9 of the information of a train-free hour for Z_{xx} , 0.4 for Z_{xy} , 0.2 for Z_{yx} and about 0.1 for Z_{yy} and the tipper. With six or seven estimates left out, these ratios are rough.

For a three-dimensional inversion the full tensor is supplied with its covariance. To reproduce the root of Section 3.4, an inversion would need the kernel Γ and d as well. The files of the EDI format carry the variance of each element, and the covariance between elements and periods is stored beside them. That covariance, formed from the scores of 39 blocks (33 for the service hours), has rank 38 (32) for 84 real quantities and cannot be inverted as it stands. An inversion needs it formed from shorter blocks or propagated from the spectra of the residual. The level of the E_y row rests on the electrode separations of the record header, 50 m for both dipoles, as in the contractor's file. One caution applies. The sandwich errors fall short of the scatter from day to day by up to a factor of four at 16 s and of two at 22–90 s. The conservative band is therefore 22–64 s, or 16–64 s with the errors at 16 s scaled by that factor.

8 Testing the remote sites of a railway survey

The consistency test of Section 3.6 is applied here to the records of a survey near 25-kV railways. It asks which recorded magnetic fields can serve as remote sites there, and whether the distant Sawauchi field used in Section 7 is valid.

8.1 Design

The two Gyeongju records belong to a survey of 29 sites along a profile of about 70 km in southeastern Korea that ends at the east coast, recorded in July and August 2023 by four instruments of the same type that were moved about once a week. The sites are numbered from west to east, and the two Gyeongju records are those of sites 26 and 23. Twenty-nine of its records, at 28 sites, are 24 h or longer and overlap a file of one week of the Sawauchi recorder by 24 h or more, and each ran together with two to five others. For each such record L and each record B that ran at the same time at another site, 96 ordered pairs in all, two roots are computed on the same 1-h blocks with ramps of 150 s: one with the Sawauchi field as instrument (A), and one with the magnetic field at the site of B. The roots are two-input free transfer functions of E_x and E_y on half-octave nodes in the octaves of 11–90 s, as in Section 7.4, so that the test does not depend on an earth model of the site. Each has its projection fitted under the weights of the blocks. The statistic of equation 20 is formed from the differences of $\log_{10} \rho_a$ of Z_{xy} and Z_{yx} at the seven periods of Figure 7, with the covariance of equation 19 from the block scores of the two roots together. A component enters only if both instruments are relevant to its driving field, with ϕ_b of equation 17 below 0.02 for H_y (Z_{xy}) or H_x (Z_{yx}), so that the pull of a weak instrument differs little between the two roots. A pair without such a component is not tested. The move μ of equation 21 is taken with the single-site estimate on the same blocks. The test is run on the service hours (06–24 h) and, as a control in which the noise is at its night level, on the train-free hours (01–05 h). It is repeated with blocks of 30 and 120 min, with thresholds of 0.01 and 0.05 on ϕ_b and with a Gaussian null in place of the multivariate t . The move μ is also predicted from equations 8 and 18. For this,

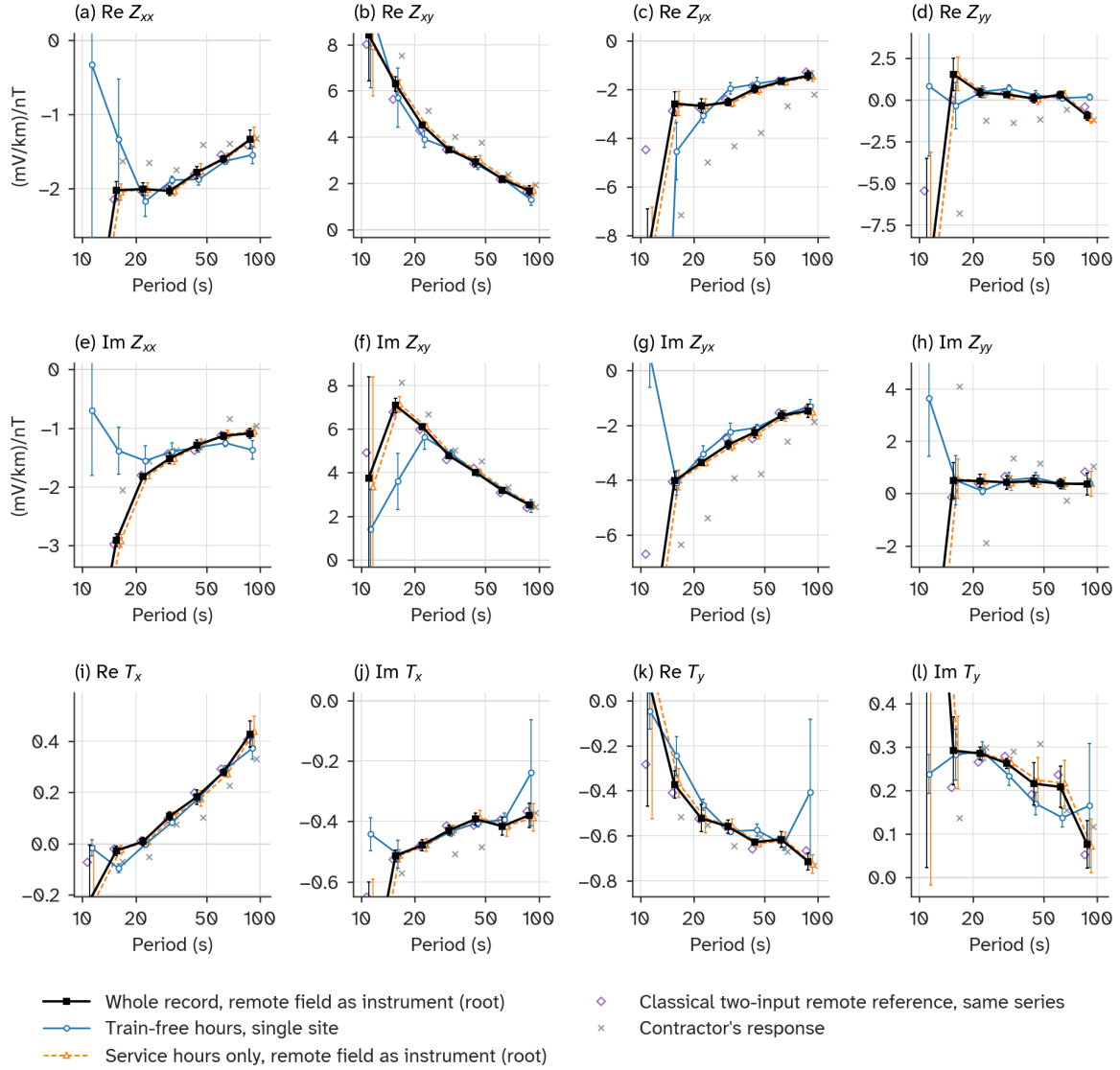


Figure 7. The full impedance tensor and the tipper at Gyeongju site 26 from free transfer functions with the Sawauchi field as instrument, on the whole record and on the service hours alone, beside the single-site estimate of the train-free hours, the classical remote-reference estimate and the contractor's response. (a)–(d) Real parts of Z_{xx} , Z_{xy} , Z_{yx} and Z_{yy} , in (mV/km)/nT (Section 7.4). Error bars, sandwich standard errors from the scores of 4-h blocks; the contractor's response, drawn at the nearest period within 20 percent, and the classical estimate without error bars; axes set by the points at 16–90 s. (e)–(h) Imaginary parts. (i, j) Real and imaginary parts of T_x . (k, l) The same for T_y .

α , ρ and γ are estimated from the band powers of the local field, of the field at B, of their projections on the Sawauchi field and of the cross powers of the remainders, with the train-free hours taken as free of the noise.

The Sawauchi field is tested in the same way against the 1-s records of four magnetic observatories, with each of the 29 records as L, the Sawauchi field as A and an observatory field as B. The observatories are Cheongyang (CYG) of the Korea Meteorological Administration, 196 to 247 km from the sites, and Kanoya (KNY), Kakioka (KAK) and Memambetsu (MMB) of the Japan Meteorological Agency, at 495 to 497, 964

to 1032 and 1546 to 1612 km. The records of the Japanese observatories are the definitive data of the Japan Meteorological Agency (Kakioka Magnetic Observatory, 2013a,b,c) served by the World Data Center for Geomagnetism, Kyoto, and that of CYG is the INTERMAGNET record (Love and Chulliat, 2013). Each is interpolated from its 1-s samples to the time grid of the local record with a cubic spline, and blocks that contain a missing value are left out. In the train-free hours the horizontal fields of the observatories and of the Sawauchi recorder correlate best at 20–100 s at lags of -0.8 to 3.0 s. A delay of 3 s changes the phase by less than 0.4 rad across the third of an octave over which the projection is fitted. With a clean observatory field, equation 22 turns μ into the move λ_A/λ_1 of the root with the Sawauchi field toward the relation of the noise, as a fraction of the move of the single-site estimate.

8.2 The other sites of the survey as second remote sites

In the service hours 61 of the 96 pairs could be tested, 34 pairs of records, with a median of 14 components. Of these, 49 were rejected at $p < 0.001$ (Table 7, Figure 8). In the train-free hours 63 pairs could be tested and none was rejected at that level. The control rejected 5 of 63 pairs at the five percent level and 1 at one percent, and 8 at the five percent level with the Gaussian null. With blocks of 30 and 120 min the service hours gave 48 of 61 and 48 of 63 rejections at $p < 0.001$. The control gave none of 69 and none of 56 at that level, but 9 and 6 at the five percent level, above the nominal share, so the counts at 0.1 percent are the ones to read. With thresholds of 0.01 and 0.05 on ϕ_b the service hours gave 41 of 52 and 50 of 61. The rejected pairs span the tested distances from 1.5 to 12.5 km, and the 12 tested pairs that were not rejected lie between 1.7 and 9.4 km.

The ordered pairs are not independent tests. The two orders of a pair of records share both records, a record enters several pairs, and the records of one week share the same days of train service and the same Sawauchi file. Counted by these units, the service hours give a rejection for 32 of the 34 pairs of records, for 18 of the 21 local records tested and in 6 of the 7 weeks with a testable pair. The train-free hours give none, for 32 pairs of records, 25 local records and 8 weeks. Of the 49 ordered pairs tested in both periods, 38 were rejected in the service hours alone and none at night alone. With the week taken as the independent unit, all six weeks whose counts differ between the two periods have more rejections in the service hours, a sign test with $p = 0.03$. Resampling the weeks gives a 95 percent interval of 0.58 to 0.97 for the share of tested pairs rejected in the service hours, 0.80.

In 47 of the 49 rejected pairs μ is positive, with a median of 0.97, so that the root with the Korean field lies about as far toward the single-site estimate as that estimate itself. By the selection both fractions ϕ_b are below 0.02, and that of the Korean field exceeds that of the Sawauchi field by 0.011 at most in the median over the components of a pair. In the scalar model a positive μ points to the Korean field as the invalid instrument, unless the Sawauchi field carried the noise in a larger proportion than the local record. Its distance of 1090 km makes that implausible, and the test against the observatories below does not find it. At night the two roots differ by a median of 0.05 in $\log_{10} \rho_a$, against 0.7 in the service hours, and none is rejected, although μ is still positive in 59 of the 63 pairs. For sites 21 and 16, 9.7 km apart, the root with the field of site 16 gives a $\log_{10} \rho_a$ of Z_{xy} 0.95 to 2.94 above the root with the Sawauchi field and 0.26 to 0.48 above the single-site estimate. For Z_{yx} it lies 0.05 to 0.66 above the root with the Sawauchi field (Figure 8d). The largest difference, at 16 s, coincides with a drop of the root with the Sawauchi field by 2.1 from 11 s, and without that period the largest standardized difference is 11.6. The root with the Sawauchi field explains 1.5 to 6.7 percent of the variance of E_x at site 21 in these hours. Predicted from the band powers, the move

Table 7. The other sites of the 2023 survey as second remote site B, with the Sawauchi field as A, over 96 ordered pairs (Section 8.2). Components, median number used (at most 14). Blocks, range of the number of 1-h blocks. Rejected, pairs whose p (multivariate t null) is below the level. μ , median over the pairs rejected at $p < 0.001$ of the median over their components, with the number of pairs in which it is positive. $\max \Delta \phi_b$, largest $\phi_b(B) - \phi_b(A)$ over the tested pairs, each a median over its components. $\max t/t_{95}$, largest statistic relative to its simulated 95 percent point.

hours	pairs tested	components	blocks	rejected at $p <$			μ	$\max \Delta \phi_b$	$\max t/t_{95}$
				0.05	0.01	0.001			
service, 06–24 h	61 of 96	14	24–102	52	52	49	0.97 (47 of 49)	0.011	8.2
train-free, 01–05 h	63 of 96	14	8–25	5	1	0	–	0.011	1.3

μ is 0.80 against 0.87 measured, in the median over 207 cells of element and octave of the tested pairs. The two are unrelated from cell to cell, with a rank correlation of -0.06 , and 47 percent of the cells agree within a factor of two.

Of the 35 pairs that could not be tested in the service hours, the Sawauchi field is the weak instrument in 13, the field at B in 8 and both in 13. In the remaining one, site 26 with site 28, the two fields are relevant in different components only. Because leakage raises the share of the local field that the field at B explains (Section 3.6), the selection favours pairs in which B carries the noise. The rate of rejection applies to the pairs tested.

8.3 The distant field against four observatories

Against the observatories the Sawauchi field passes the test (Table 8, Figure 8). In the service hours 71 of the 116 combinations of record and observatory could be tested, at 19 of the 29 records. One was rejected at the five percent level, CYG with $p = 0.025$, and none at one percent. In the train-free hours 98 could be tested, and 2 were rejected at five percent, 1 at one percent and none at 0.1 percent. In the service hours the roots with the observatory fields differ from the root with the Sawauchi field by a median of 0.07 to 0.14 in $\log_{10} \rho_a$, against 0.67 for the survey sites. The median of μ is 0.03, with 10th and 90th percentiles of -0.05 and 0.15 , against 0.82 for the survey sites. By equation 22 the 10th percentile places λ_A/λ_1 at about 0.05 or less in nine of ten tests. Sixteen of the 18 records at which a survey site was rejected are among those tested. CYG, 196 to 247 km from the sites, passes as well. Of the 45 combinations not tested in the service hours, both fields are weak instruments in 34, the Sawauchi field alone in 6 and the observatory field alone in 5. The p values of the observatory tests lie above the uniform distribution (Figure 8c), so the test is conservative here. For the Gyeongju record of Section 7.3 the Sawauchi field passes against all four observatories in the service hours, with p of 0.17 to 0.94 on five to seven components of Z_{xy} , while neither of the two survey sites that ran with it gave a component in which both fields were relevant.

9 Discussion

9.1 What the instrument in model space adds

Two comparisons with a standard robust remote-reference code bound what the method adds. With railway noise added over the whole day, the root and the code followed by a layered model recovered the response on nearly the same number of records (Section 6.3). On the daily records of two observatories the root with weights scattered less from day to day than each least-squares and M-estimate setting of the code followed

Table 8. The Sawauchi field as A tested against four observatories as B for the 29 records of the survey (Section 8.3), with the survey sites of Table 7 for comparison. L–B, range of distances over the tested records or pairs. Tested and rejected as in Table 7. $|\Delta|$, median over the components of the absolute difference of $\log_{10} \rho_a$ between the two roots. μ , median over the tested records or pairs, with its 10th and 90th percentiles.

B	L–B (km)	hours	tested	rejected at $p <$			$ \Delta $	μ	μ , 10th to 90th percentile
				0.05	0.01	0.001			
CYG	199–242	service	16 of 29	1	0	0	0.10	0.02	–0.07 to 0.18
	199–246	train-free	23 of 29	0	0	0	0.06	0.33	0.04 to 1.70
KNY	495–497	service	18 of 29	0	0	0	0.14	0.06	–0.02 to 0.16
	495–497	train-free	25 of 29	1	1	0	0.05	0.23	–0.16 to 0.72
KAK	970–1027	service	19 of 29	0	0	0	0.10	0.03	–0.06 to 0.15
	965–1032	train-free	25 of 29	0	0	0	0.04	0.09	–0.13 to 0.57
MMB	1552–1607	service	18 of 29	0	0	0	0.07	0.01	–0.03 to 0.04
	1547–1612	train-free	25 of 29	1	0	0	0.04	–0.07	–0.42 to 0.14
survey sites	1.5–12.5	service	61 of 96	52	52	49	0.67	0.82	0.26 to 2.47
	1.5–9.7	train-free	63 of 96	5	1	0	0.05	0.46	0.03 to 0.94

by the model in a robust measure of scatter, but band estimates without a model did not differ, and the advantage disappeared in the root-mean-square over all complete days (Section 5.3). The remote reference in model space is therefore not a better remote reference in itself. Both comparisons are with the usual practice, a robust estimate band by band followed by a fit with error bars, which is a different estimator from the root (Section 3.4), so they compare routes and not the reduced form. What the method adds lies elsewhere.

First, it is the remote reference of the time-series likelihood. The sample weights, the band variances and the counted degrees of freedom of Oh (2026) enter the instrument fit unchanged, so that a remote field and weights inferred from the residuals act in one fit, and that combination was the steadiest of the instrument routes. Second, it is a root and not a minimum. The two-stage minimum, the obvious way to put a projected field into an inversion, carries an error of construction of the order of the repeatability of the response, which its standard errors do not contain (Sections 5.2 and 5.4). Third, the scores of the equations over blocks of time give both the sandwich covariance of the root and the covariance of the difference of two roots. The first states the noise within a record to within a factor of 1.4 for the first element of both observatories, where the standard errors of the minimum fall short by up to a factor of three. The second makes the consistency test of two remote sites possible, a test of the remote site where no record is known to be clean. Fourth, the same equations with a free transfer function deliver the full tensor and the tipper from the service hours of a railway record, with a covariance between elements and periods and the share S_b attached to every octave.

9.2 Failure modes and their diagnostics

The instrument has two conditions and the estimate a third. The source must not reach the remote site, the remote field must explain the local one, and the equations must be solved. Table 9 sets out how each fails and how each is seen, together with the variation between records, which is not a failure of the estimate but which no single record states either. The standard error of one record shows none of the four.

Validity needs the most care, because a remote record that carries part of the local noise raises S , leaves the fixed point and the explained variance unchanged or improved, and brings the root closer to the single-site estimate, while the coverage of the stated intervals falls to near zero. Near the railways of the survey the

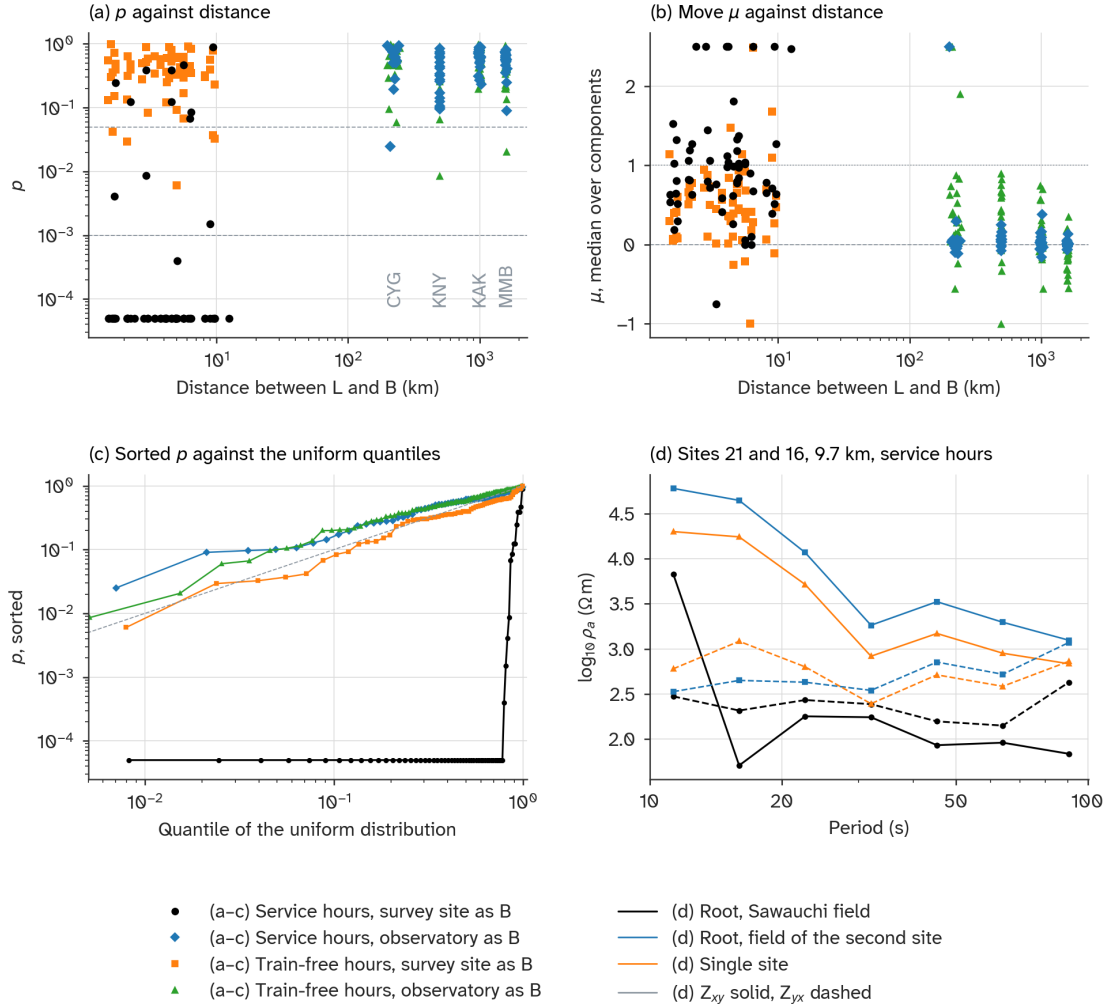


Figure 8. The consistency test of two remote sites on the records of the 2023 survey, with the Sawauchi field as one remote site and another survey site or an observatory as the second, in the service and train-free hours, with one pair of sites as an example. (a) p of the statistic of equation 20 (Section 8) against the distance between L and B, with the Sawauchi field as A, in the service (06–24 h) and train-free (01–05 h) hours; values below $1/20000$ drawn at that value; dashed lines, $p = 0.05$ and 0.001 . (b) Median over the tested components of μ against the same distance, drawn within -1 to 2.5 ; dashed line, no move; dotted line, the single-site estimate. (c) Sorted p of the four sets of (a) against uniform quantiles; dashed line, a test at its nominal level. (d) $\log_{10} \rho_a$ at site 21 in the service hours, for the roots with the Sawauchi field and with the field of site 16 and for the single-site estimate on the same blocks.

other sites, 1.5 to 12.5 km away, failed the consistency test while the trains ran, and the roots with their fields moved most of the way to the single-site response. The Sawauchi field passed against four observatories 196 to 1612 km away, among them CYG, about 200 km from the sites. No remote site between 12.5 and about 200 km was recording, so the distance beyond which the noise of these railways no longer reaches a remote site is not located more closely.

The test with recorded railway noise (Section 6) fulfilled the validity condition by construction. Egbert et al. (2000) found the signals of the direct-current railway of the Parkfield region at both stations, and a source of that kind is outside the test. At Gyeongju, 1090 km from the remote site and beside a railway fed

Table 9. Failure modes of the remote reference in model space: cause, response of the standard error of one record, diagnostic, evidence in this paper and remedy.

failure	cause	standard error	diagnostic	evidence	remedy
weak instrument	the remote field explains little of the local field, or the projection carries local noise	stays small while the root moves toward the single-site estimate	share S_b band by band and fraction ϕ_b (equations 16 and 17)	Haenam station A (Section 7.2); whole-day railway noise, 36 of 37 records recovered with $S > 0.3$ and 25 of 43 below (Section 6.4)	octaves with large S_b ; another remote site; a cross-fitted projection as a check
invalid instrument	the noise reaches the remote site	grows, but more slowly than the bias, and the coverage falls to near zero	consistency test of two remote sites and the move μ (equations 20 and 21)	leakage of 1 to 5 percent of the noise amplitude detected on synthetic records (Section 4.4); survey sites rejected in 49 of 61 pairs by day and in none of 63 at night (Section 8)	a distant remote site tested against another; weights from the residuals where the noise is localized in time
equations not solved	the two-stage minimum, or a fixed point that stops short of the root	does not contain the error	relative residual ε (equation 13)	error of construction up to 0.9 percent at the observatories (Section 5.2); four passes 20 standard errors short of the root on 16 January 2004 (Section 5.1); 54 of 526 weighted solutions on the excluded days (Section 5.3)	fixed point completed by Newton's method; a root only at $\varepsilon < 10^{-4}$
variation between records	the response contained in a record changes from one record to the next	states the noise within one record only	scatter of repeated records or of parts of one	1.9 to 3.3 times the sandwich error at Parkfield; halves at Haenam 2 to 3 times; up to 4 times at Gyeongju (Sections 5.4, 7.2 and 7.4)	repeat or divide the record

with alternating current, leakage into the remote record is not plausible. In the service hours no component of site 26 had both the Sawauchi field and the field of another survey site relevant, so the survey sites could not test it (Section 8.2). Against the four observatories the Sawauchi field passed the test for site 26 with p of 0.17 to 0.94 on five to seven components of Z_{xy} (Section 8.3), so that its validity there rests on a test as well as on its distance. Noise that reached both remote sites in the same proportion, which the test does not see, would here have to reach sites 200 to 1600 km away alike.

The uncertainty of a field response has two components. The sandwich covariance states the first, conditional on the record, its weights and the model, and simulation and transplanted residuals verify it for the root. The second is the variation between records. It is shared by the instrument response and the response with the local field (Section 5.5), so it belongs to the record and not to the estimator. Where a record can be divided, the scatter of its parts states both components together, and where the parts are few, as for the Gyeongju responses, the range obtained by leaving one block out serves instead.

9.3 Limits

The method needs a remote record. Without one, the errors-in-variables likelihood of equation 23 removes the bias of magnetic noise only with a noise ratio known independently (Section 4.3), and a single site rejects contamination localized in time (Oh, 2026) but not a source that drives the electric and magnetic channels together at all hours. In the railway test with noise over the whole day the single-site routes recovered the response on at most one of 80 records. The field tests use layered models with fixed interfaces, and the Korean records carry static effects that a layered model does not remove. The railway test idealizes the coupling of the noise (Section 6.1), and what the recorded noise contributes is its waveform and its spectrum. The Korean references are themselves uncertain: the contractor’s responses are edited estimates whose levels we could not trace, and the classical estimate at Gyeongju shares its remote record with the instrument, so that agreement between the two shows that two estimators with one remote field agree, and no more. The Sawauchi field was tested against the observatories at 19 of the 29 records of the survey, and the bound on its leakage holds for those. With the survey sites as second remote sites, the control in the train-free hours rejected 8 to 13 percent of the pairs at the five percent level and none at 0.1 percent. The closed form of equation 18, applied with one noise source and band powers, agreed with the field data only in the median (Section 8). Cross-fitting leaves a positive bias in the two longest bands of the two-input reduction of the scalar test, whose origin we have not identified (Appendix A). The weights of the daily instrument fits are one scale per octave band, and weights that differ between channels were not tested.

10 Conclusions

A remote magnetic record enters an inversion of the time series whose unknown is an earth model as an instrumental variable, through estimating equations in which the residual formed with the local field is orthogonal to the sensitivities driven by the projection of the local field on the remote one. Remote referencing is then a procedure whose solution, relevance and validity have to be checked separately, and the scores of the equations over blocks of time supply both the covariance of the root and the test of the remote site. Driving the prediction with the projected field and minimizing the misfit does not solve the equations and carries an error of construction, of up to 0.9 percent of the impedance at two observatories and 2.8 percent on Korean records, which no noise causes and no standard error contains. A fixed point iteration completed by Newton’s method solves the equations, and the sandwich covariance formed with their Jacobian states the noise within a record. A transfer function followed by an inversion reproduces the root if it is a root of the same equations, is carried with their kernel and contains the responses of the model in its basis.

The instrument can fail in two ways that its standard error does not show. A projection that fits part of the local noise draws the root toward the single-site response by a fraction that the share of the local field explained by the projection predicts, and cross-fitting the projection reduces it only in part. A remote site that the noise reaches moves the root toward the relation of the noise by a fraction with a closed form in a scalar model. No diagnostic of a single record shows it, and the consistency test of two remote sites, which judges the difference of their roots with the covariance of their common blocks, detects it. Near 25-kV railways this test rejected the fields of other survey sites 1.5 to 12.5 km away in 49 of 61 pairs, covering 32 of 34 pairs of records, while the trains ran and in none of 63 at night. It passed the remote field 1090 km away against four observatories in all 71 testable cases at the one percent level, which bounds its leakage to about five percent of the move of the single-site estimate in nine of ten tests.

On the daily records of the Parkfield and Hollister observatories the root with weights repeated from day to day better than each least-squares and M-estimate setting of a standard robust remote-reference code followed by the same model, and as well as its best setting chosen after the fact, in a robust measure of scatter. Band estimates did not differ, so the gain comes with the model and the weights. With recorded railway noise added over the whole day to ten days of the Parkfield record, single-site routes recovered the response on at most one of 80 records, and the root and the code followed by a model on 61 and 58, each measured against itself without the noise. At Haenam the instrument removed the near-field form of a single-site response. At Gyeongju, 7 km from a 25-kV railway, a remote field 1090 km away gave from the whole record the response of a classical remote-reference estimate of the same series to 0.04–0.05 in $\log_{10} \rho_a$ at 16–90 s, and from the service hours alone the full impedance tensor and the tipper, within two standard errors of the estimate from the train-free hours in 15 of 18 cells of element and octave. The service hours gave 2.2 times the information of the train-free hours for Z_{xy} and half of it for the tipper.

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Data and code availability

Waveform data and metadata of the Parkfield and Hollister stations (network BK, stations PKD and SAO) are openly available from the Northern California Earthquake Data Center (NCEDC, 2014; BDSN, 2014). The 1-s records of the observatories Kakioka, Kanoya and Memambetsu (Kakioka Magnetic Observatory, 2013a,b,c) are openly available from the World Data Center for Geomagnetism, Kyoto, and that of Cheongyang through INTERMAGNET (Love and Chulliat, 2013). The Korean field records (Haenam, and the 2023 survey to which the Gyeongju records belong) and the Japanese remote record (Sawauchi) are not public. They are available from the author on request, subject to the permission of the data providers. The code of the likelihood, the estimating equations and every experiment of this paper, with the synthetic records and the result files from which the figures and tables are drawn, is available from the author on request and will be archived with a DOI at the time of journal publication.

A Cross-fitting the projection

Design. The loading of equation 17 and its removal are measured in two settings. In the simplest one, each band has one real coefficient, the local magnetic noise is twice the signal ($S_b^\circ = 1/3$), the remote record carries no noise and $\alpha = 0$, on 120 records. The share of the noise of the driving channel that the instrument carries is compared with $\ell = L/n_q$, and the bias of the coefficient with κ_b of equation 17. In the full procedure, five conditions with large local magnetic noise are solved again on all their records, three of them rows of Table 1 and two with records of 4096 and 86 400 samples. Every regression of the chain is fitted in six blocks from the other five, with 512 s left out on both sides of each block, and in a second run the projection alone is cross-fitted.

Results. The projection fitted within the sample carries the predicted share ℓ of the local noise, and the one-input coefficient is biased by -14.5 percent at 339 s, against -15.2 percent from κ_b of equation 17. The cross-fitted projection carries none of it. The coefficient is then within a few percent of the truth, with a positive bias in the mean, of finite-sample origin, that is small in the median. In the two-input reduction the orthogonalization of the projected driving field halves the loading to $1/n_q$, and the coefficient fitted within the sample is biased by only -3 percent at 339 s. Cross-fitting the projection alone returns a loading of $-1/n_q$, and the coefficient is then biased by $+27$ percent. Cross-fitting every regression of the chain removes the loading, but a positive bias of 7 to 12 percent remains in the two longest bands, where n_q is 20 to 45, and we have not identified its origin. In the full procedure with a local magnetic noise power of 200 percent, on all 160 records of the condition (Table 1 counts 146), cross-fitting every regression lowers the bias of the root from 3.95 to 1.83 percent in the median over the periods. It lowers the r.m.s. error to 0.81 of its value (95 percent interval 0.78–0.84) and raises the coverage from 0.80 to 0.86. With local and remote noise of 100 percent each the r.m.s. error falls to 0.91, and where the local noise is moderate or the record short or long the error does not change. Contrary to the scalar test, cross-fitting the projection alone gave a still lower bias in the two conditions with the largest effect, 1.22 and 2.06 percent against 1.83 and 2.86. In the records of Section 4.4 without leakage and with the prior used there, cross-fitting raised the bias floor from 0.027 to 0.049 at $\rho = 1$ and from 0.061 to 0.115 at $\rho = 3$, because the loading within the sample had partly offset the pull of the prior. With a prior ten or a hundred times weaker it lowered the floor at $\rho = 0.3$ and 1. Cross-fitting therefore removes the loading of the local noise, but the bias that remains depends on the prior and on which regressions are cross-fitted. It does not touch the bias of leakage, which stays at 0.99 to 1.04 of its value within the sample, because a remote record that carries the noise is as correlated with it out of sample as in sample.

B Single-site routes over part of the day

Records, routes and measure are those of Section 6.1. With the blocks excised by the true mask, the single-site response moves by 0.002 to 0.005 in the median of every case and stays below 0.02 on 158 of the 160 records with coherent noise. The same mask costs 0.002 and 0.006 when nothing is added, for 30 and 60 percent of the day. Without a mask the fitted response takes the form of the added relation, 0.18 to 0.92 from the response of the day for noise at all periods and 0.03 to 0.76 for noise at 4–32 s. Noise in the electric channel alone moves it by 0.01. The mask has to be found, and there the routes differ (Table 10). The envelope detector recovers no case, and the amplitude detector applied band by band recovers strong noise over 30 percent of the day only. Weights inferred from the residual, started from that mask, complete it over

Table 10. Recorded railway noise over part of the day, single-site routes (Appendix B). Days, of ten, recovered against the single-site response of the day as recorded. Share, percent of the day that the noise covers. Band by band, the amplitude detector applied to each octave. Residual of a free transfer function (TF) or of the earth model, started from the mask of that detector or from uniform weights (asterisk). Code, the M-estimate of the robust code at a single site without a mask, and least squares given the true mask.

noise	share (%)	energy ratio	band by band	residual of free TF	residual of model	residual of free TF*	residual of model*	true mask	code, no mask / true mask
all periods	30	100	9	10	10	0	2	10	0 / 10
all periods	30	7	7	10	10	3	1	10	0 / 10
all periods	30	2	3	10	10	8	5	10	0 / 10
all periods	30	0.5	2	10	10	10	10	10	0 / 10
all periods	60	100	4	3	3	0	0	10	0 / 9
all periods	60	7	0	0	0	0	0	10	0 / 10
all periods	60	2	0	0	0	0	0	9	0 / 10
all periods	60	0.5	0	0	0	0	0	9	0 / 10
4–32 s	30	100	9	10	10	0	10	10	0 / 9
4–32 s	30	7	5	9	10	7	10	10	0 / 10
4–32 s	30	2	1	10	10	9	10	10	0 / 10
4–32 s	30	0.5	3	10	10	10	10	10	1 / 10
4–32 s	60	100	1	1	3	0	0	10	0 / 10
4–32 s	60	7	0	0	4	0	5	10	0 / 10
4–32 s	60	2	0	0	6	0	6	10	0 / 9
4–32 s	60	0.5	0	0	7	0	8	10	0 / 10

30 percent of the day at all four strengths, with the residual of a free transfer function as with that of the earth model (0.004 to 0.006). They cost 8 to 16 percent of the uncontaminated samples, more often in the intervals of strong natural activity.

Over 60 percent of the day the two residuals part. For noise at 4–32 s the residual of the earth model recovers the response on 20 of the 40 records and that of the free transfer function on one. The transfer function follows the added relation in the three octaves that the noise occupies, whereas the model, held by the octaves outside them, does not. This is the third circumstance of Oh (2026), in which no summary can be fixed before the inversion, on recorded noise. Three controls delimit it. A smooth transfer function of three complex coefficients inside the loop recovers 18 of the 40 records. An earth model fitted at every pass to a summary estimated anew under the current weights recovers 22, among them the twenty of the one-step fit. The standard robust code of Section 5.1, given the mask that the residual of the earth model found, recovers the same twenty days. For noise at all periods over 60 percent of the day, which leaves the model no octave to hold on to, no weights derived from the record recover the response below a hundred times its energy. At a single site and without a mask the standard code recovers none of the sixteen cases with coherent noise (one record of 160). It removes noise in the electric channel alone. Given the true mask it recovers all sixteen (0.002 to 0.006), as the likelihood does.

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