Geophys. J. Int. (2005) **160,** 721–728 doi: 10.1111/j.1365-246X.2004.02495.x

Electromagnetic torques in the core and resonant excitation of decadal polar motion

Jon E. Mound

Department of Physics, University of Toronto, Toronto, ON, M5S 1A7, *Canada. E-mail: jon@sphere.physics.utoronto.ca*

Accepted 2004 October 6. Received 2004 September 30; in original form 2003 December 16

SUMMARY

Motion of the rotation axis of the Earth contains decadal variations with amplitudes on the order of 10 mas. The origin of these decadal polar motions is unknown. A class of rotational normal modes of the core–mantle system termed torsional oscillations are known to affect the length of day (LOD) at decadal periods and have also been suggested as a possible excitation source for the observed decadal polar motion. Torsional oscillations involve relative motion between the outer core and the surrounding solid bodies, producing electromagnetic torques at the inner-core boundary (ICB) and core–mantle boundary (CMB). It has been proposed that the ICB torque can explain the excitation of the approximately 30-yr-period polar motion termed the Markowitz wobble. This paper uses the results of a torsional oscillation model to calculate the torques generated at Markowitz and other decadal periods and finds, in contrast to previous results, that electromagnetic torques at the ICB can not explain the observed polar motion.

Key words: core dynamics, Earth rotation, Markowitz wobble, torsional oscillations.

1 INTRODUCTION

Variations in the rotation of the Earth occur over a wide range of timescales. Both the orientation and magnitude of the planetary rotation vector are known to vary on timescales ranging from days to millions of years. Some changes are driven by external forces (e.g. lunisolar tides) and others by internal mass redistributions (e.g. mantle convection). The advent of space geodetic techniques has greatly increased the accuracy and resolution of planetary rotation measurements, resulting in a corresponding increase in our knowledge of the planetary interior. When the forcing is well known, as is the case for lunisolar tides, the observed nutations of the planet can be used to constrain physical parameters such as the dynamic ellipticity of the core and of the whole Earth (e.g. Mathews *et al.* 2002). When external forcings alone can not produce the observed rotational variations, an excitation within the Earth system (including the core, oceans and atmosphere) must exist and thus constraints can be placed on the dynamics of these regions.

Torsional oscillations are a class of rotational normal modes associated with the resonance of shear hydromagnetic waves in the fluid core. Flow in the fluid core may generate torques at both the core–mantle boundary (CMB) and inner-core boundary (ICB). Torques aligned with the rotation axis (henceforth axial torques) alter the magnitude of the rotation vector producing a change in the length of day (LOD). Non-axial torques will not appreciably influence LOD, instead they alter the orientation of the rotation vector, that is, they excite polar motion. Torsional oscillations have long been associated with decadal fluctuations in LOD through angular momentum arguments (e.g. Braginsky 1970, 1984; Jault *et al.* 1988; Jackson *et al.* 1993). Torsional oscillations may exchange angular momentum with the mantle through electromagnetic (e.g. Bullard *et al.* 1950; Rochester 1960; Holme 1998), topographic (e.g. Hide 1969; Jault *et al.* 1996; Kuang & Chao 2001), viscous (e.g. Kuang & Bloxham 1993) or gravitational (e.g. Buffett 1996a,b) couplings. Of these candidates, gravitational coupling between the ellipsoidal mantle and inner core is the mechanism most likely to produce the axial torque required to explain decadal LOD variations. Core flow may also produce non-axial torques at the CMB and ICB and thus influence polar motion as well as LOD (e.g. Greff-Lefftz & Legros 1995; Hide *et al.* 1996; Hulot *et al.* 1996; Dumberry & Bloxham 2002; Greiner-Mai *et al.* 2003).

This paper focuses on an observed set of polar motions with decadal periods. Decadal variations were first claimed by Markowitz (1960) and such polar motions with an approximately 30-yr period are termed the Markowitz wobble (Rochester 1970). More recent analysis of Earth rotation time-series has discerned a suite of polar motion variations with periods ranging from 7 to 86 yr (Schuh *et al.* 2001), indicating that the Markowitz wobble is but one example of a related class of decadal polar motions. The lack of external forcings at the observed periods implies that the excitation of these motions must be the result of a process internal to the Earth system. The Markowitz wobble may represent a normal mode of the coupled earth–ocean system (Dickman 1983), although there is debate as to the physical significance of this mode (Wahr 1984; Dickman 1985; Wahr 1985). More recent work suggests that processes occurring at the surface of the solid Earth, including changes in groundwater, oceans or atmosphere, do not appear capable of exciting the observed decadal polar motion (e.g. Celaya *et al.* 1999;

Jochmann 1999; Greiner-Mai *et al.* 2003). The excitation source for the Markowitz and other decadal wobbles may therefore reside within the core. Support for a core origin may be found from the observed correlation between decadal changes in polar motion and LOD (Poma & Proverbio 1980). As noted by Dumberry & Bloxham (2002), decadal changes in LOD are the result of angular momentum exchange between the mantle and core and thus the correlation between the two data sets suggests that the core also plays a role in the excitation of decadal polar motion.

The observed polar motion corresponds to a change in the angular momentum of the mantle. If angular momentum within the whole Earth system is conserved then the observed change in mantle angular momentum can be interpreted as an equal and opposite change in core angular momentum, however the mechanism that couples the core and mantle must also be determined. Electromagnetic and topographic coupling between the fluid core and mantle have been investigated and found to be unable to provide the torque required to produce the observed polar motions (e.g. Greff-Lefftz & Legros 1995; Hide *et al.* 1996; Hulot *et al.* 1996; Greiner-Mai *et al.* 2003). Investigations that assumed that inertial forces supply the restoring torque suggested that the inner core might play a role in the generation of the Markowitz wobble (Busse 1970; Kakuta *et al.* 1975). However, it was shown that the gravitational forces associated with the flattening of the inner core would dominate the inertial forces greatly altering the period of the inner-core motion (Szeto & Smylie 1984a,b, 1989). Following on from this work, it has been suggested that the observed polar motions might be explained if the figure axis of the inner core is tilted by $\sim 1^\circ$ and is experiencing quasi-periodic decadal fluctuations about a mean eastward drift of $\sim 0.7^\circ$ yr^{-1} (Greiner-Mai *et al.* 2000; Greiner-Mai & Barthelmes 2001; Greiner-Mai *et al.* 2003). As noted by those authors, this inner-core tilt would produce a gravity signature that should soon be detectable by the GRACE and CHAMP satellite missions. However, this work did not include the effects of the fluid core on the rotational dynamics; the influence of the fluid core was included in the work of Dumberry & Bloxham (2002) who found that the observed decadal polar motions could be produced by torques at the ICB with a resultant inner-core tilt of only 0.07°.

Dumberry & Bloxham (2002) considered the effect of electromagnetic coupling at the ICB and determined that a non-axial torque of 10^{20} N m could produce the observed magnitude of the Markowitz wobble and perhaps the ellipticity of that polar motion as well. They further argued that the required torque could be produced by the fluid motion associated with torsional oscillations. Torsional oscillations are azimuthal oscillations of the fluid core in which cylinders of fluid undergo effectively rigid-body rotation (Braginsky 1970). These oscillations occur as the inertial response to fluctuations in Lorentz forces on these cylinders in accordance with Taylor's constraint that a net axial Lorentz torque can not be maintained on such cylinders (Taylor 1963). Torsional oscillations are known to contain a number of decadal resonances (e.g. Braginsky 1970) and enhanced torques on the inner core would be expected at these periods. Torsional oscillations are believed to be responsible for the decadal fluctuations in LOD (Jault *et al.* 1988; Jackson *et al.* 1993; Zatman & Bloxham 1997) and it is therefore reasonable to suppose that torsional oscillations may also be responsible for decadal polar motion, especially in light of the observed correlation between the two sets of rotational variation (Poma & Proverbio 1980; Dumberry & Bloxham 2002).

The model of Dumberry & Bloxham (2002) assumed the amplitude of the inner-core torque to be period independent and the Markowitz wobble to be a purely forced motion. As a result, their model does not produce a peak in the polar motion spectrum at the Markowitz period. Schuh *et al.* (2001) have shown that the spectrum of polar motion contains several oscillations with periods ranging up to 86 yr. This observation is likely compatible with the model of Dumberry & Bloxham (2002) as a result of the likelihood of increased torques at periods corresponding to the suite of torsional oscillation normal modes. This paper revisits the question of the non-axial electromagnetic torque produced on the inner core by torsional oscillations. A finite-volume formulation of the core–mantle dynamics is used to determine a possible period dependence of the excitation and hence the associated spectrum of polar motions.

2 THEORY

Relative motion between the solid core and the overlying fluid will shear the magnetic field crossing the ICB resulting in an electromagnetic friction that couples the fluid and solid cores. The amplitude and orientation of the resulting torque depend on the geometry of both the core flow and the magnetic field at the ICB. This torque can be expressed as (following Dumberry & Bloxham 2002)

$$\Gamma^{\mathrm{ICB}} = \frac{1}{\mu}\int_{\mathrm{ICB}} (\mathbf{r}\times\mathbf{b})\mathbf{B}\cdot d\mathbf{S}, \tag{1}$$

where μ is the permeability of free space, $\mathbf{r}$ the position vector on the ICB, $\mathbf{B}$ the primary magnetic field, $\mathbf{b}$ the magnetic perturbation resulting from the shear flow and $d\mathbf{S} = r_i^2 \sin\theta \, d\phi \, d\theta \, \hat{\mathbf{r}}$ is the surface element on the assumed spherical ICB.

The perturbation to the radial magnetic field that arises as the result of a purely azimuthal shear flow at the ICB, as is the case for torsional oscillations, has been derived by Buffett (1992) as

$$\mathbf{b} = b_\phi \hat{\boldsymbol{\phi}} = \frac{1}{4}\mu\sigma_f\delta_f(1+i)B_r \, \Delta V_{\mathrm{ICB}} \, \hat{\boldsymbol{\phi}}, \tag{2}$$

where δ_f is the skin depth of the core, σ_f is the core conductivity and ΔV_{ICB} is the difference in linear velocity across the ICB. For torsional oscillations $\Delta V_{\mathrm{ICB}} = s[u_f(s) - u_i]$, where s is the distance from the planetary rotation axis and $u_f(s)$ and u_i are the angular velocity variations of the fluid and solid cores, respectively. Following Dumberry & Bloxham (2002), the magnetic field at the ICB is assumed to consist of axial and equatorial dipole components described in terms of Gauss coefficients defined at the ICB such that

$$B_r = -2g_{1,0}^{\mathrm{ICB}}\cos\theta - 2g_{1,1}^{\mathrm{ICB}}\cos\phi\sin\theta. \tag{3}$$

A global Cartesian coordinate system is used and oriented such that the equatorial dipole moment is oriented in the $\hat{\mathbf{x}}$ direction ($\hat{\mathbf{z}}$ is aligned with the equilibrium planetary rotation axis).

Evaluation of eq. (1), making use of eqs (2) and (3), yields the net torque on the solid core as a result of electromagnetic friction. In the chosen Cartesian coordinate system

$$\Gamma_x^{\mathrm{ICB}} = -\frac{1}{2}\pi\sigma_f\delta_f(1+i)\int_0^{r_i}\left(8g_{1,0}^{\mathrm{ICB}}g_{1,1}^{\mathrm{ICB}}\right) \times \left(\frac{z_i}{r_i}\right)s^3[u_f(s)-u_i]\,ds, \tag{4a}$$

$$\Gamma_y^{\mathrm{ICB}} = 0, \tag{4b}$$

$$\Gamma_z^{\mathrm{ICB}} = \pi\sigma_f\delta_f(1+i)\int_0^{r_i}\left[4\left(g_{1,0}^{\mathrm{ICB}}\right)^2 + 2\left(g_{1,1}^{\mathrm{ICB}}\right)^2\frac{s^2}{z_i^2}\right] \times \left(\frac{z_i}{r_i}\right)s^3[u_f(s)-u_i]\,ds, \tag{4c}$$

where $z_i = \sqrt{r_i^2 - s^2}$. $\Gamma_x^{\rm ICB}$ is the component of the torque responsible for inducing polar motion, whereas $\Gamma_z^{\rm ICB}$ is the component related to changes in LOD. Note that although this latter component contains a singularity at $s = r_i$ it is an integrable singularity, thus despite the appearance of $1/z_i$ in eq. (4c) the net torque remains bounded.

Whereas Dumberry & Bloxham (2002) assumed a constant value for $\Delta V_{\rm ICB}$, enabling a direct evaluation of eq. (4a), in this paper a finite volume torsional oscillation model (Buffett & Mound 2005) is used to solve for the frequency dependent fluid and solid core angular velocities, and hence the torque at the ICB. In this model, the outer core is described by a large number of nested cylinders aligned parallel to the planetary rotation axis (in particular, this work uses 80 cylinders within the tangent cylinder and 160 in the remainder of the fluid core). The coupled angular momentum equations of each cylinder as well as the inner core and mantle form an eigenvalue problem, which is solved to determine both the frequency and spatial structure of the torsional normal modes of the core–mantle system. Additionally, the response of the system to an applied periodic forcing can be determined. The resultant solutions for the velocities of the fluid cylinders, solid core and mantle can then be used to calculate torques at the ICB and CMB. In particular, eq. (4a) is evaluated for two different core models that simultaneously satisfy the general observational constraints on torsional oscillations. These constraints are decadal wave-like motion of the fluid core with amplitudes of a few tenths of a millimeter per second (e.g. Bloxham & Jackson 1991) producing LOD changes of a few milliseconds (e.g. McCarthy & Babcock 1986).

The electromagnetic torque generated at the CMB by flow within the core has been previously investigated and found to be too small to explain the observed magnitude of decadal polar motions (e.g. Greff-Lefftz & Legros 1995; Greiner-Mai *et al.* 2003). A determination of this torque will be included here for greater completeness and as a check of the approach against established results. Calculation of the torque at the CMB proceeds in the same fashion as for the ICB torque. The conductivity of the mantle is assumed to be confined to a thin layer just above the CMB such that $\sigma = \sigma_m \exp[(r_f - r)/\Delta]$, where σ_m is the conductivity on the mantle side of the CMB, r_f is the radius of the CMB and Δ is the effective thickness of the layer. In this case, the magnetic perturbation induced by azimuthal flow is (Buffett 1992)

$$\mathbf{b} = b_\phi\,\hat{\phi} = -\mu\sigma_m\Delta B_r\,\Delta V_{\rm CMB}\,\hat{\phi}, \tag{5}$$

where the difference in linear velocity across the CMB is given by $\Delta V_{\rm CMB} = s[u_f(s) - u_m]$, with u_m denoting the angular velocity variation of the mantle. Assuming a magnetic field that is a mixture of axial and equatorial dipole components gives the net torque on the mantle as

$$\Gamma_x^{\rm CMB} = -2\pi\sigma_m\Delta\int_0^{r_f}\left(8g_{1,0}^{\rm CMB}g_{1,1}^{\rm CMB}\right)\times\left(\frac{z_f}{r_f}\right)s^3[u_f(s) - u_m]\,ds, \tag{6a}$$

$$\Gamma_y^{\rm CMB} = 0, \tag{6b}$$

$$\Gamma_z^{\rm CMB} = 4\pi\sigma_m\Delta\int_0^{r_f}\left[4\left(g_{1,0}^{\rm CMB}\right)^2 + 2\left(g_{1,1}^{\rm CMB}\right)^2\frac{s^2}{z_f^2}\right]\times\left(\frac{z_f}{r_f}\right)s^3[u_f(s) - u_m]\,ds, \tag{6c}$$

where $z_f = \sqrt{r_f^2 - s^2}$. Note that the gauss coefficients here are for the field at the CMB and are not the same as the coefficients in eqs (3)–(4c), which are for the ICB field. In general, the orientation of the equatorial dipole components at the ICB and CMB will not be the same and, in fact, the ICB and CMB magnetic fields will be much more complex. However, for convenience, simple dipolar geometries are assumed for the field at both the CMB and ICB.

The equations governing the nutations of the Earth, including motions of the fluid and solid cores, have been previously developed and in their simplest form can be expressed by the matrix equation (e.g. Mathews *et al.* 1991, 2002)

$$\mathbf{M}\cdot\mathbf{x} = \mathbf{N}, \tag{7}$$

where $\mathbf{x} = [\tilde{m}\quad \tilde{m}_f\quad \tilde{m}_s\quad \tilde{n}_s]^{\rm T}$ is a column vector consisting of the amplitudes of polar motion of the whole Earth, fluid core and solid core, and the amplitude of the solid core tilt, respectively. **M** gives the algebraic relationships between the components of **x**; and **N** describes the forcing. In this work, the prescribed forcing is the non-axial torques at the ICB and CMB determined from the torsional oscillation model and the equations are to be solved for the resultant polar motion. (For further details on this system of equations and their solution see e.g. Mathews *et al.* 1991, 2002; and also Dumberry & Bloxham 2002 for the case of a forcing at an internal boundary.)

3 RESULTS

The finite volume model can be used to solve the torsional oscillation equations for a general suite of core–mantle properties: the particular parameter values used in this study are listed in Table 1. The values for the magnetic field components at the ICB are chosen to match Dumberry & Bloxham (2002); both the ICB and CMB dipole field strengths are in agreement with constraints obtained from analyses of tidally forced nutation series (Mathews *et al.* 2002). Relaxation of the inner-core shape is also included in the model with characteristic relaxation times, Υ, of either 1 or 100 yr. The inner-core relaxation is incorporated by the inclusion of a frequency dependent factor (see e.g. Buffett 1998), which is applied to both the axial and non-axial torques when determining the gravitational

Table 1. Physical parameters used in torsional oscillation model.

Parameter	Symbol	Value
Radius of ICB	r_i	1.22×10^6 m
Radius of CMB	r_f	3.48×10^6 m
Fluid core density	ρ_f	1.2×10^4 kg m^{-3}
Inner-core moment of inertia	C_i	5.87×10^{34} kg m^2
Mantle moment of inertia	C_m	7.12×10^{37} kg m^2
Radial magnetic field in fluid	B_s	0.165 mT
ICB magnetic field components	$g_{1,0}^{\rm ICB}$	6.3 mT
	$g_{1,1}^{\rm ICB}$	3.0 mT
CMB magnetic field components	$g_{1,0}^{\rm CMB}$	0.25 mT
	$g_{1,1}^{\rm CMB}$	0.025 mT
Core conductivity	σ_f	5×10^5 S m^{-1}
Mantle conductance	$\sigma_m\Delta$	10^8 S
Fluid core skin depth	δ_f	31.0 km†
Gravitational coupling constant	Γ_g	2×10^{20} N m

†Calculated for a period of 60 yr.

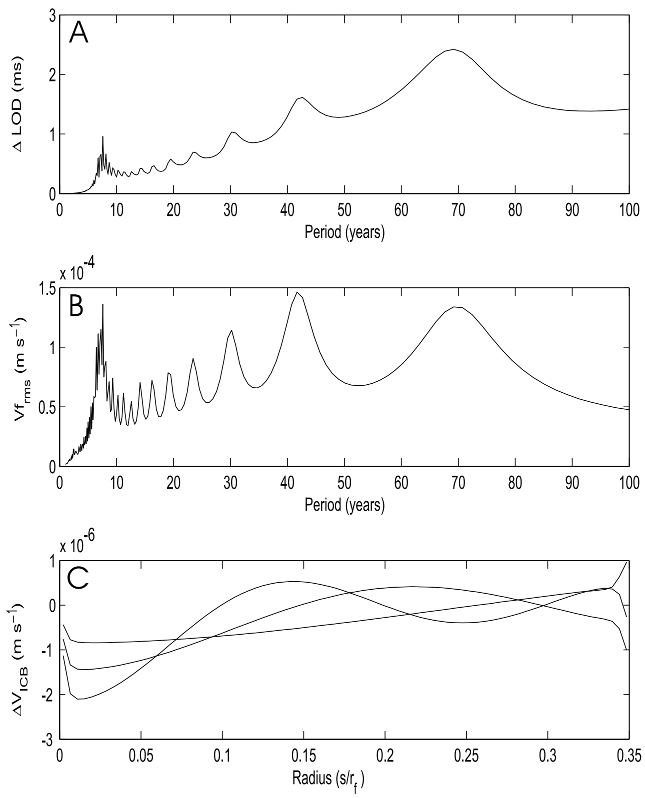

Figure 1. Model predictions for the period dependence of (a) LOD change and (b) average fluid velocity; and (c) representative examples of ΔV_{ICB}. $\Upsilon = 100$ yr; other physical parameters as in Table 1.

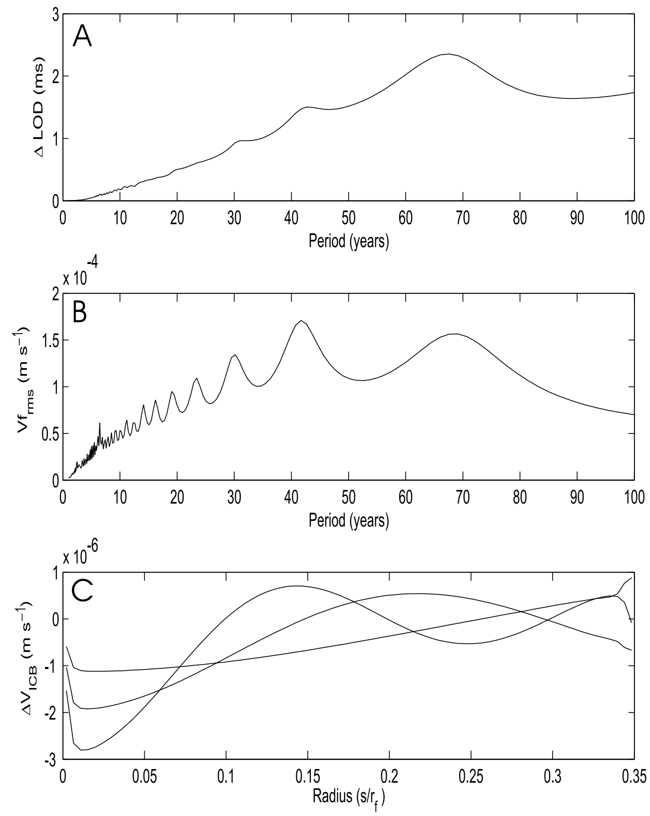

Figure 2. Model predictions for the period dependence of (a) LOD change and (b) average fluid velocity; and (c) representative examples of ΔV_{ICB}. $\Upsilon = 1$ yr; other physical parameters as in Table 1.

effect of inner-core displacement on the rotation of the mantle. If the relaxation is assumed to occur as a result of viscous deformation throughout the entire solid core, these relaxation times correspond to inner-core viscosities of 5×10^{16} and 5×10^{18} Pa s, respectively (following Buffett 1997).

Fig. 1 shows the period dependence of the amplitudes of ΔLOD and the fluid velocity oscillations obtained from the model when $\Upsilon = 100$ yr. Resonances in these quantities are found at a number of decadal periods that correspond to the torsional oscillation normal modes. As the amplitude of the fluid oscillations varies with position in the core, the root-mean-square average of the oscillation amplitude of all fluid cylinders within the model is plotted; at any period the maximum fluid oscillation amplitude is roughly a factor of 2 greater than the average. Within the tangent cylinder the fluid is strongly coupled to the inner core by the electromagnetic friction and hence the fluid within the tangent cylinder rotates with the inner core very nearly as a rigid body. The difference between the linear velocity of the fluid and solid cores as a function of radius is plotted in Fig. 1(c) for three representative examples. Note that the maximum difference in the linear velocities is 2 orders of magnitude less than the velocities themselves. Since the model considers purely rotational motion, the linear velocities (and hence their difference) must vanish as $s \rightarrow 0$. Away from the rotation axis, the model produces variations in fluid velocity that are roughly sinusoidal in appearance.

Similar results are obtained when $\Upsilon = 1$ yr (Fig. 2), although in this case there are weaker resonances in the predicted LOD spectrum and there is no peak in the model response near 6 yr. Also note that at decadal periods fluid velocities are slightly higher than in the $\Upsilon = 100$ yr case. Gravitational coupling between the inner core and mantle is the mechanism most likely to be responsible for the exchange of angular momentum associated with the observed decadal LOD variations and the observed 6–7 yr resonance (Buffett 1996a,b; Mound & Buffett 2003). A relatively rapid rate of inner-core relaxation would prevent a large offset between the inner-core and mantle density fields and reduce both the amplitude and quality factor of oscillations involving the gravitational coupling. The observed 6–7 yr oscillation in LOD therefore implies that the characteristic relaxation time of the inner core is a few years or more (Mound & Buffett 2003) and that gravitational coupling plays an important role in the exchange of angular momentum between the mantle and core.

The torques at the ICB and CMB can be computed using the determined velocities of the fluid core, solid core and mantle. The resonances in fluid velocity produce a set of peaks in the resultant torque spectra (Figs 3 and 4). This suggests that the mechanism proposed by Dumberry & Bloxham (2002) may be able to explain the entire suite of observed decadal oscillations of polar motion, including the Markowitz wobble. However, although the model fluid velocities and LOD amplitudes match the general observational constraints, the amplitude of the ICB and CMB torques are approximately 3 orders of magnitude too small to excite the observed polar motions (torques of the order of 10^{20} N m are required).

The magnitude of the non-axial torques on the mantle and inner core are found to be relatively insensitive to the choice of inner-core viscosity, except near the six-to-seven year resonance arising from the mantle inner-core gravitational normal mode. In general

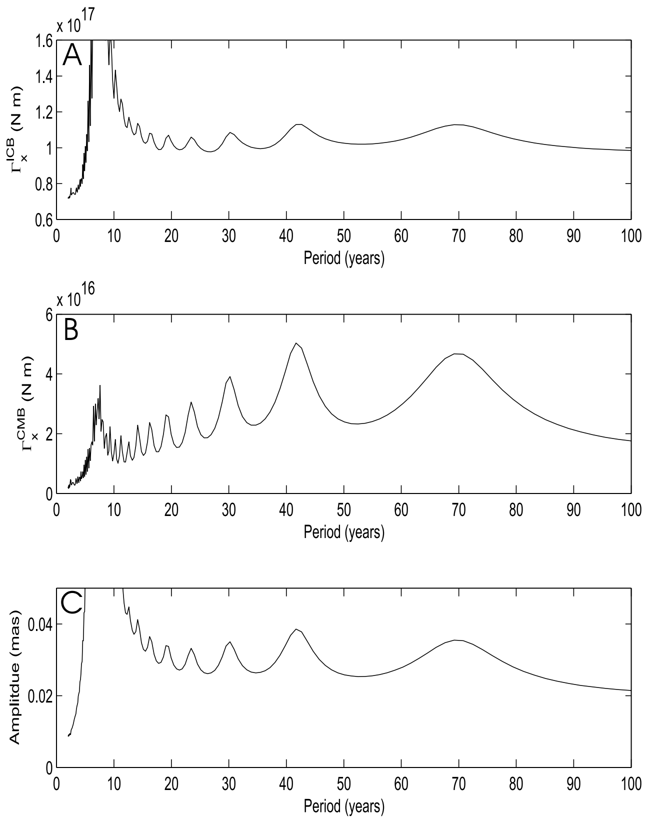

Figure 3. Model predictions for the period dependence of the amplitude of (a) non-axial torque at the ICB, (b) non-axial torque at the CMB and (c) resultant polar motion variation (the peak near 6.6 yr rises to ~8 mas). $\Upsilon = 100$ yr; other physical parameters as in Table 1.

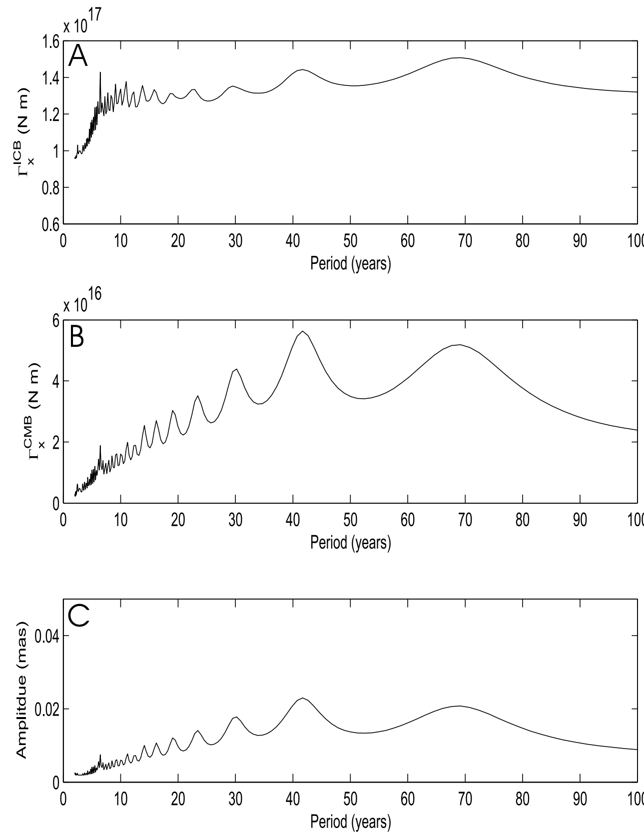

Figure 4. Model predictions for the period dependence of the amplitude of (a) non-axial torque at the ICB, (b) non-axial torque at the CMB and (c) resultant polar motion variation. $\Upsilon = 1$ yr; other physical parameters as in Table 1.

the slightly smaller fluid velocities in the $\Upsilon = 100$ yr case produce slightly smaller torques on the surrounding solid bodies. However, for periods near six years the resonance associated with the mantle inner-core gravitational normal mode produces a torque on the inner core that is approximately four times greater than the decadal torques in the $\Upsilon = 100$ yr model. This normal mode is highly damped by inner-core deformation when $\Upsilon = 1$ yr and thus in that model its resonant affect on the non-axial torques is much smaller than in the model with $\Upsilon = 100$ yr. In both cases the non-axial torques at the CMB are of the same order of magnitude as those found in previous studies (e.g. Greff-Lefftz & Legros 1995; Greiner-Mai *et al.* 2003). This agreement both supports the validity of the present approach and reaffirms the established view that CMB electromagnetic torques are insufficient to explain the observed decadal motions. The non-axial ICB torques found by the present study are approximately 3 orders of magnitude smaller than the estimate of Dumberry & Bloxham (2002).

The values of Γ_x^{ICB} found by the present model are smaller than those found previously for two reasons. Most importantly, Dumberry & Bloxham (2002) assumed that ΔV_{ICB} was equal to the observed fluid velocity, that is, they considered $u_i = 0$. However, the inner core will also undergo rotational variations so that the difference in linear velocity across the ICB is in fact much smaller than V_f. Indeed, the strong electromagnetic friction at the ICB causes the overlying fluid to be tightly coupled to the solid core such that the maximum values of ΔV_{ICB} are of the order of a few per cent of the observed fluid velocity. This explains two of the 3 orders of magnitude disagreement between the current results and those of Dumberry & Bloxham (2002). The remaining difference is attributable to the assumption of a constant ΔV_{ICB} value by Dumberry & Bloxham (2002). Although tightly coupled to the inner core, there is spatial variation in fluid velocity within the tangent cylinder (recall Figs 1c and 2c). When integrated from $s = 0 \rightarrow r_i$ geometric cancellation reduces Γ_x^{ICB} by a further order of magnitude. The particular spatial form of ΔV_{ICB} to some extent reflects the geometry chosen for the excitation source, however a variety of excitation source geometries have been considered and found to produce little change in the resultant non-axial torques.

The amplitude of polar motion induced by the torques determined from the torsional oscillation model for $\Upsilon = 100$ and 1 yr are plotted in Figs 3(c) and 4(c), respectively. In both cases, there are maxima at a set of decadal periods but with amplitudes that are 3 orders of magnitude smaller than observed. The magnitude of the computed torques at decadal periods were found to be insensitive to the characteristic relaxation time of the inner core, however their resulting influence on the induced polar motion depends more strongly on Υ. In fact, although the torques were found to be slightly larger, the induced polar motion was smaller in the $\Upsilon = 1$ yr model. Relaxation of the inner core may greatly influence the efficiency with which torques at the ICB excite polar motion.

In the model with $\Upsilon = 100$ yr, the polar motion induced by the torques at the ICB and CMB are of the same order of magnitude, except near the resonance associated with the free inner-core wobble, which has a period of approximately 6.6 yr. If the inner-core relaxation time is relatively long then a torque at the ICB efficiently excites the inner-core wobble, which amplifies the induced polar motion near this period. The torque at the CMB does not excite the inner-core wobble as strongly and thus at periods near 6.6 yr the polar motion induced by the CMB torque is 2 orders of magnitude smaller than that induced by the ICB torque. If the inner core relaxes more rapidly then an ICB torque excites polar motion less effectively. In particular, excitation of the inner-core wobble is greatly reduced as is the associated resonance in polar motion. The effect of rapid inner-core relaxation is also seen at longer periods. When $\Upsilon = 1$ yr, the polar motion induced by the ICB torque is an order of magnitude less than that induced by the CMB torque at interannual to decadal periods. Thus the total induced polar motion is approximately halved in this model in comparison to the model with $\Upsilon = 100$ yr in which the polar motion induced by the ICB and CMB torques are of the same order of magnitude.

4 DISCUSSION

No attempt was made to specifically match the observed decadal periods of polar motion. The periods of the torsional oscillation normal modes are determined by physical properties of the core–mantle system including the magnetic field inside the core and the inner-core viscosity. The observed periods might be matched by varying these physical parameters, however such attempts are premature until the core–mantle coupling mechanism and the underlying excitation source are determined. For the chosen model parameters, a sufficiently large excitation can produce electromagnetic torques that excite decadal polar motions to the observed amplitude, however in these cases the fluid velocity and LOD constraints are not met. Reasonable variations in the model parameters can not reconcile the polar motion and LOD constraints. For example, a larger magnetic field strength at the ICB or CMB would increase the magnitude of the electromagnetic torque for a given value of ΔV, however the increased coupling would also reduce this velocity difference, and as a result the net effect on the non-axial torque would be small. Similarly, changing the strength of the gravitational coupling has little effect on the non-axial torque at the ICB. The mantle–inner-core gravitational coupling provides a mechanism to transfer electromagnetic torques on the inner core to the mantle. To explain the LOD signal requires a torque of the order of 10^{18} N m and hence both the gravitational and electromagnetic axial torques on the inner core will be of this order regardless of the chosen value for Γ_g (Mound & Buffett 2003). Although the amplitude of inner-core motion varies when Γ_g is altered, the velocity difference across the ICB and hence the axial and non-axial ICB electromagnetic torques are unaffected.

Therefore, although resonant excitation of torsional oscillations provides a possible mechanism for producing the observed suite of decadal polar motions, it seems that electromagnetic couplings alone can not provide the necessary torque for models consistent with observed LOD variations. This difficulty arises from the fact that to produce the observed decadal variations in the magnitude of the planetary rotation vector (i.e. LOD change) requires torques on the mantle of the order of 10^{18} N m whereas production of the observed variations in the orientation of the planetary rotation vector (i.e. polar motion) requires torques of the order of 10^{20} N m (e.g. Dickey *et al.* 1990). Thus, despite the observed correlation between LOD and polar motion (Poma & Proverbio 1980), the question remains as to the single process that can produce both axial and non-axial torques whose magnitudes differ by 2 orders of magnitude.

As noted by Dumberry & Bloxham (2002), torsional oscillations may be quite efficient at exciting the inner-core wobble, which has a natural frequency of $\sim$ 6.6 yr. For relatively long inner-core relaxation times, the resonance associated with the inner-core wobble should be clearly visible relative to the longer period decadal polar motions (Fig. 3c), provided a sufficient excitational torque exists at the ICB. The lack of such a distinctive resonance in the observed spectrum (Schuh *et al.* 2001) suggests that either ICB torques resulting from torsional oscillations are not responsible for the observed polar motions or that the relaxation time of the inner core is relatively short (not more than a few decades). On the other hand, the observed 6–7 yr oscillation in LOD suggests that the relaxation time of the inner core is at least a few years (Mound & Buffett 2003). A more detailed comparison of theory and observations should be able to better establish these upper and lower bounds. Thus, if the observed rotational variations (both LOD and polar motion) are the result of gravitational coupling between the mantle and inner core then it may be possible to place fairly stringent constraints on the characteristic relaxation time of the inner core. However, any interpretation must be treated with caution until the origin of the decadal polar motion is determined.

The observed decadal variations in rotation are not compatible with the hypothesis investigated here, namely a period independent excitation source that is resonantly amplified by torsional oscillation normal modes. However it remains probable that, like the LOD variations, the decadal fluctuations in polar motion arise as a result of core processes. If so then the cause of the observed decadal variations must lie in some aspect of the core dynamics not included here. The model used in this work considers only the component of the core flow that can be attributed to torsional oscillations; the actual core flow is much more complex. Additionally, the torsional oscillation excitation source, which must depend on the full dynamics of the core flow, is yet to be determined. Numerical dynamo modelling suggest that the convective flow in the outer core can produce time-varying torques capable of generating fluid motions similar to torsional oscillation; because these models can not be run at entirely Earth-like conditions, it is not clear that these motions are strictly analogous with torsinal oscillations (Dumberry & Bloxham 2003). In the model of Dumberry & Bloxham (2003), the fluid motion represents a combination of forced and free oscillations. Observed LOD variations also likely reflect both the resonances associated with torsional oscillation normal modes as well as the dynamics of the complex core flow that excites the oscillations.

Similarly, if the observed decadal polar motion can not be explained solely in terms of the resonant effects of torsional oscillations then the polar motions should be interpreted as a combination of forced and free oscillations. Perhaps non-zonal flows near the ICB generate the required non-axial torque and torsional oscillations only weakly modulate this torque. Electromagnetic friction at the ICB can provide significant coupling between the fluid and solid cores and the presence of a tilted dipole field at the ICB would allow even an axisymmetric meridional flow to generate a non-axial torque on the inner core. Time variability in the flow or in the magnetic field at the ICB might then produce the fluctuations in the torque required to explain the observed polar motion spectrum. In any case, it appears that the explanation of the observed decadal rotational variations involves a dynamic process beyond resonant amplification by torsional oscillations.

5 FINAL REMARKS

The calculations described above indicate that resonances associated with torsional oscillations may represent a means of exciting a suite of decadal polar motions such as have been observed. This idea supports the previously noted connection between polar motion and core flow. Torsional oscillations may influence both the observed decadal LOD and polar motion variations. However electromagnetic couplings at the ICB do not provide a sufficient torque to match the observed amplitudes of polar motion when the oscillations are also constrained to match the amplitude of the observed decadal LOD fluctuations. This result is in disagreement with the findings of Dumberry & Bloxham (2002) as a result of the adoption here of a more realistic model of the velocity differences across the ICB. The observed polar motion spectrum may therefore reflect the dynamics of the excitation source rather than the normal modes of torsional oscillation. If a torque compatible with the observed properties of torsional oscillations could be found then the observed polar motions might yield constraints on the dynamics of the fluid core and on physical properties such as magnetic field strength within the core and inner-core viscosity. However, at present any such inferences appear premature as the origin of the Markowitz and other decadal wobbles remains unknown.

ACKNOWLEDGMENTS

Thanks to Steve Dickman and Mathieu Dumberry for constructive reviews that greatly improved this paper.

REFERENCES

Bloxham, J. & Jackson, A., 1991. Fluid flow near the surface of Earth's outer core, *Rev. Geophys.,* **29,** 97–120.

Braginsky, S., 1970. Torsional magnetohydrodynamic vibrations in the Earth's core and variations in length of day, *Geomag. Aeron., Engl. Transl.,* **10,** 1–8.

Braginsky, S., 1984. Short-period geomagnetic secular variation, *Geophys. Astrophys. Fluid Dyn.,* **30,** 1–78.

Buffett, B., 1992. Constraints on magnetic energy and mantle conductivity from the forced nutations of the Earth, *J. geophys. Res.,* **97,** 19 581–19 597.

Buffett, B., 1996a. Gravitational oscillations in the length of day, *Geophys. Res. Lett.,* **23,** 2279–2282.

Buffett, B., 1996b. A mechanism for decade fluctuations in the length of day, *Geophys. Res. Lett.,* **23,** 3803–3806.

Buffett, B., 1997. Geodynamic estimates of the viscosity of the Earth's inner core, *Nature,* **388,** 571–573.

Buffett, B., 1998. Free oscillations in the length of day: inferences on physical properties near the core–mantle boundary, *Geodynamics,* **28,** 153–165.

Buffett, B. & Mound, J., 2005. A Green's function for the excitation of torsional oscillations in the Earth's core, *J. geophys. Res.,* submitted.

Bullard, E., Freedman, C., Gellman, H. & Nixon, J., 1950. Westward drift of the Earth's magnetic field, *Phil. Trans. R. Soc. Lond.,* A., **243,** 67–92.

Busse, F., 1970. The dynamical coupling between inner core and mantle of the Earth and the 24-year libration of the pole, in, *Earthquake Displacement Fields and the Rotation of the Earth,* pp. 88–98, eds Mansinha, L., Smylie, D. & Beck, A., D. Reidel Publishing Company, Dordrecht, the Netherlands.

Celaya, M., Wahr, J. & Bryan, F., 1999. Climate-driven polar motion, *J. geophys. Res.,* **104,** 12 813–12 829.

Dickey, J., Eubanks, T. & Hide, R., 1990. Interannual and decade fluctuations in the Earth's rotation, *Geophys. Monogr.,* **59**, 157–161.

Dickman, S., 1983. The rotation of the ocean-solid earth system, *J. geophys. Res.,* **88,** 6373–6394.

Dickman, S., 1985. Comments on 'Normal modes of the coupled earth and ocean system' by John M. Wahr, *J. geophys. Res.,* **90,** 11 553–11 556.

Dumberry, M. & Bloxham, J., 2002. Inner core tilt and polar motion, *Geophys. J. Int.,* **151,** 377–392.

Dumberry, M. & Bloxham, J., 2003. Torque balance, Taylor's constraint and torsional oscillations in a numerical model of the geodynamo, *Phys. Earth planet. Int.,* **140,** 29–51, doi:10.10161/j.pepi.2003.07.012.

Greff-Lefftz, M. & Legros, H., 1995. Core–mantle coupling and polar motion, *Phys. Earth planet. Int.,* **91,** 273–283.

Greiner-Mai, H. & Barthelmes, F., 2001. Relative wobble of the Earth's inner core derived from polar motion and associated gravity variations, *Geophys. J. Int.,* **144,** 27–36.

Greiner-Mai, H., Jochmann, H. & Barthelmes, F., 2000. Influence of possible inner-core motions on the polar motion and the gravity field, *Phys. Earth planet. Int.,* **117,** 81–93.

Greiner-Mai, H., Jochmann, H., Barthelmes, F. & Ballani, L., 2003. Possible influences of core processes on the Earth's rotation and the gravity field, *J. Geodyn.,* **36,** 343–358.

Hide, R., 1969. Interactions between the Earth's liquid core and mantle, *Nature,* **222,** 1055–1056.

Hide, R., Boggs, D., Dickey, J., Dong, D., Gross, R. & Jackson, A., 1996. Topographic core–mantle coupling and polar motion on decadal timescales, *Geophys. J. Int.,* **125,** 599–607.

Holme, R., 1998. Electromagnetic core–mantle coupling—I. Explaining decadal changes in the length of day, *Geophys. J. Int.,* **132,** 167–180.

Hulot, G., Huy, M.L. & Le Mouël, J.-L., 1996. Influence of core flows on the decade variations of the polar motion, *Geophys. astrophys. Fluid Dyn.,* **82,** 35–67.

Jackson, A., Bloxham, J. & Gubbins, D., 1993. Time-dependent flow at the core surface and conservation of angular momentum in the coupled core–mantle system, *Geophys. Monogr.,* **72**, 97–107.

Jault, D., Gire, C. & Le Mouël, J.-L., 1988. Westward drift, core motions and exchange of angular momentum between core and mantle, *Nature,* **333,** 353–356.

Jault, D., Hulot, G. & Le Mouël, J.-L., 1996. Mechanical core–mantle coupling and dynamo modelling, *Phys. Earth planet. Int.,* **98,** 187–191.

Jochmann, H., 1999. The influence of continental water storage on the annual wobble of polar motion, estimated by inverse solution, *J. Geodyn.,* **27,** 147–160.

Kakuta, C., Okamoto, I. & Sasao, T., 1975. Is the nutation of the solid inner core responsible for the 24-year libration of the pole?, *Publ. Astron. Soc. Jpn.,* **27,** 357–365.

Kuang, W. & Bloxham, J., 1993. The effect of boundary topography on motions in the Earth's core, *Geophys. Astrophys. Fluid Dyn.,* **72,** 161–195.

Kuang, W. & Chao, B., 2001. Topographic core–mantle coupling in geodynamo modeling, *Geophys. Res. Lett.,* **28,** 1871–1874.

McCarthy, D. & Babcock, A., 1986. The length of day since 1656, *Phys. Earth planet. Int.,* **44,** 281–292.

Markowitz, W., 1960. Latitude and longitude and the secular motion of the pole, in *Methods and Techniques in Geophysics,* pp. 325–361, ed. Runcorn, S., Interscience Publishers, London, UK.

Mathews, P., Buffett, B., Herring, T. & Shapiro, I., 1991. Forced nutations of the Earth: Influence of inner core dynamics 1. Theory, *J. geophys. Res.,* **96,** 8219–8242.

Mathews, P., Herring, T. & Buffett, B., 2002. Modeling of nutation and precession: New nutation series for nonrigid Earth and insights into the Earth's interior, *J. geophys. Res.,* **107,** 10.1029/2001JB000390.

Mound, J. & Buffett, B., 2003. Interannual oscillations in length of day: Implications for the structure of the mantle and core, *J. geophys. Res.,* **108,** 2334, doi:10.1029/2002JB002054.

Poma, A. & Proverbio, E., 1980. Astronomical evidence of relationships between polar motion, Earth rotation and continental drift, in *Mechanisms of Continental Drift and Plate Tectonics,* pp. 345–357, eds Davies, P. & Runcorn, S., Academic Press, London, UK.

Rochester, M., 1960. Geomagnetic westward drift and irregularities in the Earth's rotation, *Phil. Trans. R. Soc. Lond.,* A., **252,** 531–555.

Rochester, M., 1970. Polar wobble and drift: A brief history, in *Earthquake Displacement Fields and the Rotation of the Earth,* pp. 3–13, eds Mansinha, L., Smylie, D. & Beck, A., D. Reidel Publishing Company, Dordrecht, the Netherlands.

Schuh, H., Nagel, S. & Seitz, T., 2001. Linear drift and periodic variations observed in long time series of polar motion, *J. Geodesy,* **74,** 701–710.
Szeto, A. & Smylie, D., 1984a. Coupled motions of the inner core and possible geomagnetic implications, *Phys. Earth planet. Int.,* **36,** 27–42.
Szeto, A. & Smylie, D., 1984b. The rotation of the Earth's inner core, *Phil. Trans. R. Soc. Lond.,* A., **313,** 171–184.
Szeto, A. & Smylie, D., 1989. Motions of the inner core and mantle coupled via mutual gravitation: regular precessional modes, *Phys. Earth planet. Int.,* **54,** 38–49.
Taylor, J., 1963. The magneto-hydrodynamics of a rotating fluid and the Earth's dynamo problem, *Proc. R. Soc. London,* A., **274,** 274–283.
Wahr, J., 1984. Normal modes of the coupled earth and ocean system, *J. geophys. Res.,* **89,** 7621–7630.
Wahr, J., 1985. Reply to 'Comments on 'Normal modes of the coupled earth and ocean system' by John M. Wahr', *J. geophys. Res.,* **90,** 11 557.
Zatman, S. & Bloxham, J., 1997. Torsional oscillations and the magnetic field within the Earth's core, *Nature,* **388,** 760–763.