

A Physics-Based Decorrelation Phase Covariance Model for Noise Reduction in Unwrapped Interferometric Phase Stacks

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Abstract—The accuracy of geophysical parameter estimation made with Interferometric Synthetic Aperture Radar (InSAR) time-series techniques can be improved with rapidly increasing available data volumes, and with the development of noise covariance matrices applicable to joint analysis of networks of interferograms. Here we present a physics-based decorrelation phase covariance model and discuss its role in noise reduction in unwrapped interferometric phase stacks. We demonstrate with an example wherein we average unwrapped interferogram phase stacks that span over a transient event how a noise covariance model can aid in noise reduction. Our model suggests that, for rapidly decorrelating surfaces (i.e., surfaces with much shorter correlation time than SAR acquisition intervals), it is preferable to incorporate all available interferograms from long observation windows. For slowly decorrelating surfaces (i.e., surfaces with longer correlation time than SAR acquisition intervals), our model suggests that a small subset of interferometric pairs is sufficient. We validate our model and three existing models of decorrelation phase covariance matrices in both Cascadia – a region with heavy vegetation cover, and Death Valley – a desert region, with C-band Sentinel-1 A observations. Our proposed model matches observations with the smallest average discrepancy between theory and observations.

Index Terms—Decorrelation noise, Covariance matrix, InSAR noise reduction.

I. INTRODUCTION

INTERFEROMETRIC Synthetic Aperture Radar (InSAR) is a widely used remote sensing technique that combines coherent radar images to form interferograms, which can be used to generate high-precision measurements of surface topography or crustal deformation over large areas with meter-level resolution [1]–[4]. In the past few years, three new InSAR satellites – Sentinel-1A/B and ALOS-2, became

operational. In the near future, additional InSAR satellites such as NISAR and ALOS-4 are anticipated to come online. Featuring higher temporal sampling rates and wider spatial coverage, this new generation of satellites provides large volumes of high-quality radar measurements and thereby makes observations of mm-level signals such as interseismic velocities possible [5].

The accuracy of measured geophysical signals with InSAR is inherently limited by atmospheric noise and decorrelation. Atmospheric noise is caused by fluctuations in wave propagation delays through the atmosphere due to the presence of water vapor. Since the revisit time of SAR satellites is on the order of days, atmospheric noise is essentially uncorrelated in time but correlated in space and is often modeled as a long-wavelength artifact in individual interferograms [6]–[9]. Decorrelation, on the other hand, can be related to changes between radar measurements in surface scattering properties, imaging geometries and thermal noise among others [10], [11]. The level of temporal decorrelation is often quantified by the correlation coefficient ρ . Decorrelation noise is generally uncorrelated in space but can be correlated in time [12]–[14].

InSAR time-series techniques are methodologies that exploit interferogram stacks with the aim of retrieving desired geophysical signals whilst minimizing effect of decorrelation. There are two broad categories of InSAR time-series algorithms. One involves identifying “persistent scatterer” (PS) pixels with highly stable scattering mechanisms [15], [16]. The PS pixels are minimally affected by decorrelation but are mostly limited to man-made structures and cities. The other approach involves exploitation of pixels that are affected by decorrelation, known as “distributed scatterers” (DS) pixels. The DS method can be further divided into two categories. The first category of DS methods

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is based on analysis of unwrapped interferometric phase stacks. For example, the conventional Small Baseline Subset (SBAS) technique [17] obtains phase time-series by solving a linear system of equations of unwrapped phases. The SBAS method limits the effect of decorrelation by restricting use of interferometric measurements to a subset of interferograms with small spatial and temporal baselines. The second category of DS methods performs time-series estimation based on analysis of SAR correlation matrices before phase unwrapping. Accounting for target statistics of SAR measurements [12], [18], approaches such as SqueeSAR [19] and its ensuing algorithms [20]–[22] relax the coherence constraint imposed in SBAS and allow for use of all interferograms [12], [18]. In this paper, we focus on the first category of DS methods that operates on networks of unwrapped interferometric phases. Hereinafter we refer to the first category of DS methods as “SBAS-like time-series algorithms”.

Phase statistics of multi-interferometric measurements play a critical part in improving the performance and design of SBAS-like time-series algorithms. Several studies have evaluated covariance between interferometric phases. In the simple mathematical framework developed in [6] to describe common noise sources in InSAR, decorrelation phase is modeled as an independent noise term, uncorrelated between interferograms. More recent work such as [13], [14], [23]–[25] show that interferometric phases are partially correlated between interferograms and present analytical models with various forms. With respect to these works, we propose a new covariance model for decorrelation phase based on surface scattering characteristics. We compare and validate our model with previously published models against Sentinel-1 data collected over both low coherence and medium-to-high coherence areas. We also demonstrate with an example how a network noise covariance model can facilitate decorrelation noise reduction.

This paper is organized as follows. In section II, we review existing decorrelation phase covariance models and then present the proposed model. In section III, we demonstrate how decorrelation covariance models can be incorporated into SBAS-like time-series algorithms to great effect and provide theoretical comparisons between the existing and the proposed models. In section IV, we validate all models using data collected by Sentinel-1. We conclude with a summary in section V.

II. DECORRELATION PHASE COVARIANCE MODELS

Decorrelation is one of the main limiting factors for InSAR observations. The mechanisms of signal decorrelation have been well studied, and can mainly be attributed to changes of surface scattering properties during the observation time span, geometrical decorrelation, and system and other thermal noises [10]. In this paper, we focus on temporal decorrelation as it is the dominating source of decorrelation for modern SAR sensors.

Similar to the definition of the correlation coefficient ρ between radar measurements, we define γ as the correlation coefficient between interferometric measurements. Let $\sigma_{x,ij}^2$ denote the variance associated with $\phi_{x,ij}^{decor}$ (by convention, $i < j$), the decorrelation phase component of the interferometric measurement between SAR acquisitions with indices i and j for pixel x . The covariance of $\phi_{x,ij}^{decor}$ and $\phi_{x,kl}^{decor}$ is

$$\text{cov}(\phi_{x,ij}^{decor}, \phi_{x,kl}^{decor}) = \gamma(\phi_{x,ij}^{decor}, \phi_{x,kl}^{decor}) \cdot \sigma_{x,ij} \sigma_{x,kl} \quad (1)$$

Phase variance $\sigma_{x,ij}^2$ have been comprehensively studied in literature and can either be derived from a probability distribution function (PDF) of interferometric phases under the assumption of a distributed scatterer mechanism [2], [26]–[29] or be approximated by the Cramer-Rao bound [30] when the correlation coefficient ρ between radar measurements is close to 1. The main focus of this paper is therefore the derivation of the correlation coefficient between interferometric phases $\gamma(\phi_{x,ij}^{decor}, \phi_{x,kl}^{decor})$. In this section, we first review the existing covariance models. Then we introduce the proposed physics-based covariance model.

It is worth emphasizing that the focus of this paper is the covariance models for multi-interferometric phase measurements of a single pixel. Hereinafter we omit the subscript x to simplify mathematical notations.

A. Existing Models for $\gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor})$

The first statistical evaluation of temporal decorrelation noise is given by [6], which models decorrelation as a fully independent noise term in each interferogram in a network:

$$\gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor}) = \delta_{ik} \delta_{jl} \quad (2)$$

where $\delta_{ij} = 1$, if $i = j$, and $\delta_{ij} = 0$ if otherwise.

In contrast, more recent work such as [13], [14], [23]–[25] argue that decorrelation noise is correlated between interferograms. [13] and [23] provide estimations under

the simplified assumption that interferometric measurements can be described as circular complex Gaussian random variables. [24] later notes that interferometric phase generally does not follow a circular Gaussian distribution and provides a closed-form analytical approximation of the correlation between interferometric decorrelation phases using the method of nonlinear error propagation:

$$\text{cov}(\phi_{ij}^{decor}, \phi_{kl}^{decor}) \approx \frac{\rho_{ik}\rho_{jl} - \rho_{il}\rho_{jk}}{2L\rho_{ij}\rho_{kl}} \quad (3)$$

where L is the number of looks. Note that when $i = k, j = l$, (3) becomes the Cramer-Rao bound [30], which is the lower bound for decorrelation phase variance. [25] also presents a similar equation with (3). We can derive $\gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor})$ from (3):

$$\gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor}) = \frac{\rho_{ik}\rho_{jl} - \rho_{il}\rho_{jk}}{\sqrt{(1 - \rho_{ij}^2)(1 - \rho_{kl}^2)}} \quad (4)$$

Since (4) has been derived with an approximation that only holds for a large coherence, (4) is not a complete covariance model for decorrelation phase noise.

Attempting to extend the covariance model from high to moderate coherence level, [14] starts with a pseudocovariance matrix $\tilde{\Omega}_{ifg}$ that is derived from a SAR coherence matrix Ω_{sar} and the InSAR incidence matrix A (Definition of A can be found in [17]): $\tilde{\Omega}_{ifg} = \frac{1}{2}A\Omega_{sar}A^T$. The pseudocovariance of ϕ_{ij}^{decor} and ϕ_{kl}^{decor} is then expressed as

$$\widetilde{\text{cov}}(\phi_{ij}^{decor}, \phi_{kl}^{decor}) = \frac{\rho_{ik} + \rho_{jl} - \rho_{il} - \rho_{jk}}{2} \quad (5)$$

where ρ_{ij} is the correlation coefficient between radar measurements s_i and s_j . The pseudocovariance is then used to approximate the true covariance of ϕ_{ij}^{decor} and ϕ_{kl}^{decor} :

$$\text{cov}(\phi_{ij}^{decor}, \phi_{kl}^{decor}) = \frac{\sigma_{ij}}{\sqrt{1 - \rho_{ij}}} \cdot \widetilde{\text{cov}}(\phi_{ij}^{decor}, \phi_{kl}^{decor}) \cdot \frac{\sigma_{kl}}{\sqrt{1 - \rho_{kl}}} \quad (6)$$

where the scaling factor $\sigma_{ij}/\sqrt{1 - \rho_{ij}}$ implies that $\text{cov}(\phi_{ij}^{decor}, \phi_{ij}^{decor}) = \sigma_{ij}^2$. We can also derive $\gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor})$ from (6):

$$\gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor}) = \frac{\rho_{ik} + \rho_{jl} - \rho_{il} - \rho_{jk}}{2\sqrt{1 - \rho_{ij}}\sqrt{1 - \rho_{kl}}} \quad (7)$$

For ease of discussion, hereinafter we refer to the model suggested by [6] as the "Hanssen model", the model suggested by [14] as the "Agram-Simons model", and the model suggested by [24] as the "Samiei-Esfahany

model". We omit discussions of models suggested in [13], [23], [25] to avoid redundancy as these models are well represented by the aforementioned three models.

B. A New Physics-based Model for $\gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor})$

Consider a series of radar signals s_1, s_2, \dots, s_n acquired at different times observing the same target. We adopt the style of analysis presented in [10]:

$$s_i = (\sqrt{\rho_\infty}C + \sqrt{1 - \rho_\infty}D_i)e^{i\psi_i} \quad (8)$$

where ψ_i represents propagation phases (e.g., deformation signal, atmospheric delay) at time t_i , C represents persistent scatterers that remains coherent, and D_i represents distributed scatterers at time t_i that decorrelate gradually over time. For the sake of simplicity and without loss of generality, we let $C = 1$, $E[D_i] = 0$ and $E[|D_i|^2] = 1$, so that the expected intensity of s_i is unity. ρ_∞ acknowledges the contribution from persistent scatterers [31]. Examining the properties of the distributed scatterers in detail,

$$D_i D_j^* = \rho_{ij}^d + R_{ij} \quad (9)$$

$$E[D_i D_j^*] = \rho_{ij}^d \quad (10)$$

where ρ_{ij}^d describes the correlation between distributed scatterers D at times t_i and t_j and which can be modeled as a decaying exponential function of time. R_{ij} describes the remaining uncorrelated part of $D_i D_j^*$ and has an expected value of zero. Let z_{ij} represent the interferometric measurement between signals s_i and s_j :

$$\begin{aligned} z_{ij} &= s_i s_j^* \\ &= \left(\rho_\infty + (1 - \rho_\infty) D_i D_j^* \right. \\ &\quad \left. + \sqrt{\rho_\infty(1 - \rho_\infty)}(D_i + D_j^*) \right) e^{i(\psi_i - \psi_j)}. \end{aligned} \quad (11)$$

The correlation ρ_{ij} between signals s_i and s_j is

$$\begin{aligned} \rho_{ij} &= \frac{|E[z_{ij}]|}{\sqrt{E[|s_i|^2]E[|s_j|^2]}} \\ &= \rho_\infty + (1 - \rho_\infty)\rho_{ij}^d. \end{aligned} \quad (12)$$

Note that ρ_{ij}^d can be of any generic form that describes temporal decorrelation of distributed scatterers. For example, with an exponential decay model $\rho_{ij}^d = \exp(-|t_i - t_j|/\tau)$, (12) takes the form of the generic decorrelation model presented in [31], [32].

$$\rho(t) = \rho_\infty + (1 - \rho_\infty)e^{-\frac{t}{\tau}} \quad (13)$$

where τ is the characteristic correlation time of the surface.

Substitute (9) and (12) into (11),

$$z_{ij} = \left(\rho_{ij} + (1 - \rho_{\infty})R_{ij} + \sqrt{\rho_{\infty}(1 - \rho_{\infty})}(D_i + D_j^*) \right) e^{i(\psi_i - \psi_j)} \quad (14)$$

The first term of (14) represents coherent signals between time t_i and t_j , which is the expected value of z_{ij} , the second and third terms are associated with decorrelation during this time period:

$$z_{ij} - E[z_{ij}] = \left((1 - \rho_{\infty})R_{ij} + \sqrt{\rho_{\infty}(1 - \rho_{\infty})}(D_i + D_j^*) \right) e^{i(\psi_i - \psi_j)} \quad (15)$$

where $z_{ij} - E[z_{ij}]$ represents the zero-mean complex decorrelation noise component in the interferometric measurement z_{ij} .

The covariance of the complex decorrelation noise components in z_{ij} and z_{kl} is then

$$\begin{aligned} cov(z_{ij}, z_{kl}) &= E[(z_{ij} - E[z_{ij}])(z_{kl} - E[z_{kl}])^*] \\ &= (\rho_{ik}\rho_{jl} - \rho_{\infty}^2) e^{i(\psi_i - \psi_j - \psi_k + \psi_l)} \end{aligned} \quad (16)$$

Detailed derivation of (16) is shown in the Appendix A. The correlation coefficient between complex interferometric measurements is therefore:

$$\begin{aligned} \gamma(z_{ij}, z_{kl}) &= \frac{|cov(z_{ij}, z_{kl})|}{\sqrt{E[|z_{ij} - E[z_{ij}]|^2] E[|z_{kl} - E[z_{kl}]|^2]}} \\ &= \frac{\rho_{ik}\rho_{jl} - \rho_{\infty}^2}{1 - \rho_{\infty}^2} \end{aligned} \quad (17)$$

We find via numerical simulation that the relation between $\gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor})$ and $\gamma(z_{ij}, z_{kl})$ exhibits a power-law behavior (See Appendix B)

$$1 - \gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor}) \approx [1 - \gamma(z_{ij}, z_{kl})]^{\frac{1}{2}} \quad (18)$$

Combining (17) and (18), we have

$$\gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor}) = 1 - \sqrt{\frac{1 - \rho_{ik}\rho_{jl}}{1 - \rho_{\infty}^2}} \quad (19)$$

Note that since we need ρ_{ij} in (19), this model is only practically applicable to multi-looked interferograms in which estimates of correlation coefficients ρ_{ij} can be obtained.

III. DECORRELATION REDUCTION IN UNWRAPPED INTERFEROMETRIC PHASE STACKS: A STACKING EXAMPLE

Typical SBAS-like time-series algorithms estimate desired geophysical parameters by linearly combining a set of unwrapped interferometric phase measurements over the same resolution unit on the ground:

$$\mathbf{P} = W\Phi \quad (20)$$

where P represents the estimated geophysical parameters, W is the weighting or inversion matrix, and $\Phi = \{\phi_{ij}\}$ is the set of unwrapped interferometric phases involved in the estimation.

Each ϕ_{ij} can be represented as the sum of deformation, atmospheric noise and decorrelation noise [4], [16], [17]

$$\phi_{ij} = \phi_{ij}^{def} + \phi_{ij}^{atm} + \phi_{ij}^{decor} \quad (21)$$

Note that we have not included phase noise terms such as digital elevation model (DEM) error or thermal noise because they are either deterministic terms that can be reasonably well modeled and removed, or typically negligible uncorrelated noise terms. Deformation is a deterministic process. Atmospheric noise and decorrelation noise, on the other hand, are stochastic variables with assumed zero means and are independent of each other because they represent unrelated physical processes. Therefore, the covariance matrix for a set of interferometric measurements $\Phi = \{\phi_{ij}\}$ can be expressed as:

$$Cov(\Phi) = Cov(\Phi^{atm}) + Cov(\Phi^{decor}) \quad (22)$$

The covariance matrix for atmospheric noise is well researched [7], [9], [33]. In this paper, we focus on the covariance matrix for decorrelation phase $Cov(\Phi^{decor})$. Assuming no atmospheric noise is present, we can propagate measurement uncertainties to uncertainties in the estimated parameters:

$$Cov(P) = W Cov(\Phi^{decor}) W^T \quad (23)$$

So far we have described four models for $Cov(\Phi^{decor})$: the Hanssen model, the Agram-Simons model, the Samiei-Esfahany model and the proposed physics-based model. In this section, we illustrate their differences by feeding each model into (23) and then comparing their respective predictions of $Cov(P)$. To achieve that, we need to first specify P and W . While there are many different deformation scenarios and time-series processing schemes in real applications, we find that a simple stacking exercise over a transient event is sufficient for our purposes.

A. Simulation Setting: stacking over a transient event

Consider a radar target that decorrelates over time. Assume that a transient deformation event took place and we have M consecutive radar measurements acquired over the surface before the event s_1, \dots, s_M and M measurements acquired after the event s_{M+1}, \dots, s_{2M} . Therefore, we have a total of M^2 interferograms that span the transient event:

$$\{\phi_{1,M+1}, \dots, \phi_{1,2M}, \dots, \phi_{M,M+1}, \dots, \phi_{M,2M}\}$$

Out of these M^2 interferograms, there is a maximum of M interferograms formed using unique pairs of radar measurements. One such combination is

$$\{\phi_{1,M+1}, \phi_{2,M+2}, \dots, \phi_{M,2M}\}.$$

The M interferograms with unique pairs of radar measurements are often referred to as “independent interferograms” in literature in the context of atmospheric noise mitigation [7], [33], [34]. Since these interferograms are not necessarily truly independent, we adopt the term “non-repeating stack” instead. In contrast, we refer to the previous group of measurements with all possible measurements as a “repeating stack”. Note that in practice, a repeating stack does not have to include all available interferograms. Assuming no atmospheric noise is present,

$$\phi_{ij} = \phi^{def} + \phi_{ij}^{decor} \quad (24)$$

where ϕ^{def} corresponds to phase change caused by the transient event and is present in every interferogram that span the event.

To retrieve signals associated with the transient deformation event ϕ^{def} , we can average either stack. Both stacking strategies are widely adopted in literature [34]–[38]. By stacking a non-repeating stack, we have

$$\begin{aligned} \Phi_{nrp} &= [\phi_{1,M+1}, \phi_{2,M+2}, \dots, \phi_{M,2M}]' \\ W &= \left[\frac{1}{M} \quad \frac{1}{M} \dots \frac{1}{M} \right] \\ P &= W\Phi_{nrp} \end{aligned} \quad (25)$$

Similarly, by stacking a repeating stack, we have

$$\begin{aligned} \Phi_{rp} &= [\phi_{1,M+1}, \dots, \phi_{1,2M}, \dots, \phi_{M,M+1}, \dots, \phi_{M,2M}]' \\ W &= \left[\frac{1}{M^2} \quad \frac{1}{M^2} \dots \frac{1}{M^2} \right] \\ P &= W\Phi_{rp} \end{aligned} \quad (26)$$

B. Construction of $Cov(\Phi^{decor})$

We are interested in determining $\sigma^2(P)$ for both stacks. Adopting the Cramer-Rao bound [30]

$$\sigma^2(\phi_{ij}) = \frac{1 - \rho_{ij}^2}{2\rho_{ij}^2}$$

as phase variances, we construct $Cov(\Phi^{decor})$ using (1) with $\gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor})$ given by each of the four models.

We also need to specify a temporal decorrelation model $\rho(t)$. Here we choose the generic decorrelation model (13). The parameters in (13) are highly dependent on land covers, wavelength and climate [31], [32]. In this simulation, we examine the impacts of correlation time constant τ and persistent correlation ρ_∞ on $Cov(\Phi^{decor})$. We also examine the impacts of different numbers of radar measurements ($2M$), and acquisition intervals on the performance of decorrelation noise reduction.

C. Comparison Between Existing And The Proposed Decorrelation Covariance Models

Fig. 1 depicts the predictions of $\sigma^2(P)$ from both the existing and the proposed models for surfaces with varying decorrelation rates and fixed persistent coherence ($\rho_\infty = 0.1$). We use the ratio between characteristic correlation time τ and SAR acquisition interval Δt to represent apparent surface correlation time – higher ratio means slower apparent decorrelation. We set $M = 25$. The Hanssen model (green lines) predicts larger phase variance with non-repeating stacking regardless of surface decorrelation rates; the Agram-Simons model (blue lines) predicts zero or small differences between two stacking strategies with either rapidly or very slowly decorrelating surfaces; and both the Samiei-Esfahany model (magenta lines) and the proposed model (red lines) predict significantly higher noise reduction from repeating stacking when $\tau/\Delta t < 15$ and similar noise reduction performances from either stacking strategy when $\tau/\Delta t > 15$. Additionally, we observe that:

1. The Agram-Simons model predicts the highest phase noise (i.e., the least reduction in decorrelation noise) for both non-repeating and repeating stacking. This means that the Agram-Simons model suggests high correlation between decorrelation noise components. In contrast, the Hanssen model, which assumes independence between decorrelation noise components, predicts the lowest uncertainty. The Samiei-Esfahany

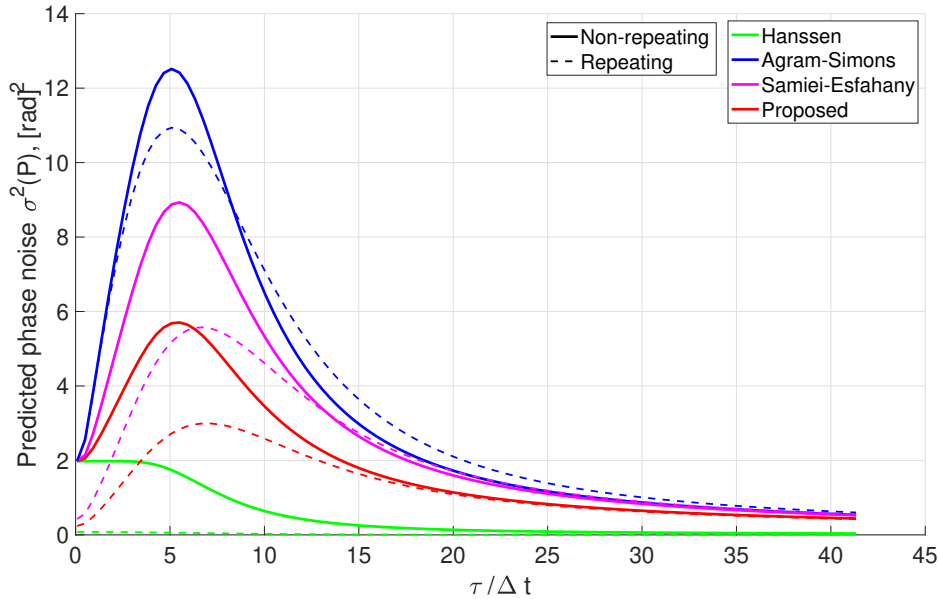


Fig. 1. Predicted uncertainty associated with decorrelation noise after non-repeating stacking and repeating stacking using the Hanssen model, the Agram-Simons model, the Samiei-Esfahany model and the proposed model.

model suggests higher correlation than that suggested by the proposed model, but lower than that suggested by the Agram-Simons model (within the range of $\tau/\Delta t$ plotted in Fig.1).

2. Predictions from the Samiei-Esfahany, the Agram-Simons model and the proposed model all show a peak around $\tau/\Delta t \approx 6$ while predictions from the Hanssen model shows a steady decrease with increasing apparent correlation time $\tau/\Delta t$. Two factors influence the predicted phase variance: a) the average phase noise level in interferograms and b) the degree of correlation between decorrelation noise components. The higher the average phase noise, the higher the predicted phase variance; the higher the degree of correlation between decorrelation noise terms, the higher the predicted phase variance. When $\tau/\Delta t$ is small, correlation between decorrelation noise terms are negligible in all three models, and when $\tau/\Delta t$ is large, the average phase noise level is low. Therefore the predicted phase variances in all three models are low at both ends of $\tau/\Delta t$ but high in the middle. Since there is no correlation in the Hanssen model, the predicted phase variance decreases monotonically with $\tau/\Delta t$.

Fig. 2 depicts how varying ρ_∞ or the number of SAR measurements ($2M$) impacts predicted phase noises with respect to different apparent decorrelation rates $\tau/\Delta t$. Fig. 2 (a) shows that increasing ρ_∞

decreases the dependency of predicted uncertainty on surface decorrelation rate as contributions from persistent scatterer increases. Fig. 2 (b) illustrates that for a slowly decorrelating surface ($\tau/\Delta t = 10$), increasing M increases predicted uncertainties while for a rapidly decorrelating surface ($\tau/\Delta t = 0.1$), increasing M reduces predicted uncertainty. Increasing M is equivalent to increasing the observation window length. For rapidly decorrelating surfaces, longer observational window provides additional measurements that are at a comparable noise level (determined by ρ_∞) with the original stack. Since these measurements are almost independent with the original stack, increasing M reduces noise. In contrast, for slowly decorrelating surfaces, long temporal span interferograms provide highly correlated measurements at a higher noise level. Therefore increasing M increases noise. The contrasting impacts of M with respect to surface decorrelation rates again reflect the two factors that influence the predicted phase variance: 1) the average phase noise level in interferograms and 2) the degree of correlation between decorrelation noise terms. Increasing M raises average noise level in interferograms but reduces the degree of correlation between decorrelation noise components. Therefore, when SAR measurements are sampled at a comparable interval with the surface correlation time ($\tau/\Delta t = 1$), the predicted uncertainty first increases then decreases when increasing M .

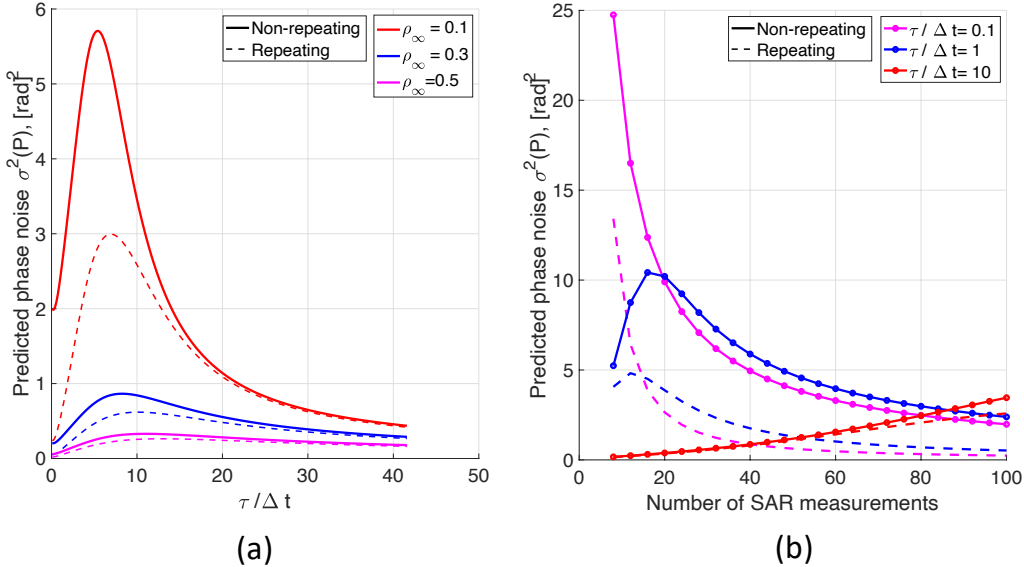


Fig. 2. The influence of (a) ρ_∞ and (b) the number of SAR measurements with respect to different apparent decorrelation rates on predicted uncertainty using the proposed model.

D. Implication for stacking strategies

Ultimately our goal is to effectively reduce decorrelation noise. Knowledge of $Cov(\Phi^{decor})$ can help us determine optimal processing strategies. In our simple example of stacking over a transient event, we compared the performances of two common stacking practices: non-repeating stacking and repeating stacking. If the target decorrelates exponentially over time, Fig.1 provides predictions of performance of either stack on decorrelation reduction with the given configuration of $M = 25$ and $\rho_\infty = 0.1$. For example, if radar measurements are not sampled 10 times the rate of surface decorrelation ($\tau/\Delta t < 10$), the proposed model suggests use of repeating stack because the performance differences between the two stacks are significant. On the other hand, if $\tau/\Delta t > 10$, the performance differences between the two stacks is insignificant, non-repeating stack becomes a better choice because it involves computation of fewer interferograms and hence is more efficient. Fig. 2 (b) demonstrates that for rapidly decorrelating surfaces, it is preferable to have long observation windows, though the rate of noise reduction declines with increasing M . In comparison, for surfaces that exhibit slow decorrelation rates, a small subset of interferometric pairs is sufficient to achieve satisfactory results.

In real applications, the weighting matrix W can take on more complicated forms than the ones shown in (25)

and (26). Moreover, there are often constraints on M . Nevertheless the same procedure as described above can be followed to guide the choice of W and the set of interferometric pairs to be used.

IV. VALIDATION WITH REAL DATA

In this section, we compare and assess all four decorrelation phase covariance models – the Hanssen model, the Samiei-Esfahany model, the Agram-Simons model, and the proposed model – with C-band Sentinel-1 data collected in both Cascadia and Death Valley. Similar to Section III, we construct $Cov(\phi_{ij}^{decor}, \phi_{kl}^{decor})$ using (1) with $\gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor})$ given by each of the four models. Note that we adopt the correlation forms of the models to avoid estimation bias caused by phase variance estimation – the Samiei-Esfahany model uses the Cramer-Rao bound to estimate phase variances while all three other models use observed phase variances. Since the Cramer-Rao bound only holds for high coherence targets and hence underestimate phase variances of the stacks, we modify the original Samiei-Esfahany model (3) by adopting observed phase variances instead.

Similar to section III, we form both non-repeating and repeating stacks that span over a given time interval in both regions, and predict residual decorrelation noise after averaging each stack using both the proposed and existing decorrelation phase covariance models. We

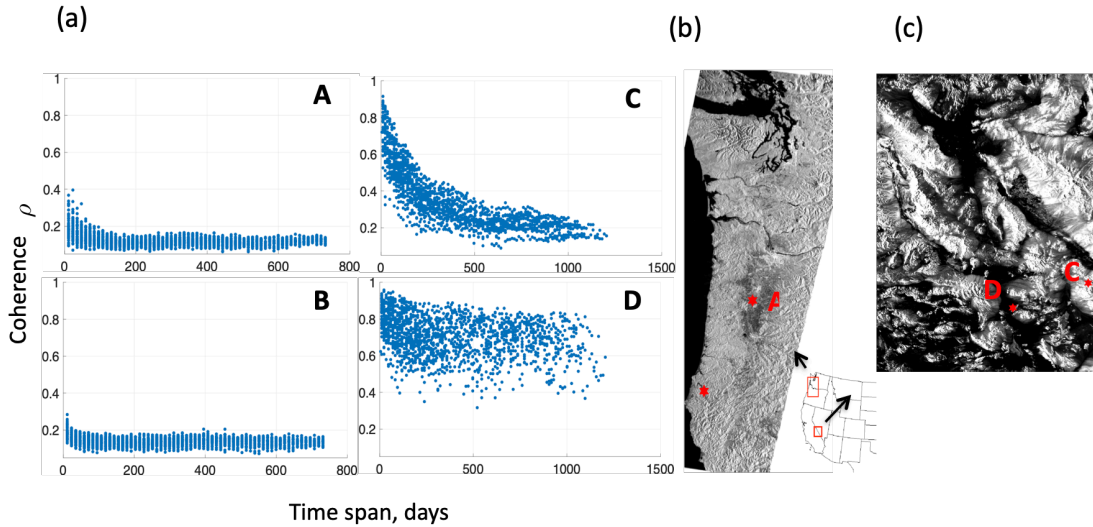


Fig. 3. Examples of observed temporal variations of correlation in the Cascadia and the Death Valley regions. (a) Scatter plots of correlation vs time in locations A, B, C and D. (b) A and B are located in the heavily vegetated Cascadia region, (c) C and D are located in the desert of Death Valley. A and B both exhibit a characteristic decorrelation time of roughly 30 days and an asymptotic coherence ρ_∞ of 0.1. C and D, on the other hand, show much longer decorrelation time of approximately 400 days and 200 days and higher asymptotic coherence ρ_∞ of 0.2 and 0.6, respectively.

estimate ρ_{ij} using sampled local averages over 300 meter by 300 meter windows. We estimate decorrelation phase variances of individual interferograms $\sigma^2(\phi_{ij})$, and of averaged stacks $\sigma^2(\phi_{nrp}^{decor})$, $\sigma^2(\phi_{rp}^{decor})$ by calculating phase variances inside 50 pixel by 50 pixel boxes (15 by 15 km^2 in area), assuming the spatial variance of atmospheric noise is negligible inside each box. We then compare predicted residual decorrelation noise with observed residual decorrelation noise.

Both Cascadia and Death Valley exhibit exponential decay in correlation with time, but with contrasting decorrelation rates (Fig. 3). For the Cascadia region, we form a non-repeating stack consisting of 40 interferograms and a repeating stack consisting of 1600 interferograms. For the Death Valley region, we form a non-repeating stack consisting of 28 interferograms and a repeating stack consisting of 784 interferograms. By averaging the respective stacks, we obtained ϕ_{nrp} and ϕ_{rp} for both regions.

Comparisons between phase variances associated with ϕ_{nrp} and ϕ_{rp} are shown in Fig. 4 (a) and (b). We obtained $\sigma^2(\phi_{nrp}^{decor})$ and $\sigma^2(\phi_{rp}^{decor})$ at evenly distributed image grid points. For the Cascadia region ($\tau/\Delta t \approx 1$ or 2), the repeating stack yields smaller uncertainties – on average $0.037 rad^2$) than the non-repeating stack – on average $0.284 rad^2$ (Fig. 4 (a), beige circles), confirming predictions from the proposed model. For the Death Valley region where

TABLE I
AVERAGE PREDICTION ERROR FOR EACH MODEL OVER TWO SURFACES. WE DEFINE PREDICTION ERRORS AS THE DISTANCES (UNIT: $[rad]^2$) BETWEEN POSITIONS OF DATA POINTS (BEIGE CIRCLES) AND POSITIONS OF THEIR CORRESPONDING PREDICTION POINTS (PURPLE STARS, ORANGE SQUARES, CYAN DIAMONDS OR MAGENTA INVERTED TRIANGLES) IN FIG. 4.

Models	Cascadia	Death Valley
Agram-Simons	0.300	0.368
Hanssen	0.097	0.114
Samiei-Esfahany (Modified)	0.096	0.231
Proposed	0.092	0.105

the apparent correlation time $\tau/\Delta t > 20$, non-repeating and repeating stacking produce comparable results – on average $0.105 rad^2$ and $0.102 rad^2$, respectively (Fig. 4 (b), beige circles), which are also consistent with predictions from the proposed model.

Predictions from both existing and the proposed models are shown in Fig. 4 and Table. I. Predictions from the proposed model (purple stars) match best with actual observations (beige circles). Predictions from existing models are consistent with the simulation results in Section III. For example, in Cascadia where the decorrelation rate is rapid, all models give comparable estimates for the non-repeating stack, while the Agram-Simons model (cyan diamonds) and the modified Samiei-Esfahany model (magenta triangles) give higher, the Hanssen model (orange squares) gives lower estimates of phase variances for the repeating

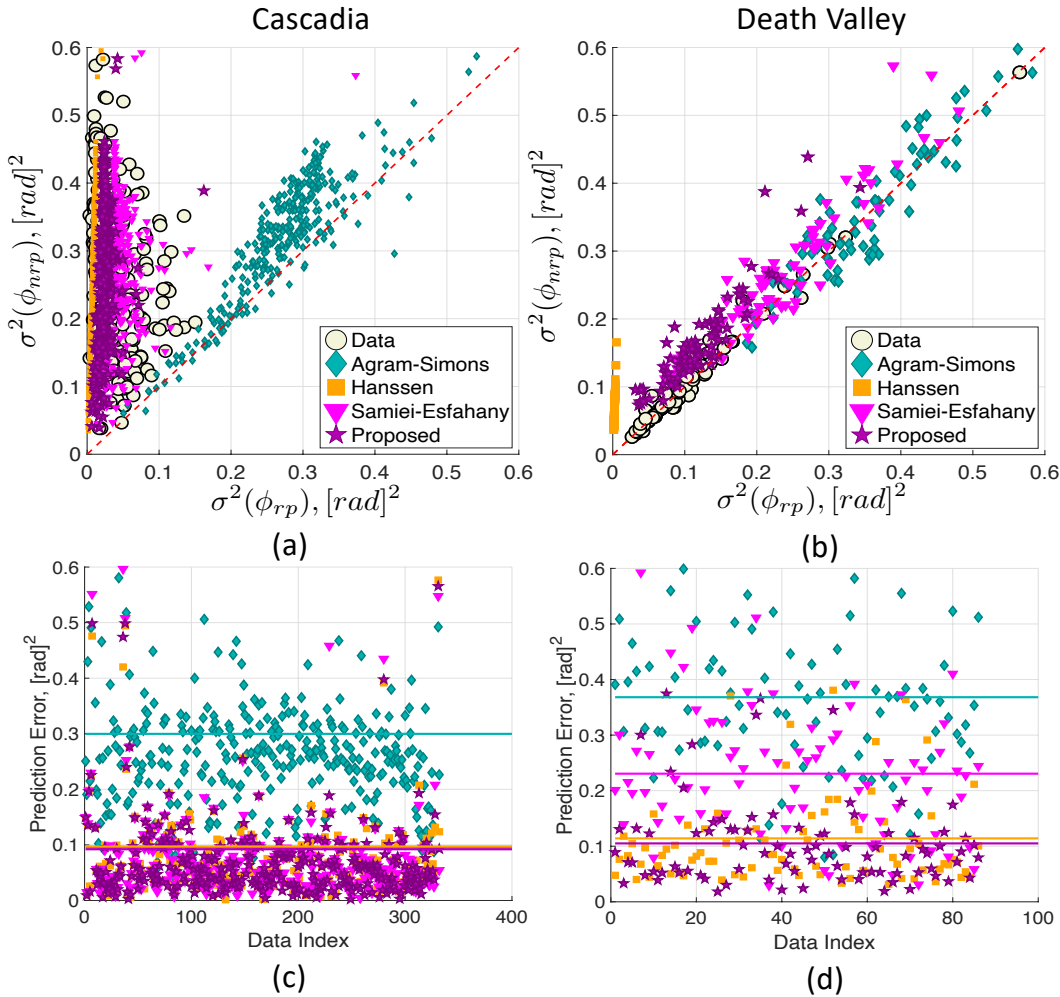


Fig. 4. Predicted phase variance and prediction errors after non-repeating stacking vs repeating stacking in (a) (c) the Cascadia region and (b) (d) the Death Valley region. Data (circles) show that for the Cascadia region, repeating stacking produces smaller phase variances and for the Death Valley region, similar phase variances, as compared to non-repeating stacking. Model predictions are shown in (a) and (b) while their respective errors are shown in (c) and (d) with the average prediction errors marked by solid lines. Predictions from the proposed model (stars) match the data best in both cases. The accuracy of each model prediction is summarized in Table I.

stack than the proposed model. In the Death Valley region where the decorrelation rate is on the order of years, all models except for the Hanssen model predict comparable phase variances between the non-repeating and the repeating stack.

Finally, we use an example to illustrate the significant reduction in decorrelation noise offered by including repeating interferograms in the stack over rapidly decorrelating areas. Fig. 5 depicts stacks over a slow slip deformation event [39], [40] that occurred in February 2016 in the Cascadia region. Slow slip events are usually hard to capture in the Cascadia region with InSAR due to extremely low signal to noise ratios. Atmospheric noise and decorrelation are the two main limitation factors.

With 10 Sentinel-1 SAR acquisitions before the slow slip event and 10 Sentinel-1 SAR acquisitions after, we formed a non-repeating interferogram stack consisting of 10 interferograms and a repeating stack consisting of 100 interferograms. It is apparent that the result from using repeating stacking yields a much cleaner signal pattern. Fig. 6 compares phase measurements along four profile lines between non-repeating and repeating stacking. Again, it is clear that while both stacking strategies produce measurements of the same expected signal, redundant stacking produces measurements with 80% less phase variance – the uncertainty associated with decorrelation reduces from 0.33 cm to 0.13 cm in the line-of-sight direction. With the same number of independent acquisitions, atmospheric noise is reduced

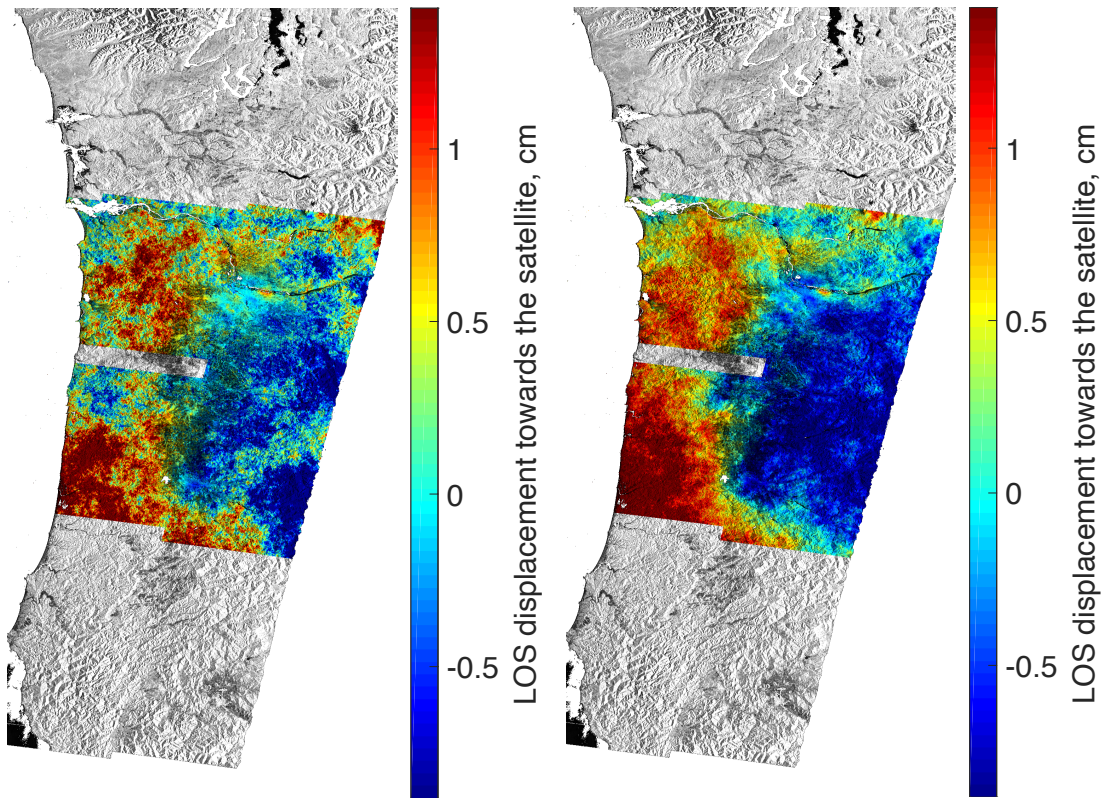


Fig. 5. Results from (left) non-repeating and (right) repeating stacking over the February 2016 slow slip event in the Cascadia region. The result from repeating stacking suffers from less decorrelation noise than the result from non-repeating stacking. Areas with less coverage than 20 acquisitions are masked. Phase measurements are converted to radar line-of-sight (LOS) measurements.

to the same extent after either non-repeating stacking or repeating stacking. Therefore, the significant reduction in phase noise reflects reduced decorrelation noise in the repeating stack.

V. DISCUSSIONS AND CONCLUSIONS

We have described a physics-based decorrelation phase covariance model for unwrapped interferogram phase stacks and compared the proposed model with three existing models – the Hanssen model, the Samiei-Esfahany model and the Agram-Simons model. Validations with Sentinel-1 data collected in both the Cascadia region and Death Valley region show that our model best captures decorrelation noise propagation in interferogram stacks. We demonstrated with a simple stacking exercise that the proposed decorrelation phase covariance model can facilitate choice of SBAS-like time-series processing strategies. The workflow applied to the stacking example can be easily adapted to general SBAS-like time-series algorithms in three steps:

- 1 Obtain the temporal decorrelation characteristics $\rho(t)$ in the area of interest. In some cases we may

have a prior knowledge or a good estimation of $\rho(t)$. Otherwise, estimate $\rho(t)$ from the processed stack.

- 2 Construct the decorrelation phase covariance matrix with (1) and (19) for the entire interferometric phase stack Φ . Phase variances can be estimated using the PDF of interferometric phases [2], [26]–[29].

- 3 Estimate residual decorrelation noise $COV(P)$ using (23) with different weighting matrices W . Determine the optimal W and subset of interferometric pairs Φ in terms of effective noise reduction and efficiency.

As we head into an era with an ever-growing SAR archive, understanding and quantifying uncertainties associated with decorrelation noise is of critical importance. A rigorous, comprehensive noise covariance model allows the InSAR community to better assess uncertainties with InSAR measurements, and to extend InSAR applications from mid-to-high correlation areas to low-correlation areas.

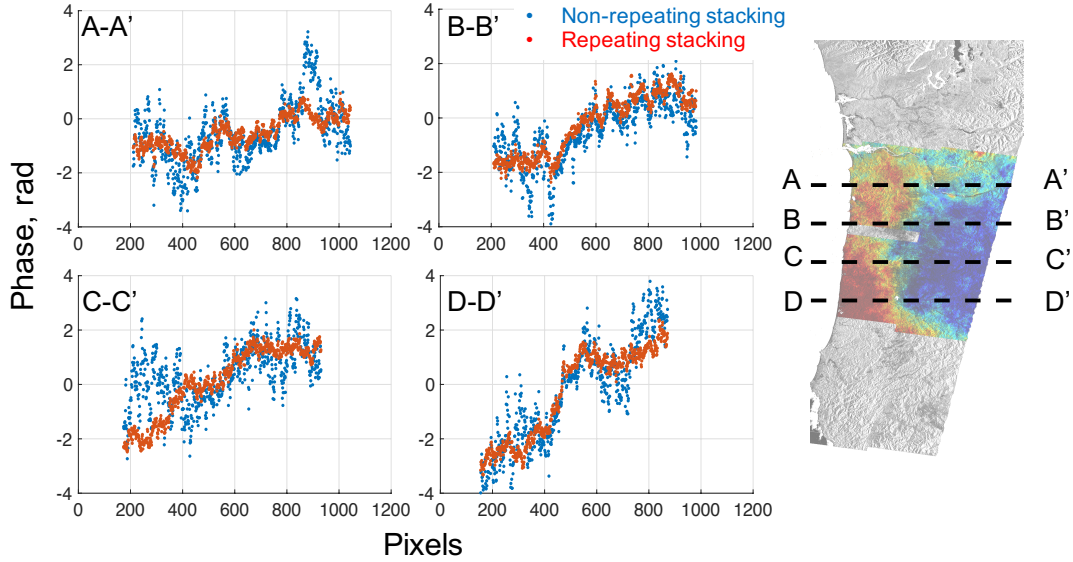


Fig. 6. Comparisons between non-repeating and repeating stacking results over profile lines A-A', B-B', C-C' and D-D'. Repeating stacking (red dots) produces similar mean measurements as with non-repeating stacking (blue dots), but with much 83% less phase variances. Equivalently, repeating stacking produces measurements with 0.33 cm line-of-sight uncertainties compared to 0.13 cm after non-repeating stacking.

APPENDIX A DERIVATION OF (16)

We model the distributed scatterers D_i as complex Gaussian random variables. D_i 's can be represented as

$$D_i = \rho_{i-1,i}^d D_{i-1} + \sqrt{1 - \rho_{i-1,i}^d{}^2} n_i \quad (27)$$

where n_i is a complex Gaussian random variables with an expected intensity of unity, and that is uncorrelated with D 's and n 's other than itself. With (27), we can derive that

$$\rho_{ij}^d \rho_{jk}^d = \rho_{ik}^d \quad (28)$$

$$E(D_i D_j^*) = \rho_{ij}^d \quad (29)$$

$$E(|D_i|^2 |D_j|^2) = 1 + \rho_{ij}^d{}^2 \quad (30)$$

Now we can derive (16):

$$\begin{aligned} & cov(z_{ij} - E[z_{ij}], z_{kl} - E[z_{kl}]) \\ &= E[(z_{ij} - E[z_{ij}])(z_{kl} - E[z_{kl}])^*] \\ &= E\left\{ (1 - \rho_\infty)^2 R_{ij} R_{kl}^* \right. \\ &+ \rho_\infty(1 - \rho_\infty)(D_i D_k^* + D_i D_l + D_j^* D_k^* + D_j^* D_l) \\ &+ (1 - \rho_\infty)\sqrt{\rho_\infty(1 - \rho_\infty)}(R_{ij}(D_k^* + D_l) \\ &+ R_{kl}^*(D_i + D_j^*)) \left. \right\} e^{i(\psi_i - \psi_j - \psi_k + \psi_l)} \\ &= \left((1 - \rho_\infty)^2 E(R_{ij} R_{kl}^*) \right. \end{aligned}$$

$$\begin{aligned} &+ \rho_\infty(1 - \rho_\infty)(\rho_{ik}^d + \rho_{jl}^d) \left. \right) e^{i(\psi_i - \psi_j - \psi_k + \psi_l)} \\ &= \left((1 - \rho_\infty)^2 \rho_{ik}^d \rho_{jl}^d \right. \\ &+ \rho_\infty(1 - \rho_\infty)(\rho_{ik}^d + \rho_{jl}^d) \left. \right) e^{i(\psi_i - \psi_j - \psi_k + \psi_l)} \\ &= (\rho_{ik} \rho_{jl} - \rho_\infty^2) e^{i(\psi_i - \psi_j - \psi_k + \psi_l)} \quad (31) \end{aligned}$$

APPENDIX B NUMERICAL SIMULATION OF (18)

To find the relation between $\gamma(z_{ij}, z_{kl})$ and ϕ_{ij}, ϕ_{kl} , we simulate a series of complex Gaussian random variables with decreasing correlation from 1 to 0. We then compute the corresponding correlations between phases of these complex Gaussian random variables (Fig. 7).

ACKNOWLEDGMENT

This work was supported by NASA Earth Science and Interior Grant NNX 17AE036. Copernicus Sentinel-1 data 2015-2018 was retrieved from ASF DAAC 7 May 2018, processed by ESA, <https://www.asf.alaska.edu>.

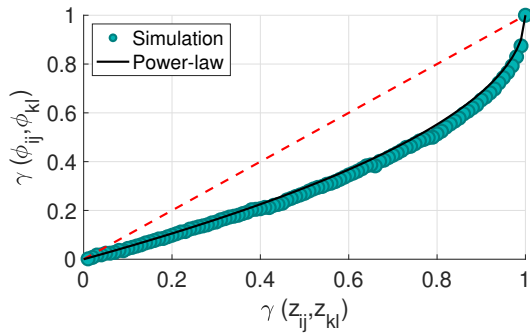


Fig. 7. The relation between $\gamma(z_{ij}, z_{kl})$ and $\gamma(\phi_{ij}, \phi_{kl})$ exhibits a power-law behavior $1 - \gamma(\phi_{ij}^{decor}, \phi_{kl}^{decor}) \approx [1 - \gamma(z_{ij}, z_{kl})]^{\frac{1}{2}}$.

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