The precession constant and its long-term variation

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The dynamical flattening of the Earth, H, related to the precession constant, is a fundamental astro-geodetic parameter that appears in studies of the Earth’s rotation and orbital evolution. We present numerical predictions and observations of the variation in H over time scales ranging from tens of millions of years to decades. The geophysical processes controlling this variation include solid-state convection in the rocky mantle of the Earth that drives plate tectonics, isostatic adjustments due to ice age loading, and ice-ocean mass transfer linked to modern global climate change. The time dependence of H is complex and non-linear, and thus, in contrast to previous suggestions, cannot be captured by a constant rate parameter.

Key words: Precession constant; dynamical flattening; mantle convection; glacial isostatic adjustment; climate change; Earth rotation

1. Introduction

The dynamic flattening of the Earth, H, is a measure of the difference between the polar moment of inertia (C) and the mean of the equatorial moments of inertia (A, B) of the planet:

\[ H = \frac{1}{C} \left[ C - \frac{1}{2} (A + B) \right] \]  
(1)

H is a fundamental parameter in procession and nutation theories of the Earth, as well as a series of other rotational normal modes of widely varying frequency (e.g., Wahr, 1981; Dehant & Capitaine, 1996; Chao, 2017). Dynamic flattening also plays an important role in a range of global geophysical studies – either explicitly or implicitly – through its connection to changes in the planetary spin rate (or, alternatively, “length-of-day” in geodesy) or dynamical form factor, J2. For example: (1) satellite-based estimates of the secular rate of change of J2 after ~1990 are thought to be impacted by the onset of significant polar ice sheet melting (Cox & Chao, 2002); (2) variations in J2 associated with ongoing, residual effects of the last ice age, as well as with tidal dissipation and other factors, combine to explain the slowing of the Earth’s rotation rate over the past three millennia that has been estimated from ancient eclipse observations (Stephenson & Morrison, 1984, 1995; Stephenson, 2003; Mitrovica et al., 2015); (3) perturbations in the dynamical flattening driven by mass changes arising from ice age effects and solid-state convective mantle flow after Milankovitch (precession, obliquity) band variations in climate proxy records (Laskar et al., 1993; Forte & Mitrovica, 1997; Mitrovica et al., 1997; Pälike & Shackleton, 2000; Lourens et al., 2001; Morrow et al., 2012); and (4) geological measurements of the period of Earth’s rotation during the Proterozoic Eon (~620 Ma) that are based on tidal rhythmites reflect long-term tidal braking and dissipation in the Earth-Moon-Sun system (Williams, 1997), which would also be manifest as a trend in H.

The above discussion raises the question: Is the rate of change of dynamical flattening constant and, if not, what is the temporal structure of its variability? Bursa et al. (2008) estimated that the long-term variation in \( dH/dt = -8.45 \times 10^{-11} \text{ yr}^{-1} \) from satellite data over the period 1979–2002 (Cox & Chao, 2002). They argue that this rate should be treated as a fundamental astro-geodetic parameter and suggest that the trend may remain valid for the past 650 Myr; this argument is based on the fact that the current rate of tidal braking of the Earth’s rotation rate would, if applied over this long time period, lead to a rotation period at 650 Ma (henceforth “Ma” denotes “million years ago”) relatively close to the geological inference of ~21.8 hours. Putting aside geophysical modeling of variations in H, there are a variety of reasons to be sceptical of this argument. First, as noted above, the trend in the dynamical form factor, or \( dJ_2/dt \), has varied significantly across the satellite period. For example, Roy & Peltier (2011) estimate rates of \(-3.7 \times 10^{-11} \text{ yr}^{-1}\) for the period 1976–1992 and \(-0.9 \times 10^{-11} \text{ yr}^{-1}\) for 1992–2009 (equivalent values for \( dH/dt \) are \(-1.1 \times 10^{-11} \text{ yr}^{-1}\) and \(-2.7 \times 10^{-11} \text{ yr}^{-1}\), respectively). The estimate of \( dH/dt = -8.45 \times 10^{-11} \text{ yr}^{-1}\) in Bursa et al. (2008), based on the results of Cox & Chao (2002), is thus a time-weighted average of these two values. Second, ongoing isostatic adjustment in response to ice age loading over the last few million years dominates the pre-1992 variation in H, and thus any trend in the precession constant over this period cannot be constant, but will instead reflect the time scales of ice age cyclicity. Third, the current dynamic flattening of the Earth is known to exceed the form of a rotating planet in hydrostatic equilibrium by ~ 1% (Nakiboglu, 1982; Chambat et al., 2010). This excess flattening is driven by convective mass flow in the mantle and will thus vary over the timescale associated with that process, which is tens of millions of years. Finally, the present rate of tidal braking of the Earth’s rotation must be anomalously high, because a back-projection of that rate leads to the so-called “time-scale problem” of Lunar origin (i.e., the Moon’s orbital radius would place it at the Roche limit only ~2 billion years ago; Kaula & Harris, 1975). The likely resolution of this problem is that ocean tidal dissipation would have been lower during the time of the Pangean supercontinent (~340–170 Ma), and during previous supercontinent periods, with a consequent reduction in rates of change of both H and Earth’s rotation period during these times (Hansen, 1982).

In this article, we describe new predictions of the variation in the dynamical ellipticity over time scales ranging from
tens of millions of years to centuries, based on geophysical modeling of changes in Earth’s shape associated with mantle convective flow over the past 50 million years and ice mass flux across the Plio-Pleistocene glacial cycles (i.e., the past 3 Myr). We also map a recent, satellite-derived time series of $J_2$ into a variation in $H$ from 1976–2012. The predictions, together with the satellite derived time series, provide a measure of the natural — and human-induced — variability in Earth’s dynamical flattening. This variability is complex, and it cannot be captured by a constant rate term.

2. Mathematical background

Perturbing Equation (1), and using the fact that the trace of the inertia tensor is invariant during non-uniform deformation (e.g. Rochester & Smylie, 1974), i.e.,

$$\delta A + \delta B + \delta C = 0$$  \hspace{1cm} (2)

yields the following expression for the variation in $H$

$$\delta H = \left[ \frac{3}{2} - H \right] \frac{\delta C}{C}$$  \hspace{1cm} (3)

(NB. we comment in Section 3 on the impact of uniform deformation on these equations). Since $H \sim 0.00327$, this expression can be approximated as

$$\delta H \approx \frac{3}{2} \frac{\delta C}{C}$$  \hspace{1cm} (4)

The dynamical form factor, $J_2$, is defined as

$$J_2 = \frac{1}{M_e a^2} \left[ C - \frac{1}{2} (A + B) \right]$$  \hspace{1cm} (5)

where $M_e$ and $a$ are the mass and radius of the Earth, respectively. Combining equations (1) and (5) yields the following relationship

$$H = \frac{M_e a^2}{C} J_2$$  \hspace{1cm} (6)

Taking the first variation of this expression, and once again using the fact that $H < < 1$, yields

$$\delta H = \frac{M_e a^2}{C} \delta J_2$$  \hspace{1cm} (7)

Since $C \sim \frac{1}{2} M_e a^2$, the scaling factor on the right-hand side of Equation (7) is $\sim 3$. This simple relationship was applied to relate expressions for $dJ_2/dt$ and $dH/dt$ that were used in Section 1.

In the results below, we will consider predictions and observations of the relative perturbation in the dynamic flattening, $\delta H/\delta H_0$, where $H_0$ is the present-day value (0.003274) and the perturbation $\delta$ is defined relative to this value (Figure 1). As a guide to interpreting the impact of such signals, if a dynamic ellipticity of value $H'$ is connected to an orbital frequency (for example, of precession or obliquity variations) of $f'$, then, in the absence of resonance effects, the perturbation $\delta H/\delta H'$ would yield a proportional perturbation in the associated frequency of $f' \delta H/\delta H'$ (Laskar et al., 1993; Williams, 1994).

3. Results

Plate tectonics is driven by thermochemical convection within the Earth’s mantle, a process that also leads to perturbations in the shape of the solid surface, core-mantle boundary, and gravitational field of the planet on a wide range of spatial scales. In the 1980s, global geophysical research focused on numerical and theoretical modeling of the process using constraints from satellite-derived estimates of Earth’s long-wavelength gravity field (e.g., Richards & Hager, 1984; Ricard et al., 1984; Hager et al., 1985). These efforts combined tomographic models of seismic velocity variations in the mantle with experimental constraints from mineral physics on the mapping between these velocities and density (or, equivalently, buoyancy), with the goal of constraining the depth-dependent variation of mantle viscosity. While this approach provides invaluable insights on mantle dynamics, trade-offs between mantle buoyancy and viscosity render results subject to considerable uncertainty (Thoraval & Richards, 1997).

Subsequent work therefore extended these studies to consider a wider range of present-day observations, including plate velocities, perturbations to surface topography, and excess ellipticity of the core-mantle boundary as inferred from the period of the Earth’s free core rotation (e.g. Forte & Peltier, 1987; Lithgow-Bertelloni & Richards, 1998; Gurnis et al., 2000; Forte & Mitrovica, 2001; Simmons et al., 2006).

A number of studies have extended the present-day snapshot of mantle dynamics, the focus of the above analyses, to model the time history of the system. These analyses were generally based on “backward advection” of the governing field equations under the caveat that thermal diffusion is treated as negligible, since it is not temporally reversible in a unique sense and is not tractable due to numerical instabilities (Steinberger & O’Connell, 1997; Conrad & Gurnis, 2003; Moucha et al., 2008). A major limitation of this approach is that it produces transient behaviour within the thermal boundary layers (regions at the base and top of the convecting mantle, which are dominated by conductive heat transport), resulting in model simulations undergoing an initial jump prior to reaching steady-state; this jump contaminates the most recent period of model evolution. These issues are avoided in more sophisticated adjoint treatments that solve the full field equations in a forward sense and therefore rigorously incorporate thermal diffusion (e.g., Bunge et al., 2003; Ismail-Zadeh et al., 2004; Zhou & Liu, 2017; Li et al., 2017; Price & Davies, 2018; Ghelichkhani & Bunge, 2018).

Here, we adopt the adjoint methodology of Ghelichkhani & Bunge (2016) to track relative changes in the dynamical ellipticity driven by mantle convection over the past 50 Myr (Figure 1A). Details of the calculation are provided in the Appendix. Our simulation yields a perturbation in the magnitude of $H$ of order 0.1% since 50 Ma, with an increase in dynamic flattening until 15 Ma, followed by a decrease of comparable magnitude in the subsequent 15 Myr (Figure 1A). Our investigation of the evolving mantle heterogeneity in the adjoint model indicates that the increase in $\delta H/H$ from 50 Ma to 20 Ma is driven by an increase in the amplitude of long-wavelength density anomalies at the base of the upper mantle (the so-called transition zone) and the base of the lower mantle (i.e., above the fluid outer core). The subsequent change in trend reflects a progressive weakening of the transition zone signal after 20 Ma.

This variation in $H$ is significantly smaller than predicted by a previous backward advection simulation (Forte & Mitrovica, 1997), and it has significant implications for the stability of Earth’s precession and obliquity parameters. In particular, Laskar et al. (1993) has shown that if the dynamic flattening were perturbed downward by $\sim 0.2$% relative to the present day value, these parameters would experience a non-linear perturbation due to a passage through the $s_0 - g_0 + g_0$ resonance that is associated with perihelion of Jupiter and Saturn and the node of Saturn. We conclude that such a passage is unlikely to have occurred over the past 50 Myr (c.f. Forte & Mitrovica, 1997).

Next, we turn to variability in the dynamical ellipticity...
associated with ice age dynamics over the past 3 Myr. Over this period, the Earth was subject to glacial cycles of increasing magnitude, and an obliquity-paced periodicity of this period, the Earth was subject to glacial cycles of increasing magnitude, and an obliquity-paced periodicity of 40 kyr. In the period after 1990, the larger ice age-corrected signal (4.3 × 10⁻⁸ yr⁻¹) reflects the onset of major melting from the polar ice sheets (Cox & Chao, 2002; Cheng et al., 2013). The best fit linear form across the full time series, i.e., 1976–2012, is characterized by a rate of change of δH/Δt of ~2.1 × 10⁻⁸ yr⁻¹.

The calculations in Figure 1, since they are based on Equation (2), do not include the impact on the inertia tensor of a uniform, degree-0 deformation of the Earth. If we included this spatially uniform signal in the theory, Equation (3) would be revised to

$$\delta H = \left[ \frac{3}{2} - H \left( 1 + \frac{\delta C(0)}{\delta C(2)} \right) \right] \frac{\delta C(2)}{C} \quad (8)$$

where the subscripts denote the spherical harmonic degree and order of the structure contributing to the inertia perturbation. In our calculations of perturbations in δH/Δt due to ice mass changes (Figure 1B), we include complementary sea level changes and the total mass of the surface load is conserved (i.e., it has no degree-0 component).

The same must be true for the processes responsible for the observations that form the basis of Figure 1C. Thus, in these cases, there is no degree-0 deformation, δC(0) = 0, and the above expression collapses to that in Equation (3).
While our calculations of the perturbation to $\delta H/H$ driven by mantle convection adopt a compressible flow model, any changes in the volume of the Earth are negligible. However, these calculations do not account for secular cooling and thermal contraction of the Earth. Estimates of this latter process suggest that the reduction in Earth radius over the past 50 Myr due to thermal contraction has been $\sim 80$ m (Tsuchiya et al., 2013). Using this value and the results in Figure 1A yields an estimate of $\delta C(0)/\delta C(2,0) \sim 5$, and therefore $H(1 + 8\delta C(0)/\delta C(2,0)) \sim 6H$. This value is of order 1% of the leading term of $\frac{d}{dt}$ in Equation (8), and neglecting it in adopting Equation (4) remains justified.

The results in Figure 1 do not include the impact on the dynamical flattening of an additional process mentioned in the introductory section, namely, tidal dissipation. The present level of tidal dissipation is slowing the Earth’s rotation at a rate of $(d\Omega/dt)/\Omega = 8.8 \times 10^{-18}$ s$^{-1}$ (e.g. Quinn et al., 1991), and the dynamical flattening will be approximately proportional to $\Omega^2$. While the variation of tidal dissipation over time is uncertain, any effort to estimate the total change in dynamical flattening from all geophysical processes must include this contribution.

### 4. Final remarks

The dynamical flattening of the Earth, a parameter associated with the precession constant, plays an important role in a wide range of applications in astronomy, geodesy and geophysics, including astronomical observations of mutations, investigations of the stability of the orbital elements (precession, obliquity) controlling Milankovitch forcing of ice age climate, and the evolution of the Earth-Moon-Sun system extending from 1979–2002 – they suggested that the variation in precession constant within astronomical analyses. However, the importance of recognizing the time dependence in the precession might be of order 1% of the leading term of $\frac{d}{dt}$ in Equation (8), and neglecting it in adopting Equation (4) remains justified.

In Figure 1A, the change in Earth radius over a billion-year time scale, is characterized by a three order of magnitude increase in viscosity from the shallow mantle beneath the lithosphere ($10^{20}$ Pa s) to 2000 km depth ($10^{23}$ Pa s), followed by a reduction of comparable magnitude toward the core-mantle boundary.

To construct the present-day mantle buoyancy field, we use lower mantle shear wave velocities from the recent tomography model L1M-G3D (Simmonet et al., 2012). Upper mantle velocity structure is prescribed from the higher resolution surface wave tomography model SL2013sv (Schaeffer & Lebedev, 2013), smoothly blended into the deeper mantle model over the depth range 250–350 km. To convert seismic velocities into density, we first calculate anharmonic velocities and densities as a function of pressure and temperature for a pyroclastic mantle composition using the thermodynamic database of Stixrude & Lithgow-Bertelloni (2011) and the PerpleX Gibbs-free-energy minimisation software (Connolly, 2005). Anharmonic velocities variation as a function of depth are then used to query the resultinglookup table and extract corresponding values of density. To prevent the continental lithosphere from actively participating in convection, densities within the lithosphere are set to the radial average using the lithosphere-asthenosphere boundary map of Hoggard et al. (2020).

Once the temporal evolution of the mantle flow field has been successfully reconstructed, we calculate the time history of dynamic ellipticity, $H$. For this purpose, we solve the governing, coupled system of Stokes and Poisson’s equations using an instantaneous flow methodology that includes the effects of self-gravitation and compressibility and assumes a free-slip (no tangential stress) boundary condition (Corrieu et al., 1995).

### Appendix

Time-evolution in our adjoint treatment (Ghelichkhan & Bunge, 2016) is constrained by assimilating a history of plate motions (Young et al., 2019), and the initial buoyancy field (i.e., at 50 Ma) is iteratively optimized through comparison of the final, present-day buoyancy field predicted by the flow model with the buoyancy field inferred from seismic tomography. This procedure typically converges after 12-15 iterations.

Two other fields need to be prescribed in this procedure, the radial viscosity structure used in the flow calculation and the present-day mantle buoyancy field to which the prediction of the flow model at the present day is compared. We consider each, in turn. All other material parameters and boundary conditions are adopted from Colli et al. (2018).

We use a radial viscosity profile derived from a joint inversion of data related to mantle convection and ice age dynamics (Mitrovica & Forte, 2004). The viscosity model, which we also adopt in the ice age calculations described in the main text, is characterized by a three order of magnitude increase in viscosity from the shallow mantle beneath the lithosphere ($10^{20}$ Pa s) to 2000 km depth ($10^{23}$ Pa s), followed by a reduction of comparable magnitude toward the core-mantle boundary.

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