

16 Recrystallization of ice enhances the creep and
17 vulnerability to fracture of ice shelves

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19 **Abstract**

The initiation of fractures and fast flow in floating regions of Antarctica have the potential to destabilize large regions of the grounded ice sheet, leading to rapid sea-level rise. While observations have shown rapid, localized deformation and damage in the margins of fast-flowing glaciers, there remain gaps in our understanding of how rapid deformation affects the viscosity and toughness of ice. Here we derive a model for dynamic recrystallization of ice that includes a novel representation of migration recrystallization. This mechanism is absent from existing models and is likely dominant in warm areas undergoing rapid deformation, such as shear margins in ice sheets. While solid earth studies find fine-grained rock in shear zones, here we find elevated ice grain sizes (> 10 mm) due to warmer temperatures and high strain rates activating migration recrystallization. Large grain sizes imply that ice in shear margins deforms primarily by dislocation creep, suggesting a flow-law stress exponent of $n \approx 4$ rather than the canonical $n = 3$. Further, we find that this increase in grain size results in a decrease in tensile strength of ice by $\sim 75\%$ in the margins of glaciers. Thus, this increase in grain size

softens the margins of fast-flowing glaciers and makes ice shelf margins more vulnerable to fracture than previously supposed. These results also suggest the need to consider the effects of dynamic recrystallization in large-scale ice-sheet modeling.

20 *Keywords:* glaciers, fracture, recrystallization, creep

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23 **1. Introduction**

24 Ice shelves, the floating regions of large ice sheets, provide a significant
25 control on the evolution of ice sheets and their contributions to sea-level rise.
26 Ice shelves restrain (i.e., buttress) the upstream grounded portions of the
27 ice sheet, preventing rapid flow of grounded ice towards the ocean. Calving
28 events and dynamic thinning reduce the buttressing that ice shelves provide
29 to the grounded ice, resulting in accelerated flow and possible instability of
30 the ice sheet. Thus, a combination of ice fracture and accelerated flow may
31 play a significant role in controlling the stability of the West Antarctic Ice
32 Sheet (Thomas and Bentley, 1978; Wingham et al., 2009; Pollard et al., 2015;
33 Gudmundsson et al., 2019).

34 Fracture and flow generally occur in areas of rapid deformation, which
35 appears in the margins of fast-flowing glaciers and ice shelves (known as *shear*
36 *margins*). A significant concentration of fractures and damage on ice shelves
37 are found in the margins, which may have implications for the stability of the
38 ice shelf (Lhermitte et al., 2020). Further, the lateral shearing that occurs in
39 shear margins of grounded glaciers provides a control on flow speed and con-

40 tributes to the buttressing effect (MacAyeal, 1989; Ranganathan et al., 2021).
41 While this has been well-observed, there remains uncertainty in the physical
42 processes underlying fracturing and accelerated flow in shear margins.

43 Fundamentally, the creep and fracture of ice are dictated by the grain-
44 scale microstructure of the ice. It is well-known from solid earth studies that
45 the physical properties of the crystalline microstructure - including grain
46 size and grain orientation - affect the rates of creep and fracture of rocks
47 significantly (Van der Wal et al., 1993; De Bresser et al., 2001; Montési and
48 Hirth, 2003) and modeling and laboratory studies have proposed similar ef-
49 fects in ice (e.g. (Currier and Schulson, 1982; Cuffey et al., 2000; Goldsby and
50 Kohlstedt, 2001; Hruby et al., 2020; Behn et al., 2020)). However, the physics
51 of the microstructure of ice has rarely been applied to the question of how
52 rapid deformation induces positive feedbacks on flow and how areas of rapid
53 deformation fracture. Here, we study the effect that deformation-induced
54 grain size evolution may have on flow and fracture of ice.

55 Observations show that grains are large in areas of glaciers where ice is
56 warm and being sheared. Measurements of grain size in the GRIP (Greenland
57 Ice Core Project) ice core and GISP2 (Greenland Ice Sheet Project 2) ice core
58 shows that grain sizes increase rapidly with depth near the base, where the
59 ice is frozen to the bed and thus strain rates are relatively large and the ice
60 is warm (Thorsteinsson et al., 1997; Gow et al., 1997). We would therefore
61 expect grains to be large in shear margins, where strain rates are quite high
62 (Gardner et al., 2018) and consequently the ice is warmed, sometimes to
63 the melting point, through viscous dissipation (Meyer and Minchew, 2018).
64 While there are no observations of grain size at depth in shear margins,

65 measurements made in shallow boreholes (Jackson and Kamb, 1997) and
66 observations of grain size in temperate glaciers (Tison and Hubbard, 2000)
67 support the suggestion that grains are likely large in shear margins.

68 Grain size influences the mechanisms of creep that allow ice to flow as
69 a viscous fluid (Goldsby and Kohlstedt, 2001). Most known creep mecha-
70 nisms, such as diffusion creep and grain-boundary sliding, have explicit and
71 well-tested grain size dependencies. On the other hand, numerous laboratory
72 experiments have shown that dislocation creep is practically independent of
73 grain size (Duval and Gac, 1980; Jacka, 1984). For grain-size-dependent
74 mechanisms, creep deformation is enhanced as grain sizes get smaller and di-
75 minished as grain sizes grow. Therefore, the relative influence of dislocation
76 creep increases as grains grow and we may expect that areas of large grain
77 sizes will deform primarily by dislocation creep, a consideration with impor-
78 tant implications for the viscosity of ice. Since rates of shearing in shear
79 margins affect the flow speed of grounded ice and may affect the buttressing
80 of ice shelves, grain size in shear margins may also affect ice shelf evolution.

81 Furthermore, the tendency for ice to fracture is a function of the size
82 and distribution of flaws, where stresses intensify. Larger flaw sizes tend to
83 increase the stress intensity, implying that in general, the tensile strength
84 of ice decreases as the flaw size increases. For intact or pristine ice, the
85 flaw size is set by the grain size, and therefore the tensile strength of ice
86 decreases as grain size increases, consistent with laboratory studies (Figure
87 3a) (Currier and Schulson, 1982; Nixon and Schulson, 1987, 1988). Thus, we
88 might suppose that glacier shear margins are likely to have relatively large
89 grain sizes that will decrease the tensile strength of the ice and could explain

90 the observations of crevassing and fracture (e.g. Lhermitte et al. (2020)).
91 Here, we derive a model for steady-state grain size in deforming glacier ice
92 to consider the effect that grain size may have on the creep and vulnerability
93 of ice to fracture in shear margins of rapidly-deforming glaciers.

94 **2. A Steady-State Grain Size Model**

95 Recrystallization processes alter the orientation and size of ice grains both
96 in the absence of and in response to deformation. While there are many
97 mechanisms of recrystallization, three main processes likely dominate the
98 evolution of grain size in ice: normal grain growth, grain-size reduction, and
99 migration recrystallization (Duval and Castelnau, 1995). Thus the net rate
100 of change in grain size can be described as the sum of the contributions from
101 all mechanisms, assuming that these mechanisms operate independently, as
102 past work has assumed (Austin and Evans, 2007):

$$\dot{d} = \dot{d}_{\text{red}} + \dot{d}_{\text{mig}} + \dot{d}_{\text{nor}} \quad (1)$$

103 where overdots represent time derivatives, \dot{d}_{nor} is the rate of change in grain
104 size due to normal grain growth, \dot{d}_{red} is the rate of change in grain size due to
105 grain-size reduction, and \dot{d}_{mig} is the rate of change in grain size due to migra-
106 tion recrystallization. We note that there are multiple proposed mechanisms
107 for grain size reduction (subgrain rotation by rotation recrystallization is well-
108 known in studies of ice (Derby and Ashby, 1987; Duval and Castelnau, 1995;
109 De La Chapelle et al., 1998; De Bresser et al., 1998; Montagnat and Duval,
110 2000), and other mechanisms include nucleation of grains by bulging) (De La

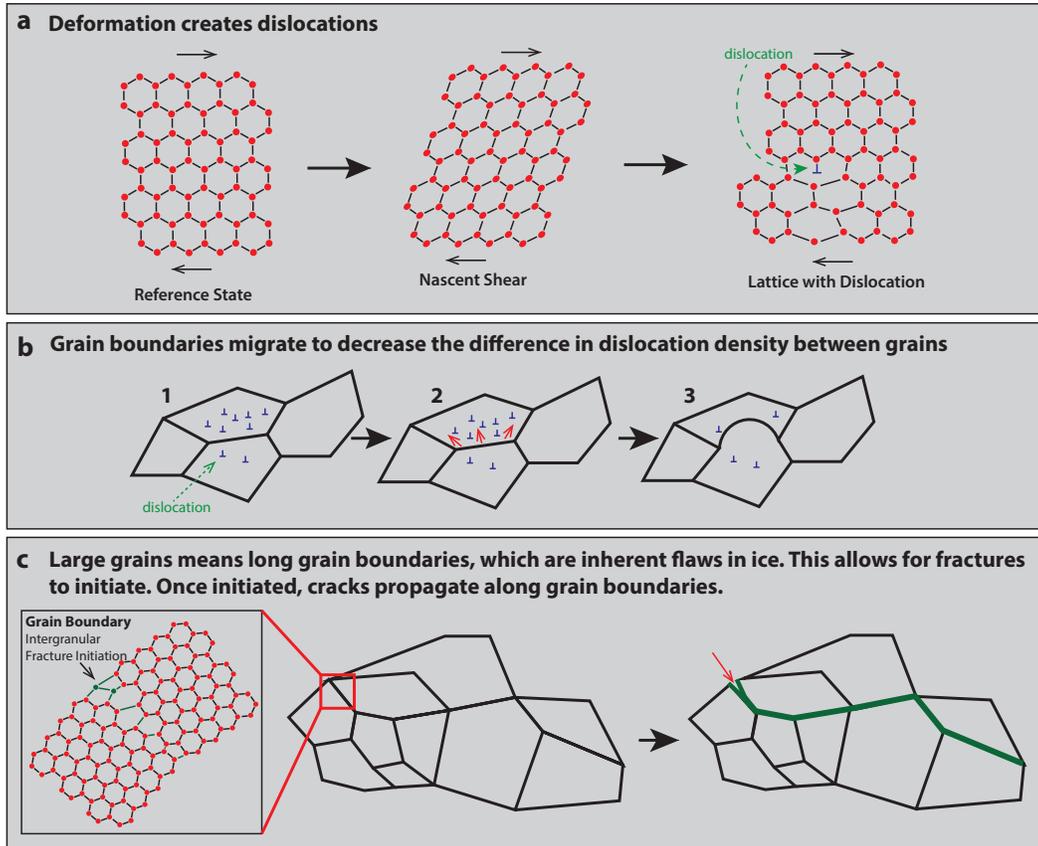


Figure 1: Schematic of migration recrystallization and its effect on ice strength. (a) In response to stress (in Antarctic glaciers, this stress arises from the ice sheet deforming under its own weight), the ice shears, creating dislocations. (b) A hypothetical polycrystalline ice of four grains. Due to local heterogeneities in stress, the density of the resulting dislocations are also heterogeneous [1]. To relieve stresses created by the difference in dislocation density between two grains, the grain boundary migrates towards the area of higher dislocation density [2], absorbing the dislocations and leaving behind a region of zero dislocation density [3]. The fact that the boundary leaves behind a region of no dislocation density may create more heterogeneities in dislocation density, driving further grain boundary migration. (c) Schematic that illustrates the role grain boundaries play in fracture. This shows a theoretical polycrystalline ice of 10 grains. Grain boundaries are inherent flaws in the ice because they interrupt the ordered structure of the lattice (inset). This enables initiation of intergranular fracture in response to stresses. Once the fracture is initiated, cracks propagate along grain boundaries because they are the weakest part of the ice. Outlined in green is a potential path a fracture may take.

111 Chapelle et al., 1998; De Bresser et al., 2001; Rios et al., 2005; Chauve et al.,
 112 2017). For many of these mechanisms, there are not explicit models or clear
 113 understanding of the physical processes. In this model, we parameterize the
 114 energy changes that occur during grain-size reduction and do not explicitly
 115 model specific mechanisms of grain-size reduction, as previous studies have
 116 done (Austin and Evans, 2007; Behn et al., 2020). Therefore, the estimates
 117 presented in this study may account for multiple physical processes of grain
 118 size reduction.

119 In the absence of deformation (static recrystallization), normal grain
 120 growth dominates, meaning that grain boundaries migrate outwards, leading
 121 to an increase in grain size (Alley, 1992). This migration is driven partially
 122 by grain boundary energy γ , which represents the change in free energy
 123 per change in unit area of the grain (Alley et al., 1986a,b). In contrast,
 124 deformation activates the two other recrystallization mechanisms (dynamic
 125 recrystallization) through the introduction of dislocations into the ice crys-
 126 talline lattice. In an incompressible material such as ice, the rate of work
 127 done during deformation is defined as the double inner product $\tau_{ij}\dot{\epsilon}_{ij}$ (in
 128 summation notation), where τ_{ij} is the deviatoric stress tensor and $\dot{\epsilon}_{ij}$ is the
 129 strain rate tensor. The work rate is a combination of the change in internal
 130 energy from migration recrystallization and grain-size reduction, described
 131 mathematically as

$$(1 - \Theta)\tau_{ij}\dot{\epsilon}_{ij} = \dot{E}_{\text{red}} - \dot{E}_{\text{mig}} \quad (2)$$

132 where Θ represents the fraction of the work rate that is dissipated as heat,
 133 \dot{E}_{red} is the rate of change in internal energy due to grain-size reduction,

134 and \dot{E}_{mig} is the rate of change in internal energy due to migration recrystallization. While grain-size reduction reduces grain size, migration recrystallization grows grains: grain-scale stress gradients cause heterogeneity in dislocation density within the grain, which result in stress gradients that drive the outward migration of boundaries. This mechanism is dominant at high temperatures and high strain, where dislocation density is likely to be most heterogeneous (Duval, 1985; Alley, 1988). Since grain-size reduction and migration recrystallization have opposite effects on surface energy, the two energy rates have opposite signs (discussed more in detail in Supplement Section A).

144 Here, we build upon the steady-state grain size model from Austin and Evans (2007) by adding a parameterization for migration recrystallization, allowing us to predict grain size in shear margins. Migration recrystallization occurs when the temperature of the material approaches the melting temperature (Duval and Castelnau, 1995; Montagnat and Duval, 2000). Current steady-state grain size models, such as those derived by Derby and Ashby (1987), De Bresser et al. (1998), Hall and Parmentier (2003), and Austin and Evans (2007), were developed for solid earth studies and do not incorporate effects of migration recrystallization because rocks tend to deform at temperatures well below their melting temperatures. Ice on Earth is never more than a few tens of degrees colder than its melting temperature and thus deformation can warm ice to within a few degrees or less of its melting temperature (Meyer and Minchew, 2018), where we'd expect migration recrystallization to be most active.

158 *2.1. Migration Recrystallization*

159 The driving forces for migration recrystallization are the stress gradients
160 created by heterogeneities in dislocation density that drive the outward mi-
161 gration of grain boundaries (Figure 1) (Derby and Ashby, 1987). Once the
162 strain energy of grains exceeds the surface energy of the grain boundaries of
163 an individual grain, recrystallization begins in a wave from regions of high
164 strain energy and large gradients in strain energy (Duval et al., 1983; Alley,
165 1992). The grain boundaries of an individual grain migrate outwards to re-
166 duce the lattice strain energy. Recrystallization ceases when the boundary
167 energy of the grain exceeds the lattice strain energy of the grain (Duval and
168 Castelnau, 1995).

169 In this study, we derive a steady-state model and thus we consider the
170 bulk properties of a macroscopic parcel of ice, rather than any localized
171 discontinuities, when determining when migration recrystallization occurs.
172 Since strain must be accumulated to generate dislocations, previous studies
173 have assumed that this criterion is fulfilled for strains larger than 1 – 10%
174 (Duval and Castelnau, 1995). Strains of this magnitude are likely in shear
175 margins of fast-flowing glaciers and we can expect that once ice has deformed
176 sufficiently to warm the ice to -10°C , the ice has achieved strains of 1 – 10%.
177 Thus, here we let temperature be a proxy for strain and assume migration
178 recrystallization occurs for temperatures that exceed approximately -10°C ,
179 as suggested by previous works (Duval, 1981; Duval and Castelnau, 1995).

180 The temperature dependence of recrystallization kinetics are represented
181 by the activation energies. Previous studies have shown that at temperatures
182 above -10°C , the kinetics of creep and grain growth change discontinuously

183 due to the formation of pre-melt film and the proximity to the melting point
 184 (Jacka and Li Jun, 1994; Dash et al., 2006). Here, we set the temperature
 185 dependence of activation energies for creep and grain growth accordingly,
 186 such that temperature plays a significant role in determining which creep
 187 mechanism is dominant.

188 Ice sheet-scale shear stresses drive deformation in lateral shear margins,
 189 which consequently increases the density of dislocations within grains (Figure
 190 1). We can represent the driving force of migration recrystallization as the
 191 difference of energy associated with a dislocation density ρ_d (defined as the
 192 number of dislocations per unit surface area) between neighboring grains,
 193 expressed as (Duval et al., 1983; Derby and Ashby, 1987; Derby, 1992)

$$\Delta E_{\text{dis}} = \frac{1}{2} \mu b^2 \Delta \rho_d \quad (3)$$

194 where μ is the shear modulus and b is the magnitude of the Burger's vector.
 195 We express the change in dislocation density as $\Delta \rho_d \approx (\frac{D}{d})^q \rho_d$, where q is an
 196 exponent to be defined, and D is the characteristic length scale over which
 197 we consider the change in dislocation density. This expression is physically
 198 justified by the fact that the length scale over which we consider changes in
 199 dislocation density is approximately the grain size d (Duval et al., 1983; Alley,
 200 1992). The scaling of grain size by the characteristic length scale D gives
 201 us a term physically comparable to strain. We relate dislocation density to
 202 the applied shear stress τ_s as $\rho_d \approx \frac{\tau_s^2}{\mu^2 b^2}$. This relationship can be understood
 203 theoretically by equating the internal stress from dislocation density ρ_d with
 204 the stress applied to the material (Alley, 1992) and has been derived and
 205 applied in metals and ceramics studies (Duval et al., 1983).

206 Applying these expressions for the change in dislocation density and for
 207 dislocation density to Equation 3, we can find the change in energy associated
 208 with dislocation density, which is the driving force for migration recrystal-
 209 lization (F_{mig}):

$$F_{mig} = \Delta E_{dis} \approx \frac{1}{2} \left(\frac{D}{d} \right)^q \frac{\tau_s^2}{\mu} \quad (4)$$

210 We can find an expression for the change of grain size by considering the
 211 growth rate for grain boundary migration, which is equal to the velocity
 212 of migration, $v = MF_{mig}$, where M is the mobility of the grain boundary
 213 (Duval et al., 1983; Derby and Ashby, 1987; Derby, 1992). The mobility
 214 of grain boundaries is expressed as $M = M_0 \exp \left[-\frac{Q_m}{RT} \right]$, where Q_m is the
 215 activation energy for grain boundary mobility, R is the ideal gas constant,
 216 T is temperature, and M_0 is the intrinsic mobility (Higashi, 1978), defined
 217 here as $M_0 = 0.023 \text{ m}^4 \text{ J}^{-1} \text{ s}^{-1}$ (Llorens et al., 2017). The rate of change in
 218 internal strain energy due to migration recrystallization, \dot{E}_{mig} (Equation 5),
 219 is the time derivative of Equation 4, represented as

$$\dot{E}_{mig} = -\frac{1}{2} \frac{\tau_s^2}{\mu} q \frac{D^q}{d^{q+1}} \dot{d}_{mig} \quad (5)$$

$$\dot{d}_{mig} = MF_{mig} = \frac{1}{2} \frac{\tau_s^2}{\mu} \frac{D^q}{d^q} M \quad (6)$$

220 with the corresponding rate of change in grain size given by Equation 6.

221 2.2. Normal Grain Growth

222 The expression for the increase in grain size from normal grain growth is
 223 well-established and derived from the change in surface energy that occurs

224 due to the migration of a grain boundary (Alley et al., 1986a):

$$d^p = d_0^p + kt \quad (7)$$

225 where p is the grain-growth exponent (to be constrained), d_0 is the initial
 226 grain size, and k is the grain growth rate factor. The grain growth factor
 227 is parameterized by $k = k_0 \exp \left[-\frac{Q_{gg}}{RT} \right]$, where k_0 is an empirical prefactor
 228 and Q_{gg} is the activation energy for normal grain growth (Duval, 1985; Alley
 229 et al., 1986a; Jacka and Li Jun, 1994). The rate of change in grain size due
 230 to normal grain growth \dot{d}_{nor} is the time-derivative of Equation 7.

231 *2.3. Grain-size reduction*

232 Grain-size reduction increases surface energy within a volume of a polycrys-
 233 talline material (Duval and Castelnau, 1995). This change in surface energy
 234 is related to a geometric constant that represents the characteristic shape of
 235 grains, grain size, and grain boundary energy γ (Alley et al., 1986a; Austin
 236 and Evans, 2007). Grain boundary energy γ represents the change in free
 237 energy resultant from a change in area of the grain (Derby and Ashby, 1987),
 238 and laboratory experiments has found the value to be $\gamma = 0.065 \frac{\text{J}}{\text{m}^2}$ (Ketcham
 239 and Hobbs, 1969). From this, the rate of change in internal energy density
 240 to grain-size reduction is given as the change in surface energy, as shown in
 241 Austin and Evans (2007):

$$\dot{E}_{\text{red}} = \frac{-c\gamma}{d^2} \dot{d}_{\text{red}} \quad (8)$$

242 *2.4. Steady-State Grain Size*

243 Grain size evolution is a function of current grain size for all three recrystallization mechanisms. In the case of normal grain growth and migration
 244 tallization mechanisms. In the case of normal grain growth and migration
 245 recrystallization, the exponents p and q respectively govern the rate of grain
 246 growth. We note that both normal grain growth and migration recrystalliza-
 247 tion occur by grain boundary migration. Since both recrystallization pro-
 248 cesses occur by the same process, with different driving forces, the change in
 249 grain size due to migration recrystallization and normal grain growth should
 250 have the same grain-size dependence. To represent this condition and to
 251 derive an expression for the steady-state grain size, we thus assume $q = \frac{p}{2}$.
 252 We then define the expression for steady-state grain size, accounting for the
 253 contribution of all mechanisms to grain size (Equation 1) and the mechanical
 254 work that goes into recrystallization (Equation 2):

$$d_{ss} = \left[\frac{\overbrace{4kp^{-1}c\gamma\mu^2}^{\text{Normal grain growth}} + \overbrace{\tau_s^4 D^p \left(\frac{p}{2}\right) M}^{\text{Migration recrystallization}}}{\underbrace{8(1-\Theta)\tau_s\dot{\epsilon}_s\mu^2}_{\text{Grain-Size Reduction}}} \right]^{\frac{1}{1+p}} \quad (9)$$

255 where $\dot{\epsilon}_s$ is the shear strain rate. The full derivation is found in Supplement
 256 Section A. The numerator consists of both grain growth mechanisms and
 257 the denominator describes the contribution of grain reduction, similar to
 258 relations derived previously (Derby and Ashby, 1987). Without any clear
 259 estimates for Θ , we assume $\Theta \approx 1$, implying that most of the work done
 260 during deformation drives changes in thermal energy that warm the ice, a
 261 common assumption made when studying shear margins of glaciers (Jacobson
 262 and Raymond, 1998; Suckale et al., 2014; Meyer and Minchew, 2018).

263 *2.5. Model Validation*

264 We use GRIP ice core temperature and grain size datasets (Gundestrup
265 et al., 1993; Thorsteinsson et al., 1997; Johnsen et al., 1997) to benchmark our
266 model due to the availability of grain size and temperature data. In bench-
267 marking our model against ice cores, we focus on the lower ~ 500 m of the ice
268 column where we expect vertical shearing to be the dominant component of
269 deformation, as these are conditions that most closely match those of shear
270 margins and it is the region in which migration recrystallization is expected
271 to be most active. Since the parameterizations for normal grain growth and
272 grain-size reduction are well-established (Alley et al., 1986a,b; Austin and
273 Evans, 2007; Behn et al., 2020), the term for migration recrystallization is
274 the main piece of the model that requires benchmarking. Therefore, possi-
275 ble inconsistencies between our model setup and the conditions at shallow
276 depths (< 500 m) in GRIP do not adversely affect the comparison of our
277 model to the data.

278 The depth profile of shear strain rate and shear stress come from a nonzero
279 surface slope α , which drives ice deformation. The region of GRIP is approx-
280 imately 3 – 4 km away from an ice divide, whose position we estimated using
281 a digital elevation model (ArcticDEM; (Porter et al., 2018)) and validated
282 by previous work that used GPS data (Hvidberg et al., 1997). Close to ice
283 divides (less than an ice thickness away from the ice divide; in the case of
284 GRIP, 3 km), the strain rate is dominated by (normal) longitudinal strain,
285 whereas further away from ice divides (more than an ice thickness from the
286 divide), the strain rate becomes dominated by the vertical shear strain rate
287 due to the ice being frozen to the bed (Raymond, 1983; Gundestrup et al.,

288 1993; Hvidberg et al., 1997). Therefore, we take the vertical shear strain rate
 289 to be the dominant component of the strain rate tensor in the lower portion
 290 of the ice column and compute it from temperature and shear stress (Figure
 291 2b). We compute vertical shear stress (taken to be equal to the gravitational
 292 driving stress) for $\alpha = 0.01^\circ$ and $\alpha = 0.05^\circ$, reasonable bounds on the surface
 293 slope in the region of the GRIP ice core (Helm et al., 2014). The grey shading
 294 represents the depth at which the ice has not yet reached steady state (dark
 295 grey for $\alpha = 0.05^\circ$, light grey for $\alpha = 0.01^\circ$), and therefore the models should
 296 not predict the correct grain sizes (Figure 2c). The independence of grain
 297 size model to conditions (temperature, shear strain rate, stress, grain size) at
 298 all other depths (Equation 9) prevents errors at shallower depths that may
 299 be attributable to unmodeled longitudinal strain rates or lack of steady state
 300 from propagating to deeper depths, which are being used to benchmark the
 301 model.

302 Our model is largely consistent with the grain size data from the GRIP ice
 303 core (Figure 2c). Near the bed, migration recrystallization is the dominant
 304 mechanism and thus responsible for the rapid increase in grain size. When
 305 applying our model, which incorporates the contributions of migration re-
 306 crystallization, we see a reasonable fit to the GRIP ice core data near the
 307 bed. The depth at which grains begin to grow is largely dictated by temper-
 308 ature. At temperatures of approximately $-10^\circ C$, grain boundaries become
 309 more mobile, enabling high-velocity grain boundary migration (Duval and
 310 Castelnau, 1995; Urai et al., 1995). This critical temperature T_c at which
 311 this change in activation energy occurs has been experimentally determined.
 312 However, studies have shown that critical temperatures between $-8^\circ C$ and

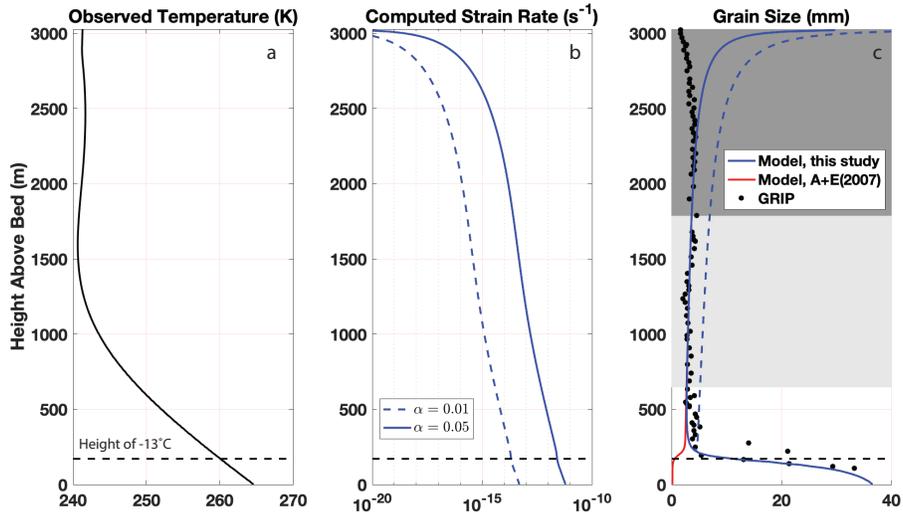


Figure 2: Results of a steady-state grain size model: (a) Temperature measured from the GRIP ice core, (b) strain rate computed from shear stress using the constitutive relation (Glen’s Flow Law) for ice (where the flow-rate parameter is found from temperature by the Arrhenius relation and the flow-law exponent is taken to be $n = 3$ (Jezek et al., 1985) for surface slopes of 0.05° (solid line) and 0.01° (dashed line), (c) grain size computed from the model presented in this study from surface slopes of 0.05° (solid blue line) and 0.01° (dashed blue line), reasonable surface slopes for this region (Helm et al., 2014), the model presented in Austin and Evans (2007) (red line), and measured from the GRIP ice core (black circles). The grey shading represents the depths at which the ice has not yet reached steady-state (dark grey for a surface slope of 0.05° and light grey for a surface slope of 0.01°) and may be contaminated by firn processes. For shear margins, the most relevant areas are those that are in steady state and thus outside the grey shaded boxes (discussed further in Supplement Section D).

313 $-15^{\circ}C$ may apply to natural conditions (Barnes et al., 1971; Goldsby and
 314 Kohlstedt, 2001; Kuiper et al., 2020). We show model estimates of grain size
 315 for a critical temperature of $T_c = -13^{\circ}C$ (Figure 2), to demonstrate that
 316 defining a critical temperature within reasonable bounds of the canonical
 317 value of $-10^{\circ}C$ produces an accurate estimate of the grain size profile. How-
 318 ever, for the remainder of this study, we use the canonical value $T_c = -10^{\circ}C$
 319 for consistency with much of the salient literature referenced here. We show
 320 in Supplement Section B that the model provides a good fit to both GISP2
 321 ice core data and WAIS Divide ice core data as well, showing that the model
 322 is applicable to different ice sheets and different regions.

323 The magnitude of the change in grain size with depth is controlled primar-
 324 ily by two parameters: the characteristic length-scale D and the grain growth
 325 exponent p (Equation 7). These two parameters are poorly constrained in
 326 natural deforming glacier ice. Traditionally, the grain growth exponent is
 327 taken to be $p = 2$ in glacier ice, from a fit to laboratory data and borehole
 328 measurements (Duval, 1985; Alley et al., 1986a,b). Recent work has shown
 329 that this value of the grain growth exponent best fits bubble-free glacier ice
 330 and that bubbled ice more likely has a higher grain growth exponent (Azuma
 331 et al., 2012). Since GRIP ice core is in a slowly-deforming region that is likely
 332 to have a higher concentration of bubbles, we use $p = 9$ for that fit. On the
 333 other hand, we are interested in rapidly-deforming regions that likely have
 334 a low concentration of bubbles, so we use $p = 2$ for the remainder of this
 335 study. We reserve for future work a complete exploration of the effect of
 336 varying grain growth exponents. The characteristic grain size D is uncertain
 337 as well, given that this is a scaling factor and the average grain size can vary

338 widely in different parts of Antarctica. In the Supplement Section C, we
339 show that values of D between 50 mm and 100 mm best represent the ice
340 core data we use here, and we take $D = 50$ mm to approximate the best fit.

341 **3. Model Results in Shear Margins**

342 We first apply this model to a single column of an idealized shear margin
343 in which the strain rate is constant with depth. We compute grain size
344 from three different strain rates, representing a reasonable range of strain
345 rates seen in shear margins of Antarctic ice streams (Alley et al., 2018).
346 We compute ice temperature from strain rate using the thermomechanical
347 model developed by Meyer and Minchew (2018) (Figure 3b) (with vertical
348 accumulation accounted for in the Peclet number, where $Pe = 2$).

349 For a low strain rate ($\dot{\epsilon} = 6 \times 10^{-10} \text{ s}^{-1}$), temperature increases only
350 slightly with depth and thus grain size remains relatively constant with depth.
351 For an intermediate strain rate ($\dot{\epsilon} = 1.3 \times 10^{-9} \text{ s}^{-1}$), comparable to that found
352 in shear margins of most ice streams in Antarctica, temperature increases
353 significantly with depth, reaching the melting temperature approximately
354 100 m from the bed. Grains grow with depth until the critical temperature
355 of -10°C , where there is a decrease in grain sizes due to an increase in the
356 prevalence of grain-size reduction. There is then a rapid growth of grains
357 due to temperatures approaching -10°C , when enough strain energy has
358 built for grain boundaries to migrate through migration recrystallization.
359 Below approximately 500 meters above the bed, grain sizes become roughly
360 constant with depth due to strain rate and temperature increasing enough
361 such that creep and subsequent grain reduction becomes more active and

362 balances the contribution of migration recrystallization. For a high strain rate
 363 ($\dot{\epsilon} = 6 \times 10^{-8} \text{ s}^{-1}$), temperatures increase dramatically, reaching the melting
 364 point approximately 700 m above the bed. The ice remains temperate for the
 365 remainder of the ice column. Due to the dramatic increase in temperature
 366 in the first few hundred meters, grain size increases from ~ 2 mm at the
 367 surface to ~ 13 mm approximately 200 m from the surface. Grain sizes then
 368 remain roughly constant with depth for the remainder of the ice column. The
 369 estimate that grains are large in shear margins and regions where the ice is
 370 warm is supported by observations from Antarctic ice streams (Jackson and
 371 Kamb, 1997) and from temperate glaciers (Tison and Hubbard, 2000).

372 In contrast to our results, studies in the solid earth community have con-
 373 sidered the effect of recrystallization on grain sizes in shear zones and found
 374 that grain size reduces in shear zones due to the dominance of grain-size
 375 reduction in regions with high strain rate (e.g. De Bresser et al. (2001);
 376 Montési and Hirth (2003)). Rocks in deformational zones are often far be-
 377 low their melting temperature, so a temperature increase by shear heating
 378 would have to be much larger than that for ice, which is everywhere close
 379 to its melting temperature. Ice temperatures near the melting point drive
 380 migration recrystallization on Earth, which results in a growth in grains in
 381 shear margins rather than a reduction in grain size.

382 *3.1. Effect of Grain Size on Ice Rheology*

383 Grain size affects the rheology of ice. Typically, ice rheology is described
 384 through a power-law relationship (Glen's flow law), which relates strain rate
 385 to stress raised to a power n , $\dot{\epsilon} = A\tau^n$. The value of n reflects the creep
 386 mechanism that ice deforms by and thus the choice of n in ice-flow model-

387 ing significantly affects the behavior of deforming ice. Uncertainties in the
388 parameters of this flow law contribute significantly to uncertainties in large-
389 scale ice-flow modeling, and constraining values of n is critical to making
390 projections of ice sheet behavior.

391 Values of $n = 3$ are commonly used because this value fits laboratory
392 data for the creep of ice (Jezek et al., 1985). However, a value of $n = 3$ does
393 not clearly match with one creep mechanism. Instead, a flow law exponent
394 of $n \approx 3$ may describe creep by a combination of dislocation creep ($n \approx 4$),
395 which is grain-size-independent, and grain-boundary sliding ($n \approx 2$), which is
396 grain-size-dependent (Montagnat and Duval, 2000; Goldsby and Kohlstedt,
397 2001; Behn et al., 2020). Deformation of ice with large grain sizes generally
398 favors dislocation creep as the dominant deformation mechanism.

399 Dislocation creep occurs through dislocations, line defects in the ice,
400 which enable planes of the ice crystalline lattice to move past each other.
401 Migration recrystallization annihilates dislocations through the migration of
402 grain boundaries, further increasing grain size and producing space for new
403 dislocations to move through, which allows for continued dislocation creep.
404 The rate of creep for grain-size-dependent deformation mechanisms (all ex-
405 cept dislocation creep) is inversely related to grain size, so in ice with large
406 grains, the rate of grain-size-dependent creep is likely to be low. Thus, as
407 grains grow, the flow law tends to a power-law relationship with $n = 4$,
408 describing dislocation creep as the sole creep mechanism.

409 This suggests that in areas of rapid deformation, such as the margins of
410 ice streams, modeling ice flow with a flow-law exponent of $n \approx 4$ (dislocation-
411 creep-dominant flow) may more accurately capture the dynamics occurring

412 as the ice deforms, a result also estimated using satellite observations of ice
 413 shelves (Millstein and Minchew, 2020). In Supplement Section C, we show
 414 these results from our model for varying values of n . The value of n directly
 415 affects the rate of flow of ice, as viscosity scales with strain rate to the power
 416 of $\frac{1-n}{n}$. Thus, a value of $n = 4$ implies a lower viscosity for a given strain
 417 rate, suggesting that models may be overestimating the viscosity of ice in
 418 areas of rapid deformation.

419 *3.2. Effect of grain size on fracture vulnerability*

420 In the absence of pre-existing macro-scale fractures, the size of grains
 421 has a significant effect upon the strength of ice because grain boundaries are
 422 themselves flaws in the ice along which cracks can propagate (Schulson and
 423 Hibler, 1991). Intuitively, an increase in grain size translates to an increase
 424 in the length of grain boundaries, resulting in an increase in vulnerability
 425 to fracture (Figure 3a). Laboratory studies have similarly found that the
 426 tensile strength of ice σ_t , defined as the total stress required to fracture ice
 427 in tension, decreases with increasing grain size according to the following
 428 relationship: (Currier and Schulson, 1982; Schulson et al., 1984; Nixon and
 429 Schulson, 1988)

$$\sigma_t = K d^{-\frac{1}{2}} \quad (10)$$

430 where K is a constant. While this is an empirical relationship, studies have
 431 developed theoretical bases for this relationship. The most prevalent ex-
 432 planation is the dislocation pileup mechanism, which explains deformation
 433 through the pileup of dislocations at the edge of a grain that then induces

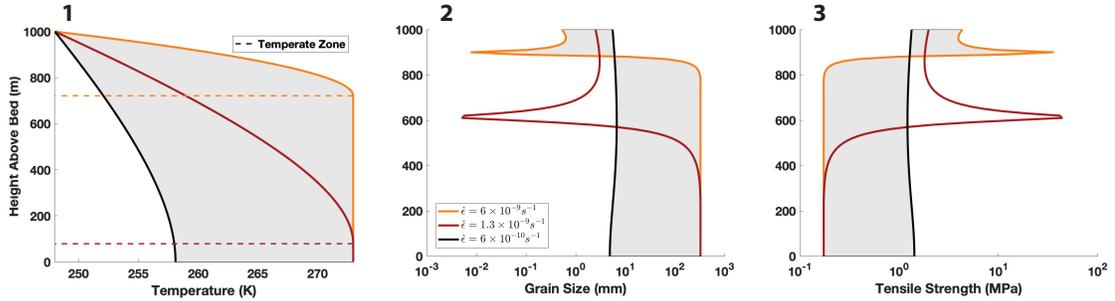


Figure 3: Results from an idealized model showing the relationship between ice temperature, grain size, and tensile strength. (1) Ice temperature computed from the thermomechanical model presented in Meyer and Minchew (2018), (2) Grain size computed from the steady-state grain size model developed here (Equation 9), (3) Tensile strength computed from Equation 10, for 3 strain rates.

434 deformation in a neighboring grain (Li and Chou, 1970). Fractures initiate
 435 to reduce the stress that forms due to this dislocation pileup. The stress
 436 required for this to occur has the same grain size dependence as that in
 437 Equation 10 (Li and Chou, 1970; Schulson et al., 1984).

438 We apply Equation 10 to compute the tensile strength of ice as a function
 439 of grain size (setting $K = 52 \text{ kPa m}^{\frac{1}{2}}$ (Lee and Schulson, 1988)) for the
 440 case of the idealized shear margin (Figure 3b). For a low strain rate, since
 441 grain sizes remain approximately constant with depth, tensile strength also
 442 remains roughly constant with depth and $\sigma_t \approx 1.2 \text{ MPa}$. For an intermediate
 443 strain rate, grain sizes grow between approximately 400 and 600 m above
 444 the bed before reaching a steady-state grain size of approximately 15 mm
 445 and then remaining constant with depth for the remainder of the ice column.
 446 Similarly, tensile strength remains constant until approximately 600 m above
 447 the bed. At this depth, tensile strength increases sharply due to a decrease

448 in grain size, and then tensile strength decreases to approximately 0.4 MPa
449 and remains constant with depth to the bed. At a high strain rate, tensile
450 strength follows a similar pattern as that for intermediate strain rates, though
451 the decrease in tensile strength occurs closer to the surface (~ 900 m height).

452 In locations of ice sheets in which the ice is frozen to the bed, a similar
453 decrease in tensile strength will be likely near the bed due to an increase
454 in grain size caused by migration recrystallization, as seen in the GRIP ice
455 core (Figure 2). However, that decrease in tensile strength would be coupled
456 with an increase in the overburden pressure, preventing tensile fractures from
457 forming. In the case of shear margins, however, we observe a decrease in
458 tensile strength to approximately 25% of the tensile strength a few hundreds
459 of meters below the surface. With relatively low overburden pressure at these
460 depths, this leaves a significant depth of the shear margin vulnerable to the
461 propagation of microcracks along grain boundaries and thus the nucleation
462 of large-scale fractures. Though not explicitly represented in these models,
463 we would expect the water pressure at the base of ice shelves to facilitate the
464 opening of tensile fractures, which renders the deeper portions of the shear
465 margins on ice shelves, where tensile strength is lowest, quite vulnerable to
466 fracture.

467 **4. Application to Pine Island Glacier, West Antarctica**

468 We apply our model to Pine Island Glacier in West Antarctica because
469 of its rapid deformation and potential for large-scale implications for the
470 Antarctic Ice Sheet (Wingham et al., 2009). The yearly velocity of Pine Is-
471 land Glacier is found from LANDSAT 8 satellite imagery (Figure 4b) (Gard-

472 ner et al., 2018), ice thickness is calculated from basal topography from
473 BedMachine (Morlighem et al., 2020), and surface elevation from the Ref-
474 erence Elevation Model of Antarctica (Howat et al., 2019). We use surface
475 mass balance, averaged over the years 1979-2019, from the RACMO model
476 of Antarctica to set the rate of vertical advection in the thermomechanical
477 model (Van Wessem et al., 2014). Results for other outlet glaciers in
478 Antarctica are shown in Supplement Section F.

479 We compute grain size from surface strain rates (calculated from surface
480 velocity; Figure 4b), ice temperature (calculated from surface strain rates),
481 and ice thickness. Grain size is also dependent upon Θ , the fraction of work
482 dissipated as heat. Commonly, it is assumed that all the work done during
483 deformation is dissipated as heat, $\Theta \approx 1$ (Jacobson and Raymond, 1998;
484 Suckale et al., 2014; Meyer and Minchew, 2018). However, the value has not
485 been experimentally or theoretically constrained. Here, we present results
486 for $\Theta \approx 1$ and in the Supplement Section F we present results with $\Theta = 0.5$
487 and $\Theta = 0.25$. The tensile strength of ice is then computed from grain size.
488 We show three slices of the ice column: the grain size and tensile strength
489 at 25% of the ice thickness, at 50% of the ice thickness, and 75% of the ice
490 thickness (Figure 4c).

491 Grains are large in the shear margins of Pine Island Glacier (~ 12 mm)
492 relative to the rest of the glacier and ice core data. This is likely due to high
493 strain rates resulting in elevated ice temperatures (at or near the melting
494 point). Previous studies show extensive zones of temperate ice in the shear
495 margins of Pine Island Glacier (Meyer and Minchew, 2018), and this drives
496 migration recrystallization and increases the size of grains. The depth profile

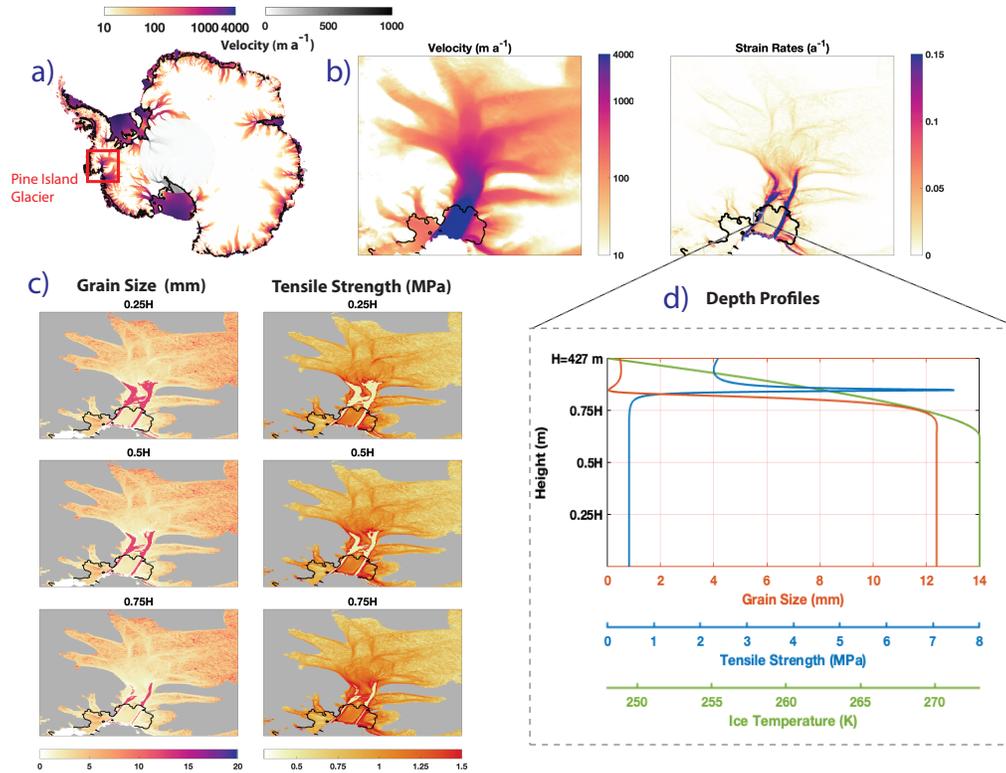


Figure 4: (a) Surface velocity of Antarctica from Landsat 7 and 8 (yellow to purple scale bar) (Gardner et al., 2018), with the pole hole filled in from NASA MEaSUREs (grey scale bar) (Mouginot et al., 2012; Rignot et al., 2017), with the region of Pine Island Glacier outlined in red. (b) Surface velocity and surface strain-rates of Pine Island Glacier. (c) Estimated grain sizes and tensile strength at varying depths: 25% of ice thickness (H) from the bed, 50% of ice thickness from the bed, and 75% of ice thickness from the bed. Areas where the model is not valid (flow speed $< 30 \text{ m a}^{-1}$) are shown in grey. Here we show results for $\Theta \approx 1$, the assumption used in thermomechanical models of ice (Jacobson and Raymond, 1998; Suckale et al., 2014; Meyer and Minchew, 2018). Results using other values of Θ are shown in Supplement Section F. (d) Depth profiles of grain size, tensile strength, and ice temperature for a single point of the shear margin of the Pine Island Glacier ice shelf.

497 largely mirrors that seen in the idealized case (Figure 3b): at the bed, most
498 of the margin contains coarse-grained ice. A similar area of coarse-grained
499 ice exists at 25% of the ice thickness. In the middle of the ice column (50%),
500 the area of coarse-grained ice thins but still spans a significant portion of the
501 margin, especially upstream. Finally, near the surface (75% of ice thickness),
502 the area of large grains thins even more but still dominates the shear margin
503 (Figure 4c,d). In general, as seen in Figure 4d, grain sizes remain constant
504 with depth beyond the region near the surface and therefore the profiles in
505 Figure 4c show the region of grain growth expanding as temperatures increase
506 with depth. The difference between grain size in the margins and grain size in
507 the trunk of the ice stream decreases as Θ decreases (as less work is dissipated
508 as heat). Even at low Θ , grains are still larger in the margins (Supplement
509 Section F). This may imply that, in the margins, dislocation creep is the
510 dominant deformation mechanism and thus modeling the evolution of Pine
511 Island Glacier using $n = 4$ in the margins is most accurate.

512 Large grain sizes in the margins translate to relatively low values of tensile
513 strength. Tensile strength drops from ~ 1.5 MPa in the fine-grained regions
514 to ~ 0.2 MPa in the coarse-grained regions. These values are significantly
515 lower than some estimated tensile strength values for relatively pristine and
516 undeformed ice (Ultee et al., 2020) and within the range of reasonable values
517 found by other studies (Vaughan, 1993). Furthermore, there is a significant
518 portion of the shear margin that has very low tensile strength near the surface
519 (75% of ice thickness). A reduction in tensile strength occurs for low values
520 of Θ as well, though the reduction is not as significant and does not extend as
521 far up the ice column (Supplement Section F). This dramatic drop in tensile

522 strength, particularly near the surface, may increase the vulnerability of the
523 shear margin to fracture and is positioned approximately where significant
524 damage and fracturing in Pine Island Glacier have been observed (Lhermitte
525 et al., 2020). Ice shelves are particularly vulnerable to changes in tensile
526 strength because basal crevasses are more easily formed than in grounded ice
527 due to the fact that the cracks are water-filled. A reduction in the strength of
528 ice at the base of the ice column may increase the vulnerability of ice shelves
529 significantly relative to grounded ice since it allows for cracks to propagate
530 from the base of the ice shelf and may allow for full-thickness fractures to
531 develop. This drop in tensile strength is due to the rate of deformation
532 in shear margins, and so as Pine Island Glacier accelerates in a changing
533 climate, the ice shelf of Pine Island Glacier may become more vulnerable to
534 fracture and calving events (Wingham et al., 2009).

535 **5. Conclusions**

536 In this study, we show that grain sizes in shear margins are large relative
537 to slower deforming regions, which influences the rate of creep and vulner-
538 ability to fracture of the ice and may contribute to accelerated flow and
539 instability of ice shelves. To show this, we derive a new model for steady-
540 state grain size that accounts for migration recrystallization, a mechanism
541 for recrystallization that is dominant at high strain rate and high tempera-
542 ture and results in an increase in grain size. Our model demonstrates that
543 migration recrystallization is dominant in shear margins and thus ice grains
544 in shear margins are large (~ 12 mm), compared to grain sizes of $\sim 2 - 7$ mm
545 in surrounding regions. This is a significant deviation from previous work in

546 solid earth recrystallization studies that have shown shear zones of rock to
547 be fine-grained. This distinction arises because ice in terrestrial glaciers and
548 ice sheets is close to its melting temperature and thus migration recrystal-
549 lization can outpace grain-size reduction, resulting in coarse grains in shear
550 zones. We show here that this result has implications for the vulnerability
551 of shear margins to fracture and the rheology of ice in shear margins.

552 The flow of ice is described by a constitutive relation that relates strain
553 rate and stress through a power law, with a flow exponent n . The value of $n =$
554 3 has been found to match laboratory data and is commonly used in ice sheet
555 and ice flow models. However, we suggest here that in shear margins where
556 grain sizes are large, dislocation creep ($n = 4$) is likely to be the dominant
557 deformation mechanism, since large grain sizes give more area for slip to
558 occur through dislocations and large grain sizes also reduce the rates of creep
559 by mechanisms such as grain-boundary sliding and diffusion creep. Thus, a
560 flow law exponent of $n \approx 4$ may be more appropriate than the commonly-
561 used $n = 3$ for rapidly-deforming regions of ice streams, such as the lateral
562 margins. This may imply that, by using the traditional Glen's flow law
563 with $n = 3$ in large-scale ice flow models, we are underestimating the rate of
564 creep, and consequently the acceleration of flow, in key regions of Antarctica.
565 While we do not directly model the effects of dynamic recrystallization on
566 fabric development here, including fabric is likely to strengthen this result
567 due to the creation of a single-maximum fabric that softens the ice and allows
568 for higher rates of deformation. Further, it is well known that an increase
569 in grain size reduces the strength of polycrystalline materials. Here, we
570 show that the tensile strength of ice in shear margins of Pine Island Glacier,

571 West Antarctica are approximately 25% of the tensile strength of ice in the
572 centerline of the glacier. This decrease in tensile strength may give rise to
573 damage and fracture that previous studies have identified in Pine Island
574 Glacier (Lhermitte et al., 2020). Further, this model produces predictions of
575 grain size that can be tested by observations of grain size in shear margins.

576 This new understanding of recrystallization in shear zones may provide
577 a way to estimate more accurately the vulnerability of rapidly deforming
578 glaciers to instability by parameterizing the effect of dynamic recrystalliza-
579 tion processes in large-scale ice flow models. This work provides inroads
580 into thinking about how to represent different types of flow in large-scale ice
581 flow models with a spatially varying flow exponent n . Finally, this work sug-
582 gests that dynamic recrystallization processes significantly affect the physical
583 properties and dynamics of rapidly-deforming glaciers, and further work will
584 consider the role that dynamic recrystallization and grain-scale processes play
585 in the large-scale dynamics and energetics of shear margins.

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595 code for the model and that generates the figures in this paper can be found
596 at: <https://github.com/megr090/grain-size-tensile-strength-model>.

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