Probabilistic soil moisture dynamics of water- and energy-limited ecosystems

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Abstract

This paper presents an extension of the stochastic ecohydrological model for soil moisture dynamics at a point of Rodríguez-Iturbe et al. (1999) and Laio et al. (2001). In the original model, evapotranspiration is a function of soil moisture and vegetation parameters, so that the model is suitable for water-limited environments. Our extension introduces a dependence on maximum evapotranspiration of available solar radiation, and thus our extended model is suitable for both water- and energy-limited environments. Furthermore, an analysis of the daily relationship between available energy for photosynthesis and transpiration through the stomatal conductance is carried out. This study regards the Penman-Monteith equation to model transpiration, the Leuning’s stomatal conductance approach, the $C_3$ photosynthesis model of Farquhar et al., and the FLUXNET database. Results are upscaled from half-hourly to daily scale, introducing an expression of transpiration in terms of the available radiation. The sensitivity of the model is analyzed using four dimensionless groups, and the long-term water balance is evaluated for distinct values of available energy.

Keywords:
Transpiration, photosynthesis, PAR, stomatal conductance, radiation

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1. Introduction

The soil water content ($s$) is a key player in the climate-soil-vegetation system (Entekhabi and Brubaker, 1995; Porporato and Rodríguez-iturbe, 2002; Rodríguez-Iturbe and Porporato, 2004). This system involves many variables and processes with high spatial and temporal variability, feedbacks and non-linear relations. Furthermore, soil moisture depends critically on the physiological characteristics of vegetation, pedology and climate (Entekhabi and Rodríguez-Iturbe, 1994; Rodríguez-Iturbe et al., 1999; Rodriguez-Iturbe et al., 2001). Climate and weather patterns determine the amount of water and energy available, crucially impacting the evapotranspiration process (Leuning, 1995; De Pury and Farquhar, 1997; Stoy et al., 2009; Manzoni et al., 2011). Soil texture, its mineralogical composition, and the particle size distribution determine the storage capacity of the soil. Vegetation controls the energy and water fluxes, linking the soil and the atmosphere (Feddes et al., 2001; Rodriguez-Iturbe et al., 2001).

Climate, soil, and vegetation are related through physical, chemical and biological processes, which lead to the mass and energy transport between land and atmosphere (Eagleson, 1978). Actual evapotranspiration couples water and energy balances. There are two evapotranspiration ($ET$) regimes related to soil moisture, $s$: an energy-limited regime and a water-limited regime. Between these two regimes, there are seasonal environments, in which the availability of water and energy fluctuates.

Among the approaches to modeling soil moisture are biophysical process-based, physical-based and statistical models (Wang et al., 2019). These models mostly feed on in-situ (e.g. Korres et al., 2015; Noh et al., 2015; Pirone et al., 2015; Gevaert et al., 2018) and remote sensing (e.g. Wagner et al., 1999; Kim and Barros, 2002; Fang and Lakshmi, 2014; Zehe et al., 2018) data or involve numerical simulations (e.g. Mtundu and Koch, 1987; Brubaker, 1995; Brubaker and Entekhabi, 1996; Albertson and Montaldo, 2003; Ridolfi et al., 2003; Rigon et al., 2006; Margulis and Entekhabi, 2001; Sela et al., 2012; Chen et al., 2017; de Assunção et al., 2018). In-situ data are not easy to extrapolate to spatial scales that allow hydrological applications, remote sensing methods measure continuous spatiotemporal information but only comprise the most superficial centimeters of the soil (Niemann, 2004), and numerical simulations do not permit to generalize the results (Ogren, 1993). Daly and Porporato (2005), Seneviratne et al. (2010), Asbjørnsen et al. (2011), Legates et al. (2011) and Wang et al. (2019) present some complete reviews of the
state of the art of soil moisture modeling.

Eagleson (1978), Cordova and Bras (1981), Hosking and Clarke (1990), and Milly (1993) initiate a biophysical based approach that comprises simplified but realistic conceptual models that analytically describe the phenomena taking place in the climate-soil-vegetation system. This approach involves stochastic components that take into account the randomness of precipitation and the inherent variability of soil and vegetation properties. Some models have been developed following this approach (e.g. Rodríguez-Iturbe et al., 1999; D’Odorico et al., 2000; Laio et al., 2001; Milly, 2001; Laio et al., 2002; Porporato et al., 2003; D’Odorico and Porporato, 2004; Daly and Porporato, 2006; De Michele et al., 2008; Laio et al., 2009), modeling precipitation as a stochastic process and deriving analytical expressions of soil moisture dynamics from the soil, climate and vegetation parameters. These models have been developed for arid and semi-arid environments, characterized by scarce rainfall, low soil moisture, recurrent water stress, and deep water table (Laio et al., 2009). Since the available energy is not directly considered, they are not suitable to be applied in energy-limited environments.

Photosynthetically active radiation (PAR) is the energy source of biophysical processes, such as photosynthesis, stomatal conductance, transpiration, evaporation, leaf temperature, plant growth, seedling generation, biochemical cycling, and atmospheric chemistry (Thorpe et al., 1978; Baldocchi and Meyers, 1991; Baldocchi and Collineau, 1994; Ballaré, 1994; Hansen, 1999; Yu et al., 2004; Daly et al., 2004; Ge et al., 2011), which are directly or indirectly related to \( s \). On the other hand, the stomata movement regulates simultaneously the water and CO\(_2\) fluxes during transpiration and photosynthesis (Collatz et al., 1991; Yu et al., 2004; Medlyn et al., 2017; Shan et al., 2019), being necessary to model photosynthesis and transpiration coupled with the stomatal conductance (\( g_s \)).

In this study, we propose an extension of the model by Rodríguez-Iturbe et al. (1999) and Laio et al. (2001) towards the representation of the stochastic behavior of soil moisture in both water- and energy-limited environments. The moisture loss model proposed by Laio et al. (2001) is modified in such a way that actual \( ET \) becomes a function of soil moisture and available radiation. Then, we analyze the relations of transpiration (\( T \)) and available radiation, and transpiration and soil moisture when radiation is the limiting variable. Stomatal conductance is modeled using the Leuning’s approach (Leuning, 1990, 1995), and transpiration using the Penman-Monteith equation. Net assimilation of CO\(_2\) (\( A_n \)) is determined with the Farquhar model
Table 1: Parameters for the stomatal and transpiration models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>18</td>
<td>Eq. 3</td>
</tr>
<tr>
<td>$c_a$ [µmol mol$^{-1}$]</td>
<td>350</td>
<td>Atmospheric CO$_2$ concentration</td>
</tr>
<tr>
<td>$c_p$ [J kg$^{-1}$ K$^{-1}$]</td>
<td>1013</td>
<td>Specific heat of air</td>
</tr>
<tr>
<td>$D_x$ [Pa]</td>
<td>300</td>
<td>Eq. 3</td>
</tr>
<tr>
<td>$e$</td>
<td>0.622</td>
<td>Ratio molecular weight of water vapour/dry air</td>
</tr>
<tr>
<td>$g_a$ [mm s$^{-1}$]</td>
<td>20</td>
<td>Atmospheric conductance</td>
</tr>
<tr>
<td>$g_b$ [mm s$^{-1}$]</td>
<td>20</td>
<td>Leaf boundary layer conductance</td>
</tr>
<tr>
<td>LAI [m m$^{-1}$]</td>
<td>1.4</td>
<td>Leaf area index</td>
</tr>
<tr>
<td>$\lambda_w$ [J kg$^{-1}$]</td>
<td>$2.26 \times 10^6$</td>
<td>Latent heat of water vaporization</td>
</tr>
<tr>
<td>$\rho_a$ [kg m$^{-3}$]</td>
<td>1.2</td>
<td>Air density</td>
</tr>
<tr>
<td>$\rho_w$ [kg m$^{-3}$]</td>
<td>997</td>
<td>Water density</td>
</tr>
</tbody>
</table>

and information from the FLUXNET database. The dependence of $g_s$ and $T$ on available PAR is integrated at the daily level, relating $T$ and PAR through a simple expression. Finally, we analyze the sensitivity of the probability density distribution (pdf) to the available energy and the long-term water balance.

2. Data

Half hourly resolution data of air temperature ($T_a$), atmospheric pressure ($P_a$), vapor pressure deficit ($\Delta_e$), photosynthetic photon flux density (PPFD), net ecosystem CO$_2$ exchange (NEE), CO$_2$ air concentration, and soil moisture in 28 sites around the world are taken from the FLUXNET dataset (Baldocchi et al., 2001; Olson et al., 2004). NEE data contain positive values during the day (assimilation), and negative values during the night (respiratory loss) (Drake and Read, 1981), therefore the positive values of these series are used as $A_n$. Table 1 shows the parameters for applying Penman-Monteith and Leuning equations, and Table 2 those for applying the Farquhar model. These values are the same published by Daly et al. (2004).

3. Transpiration dynamics

The major components in the earth’s hydrological cycle are transpiration and evaporation. Their analysis and understanding are fundamental
Table 2: Parameters for the C₃ photosynthesis model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{Kc}$ [J mol⁻¹]</td>
<td>59430</td>
<td>Activation energy for $K_c$</td>
</tr>
<tr>
<td>$H_{Ko}$ [J mol⁻¹]</td>
<td>36000</td>
<td>Activation energy for $K_o$</td>
</tr>
<tr>
<td>$H_{vV}$ [J mol⁻¹]</td>
<td>116300</td>
<td>Activation energy for $V_{c,max}$</td>
</tr>
<tr>
<td>$H_{dV}$ [J mol⁻¹]</td>
<td>202900</td>
<td>Deactivation energy for $V_{c,max}$</td>
</tr>
<tr>
<td>$H_{vJ}$ [J mol⁻¹]</td>
<td>79500</td>
<td>Activation energy for $J_{max}$</td>
</tr>
<tr>
<td>$H_{dJ}$ [J mol⁻¹]</td>
<td>201000</td>
<td>Deactivation energy for $J_{max}$</td>
</tr>
<tr>
<td>$J_{max0}$ [µmol m⁻² s⁻¹]</td>
<td>$2 \times V_{c,max0}$</td>
<td>Eq. A.5 (Kattge and Knorr, 2007)</td>
</tr>
<tr>
<td>$K_{c0}$ [µmol mol⁻¹]</td>
<td>302</td>
<td>Michaelis constant for CO₂ at $T_0$</td>
</tr>
<tr>
<td>$K_{o0}$ [µmol mol⁻¹]</td>
<td>256</td>
<td>Michaelis constant for O₂ at $T_0$</td>
</tr>
<tr>
<td>$o_i$ [mol mol⁻¹]</td>
<td>0.209</td>
<td>Oxygen concentration</td>
</tr>
<tr>
<td>$R_g$ [J mol⁻¹ K⁻¹]</td>
<td>8.31</td>
<td>Universal gas constant</td>
</tr>
<tr>
<td>$S_v$ [J mol⁻¹ K⁻¹]</td>
<td>650</td>
<td>Entropy term</td>
</tr>
<tr>
<td>$T_0$ [K]</td>
<td>293.2</td>
<td>Reference temperature</td>
</tr>
<tr>
<td>$V_{c,max0}$ [µmol m⁻² s⁻¹]</td>
<td>50</td>
<td>Eq. A.3</td>
</tr>
<tr>
<td>$\gamma_0$ [µmol mol⁻¹]</td>
<td>34.6</td>
<td>CO₂ compensation point at $T_0$</td>
</tr>
<tr>
<td>$\gamma_1$ K⁻¹</td>
<td>0.0451</td>
<td>Eq. 4</td>
</tr>
<tr>
<td>$\gamma_2$ K⁻²</td>
<td>0.000347</td>
<td>Eq. 4</td>
</tr>
</tbody>
</table>
in applications associated with biogeochemical cycles, nutrient losses, salt
accumulations of soil, production efficiency, etc. (Schulze et al., 1995). Trans-
spiration couples water and carbon cycles (Miner et al., 2017; Shan et al.,
2019), while evapotranspiration couples water and land-surface energy bal-
ances (Fisher et al., 2009; Seneviratne et al., 2010; Zhang et al., 2016). These
links are driven by vegetation, climate, and soil, existing a close dependence
between atmosphere and vegetation. The sensible and latent heat fluxes from
vegetation cause changes in the atmosphere state and, at the same time, veg-
etation responds to changes in air temperature and humidity (Monteith and
Unsworth, 2013). Vegetation closes its stomata in the absence of light or wa-
ter in the soil so that both radiation and soil moisture are variables directly
related to transpiration (Monteith, 1995).

Although transpiration \( T \) responds to a wide variety of complex envi-
ronmental and physiological factors (Cowan and Farquhar, 1977; Tuzet et al.,
2003), here it is assumed that \( T \) can be limited by three factors: soil water,
energy, and vegetation capacity (physiology) (see Fig. 1). The maximum
rate at which vegetation can transpire when it has no external limitations
depends on the maximum stomatal conductance, which is directly propor-
tional to pore width (Larcher, 1995). This rate is \( T_{\text{maxmax}} \) and is represented
by the red line in Fig. 1. The left panel of Fig. 1 shows the relationship
of transpiration rate and available radiation \( (R) \) when there are no water
limitations (green line). This relationship is direct until a value of \( R \) where
transpiration ceases increasing. This dependence is analyzed in detail in sec-
tion 3.3. The right panel in Fig. 1 indicates the relationship of transpiration
and soil moisture. The dark blue line shows the transpiration rate when
it is limited by soil moisture and vegetation physiology, but not by energy.
Transpiration is maximum for values of \( s \) greater than the incipient stomata
closure \( (s^*) \) \( (T \) is equal to \( T_{\text{maxmax}}) \). For values lower than \( s^* \), \( T \) begins to
decrease because vegetation closes its stomata to avoid internal losses of wa-
ter. Transpiration continues to reduce until the wilting point \( (s_w) \) where it
becomes zero. When considering both water and energy limitations, energy
limits transpiration for values above \( s^* \) (see the plateau of the right graph in
Fig. 1), while soil moisture limits for values below \( s^* \) (Petersen et al., 1992).

High values of available energy \( (R) \) result in higher maximum transpira-
tion rates \( (T_{\text{max}}) \). For example, as shown by the light blue lines in Fig. 1,
a high available energy value \( (R_1) \) derives in a higher transpiration rate for
\( s > s^* \) \( (T_{\text{max}1}) \) than a low available energy value \( (R_2) \) that results in a lower
value of transpiration \( (T_{\text{max}2}) \). In this case, both \( T_{\text{max}1} \) and \( T_{\text{max}2} \) are lower
Figure 1: Limitations of transpiration. Left (right) graph illustrates the dependence of transpiration on available energy (water). $T_{\text{maxmax}}$ indicates the maximum transpiration rate when there are no external limitations.

than $T_{\text{maxmax}}$, therefore, the plateaus of both light blue lines are determined by the available radiation. Energy also influences the response of the plant to water stress (Petersen et al., 1991, 1992). The rate of water loss is proportional to the water vapor concentration gradient within the vegetation and the bulk atmosphere (Pallardy, 2008), and high radiation values result in high vapor-pressure deficit in the air. When there is much energy in the atmosphere, the vegetation must react more drastically to the water stress ($s < s^*$), because it can lose water at a high rate (see the steeper light blue line 1 from $s^*$ to $s_w$ in the right panel of Fig. 1). Vegetation begins to rapidly close their stomata as soil moisture decreases, reducing its transpiration from $T_{\text{max}1}$ when $s > s^*$ to zero when $s < s_w$. On the other hand, when energy demand is low ($R_2$), vegetation can also suffer water stress, but its reaction may be slighter (Kaufmann, 1976), as shown in the light blue line 2 with $T_{\text{max}}$ equal to $T_{\text{max}2}$.

3.1. Water-limited ecosystems

The water-limited regime occurs when $ET$ is very sensitive to $s$. This regime is associated with arid and semi-arid ecosystems (Budyko, 1974; Eagleson, 1982; Seneviratne et al., 2010). Water restricts $ET$ by its scarcity, intermittency, and unpredictability (Porporato and Rodríguez-iturbe, 2002), and photosynthesis is controlled by soil moisture (Porporato and Rodríguez-iturbe, 2002; Daly et al., 2004).

When soil moisture decreases, vegetation reduces its stomata aperture avoiding changes in its internal water status (Cowan and Farquhar, 1977;
Lhomme, 2001). Stomata close as a response to a signal from the roots when the soil is dry before leaf wilting (Schulze, 1986). This phenomenon is known as vegetation water stress and occurs because vegetation needs an adequate level of humidity in their tissues to growth and survival (Davies et al., 1990; Lhomme, 2001). The description and effects of water stress are widely explained by Schulze (1986); Davies et al. (1990); Flexas and Medrano (2002); Chaves et al. (2003); Xu et al. (2010); Tardieu et al. (2018), among others. Laio et al. (2001) proposed a transpiration model as a function of soil moisture for arid and semi-arid regions. In this model, there are no energy limitations, and it is expressed as:

\[
T(s) = \begin{cases} 
0, & 0 < s \leq s_w \\
\frac{T_{\text{max}}(s<s_w)}{s-w}, & s_w < s \leq s^* \\
T_{\text{max}}, & s^* < s \leq 1.
\end{cases}
\]  

(1)

The term \(T_{\text{max}}\) represents the maximum evapotranspiration for the vegetation in the presence of unlimited water and energy. When \(s < s^*\), \(T\) is assumed to decrease linearly because of the limitations of soil moisture until it reaches the wilting point, \(s_w\). Below \(s_w\) transpiration ceases. The right panel of Fig.1 represents the behavior of transpiration as modeled by Eq. 1.

3.2. Energy-limited and seasonal ecosystems

The energy-limited regime occurs when soil moisture is most of the time greater than a critical value, with \(ET\) weakly dependent on \(s\) (Budyko, 1974; Seneviratne et al., 2010). This regime is associated with wet ecosystems. Light limits by its high spatiotemporal variability, that is related to structural and environmental heterogeneity (gapping and clumping of foliage, gaps in the canopy, leaf orientation, type and distribution of clouds, topography, seasonal trends in plant phenology, and seasonality movements of the sun) (Baldocchi and Collineau, 1994).

Radiation in the spectral band of photosynthetically active radiation (PAR) directly drives the fundamental plant physiological processes involving in transpiration, i.e., photosynthesis, stomatal conductance, and leaf temperature. Besides, it indirectly influences secondary processes such as plant growth, seedling generation, structure, and gas emission (Monteith, 1965; Baldocchi and Meyers, 1991; De Pury and Farquhar, 1997).

Transpiration and photosynthesis are processes taking place simultaneously since vegetation loses water through transpiration when take up \(\text{CO}_2\) to photosynthesis (Daly et al., 2004; Yu et al., 2004). Photosynthetic rate is
a function of irradiance, \(\text{CO}_2\) concentration, temperature, nutrient and, water supply (Luoma, 1997). However, under well-watered conditions, PAR is one of the major environmental factors controlling photosynthesis, stomatal conductance, and consequently, transpiration, in a great number of species (Kaufmann, 1976; Schulze et al., 1995; Mielke et al., 1999; Gao et al., 2002).

Stomatal conductance and transpiration increase with PAR (Gao et al., 2002; Pieruschka et al., 2010), as shown in the left graph of Fig. 1. This can be explained by the proportionality between the potassium cation concentration in guard cells and PAR. An increase in the potassium cation concentration causes a decreasing in the osmotic potential of guard cells, resulting in additional water leaves epidermal cells and enter guard cells. This provokes great turgor pressure inside guards and reduces turgor on subsidiary cells so that the vegetation opens its stomata, rising thus its conductance and transpiration (Cooke et al., 1976; Gao et al., 2002; Yu et al., 2004). In seasonal ecosystems, the availability of water and energy fluctuates, and vegetation can present unique adaptations and effects on the hydrological cycle that differ from water and energy limited ecosystems (Asbjornsen et al., 2011).

The expression of transpiration of Laio et al. (2001) manages to describe the daily \(ET\) dynamics in energy-limited and seasonal ecosystems provided that \(E_{\text{max}}\) is defined taking into account the available energy, and stationarity is maintained both in the parameters that describe rainfall and radiation. Fig. 2 represents transpiration as a function of soil moisture and available energy \((T(s,R))\) for a particular set of parameter values. The Penman-Monteith equation is used to relate radiation and \(T_{\text{max}}\), varying radiation from 0 to 18 MJ m\(^{-2}\) (for a fixed stomatal conductance). This figure shows that when the available radiation is high, the rate at which transpiration decreases with \(s\) is much steeper than when radiation is low, representing the response of vegetation to atmospheric demand. We notice that for \(R_n = 0\) there is still minimal evapotranspiration due to the non-zero value of the adiabatic term.

### 3.3. Transpiration and available energy

Available energy affects transpiration, stomatal aperture and photosynthesis through light receptors driving \(\text{CO}_2\) fixation and lower intercellular \(\text{CO}_2\) concentration (Yu et al., 2004), and determining the diabatic component of transpiration (Monteith and Unsworth, 2013). Hence, to properly study the effects of radiation on transpiration \((T)\), the relations among carbon assimilation \((A_n)\), stomatal conductance \((g_s)\) and transpiration must be
Figure 2: Transpiration as a function of soil moisture and available radiation according to the Penman-Monteith equation and Laio et al. (2001) model. Each horizontal line represents an available radiation value. The parameters used in this figure are $Z_r = 90 \text{ cm}$, $\lambda = 0.1 \text{ d}^{-1}$, $\alpha = 1.5 \text{ cm}$, $\Delta = 0 \text{ cm}$, $E_w = 0.05 \text{ cm d}^{-1}$, $E_{\text{max}} = 0.43 \text{ cm d}^{-1}$, $s_b = 0.1$, $s_w = 0.24$, $s^* = 0.57$, $T_{\text{min}} = 17.1^\circ C$, $T_{\text{max}} = 28.1^\circ C$, $r_a = 20.76 \text{ s m}^{-1}$, $r_c = 69.4 \text{ s m}^{-1}$ and $G = 0 \text{ MJ m}^{-2}$.
taken into account. For this, the Penman-Monteith equation, the Leuning’s
stomatal conductance model, the Farquhar model, and a simplified energy
balance model are solved numerically and simultaneously. This solution is
at a half-hourly scale since the information from the FLUXNET database
has this resolution, but as this analysis is carried out to use the Laio et al.
(2001) model, these results are integrated on the daily scale. Bartlett et al.
(2014), Daly et al. (2004) and Leuning et al. (1995) present methodologies
to solve simultaneously stomatal conductance, CO₂ assimilation, and energy
balance.

Penman-Monteith equation (Monteith, 1965; Monteith and Unsworth,
2013) is adopted because it is widely used in hydrology, and relates tran-
spiration and stomatal conductance. It is expressed as:

\[
T = \frac{(\rho_a c_p D g_{ba} + \Delta_e R) g_s LAI}{\rho_w \lambda_v [\Delta_e g_s LAI + \gamma_p (g_{ba} + g_s LAI)]},
\]  

(2)

where \(\lambda_v\) is the latent heat of vaporization (2.26 MJ kg\(^{-1}\)), \(\rho_w\) and \(\rho_a\) are the water (998.2 kg m\(^{-3}\)) and air (1.2 kg m\(^{-3}\)) densities, respectively, \(c_p\) is the specific heat of air (1.013 \(\cdot\) 10\(^{-3}\) MJ kg\(^{-1}\) K\(^{-1}\)), \(\Delta_e\) is the slope of
saturation of vapor pressure, \(\gamma_p\) is the psychometric constant, \(D\) is the satu-
ration vapor pressure deficit, LAI is the leaf area index, and \(g_{ba}\) is the series
of leaf boundary conductance \((g_b)\) and atmospheric boundary layer conduc-
tance \((g_a)\). Both \(g_a\) and \(g_b\) are assumed to be constant. The first term in
Eq. 2 is the adiabatic component which accounts for the atmospheric satu-
ration deficit, and the second term is the diabatic component of latent heat
loss, related to radiation supply. According to the Penman-Monteith equa-
tion, \(T\) increases linearly with \(R\) and with the atmospheric saturation deficit.
As \(g_{ba}\) is strongly related to wind speed, when it increases, \(T\) also increases,
and when variables in the numerator remain constant, \(\Delta_e\) increases with
temperature.

3.3.1. Stomatal conductance

Stomatal conductance \((g_s)\) can be calculated using physiological and bio-
chemical models (e.g. Jarvis, 1976; Farquhar et al., 1980; Ball et al., 1987;
Farquhar, 1989; Collatz et al., 1991; Leuning, 1995; Gao et al., 2002; Dewar,
2002; Tuzet et al., 2003; Yu et al., 2004). The models most widely used are
those based on Jarvis (1976) (e.g. Baldocchi and Meyers, 1991; Peters-Lidard
et al., 1997; Daly et al., 2004; Yu et al., 2004) and Ball et al. (1987) (e.g.
Leuning, 1990, 1995; Leuning et al., 1995; Daly et al., 2004) approaches.
Net assimilation and transpiration are processes coupled with the stomatal aperture. Therefore, a stomatal conductance model that relates transpiration to net assimilation is necessary to analyze the dynamics of transpiration. For this purpose, we use the semi-empirical formulation given by Ball et al. (1987) and improved by Leuning (1990, 1995), expressed as:

\[ g_s = 1.6a_1 \frac{A_n}{(c_s - \Gamma^*) \left(1 + \frac{D}{D_x}\right)}. \]  

(3)

This equation gives \( g_s \) in terms of carbon assimilation (\( A_n \)), water vapor saturation deficit (\( D \)), CO\(_2\) compensation point (\( \Gamma^* \)), carbon concentration at the leaf surface (\( c_s \)), a fitted parameter representing the sensitivity of stomata to changes in \( D \) (\( D_x \)), and an empirical constant with a typical value around 15 (\( a_1 \)). The CO\(_2\) compensation point is the CO\(_2\) concentration at which the CO\(_2\) uptake rate in the photosynthesis equals the CO\(_2\) loss rate of respiration (Birmingham and Colman, 1979). \( \Gamma^* \) is significantly affected by leaf temperature, and according to Brooks and Farquhar (1985), they can be related by:

\[ \Gamma^* = \gamma_0 + \left[1 + \gamma_0 (T_l - T_0) + \gamma_2 (T_l - T_0)^2\right], \]

where \( \gamma_0, \gamma_1 \) and \( \gamma_2 \) are empirical constants, \( T_0 \) is the reference temperature, and \( T_l \) is the leaf temperature.

3.3.2. Energy balance

Since when solving Eqs. 2 and 3 there are three unknowns (\( T, g_s \) and \( T_l \)), it is mandatory to couple another equation that allows solving the system, in this case the energy balance equation:

\[ T_l = T_a + \frac{R - \rho_w \lambda_w T}{c_p \rho_a g_a}. \]  

(5)

3.3.3. Net carbon assimilation

The Farquhar model (Farquhar, 1973; Cowan and Farquhar, 1977; Farquhar et al., 1980) is applied to calculate \( A_n \) in sites where there are no measurements of it. This is the most frequently used model to quantify the responses of C\(_3\) plants to external perturbations under well-watered conditions. The biochemical demand for CO\(_2\) is determined as a function of the photosynthetic photon flux density (\( Q \)), CO\(_2\) concentration in the mesophyll cytosol (\( c_i \)) and leaf temperature (\( T_l \)), and expressed as:
where $A_c$ and $A_q$ are the photosynthesis rates limited by the Ribulose bisphosphate carboxylase-oxygenase (Rubisco) activity, and by the Ribulose bisphosphate (RuP$_2$) regeneration through electron transport, respectively (see Appendix A for more details).

### 3.3.4. Upscaling from half-hourly to daily timescale

The results obtained with the models of transpiration, stomatal conductance, and net assimilation have the temporal resolution of FLUXNET data, i.e., half-hour. To evaluate the daily dynamics of transpiration, we integrate both the calculated results and the information from the FLUXNET database at this time scale. The daily values of $s$, $T$ and $g_s$ correspond to the average during the day, while $PAR$ and $A_n$ are the cumulative sub-daily values.

### 4. Soil moisture dynamics

Rodríguez-Iturbe et al. (1999) proposed a daily stochastic zero-dimensional model for soil moisture dynamics at a point in terms of climate-soil-vegetation interactions, under seasonally fixed conditions. The stochastic behavior of rainfall propagates through interception, evapotranspiration, runoff, leakage and soil moisture. Rainfall is modeled as a marked Poisson process that generates infiltration into the soil as a function on the existing soil water content until it reaches saturation. Soil water losses are due to evapotranspiration and leakage, which also depend on the soil moisture state. Soil moisture dynamics is the result of the water mass balance over the plant’s rooting depth, expressed by the stochastic differential equation:

$$nZ_r \frac{ds(t)}{dt} = \varphi[s(t), t] - \chi[s(t), R(t)],$$

where $n$ is the soil porosity, $Z_r$ is the rooting depth, $s$ is the soil water content, $R$ is the available radiation, $\varphi[s(t), t]$ is the infiltration rate, and $\chi[s(t), R(t)]$ is the soil moisture loss rate.

Infiltration is a stochastic component, expressed as:

$$\varphi[s(t), t] = P(t) - I(t) - Q[s(t), t],$$

where $P(t)$ is the rainfall rate, $I(t)$ is the rainfall rate intercepted by the canopy, and $Q[s(t), t]$ is the rate of surface runoff generation.
Soil water losses are evaporation, transpiration and, leakage, thus the total water loss rate ($\chi$) is given by:

$$\chi [s(t), R(t)] = ET [s(t), R(t)] + L [s(t)],$$

(9)

where $ET [s(t), R(t)]$ and $L [s(t)]$ are the evapotranspiration and leakage rates, respectively.

$ET$ is modeled as the sum of evaporation ($E$) and transpiration ($T$). $E$ is a fixed rate equal to $E_w$ when $s_w \leq s \leq 1$, which decreases from $s_w$ until it reaches the hygroscopic point ($s_h$), where it becomes zero. Transpiration is modeled as Eq. 1, being $ET$ given by:

$$ET (s) = \begin{cases} 
0, & 0 < s \leq s_h \\
E_w \frac{s-s_h}{s_w-s_h}, & s_h < s \leq s_w \\
E_w + (E_{\text{max}} - E_w) \frac{s-s_w}{s^*-s_w}, & s_w < s \leq s^* \\
E_{\text{max}}, & s^* < s \leq 1.
\end{cases}$$

(10)

$E_{\text{max}}$ is equal to $T_{\text{max}} + E_w$. AppendixB describes the modeling of the other variables in Eqs. 8 and 9.

Following Rodríguez-Iturbe et al. (1999) and Laio et al. (2001), the probability density function (pdf) of soil moisture under steady-state conditions may be derived from the Chapman-Kolmogorov forward equation. The general form of the solution is:

$$p(s) = \frac{C}{\rho (s, R_n)} e^{-\gamma s + \lambda' \int_{s_h}^s \frac{ds}{\rho(s)}}, \text{ for } s \geq s_h,$$

(11)

where $\lambda'$ is the mean time between rainy days, and $C$ is a constant that can be obtained by imposing the normalized condition $\int_{s_h}^1 p(s) ds = 1$. This constant is easily obtained numerically, although its analytical expressions are given in Laio et al. (2001) and Rodríguez-Iturbe and Porporato (2004). Details of the derivation of $p(s)$ can be found in Rodríguez-Iturbe and Porporato (2004); Laio et al. (2001); and Rodríguez-Iturbe et al. (1999). The general solution is:
\[ p(s) = \begin{cases} 
\frac{C}{\eta_w} \left( \frac{s-s_h}{s_w-s_h} \right) T_1 e^{-\gamma s} & \text{for } s_h < s \leq s_w \\
\frac{C}{\eta_w} \left[ 1 + \left( \frac{\eta}{\eta_w} - 1 \right) \frac{s-s_w}{s^*-s_w} \right] T_2 e^{-\gamma s} & \text{for } s_w < s \leq s_{cr} \\
\frac{C}{\eta} e^{-\gamma s + \frac{\lambda}{\eta}(s-s^*)} \left( \frac{\eta}{\eta_w} \right)^T_2 & \text{for } s_{cr} < s \leq s_{fc} \\
\left( \frac{\eta}{\eta_w} \right) T_2^{-1} e^{T_4} & \text{for } s_{fc} < s \leq 1,
\end{cases}\]

where

\[ T_1 = \lambda \frac{s_w-s_h}{\eta_w}, \quad T_2 = \lambda^* \frac{s^*-s_w}{\eta-\eta_w}, \quad T_3 = \frac{\lambda^*}{\beta (\eta-m)}, \quad T_4 = \frac{\lambda^* s_{fc} - s^*}{\eta}\]

\[ \eta_w = \frac{E_w}{nZ_r}, \quad \eta = \frac{E_{max}}{nZ_r}, \quad m = \frac{K_s}{nZ_r \left[ e^{\beta(1-s_{fc})} - 1 \right]}\]

As mentioned before, the transpiration model of Laio et al. (2001) manages to describe the daily \( T \) dynamics in energy-limited ecosystems. Consequently, Eq. 10 manages to represent the evapotranspiration dynamics, and Eq. 12 the dynamics of soil moisture. This is proper as long as \( T_{max} \) (or \( E_{max} \)) is defined as a function of the available energy, and the stationarity of the parameters describing rainfall and radiation is preserved. It is noted that considerations in the model of Rodríguez-Iturbe et al. (1999) must continue to be taken into account, e.g., deep water table, soil homogeneity, distribution of infiltration volume into the rooting depth, etc. Interactions between vegetation and water table are not considered. This is a realistic assumption for water-controlled arid and semi-arid ecosystems, but may be a questionable one for energy-limited ecosystems. In the latter case, there may exist a close interaction between transpiration and the water table level (Tamea et al., 2009), but this may or not may impact heavily the pdf of soil moisture in systems that are both water- and energy-limited.

5. Daily dynamics

Fig. 3 shows the relationship between available energy and \( \text{CO}_2 \) assimilation, and available energy and the stomatal conductance in two sites,
Figure 3: Relationship between daily PAR and CO$_2$ assimilation (left panel) and daily PAR and stomatal conductance (right panel) at (a,b) an extratropical site in Germany and (c,d) a tropical site in French Guiana.

one located in the extratropics (Germany) and other in the tropics (French Guiana). In the extratropics (Fig. 3(a,b)), the relationships of PAR and $A_n$, and PAR and $g_s$ are positive for low values of PAR ($\approx 4$ MJ m$^{-2}$) and negative for high values. The photo-inhibition phenomenon, that occurs under strong light since it can destroy the plant tissues, can explain the above. This phenomenon involves the direct diversion of the superfluous radiation energy from the photosystems via fluorescence, and above as heat (Larcher, 1995). Nonetheless, at sites in tropics (see Fig. 3(c-d)), the relationships of PAR with $g_s$ and $A_n$ seem more random, which can be explained by the adaptation and the strategies developed by the plants at sites where they usually receive high radiation. We recalled that the PAR values analyzed correspond to those reaching the ground surface, and not those absorbed by the plant.

Fig. 4 shows the relationship between PAR and transpiration at the same sites in Fig. 3. In both types of ecosystems the relationship is direct since when PAR increases, both adiabatic and diabatic terms of Penman-Monteith increase. Radiation affects temperature, and this, in turn, modifies the vapor
saturation deficit. Furthermore, if there is available energy, the stomata open up as they can fix more CO\textsubscript{2}, leading to the plant loses water. However, as shown in Fig. 3, the relation of PAR and \(g_s\) is not always direct, but \(g_s\) stabilizes (light-saturated plateau) at a point (Lambers et al., 2008), and may even decrease. The effect of light-saturation is also observed on \(T\), but not that of the photo-inhibition, at least for the values of PAR measured at the sites studied.

Since transpiration is modeled using measured data, many factors may be limiting \(A_n\), and consequently \(g_s\) and \(T\), so a link between PAR and \(T\) must involve the envelope of simulated points relating these variables (see Fig. 4). For most sites, the envelope fits well to the expression:

\[
T_{\text{max}} (\text{PAR}) = T^* \left( 1 - e^{-a\text{PAR}} \right). \tag{13}
\]

This expression is a function of the maximum possible value of transpiration (light-saturation) (\(T^*\)) and a fitting parameter that determines the shape of the curve (\(a\)). This relationship avoids considering the indirect effects of radiation in transpiration (\(g_s, T_a, D\), etc.). Fig. 4 shows the PAR-\(T\) curves (black lines) and their expressions in the sites in Germany and French Guiana. From Eq. 13 and considering the transpiration rate given by the vegetation physiology (\(T_{\text{maxmax}}\), \(T_{\text{max}}\) can be defined as:

\[
T_{\text{max}} (R) = \begin{cases} 
T^* \left( 1 - e^{-aR} \right), & T_{\text{max}} (R) < T_{\text{maxmax}} \\
T_{\text{maxmax}}, & T_{\text{max}} (R) \geq T_{\text{maxmax}}. 
\end{cases} \tag{14}
\]

We noticed that available energy is considered as a constant since its stochasticity at the daily scale does not play a fundamental role in soil mois-
ture dynamics under the assumptions of the Rodríguez-Iturbe et al. (1999) model, as shown by Muñoz (2019).

6. Analysis of sensitivity

Fig. 5 shows the response of soil water dynamics to PAR when other parameters of the Rodríguez-Iturbe et al. (1999) and Laio et al. (2001) model vary following the dimensionless groups:

\[
\pi_1 = \frac{E_{max}}{\alpha \lambda}, \quad \pi_2 = \frac{nZ_r}{\alpha}, \quad \pi_3 = \frac{k_s}{\alpha \lambda}, \quad \pi_4 = \frac{k_s}{E_{max}}.
\] (15)

These dimensionless groups are used because they simplify the interpretation and visualization of the results (Bridgman, 1922; Barenblatt, 1996; Gorokhovski and Hosseinipour, 1997; Butterfield, 1999; Barenblatt and Isaakovich, 2003). The sensitivity of the model output to each parameter is evaluated by moving the input parameter within an appropriate range and keeping the other parameters fixed. \( \pi_1 \) and \( \pi_2 \) groups have been adopted in previous works to analyze the soil moisture response to rainfall forcing, soil and vegetation changes (e.g. Li, 2014; Feng et al., 2012; Daly and Porporato, 2006; Porporato et al., 2004; Rodríguez-Iturbe and Porporato, 2004; Guswa et al., 2002; Milly, 2001; Rodríguez-Iturbe et al., 1999; Milly, 1993). \( \pi_1 \) is the dryness index of Budyko (1974) and represents the ratio between the maximum evapotranspiration rate and the long-term mean rainfall rate. \( \pi_2 \) is called the storage index and is the ratio between the amount of water that can be stored in the soil (until the rooting depth) and the long-term mean rainfall depth (Feng et al., 2012). \( \pi_3 \) and \( \pi_4 \) are proposed by Guswa et al. (2002). \( \pi_3 \) is the runoff index and relates the saturated hydraulic conductivity coefficient and the long-term mean rainfall rate and, \( \pi_4 \) is the infiltration index, relating the saturated hydraulic conductivity and the maximum evapotranspiration rate.

For this analysis, we consider a loamy sand soil and a grass cover with the parameters in the caption of Fig. 5, where are the results of the four dimensionless groups are shown. In this, each color corresponds to a value of \( \pi \), solid lines represent a low value of PAR (3 MJ m\(^{-2}\)), and dotted lines a high value (15 MJ m\(^{-2}\)). Fig. 5(a) shows the pdf of \( s (f(s)) \) for \( \pi_1 \) values between 0.1 and 1.4. As the value of \( \pi_1 \) increases, \( f(s) \) moves to the left. Higher \( \pi_1 \) results in lower soil moisture values in the long-term, since the losses due to evapotranspiration are greater than soil water gains due to
rainfall. High values of available energy result in lower modes and greater
dispersion than low PAR values. Fig. 5(b) shows $f(s)$ for $\pi_2$ varying between
4 and 20, since natural ecosystems tend to have root zones deep enough to
result in values of $\pi_2$ larger than 1.0 (Milly, 2001). The higher the value of $\pi_2$,
the lower the soil moisture. For large values of $nZ_r$, characteristic of plants
with deeper roots such as trees, the amount of rainfall reaching the soil is
distributed into a larger volume (according to the model), resulting in smaller
increases in $s$. For lower values of $nZ_r$, rainfall is uniformly distributed in a
smaller volume, increasing soil moisture rapidly. Very high and very low $\pi_2$
values occur when soil storage capacity is much larger or smaller than the
rainfall amount, respectively. High PAR changes the dynamics of $s$, notably
for high values of $\pi$ related to large soil water storage or very small rainfall.
Fig. 5(c) shows the results for $\pi_3$ values varying between 50 and 400. As the
runoff index increases, the water moves rapidly out of the soil, decreasing
$s$. As for $\pi_2$, the differences in available energy give very different dynamics
of soil moisture for $\pi_3$, especially for high values of it, occurring when the
amount of water flowing out the soil is much greater than the rainfall rate.
Fig. 5(d) shows $f(s)$ for $\pi_4$ values between 100 and 1000. For low values
of $\pi_4$, $s$ remains high because water losses are minor. For high values of $\pi_4$
(greater than 550), the mode of the pdfs stabilizes near the field capacity
point, changing only its frequency, and consequently, the dispersion. When
$k_s$ is much larger than $E_{max}$, soil loses water by leakage at a very high rate,
being the evapotranspiration and its variability less relevant. High values of
PAR result in curves more pulled to the left than low values of PAR.

If the available energy is high (dotted lines), the curves of $p(s)$ for all $\pi$
groups move more rapidly to the left than when it is low (solid lines), since
vegetation transpires at higher rates, maintaining soil moisture lower. The
sensitivity of $s$ is more noticeable for $\pi$ values related to lower soil moisture
because the demand of energy in the atmosphere changes the rate at which
vegetation decreases its transpiration when it is under water stress. The
dimensionless groups that consider $E_{max}$ ($\pi_1$ and $\pi_4$) show less sensitivity
to PAR and the modes always a minor frequency for high available energy.
The other dimensionless groups ($\pi_2$ and $\pi_3$) show a more noticeable variation
with PAR, completely changing the dynamics of $s$ for some $\pi$ values (e.g.,
$\pi_2=16$ and $\pi_3=225$). Furthermore, the mode has a high (low) frequency for
low values of PAR when it is greater (lower) than $s^*$, decreasing (increasing)
the dispersion.
Figure 5: Dimensionless sensitivity analysis of soil water dynamics conditioned by available energy. Parameters in this figure are $\alpha=2$ cm, $\lambda=0.5$ d$^{-1}$, $\Delta=0$ cm, $Z_r=30$ cm, $T_{max}=0.47$ cm d$^{-1}$, $a=0.384$ m$^2$ MJ$^{-1}$, $b=4.48$, $\beta=12.7$, $n=0.42$, $k_s=100$ cm d$^{-1}$, $s_h=0.08$, $s_w=0.10$, $s^*=0.24$, and $s_{fc}=0.52$. 
7. Water balance

Fig. 6 shows the behavior of the components of the water balance normalized by the average rainfall rate for a loamy sand soil. The expression of each component can be consulted in Laio et al. (2001) and Rodríguez-Iturbe and Porporato (2004). Figs. 6 (a,b) show the influence of rainfall events frequency ($\lambda$) for PAR equal to 3 and 15 MJ m$^{-2}$, respectively. In both cases, the fraction of intercepted water ($I$) is constant and equal, since it changes in proportion to the rainfall rate. The percentage of runoff ($Q$) increases with $\lambda$ in a similar proportion for both cases. The fraction of water transpired under stressed conditions ($E_s$) decreases rapidly until $\lambda \approx 0.3$ d$^{-1}$ for PAR=3 MJ m$^{-2}$ and until $\lambda \approx 0.5$ d$^{-1}$ for PAR=15 MJ m$^{-2}$, being in the first case much lower. The same behavior is observed in the fraction of water transpired under non-stressed conditions ($E_n$). When PAR is low, the percentage of leakage is higher than when PAR is high, and the percentage of evapotranspired water is significantly lower. This suggests that more water reaching the soil is lost by evapotranspiration in water-limited regions than in energy-limited regions (for these parameter values), becoming $Q$ and $L$ more important in energy-limited ecosystems. These results are in agreement with field observations and results found in previous studies (e.g. Sala et al., 1992; Entekhabi and Rodríguez-Iturbe, 1994; Golubev et al., 2001; Rodríguez-Iturbe and Porporato, 2004; Robock and Li, 2006; Roderick et al., 2009).

Figs. 6(c,d) show the behavior of the water balance when $\lambda$ and $\alpha$ are varied while maintaining constant the total amount of precipitation during a season $\Theta$ ($\Theta = \alpha \cdot \lambda \cdot nd$, being $nd$ the number of days of the growing season) for PAR equal to 3 and 15 MJ m$^{-2}$, respectively. For this figure $\Theta = 60$ cm and $nd = 200$ d. Interception increases almost linearly with $\lambda$ while runoff decreases rapidly. According to Laio et al. (2001), this decreasing depends strongly on the ratio between soil depth and mean depth of rainfall events. The opposite behavior of interception and runoff determines a maximum of evapotranspiration at certain values of $\lambda$. As when only $\lambda$ is varied, the main difference in the behavior of the water balance components for high and low PAR is observed in the percentage of evapotranspiration, being remarkably lower in the first case.
Figure 6: Examples of the behavior of the components of the water balance normalized by the total rainfall $\langle P \rangle$ for loamy sand soil, grass vegetation, and (a,c) PAR=3 MJ m$^{-2}$ and (b,d) PAR=15 MJ m$^{-2}$. The parameters are shown in caption of Fig. 5.
8. Conclusions

In this paper, we have presented an analysis of transpiration as a function of available soil water and energy, extending the model of Rodríguez-Iturbe et al. (1999) and Laio et al. (2001), originally introduced to represent the pdf of soil moisture dynamics at a point in water-limited ecosystems, to the general case of ecosystems ranging from arid (water-limited) to humid (energy-limited). This model manages to describe the stochastic behavior of soil water content in environments limited by both energy and water, since evapotranspiration is expressed as a function of soil moisture and net radiation. This extension is valid as long as the $E_{\text{max}}$ parameter is calculated taking into account the available energy, the parameters of both rainfall and radiation are stationarity, and considerations of the water-limited model are preserved, such as a deep water table, stationarity, homogeneous soil, and vegetation, etc.

We also analyzed the daily relationship of transpiration and photosynthetic active radiation by coupling the water and CO$_2$ fluxes through the leaf. As transpiration is directly related to the stomatal conductance, the relation between PAR and $T$ is positive until a certain point where transpiration ceases to increase. We proposed an expression to parameterize the link between these two variables. This expression allows calculating the daily maximum transpiration rate from the value of daily available energy.

Several examples are presented exhibiting the influence of radiation on $s$, noticing that the available energy can notoriously change the soil moisture dynamics, and that evapotranspiration plays a more important role in water-limited than in energy-limited ecosystems. We note that these results are only valid on a daily scale since soil-climate-vegetation system dynamics change in more detailed temporal scales.

Appendix A. Assimilation model for C$_3$ plants

The photosynthesis rates limited by the Ribulose bisphosphate carboxylase-oxygenase (Rubisco) activity ($A_c$), and by the Ribulose bisphosphate (RuP$_2$) regeneration through electron transport ($A_q$) are given by:

$$A_c = V_{c,max} (T_i) \frac{c_i - \Gamma^*}{c_i + K_c (1 + o_i/K_o)}, \quad (A.1)$$

$$A_q = \frac{J}{4} \frac{c_i - \Gamma^*}{c_i - 2\Gamma^*}, \quad (A.2)$$
where $\Gamma^*$ is the CO$_2$ compensation point (see Eq. 4), $o_i$ is the intercellular oxygen concentration, $V_{c,max}$ is the maximum catalytic activity of Rubisco in the presence of saturating levels of RuP$_2$ and CO$_2$ (Eq. A.3), and $K_c$ and $K_o$ are Michaelis coefficients for CO$_2$ and O$_2$, respectively, given by Eq. A.4.

$$V_{c,max} (T_l) = V_{c,max_0} \frac{\exp \left[ \frac{H_{vJ}}{R_g T_0} \left( 1 - \frac{T_0}{T_l} \right) \right]}{1 + \exp \left[ \frac{S_T T_l - H_{vJ}}{R_g T_l} \right]},$$  \hspace{1cm} (A.3)

$$K_x (T_l) = K_{x_0} \exp \left[ \frac{H_{Kx}}{R_g T_0} \left( 1 - \frac{T_0}{T_l} \right) \right].$$  \hspace{1cm} (A.4)

$J$ is the electron transport for a given absorbed photon irradiance, and is equal to min $[J_{max} (T_l), Q]$, being $J_{max}$ equal to:

$$J_{max} (T_l) = J_{max_0} \frac{\exp \left[ \frac{H_{sd}}{R_g T_0} \left( 1 - \frac{T_0}{T_l} \right) \right]}{1 + \exp \left[ \frac{S_T T_l - H_{sd}}{R_g T_l} \right]}.$$  \hspace{1cm} (A.5)

The parameters not mentioned here are described in Table 2.

Appendix B. Soil moisture model

The variables involved in Eq. 7, except the evapotranspiration (see Eq. 10 in section 4), are modeled as Rodríguez-Iturbe et al. (1999) and Laio et al. (2001).

Appendix B.1. Rainfall and interception

Daily precipitation is modeled through a marked Poisson process with arrival rate $\lambda$ (Eagleson, 1972). The pdf of time intervals between rainy days $\tau$ is exponential with mean $\lambda^{-1}$:

$$f_T (\tau) = \lambda e^{-\lambda \tau}, \text{ for } \tau \geq 0.$$  \hspace{1cm} (B.1)

The marks correspond to the rainfall depth of rainy days, $h$, modeled as an independent exponentially distributed random variable with mean $\alpha$

$$f_H (h) = \frac{1}{\alpha} e^{-\frac{1}{\alpha} h}, \text{ for } h \geq 0.$$  \hspace{1cm} (B.2)
The values of $\alpha$ and $\lambda$ are assumed to be time-invariant quantities during the modeling period (growing season or climate season), i.e. rainfall is considered as a stationary stochastic process.

Rainfall rate is linked to the probability distributions expressed by Eqs. B.1 and B.2 as the marked Poisson process (Rodríguez-Iturbe and Porporato, 2004):

$$P(t) = \sum_1 \delta(t - t_i), \quad (B.3)$$

where $\delta(\cdot)$ is the Dirac delta function, $h_i$ is the sequence of random rainfall depths distributed as eqn. B.2 and $[\tau_i = t_i - t_{i-1}, i = 1,2,3...]$ is the interarrival time sequence of a stationary Poisson process of frequency $\lambda$.

Following Rodríguez-Iturbe et al. (1999), interception is modeled through a threshold, $\Delta$, such that only rainfall above $\Delta$ reaches the soil. The censored rainfall process is thus Poissonian with rate $\lambda'$:

$$\lambda' = \lambda \int_{\Delta}^\infty f_H(h)\,dh = \lambda e^{-\Delta}. \quad (B.4)$$

The depths $h'$ of the censored rainfall process have the same exponential distribution as the original marks $h$ (Rodríguez-Iturbe et al., 1999). Then, the new Poisson process is:

$$P(t) - I(t) = \sum_1 h'_i \delta(t - t'_i), \quad (B.5)$$

where $[\tau'_i = t'_i - t'_{i-1}, i = 1,2,3...]$ is the interarrival time sequence of a stationary Poisson process with frequency $\lambda'$.

Appendix B.2. Infiltration and runoff

Surface runoff is generated via saturation excess (Dunne mechanism) that occurs when the infiltrated water saturates the soil profile. When rainfall depth is less than or equal to the available soil water storage, all the water from rainfall infiltrates. Infiltration is thus a function of the amount of rainfall and soil moisture, being a stochastic and state-dependent component. Its magnitude and temporal occurrence are controlled by soil moisture dynamics (Rodríguez-Iturbe and Porporato, 2004). The probability distribution of the infiltration may then be written as (Rodríguez-Iturbe et al., 1999):
\begin{equation}
    f_Y(y, s) = \gamma e^{-\gamma y} + \delta (y - 1 - s) \int_{1-s}^{\infty} \gamma e^{-\gamma u} du, \text{ for } 0 \leq y \leq 1 - s,
\end{equation}

where \( \gamma = \frac{nZr}{\alpha} \) and \( y \) is the dimensionless infiltration normalized by \( nZr \).

Infiltration from rainfall can be written as:

\begin{equation}
    \varphi[s(t), t] = nZr \sum_{i} y_i \delta(t - t_i'),
\end{equation}

where \([y_i, i = 1, 2, 3, ...]\) is the sequence of random infiltration events whose distribution is represented by Eq. B.6.

Appendix B.3. Leakage

Losses by leakage or deep infiltration, \( L \), occur when soil water content is higher than field capacity, \( s_{fc} \). The maximum percolation rate equals the saturated hydraulic conductivity, \( K_s \), and decreases rapidly when the soil begins to dry, as expressed by (Laio et al., 2001):

\begin{equation}
    L(s) = K(s) = \frac{K_s}{e^{\beta(1-s_{fc})} - 1} \left[e^{\beta(s-s_{fc})} - 1 \right], \text{ for } s_{fc} < s \leq 1.
\end{equation}

Appendix B.4. Soil-drying process

During no-rain periods, soil moisture decays are deterministically modeled from initial values that depend on the the previous history of the entire soil-drying-wetting process. The soil moisture losses normalized by \( nZr \) are:

\begin{equation}
    \rho(s, R_n) = \frac{\chi(s, R_n)}{nZr} = \frac{E(s, R_n) + L(s)}{nZr} = \begin{cases}
    0, & 0 < s \leq s_h \\
    \eta w \frac{s-s_h}{s_w-s_h}, & s_h < s \leq s_w \\
    \eta w + (\eta - \eta w) \frac{s-s_w}{s^*-s_w}, & s_w < s \leq s^* \\
    \eta, & s^* < s \leq s_{fc} \\
    \eta + m \left[e^{\beta(s-s_{fc})} - 1 \right], & s_{fc} < s \leq 1.
\end{cases}
\end{equation}


