

The influence of spreading rate and permeability on melt focusing beneath mid-ocean ridges

Shi J. Sim^{a,d}, Marc Spiegelman^{b,c}, Dave R. Stegman^a, Cian Wilson^d

^a*Institute for Geophysics and Planetary Physics, Scripps Institution of Oceanography, University of California San Diego, La Jolla, California, USA*

^b*Department of Earth and Environmental Sciences, Columbia University, New York, New York, USA*

^c*Applied Physics and Applied Mathematics, Columbia University, New York, New York, USA*

^d*Department of Terrestrial Magnetism, Carnegie Institution for Science, Washington, District of Columbia, USA*

Abstract

At mid-ocean ridges, oceanic crust is emplaced in a narrow neo-volcanic region on the seafloor, whereas basaltic melt that forms this oceanic crust is generated in a wide region beneath as suggested by a few geophysical surveys. The combined observations suggest that melt generated in a wide region at depths has to be transported horizontally to a small region at the surface. We present results from a suite of two-phase models applied to the mid-ocean ridges, varying half-spreading rate and intrinsic mantle permeability using new openly available models, with the goal of understanding melt focusing beneath mid-ocean ridges and its relevance to the lithosphere-asthenosphere boundary (LAB). Three distinct melt focusing mechanisms are recognized in these models: 1) melting pressure focusing, 2) decompaction layers and 3) ridge suction, of which the first two play dominant roles in focusing melt. All

Email address: ssim@carnegiescience.edu ()

three of these mechanisms exist in the fundamental two phase flow formulation but the manifestation depends largely on the choice of rheological model. The models also show that regardless of spreading rates, the amount of melt and melt transport patterns are sensitive to changes in intrinsic permeability, K_0 . In these models, the LAB is delineated by the melt-rich decompaction layers, which are essentially defined by the temperature dependent rheological and freezing boundaries. Geophysical observations place the LAB at a steeper incline as compared to the gentler profile suggested by most of our models. The models suggest that one way to reconcile this discrepancy is to have stronger melting pressure focusing mechanism as it is the only mechanism in these models that can focus melt before reaching the typical model thermal LAB. The apparent lack of observable decompaction layers in the geophysical observations hints at the possibility that melting pressure focusing could be significant. These models help improve our understanding of melt focusing beneath mid-ocean ridges and could provide new constraints for mantle rheology and permeability.

Keywords: two-phase flow, mid-ocean ridges, geodynamics, melt transport in the mantle, melt focusing, lithosphere asthenosphere boundary

1. Introduction

Mid-ocean ridges are a prominent feature of plate tectonics that run for more than 60,000 km on the ocean floor. Tectonic forces pull oceanic plates apart at ridges, allowing mantle to upwell and generate melt due to decompression melting. New oceanic crust is created at a narrow neo-volcanic region a few kilometers across, accounting for 60% of global magmatism

7 (Macdonald, 1982; Vera et al., 1990; Carbotte et al., 2016). Geophysical
8 observations at specific mid-ocean ridges suggest that there are regions of
9 partially molten mantle a few hundred kilometers wide at ~ 50 km depths
10 (Forsyth et al., 1998b,a; Key et al., 2013). Without any external forcing,
11 buoyant melt will rise vertically due to the density contrast with the back-
12 ground mantle. The combined observations of a wide region of melt gener-
13 ation and narrow volcanic zone suggest that mechanisms have to be present
14 to move melt horizontally (or simply melt focusing).

15 Several processes have been proposed to explain melt focusing, of which
16 ridge suction and decompaction layers are well established. ‘Ridge suction’ is
17 the melt flow driven by a reduction in dynamic pressure due to incompress-
18 ible shear near the ridge axis which pulls melt towards the ridge axis from the
19 surrounding region (Spiegelman and McKenzie, 1987; Morgan, 1987). This
20 pressure gradient becomes strong when the solid mantle is assumed to be
21 highly viscous. For melt focusing by decompaction layers (Sparks and Par-
22 mentier, 1991; Spiegelman, 1993c; Hebert and Montési, 2010; Keller et al.,
23 2017), buoyant melt is generated in a wide melting region and travels upwards
24 until it encounters an impermeable barrier formed by the thermal boundary
25 layer of the cold and stronger lithosphere. Compaction pressure opens up
26 pore space beneath this impermeable barrier, where melt can accumulate
27 and pond. The melt in these decompaction layers or high porosity “chan-
28 nels” then travels along the slope of the lithosphere towards the ridge axis.
29 Melting pressure focusing is a recently recognized mechanism (Turner et al.,
30 2017), in which compaction pressure focuses melt if bulk viscosity and melt-
31 ing rates are sufficiently large in quasi-steady state. This study focuses on

32 these three melt focusing mechanisms. Other focusing mechanisms have also
33 been proposed that we do not consider. Anisotropic permeability generated
34 by dike propagation (Morgan, 1987) could focus melt, although the mode of
35 dike propagation proposed would emplace melt away from the ridge axis. A
36 related mechanism to dike propagation is the formation of shear driven melt
37 bands from localization instabilities that align melt bands in accordance to
38 the shear direction (Holtzman et al., 2003b,a; Katz et al., 2006; Kohlstedt
39 and Holtzman, 2009).

40 In this study, we not only highlight and explain the three melt focus-
41 ing mechanisms as laid out above, but also demonstrate the robustness of
42 the melting pressure mechanism without any complexity of grain size evolu-
43 tion as included in Turner et al. (2017). Studies of melting and melt trans-
44 port at mid-ocean ridges have used one-dimensional column modeling (Ribe,
45 1985; Asimow and Stolper, 1999; Hewitt and Fowler, 2008) whereas two-
46 dimensional modeling efforts (including work presented here) show that hor-
47 izontal melt transport is inherent to the magma migration formulation in the
48 mid-ocean ridge setting (Buck and Su, 1989; Spiegelman, 1993c; Hebert and
49 Montési, 2010; Keller et al., 2017). Recently, other two dimensional models
50 use similar two phase formulation employing reactive transport (Keller and
51 Katz, 2016; Keller et al., 2017) to understand volatile distribution and incor-
52 porating grain size evolution to explore melt focusing (Turner et al., 2017).
53 Both melting pressure focusing and decompaction layers will not be present
54 in models that exclude pressure gradients due to viscous solid deformation
55 (e.g. Buck and Su (1989); Scott and Stevenson (1984)).

56 To investigate melt focusing mechanisms and its relation to the lithosphere-

57 asthenosphere boundary (LAB), we present a suite of new open-source two-
58 dimensional two-phase flow models and explore the primary controls of spread-
59 ing rate, U_0 , and intrinsic permeability, K_0 . These models are built using
60 TerraFERMA, the Transparent Finite Element Rapid Model Assembler (Wil-
61 son et al., 2017), and solves the melt migration equations that were derived
62 independently by several workers (McKenzie, 1984; Fowler, 1985; Scott and
63 Stevenson, 1986). The spreading rate is a fundamental observation at mid-
64 ocean ridges. The permeability of the solid mantle depends on grain size and
65 is coupled with the porosity, melt transport and length scale at which the
66 mantle promotes/resists melt transport. Larger permeability promotes faster
67 segregation of melt and therefore smaller melt retention, and vice versa. The
68 retention of melt affects the bulk viscosity, which alters the strength of the
69 melting pressure focusing mechanism, such that melting pressure focusing
70 will be weaker with more melt retained. Our new models include thermal
71 evolution, melting that evolves depending on pressure, temperature and non-
72 linear solid rheology that depends on temperature and strain-rate as com-
73 pared to recent modeling efforts with reactive transport, Newtonian rheology
74 and constant grain size (Keller et al., 2017) or with grain size evolution and
75 non-Newtonian rheology (Turner et al., 2017). We also explore how different
76 rheological choices for shear viscosity affects both decompaction layers and
77 melting pressure focusing effect. Both TerraFERMA and the model descrip-
78 tion files are open source and available at [terraferma.github.io](https://github.com/terraferma).

79 The LAB can be defined multiple ways, e.g. as a thermal, rheological,
80 permeability and melt-rich boundary etc (Fischer et al., 2010). The LAB
81 is not always clearly defined by geophysical observations. Seismic velocities

82 are sensitive to several physical properties such as temperature, melt, grain
83 size and density etc (Kawakatsu and Utada, 2017). Similarly, electromag-
84 netic methods are sensitive to temperature, hydration, melt, oxygen fugacity
85 and chemical composition (Kawakatsu and Utada, 2017). Although geophys-
86 ical inversions are often non-unique, even when the velocity or conductivity
87 structures are known exactly, the interpretation can lead to tradeoffs be-
88 tween the physical properties. Nevertheless, geophysical observations pro-
89 vide good constraints when combined with independent information. This is
90 where modeling can come in handy to understand the underlying processes
91 and properties from a different perspective. If the LAB is delineated by
92 a melt-no melt boundary, then there are some disagreements between ob-
93 servations and models about the location of the LAB at mid-ocean ridges.
94 In two-phase flow models, both rheological strengthening with temperature-
95 dependent rheology and crystallization can lead to a melt-rich decompaction
96 layer that follows the predicted shallow thermal LAB. Both the steep-sided
97 conductive partial melt region shown by electromagnetic survey at the East
98 Pacific Rise (Key et al., 2013), and the deeper than expected large seismic
99 attenuating region within 50 km of the ridge axis beneath the Juan de Fuca
100 Ridge (Eilon and Abers, 2017; Ruan et al., 2018) suggest that melt focusing
101 has to occur before melt can reach the shallower thermal LAB or the melt-
102 rich decompaction layers in our models. Melting pressure focusing is the only
103 mechanism so far that can focus melt before melt reaches the shallow LAB.

104 Section 2 describes the model set up, laying out the one-way coupled
105 non-dimensional form of the magma migration equations along with closure
106 equations (rheology, interphase exchange and permeability), the boundary

107 and initial conditions chosen, with all the parameters and variables given in
108 Tables 1 and 2. Section 3 describes the general evolution of the models and
109 first order trends such as porosity or melt fraction and melt transport with
110 respect to our variables, half spreading rates and intrinsic permeability. We
111 predict crustal thickness using proxies from the model outputs and show how
112 they fit with crustal thickness from geophysical observations. The discussion
113 section goes through the three recognized melt focusing mechanisms in terms
114 of the associated pressures. We also present models with different rheological
115 model choices for shear viscosity to illustrate melting pressure focusing mech-
116 anism. Then, we analyze the melt focusing trends versus spreading rate and
117 permeability using proportion of melt flux focused by the dominant mech-
118 anisms. We compare the location and amount of melt in the model results
119 with geophysical observations and try to reconcile the difference in the un-
120 derstanding of the LAB between the two. Finally, we discuss the limitations
121 of our model setup and conclude the findings.

122 **2. Model formulation**

123 We model melt migration beneath a mid-ocean ridge using a Darcy-like
124 melt flow in a viscously deforming solid matrix (McKenzie, 1984; Scott and
125 Stevenson, 1984, 1986; Fowler, 1985). To clearly identify the various pressure
126 gradients that affect melt flow, we decompose the total liquid pressure into
127 compaction pressure, \mathcal{P} , dynamic pressure, p^* , and lithostatic pressure, P_L
128 (Spiegelman, 1993c; Katz et al., 2007; Keller and Katz, 2016):

$$P = \mathcal{P} + p^* + P_L \quad (1)$$

129 Compaction pressure, \mathcal{P} , is defined by the divergence of solid velocity, \mathbf{v}_s ,
 130 or the compaction and decompaction of the solid matrix scaled by the bulk
 131 viscosity and is a measure of the pressure difference between the phases:

$$\mathcal{P} = \zeta \nabla \cdot \mathbf{v}_s \quad (2)$$

132 This definition serves to indicate an overpressure when the divergence of
 133 solid velocity is positive, which is consistent with the idea of melt overpres-
 134 sure driving expansion of the matrix, and underpressure causing compaction
 135 (McKenzie, 1984). The definition of compaction pressure here follows previ-
 136 ous work such as Katz (2008) while it differs in sign from Keller et al. (2017).
 137 p^* is the dynamic pressure due to incompressible solid shear and P_L is a
 138 reference lithostatic pressure:

$$P_L(z) = \rho_s g z \quad (3)$$

139 Substituting the pressure decomposition into the dimensional Darcy's equa-
 140 tion, we have:

$$\phi(\mathbf{v}_f - \mathbf{v}_s) = -\frac{K}{\mu} [\nabla \mathcal{P} + \nabla p^* + \Delta \rho \mathbf{g}] \quad (4)$$

141 where ϕ is the porosity, the volume fraction of the melt of liquid phase in the
 142 two-phase system. We assume that any pore space in the solid matrix is satu-
 143 rated with melt. \mathbf{v}_s and \mathbf{v}_f are the solid and liquid velocities, K is the matrix
 144 permeability and μ is the liquid viscosity. This equation says that melt seg-
 145regation occurs due to various pressure gradients modulated by permeability
 146 and liquid viscosity. The buoyancy term moves melt vertically. Since we
 147 are exploring melt focusing in this study, the horizontal pressure gradients
 148 would be of interest although the pressures can also transport melt vertically.

149 Compaction and dynamic pressures focus melt if their corresponding gradi-
150 ents in the horizontal direction is significant compared to buoyancy. In this
151 work, the set of equations is non-dimensionalised with velocities scaled to the
152 reference melt separation velocity, lengths scaled to the depth of the domain,
153 pressures scaled to the lithostatic pressure of the depth of the domain and
154 temperature scaled to the mantle potential temperature. The full derivation
155 and non-dimensionalization are given in the supplementary material. The
156 characteristic scales used are give in Table 1.

157 In the limit of small porosity, Spiegelman (1993a,b) show that the mo-
158 mentum and continuity equations of the solid and liquid become decoupled
159 such that the solid flow no longer depends on the liquid flow, therefore al-
160 lowing us to use the one-way coupling approach, e.g. Wilson et al. (2014).
161 First, we solve the time-independent solid Stokes flow and the steady-state
162 energy equation (Equations 5-7) once at the beginning of each model run
163 for solid velocity and dynamic pressure and temperature. Holding the solid
164 flow constant (i.e. the solid velocity and dynamic pressure solution does not
165 change with time) and using the temperature solution from the solid solve
166 as the initial condition, we then solve for the time-dependent two-phase flow
167 (Equations 15-17) for porosity, compaction pressure and temperature. Both
168 solid and two-phase systems are solved in a two-dimensional rectangular do-
169 main 200 km wide and 100 km deep as shown in Figure 1, where the center
170 line ($x = 0$) represents the ridge axis and the top of the domain ($z = 0$)
171 represents the base of the Moho.

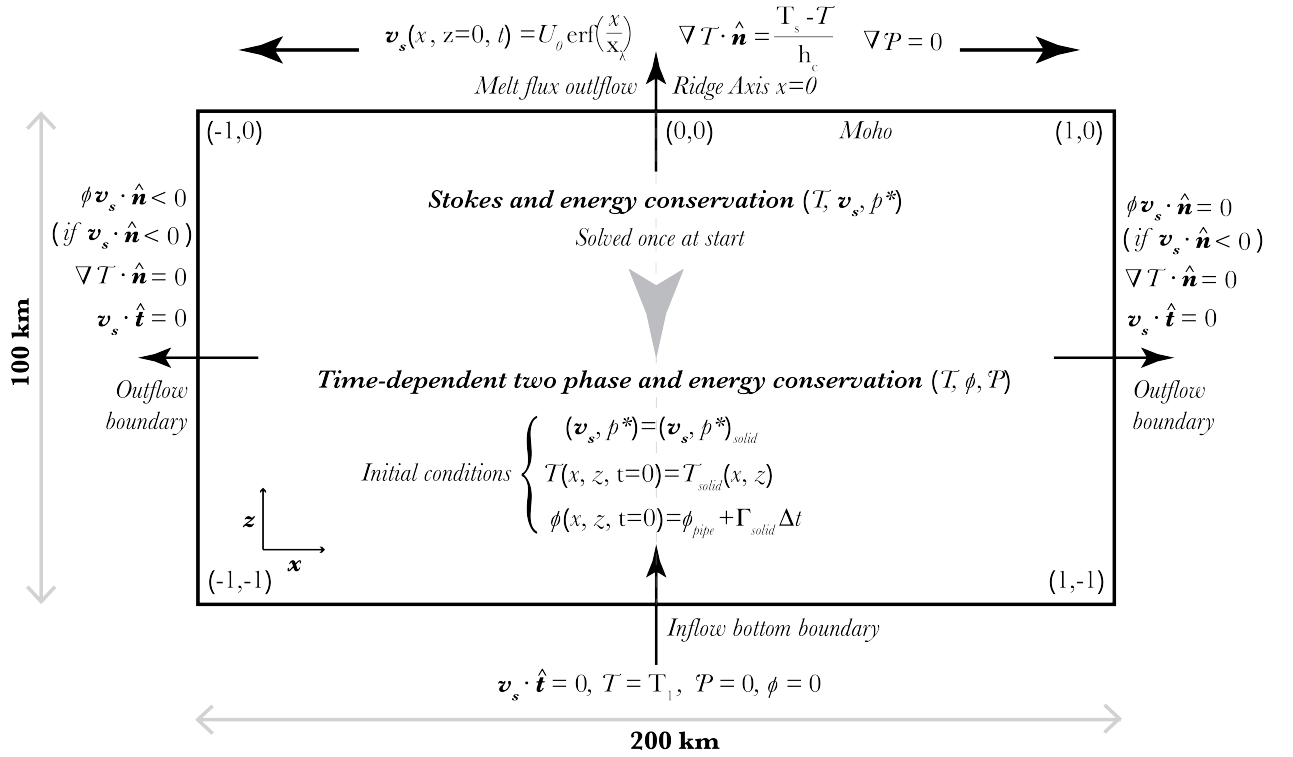


Figure 1: Schematic of two-dimensional model diagram with initial and boundary conditions.

172 *2.1. Solid Stokes system*

173 In the solid flow system, the non-dimensionalized incompressible Stokes
174 equations are solved along with the steady-state thermal energy equation:

$$\phi_0^{m-1} \frac{\delta_0^2}{h^2} \nabla \cdot 2\eta \dot{\epsilon}_d - \nabla p^* = 0 \quad (5)$$

$$\nabla \cdot \mathbf{v}_s = 0 \quad (6)$$

$$\mathbf{v}_s \cdot \nabla T - \beta T \mathbf{v}_s \cdot \hat{\mathbf{k}} + \phi_0 L_{cp} T \Gamma_{\text{Stokes}} - \frac{1}{Pe} \nabla^2 T = 0 \quad (7)$$

177 where we solve for the dynamic pressure, p^* , the velocity of the solid phase,
178 \mathbf{v}_s , and the temperature, T . m is the bulk viscosity exponent, h is the depth
179 of the domain, β is the non-dimensional adiabatic gradient, L_{cp} is the non-
180 dimensional latent heat coefficient and Pe is the Peclet number. We define
181 the reference porosity, ϕ_0 , and melt velocity, w_0 , using a buoyancy-driven
182 Darcy-flow approximation and mass conservation for a one-dimensional melt
183 column (Spiegelman and Elliott, 1993; Ribe, 1987) given by:

$$\phi_0 w_0 = \frac{K_0 \phi_0^n \Delta \rho g}{\mu_0} \quad (8)$$

184 and

$$\rho_f \phi_0 w_0 = \rho_s U_0 F_{\text{max}} \quad (9)$$

185 The reference compaction length, δ_0 , is defined as:

$$\delta_0 = \sqrt{\frac{K_0 \phi_0^n \eta_0}{\mu_0 \phi_0^m}} \quad (10)$$

186 $\dot{\epsilon}_d = \frac{1}{2}(\nabla \mathbf{v}_s + \nabla \mathbf{v}_s^T) - \frac{1}{3} \nabla \cdot \mathbf{v}_s \mathbf{I}$ is the deviatoric strain rate tensor. η is the
187 non-dimensional solid shear viscosity, given by a harmonic sum of diffusion

188 creep, dislocation creep and a small plasticity term to keep the ridge axis
 189 weak (Spiegelman et al., 2016; Tosi et al., 2015):

$$\frac{1}{\eta} = \frac{\eta_0}{\eta_{\text{diff}}(T)} + \frac{\eta_0}{\eta_{\text{disl}}(T, \dot{\epsilon}_{\text{II}})} + \frac{\eta_0}{\eta_{\text{plas}}(\dot{\epsilon}_{\text{II}})} + \frac{\eta_0}{\eta_{\text{max}}} \quad (11)$$

190 where diffusion creep, dislocation creep (Karato and Wu, 1993; Hirth and
 191 Kohlstedt, 2003) and the plasticity term are given by:

$$\eta_{\text{diff}} = A_{\text{diff}} e^{[E_{\text{diff}}/(nRT)]} \quad (12)$$

$$\eta_{\text{disl}} = A_{\text{disl}} e^{[E_{\text{disl}}/(nRT)]} \dot{\epsilon}_{\text{II}}^{\frac{1}{n_{\text{disl}}} - 1} \quad (13)$$

193 and

$$\eta_{\text{plas}} = \frac{Y}{2\dot{\epsilon}_{\text{II}}} \quad (14)$$

194 A_{diff} and E_{diff} are the constant and activation energy for diffusion creep re-
 195 spectively, and R is the universal gas constant. A_{disl} and E_{disl} are the constant
 196 and activation energy for dislocation creep respectively, $\dot{\epsilon}_{\text{II}}$ is the second in-
 197 variant of strain rate and n_{disl} is the stress exponent. $Y = C \cos \varphi + P_L \sin \varphi$
 198 is the yield criterion, C is the constant of cohesion independent of pressure,
 199 and φ is the friction angle. Γ_{Stokes} is the non-dimensional interphase mass
 200 exchange rate. Despite not including the melt phase in the solid Stokes equa-
 201 tion, the melting term in the energy equation is not dropped in the small
 202 porosity approximation since it is of order F_{max} , the maximum degree of
 203 melting, when taking into account that Γ_{Stokes} is of order F_{max}/ϕ_0 .

204 2.2. Two-phase system

205 The mantle upwelling from the solid system drives decompression melting.
 206 We model the evolution of this melt using the non-dimensionalized two-phase

Table 1: Symbols and definitions of parameters

Symbol	Formula	Definition	Value
h		reference length scale	100 km
T_0		reference mantle temperature	1623 K
ρ_s		density of solid phase	3300 kg/m ³
ρ_f		density of liquid phase	2800 kg/m ³
$\Delta\rho$	$\rho_s - \rho_f$	density difference between solid and liquid phases	500 kg/m ³
F_{\max}		maximum degree of melting	0.2
μ_0		reference liquid viscosity	1 Pa s
η_0		reference background solid shear viscosity	10 ¹⁹ Pa s
η_{\max}		maximum solid shear viscosity	10 ²³ Pa s
β	$\alpha_s g h / c_p$	non-dimensional adiabatic gradient	2.45×10^8
α_s		thermal expansion coefficient for solid phase	3×10^5 /K
g		gravitational acceleration	9.81 m/s ²
c_p		heat capacity at constant pressure for solid phase	1200 J/K
L_{cp}	$L_0 / T_0 c_p$	non-dimensional latent heat	0.205
L_0		reference latent heat of melting	4×10^5 J/kg
κ_0	$k / \rho_s c_p$	reference thermal diffusivity	7.272×10^{-7} m ² /s
A_{diff}		Diffusion creep constant	1.32043×10^9
E_{diff}		Activation energy for diffusion creep	335×10^3 J
A_{disl}		Dislocation creep constant	28968.6
E_{disl}		Activation energy for dislocation creep	540×10^3 J
R		Universal gas constant	8.314 J/K mol
n_{disl}		exponent dependence on strain rate for dislocation creep	3.5
C		constant of cohesion independent of pressure	100 MPa
φ		friction angle for yield criterion	30°
m		bulk viscosity exponent	1
ϕ_ϵ		regularization for porosity in bulk viscosity relation	0.01
A_1, A_2, A_3		constants for peridotite solidus	1085.7 °C, 132.9 °C/GPa, -5.1 °C/GPa ²
B_1, B_2, B_3		constants for peridotite liquidus	1475.0 °C, 80.0 °C/GPa, -3.2 °C/GPa ²
n		permeability exponent	3
Δt		Time step for initial porosity estimation	0.1
θ_{tr}		degree of fanning downwards for width of initial porosity transient	100
x_{width}		non-dimensional width of initial porosity transient at ridge axis	0.01
z_0		non-dimensional beginning depth of initial porosity transient	0
z_1		non-dimensional ending depth of initial porosity transient	0.1
z_λ		non-dimensional width of vertical smoothing for initial porosity transient	0.035

Table 2: Symbols for variables and their definition

Symbol	Formula	Definition	Values						Units
K_0		intrinsic permeability	4×10^{-9}			4×10^{-7}			m^2
U_0		half-spreading rate	2	4	8	2	4	8	cm/yr
ϕ_0	$\left(\frac{\rho_s F_{max} U_0 \mu_0}{\rho_f \Delta \rho K_0}\right)^{1/n}$	reference background porosity	2.0	2.5	3.1	0.4	0.5	0.6	%
w_0	$\frac{\rho_s U_0 F_{max}}{\rho_f \phi_0}$	reference melt velocity	24.0	38.0	60.4	111.2	176.5	280.2	cm/yr
w_0/U_0		“mobility”	12.0	9.5	7.55	55.6	44.1	35.0	-
δ_0	$\sqrt[n]{\frac{K_0 \phi_0^n \eta_0}{\mu_0 \phi_0^n}} = \sqrt[n]{\frac{\eta_0 w_0 \phi_0}{\Delta \rho g \phi_0^n}}$	reference compaction length	3.9	4.9	6.2	8.4	10.6	13.4	km
Pe	hw_0/κ_0	Peclet number	1044	1658	2631	4846	7693	12213	-
R_f		freezing rate constant	100	200	400	100	200	400	-

207 flow formulation including conservation of energy:

$$\frac{\partial \phi}{\partial t} + \mathbf{v}_s \cdot \nabla \phi - \frac{h^2 \mathcal{P}}{\delta_0^2 \zeta} = \Gamma \quad (15)$$

208

$$\frac{h^2 \mathcal{P}}{\delta_0^2 \zeta} - \nabla \cdot \frac{\phi^n}{\mu} [\nabla(\mathcal{P} + p^*) + \hat{\mathbf{k}}] = \frac{\Delta \rho}{\rho_f} \Gamma \quad (16)$$

209

$$\left(\frac{\rho_f}{\rho_s} \phi_0 \phi + (1 - \phi_0 \phi)\right) \frac{\partial T}{\partial t} + \frac{\rho_f}{\rho_s} \phi_0 \phi \mathbf{v}_f \cdot \nabla T + (1 - \phi_0 \phi) \mathbf{v}_s \cdot \nabla T + \beta T \mathbf{v}_s \cdot \hat{\mathbf{k}} + \phi_0 L_{cp} T \Gamma - \frac{1}{Pe} \nabla^2 T = 0 \quad (17)$$

210 where we solve for the porosity or volume fraction of melt, ϕ , the compaction
 211 pressure, \mathcal{P} , and the temperature, T . ζ is the non-dimensional bulk viscosity
 212 given by:

$$\zeta = \frac{\eta}{(\phi + \phi_\epsilon)^m} \quad (18)$$

213 where an inverse dependence on porosity for ζ was previously suggested based
 214 on homogenization theory (Simpson et al., 2010a) and m is the exponent
 215 on porosity. In this formulation of bulk viscosity, a small regularization of
 216 porosity, ϕ_ϵ , is used to avoid singularity in the limit of $\phi \rightarrow 0$.

217 Γ is the non-dimensional interphase mass exchange rate given by:

$$\Gamma = \Gamma_+ + \Gamma_- \quad (19)$$

218 where Γ_+ is the melting rate:

$$\Gamma_+ = G(F) \frac{(1 - \phi_0 \phi) D_s f}{\phi_0 Dt} \quad (20)$$

219 and Γ_- is the freezing rate:

$$\Gamma_- = \frac{1}{2} \phi R_f [T - T_{\text{liquidus}}^{\text{basalt}} - |T - T_{\text{liquidus}}^{\text{basalt}}|] \quad (21)$$

220 $f(T, P) = \left[\frac{T - T_{\text{solidus}}}{T_{\text{liquidus}} - T_{\text{solidus}}} \right]^{\beta_f}$ is the volume fraction of melt predicted in a
 221 closed system (batch melting) as a function of pressure and temperature,
 222 based on a power law parameterization of the peridotite phase diagram (Katz
 223 et al., 2003). The peridotite solidus and lherzolite liquidus are defined as
 224 functions of the lithostatic pressure (Hirschmann, 2000):

$$T_{\text{solidus}} = A_1 + A_2 P_L + A_2 P_L^2 \quad (22)$$

225

$$T_{\text{liquidus}} = B_1 + B_2 P_L + B_2 P_L^2 \quad (23)$$

226 where A_i, B_i are constants (see Table 1). $G(F)$ is a tanh function of the
 227 local degree of depletion, F , that simply ceases melting with clinopyroxene
 228 exhaustion. The local degree of depletion, F , is calculated by tracking the
 229 concentration of a completely compatible trace element, i.e an element with
 230 bulk partition coefficient $D \rightarrow \infty$, (Spiegelman, 1996). R_f is the freezing
 231 rate constant that we vary proportionally with the spreading rate such that
 232 melt is kept out of the cold, strong lithosphere. If R_f is taken to be zero,
 233 there will be no freezing. $T_{\text{liquidus}}^{\text{basalt}}$ is the basaltic liquidus (Hirschmann, 2000)
 234 taken to be the same as the peridotite solidus, T_{solidus} .

235 Permeability is typically described by $K = \frac{a^2 \phi^n}{b}$, where a is the mean grain
 236 size or reference spacing of melt rich veins, b and n are empirical constants

237 that are constrained experimentally (Zhu et al., 1995; Wark and Watson,
 238 1998). We follow Katz (2008) in our model formulation for dimensional
 239 permeability:

$$K = K_0 \phi^n \quad (24)$$

240 where the mean grain size, a , and empirical constant, b , are incorporated
 241 into K_0 , the intrinsic permeability, which is one of the variables we vary
 242 in the models (Table 2). The intrinsic permeability has units of m^2 . Non-
 243 dimensional permeability is described by

$$K = \phi^n \quad (25)$$

244 where n is an empirical constant, constrained experimentally to be between
 245 2 and 3 (Zhu et al., 1995; Wark and Watson, 1998; Miller et al., 2014). Non-
 246 dimensional liquid viscosity, μ , is taken here to be constant 1.

247 *2.3. Boundary and initial conditions*

248 Boundary and initial conditions for these models are shown schematically
 249 in Figure 1. Mantle upwelling is driven by imposing ridge-like plate motion
 250 along the top boundary such that $U_{\text{top}} = U_0 \text{erf}\left(\frac{x}{x_\lambda}\right)$, where U_0 is the half-
 251 spreading rate, x is the distance from the ridge axis, and $x_\lambda = 0.01$ (1 km) is
 252 the width of the smoothed step function, which is chosen to represent roughly
 253 the width of the neo-volcanic zone at the ridge axis. We allow solid inflow
 254 at the bottom of the domain and outflow at the sides such that solid flow
 255 tangent to the sides and bottom boundaries is zero (i.e. $\mathbf{v}_s \cdot \hat{\mathbf{t}} = 0$, where $\hat{\mathbf{t}}$ is
 256 the unit vector tangent to the boundary). The top boundary allows melt to
 257 outflow but prevents any outflow of the solid mantle (i.e. $\mathbf{v}_s \cdot \hat{\mathbf{n}} = 0$, where $\hat{\mathbf{n}}$
 258 is the unit vector normal to the boundary).

259 The top of the domain represents the Moho, where we apply a Robin
 260 condition on the temperature:

$$\nabla T \cdot \hat{\mathbf{n}} = \frac{T_s - T}{h_c} \quad (26)$$

261 This assumes a linear relationship between ocean surface temperature, $T_s =$
 262 0°C , and temperature at the Moho over $h_c = 7$ km, the crustal thickness.
 263 The temperature solution for the solid system for each model (Figure 2)
 264 is used as the initial condition for the two-phase system. Obtaining the
 265 compaction pressure involves solving for an elliptic equation that requires
 266 boundary conditions on all boundaries. We enforce that the compaction
 267 pressure is zero on the bottom boundary before melting, consistent with
 268 the fact that solid flow is incompressible outside of the melting regime (i.e.
 269 $\nabla \cdot \mathbf{v}_s = 0$ implying $\mathcal{P} = 0$). The top and sides boundaries are Neumann
 270 boundaries on the melt flux such that $\nabla \mathcal{P}$ has some value (Equation 4).

271 The initial condition for porosity consists of two parts:

$$\phi|_{t=0} = \Gamma_{\text{Stokes}} \Delta t + \phi_{\text{tr}} \quad (27)$$

272 Γ_{Stokes} is the melt production rate in the solid system, which is solved using
 273 the steady state version of Equation 20, such that only the advective part of
 274 the material derivative is used and the tanh function $G(F)$ is omitted. Δt is
 275 the time step used as an approximation to obtain the initial porosity in the
 276 melting region. ϕ_{tr} is an initial porous region to allow melt flow through the
 277 axial region during the initial part of these calculations:

$$\phi_{\text{tr}} = \frac{1}{2} e^{-\left(\frac{x}{(1+\theta_{\text{tr}}z)x_{\text{width}}}\right)^2} \left[\tanh\left(\frac{z - z_0}{z_\lambda}\right) - \tanh\left(\frac{2(z - z_1)}{z_\lambda}\right) \right] \quad (28)$$

278 where θ_{tr} , x_{width} , z_0 , z_1 , and z_λ define the shape of the initial porosity tran-
 279 sient and are constants given in Table 1.

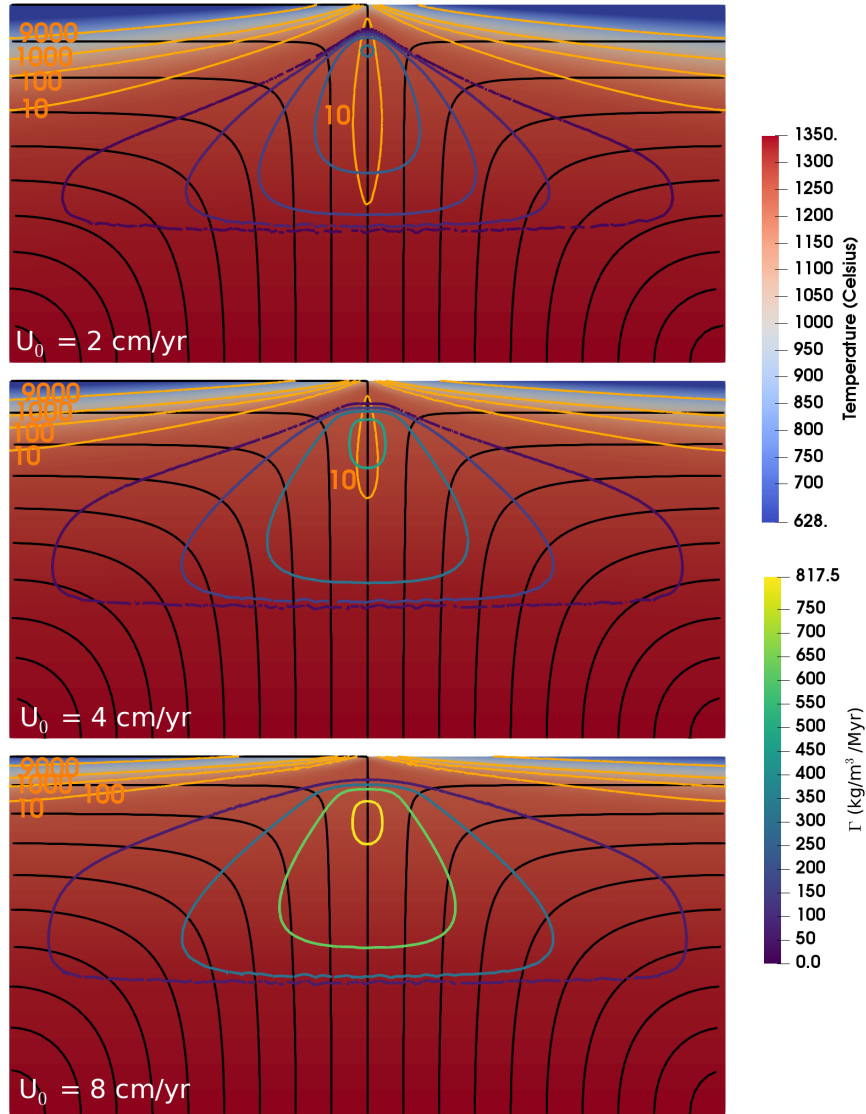


Figure 2: The temperature solutions from the solid system for half-spreading rates $U_0 = 2, 4, 8$ cm/yr. Black lines are solid flow lines. Yellow to purple contours are melting rate production, Γ . Orange labeled contours are non-dimensional viscosity contours scaled by $\eta_0 = 10^{19}$ Pa s.

280 *2.4. Numerical implementations*

281 We explore the two-phase dynamics for mid-oceanic ridges by varying
282 half-spreading rates $U_0 = 2, 4, 8$ cm/yr and intrinsic permeability, $K_0 =$
283 $4 \times 10^{-7}, 10^{-9}$ m². We discretize the non-dimensionalized set of equations
284 using finite differences in time and finite elements in space. Dynamic pres-
285 sure is chosen to be a piecewise linear function (P1) while temperature, solid
286 velocity and compaction pressure are piecewise quadratic (P2). Porosity is
287 discontinuous piecewise quadratic (P2DG). We use TerraFERMA, the Trans-
288 parent Finite Element Rapid Model Assembler (Wilson et al., 2017) to con-
289 struct and solve our non-dimensional, nonlinear system of equations in two
290 dimensions. TerraFERMA leverages open source libraries, FEniCs (Logg
291 et al., 2011, 2012), PETSc (Balay et al., 2017) and SPuD (Ham et al., 2009)
292 to provide a common interface for building custom finite element method
293 models that are transparent and reproducible. We use Gmsh (Geuzaine and
294 Remacle, 2009) to generate an unstructured triangular mesh over the rectan-
295 gular domain. The smallest element is ~ 100 m at the ridge axis and coarsens
296 away from it, with the largest element being ~ 8 km.

297 The flexibility of TerraFERMA allows us to build on this current model
298 and expand it to be fully coupled or test other parameterizations. This flex-
299 ibility is an important design feature of TerraFERMA and other software
300 such as ComSol (www.comsol.com) and Underworld2 (Moresi et al., 2007).
301 For these types of complex highly coupled problems, the flexibility gives it an
302 advantage over other software that can be purpose specific and therefore dif-
303 ficult to modify. An example of fully coupled ridge model with an isoviscous
304 rheology is included in Wilson et al. (2017). This suite of models are openly

305 available to the readers in the form of TerraFERMA markup files (.tfml) at
306 https://github.com/joycesim/M3LT_one_Uall.git.

307 **3. Results**

308 The solid Stokes system represents passively driven single phase flow with
309 thermal feedback. The temperature solutions from the solid system for all
310 the spreading rates show the cooling - hence strengthening - and thickening
311 of the oceanic lithosphere with distance away from the ridge axis (Figure 2).
312 The solid mantle velocity field resembles a typical corner flow (black flow
313 lines in Figure 2). Most of the melt is generated on the ridge axis, where
314 the solid mantle is upwelling at the fastest rates, although the melting region
315 (colored contours in Figure 2) is as wide as the width of the domain. The
316 melting rate is higher for faster spreading rates due to faster upwelling rates.

317 In the two-phase system, these models are initialized with the solid ve-
318 locity, dynamic pressure and temperature from the solid system with cor-
319 responding spreading rate (Figure 2) and initial conditions for porosity and
320 compaction pressure as described in the previous section. The models go
321 through an initial transient period before settling into a quasi-steady state
322 (Figure 3). During the transient phase, large amplitude porosity waves prop-
323 agate from the melting region through the relatively low porosity region be-
324 tween the top of the melting region and the moho. Porosity waves are a
325 natural consequences of poro-viscous flow with non-zero compaction length
326 (Spiegelman, 1993b). In these models, the porosity waves are transient fea-
327 tures except in the high permeability, low spreading rate case, in which the
328 waves persist throughout the model run. These high amplitude porosity

329 waves occur when the melt flux varies on length scales comparable to the
 330 compaction length:

$$\delta = \sqrt{\frac{K\phi^n\eta}{\mu\phi^m}} \quad (29)$$

331 and they are dependent on the initial conditions. The reference values for
 332 compaction length for all model runs are given in Table 2. At quasi-steady
 333 state, there are two main melt transport patterns. Near the ridge axis, melt
 334 moves horizontally towards the ridge axis to form a central high porosity
 335 region, which eventually moves out of the domain at the top of the ridge
 336 axis. Away from the ridge axis, buoyant melt rises up to encounter the cold,
 337 and thus stronger, sloping lithosphere, where it is either diverted to the ridge
 338 axis or freezes into the lithosphere.

339 *3.1. Crustal production versus spreading rate*

340 The variation of oceanic crustal thickness with spreading rate is a first
 341 order seismically observable feature of mid-ocean ridges (White et al., 1992).
 342 In order to validate the models with seismic observations, we use three model
 343 output proxies for crustal thickness: 1) total melt production rate in the
 344 purely solid Stokes system ignoring the effect of melt transport, 2) total melt
 345 production rate in the two-phase system and 3) melt flux through the top
 346 boundary. Using mass balance, the total amount of melt produced gives an
 347 upper bound of the amount of oceanic crust that can be produced:

$$2\rho_c h_c U_0 = \rho_s \phi_0 w_0 h \int \Gamma^* dX \quad (30)$$

348 where h_c is the oceanic crustal thickness, ρ_c is the oceanic crustal density
 349 assumed to be 2800 kg/m³, $\int \Gamma^* dX$ is the non-dimensional total melt pro-
 350 duction rate in the whole domain (dX) either in the solid system, $\Gamma^* = \Gamma_{\text{Stokes}}$,

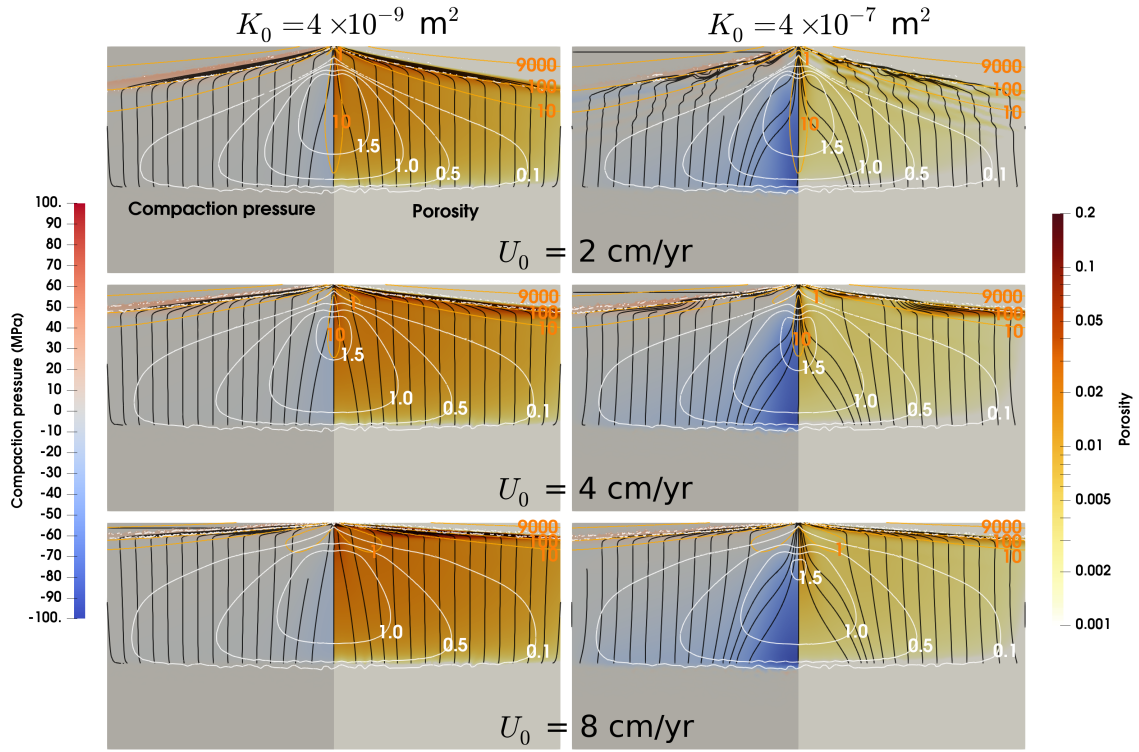


Figure 3: Snapshots of the dimensional compaction pressure and porosity field at the end of all six model runs with increasing permeabilities to the right and increasing half-spreading rates down the panels. Domain of each panel is 200 km wide and 100 km deep. The left side of each panel shows the dimensional compaction pressure. The right side of each panel shows the porosity (i.e. $\phi = 0.01$ is 1% porosity). The black lines on both sides of the each panel track the melt flow. White contours are for non-dimensional total melting rate, Γ .

351 or, in our second estimate, the two-phase system, $\Gamma^* = \Gamma_+$. The mass bal-
 352 ance for oceanic crustal production based on total melt flux through the top
 353 boundary in the two-phase system is:

$$2\rho_c h_c U_0 = \rho_f \phi_0 w_0 h \int \mathbf{q} \cdot \hat{\mathbf{n}} ds_{top} \quad (31)$$

354 where $\hat{\mathbf{n}}$ is the unit outward normal to the top boundary, ds_{top} , and $\mathbf{q} = \phi \mathbf{v}_f$
 355 is the melt flux.

356 The melt system is time-dependent, causing fluctuations in both melt
 357 production rate and melt flux at the ridge axis (Figure 4a). The crustal
 358 thickness calculated using the total melt production rate for the two-phase
 359 models are similar, regardless of permeability, whereas those calculated by
 360 integrating the melt flux through the top boundary are smaller. After an
 361 initial transient of about ~ 2 Myrs, however, the system settles into a quasi-
 362 steady state and we calculate mean melt production rates and melt fluxes
 363 over this period (Figure 4b). Only the crustal thickness estimates using melt
 364 flux from model with large permeability and slowest spreading has persistent
 365 fluctuations and this is due to the sustained porosity waves through the model
 366 run (top right panel in Figure 3).

367 Crustal thicknesses from both model estimates and geophysical observa-
 368 tions are plotted together as a function of spreading rate (Figure 4b). The
 369 crustal thickness predicted from the total melt production rate for the two-
 370 phase models are slightly larger than those from the solid model because
 371 there is more melting in the two-phase model due to a warmer mantle on the
 372 ridge axis from melt advection. Some of the melt produced freezes back into
 373 the lithosphere while most of the melt leaves the top boundary and is inter-
 374 preted here as forming oceanic crust. There is more freezing in the models

375 with smaller permeability, particularly for faster spreading rates, as shown
376 by the smaller amount of crustal thickness predicted from melt flux through
377 the top boundary (Figure 4b).

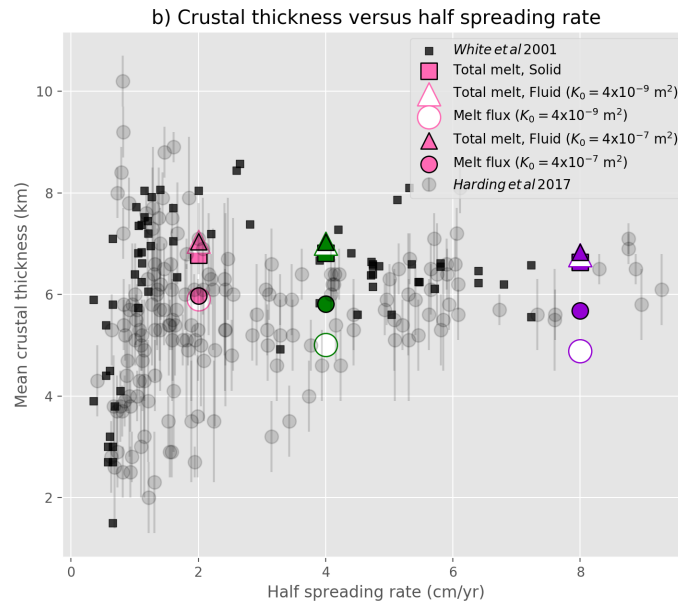
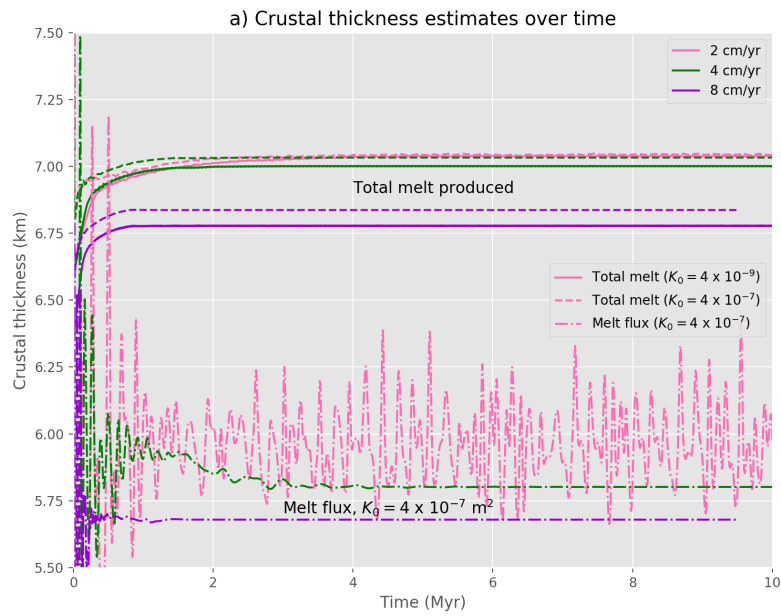


Figure 4: Model predictions of crustal thicknesses for all models compared with geophysical observations. a) Crustal thickness calculated for each model as a function of time. Crustal thickness is calculated using two different methods.

Figure 4 (continued): Solid and dashed lines represent crustal thickness predictions from total melt produced in models with smaller and larger permeability respectively. Dash-dotted lines represent crustal thickness predictions from melt flux for models with larger permeability. b) A plot of mean crustal thickness production predicted versus half-spreading rate, U_0 , for the models presented in this work along with geophysical observations (White et al., 2001; Harding et al., 2017). Colored squares are the crustal thicknesses calculated from the total melt produced in the solid system, ignoring melt transport. Triangles represent the mean crustal thicknesses predicted from the time series of total melt production rate for the two-phase system (solid and dashed lines in panel a) after the transient ~ 2 Myrs. Circles represent the mean crustal thicknesses predicted from the time series of the melt flux coming out of the top boundary of the model domain (dash-dotted lines in panel a). Filled and unfilled triangles and circles represents model predictions for models with lower and higher permeability respectively. The unfilled triangles are covered by the filled triangles since they are almost always nearly identical. Black squares are crustal thickness inferred from an active seismic data compilation (White et al., 2001). Grey circles are from a more recent compilation of active seismic data along with error bars (Harding et al., 2017).

378 Along with our mean estimates from our models in Figure 4b, a com-
 379 pilation of observed oceanic crustal thicknesses versus spreading rates from
 380 active seismic surveys is plotted in black squares (White et al., 1992; Bown
 381 and White, 1994; White et al., 2001) along with a more recent compila-
 382 tion plotted in grey circles (Harding et al., 2017). The older compilation of
 383 seismic data suggests that crustal thickness is about 7 km independent of
 384 half-spreading rate for half-spreading rate more than 1 cm/yr and rapidly
 385 decreases for slower spreading rates. The more recent compilation of Hard-
 386 ing et al. (2017) gives an average oceanic crustal thickness of ~ 6 km. The
 387 recent compilation also suggests that not only does the oceanic crustal thick-

388 nesses vary more than previously observed, this variability seems to decrease
389 as spreading rates increase. Crustal thickness estimates from our model out-
390 puts agree well with the seismic observations for half-spreading rates more
391 than 2 cm/yr, thus giving some confidence in the robustness of the models.
392 It also suggest that fluctuations in melt flux due to porosity waves could
393 contribute to the variable crustal thickness at slower spreading rate.

394 *3.2. Importance of permeability*

395 The permeability of the solid mantle matrix determines how quickly melt
396 generated can segregate from the solid mantle and controls the relative im-
397 portance of solid advection and pressure driven flow. For example, equation 4
398 shows that if the solid were impermeable, melt could not separate from the
399 solid and would solely be advected by the solid phase (and the porosity would
400 equal the degree of melting, i.e. $\phi \sim F$). Large K_0 implies that the melt
401 flow is more dominated by pressure gradients than solid advection. Since
402 the total melt production rate is bounded by the solid flow field, increasing
403 the intrinsic permeability also means that less melt can be retained in the
404 mantle, i.e. higher intrinsic permeability, K_0 , leads to a faster reference melt
405 velocity, w_0 , and lower amount of background melt or porosity, ϕ_0 (Table 2).
406 Figure 3 shows that the maximum porosity for high intrinsic permeability
407 ($K_0 = 10^{-7}$) is $\sim 2\%$ versus 20% for lower K_0 while both models have the
408 same melt production rate (Figure 2). There are two main regions of higher
409 porosity accumulation, at depths on the ridge axis and off to the sides be-
410 neath the LAB, which is defined by the temperature dependent rheological
411 and freezing boundary. The accumulation of melt beneath the LAB in these
412 models is consistent with the “decompaction melt layers” described initially

413 by Sparks and Parmentier (1991).

414 The intrinsic permeability alters melt transport patterns. At higher in-
415 trinsic permeability, a wider region of melt near the ridge axis feeds the cen-
416 tral high porosity region (Figure 3) with more pronounced horizontal melt
417 transport. This is coupled with larger magnitude and therefore gradient of
418 compaction pressure (Figure 3), which we will discuss further in the following
419 section. The “mobility”, w_0/U_0 , is the ratio of the reference melt velocity to
420 the spreading rate and is a measure of how strongly melt transport deviates
421 from solid mantle flow. If no melt segregation occurs, or $w_0/U_0 = 1$, melt
422 will simply follow the solid mantle flow. For lower intrinsic permeability,
423 the mobility is smaller (Table 2), suggesting that the melt deviates less from
424 the solid mantle flow compared to models with larger intrinsic permeability,
425 and larger mobility (Figure 3). The largest mobility is in the model case
426 with slow spreading and high intrinsic permeability, where porosity waves
427 are persistent.

428 *3.3. Melt transport due to pressure gradients*

429 Melt focuses due to horizontal pressure gradients. In the model formula-
430 tion for this study, pressure is decomposed into 1) compaction pressure, 2)
431 dynamic pressure and 3) lithostatic pressure as shown in Equation 1. The
432 horizontal gradient of dynamic pressure, p^* , is only significant near the ridge
433 axis (first column in Figure 5). The length scale over which this pressure
434 exerts itself is a balance between pressure gradients induced by incompress-
435 ible shear and those due to melt buoyancy in the vicinity of the ridge axis

436 (Spiegelman and McKenzie, 1987):

$$L_{\text{ridge}} \propto \sqrt{\frac{\eta U_0}{\Delta \rho g}} \quad (32)$$

437 L_{ridge} are small ($\sim 1 - 2$ km) in the models presented here.

438 Compaction pressure, \mathcal{P} , is largest in magnitude where the solid mantle
439 begins to melt above the dry peridotite solidus on the ridge axis. When com-
440 paction pressure is positive, the solid matrix is expanding or pore spaces are
441 opening up. One can also think of it as an overpressure of the liquid phase
442 with respect to the solid phase. When compaction pressure is negative, the
443 solid matrix is contracting or pore spaces are contracting. Here, there is an
444 underpressure of the liquid phase with respect to the solid phase. Horizontal
445 compaction pressure gradient is comparable to buoyancy within the melting
446 region and along the decompaction layers (second and third columns in Fig-
447 ure 5). The increase in intrinsic permeability between model runs correlates
448 with an order of magnitude increase in horizontal compaction pressure gradi-
449 ent. Spreading rate does not change the magnitude of the pressure gradients.
450 Away from the ridge axis where the horizontal compaction pressure is larger,
451 melt flow is vertical due to buoyancy. The horizontal compaction pressure
452 gradient is also significant where the decompaction layers are, pushing melt
453 away from the cold and therefore strong lithosphere, thus focusing melt to-
454 wards the ridge axis. Bands of positive and negative horizontal compaction
455 pressure gradients are present where there are porosity waves in the model
456 run with slowest half-spreading rate and larger intrinsic permeability (top
457 right in Figure 5).

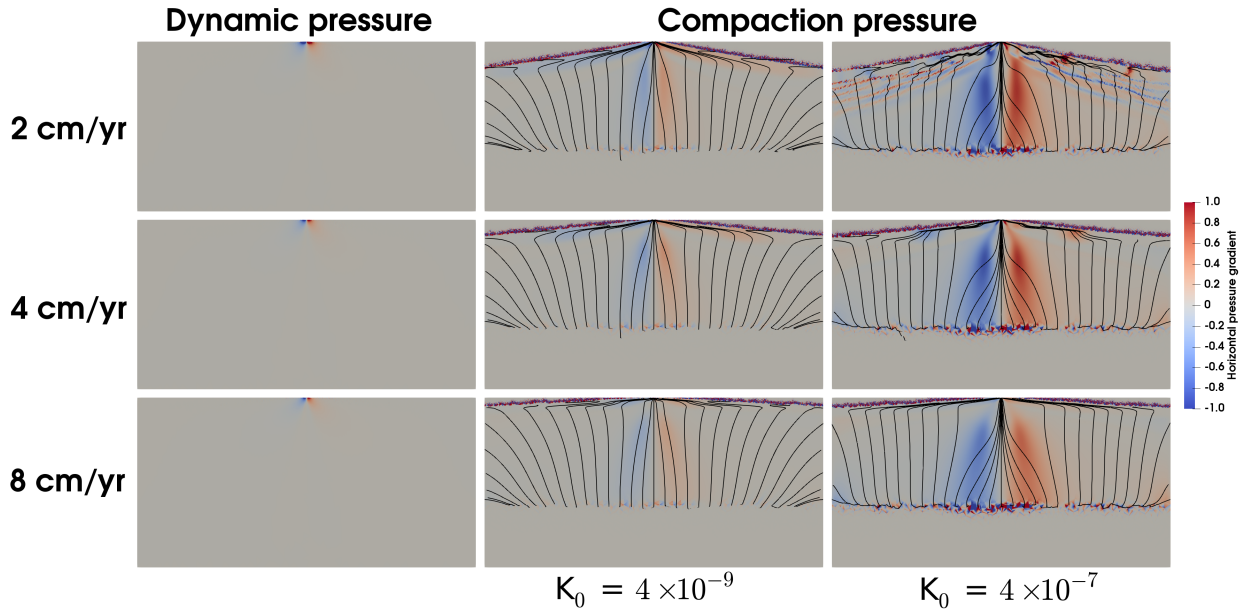


Figure 5: Comparison of non-dimensionalized horizontal pressure gradients across all models. All pressures are non-dimensionalized using $\Delta\rho gh \sim 500$ MPa.

458 4. Discussion

459 4.1. Melt focusing

460 There are three melt focusing mechanisms that we identify in our models:
 461 1) Melting pressure focusing, 2) Decompaction layers and 3) Ridge suction
 462 (Figure 6). Melting pressure focusing is a dominant feature in these models
 463 that was recently recognized (Turner et al., 2017). In these models, melt
 464 generated deep in the melting region near the ridge axis is focused towards
 465 the ridge axis (Figure 3) due to the horizontal compaction pressure gradi-
 466 ent (Figure 5). Turner et al. (2017) discusses this effect but also explored
 467 how grain growth evolution and grain size dependent viscosity affects melt
 468 dynamics in a two-phase flow model applied to mid-ocean ridges. We want

469 to emphasize that this mechanism is robust. In our models, melting pressure
 470 focusing is observed with a variable viscosity without any grain size depen-
 471 dence and we will show that it can develop even in an isoviscous model with
 472 sufficiently large shear viscosity.

473 In quasi-steady state in the frame of the solid and with the small porosity
 474 approximation, the dimensional conservation of mass for porosity dictates
 475 that the volumetric strain-rate of the solid (compaction rate) balances melt
 476 production rate:

$$\nabla \cdot \mathbf{v}_s \approx -\frac{\Gamma}{\rho_s} \quad (33)$$

477 This balance assumes that melt segregation is efficient and that the solid
 478 advection of melt is negligible, which is similar to Turner et al. (2017). Be-
 479 cause the compaction pressure is related to the volumetric strain rate, i.e.
 480 $\mathcal{P} = \zeta \nabla \cdot \mathbf{v}_s$ (Equation 2), it follows that in quasi-steady state,

$$\mathcal{P} \approx -\zeta \frac{\Gamma}{\rho_s} \quad (34)$$

481 Therefore, the compaction pressure assumes the shape of the total interphase
 482 exchange rate field, Γ , with amplitude controlled by the bulk viscosity, ζ .
 483 For adiabatic melting beneath ridges, Γ is always roughly triangular shaped
 484 with a maximum melting rate on the ridge axis, and therefore, a minimum
 485 compaction pressure on axis (Figure 3). The bulk viscosity, ζ , depends on
 486 the shear viscosity, η , and is inversely proportionate to the porosity, ϕ , since
 487 the bulk viscosity formulation used in this work is $\zeta \sim \frac{\eta}{\phi}$ (Equation 18). The
 488 bulk viscosity can be significantly larger than the shear viscosity for small
 489 porosities. In particular, higher permeability systems should show greater
 490 melting pressure focusing effects for the same shear viscosity, η , since higher
 491 permeabilities lead to smaller retained porosities (Figure 5 and Table 2).

492 Decompaction layers are melt rich layers along which melt is focused
493 towards the ridge axis (Sparks and Parmentier, 1991; Spiegelman, 1993c;
494 Hebert and Montési, 2010; Keller et al., 2017). Melt generated in the melting
495 region is more buoyant than the surrounding mantle and segregates upwards
496 to the LAB. While melt cannot permeate a rheologically strong lithosphere,
497 it can also begin to freeze as it encounters a colder lithosphere, making an
498 impermeable layer. The compaction overpressure opens up pore space below
499 this impermeable layer, thus forming the decompaction layers that deflect
500 melt towards the ridge axis. The balance of melt deflection versus freezing
501 is delicate and depends on both the rheological and freezing rate parameter-
502 ization (Spiegelman, 1993c). Decompaction channels can be present even in
503 an isoviscous case as long as freezing occurs on a length scale comparable
504 to the compaction length scale (Spiegelman, 1993c) as given in Equation 29,
505 which depends on the rheological models. The cold and therefore strong
506 lithosphere increases the compaction length scale. Turner et al. (2017) have
507 decompaction layers that are less prominent on the freezing front, since it
508 is below the rheologically stronger lithosphere, which would make the com-
509 paction length larger.

510 As tectonic forces pull oceanic plates apart, the dynamic pressure draws
511 melt towards the ridge axis from the surrounding region due to incompress-
512 ible shear (Spiegelman and McKenzie, 1987; Morgan, 1987). This ridge
513 suction focusing mechanism depends on spreading rates and shear viscos-
514 ity (Equation 32) such that the dynamic pressure becomes larger in mag-
515 nitude as spreading rate increase for the same shear viscosity, η . However,
516 faster spreading rate leads to a hotter sub-Moho mantle and therefore, lower

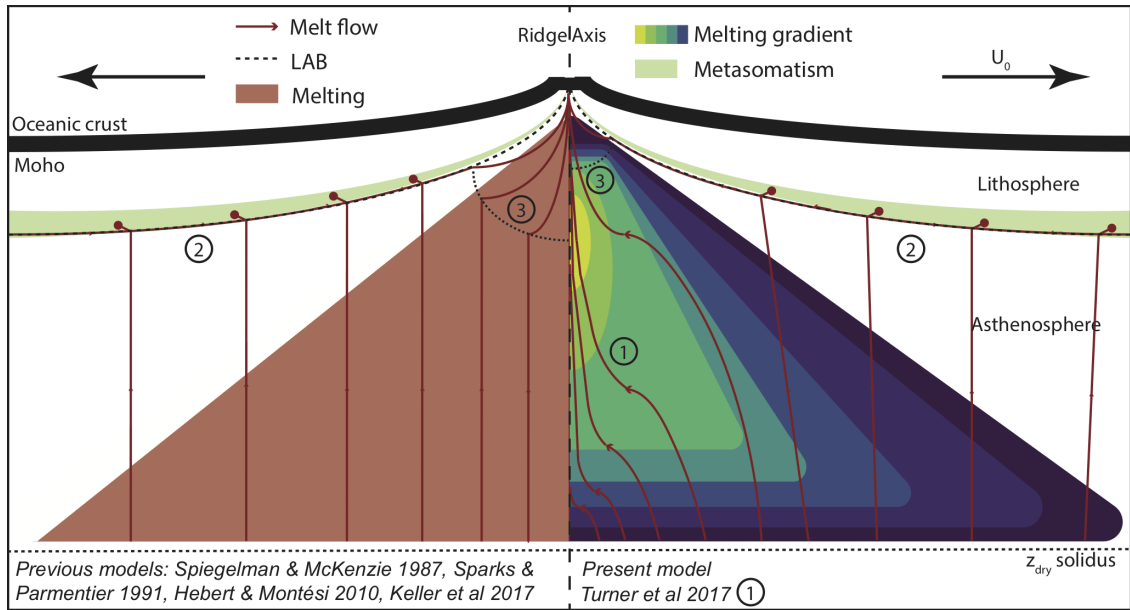


Figure 6: Illustration of melt focusing mechanisms from past and present work based on Keller et al. (2017). The three melt focusing mechanisms are numbered: 1) Melting pressure focusing 2) Decompaction layers and 3) Ridge suction. The dashed black line down the center represents the ridge axis. The thick black curved lines that connect at the highest point at the ridge axis represent the oceanic crust. The Moho is the bottom of the oceanic crust. Modeled or hypothesized melting is represented as the half triangle on the left for previous work while it is represented by a lime green to dark violet melting triangle on the right for these models presented. Red lines and arrows indicate melt flow and direction. Red circles indicate where melt freezes into the lithosphere in the green region of metasomatism above the black dashed line for the lithosphere-asthenosphere boundary (LAB).

517 shear viscosity, which balances with spreading rate such that the product
518 ηU_0 related to shear stress (Equation 32) actually remains roughly the same
519 regardless of spreading rate. In addition, advection of heat by the melt will
520 weaken the solid in a fully coupled model with temperature-dependent shear
521 viscosity as compared to the one-way coupled models in this work. With a
522 weaker ridge axis, the already small ridge suction focusing effect will be even
523 weaker relative to the other two dominant melt focusing mechanisms.

524 All the focusing mechanisms illustrated in Figure 6 exist in any two-phase
525 flow model that include pressure gradients due to both incompressible and
526 compressible viscous deformation. However, the different mechanisms may
527 not manifest themselves due to specific model choices for the constitutive
528 rheological relations (i.e shear and bulk viscosities). Melting pressure fo-
529 cusing was generally insignificant and therefore not recognized in previous
530 models because of different model choices for mantle rheology. In particu-
531 lar, Spiegelman and McKenzie (1987) shows the ridge suction effect clearly
532 but would not have this melting pressure focusing effect since their models
533 have no melting and therefore no volumetric deformation. In contrast, the
534 ridge suction effect would be smaller for a smaller background shear viscosity
535 ($\eta \sim 10^{19}$) as used in this study.

536 To understand how melting pressure focusing is affected by rheological
537 choices for shear viscosity, we present five models with different shear viscos-
538 ity models, for half spreading rate, $U_0 = 4$ cm/yr, and intrinsic permeability,
539 $K_0 = 4 \times 10^{-7}$ m² (Figure 7). We have three constant shear viscosity models,
540 $\eta = 10^{19}, 10^{20}, 10^{21}$ Pa s, with a weak ridge axis ~ 1 km radius (Figure 7a, b
541 and c). To clarify the role of melting pressure focusing, these models do not

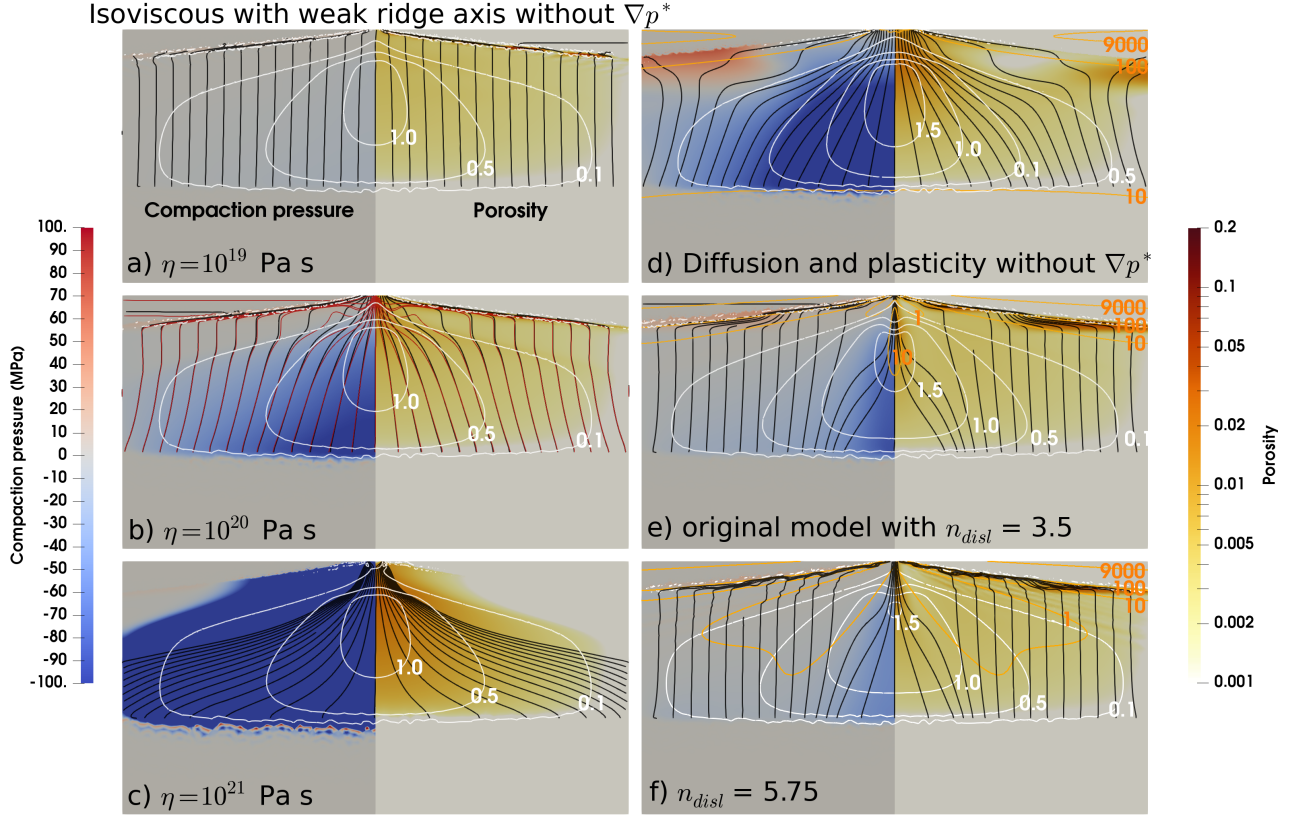


Figure 7: Snapshots of dimensional compaction pressure and porosity at the end of model runs with different rheological choices made. All models have half spreading rate, $U_0 = 4$ cm/yr, and intrinsic permeability, $K_0 = 4 \times 10^{-7}$ m². Domain of each panel is 200 km wide and 100 km deep. The left side of each panel shows the compaction pressure in MPa. The right side of each panel shows the porosity (i.e. $\phi = 0.01$ is 1% porosity). The black lines are melt streamlines. The white contours are melting rate contours. The orange contours are the non-dimensional shear viscosity with labels. On the left column, we show three models with increasing constant shear viscosity, $\eta =$ a) 10^{19} Pa s b) 10^{20} Pa s and c) 10^{21} Pa s. The isoviscous model cases have a ridge axis weaker by ~ 100 orders of magnitude, which is ~ 1 km in radius and have no ∇p^* included in Equation 4. Red lines in panel b) are melt streamline for the model case with grad ∇p^* included. On the right column, we present models with d) diffusion creep and plasticity without ∇p^* included, e) the original model as in Figure 3e) and f) the original model with strain rate exponent $n_{disl} = 5.75$.

542 include the the ridge suction effect by neglecting the dynamic pressure ∇p^*
 543 in the melt flow field. This effect should only be important with a suction
 544 length, L_{ridge} (Equation 32), near the axis as shown in Figure 7b, which com-
 545 pares melt flows with and without the ∇p^* term in Equation 4 at $\eta = 10^{20}$
 546 Pa s. Figure 7 also compares the behavior of the original model (Figure 3)
 547 with two more models with composite rheologies. The first just uses a mix-
 548 ture of diffusion creep and plasticity (Equations 12 and 14) without ∇p^*
 549 (Figure 7d). The second combines diffusion creep, plasticity and dislocation
 550 creep but with a larger stress exponent $n_{\text{disl}} = 5.75$ to mimic the effects of
 551 grain-size reduction (Figure 7f) as suggested by Turner et al. (2017). The
 552 behavior of all of these models can be readily understood given the structure
 553 of the compaction pressure (Equation 34). The shape of the pressure field
 554 is controlled by the structure of Γ which is roughly the same for all models,
 555 while the magnitude of the pressure drop depends on choice of bulk viscosity
 556 models.

557 The isoviscous cases clearly illustrate how melting pressure focusing works.
 558 All three cases have identical melting rate fields, Γ . The compaction pres-
 559 sure is primarily controlled by the shear viscosity which is spatially constant.
 560 These calculation suggest that the melt pressure focusing becomes important
 561 for mean shear viscosities of order 10^{20} Pa s. At shear viscosities less than
 562 10^{20} Pa s melt pressure focusing is negligible and melt transport is primar-
 563 ily vertical. At higher shear viscosities, the melting pressure focusing effect
 564 dominates over buoyancy, causing significant lateral melt transport deep in
 565 the melting region. It should be noted that the variation in bulk viscosity
 566 in between these isoviscous models is actually smaller than the variation in

567 shear viscosity as the porosity is concentrated in specific regions.

568 As stated, these calculations do not include the ridge suction effect which
569 is expected to only be important within a suction length, L_{ridge} (Equation 32),
570 around the ridge axis. For $\eta = 10^{20}$ Pa s, L_{ridge} is small (~ 3 km) and ridge
571 suction effect is unimportant (Figure 7b). For shear viscosity, $\eta = 10^{21}$ Pa s,
572 L_{ridge} is larger (~ 10 km) and ridge suction effect could contribute to ridge
573 focusing (Spiegelman and McKenzie, 1987).

574 While the isoviscous model cases serve to help us understand the melt-
575 ing pressure focusing mechanism, models with more realistic rheologies (e.g.
576 presence of lithosphere) help us understand the mechanism in the context of
577 the Earth. The model with diffusion creep and plasticity (Figure 7d) with-
578 out ridge suction effect, has a mean shear viscosity in the melting region
579 of $\sim 10^{20}$ Pa s (for this choice of rheological parameters) and shows signif-
580 icant melting pressure focusing in the melting region. Adding dislocation
581 creep mechanisms (Figure 7d and f) weakens the shear viscosity in the re-
582 gions flanking the ridge axis where the shear strain is greatest. This leads to
583 additional spatial gradients in the compaction pressure which weakens the
584 melting pressure focusing effect. Models with higher powerlaw exponent as
585 suggested by Turner et al. (2017) do not alter the extent of the melting pres-
586 sure focusing but rather the melt pathways due to the varying strength of
587 horizontal compaction gradients with depth (Figure 7e and f).

588 The magnitude of the compaction pressure due to melting, depends on
589 the bulk viscosity which is an effective property of the two phase medium. In
590 most early two phase formulations, the bulk viscosity relation was chosen as
591 η/ϕ^m with m as a free parameter usually chosen to be 0 or 1. In these mod-

592 els, we use $m = 1$, which was shown to be consistent with the two-phase flow
593 equations using homogenization theory (Simpson et al., 2010a,b). This bulk
594 viscosity formulation, $\zeta \sim \eta/\phi$, amplifies the bulk viscosity since porosities
595 are much smaller than 1, thus affecting the compaction pressure term. Re-
596 cent work on homogenization of other viscosity mechanisms (Rudge, 2018a,b)
597 provide additional functional relationships between ζ , η and ϕ , which give
598 a bulk viscosity that is more comparable to the shear viscosity, potentially
599 weakening the melting pressure focusing mechanism.

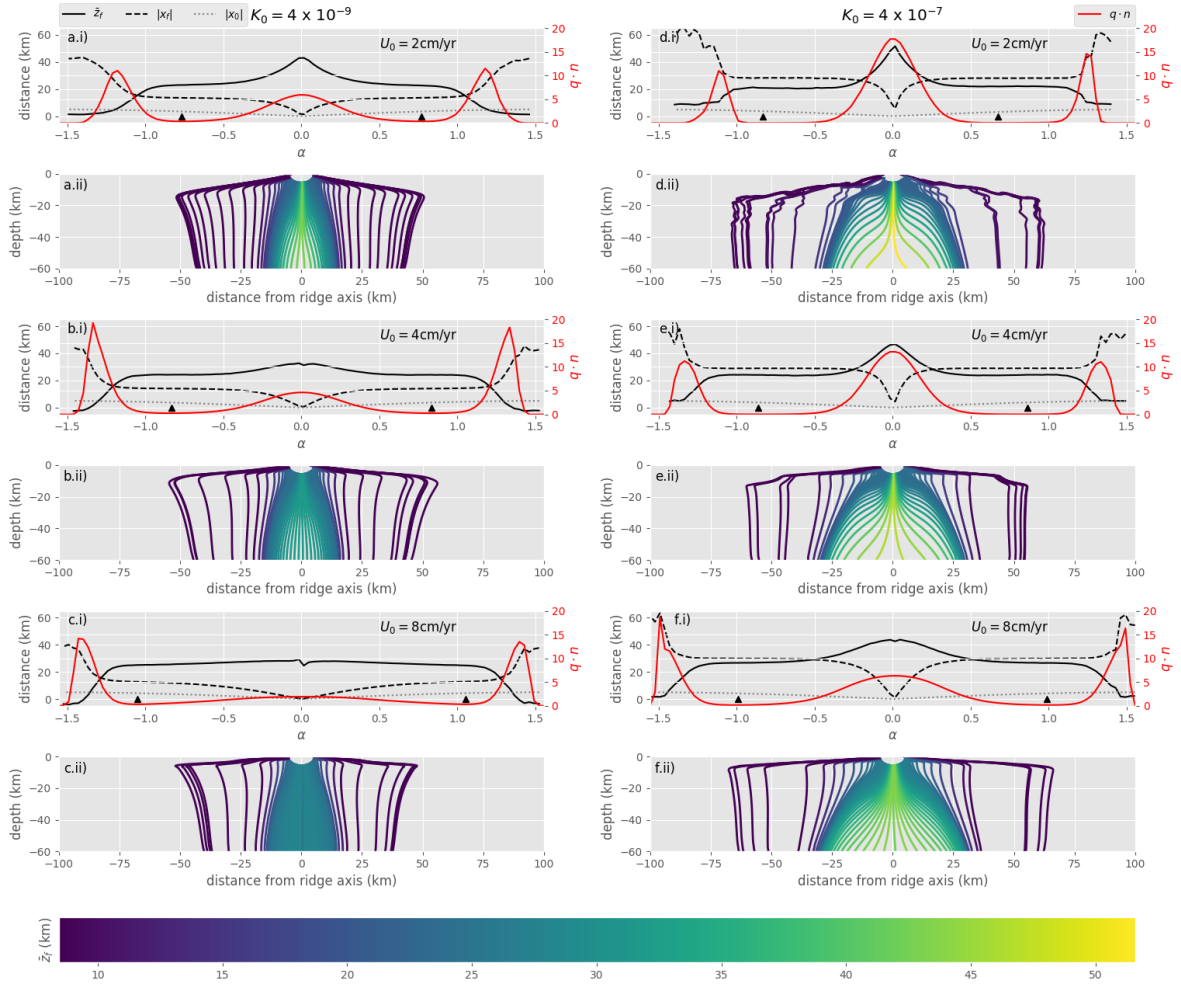


Figure 8: Mean focusing depth, horizontal melt transport distance, non-dimensional melt flux and melt streamlines based on an arc of 5 km radius centered at the ridge axis at the end of all model runs in Figure 3 with increasing permeabilities to the right and increasing half-spreading rates down the panels. Each model run has two corresponding panels, panels i) with plots of values along the arc and panels ii) showing melt streamlines emanating from the arc.

Figure 8 (continued): In panels i) of each model set, the black solid line is the mean focusing depth of each streamline, \bar{z}_f , the black dashed line is the horizontal distance to the origin of melt at the end of each streamline along the arc from the ridge axis, $|x_f|$, the grey dotted line is the horizontal distance to the arc from the ridge axis, $|x_0|$, and the red line is the non-dimensional melt flux normal to the arc. The black triangles represent the locations of the minimum normal melt flux on the arc. The x and y axis correspond to distance (km) and radians along the arc. In panels ii) of each model set, melt flux streamlines emanating from the arc of 5 km radius are plotted along with the colors representing the mean focusing depth, \bar{z}_f . The x and y axis correspond to distance from the ridge axis (km) and depth (km).

Table 3: Melt focusing analysis

Symbol	Formula/Definition	Values						Units
K_0	intrinsic permeability	4×10^{-9}			4×10^{-7}			m^2
U_0	half-spreading rate	2	4	8	2	4	8	cm/yr
z_{focus}	Equation 35	22.8	24.1	24.9	21.3	24.1	26.6	km
L_{melting}	Equation 36	13.5	14.0	12.7	29.0	28.9	29.8	km
Q_{focus}	Equation 37	0.33	0.26	0.33	0.43	0.49	0.49	-

600 4.2. Melt transport analysis

601 To quantify the relative contributions of the different focusing mechanisms
602 in the models (Figure 3), we analyze the non-dimensional melt flux profiles
603 across an arc of 5 km radius centered at the ridge axis and the instantaneous
604 flow lines emanating from it (Figure 8). The melt flux, $\mathbf{q} = \phi \mathbf{v}_f$, is obtained
605 using Equation 4 and $\hat{\mathbf{r}}$ is the unit normal to the arc. Together, the non-
606 dimensional melt flux normal to the arc, $\mathbf{q} \cdot \hat{\mathbf{r}}$ (red lines in all the i) panels in
607 Figure 8), suggests two distinct sources of melt supply to the ridge axis: melt

608 flux from the central region and melt flux from the decompaction layers on the
609 sides. However, melt flux is only a local measure and it is not obvious given
610 only \mathbf{q} whether the melt has been focused at depth or traveled in shallow
611 decompaction channels. To better quantify these ideas, we also calculate the
612 instantaneous flow lines (all the ii) panels in Figure 8) that arrive at the
613 sampling arc of 5 km radius centered at the ridge axis and calculate several
614 metrics for each flow line that allow us to distinguish between deep and
615 shallow melt focusing. The first measure is the extraction width, $|x_f|$, which
616 is the distance from the axis that a melt streamline originates from at the
617 solidus ~ 60 km (dashed black lines in all the i) panels in Figure 8). We also
618 plot the x-coordinate on the arc of 5 km radius, $|x_0|$. The second measure
619 is the integrated mean depth of focusing, \bar{z}_f , which is the depth weighted by
620 the magnitude of the horizontal flux:

$$\bar{z}_f = \frac{\int_{\text{flowline}} z q_x d\tau}{\int_{\text{flowline}} q_x d\tau} \quad (35)$$

621 where z and q_x are the depth and the horizontal component of the melt
622 flux at any point of the flow line and τ is the time travelled along the flow
623 line, which goes to the dry solidus at about 60 km depths. The instantaneous
624 flow lines are colored by \bar{z}_f (all the ii) panels in Figure 8). Shallow horizontal
625 transport in decompaction channels should lead to smaller \bar{z}_f etc.

626 We compare \mathbf{q} , $|x_f|$, $|x_0|$ and \bar{z}_f for all the model runs (Figure 3) in the
627 i) panels in Figure 8. Melt flux coming from either the central region and
628 decompaction channels is clearly identifiable in all measures. In general, for
629 these model runs, \mathbf{q} is bi-modal with a central peak coming from deeper
630 within the melting region and two side peaks coming from the shallower
631 decompaction channels. Melt that ends up in the central region comes from

632 a shorter horizontal distance from the ridge axis and has deeper mean melt
 633 focusing depth. On the other hand, melt that ends up in the decompaction
 634 channels come from further off the ridge axis and has shallower mean focusing
 635 depth. The peaks in flux at the sides become shallower with increasing
 636 spreading rate as expected with plate cooling.

637 In all these model cases, there is very little melt flux between the two
 638 peaks and a simple discriminate is to use the two minima, α_- and α_+ , in
 639 melt flux on the arc (black triangles in all the i) panels in Figure 8). The
 640 “central melts” come from between these two minima while the “channel
 641 melts” come from beyond either minima. Given this separation, we can
 642 further quantify the width of melting pressure focusing in the central region:

$$L_{\text{melting}} = \frac{|x_{f\alpha_-}| + |x_{f\alpha_+}|}{2} \quad (36)$$

643 which is the length scale over which melting pressure focusing operates in
 644 these models. To quantify the melt flux coming from the decompaction
 645 channels or the central region, we define the melt flux quotient, Q_{focus} :

$$Q_{\text{focus}} = \frac{\int_{\alpha_-}^{\alpha_+} \mathbf{q} \cdot \hat{\mathbf{r}} d\alpha}{\int_{\alpha} \mathbf{q} \cdot \hat{\mathbf{r}} d\alpha} \quad (37)$$

646 which tells us the proportions of total melt flux coming from the central
 647 region. Since this quotient does not discriminate how much this melt have
 648 been focused, we also define the mean focusing depth, z_{focus} , to be given by
 649 average of the two \bar{z}_f at the two minima of normal melt flux, α_- and α_+
 650 (black triangles in all the i) panels in Figure 8).

651 Table 3 shows how these different measures for melt focusing change with
 652 intrinsic permeability and spreading rate. Melting pressure focusing acts over
 653 the length scale, L_{melting} , that is roughly twice the width at about 30 km for

654 the model runs with larger intrinsic permeability while remaining unchanged
655 with spreading rates (Table 3). The proportions of total melt flux coming
656 from the central region, Q_{focus} , increases with intrinsic permeability and does
657 not change significantly with spreading rates (Table 3). The mean focus-
658 ing depth, z_{focus} , does not change significantly with intrinsic permeability or
659 spreading rate (Table 3).

660 A corresponding figure for the rheological models shown in Figure S1 is
661 given in the supplementary material. In general, there is a transition from
662 decompaction layer focusing to melting pressure focusing in the isoviscous
663 model cases consistent with Figure 7. There is stronger melting pressure
664 focusing when dislocation creep is excluded (panel d) in Figure S1).

665 *4.3. Comparison with geophysical observations with implications for the litho- 666 sphere asthenosphere boundary (LAB)*

667 The porosities, melt velocities and melt transport patterns from these
668 models can be compared with geophysical observations from both seismic
669 and electromagnetic surveys at various mid-ocean ridges. The models predict
670 two high porosity regions, one on the ridge axis at depths (between 10 to
671 40 km depths) and another in the decompaction layers, following the LAB
672 roughly. Geophysical estimates of porosity beneath the fast spreading East
673 Pacific Rise (EPR) range from $<1\%$ to $>10\%$, although these estimates are
674 sensitive to assumptions about how porosity affects observable seismic and
675 electromagnetic properties (Forsyth et al., 1998b,a; Baba et al., 2006; Key
676 et al., 2013). For faster spreading rate, the models predict up to 20% porosity
677 for models with lower permeability and an order of magnitude lower for
678 models with higher permeability. A model with lower permeability generally

679 leads to slower melt velocities and hence larger amount of melt retention.

680 Both magnetotelluric (MT) imaging of the EPR and seismic attenuation
681 studies at Juan de Fuca ridge suggest that melt may be focused deeper even
682 before reaching the theorized decompaction layers (Key et al., 2013; Eilon
683 and Abers, 2017; Ruan et al., 2018). The MT inversions show that the shal-
684 low upper mantle is resistive and devoid of melt (Key et al., 2013) where the
685 decompaction layers are predicted to be in existing models while the seismic
686 attenuating regions are much deeper than the expected decompaction layers
687 as well (Eilon and Abers, 2017; Ruan et al., 2018). The melt rich decom-
688 paction layers essentially delineates the LAB in the models, which would
689 otherwise be defined by both the temperature dependent rheological or
690 freezing boundaries. The prominent decompaction layers in the models with
691 smaller intrinsic permeability, $K_0 = 4 \times 10^{-9} \text{ m}^2$ (Figure 3), make them per-
692 haps less plausible since higher porosities are better detected by geophysical
693 methods. It is also possible that there exist melt channels that transport
694 melt rapidly along this decompaction layer that are too small and narrow
695 to be detected by geophysical observations since the porosity shown in our
696 models are volumetric averages of each mesh element ($\sim 1 \text{ km}$ along decom-
697 paction layers). Although the MT method is sensitive to conductors such
698 as melt, the ocean is highly conductive and attenuates the high frequency
699 natural source field such that seafloor MT data may not be able to resolve
700 narrow and shallow melt channels. However, if the interpretations of the MT
701 surveys are accurate, one possibility to reconcile this discrepancy would be
702 that the melting pressure focusing is even stronger in reality compared to our
703 models, since it is the only mechanism in our models thus far that focuses

704 melt in the deeper melting region before it reaches the thermal boundary
705 layer.

706 To strengthen the melting rate focusing mechanism, bulk viscosity has to
707 be larger since melting is mainly controlled by solid mantle upwelling, which
708 is better constrained. In this study, bulk viscosity depends on both shear vis-
709 cosity and permeability, which sets the mean porosity retained (Equations 18
710 and 24). Similar models with grain size evolution considerations could recon-
711 cile this discrepancy (Turner et al., 2017) as there is a steeper freezing front
712 in those models. Another way to reconcile this could be that the lithosphere
713 is indeed thicker than previously thought. Turner et al. (2017) has shown
714 that using different choices for rheological flow laws could lead to a thicker
715 lithosphere.

716 *4.4. Model caveats*

717 Our models are one-way coupled so that the solid velocity and dynamic
718 pressure in the solid system are not affected by the melt flow system. This
719 problem is less computationally expensive since we avoid the need to solve
720 the full Stokes equation at every time-step, while still capturing the main
721 dynamics of the system. However, one-way coupling leads to an inconsistency
722 in mass conservation. The solid velocity is not adjusted to move up faster as
723 melt is produced and extracted/segregated. This is a minor contribution and
724 may allow slightly more melt to be produced than predicted in Figure 4a.

725 Our numerical models are passively driven by a Dirichlet condition that
726 sets the solid velocity at the top boundary to be plate spreading rate. Geo-
727 physical observations of asymmetric upwelling could suggest buoyancy driven
728 flow beneath some mid-ocean ridges (Hammond and Toomey, 2003; Dunn

729 and Forsyth, 2003; Baba et al., 2006). However, a combined geochemical and
730 dynamical study suggests that passively driven upwelling seems more con-
731 sistent with indirect geochemical observations (Spiegelman and Reynolds,
732 1999). Furthermore, symmetry in a highly conductive region beneath the
733 East Pacific Rise is interpreted to be evidence for passively driven flow (Key
734 et al., 2013).

735 5. Conclusion

736 We describe and present an openly available suite of two-phase flow mod-
737 els applied to mid-ocean ridge setting, varying half-spreading rates and in-
738 trinsic permeability to understand melt focusing. Three distinct melt focus-
739 ing mechanisms are recognized in the models: 1) Melting pressure focusing,
740 2) Decompaction layers and 3) Ridge suction, of which the first two are
741 dominant. Our models suggest that even with similar melting patterns, the
742 amount of melt and melt transport patterns can be significantly different
743 due to changes in intrinsic permeability, K_0 , regardless of spreading rates: 1)
744 increasing intrinsic permeability increases melt velocity, therefore decreasing
745 porosity or melt fractions due to efficient melt transport, 2) this reduction
746 in porosity then leads to an increase in bulk viscosity since the bulk viscos-
747 ity model used here assumes $\zeta \approx \eta/\phi$, . This increases the magnitude of
748 the compaction pressure at quasi-steady state (Equation 34). In particular,
749 the increase in compaction pressure and its gradient in the axial melting re-
750 gion pulls melt from a wider region towards the axis, which is the basis for
751 melting pressure focusing. These melt focusing mechanisms are a natural
752 consequence of the two phase flow formulation with viscous deformation and

753 their manifestation depends largely on the rheological model choices made.
754 Melting-pressure focusing is a consequence of the compaction pressure field
755 mapping to the melting rate field modulated by the bulk viscosity in quasi-
756 steady state (Equation 34). The geometry of the melting rate field will always
757 be roughly triangular in a ridge setting. However, the magnitude of the com-
758 paction pressure depends on the rheological model used for bulk viscosity
759 and therefore shear viscosity in our models. Stronger overall shear viscosity
760 in the melting region leads to larger melting pressure focusing effect. The
761 length scale associated with melting pressure focusing is about two times
762 wider for larger intrinsic permeability and the consequent increase in bulk
763 viscosity. The dominance and strength of these melt focusing mechanisms
764 affect the locality of melt rich regions and also melt transport, thus can affect
765 the interpreted position of the LAB. To reconcile the models with geophysi-
766 cal observations with regards to the LAB, stronger melting pressure focusing
767 might be needed to focus melt before it reaches the lithosphere.

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